Documentation of modeling:

(Dikshant and Madhumita)

The trajectory tracking control of an autonomous vehicle is one of the most difficult automation challenges due to its motion constraints: linear and angular speeds, linear and angular accelerations, etc; and there exist a constant environment interaction.

There are many approaches when one wants to model the dynamics of a vehicle. In the literature, it is very common to find three models: the kinematic bicycle model, the dynamic bicycle model, and the four wheels dynamic model.

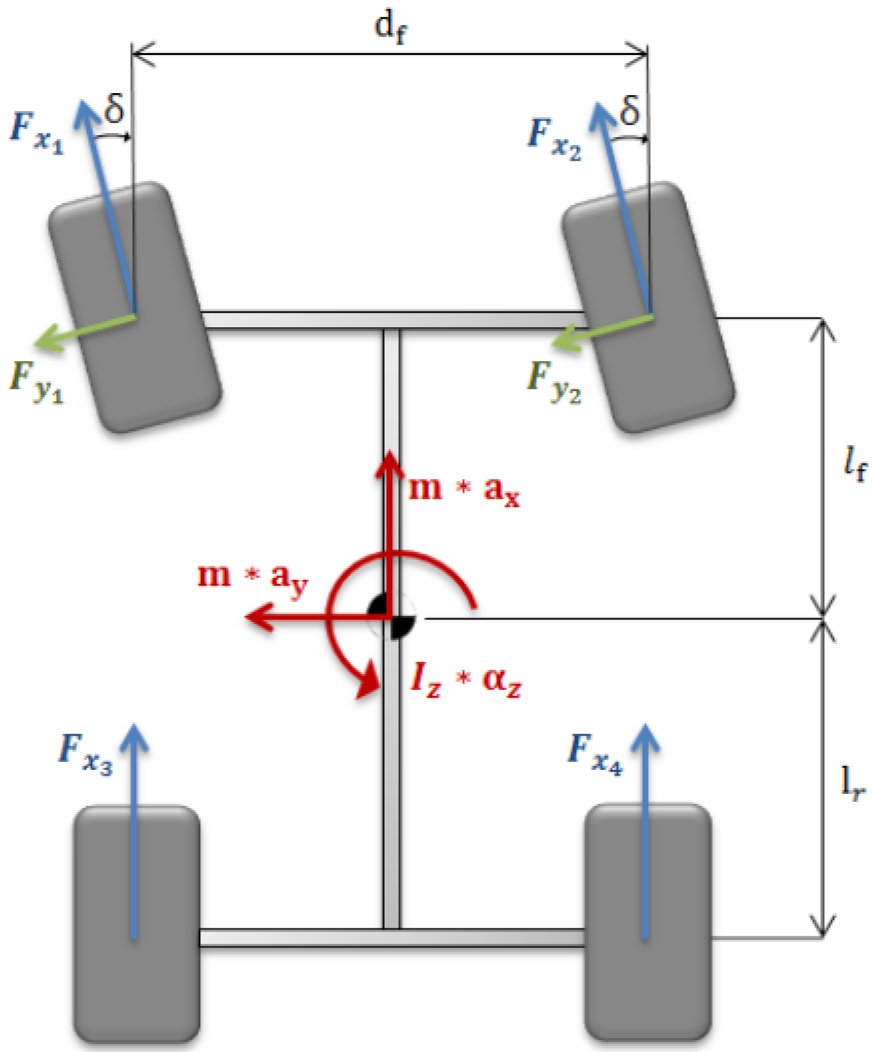
The way to achieve the dynamic equations is by looking for the force and torque equilibrium in every axis. We are going to denote the X-axis as the longitudinal car axis, the Y-axis as the lateral one, and the Z-axis as the perpendicular to the road. Such an XYZ frame is in the center of gravity of the vehicle which from now will be named CoG. The equations that describe the equilibrium of longitudinal forces are provided below:

$$F_{x1} \cdot \cos(\delta) - F_{y1} \cdot \sin(\delta)$$
$$+F_{x2} \cdot \cos(\delta) - F_{y2} \cdot \sin(\delta)$$
$$+F_{x3} + F_{x4} = m \cdot a_x$$

where the forces F_xi is the longitudinal wheel forces represented as blue force vectors, the forces F_yi are the lateral wheel forces represented as green force vectors, the term δi represents the steering angle of front wheels, a_x is the inertial acceleration along the CoG X-axis. (See the diagram below)

Regarding the equations that define the lateral dynamics, they are similar to the longitudinal ones. The equilibrium of lateral forces is given by:

$$F_{x1} \cdot \sin(\delta) - F_{y1} \cdot \cos(\delta)$$
$$+F_{x2} \cdot \sin(\delta) - F_{y2} \cdot \cos(\delta)$$



Current model:

La is the distance between the centers of the front wheels and the vehicle's center of mass, Lb is the distance between the centers of the rear wheels and the vehicle's center of mass, and J is the rotational momentum. Fy,f is the front tire lateral force, Fy,r is the rear tire lateral force. These forces can be computed from:

$$F_{y,f} = C_y\,(\delta - La\phi / v)$$

$$F_{y,r} = C_y * Lb\phi / v$$

where Cy is the lateral tire stiffness