CHAPTER- 3

TIME VALUE OF MONEY

By Sushma Maharjan

Introduction : Time Value of Money

Definition:

Time value of money is the premise that an investor prefers to receive a payment of a fixed amount of money today, rather than an equal amount in the future, all else being equal.

Source: http://en.wikipedia.org/wiki/Time_value_of_money

Time Value of Money

- Which would you rather have –RS.1,000 today or RS.1,000 in 5 years?
- Money received sooner rather than later allows one to use the funds for investment or consumption purposes.
- All other factors being equal, it is better to have RS.1,000 today.
- Simply this is the concept of the time value of money.

The Interest Rate:

Which would you prefer -- RS..10,000 today or RS.10,000 in 5 years?

Obviously, RS.10,000 today.

You already recognize that there is ***TIME VALUE TO MONEY***!!

Time preference for money

- Time preference for money is an individual's preference for possession of a given amount of money now, rather than the same amount at some future time.

- Three reasons may be attributed to the individual's time preference for money:

  - Risk
  - preference for consumption
  - investment opportunities

Notations

- PV (Present Value) is the value at time = 0
- FV (Future Value) is the value at time = n
- ‘i’ is the rate at which the amount will be compounded each period
- ‘n’ is the number of periods

Notations

- PV(A) the value of the annuity at time = 0
- FV(A) the value of the annuity at time = n
- ‘A’ the value of the individual payments in each compounding period

Time Value Adjustment

Two most common methods of adjusting cash flows for time value of money:

- **Compounding—the process of calculating future values of cash flows and**
- **Discounting—the process of calculating present values of cash flows.**

Future Value

- **Compounding** is the process of finding the future values of cash flows by applying the concept of compound interest.
- **Compound interest** is the interest that is received on the original amount (principal) as well as on any interest earned but not withdrawn during earlier periods.
- **Simple interest** is the interest that is calculated only on the original amount (principal), and thus, no compounding of interest takes place.

Future Value

- The general form of equation for calculating the future value of a lump sum after *n* periods may, therefore, be written as follows:

$$FV = PV(1 + i)^n$$

- The term $(1 + i)^n$ is the **compound value factor or future value factor** (*FVF*) of a lump sum of Re 1, and it always has a value greater than 1 for positive *i*, indicating that *FVF* increases as *i* and *n* increase.
- $FV_n = P \times \mathrm{FVF}_{n,i}$

Example

If you deposited Rs 55,650 in a bank, which was paying a 15 per cent rate of interest on a ten-year time deposit, how much would the deposit grow at the end of ten years?

We will first find out the compound value factor at 15 percent for 10 years which is 4.046. Multiplying 4.046 by Rs 55,650, we get Rs 225,159.90 as the compound value:

- $FV = 55{,}650 \times FVF_{10,\,0.15} = 55{,}650 \times 4.046 = \text{Rs } 225{,}159.90$

Table 3:

Future value interest factor of Re 1 at 1% at the end of n periods

$FVIF_{i,n} = (1+i)^n$

n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.8347
26	1.2953	1.6734	2.1566	2.7725	3.5557	4.5494	5.8074	7.3964	9.3992	11.9182
27	1.3082	1.7069	2.2213	2.8834	3.7335	4.8223	6.2139	7.9881	10.2451	13.1100
28	1.3213	1.7410	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.1671	14.4210
29	1.3345	1.7758	2.3566	3.1187	4.1161	5.4184	7.1143	9.3173	12.1722	15.8631
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.0627	13.2677	17.4494
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.2857	14.9745	21.7245	31.4094	45.2593
50	1.6446	2.6916	4.3839	7.1067	11.4674	18.4202	29.4570	46.9016	74.3575	117.3909
60	1.8167	3.2810	5.8916	10.5196	18.6792	32.9877	57.9464	101.2571	176.0313	304.4816

Table 3: (Cntd...)

Future value interest factor of Re 1 at 1% at the end of n periods

$FVIF_{i,n} = (1+i)^n$

n	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	1.1100	1.1200	1.1300	1.1400	1.1500	1.1600	1.1700	1.1800	1.1900	1.2000
2	1.2321	1.2544	1.2769	1.2996	1.3225	1.3456	1.3689	1.3924	1.4161	1.4400
3	1.3676	1.4049	1.4429	1.4815	1.5209	1.5609	1.6016	1.6430	1.6852	1.7280
4	1.5181	1.5735	1.6305	1.6890	1.7490	1.8106	1.8739	1.9388	2.0053	2.0736
5	1.6851	1.7623	1.8424	1.9254	2.0114	2.1003	2.1924	2.2878	2.3864	2.4883
6	1.8704	1.9738	2.0820	2.1950	2.3131	2.4364	2.5652	2.6996	2.8398	2.9860
7	2.0762	2.2107	2.3526	2.5023	2.6600	2.8262	3.0012	3.1855	3.3793	3.5832
8	2.3045	2.4760	2.6584	2.8526	3.0590	3.2784	3.5115	3.7589	4.0214	4.2998
9	2.5580	2.7731	3.0040	3.2519	3.5179	3.8030	4.1084	4.4355	4.7854	5.1598
10	2.8394	3.1058	3.3946	3.7072	4.0456	4.4114	4.8068	5.2338	5.6947	6.1917
11	3.1518	3.4785	3.8359	4.2262	4.6524	5.1173	5.6240	6.1759	6.7767	7.4301
12	3.4985	3.8960	4.3345	4.8179	5.3503	5.9360	6.5801	7.2876	8.0642	8.9161
13	3.8833	4.3635	4.8980	5.4924	6.1528	6.8858	7.6987	8.5994	9.5964	10.6993
14	4.3104	4.8871	5.5348	6.2613	7.0757	7.9875	9.0075	10.1472	11.4198	12.8392
15	4.7846	5.4736	6.2543	7.1379	8.1371	9.2655	10.5387	11.9737	13.5895	15.4070
16	5.3109	6.1304	7.0673	8.1372	9.3576	10.7480	12.3303	14.1290	16.1715	18.4884
17	5.8951	6.8660	7.9861	9.2765	10.7613	12.4677	14.4265	16.6722	19.2441	22.1861
18	6.5436	7.6900	9.0243	10.5752	12.3755	14.4625	16.8790	19.6733	22.9005	26.6233
19	7.2633	8.6128	10.1974	12.0557	14.2318	16.7765	19.7484	23.2144	27.2516	31.9480
20	8.0623	9.6463	11.5231	13.7435	16.3665	19.4608	23.1056	27.3930	32.4294	38.3376
21	8.9492	10.8038	13.0211	15.6676	18.8215	22.5745	27.0336	32.3238	38.5910	46.0051
22	9.9336	12.1003	14.7138	17.8610	21.6447	26.1864	31.6293	38.1421	45.9233	55.2061
23	11.0263	13.5523	16.6266	20.3616	24.8915	30.3762	37.0062	45.0076	54.6487	66.2474
24	12.2392	15.1786	18.7881	23.2122	28.6252	35.2364	43.2973	53.1090	65.0320	79.4968
25	13.5855	17.0001	21.2305	26.4619	32.9190	40.8742	50.6578	62.6686	77.3881	95.3962
26	15.0799	19.0401	23.9905	30.1666	37.8568	47.4141	59.2697	73.9490	92.0918	114.4755
27	16.7386	21.3249	27.1093	34.3899	43.5353	55.0004	69.3455	87.2598	109.5893	137.3706
28	18.5799	23.8839	30.6335	39.2045	50.0656	63.8004	81.1342	102.9666	130.4112	164.8447
29	20.6237	26.7499	34.6158	44.6931	57.5755	74.0085	94.9271	121.5005	155.1893	197.8136
30	22.8923	29.9599	39.1159	50.9502	66.2118	85.8499	111.0647	143.3706	184.6753	237.3763
40	65.0009	93.0510	132.7816	188.8835	267.8635	378.7212	533.8687	750.3783	1051.6675	1469.7716
50	184.5648	289.0022	450.7359	700.2330	1083.6574	1670.7038	2566.2153	3927.3569	5988.9139	9100.4382
60	524.0572	897.5969	1530.0535	2595.9187	4383.9987	7370.2014	12335.3565	20555.140	34104.9709	56347.5144

Present Value

- **Present value** is the current value of a future amount of money, or a series of payments, evaluated at a given interest rate.
- **Discounting** is the process of determining present value of a series of future cash flows.
- The *interest rate* used for discounting cash flows is also called the *discount rate*.

Present Value of a Single Cash Flow

- The following general formula can be employed to calculate the present value of a lump sum to be received after some future periods: **Discounting** is the process of determining present value of a series of future cash flows.

$$P = \frac{F}{(1+i)n}$$

- The term in parentheses is the **discount factor** or **present value factor** (*PVF*), and it is always less than 1.0 for positive *i*, indicating that a future amount has a smaller present value.

$$PV = F_n \times \mathrm{PVF}_{n,i}$$

Example

- Suppose that an investor wants to find out the present value of Rs 50,000 to be received after 15 years. Here interest rate is 9 per cent.

- First, we will find out the present value factor, which is 0.275.

- Multiplying 0.275 by Rs 50,000, we obtain Rs 13,750 as the present value:

$$PV = 50{,}000 \times PVF_{15,\ 0.09} = 50{,}000 \times 0.275 = \text{Rs } 13{,}750$$

Table 1:

Present value of one rupee due at the end of n years:

$$PVIF_{i,n} = \frac{1}{(1+i)^n}$$

n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923
26	0.7720	0.5976	0.4637	0.3607	0.2812	0.2198	0.1722	0.1352	0.1064	0.0839
27	0.7644	0.5859	0.4502	0.3468	0.2678	0.2074	0.1609	0.1252	0.0976	0.0763
28	0.7568	0.5744	0.4371	0.3335	0.2551	0.1956	0.1504	0.1159	0.0895	0.0693
29	0.7493	0.5631	0.4243	0.3207	0.2429	0.1846	0.1406	0.1073	0.0822	0.0630
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573
35	0.7059	0.5000	0.3554	0.2534	0.1813	0.1301	0.0937	0.0676	0.0490	0.0356
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221
45	0.6391	0.4102	0.2644	0.1712	0.1113	0.0727	0.0476	0.0313	0.0207	0.0137
50	0.6080	0.3715	0.2281	0.1407	0.0872	0.0543	0.0339	0.0213	0.0134	0.0085
55	0.5785	0.3365	0.1968	0.1157	0.0683	0.0406	0.0242	0.0145	0.0087	0.0053

Table 1: (Cntd....)

Present value of one rupee due at the end of n years:

$$PVIF_{i,n} = \frac{1}{(1+i)^n}$$

n	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	0.9009	0.8929	0.8850	0.8772	0.8696	0.8621	0.8547	0.8475	0.8403	0.8333
2	0.8116	0.7972	0.7831	0.7695	0.7561	0.7432	0.7305	0.7182	0.7062	0.6944
3	0.7312	0.7118	0.6931	0.6750	0.6575	0.6407	0.6244	0.6086	0.5934	0.5787
4	0.6587	0.6355	0.6133	0.5921	0.5718	0.5523	0.5337	0.5158	0.4987	0.4823
5	0.5935	0.5674	0.5428	0.5194	0.4972	0.4761	0.4561	0.4371	0.4190	0.4019
6	0.5346	0.5066	0.4803	0.4556	0.4323	0.4104	0.3898	0.3704	0.3521	0.3349
7	0.4817	0.4523	0.4251	0.3996	0.3759	0.3538	0.3332	0.3139	0.2959	0.2791
8	0.4339	0.4039	0.3762	0.3506	0.3269	0.3050	0.2848	0.2660	0.2487	0.2326
9	0.3909	0.3606	0.3329	0.3075	0.2843	0.2630	0.2434	0.2255	0.2090	0.1938
10	0.3522	0.3220	0.2946	0.2697	0.2472	0.2267	0.2080	0.1911	0.1756	0.1615
11	0.3173	0.2875	0.2607	0.2366	0.2149	0.1954	0.1778	0.1619	0.1476	0.1346
12	0.2858	0.2567	0.2307	0.2076	0.1869	0.1685	0.1520	0.1372	0.1240	0.1122
13	0.2575	0.2292	0.2042	0.1821	0.1625	0.1452	0.1299	0.1163	0.1042	0.0935
14	0.2320	0.2046	0.1807	0.1597	0.1413	0.1252	0.1110	0.0985	0.0876	0.0779
15	0.2090	0.1827	0.1599	0.1401	0.1229	0.1079	0.0949	0.0835	0.0736	0.0649
16	0.1883	0.1631	0.1415	0.1229	0.1069	0.0930	0.0811	0.0708	0.0618	0.0541
17	0.1696	0.1456	0.1252	0.1078	0.0929	0.0802	0.0693	0.0600	0.0520	0.0451
18	0.1528	0.1300	0.1108	0.0946	0.0808	0.0691	0.0592	0.0508	0.0437	0.0376
19	0.1377	0.1161	0.0981	0.0829	0.0703	0.0596	0.0506	0.0431	0.0367	0.0313
20	0.1240	0.1037	0.0868	0.0728	0.0611	0.0514	0.0433	0.0365	0.0308	0.0261
21	0.1117	0.0926	0.0768	0.0638	0.0531	0.0443	0.0370	0.0309	0.0259	0.0217
22	0.1007	0.0826	0.0680	0.0560	0.0462	0.0382	0.0316	0.0262	0.0218	0.0181
23	0.0907	0.0738	0.0601	0.0491	0.0402	0.0329	0.0270	0.0222	0.0183	0.0151
24	0.0817	0.0659	0.0532	0.0431	0.0349	0.0284	0.0231	0.0188	0.0154	0.0126
25	0.0736	0.0588	0.0471	0.0378	0.0304	0.0245	0.0197	0.0160	0.0129	0.0105
26	0.0663	0.0525	0.0417	0.0331	0.0264	0.0211	0.0169	0.0135	0.0109	0.0087
27	0.0597	0.0469	0.0369	0.0291	0.0230	0.0182	0.0144	0.0115	0.0091	0.0073
28	0.0538	0.0419	0.0326	0.0255	0.0200	0.0157	0.0123	0.0097	0.0077	0.0061
29	0.0485	0.0374	0.0289	0.0224	0.0174	0.0135	0.0105	0.0082	0.0064	0.0051
30	0.0437	0.0334	0.0256	0.0196	0.0151	0.0116	0.0090	0.0070	0.0054	0.0042
35	0.0259	0.0189	0.0139	0.0102	0.0075	0.0055	0.0041	0.0030	0.0023	0.0017
40	0.0154	0.0107	0.0075	0.0053	0.0037	0.0026	0.0019	0.0013	0.0010	0.0007
45	0.0091	0.0061	0.0041	0.0027	0.0019	0.0013	0.0009	0.0006	0.0004	0.0003
50	0.0054	0.0035	0.0022	0.0014	0.0009	0.0006	0.0004	0.0003	0.0002	0.0001
55	0.0032	0.0020	0.0012	0.0007	0.0005	0.0003	0.0002	0.0001	.	.

Example

Consider 2 situations

- Option A: You receive Rs. 10,000 today.
- Option B: You receive Rs. 10,000 in 3 years time

Assume no inflation

Assume interest rate 10% (Compound Interest)

Assume no change in any other financial situation

Future Value Calculation

- Consider Option A

  Let's calculate the future value of Rs. 10,000 received at the present time.

Present Value = Rs. 10,000

$FV = PV \times (1 + r)^n$

For Year 1:

$FV = 10{,}000 \times (1 + 0.1)^1$

$\therefore FV = 11{,}000$

For Year 2:

$FV = 10{,}000 \times (1 + 0.1)^2$

$FV = 12{,}100$

For Year 3:

$FV = 10{,}000 \times (1 + 0.1)^3$

$FV = 13{,}310$

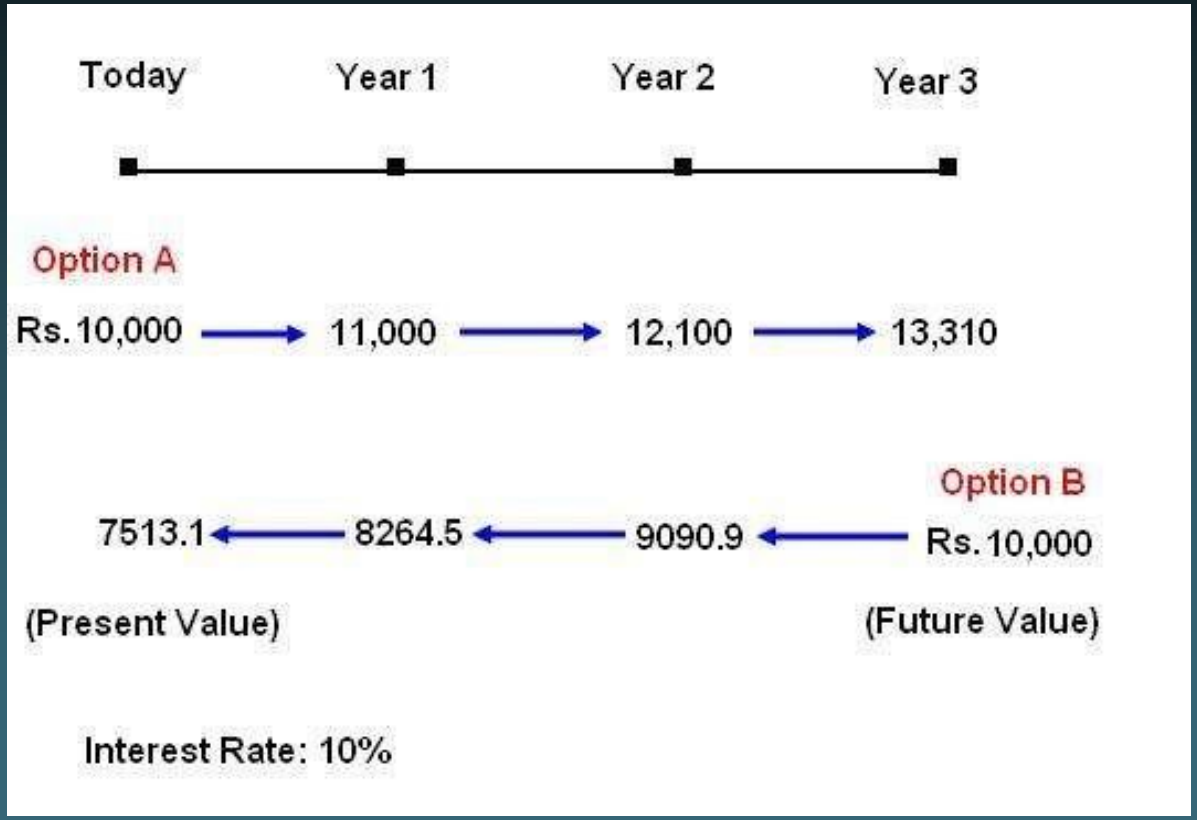
Present Value Calculation

Similarly using the equation as

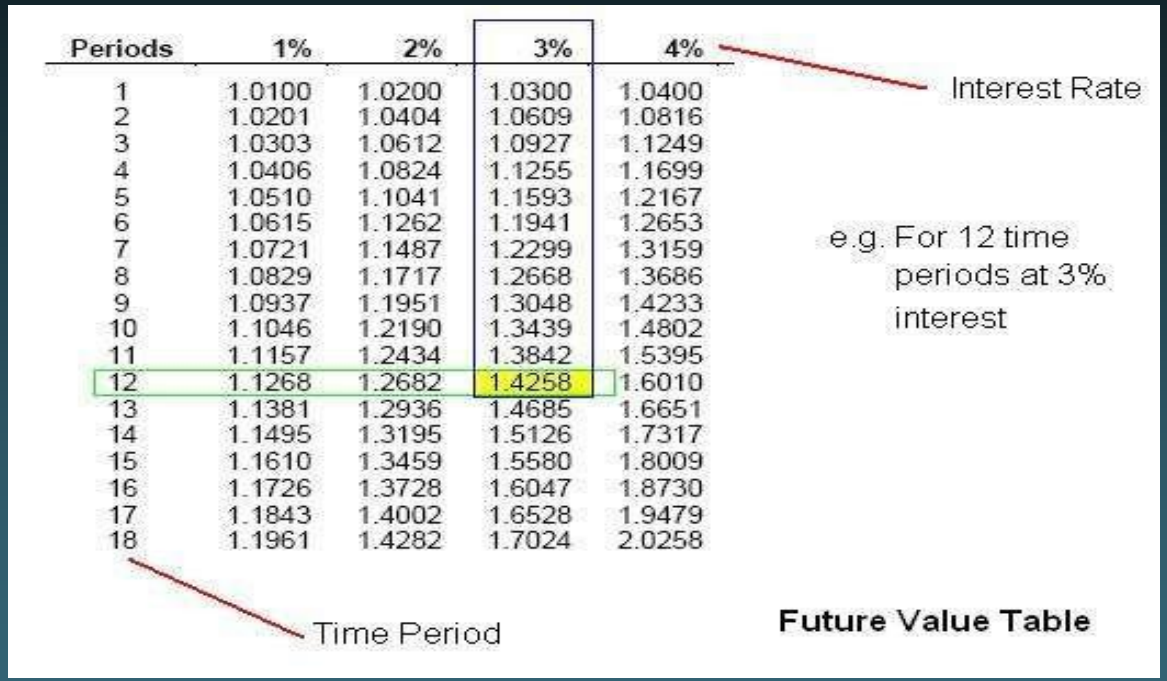
$$PV = \frac{FV}{(1 + r)^n}$$

the present value of Rs. 10,000 received in 3 years when the interest rate is 10% can be calculated as Rs. 7513.1

Time Value of Money



Time Value Calculations using Tables



Periods	1%	2%	3%	4%
1	1.0100	1.0200	1.0300	1.0400
2	1.0201	1.0404	1.0609	1.0816
3	1.0303	1.0612	1.0927	1.1249
4	1.0406	1.0824	1.1255	1.1699
5	1.0510	1.1041	1.1593	1.2167
6	1.0615	1.1262	1.1941	1.2653
7	1.0721	1.1487	1.2299	1.3159
8	1.0829	1.1717	1.2668	1.3686
9	1.0937	1.1951	1.3048	1.4233
10	1.1046	1.2190	1.3439	1.4802
11	1.1157	1.2434	1.3842	1.5395
12	1.1268	1.2682	1.4258	1.6010
13	1.1381	1.2936	1.4685	1.6651
14	1.1495	1.3195	1.5126	1.7317
15	1.1610	1.3459	1.5580	1.8009
16	1.1726	1.3728	1.6047	1.8730
17	1.1843	1.4002	1.6528	1.9479
18	1.1961	1.4282	1.7024	2.0258

Interest Rate

e.g. For 12 time periods at 3% interest

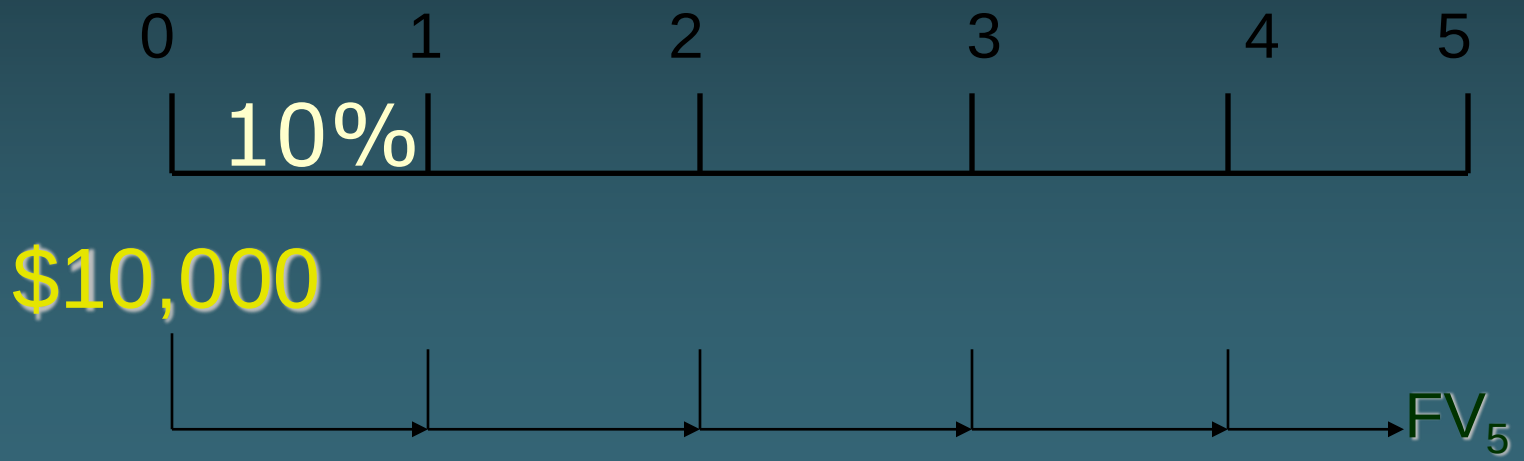
Time Period

Future Value Table

Example

Julie Miller wants to know how large her deposit of $10,000 today will become at a compound annual interest rate of 10% for 5 years.

Cash flow time line:



Solution

Calculation based on general formula:

FVn = PV (1+i)n

FV5 = $10,000 (1+ 0.10)5

=$16,105.10

Calculation based on Table I:

FV5 = $10,000 (FVIF10%, 5)

= $10,000 (1.611)

= $16,110 [Due to Rounding]

Finding the interest rate:

Tabular Solution:

The interest rate can be find out by using present value interest factor(PVIF) or future value interest factor (FVIF) table,

By using PV,

$PV = FV_n (PVIF_{i,n})$

For example,

PV= Rs. 3,000

FV= RS. 6,000

N = 4 years

I = ?

Now, substituting the respective values,

RS. 3,000 = RS. 6,000 ($PVIF_{I,4}$)

Or, RS. 3,000/RS.6,000 = $PVIF_{I,4}$

∴ $PVIF_{I,4} = 0.5$

As per PVIF table at 4 year period row, the factor 0.5 lies between 18% lower rate (LR) and 19% higher rate (HR) with factor values of 0.4987 and 0.5158 respectively.

So, by interpolation,

$$\text{Interest rate (i)} = LR + \frac{Factor\ at\ LR - Exact\ Factor}{Factor\ at\ LR - Factor\ at\ HR} \times (HR\text{-}LR)$$

$$= 18\% + \frac{0.5158 - 0.50}{0.5158 - 0.4987} \times (19\% - 18\%)$$

$$= 18.90\%$$

Table 1: (Cntd....)

Present value of one rupee due at the end of n years:

$$PVIF_{i,n} = \frac{1}{(1+i)^n}$$

n	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	0.9009	0.8929	0.8850	0.8772	0.8696	0.8621	0.8547	0.8475	0.8403	0.8333
2	0.8116	0.7972	0.7831	0.7695	0.7561	0.7432	0.7305	0.7182	0.7062	0.6944
3	0.7312	0.7118	0.6931	0.6750	0.6575	0.6407	0.6244	0.6086	0.5934	0.5787
4	0.6587	0.6355	0.6133	0.5921	0.5718	0.5523	0.5337	0.5158	0.4987	0.4823
5	0.5935	0.5674	0.5428	0.5194	0.4972	0.4761	0.4561	0.4371	0.4190	0.4019
6	0.5346	0.5066	0.4803	0.4556	0.4323	0.4104	0.3898	0.3704	0.3521	0.3349
7	0.4817	0.4523	0.4251	0.3996	0.3759	0.3538	0.3332	0.3139	0.2959	0.2791
8	0.4339	0.4039	0.3762	0.3506	0.3269	0.3050	0.2848	0.2660	0.2487	0.2326
9	0.3909	0.3606	0.3329	0.3075	0.2843	0.2630	0.2434	0.2255	0.2090	0.1938
10	0.3522	0.3220	0.2946	0.2697	0.2472	0.2267	0.2080	0.1911	0.1756	0.1615
11	0.3173	0.2875	0.2607	0.2366	0.2149	0.1954	0.1778	0.1619	0.1476	0.1346
12	0.2858	0.2567	0.2307	0.2076	0.1869	0.1685	0.1520	0.1372	0.1240	0.1122
13	0.2575	0.2292	0.2042	0.1821	0.1625	0.1452	0.1299	0.1163	0.1042	0.0935
14	0.2320	0.2046	0.1807	0.1597	0.1413	0.1252	0.1110	0.0985	0.0876	0.0779
15	0.2090	0.1827	0.1599	0.1401	0.1229	0.1079	0.0949	0.0835	0.0736	0.0649
16	0.1883	0.1631	0.1415	0.1229	0.1069	0.0930	0.0811	0.0708	0.0618	0.0541
17	0.1696	0.1456	0.1252	0.1078	0.0929	0.0802	0.0693	0.0600	0.0520	0.0451
18	0.1528	0.1300	0.1108	0.0946	0.0808	0.0691	0.0592	0.0508	0.0437	0.0376
19	0.1377	0.1161	0.0981	0.0829	0.0703	0.0596	0.0506	0.0431	0.0367	0.0313
20	0.1240	0.1037	0.0868	0.0728	0.0611	0.0514	0.0433	0.0365	0.0308	0.0261
21	0.1117	0.0926	0.0768	0.0638	0.0531	0.0443	0.0370	0.0309	0.0259	0.0217
22	0.1007	0.0826	0.0680	0.0560	0.0462	0.0382	0.0316	0.0262	0.0218	0.0181
23	0.0907	0.0738	0.0601	0.0491	0.0402	0.0329	0.0270	0.0222	0.0183	0.0151
24	0.0817	0.0659	0.0532	0.0431	0.0349	0.0284	0.0231	0.0188	0.0154	0.0126
25	0.0736	0.0588	0.0471	0.0378	0.0304	0.0245	0.0197	0.0160	0.0129	0.0105
26	0.0663	0.0525	0.0417	0.0331	0.0264	0.0211	0.0169	0.0135	0.0109	0.0087
27	0.0597	0.0469	0.0369	0.0291	0.0230	0.0182	0.0144	0.0115	0.0091	0.0073
28	0.0538	0.0419	0.0326	0.0255	0.0200	0.0157	0.0123	0.0097	0.0077	0.0061
29	0.0485	0.0374	0.0289	0.0224	0.0174	0.0135	0.0105	0.0082	0.0064	0.0051
30	0.0437	0.0334	0.0256	0.0196	0.0151	0.0116	0.0090	0.0070	0.0054	0.0042
35	0.0259	0.0189	0.0139	0.0102	0.0075	0.0055	0.0041	0.0030	0.0023	0.0017
40	0.0154	0.0107	0.0075	0.0053	0.0037	0.0026	0.0019	0.0013	0.0010	0.0007
45	0.0091	0.0061	0.0041	0.0027	0.0019	0.0013	0.0009	0.0006	0.0004	0.0003
50	0.0054	0.0035	0.0022	0.0014	0.0009	0.0006	0.0004	0.0003	0.0002	0.0001
55	0.0032	0.0020	0.0012	0.0007	0.0005	0.0003	0.0002	0.0001	.	.

Alternative method:

$FV = PV (1+ i)^n$

Or, RS. $6,000 = RS. 3,000 (1 + i)^4$

Or, RS. $2 = (1 + i)^4$

Or,$(RS. 2)^{1/4} - 1 = i$

$\therefore i = 18.92\%$

Finding the number of periods:

Tabular Solution:

The number of periods can be find out by using present value interest factor(PVIF) or future value interest factor (FVIF) table,

By using PV,

$PV = FV_n (PVIF_{i,n})$

For example,

PV= Rs. 3,000

FV= RS. 6,000

I = 10%

N = ?

Now, substituting the respective values,

RS. 3,000 = RS. 6,000 ($PVIF_{10\%,n}$)

Or, RS. 3,000/RS.6,000 = $PVIF_{10\%,n}$

∴ $PVIF_{10\%,n} = 0.5$

As per PVIF table at 10% period column, the factor 0.5 lies between 7 years lower year (LY) and 8 higher year (HY) with factor values of 0.5132 and 0.4665 respectively.

So, by interpolation,

$$\text{Interest rate (i)} = LY + \frac{Factor\ at\ LY - Exact\ Factor}{Factor\ at\ LY - Factor\ at\ HY} \times (HR\text{-}LR)$$

$$= 7 + \frac{0.5132 - 0.50}{0.5132 - 0.4665} \times (8 - 7)$$

= 7.24 years

Table 1:

Present value of one rupee due at the end of n years:

$$PVIF_{i,n} = \frac{1}{(1+i)^n}$$

n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923
26	0.7720	0.5976	0.4637	0.3607	0.2812	0.2198	0.1722	0.1352	0.1064	0.0839
27	0.7644	0.5859	0.4502	0.3468	0.2678	0.2074	0.1609	0.1252	0.0976	0.0763
28	0.7568	0.5744	0.4371	0.3335	0.2551	0.1956	0.1504	0.1159	0.0895	0.0693
29	0.7493	0.5631	0.4243	0.3207	0.2429	0.1846	0.1406	0.1073	0.0822	0.0630
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573
35	0.7059	0.5000	0.3554	0.2534	0.1813	0.1301	0.0937	0.0676	0.0490	0.0356
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221
45	0.6391	0.4102	0.2644	0.1712	0.1113	0.0727	0.0476	0.0313	0.0207	0.0137
50	0.6080	0.3715	0.2281	0.1407	0.0872	0.0543	0.0339	0.0213	0.0134	0.0085
55	0.5785	0.3365	0.1968	0.1157	0.0683	0.0406	0.0242	0.0145	0.0087	0.0053

Alternative method:

$FV = PV (1+ i)^n$

Or, RS. 6,000 = RS. 3,000 $(1 + 0.10)^n$

Or, RS. 2 = $(1 + 0.10)^n$

Or, $(1.1)^n = 2$

Taking log on both sides

N log 1.10 = log 2

N = 7.24 years