

Course: Linear Algebra
Code: MATH201
Semester: Fall 2023

Sample Questions:

1. Which of the following is a vector space?
 - a) Set of all polynomials of degree at most 2
 - b) Set of all integers
 - c) Set of all natural numbers
 - d) Set of all prime numbers
2. What is a basis of a vector space?
 - a) Linearly independent set spanning the vector space
 - b) Any subset of the vector space
 - c) Linearly dependent set spanning the vector space
 - d) Any linearly independent set
3. When are vectors linearly independent?
 - a) Only the trivial combination is zero
 - b) At least one non-trivial combination is zero
 - c) Any combination is zero
 - d) No combination is zero
4. What is the rank of a matrix?
 - a) Dimension of the column space
 - b) Dimension of the null space
 - c) Number of rows
 - d) Number of columns
5. What is the null space of a matrix?
 - a) Set of all solutions to the homogeneous equation
 - b) Set of all solutions to the non-homogeneous equation
 - c) Set of all rows of the matrix
 - d) Set of all columns of the matrix
6. What is an eigenvalue?
 - a) Scalar such that $Av = \lambda v$
 - b) Vector such that $Av = \lambda v$
 - c) Matrix such that $Av = \lambda v$
 - d) Scalar such that $A = \lambda v$
7. What is an eigenvector?
 - a) Vector such that $Av = \lambda v$

- b) Scalar such that $Av = \lambda v$
- c) Matrix such that $Av = \lambda v$
- d) Vector such that $A = \lambda v$

8. What is a linear transformation?

- a) Function preserving vector addition and scalar multiplication
- b) Function preserving vector subtraction and scalar division
- c) Function preserving matrix addition and scalar multiplication
- d) Function preserving matrix subtraction and scalar division

9. How do you calculate the determinant of a matrix?

- a) Using cofactor expansion
- b) Using matrix addition
- c) Using matrix multiplication
- d) Using scalar multiplication

10. What is a diagonal matrix?

- a) Matrix with non-zero entries only on the diagonal
- b) Matrix with all zero entries
- c) Matrix with non-zero entries off the diagonal
- d) Matrix with all non-zero entries

11. What is the trace of a matrix?

- a) Sum of the diagonal elements
- b) Product of the diagonal elements
- c) Sum of all elements
- d) Product of all elements

12. What is a symmetric matrix?

- a) $A = A^T$
- b) $A = -A^T$
- c) $A = A^{-1}$
- d) $A = -A^{-1}$

13. What is orthogonality?

- a) Vectors with zero dot product
- b) Vectors with non-zero dot product
- c) Vectors with zero cross product
- d) Vectors with non-zero cross product

14. What is the Gram-Schmidt process used for?

- a) Orthonormalizing a set of vectors
- b) Finding the determinant of a matrix
- c) Finding the inverse of a matrix

d) Solving a system of equations

15. What is a unitary matrix?

- a) $U^*U^H = I$
- b) $U^2 = I$
- c) $U^T = U$
- d) $U = -U$

16. What is a Hermitian matrix?

- a) $A = A^H$
- b) $A = A^T$
- c) $A = -A^H$
- d) $A = -A^T$

17. What is the characteristic polynomial of a matrix?

- a) $\det(A - \lambda I)$
- b) $A^2 - \lambda I$
- c) $A + \lambda I$
- d) $A - \lambda I$

18. What is the minimal polynomial of a matrix?

- a) Smallest degree monic polynomial annihilating the matrix
- b) Largest degree monic polynomial annihilating the matrix
- c) Any polynomial annihilating the matrix
- d) Polynomial with the smallest coefficients

19. How do you find the inverse of a matrix?

- a) Using the adjoint method
- b) Using matrix addition
- c) Using matrix multiplication
- d) Using scalar multiplication

20. What is the adjoint of a matrix?

- a) Transpose of the cofactor matrix
- b) Inverse of the cofactor matrix
- c) Sum of the cofactor matrix
- d) Product of the cofactor matrix

21. What is the transpose of a matrix?

- a) Matrix with rows and columns swapped
- b) Matrix with rows swapped
- c) Matrix with columns swapped
- d) Matrix with all elements swapped

22. What is a skew-symmetric matrix?

- a) $A = -A^T$
- b) $A = A^T$
- c) $A = A^{-1}$
- d) $A = -A^{-1}$

23. What is a positive definite matrix?

- a) All eigenvalues are positive
- b) All eigenvalues are negative
- c) Determinant is zero
- d) Trace is zero

24. What is the singular value decomposition (SVD)?

- a) $A = U\Sigma V^T$
- b) $A = U\Sigma V$
- c) $A = UV^T$
- d) $A = UV$

25. What is the spectral theorem?

- a) Any normal matrix can be diagonalized by a unitary matrix
- b) Any normal matrix can be diagonalized by a symmetric matrix
- c) Any normal matrix can be diagonalized by a skew-symmetric matrix
- d) Any normal matrix can be diagonalized by an orthogonal matrix

26. What is a projection matrix?

- a) $P^2 = P$
- b) $P^2 = I$
- c) $P = P^{-1}$
- d) $P = P^T$

27. What is the Moore-Penrose pseudoinverse?

- a) A^+ satisfying four Penrose conditions
- b) A^+ satisfying two Penrose conditions
- c) A^+ satisfying one Penrose condition
- d) A^+ satisfying three Penrose conditions

28. What is the Cayley-Hamilton theorem?

- a) Every square matrix satisfies its own characteristic equation
- b) Every square matrix satisfies its own minimal equation
- c) Every square matrix satisfies its own polynomial equation
- d) Every square matrix satisfies its own linear equation

29. What is a normal matrix?

- a) $A^T A = A A^T$

- b) $A = A^T$
- c) $A = A^{-1}$
- d) $A = -A^T$

30. What is a stochastic matrix?

- a) Matrix with non-negative entries and rows summing to 1
- b) Matrix with negative entries and rows summing to 1
- c) Matrix with non-negative entries and rows summing to 0
- d) Matrix with negative entries and rows summing to 0

31. What is a Markov chain?

- a) Stochastic process with the Markov property
- b) Deterministic process with the Markov property
- c) Stochastic process without the Markov property
- d) Deterministic process without the Markov property

32. What is a linear combination?

- a) Sum of scalar multiples of vectors
- b) Difference of scalar multiples of vectors
- c) Product of scalar multiples of vectors
- d) Quotient of scalar multiples of vectors

33. What is the column space of a matrix?

- a) Span of the columns
- b) Span of the rows
- c) Null space
- d) Row space

34. What is the row space of a matrix?

- a) Span of the rows
- b) Span of the columns
- c) Null space
- d) Column space

35. What is a linear equation system?

- a) Set of equations with linear terms
- b) Set of equations with non-linear terms
- c) Set of equations with quadratic terms
- d) Set of equations with cubic terms

36. What is the solution set of a linear system?

- a) Set of all possible solutions
- b) Set of all impossible solutions
- c) Set of all unique solutions

d) Set of all duplicate solutions

37. What is Gaussian elimination?

- a) Method for solving linear systems
- b) Method for solving quadratic systems
- c) Method for solving cubic systems
- d) Method for solving differential equations

38. What is LU decomposition?

- a) $A = LU$
- b) $A = LU^T$
- c) $A = L^T U$
- d) $A = U^T L$

39. What is QR decomposition?

- a) $A = QR$
- b) $A = Q^T R$
- c) $A = Q^H R$
- d) $A = RQ$

40. What is the Jordan canonical form?

- a) Diagonal form with Jordan blocks
- b) Diagonal form with null blocks
- c) Diagonal form with identity blocks
- d) Diagonal form with scalar blocks

41. What is a bilinear form?

- a) Function mapping two vectors to a scalar
- b) Function mapping one vector to a scalar
- c) Function mapping three vectors to a scalar
- d) Function mapping two scalars to a vector

42. What is an inner product space?

- a) Vector space with an inner product
- b) Vector space with an outer product
- c) Vector space with a cross product
- d) Vector space with a dot product

43. What is the Cauchy-Schwarz inequality?

- a) $|\langle u, v \rangle| \leq \|u\| \|v\|$
- b) $|\langle u, v \rangle| \geq \|u\| \|v\|$
- c) $|\langle u, v \rangle| = \|u\| \|v\|$
- d) $|\langle u, v \rangle| \neq \|u\| \|v\|$

44. What is a normed vector space?
- a) Vector space with a norm
 - b) Vector space with a length
 - c) Vector space with a width
 - d) Vector space with a height
45. What is the L2 norm?
- a) Euclidean norm
 - b) Manhattan norm
 - c) Chebyshev norm
 - d) Hamming norm
46. What is the L1 norm?
- a) Manhattan norm
 - b) Euclidean norm
 - c) Chebyshev norm
 - d) Hamming norm
47. What is a Banach space?
- a) Complete normed vector space
 - b) Incomplete normed vector space
 - c) Complete inner product space
 - d) Incomplete inner product space
48. What is a Hilbert space?
- a) Complete inner product space
 - b) Incomplete inner product space
 - c) Complete normed vector space
 - d) Incomplete normed vector space
49. What is the triangle inequality?
- a) $\|u + v\| \leq \|u\| + \|v\|$
 - b) $\|u + v\| \geq \|u\| + \|v\|$
 - c) $\|u + v\| = \|u\| + \|v\|$
 - d) $\|u + v\| \neq \|u\| + \|v\|$
50. What is a convex set?
- a) Set containing all line segments between any two points
 - b) Set containing some line segments between any two points
 - c) Set containing no line segments between any two points
 - d) Set containing one line segment between any two points
51. What is the convex hull?
- a) Smallest convex set containing a given set of points

- b) Largest convex set containing a given set of points
- c) Any convex set containing a given set of points
- d) No convex set containing a given set of points

52. What is a linear map?

- a) Function preserving vector addition and scalar multiplication
- b) Function preserving vector subtraction and scalar division
- c) Function preserving matrix addition and scalar multiplication
- d) Function preserving matrix subtraction and scalar division

53. What is the kernel of a linear map?

- a) Set of vectors mapped to the zero vector
- b) Set of vectors mapped to non-zero vectors
- c) Set of vectors mapped to positive vectors
- d) Set of vectors mapped to negative vectors

54. What is the image of a linear map?

- a) Set of vectors mapped to by the linear map
- b) Set of vectors mapped to the zero vector
- c) Set of vectors mapped to positive vectors
- d) Set of vectors mapped to negative vectors

55. What is the rank-nullity theorem?

- a) $\dim(\ker(T)) + \dim(\text{im}(T)) = \dim(V)$
- b) $\dim(\ker(T)) - \dim(\text{im}(T)) = \dim(V)$
- c) $\dim(\ker(T)) + \dim(\text{im}(T)) = \dim(W)$
- d) $\dim(\ker(T)) - \dim(\text{im}(T)) = \dim(W)$

56. What is a linear operator?

- a) Linear map from a vector space to itself
- b) Linear map from a vector space to another vector space
- c) Linear map from a vector space to a scalar field
- d) Linear map from a scalar field to a vector space

57. What is the matrix representation of a linear transformation?

- a) Matrix acting on vectors to produce the same result as the transformation
- b) Matrix acting on scalars to produce the same result as the transformation
- c) Matrix acting on vectors to produce a different result as the transformation
- d) Matrix acting on scalars to produce a different result as the transformation

58. What is a change of basis?

- a) Transition from one vector space basis to another
- b) Transition from one scalar field to another
- c) Transition from one matrix to another

d) Transition from one vector to another

59. What is a coordinate vector?

- a) Vector of coefficients representing a vector in a given basis
- b) Vector of coefficients representing a vector in the standard basis
- c) Vector of coefficients representing a vector in the zero basis
- d) Vector of coefficients representing a vector in the one basis

60. What is the orthogonal complement?

- a) Set of all vectors orthogonal to a given set
- b) Set of all vectors parallel to a given set
- c) Set of all vectors equal to a given set
- d) Set of all vectors opposite to a given set

61. What is an orthonormal basis?

- a) Basis with orthogonal unit vectors
- b) Basis with orthogonal vectors of any length
- c) Basis with parallel unit vectors
- d) Basis with parallel vectors of any length

62. What is the spectral radius?

- a) Largest absolute value of the eigenvalues
- b) Smallest absolute value of the eigenvalues
- c) Sum of the absolute values of the eigenvalues
- d) Product of the absolute values of the eigenvalues

63. What is a cyclic subspace?

- a) Smallest invariant subspace containing a vector
- b) Largest invariant subspace containing a vector
- c) Any invariant subspace containing a vector
- d) No invariant subspace containing a vector

64. What is a Jordan block?

- a) Matrix with eigenvalue on the diagonal and ones on the superdiagonal
- b) Matrix with eigenvalue on the diagonal and zeros elsewhere
- c) Matrix with ones on the diagonal and eigenvalue on the superdiagonal
- d) Matrix with ones on the diagonal and zeros elsewhere

65. What is the polar decomposition?

- a) $A = UP$
- b) $A = U\Sigma$
- c) $A = V\Sigma$
- d) $A = UV$

66. What is a quadratic form?

- a) Homogeneous polynomial of degree 2
- b) Homogeneous polynomial of degree 3
- c) Homogeneous polynomial of degree 1
- d) Homogeneous polynomial of degree 4

67. What is the Sylvester's criterion?

- a) Criterion for positive definiteness using leading principal minors
- b) Criterion for negative definiteness using leading principal minors
- c) Criterion for positive definiteness using eigenvalues
- d) Criterion for negative definiteness using eigenvalues

68. What is the characteristic equation?

- a) $\det(A - \lambda I) = 0$
- b) $\det(A + \lambda I) = 0$
- c) $A - \lambda I = 0$
- d) $A + \lambda I = 0$

69. What is the matrix exponential?

- a) $\exp(A) = I + A + A^2/2! + A^3/3! + \dots$
- b) $\exp(A) = I - A + A^2/2! - A^3/3! + \dots$
- c) $\exp(A) = I + A - A^2/2! + A^3/3! - \dots$
- d) $\exp(A) = I - A - A^2/2! - A^3/3! - \dots$

70. What is a block matrix?

- a) Matrix divided into submatrices
- b) Matrix divided into subrows
- c) Matrix divided into subcolumns
- d) Matrix divided into subdiagonals

71. What is an invariant subspace?

- a) Subspace preserved under a linear transformation
- b) Subspace not preserved under a linear transformation
- c) Subspace preserved under a scalar transformation
- d) Subspace not preserved under a scalar transformation

72. What is the singular value?

- a) Square root of the eigenvalue of $A^T A$
- b) Square root of the eigenvalue of $A A^T$
- c) Square of the eigenvalue of $A^T A$
- d) Square of the eigenvalue of $A A^T$

73. What is the Rayleigh quotient?

- a) $(x^T A x) / (x^T x)$

- b) $(x^T A x) / (x^T A)$
- c) $(x A x) / (x^T x)$
- d) $(A x) / (x^T x)$

74. What is a tridiagonal matrix?

- a) Matrix with non-zero entries only on the main diagonal, first superdiagonal, and first subdiagonal
- b) Matrix with non-zero entries only on the main diagonal
- c) Matrix with non-zero entries only on the superdiagonal
- d) Matrix with non-zero entries only on the subdiagonal

75. What is the Hessenberg form?

- a) Almost upper triangular matrix form
- b) Diagonal matrix form
- c) Lower triangular matrix form
- d) Upper triangular matrix form

76. What is the Cholesky decomposition?

- a) $A = L L^T$
- b) $A = L U$
- c) $A = Q R$
- d) $A = U V$

77. What is the Gershgorin circle theorem?

- a) Estimate the location of eigenvalues using Gershgorin discs
- b) Estimate the location of eigenvalues using Gershgorin squares
- c) Estimate the location of eigenvalues using Gershgorin triangles
- d) Estimate the location of eigenvalues using Gershgorin hexagons

78. What is a positive semi-definite matrix?

- a) All eigenvalues are non-negative
- b) All eigenvalues are non-positive
- c) Determinant is non-negative
- d) Trace is non-negative

79. What is a spectral norm?

- a) Largest singular value
- b) Smallest singular value
- c) Sum of singular values
- d) Product of singular values

80. What is the Frobenius norm?

- a) Square root of the sum of the squares of all elements
- b) Sum of the squares of all elements

- c) Square root of the sum of all elements
- d) Sum of all elements

81. What is a unit vector?

- a) Vector with length 1
- b) Vector with length 0
- c) Vector with length 2
- d) Vector with length -1

82. What is the spectral decomposition?

- a) $A = Q\Lambda Q^T$
- b) $A = Q\Lambda Q$
- c) $A = Q^T \Lambda Q$
- d) $A = Q\Lambda Q^{-1}$

83. What is a diagonalizable matrix?

- a) Matrix similar to a diagonal matrix
- b) Matrix similar to a triangular matrix
- c) Matrix similar to a square matrix
- d) Matrix similar to a rectangular matrix

84. What is the Schur decomposition?

- a) $A = UTU^H$
- b) $A = UTU^T$
- c) $A = U^T TU$
- d) $A = U^H TU$

85. What is a block diagonal matrix?

- a) Matrix with diagonal blocks
- b) Matrix with superdiagonal blocks
- c) Matrix with subdiagonal blocks
- d) Matrix with off-diagonal blocks

86. What is the rank of a linear map?

- a) Dimension of the image
- b) Dimension of the kernel
- c) Dimension of the vector space
- d) Dimension of the scalar field

87. What is the Moore-Penrose pseudoinverse used for?

- a) Solving least squares problems
- b) Solving linear systems
- c) Solving quadratic systems
- d) Solving cubic systems

88. What is the orthogonal projection?
- a) Projection onto a subspace using orthogonal vectors
 - b) Projection onto a subspace using parallel vectors
 - c) Projection onto a subspace using skew vectors
 - d) Projection onto a subspace using random vectors
89. What is a Hermitian inner product?
- a) Inner product with complex conjugate symmetry
 - b) Inner product with real symmetry
 - c) Inner product with imaginary symmetry
 - d) Inner product with no symmetry
90. What is a linear functional?
- a) Linear map from a vector space to its field of scalars
 - b) Linear map from a vector space to itself
 - c) Linear map from a vector space to another vector space
 - d) Linear map from a scalar field to a vector space
91. What is a hyperplane?
- a) Subspace of one dimension less than the ambient space
 - b) Subspace of the same dimension as the ambient space
 - c) Subspace of one dimension more than the ambient space
 - d) Subspace of two dimensions less than the ambient space
92. What is the canonical form?
- a) Simplified standard form of a mathematical object
 - b) Complex standard form of a mathematical object
 - c) Arbitrary standard form of a mathematical object
 - d) Random standard form of a mathematical object
93. What is the minimal polynomial used for?
- a) Determining the algebraic multiplicity of eigenvalues
 - b) Determining the geometric multiplicity of eigenvalues
 - c) Determining the characteristic polynomial of a matrix
 - d) Determining the determinant of a matrix
94. What is a nilpotent matrix?
- a) Matrix N such that $N^k = 0$ for some k
 - b) Matrix N such that $N^k \neq 0$ for any k
 - c) Matrix N such that $N^k = I$ for some k
 - d) Matrix N such that $N^k \neq I$ for any k
95. What is the adjugate matrix?

- a) Transpose of the cofactor matrix
- b) Inverse of the cofactor matrix
- c) Sum of the cofactor matrix
- d) Product of the cofactor matrix

96. What is the rank-nullity theorem used for?

- a) Relating the rank and nullity of a linear transformation
- b) Relating the rank and nullity of a matrix
- c) Relating the rank and nullity of a vector space
- d) Relating the rank and nullity of a scalar field

97. What is a rotation matrix?

- a) Orthogonal matrix representing a rotation
- b) Diagonal matrix representing a rotation
- c) Upper triangular matrix representing a rotation
- d) Lower triangular matrix representing a rotation

98. What is the matrix norm?

- a) Function mapping a matrix to a non-negative scalar
- b) Function mapping a matrix to a negative scalar
- c) Function mapping a matrix to a positive scalar
- d) Function mapping a matrix to a zero scalar

99. What is a permutation matrix?

- a) Matrix obtained by permuting rows of an identity matrix
- b) Matrix obtained by permuting columns of an identity matrix
- c) Matrix obtained by permuting rows and columns of an identity matrix
- d) Matrix obtained by permuting elements of an identity matrix

100. What is the least squares solution?

- a) Solution minimizing the sum of squared residuals
- b) Solution maximizing the sum of squared residuals
- c) Solution minimizing the sum of residuals
- d) Solution maximizing the sum of residuals