Dilan_Wijesinghe_Final_Project

December 5, 2022

1 Jack in the Box

[56]:
```
#########################################################################################
# If you're using Google Colab, uncomment this section by selecting the whole␣
↪section and press
# ctrl+'/' on your and keyboard. Run it before you start programming, this will␣
↪enable the nice
# LaTeX "display()" function for you. If you're using the local Jupyter␣
↪environment, leave it alone
#########################################################################################
import sympy as sym
import numpy as np
import matplotlib.pyplot as plt
from math import pi
def custom_latex_printer(exp,**options):
    from google.colab.output._publish import javascript
    url = "https://cdnjs.cloudflare.com/ajax/libs/mathjax/3.1.1/latest.js?
↪config=TeX-AMS_HTML"
    javascript(url=url)
    return sym.printing.latex(exp,**options)
sym.init_printing(use_latex="mathjax",latex_printer=custom_latex_printer)
```

--_aadaaa### Boilerplate

[57]:
```
def rk4(dxdt, x, dt, t):
  k1 = dt * dxdt(x, t)
  k2 = dt * dxdt(x + k1/2., t + dt/2)
  k3 = dt * dxdt(x + k2/2., t + dt/2)
  k4 = dt * dxdt(x + k3, t + dt)
  x_new = x + (1/6.) * (k1 + 2.* k2 + 2. * k3+k4)
  return(x_new)

def simulate2(f, x0, tspan, dt, integrate):
    """
    This function takes in an initial condition x0, a timestep dt,
    a time span tspan consisting of a list [min_time, max_time],
    as well as a dynamical system f(x) that outputs a vector of the
```

```
    same dimension as x0. It outputs a full trajectory simulated
    over the time span of dimensions (xvec_size, time_vec_size).

    Parameters
    ============
    f: Python function
        derivate of the system at a given step x(t),
        it can considered as \dot{x}(t) = func(x(t))
    x0: NumPy array
        initial conditions
    tspan: Python list
        tspan = [min_time, max_time], it defines the start and end
        time of simulation
    dt:
        time step for numerical integration
    integrate: Python function
        numerical integration method used in this simulation

    Return
    ============
    x_traj:
        simulated trajectory of x(t) from t=0 to tf
    """
    N = int((max(tspan)-min(tspan))/dt)
    x = np.copy(x0)
    tvec = np.linspace(min(tspan),max(tspan),N)
    xtraj = np.zeros((len(x0),N))
    for i in range(N):
        xtraj[:,i]=integrate(f,x,i*dt,dt)
        x = np.copy(xtraj[:,i])
    return xtraj
```

[58]:

```
# Almost a Carbon Copy of what we did in HW5
def impact_condition(s, phi_func, threshold=1e-1):
    phi_val = phi_func(*s)
    for i in range(phi_val.shape[0]):
        # print("Phi Val", phi_val[i])
        if phi_val[i] > -threshold and phi_val[i] < threshold:
            return (True, i)
    return (False, None)



def impact_update(s, imp_eqs, imp_subs):
  # Substitute First
  substitute_teacher = {xbd_dumb:s[6], ybd_dumb:s[7], tbd_dumb:s[8],
                        xjd_dumb:s[9], yjd_dumb:s[10], tjd_dumb:s[11],
                        xb_dumb:s[0], yb_dumb:s[1], tb_dumb:s[2],
```

```
                        xj_dumb:s[3], yj_dumb:s[4], tj_dumb:s[5]}

  substitute_again = {xb_plus:s[0], yb_plus:s[1], tb_plus:s[2],
                      xj_plus:s[3], yj_plus:s[4], tj_plus:s[5]}
  # Teach the impact a lesson
  impact_again = imp_eqs.subs(substitute_teacher)
  impact_again = sym.simplify(impact_again)

  impact_sub_again = impact_again.subs(substitute_again)
  # display(impact_sub_again)

  # Let the impact solve some questions on the test
  impact_sols = sym.solve(impact_sub_again, [xb_plus, yb_plus, tb_plus,
                                             xj_plus, yj_plus, tj_plus,
                                             xbd_plus, ybd_plus, tbd_plus,
                                             xjd_plus, yjd_plus, tjd_plus,
                                             lamb], dict=True)

  # Check if those solutions are good
  for sol in impact_sols:
    # display("We hit sumthin!", sol)
    lambda_update_sol = sol[lamb]
    # display(lambda_update_sol)
    if abs(lambda_update_sol) < 0.000001:
      print("Yo, that's small")
    else:
        return np.array([s[0], s[1], s[2], s[3], s[4], s[5],
                        sol[xbd_plus], sol[ybd_plus],sol[tbd_plus],
                        sol[xjd_plus], sol[yjd_plus],sol[tjd_plus], ])

def simulate_with_impact(f,x0,tspan,dt,integrate,imp_eqs,imp_subs):
    N = int ((max(tspan)-min(tspan))/dt)
    x = np.copy(x0)
    tvec = np.linspace(min(tspan), max(tspan), N)
    xtraj = np.zeros((len(x0), N))
    t = 0
    for i in range (N):
        t = t+dt
        (impact, which_imp) = impact_condition(x, phi_func, 1e-1)
        # print(f"Impact is {impact} num: {which_imp}")
        if impact is True:
            print(f"Impact is {impact} num: {which_imp}")
            x = impact_update(x, imp_eqs[which_imp], imp_subs)
            xtraj[:,i]=integrate(f, x, dt, t)
            # print("Finished Integration")
        else :
            xtraj[:,i]=integrate(f, x, dt, t)
```

```python
        x = np.copy(xtraj[:,i])
    return xtraj
```

1.0.1 Helper Functions

[59]:
```python
# Input: Rotation Angle, 2D Translation Vector
# Returns: SE(3) Matrix
def vecToSE3(theta, vec):
  return sym.Matrix([ [sym.cos(theta), -sym.sin(theta), 0,  vec[0]],
                      [sym.sin(theta),  sym.cos(theta), 0,  vec[1]],
                      [0,                0,              1,       0],
                      [0,                0,              0,       1] ])

def npvecToSE3(theta, vec):
  return np.array([ [np.cos(theta), -np.sin(theta), 0,  vec[0]],
                    [np.sin(theta),  np.cos(theta), 0,  vec[1]],
                    [0,               0,             1,     0],
                    [0,               0,             0,     1] ])

# Input: A 3x1 Matrix
# Returns: a 3x3 Matrix
def hat(x):
  return sym.Matrix([ [0, -x[2], x[1]],
                      [x[2], 0, -x[0]],
                      [-x[1], x[0], 0] ])

# Input: A 3x3 Matrix
# Returns: a 3x1 Matrix
def unhat(x):
  return sym.Matrix([ [x[2,1], x[0,2], x[1,0]] ])

# Input: A 4x4 Matrix
# Returns: A 6x1 Matrix
def unhatSE3(x):
  in_ = sym.Matrix([ [x[0,0], x[0,1], x[0,2]],
                     [x[1,0], x[1,1], x[1,2]],
                     [x[2,0], x[2,1], x[2,2]]
                   ])
  w = unhat(in_)
  v = sym.Matrix([ x[0,3], x[1,3], x[2,3] ])
  return sym.Matrix([ [v[0], v[1], v[2], w[0], w[1], w[2]] ]).T

# Input: A 6x1 Matrix
# Returns: A 4x4 Matrix
def hatSE3(x):
  v = sym.Matrix([ [x[0], x[1], x[2]] ])
```

```
  w = sym.Matrix([ [x[3], x[4], x[5]] ])
  w_hat = hat(w)
  return sym.Matrix([ [w_hat[0,0], w_hat[0,1], w_hat[0,2], v[0]],
                      [w_hat[1,0], w_hat[1,1], w_hat[1,2], v[1]],
                      [w_hat[2,0], w_hat[2,1], w_hat[2,2], v[2]],
                      [0,          0,          0,              0]
                    ])

def SE3Inv(x):
  R = sym.Matrix([ [x[0,0], x[0,1], x[0,2]],
                   [x[1,0], x[1,1], x[1,2]],
                   [x[2,0], x[2,1], x[2,2]] ])

  p = sym.Matrix([ [x[0,3]],
                   [x[1,3]],
                   [x[2,3]] ])
  rhs = R.T
  lhs = -rhs*p
  return sym.Matrix([ [rhs[0,0], rhs[0,1], rhs[0,2], lhs[0]],
                      [rhs[1,0], rhs[1,1], rhs[1,2], lhs[1]],
                      [rhs[2,0], rhs[2,1], rhs[2,2], lhs[2]],
                      [0,        0,        0,        1] ])

def SO3dot(x):
  return x.diff(t)

def SE3dot(x):
  return x.diff(t)
```

1.0.2 System Variables and Transformation Frames

[60]:
```
# Define Symbols
t = sym.symbols('t')

# Rather than define symbols, just insert the values
lb = 10
lj = 1
mb = 1000
mj = 1
g = 9.8
J_j = 1/12 * (mj*(lj)**2) # Based off moment of inertia
J_b = 1/12 * (mb*(lb)**2)

# Define the system vars (q)
# Box Vars
xb = sym.Function('x_b')(t)
```

```python
yb = sym.Function('y_b')(t)
tb = sym.Function('theta_b')(t)
# Jack Vars
xj = sym.Function('x_j')(t)
yj = sym.Function('y_j')(t)
tj = sym.Function('theta_j')(t)

# Transformation Frames
t_wj = vecToSE3(tj, [xj,yj]) # Transform from world -> jack
t_wb = vecToSE3(tb, [xb,yb]) # Transform from world -> Middle of Box

t_bb1 = vecToSE3(0, [lb/2, 0]) # Right
t_bb2 = vecToSE3(0, [0,-lb/2]) # Down
t_bb3 = vecToSE3(0, [-lb/2,0]) # Left
t_bb4 = vecToSE3(0, [0, lb/2]) # Up

t_jj1 = vecToSE3(0, [lj/2, lj/2])  # Right
t_jj2 = vecToSE3(0, [lj/2,-lj/2])  # Down
t_jj3 = vecToSE3(0, [-lj/2,-lj/2]) # Left
t_jj4 = vecToSE3(0, [-lj/2, lj/2]) # Up

# Lets define BOX in terms of WORLD FRAME
t_wb1 = t_wb * t_bb1
t_wb2 = t_wb * t_bb2
t_wb3 = t_wb * t_bb3
t_wb4 = t_wb * t_bb4

print("SE(3) transformation from {W} to {(B)ox}")
display(t_wb)

print("SE(3) transformation from {W} to {(J)ack}")
display(t_wj)

print("SE(3) transformation from {W} to {(B1)ox}")
display(t_wb1)

print("SE(3) transformation from {W} to {(B2)ox}")
display(t_wb2)

print("SE(3) transformation from {W} to {(B3)ox}")
display(t_wb3)

print("SE(3) transformation from {W} to {(B4)ox}")
display(t_wb4)

# Lets define JACK/DICE in terms of WORLD FRAME
t_wj1 = t_wj * t_jj1
```

```
t_wj2 = t_wj * t_jj2
t_wj3 = t_wj * t_jj3
t_wj4 = t_wj * t_jj4

print("SE(3) transformation from {W} to {(J1)ack}")
display(t_wj1)

print("SE(3) transformation from {W} to {(J2)ack}")
display(t_wj2)

print("SE(3) transformation from {W} to {(J3)ack}")
display(t_wj3)

print("SE(3) transformation from {W} to {(J4)ack}")
display(t_wj4)
```

```
SE(3) transformation from {W} to {(B)ox}

<IPython.core.display.HTML object>
```

$$\begin{bmatrix}\cos(\theta_b(t)) & -\sin(\theta_b(t)) & 0 & \mathrm{x_b}(t)\\ \sin(\theta_b(t)) & \cos(\theta_b(t)) & 0 & \mathrm{y_b}(t)\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

```
SE(3) transformation from {W} to {(J)ack}

<IPython.core.display.HTML object>
```

$$\begin{bmatrix}\cos(\theta_j(t)) & -\sin(\theta_j(t)) & 0 & \mathrm{x_j}(t)\\ \sin(\theta_j(t)) & \cos(\theta_j(t)) & 0 & \mathrm{y_j}(t)\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

```
SE(3) transformation from {W} to {(B1)ox}

<IPython.core.display.HTML object>
```

$$\begin{bmatrix}\cos(\theta_b(t)) & -\sin(\theta_b(t)) & 0 & \mathrm{x_b}(t) + 5.0\cos(\theta_b(t))\\ \sin(\theta_b(t)) & \cos(\theta_b(t)) & 0 & \mathrm{y_b}(t) + 5.0\sin(\theta_b(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

```
SE(3) transformation from {W} to {(B2)ox}

<IPython.core.display.HTML object>
```

$$\left[\begin{matrix}\cos{\left(\theta_{b}{\left(t \right)} \right)} & - \sin{\left(\theta_{b}{\left(t \right)} \right)} & 0 & \operatorname{x_{b}}{\left(t \right)} + 5.0 \sin{\left(\theta_{b}{\left(t \right)} \right)}\\\sin{\left(\theta_{b}{\left(t \right)} \right)} & \cos{\left(\theta_{b}{\left(t \right)} \right)} & 0 & \operatorname{y_{b}}{\left(t \right)} - 5.0 \cos{\left(\theta_{b}{\left(t \right)} \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{matrix}\right]$$

SE(3) transformation from {W} to {(B3)ox}

<IPython.core.display.HTML object>

$$\left[\begin{matrix}\cos{\left(\theta_{b}{\left(t \right)} \right)} & - \sin{\left(\theta_{b}{\left(t \right)} \right)} & 0 & \operatorname{x_{b}}{\left(t \right)} - 5.0 \cos{\left(\theta_{b}{\left(t \right)} \right)}\\\sin{\left(\theta_{b}{\left(t \right)} \right)} & \cos{\left(\theta_{b}{\left(t \right)} \right)} & 0 & \operatorname{y_{b}}{\left(t \right)} - 5.0 \sin{\left(\theta_{b}{\left(t \right)} \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{matrix}\right]$$

SE(3) transformation from {W} to {(B4)ox}

<IPython.core.display.HTML object>

$$\left[\begin{matrix}\cos{\left(\theta_{b}{\left(t \right)} \right)} & - \sin{\left(\theta_{b}{\left(t \right)} \right)} & 0 & \operatorname{x_{b}}{\left(t \right)} - 5.0 \sin{\left(\theta_{b}{\left(t \right)} \right)}\\\sin{\left(\theta_{b}{\left(t \right)} \right)} & \cos{\left(\theta_{b}{\left(t \right)} \right)} & 0 & \operatorname{y_{b}}{\left(t \right)} + 5.0 \cos{\left(\theta_{b}{\left(t \right)} \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{matrix}\right]$$

SE(3) transformation from {W} to {(J1)ack}

<IPython.core.display.HTML object>

$$\left[\begin{matrix}\cos{\left(\theta_{j}{\left(t \right)} \right)} & - \sin{\left(\theta_{j}{\left(t \right)} \right)} & 0 & \operatorname{x_{j}}{\left(t \right)} - 0.5 \sin{\left(\theta_{j}{\left(t \right)} \right)} + 0.5 \cos{\left(\theta_{j}{\left(t \right)} \right)}\\\sin{\left(\theta_{j}{\left(t \right)} \right)} & \cos{\left(\theta_{j}{\left(t \right)} \right)} & 0 & \operatorname{y_{j}}{\left(t \right)} + 0.5 \sin{\left(\theta_{j}{\left(t \right)} \right)} + 0.5 \cos{\left(\theta_{j}{\left(t \right)} \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{matrix}\right]$$

SE(3) transformation from {W} to {(J2)ack}

<IPython.core.display.HTML object>

$$\left[\begin{matrix}\cos{\left(\theta_{j}{\left(t \right)} \right)} & - \sin{\left(\theta_{j}{\left(t \right)} \right)} & 0 & \operatorname{x_{j}}{\left(t \right)} + 0.5 \sin{\left(\theta_{j}{\left(t \right)} \right)} + 0.5 \cos{\left(\theta_{j}{\left(t \right)} \right)}\\\sin{\left(\theta_{j}{\left(t \right)} \right)} & \cos{\left(\theta_{j}{\left(t \right)} \right)} & 0 & \operatorname{y_{j}}{\left(t \right)} + 0.5 \sin{\left(\theta_{j}{\left(t \right)} \right)} - 0.5 \cos{\left(\theta_{j}{\left(t \right)} \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{matrix}\right]$$

SE(3) transformation from {W} to {(J3)ack}

<IPython.core.display.HTML object>

$$\left[\begin{matrix}\cos{\left(\theta_{j}{\left(t \right)} \right)} & - \sin{\left(\theta_{j}{\left(t \right)} \right)} & 0 & \operatorname{x_{j}}{\left(t \right)} + 0.5 \sin{\left(\theta_{j}{\left(t \right)} \right)} - 0.5 \cos{\left(\theta_{j}{\left(t \right)} \right)}\\\sin{\left(\theta_{j}{\left(t \right)} \right)} & \cos{\left(\theta_{j}{\left(t \right)} \right)} & 0 & \operatorname{y_{j}}{\left(t \right)} - 0.5 \sin{\left(\theta_{j}{\left(t \right)} \right)} - 0.5 \cos{\left(\theta_{j}{\left(t \right)} \right)}\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{matrix}\right]$$

SE(3) transformation from {W} to {(J4)ack}

<IPython.core.display.HTML object>

$$\begin{bmatrix}\cos(\theta_j(t)) & -\sin(\theta_j(t)) & 0 & \mathrm{x_j}(t) - 0.5\sin(\theta_j(t)) - 0.5\cos(\theta_j(t)) \\ \sin(\theta_j(t)) & \cos(\theta_j(t)) & 0 & \mathrm{y_j}(t) - 0.5\sin(\theta_j(t)) + 0.5\cos(\theta_j(t)) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{bmatrix}$$

[61]:
```
# Inertia Matrix 1 - For The Jack
In_j = sym.Matrix([[mj,0,0,0,0,0],
                  [0,mj,0,0,0,0],
                  [0,0,mj,0,0,0],
                  [0,0,0,0,0,0],
                  [0,0,0,0,0,0],
                  [0,0,0,0,0,J_j] ])

# Inertia Matrix 2 - For the Box
In_b = sym.Matrix([[mb,0,0,0,0,0],
                   [0,mb,0,0,0,0],
                   [0,0,mb,0,0,0],
                   [0,0,0,0,0,0],
                   [0,0,0,0,0,0],
                   [0,0,0,0,0,J_b] ])



# Twists
V_wj =  unhatSE3((SE3Inv(t_wj) @ SE3dot(t_wj))) # For Jack
V_wb =  unhatSE3((SE3Inv(t_wb) @ SE3dot(t_wb))) # For Box

print("SE(3) Twist from {W} to {B}")
V_wb = sym.simplify(V_wb)
display(V_wb)

print("SE(3) Twist from {W} to {J}")
V_wj = sym.simplify(V_wj)
display(V_wj)

print("Kinetic Energy")
KE = (1/2 * (V_wj.T @ In_j @ V_wj)) + (1/2 * (V_wb.T @ In_b @ V_wb))
KE = sym.simplify(KE[0])
display(KE)

V = mb*g*yb + mj*g*yj
V = sym.simplify(V)
print("V")
display(V)

L = KE - V
```

```
L = sym.simplify(L)
print("Lagrangian")
display(L)
```

```
SE(3) Twist from {W} to {B}
<IPython.core.display.HTML object>
```

$$\left[\begin{matrix}\sin\left(\theta_b(t)\right)\frac{d}{dt}\operatorname{y_b}\left(t\right)+\cos\left(\theta_b(t)\right)\frac{d}{dt}\operatorname{x_b}\left(t\right)\\-\sin\left(\theta_b(t)\right)\frac{d}{dt}\operatorname{x_b}\left(t\right)+\cos\left(\theta_b(t)\right)\frac{d}{dt}\operatorname{y_b}\left(t\right)\\0\\0\\0\\\frac{d}{dt}\theta_b(t)\end{matrix}\right]$$

```
SE(3) Twist from {W} to {J}
<IPython.core.display.HTML object>
```

$$\left[\begin{matrix}\sin\left(\theta_j(t)\right)\frac{d}{dt}\operatorname{y_j}\left(t\right)+\cos\left(\theta_j(t)\right)\frac{d}{dt}\operatorname{x_j}\left(t\right)\\-\sin\left(\theta_j(t)\right)\frac{d}{dt}\operatorname{x_j}\left(t\right)+\cos\left(\theta_j(t)\right)\frac{d}{dt}\operatorname{y_j}\left(t\right)\\0\\0\\0\\\frac{d}{dt}\theta_j(t)\end{matrix}\right]$$

```
Kinetic Energy
<IPython.core.display.HTML object>
```

$$4166.66666666667\left(\frac{d}{dt}\theta_b(t)\right)^2+0.0416666666666667\left(\frac{d}{dt}\theta_j(t)\right)^2+500.0\left(\frac{d}{dt}\operatorname{x_b}\left(t\right)\right)^2+0.5\left(\frac{d}{dt}\operatorname{x_j}\left(t\right)\right)^2+500.0\left(\frac{d}{dt}\operatorname{y_b}\left(t\right)\right)^2+0.5\left(\frac{d}{dt}\operatorname{y_j}\left(t\right)\right)^2$$

```
V
<IPython.core.display.HTML object>
```

$$9800.0\operatorname{y_b}\left(t\right)+9.8\operatorname{y_j}\left(t\right)$$

```
Lagrangian
<IPython.core.display.HTML object>
```

$$-9800.0\operatorname{y_b}\left(t\right)-9.8\operatorname{y_j}\left(t\right)+4166.66666666667\left(\frac{d}{dt}\theta_b(t)\right)^2+0.0416666666666667\left(\frac{d}{dt}\theta_j(t)\right)^2+500.0\left(\frac{d}{dt}\operatorname{x_b}\left(t\right)\right)^2+0.5\left(\frac{d}{dt}\operatorname{x_j}\left(t\right)\right)^2+500.0\left(\frac{d}{dt}\operatorname{y_b}\left(t\right)\right)^2+0.5\left(\frac{d}{dt}\operatorname{y_j}\left(t\right)\right)^2$$

[62]:
```python
# Define Symbols for Constraints + Force
# Define lambda 1 - 16
# Define phidd_1 - 16

# Euler Lagrange Calculations
q = sym.Matrix([xb, yb, tb, xj, yj, tj])
q_dot = q.diff(t)
q_ddot = q_dot.diff(t)

L_mat = sym.Matrix([L])

dLdq = L_mat.jacobian(q).T
dLdqd = L_mat.jacobian(q_dot).T
dLdqd_dt = dLdqd.diff(t)
dLdqd_dt = sym.simplify(dLdqd_dt)

el = dLdqd_dt - dLdq
print('\n\033[1m Euler-Lagrange Eq: ')
el = sym.simplify(el)
display(el)

# Forces
Fy = mb*g # Offset Force to Keep Box Up
Ft = sym.cos(pi*t/6) # Force that spins the cup
Amp = 2
FE = sym.Matrix([0, Fy, -10000*Ft, 0, 0, 0]) # Not Iron


# Eqn for LHS and RHS
lhs = sym.Matrix([*el])
# rhs = sym.Matrix([0, 0, 0, 0, 0, 0]) # For earlier testing
rhs = FE
eqn = sym.Eq(lhs, rhs)
print('\n\033[1m Euler-Lagrange Eq LHS vs RHS: ')
display(eqn)

soln = sym.solve(eqn, q_ddot, dict=True)
```

Euler-Lagrange Eq:

<IPython.core.display.HTML object>

$$\begin{bmatrix} 1000.0\frac{d^2}{dt^2}\,\mathrm{x_b}(t) \\ 1000.0\frac{d^2}{dt^2}\,\mathrm{y_b}(t) + 9800.0 \\ 8333.33333333333\frac{d^2}{dt^2}\theta_b(t) \\ 1.0\frac{d^2}{dt^2}\,\mathrm{x_j}(t) \\ 1.0\frac{d^2}{dt^2}\,\mathrm{y_j}(t) + 9.8 \\ 0.0833333333333333\frac{d^2}{dt^2}\theta_j(t) \end{bmatrix}$$

Euler-Lagrange Eq LHS vs RHS:

<IPython.core.display.HTML object>

$$\begin{bmatrix} 1000.0\frac{d^2}{dt^2}\,\mathrm{x_b}(t) \\ 1000.0\frac{d^2}{dt^2}\,\mathrm{y_b}(t) + 9800.0 \\ 8333.33333333333\frac{d^2}{dt^2}\theta_b(t) \\ 1.0\frac{d^2}{dt^2}\,\mathrm{x_j}(t) \\ 1.0\frac{d^2}{dt^2}\,\mathrm{y_j}(t) + 9.8 \\ 0.0833333333333333\frac{d^2}{dt^2}\theta_j(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 9800.0 \\ -10000\cos(0.523598775598299t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

[63]:
```
# solver = sym.Matrix([*q_ddot]) # just Q_dot for now, but add in lambdas later
sim_funcs = []
for sol in soln:
  print('\n\033[1mSolution: ')
  for v in q_ddot:
    display(sym.Eq(v,sol[v]))
    sim_funcs.append(sol[v])
```

Solution:

<IPython.core.display.HTML object>

$$\frac{d^2}{dt^2}\,\mathrm{x_b}(t) = 0.0$$

<IPython.core.display.HTML object>

$$\frac{d^2}{dt^2}\,\mathrm{y_b}(t) = 0.0$$

<IPython.core.display.HTML object>

$$\frac{d^2}{dt^2}\theta_b(t) = -1.2\cos(0.523598775598299t)$$

<IPython.core.display.HTML object>

$$\frac{d^2}{dt^2}\,\mathrm{x_j}(t) = 0.0$$

<IPython.core.display.HTML object>

$$\frac{d^2}{dt^2}\,\mathrm{y_j}(t) = -9.8$$

<IPython.core.display.HTML object>

$$\frac{d^2}{dt^2}\theta_j(t) = 0.0$$

[64]:
```
xb_dd_sol = sim_funcs[0] # For x_box
yb_dd_sol = sim_funcs[1] # For y_box
tb_dd_sol = sim_funcs[2] # For theta_box
xj_dd_sol = sim_funcs[3] # For x_jack
yj_dd_sol = sim_funcs[4] # For y_jack
tj_dd_sol = sim_funcs[5] # For theta_jack

# Lambdified Equations
xb_dd_l = sym.lambdify([q[0], q[1], q[2], q[3], q[4], q[5], q_dot[0], q_dot[1],␣
↪q_dot[2], q_dot[3], q_dot[4], q_dot[5], t], xb_dd_sol)
yb_dd_l = sym.lambdify([q[0], q[1], q[2], q[3], q[4], q[5], q_dot[0], q_dot[1],␣
↪q_dot[2], q_dot[3], q_dot[4], q_dot[5], t], yb_dd_sol)
tb_dd_l = sym.lambdify([q[0], q[1], q[2], q[3], q[4], q[5], q_dot[0], q_dot[1],␣
↪q_dot[2], q_dot[3], q_dot[4], q_dot[5], t], tb_dd_sol)
xj_dd_l = sym.lambdify([q[0], q[1], q[2], q[3], q[4], q[5], q_dot[0], q_dot[1],␣
↪q_dot[2], q_dot[3], q_dot[4], q_dot[5], t], xj_dd_sol)
yj_dd_l = sym.lambdify([q[0], q[1], q[2], q[3], q[4], q[5], q_dot[0], q_dot[1],␣
↪q_dot[2], q_dot[3], q_dot[4], q_dot[5], t], yj_dd_sol)
tj_dd_l = sym.lambdify([q[0], q[1], q[2], q[3], q[4], q[5], q_dot[0], q_dot[1],␣
↪q_dot[2], q_dot[3], q_dot[4], q_dot[5], t], tj_dd_sol)
```

1.1 Dummy Variables for Impact

[65]:
```
xb_dumb, yb_dumb, tb_dumb, xj_dumb, yj_dumb, tj_dumb = sym.symbols(r'x_{box},␣
↪y_{box}, \theta_{box}, x_{jack}, y_{jack}, \theta_{jack}')
xbd_dumb, ybd_dumb, tbd_dumb, xjd_dumb, yjd_dumb, tjd_dumb = sym.
↪symbols(r'\dot{x_{box}}, \dot{y_{box}}, \dot{\theta_{box}}, \dot{x_{jack}},␣
↪\dot{y_{jack}}, \dot{\theta_{jack}}')

# Start with Veloicites since they always have a hard time subbing :(!
dumb_sub = {q_dot[0]:xbd_dumb, q_dot[1]:ybd_dumb, q_dot[2]:tbd_dumb,
            q_dot[3]:xjd_dumb, q_dot[4]:yjd_dumb, q_dot[5]:tjd_dumb,
            xb:xb_dumb, yb:yb_dumb, tb:tb_dumb,
            xj:xj_dumb, yj:yj_dumb, tj:tj_dumb,}

# print('\n\033[1m q and q_dots: ')
```

```python
display(dumb_sub)
# display(xb_dumb, yb_dumb, tb_dumb, xj_dumb, yj_dumb, tj_dumb)
# display(xbd_dumb, ybd_dumb, tbd_dumb, xjd_dumb, yjd_dumb, tjd_dumb)
```

<IPython.core.display.HTML object>

$$\Big\{\theta_b(t) : \theta_{box},\ \theta_j(t) : \theta_{jack},\ \mathrm{x_b}(t) : x_{box},\ \mathrm{x_j}(t) : x_{jack},\ \mathrm{y_b}(t) : y_{box},\ \mathrm{y_j}(t) : y_{jack},\ \frac{d}{dt}\theta_b(t) : \dot{\theta_{box}},\ \frac{d}{dt}\theta_j(t) : \dot{\theta_{jack}},\ \frac{d}{dt}\mathrm{x}$$

[66]:
```python
# Impact Transformation Matrices
# From Wall 1 to Jack Vertices
t_b1j1 = SE3Inv(t_wb1) @ t_wj1
t_b1j2 = SE3Inv(t_wb1) @ t_wj2
t_b1j3 = SE3Inv(t_wb1) @ t_wj3
t_b1j4 = SE3Inv(t_wb1) @ t_wj4

# From Wall 2 to Jack Vertices
t_b2j1 = SE3Inv(t_wb2) @ t_wj1
t_b2j2 = SE3Inv(t_wb2) @ t_wj2
t_b2j3 = SE3Inv(t_wb2) @ t_wj3
t_b2j4 = SE3Inv(t_wb2) @ t_wj4
# display(t_b2j1)

# From Wall 3 to Jack Vertices
t_b3j1 = SE3Inv(t_wb3) @ t_wj1
t_b3j2 = SE3Inv(t_wb3) @ t_wj2
t_b3j3 = SE3Inv(t_wb3) @ t_wj3
t_b3j4 = SE3Inv(t_wb3) @ t_wj4

# From Wall 4 to Jack Vertices
t_b4j1 = SE3Inv(t_wb4) @ t_wj1
t_b4j2 = SE3Inv(t_wb4) @ t_wj2
t_b4j3 = SE3Inv(t_wb4) @ t_wj3
t_b4j4 = SE3Inv(t_wb4) @ t_wj4
```

[67]:
```python
t_b1j1_sub = t_b1j1.subs(dumb_sub)
t_b1j2_sub = t_b1j2.subs(dumb_sub)
t_b1j3_sub = t_b1j3.subs(dumb_sub)
t_b1j4_sub = t_b1j4.subs(dumb_sub)

# Consider Impact for First Wall
phi_b1j1 = t_b1j1_sub[0,3]
phi_b1j2 = t_b1j2_sub[0,3]
phi_b1j3 = t_b1j3_sub[0,3]
phi_b1j4 = t_b1j4_sub[0,3]
# display(phi_b1j1)
```

[68]:
```
t_b2j1_sub = t_b2j1.subs(dumb_sub)
t_b2j2_sub = t_b2j2.subs(dumb_sub)
t_b2j3_sub = t_b2j3.subs(dumb_sub)
t_b2j4_sub = t_b2j4.subs(dumb_sub)

# Consider Impact for Second Wall
phi_b2j1 = t_b2j1_sub[1,3]
phi_b2j2 = t_b2j2_sub[1,3]
phi_b2j3 = t_b2j3_sub[1,3]
phi_b2j4 = t_b2j4_sub[1,3]
```

[69]:
```
t_b3j1_sub = t_b3j1.subs(dumb_sub)
t_b3j2_sub = t_b3j2.subs(dumb_sub)
t_b3j3_sub = t_b3j3.subs(dumb_sub)
t_b3j4_sub = t_b3j4.subs(dumb_sub)

# Consider Impact for Thirds Wall
phi_b3j1 = t_b3j1_sub[0,3]
phi_b3j2 = t_b3j2_sub[0,3]
phi_b3j3 = t_b3j3_sub[0,3]
phi_b3j4 = t_b3j4_sub[0,3]
# display(phi_b1j1)
```

[70]:
```
t_b4j1_sub = t_b4j1.subs(dumb_sub)
t_b4j2_sub = t_b4j2.subs(dumb_sub)
t_b4j3_sub = t_b4j3.subs(dumb_sub)
t_b4j4_sub = t_b4j4.subs(dumb_sub)

# Consider Impact for Fourth Wall
phi_b4j1 = t_b4j1_sub[1,3]
phi_b4j2 = t_b4j2_sub[1,3]
phi_b4j3 = t_b4j3_sub[1,3]
phi_b4j4 = t_b4j4_sub[1,3]
# display(phi_b4j1)
```

1.2 Calculating Impacts

- According to HW5, we started with finding dLdq_dot at - and + (pre/post impact)
- Then we found dPhidq
- Then we found Hamiltonian

[71]:
```
# Create + and - Vars
lamb = sym.symbols(r'\lambda')
xb_min, yb_min, tb_min, xj_min, yj_min, tj_min = sym.symbols(r'x_box^-,␣
↪y_box^-, \theta_box^-, x_jack^-, y_jack^-, \theta_jack^-')
```

```
xb_plus, yb_plus, tb_plus, xj_plus, yj_plus, tj_plus = sym.symbols(r'x_box^+,␣
↪y_box^+, \theta_box^+, x_jack^+, y_jack^+, \theta_jack^+')
xbd_min, ybd_min, tbd_min = sym.symbols(r'\dot{x_{box}}^-, \dot{y_{box}}^-,␣
↪\dot{\theta_{box}}^-')
xjd_min, yjd_min, tjd_min = sym.symbols(r'\dot{x_{jack}}^-, \dot{y_{jack}}^-,␣
↪\dot{\theta_{jack}}^-')
xbd_plus, ybd_plus, tbd_plus = sym.symbols(r'\dot{x_{box}}^+, \dot{y_{box}}^+,␣
↪\dot{\theta_{box}}^+')
xjd_plus, yjd_plus, tjd_plus = sym.symbols(r'\dot{x_{jack}}^+,␣
↪\dot{y_{jack}}^+, \dot{\theta_{jack}}^+')

dumb_plus = {xbd_dumb:xbd_plus, ybd_dumb:ybd_plus, tbd_dumb:tbd_plus, xjd_dumb:
↪xjd_plus, yjd_dumb:yjd_plus, tjd_dumb:tjd_plus,
             xb_dumb:xb_plus, yb_dumb:yb_plus, tb_dumb:tb_plus, xj_dumb:
↪xj_plus, yj_dumb:yj_plus, tj_dumb:tj_plus}

dumb_min = {xbd_dumb:xbd_min, ybd_dumb:ybd_min, tbd_dumb:tbd_min, xjd_dumb:
↪xjd_min, yjd_dumb:yjd_min, tjd_dumb:tjd_min,
             xb_dumb:xb_min, yb_dumb:yb_min, tb_dumb:tb_min, xj_dumb:xj_min,␣
↪yj_dumb:yj_min, tj_dumb:tj_min}

# display(xb_min, yb_min, tb_min, xj_min, yj_min, tj_min)
# display(xb_plus, yb_plus, tb_plus, xj_plus, yj_plus, tj_plus)

# display(xbd_min, ybd_min, tbd_min, xjd_min, yjd_min, tjd_min)
# display(xbd_plus, ybd_plus, tbd_plus, xjd_plus, yjd_plus, tjd_plus)
```

[72]:
```
# Make sure we have dLdq_dot
# display(dLdq)-> Doesn't depend on any of my variables, so lets not sub
dLdqd_sub = dLdqd.subs(dumb_sub)
display(dLdqd_sub)

dLdqd_min = dLdqd_sub.subs(dumb_min)
print('\n\033[1m dLdqdot_minus: ')
display(dLdqd_min)

# Now we do the same for Plus
dLdqd_plus = dLdqd_sub.subs(dumb_plus)
print('\n\033[1m dLdqdot_plus: ')
display(dLdqd_plus)
```

<IPython.core.display.HTML object>

$$\begin{bmatrix} 1000.0\dot{x_{box}} \\ 1000.0\dot{y_{box}} \\ 8333.33333333333\dot{\theta_{box}} \\ 1.0\dot{x_{jack}} \\ 1.0\dot{y_{jack}} \\ 0.0833333333333333\dot{\theta_{jack}} \end{bmatrix}$$

dLdqdot_minus:

<IPython.core.display.HTML object>

$$\begin{bmatrix} 1000.0\dot{x_{box}}^{-} \\ 1000.0\dot{y_{box}}^{-} \\ 8333.33333333333\dot{\theta_{box}}^{-} \\ 1.0\dot{x_{jack}}^{-} \\ 1.0\dot{y_{jack}}^{-} \\ 0.0833333333333333\dot{\theta_{jack}}^{-} \end{bmatrix}$$

dLdqdot_plus:

<IPython.core.display.HTML object>

$$\begin{bmatrix} 1000.0\dot{x_{box}}^{+} \\ 1000.0\dot{y_{box}}^{+} \\ 8333.33333333333\dot{\theta_{box}}^{+} \\ 1.0\dot{x_{jack}}^{+} \\ 1.0\dot{y_{jack}}^{+} \\ 0.0833333333333333\dot{\theta_{jack}}^{+} \end{bmatrix}$$

[73]:
```
# Define dPhi/dq -> Pulled from HW5
phi = sym.Matrix([[phi_b1j1], [phi_b1j2], [phi_b1j3], [phi_b1j4],
                  [phi_b2j1], [phi_b2j2], [phi_b2j3], [phi_b2j4],
                  [phi_b3j1], [phi_b3j2], [phi_b3j3], [phi_b3j4],
                  [phi_b4j1], [phi_b4j2], [phi_b4j3], [phi_b4j4],
                 ])
# display(phi)
dphidq = phi.jacobian([xb_dumb, yb_dumb, tb_dumb, xj_dumb, yj_dumb, tj_dumb])
# display(dphidq)
```

[74]:
```
# Hamiltonians
Ham = (dLdqd.T * q_dot)[0] - L
Ham = sym.simplify(Ham)

print('\n\033[1m Ham_minus: ')
Ham_dumb = Ham.subs(dumb_sub)
Ham_min = Ham_dumb.subs(dumb_min)
```

```
display(Ham_min)

print('\n\033[1m Ham_plus: ')
Ham_plus = Ham_dumb.subs(dumb_plus)
display(Ham_plus)
```

Ham_minus:

<IPython.core.display.HTML object>

$$4166.66666666667\left(\dot{\theta_{box}}^{-}\right)^2 + 0.0416666666666667\left(\dot{\theta_{jack}}^{-}\right)^2 + 500.0\left(\dot{x_{box}}^{-}\right)^2 + 0.5\left(\dot{x_{jack}}^{-}\right)^2 + 500.0\left(\dot{y_{box}}^{-}\right)^2 + 0.5\left(\dot{y_{jack}}^{-}\right)^2 + 9800.0y_{box}^{-} + 9.8y_{jack}^{-}$$

Ham_plus:

<IPython.core.display.HTML object>

$$4166.66666666667\left(\dot{\theta_{box}}^{+}\right)^2 + 0.0416666666666667\left(\dot{\theta_{jack}}^{+}\right)^2 + 500.0\left(\dot{x_{box}}^{+}\right)^2 + 0.5\left(\dot{x_{jack}}^{+}\right)^2 + 500.0\left(\dot{y_{box}}^{+}\right)^2 + 0.5\left(\dot{y_{jack}}^{+}\right)^2 + 9800.0y_{box}^{+} + 9.8y_{jack}^{+}$$

[75]:
```
impact_eqn_lhs = sym.simplify(sym.Matrix([dLdqd_plus - dLdqd_sub,  Ham_plus -␣
↪Ham_dumb]))
# display(impact_eqn_lhs)

impact_eqns = []
for i in range(phi.shape[0]):
    impact_eqn_rhs = sym.Matrix([lamb * dphidq[i, 0], lamb * dphidq[i, 1],
                                 lamb * dphidq[i, 2], lamb * dphidq[i, 3],
                                 lamb * dphidq[i, 4], lamb * dphidq[i, 5], 0])
    # display(impact_eqn_rhs)
    impact_eqns.append(sym.Eq(impact_eqn_lhs, impact_eqn_rhs))
display(impact_eqns)
```

<IPython.core.display.HTML object>

$$\Bigg[\Bigg[-4166.66666666667\dot{\theta_{box}}^2 + 4166.66666666667\left(\dot{\theta_{box}}^{+}\right)^2 - 0.0416666666666667\dot{\theta_{jack}}^2 + 0.0416666666666667\left(\dot{\theta_{ja}}\right.$$

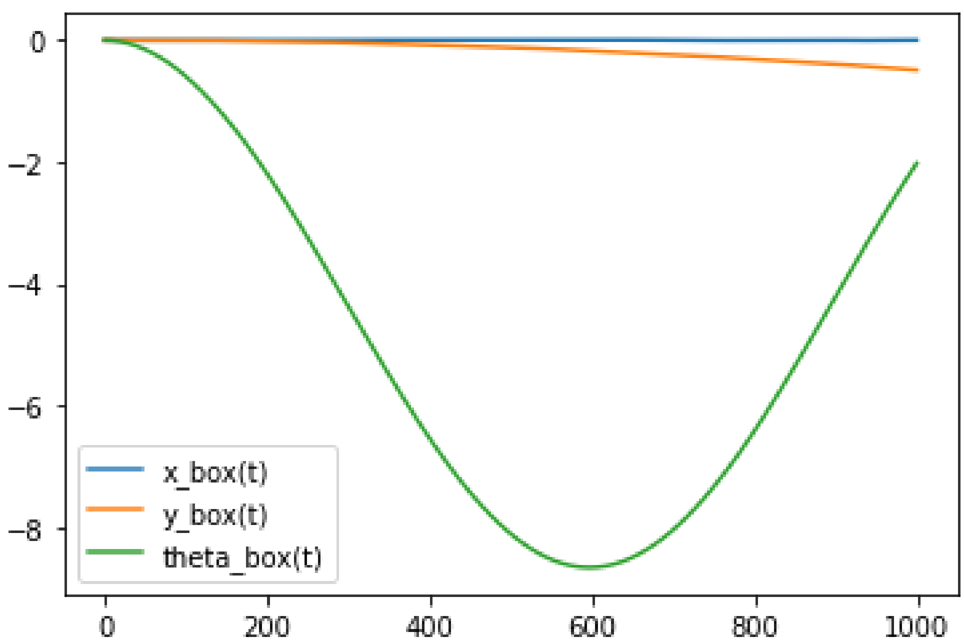
[76]:
```
impact_subs = [xbd_plus, ybd_plus, tbd_plus, xjd_plus, yjd_plus, tjd_plus]

phi_func = sym.lambdify([xb_dumb, yb_dumb, tb_dumb, \
                         xj_dumb, yj_dumb, tj_dumb, \
                         xbd_dumb, ybd_dumb, tbd_dumb,\
                         xjd_dumb, yjd_dumb, tjd_dumb], phi)

# display(impact_subs)
# display(phi_func)
```

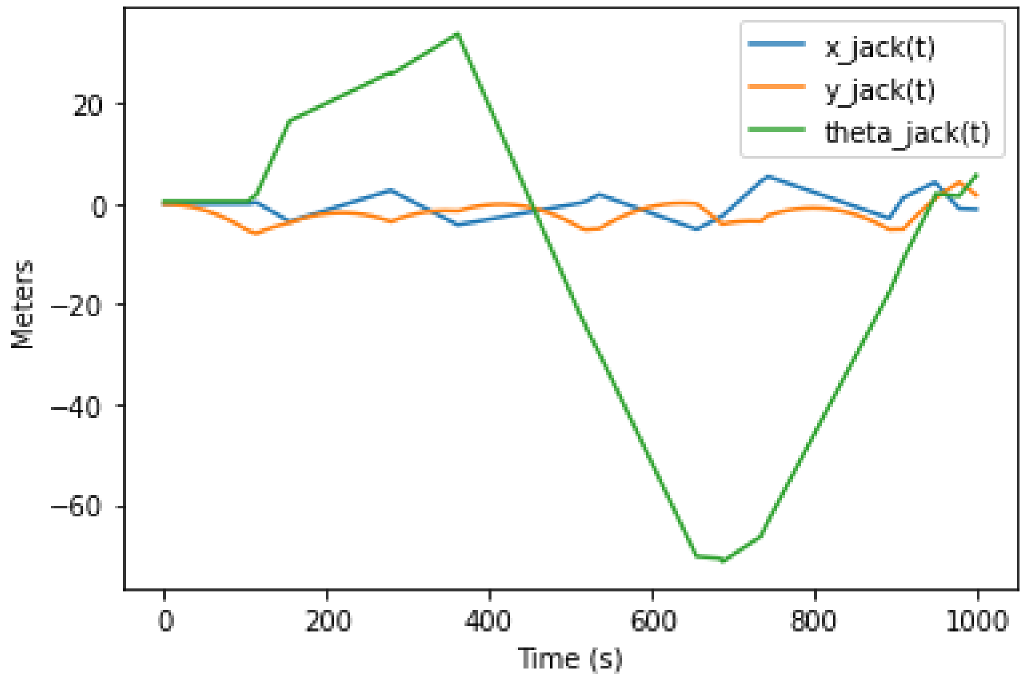
[77]:
```
# Updated dxdt, updated it from HW7 to work for Final Proj
def dxdt(s,t):
  return np.array([s[6], s[7], s[8], s[9], s[10], s[11],
                   xb_dd_l(*s, t), yb_dd_l(*s, t), tb_dd_l(*s, t),
                   xj_dd_l(*s, t), yj_dd_l(*s, t), tj_dd_l(*s, t),
                  ])

dt = 0.01
T = 10
s0 = np.array([0, 0, 0, 0, 0, pi/6, 0, 0, 0, 0, 0, 0]) # x, y, th for Box then␣
 ↪Jack then Vels
# trajectory = simulate2(dxdt, s0, [0,T], dt, rk4)
trajectory = simulate_with_impact(dxdt, s0, [0,T], dt, rk4, impact_eqns,␣
 ↪impact_subs)
print('\033[1mShape of traj: \033[0m', trajectory.shape)
plt.figure()
plt.plot(trajectory[0], label="x_box(t)")
plt.plot(trajectory[1], label="y_box(t)")
plt.plot(trajectory[2], label="theta_box(t)")
plt.legend()
plt.show()

plt.figure()
plt.plot(trajectory[3], label="x_jack(t)")
plt.plot(trajectory[4], label="y_jack(t)")
plt.plot(trajectory[5], label="theta_jack(t)")
plt.xlabel("Time (s)")
plt.ylabel("Meters")
plt.legend()
plt.show()
```

```
Impact is True num: 6
Impact is True num: 7
Impact is True num: 2
Yo, that's small
Impact is True num: 5
Impact is True num: 10
```

```
Yo, that's small
Impact is True num: 9
Yo, that's small
Impact is True num: 8
Yo, that's small
Impact is True num: 3
Impact is True num: 3
Yo, that's small
Impact is True num: 3
Impact is True num: 2
Impact is True num: 14
Yo, that's small
Impact is True num: 6
Impact is True num: 1
Yo, that's small
Impact is True num: 0
Yo, that's small
Impact is True num: 3
Impact is True num: 12
Yo, that's small
Impact is True num: 15
Impact is True num: 10
Yo, that's small
Impact is True num: 9
Impact is True num: 10
Impact is True num: 4
Yo, that's small
```

Shape of traj: (12, 1000)






```
[81]: def animate_dice(theta_array,L_box=1,L_jack=1,T=10):
          """
          Function to generate web-based animation of a dice, or jack in a cup

          Parameters:
          ================================================
          theta_array:
              trajectory of theta1 and theta2, should be a NumPy array with
              shape of (2,N)
          L_box:
              length of the sides of the box
          L_jack:
              length of the sides of the jack / dice
          T:
              length/seconds of animation duration

          Returns: None
          """

          ################################
          # Imports required for animation.
          from plotly.offline import init_notebook_mode, iplot
```

```python
from IPython.display import display, HTML
import plotly.graph_objects as go

######################
# Browser configuration.
def configure_plotly_browser_state():
    import IPython
    display(IPython.core.display.HTML('''
        <script src="/static/components/requirejs/require.js"></script>
        <script>
          requirejs.config({
            paths: {
              base: '/static/base',
              plotly: 'https://cdn.plot.ly/plotly-1.5.1.min.js?noext',
            },
          });
        </script>
        '''))
configure_plotly_browser_state()
init_notebook_mode(connected=False)

###############################################
# Getting data from dice trajectories. Borrows a lot of code from HW7
xb = theta_array[0]
yb = theta_array[1]
tb = theta_array[2]
# Data of the Jack
xj = theta_array[3]
yj = theta_array[4]
tj = theta_array[5]

xx1=(xb)
yy1=(yb)

xx2=(xj)
yy2=(yj)

# xx2=L*np.sin(theta1)
# xx3=L*np.sin(theta2)

# Denoting the following points for Box
# Point 1 x and y
pb1_x = xx1 + L_box/2*np.cos(tb) - L_box/2*np.sin(tb)
pb1_y = yy1 + L_box/2*np.sin(tb) + L_box/2*np.cos(tb)

# Point 2 x and y
pb2_x = xx1 + L_box/2*np.cos(tb) + L_box/2*np.sin(tb)
```

```
pb2_y = yy1 + L_box/2*np.sin(tb) - L_box/2*np.cos(tb)

# Point 3 x and y
pb3_x = xx1 - L_box/2*np.cos(tb) + L_box/2*np.sin(tb)
pb3_y = yy1 - L_box/2*np.sin(tb) - L_box/2*np.cos(tb)

# Point 4 x and y
pb4_x = xx1 - L_box/2*np.sin(tb) - L_box/2*np.cos(tb)
pb4_y = yy1 - L_box/2*np.sin(tb) + L_box/2*np.cos(tb)

# Denoting the following points for Jack/Dice
# Point 1 x and y
pj1_x = xx2 + L_jack/2*np.cos(tj) - L_jack/2*np.sin(tj)
pj1_y = yy2 + L_jack/2*np.sin(tj) + L_jack/2*np.cos(tj)

# Point 2 x and y
pj2_x = xx2 + L_jack/2*np.cos(tj) + L_jack/2*np.sin(tj)
pj2_y = yy2 + L_jack/2*np.sin(tj) - L_jack/2*np.cos(tj)

# Point 3 x and y
pj3_x = xx2 - L_jack/2*np.cos(tj) + L_jack/2*np.sin(tj)
pj3_y = yy2 - L_jack/2*np.sin(tj) - L_jack/2*np.cos(tj)

# Point 4 x and y
pj4_x = xx2 - L_jack/2*np.sin(tj) - L_jack/2*np.cos(tj)
pj4_y = yy2 - L_jack/2*np.sin(tj) + L_jack/2*np.cos(tj)

N = len(theta_array[0]) # Need this for specifying length of simulation

###############################################
# Define arrays containing data for frame axes
# In each frame, the x and y axis are always fixed
x_axis = np.array([0.3, 0.0])
y_axis = np.array([0.0, 0.3])
# Use homogeneous tranformation to transfer these two axes/points
# back to the fixed frame
frame_b_x_axis = np.zeros((2,N))
frame_b_y_axis = np.zeros((2,N))

frame_b1_x_axis = np.zeros((2,N))
frame_b1_y_axis = np.zeros((2,N))

frame_b2_x_axis = np.zeros((2,N))
frame_b2_y_axis = np.zeros((2,N))

frame_b3_x_axis = np.zeros((2,N))
frame_b3_y_axis = np.zeros((2,N))
```

```
frame_b4_x_axis = np.zeros((2,N))
frame_b4_y_axis = np.zeros((2,N))

frame_j_x_axis = np.zeros((2,N))
frame_j_y_axis = np.zeros((2,N))

frame_j1_x_axis = np.zeros((2,N))
frame_j1_y_axis = np.zeros((2,N))

frame_j2_x_axis = np.zeros((2,N))
frame_j2_y_axis = np.zeros((2,N))

frame_j3_x_axis = np.zeros((2,N))
frame_j3_y_axis = np.zeros((2,N))

frame_j4_x_axis = np.zeros((2,N))
frame_j4_y_axis = np.zeros((2,N))


for i in range(N): # iteration through each time step

    # evaluate homogeneous transformation
    # Would totally use 2D versions of these
    # But I made helper functions for these, so I am going to use them!
    t_wb = npvecToSE3(tb[i], [xb[i],yb[i]])
    t_wj = npvecToSE3(tj[i], [xj[i],yj[i]])

    # t_bb1 = vecToSE3(0, [lb/2, 0]) # Right
    # t_bb2 = vecToSE3(0, [0,-lb/2]) # Down
    # t_bb3 = vecToSE3(0, [-lb/2,0]) # Left
    # t_bb4 = vecToSE3(0, [0, lb/2]) # Up

####################################
# Using these to specify axis limits.
xm = -8 #np.min(xx1)-0.5
xM = 8 #np.max(xx1)+0.5
ym = -8#np.min(yy1)-2.5
yM = 8 #np.max(yy1)+1.5

###########################
# Defining data dictionary.
# Trajectories are here.
data=[
    dict(name='Cup'),
    dict(name='Jack'),
    dict(name='V1'),
```

```
        dict(name='V2'),
        dict(name='V3'),
        dict(name='V4'),
        ]

    ################################
    # Preparing simulation layout.
    # Title and axis ranges are here.
    layout=dict(autosize=False, width=1000, height=1000,
                xaxis=dict(range=[xm, xM], autorange=False,␣
↪zeroline=False,dtick=1),
                yaxis=dict(range=[ym, yM], autorange=False,␣
↪zeroline=False,scaleanchor = "x",dtick=1),
                title='Double Pendulum Simulation',
                hovermode='closest',
                updatemenus= [{'type': 'buttons',
                               'buttons': [{'label': 'Play','method': 'animate',
                                            'args': [None, {'frame':␣
↪{'duration': T, 'redraw': False}}]},
                                           {'args': [[None], {'frame':␣
↪{'duration': T, 'redraw': False}, 'mode': 'immediate',
                                            'transition': {'duration':␣
↪0}}],'label': 'Pause','method': 'animate'}
                                          ]
                              }]
               )

    ########################################
    # Defining the frames of the simulation.
    # This is what draws the lines from
    # joint to joint of the pendulum.
    frames=[dict(data=[# first three objects correspond to the arms and two␣
↪masses,
                       # same order as in the "data" variable defined above␣
↪(thus
                       # they will be labeled in the same order)

                       dict(x=[pb1_x[k], pb2_x[k], pb3_x[k], pb4_x[k],␣
↪pb1_x[k]],
                            y=[pb1_y[k], pb2_y[k], pb3_y[k], pb4_y[k],␣
↪pb1_y[k]],
                            mode='lines',
                            line=dict(color='green', width=5),
                            ),
```

```
                      dict(x=[pj1_x[k], pj2_x[k], pj3_x[k], pj4_x[k],␣
↪pj1_x[k]],
                           y=[pj1_y[k], pj2_y[k], pj3_y[k], pj4_y[k],␣
↪pj1_y[k]],
                           mode='lines',
                           line=dict(color='red', width=3),
                           ),
                     go.Scatter(
                       x=[pj1_x[k]],
                       y=[pj1_y[k]],
                       mode="markers",
                       marker=dict(color="blue", size=6)),

                     go.Scatter(
                       x=[pj2_x[k]],
                       y=[pj2_y[k]],
                       mode="markers",
                       marker=dict(color="blue", size=6)),

                     go.Scatter(
                         x=[pj3_x[k]],
                         y=[pj3_y[k]],
                         mode="markers",
                         marker=dict(color="blue", size=6)),
                     go.Scatter(x=[pj4_x[k]],
                                y=[pj4_y[k]],
                                mode="markers",
                                marker=dict(color="blue", size=6)),

                     ]) for k in range(N)]

   pj1_x = xx2 + L_jack/2*np.cos(tj) - L_jack/2*np.sin(tj)
   pj1_y = yy2 + L_jack/2*np.sin(tj) + L_jack/2*np.cos(tj)

   # Point 2 x and y
   pj2_x = xx2 + L_jack/2*np.cos(tj) + L_jack/2*np.sin(tj)
   pj2_y = yy2 + L_jack/2*np.sin(tj) - L_jack/2*np.cos(tj)

   # Point 3 x and y
   pj3_x = xx2 - L_jack/2*np.cos(tj) + L_jack/2*np.sin(tj)
   pj3_y = yy2 - L_jack/2*np.sin(tj) - L_jack/2*np.cos(tj)

   # Point 4 x and y
   pj4_x = xx2 - L_jack/2*np.sin(tj) - L_jack/2*np.cos(tj)
   pj4_y = yy2 - L_jack/2*np.sin(tj) + L_jack/2*np.cos(tj)

   #######################################
```

```
    # Putting it all together and plotting.
    figure1=dict(data=data, layout=layout, frames=frames)
    iplot(figure1)
```

[82]:
```
animate_dice(trajectory,L_box=10,L_jack=1,T=15)
```

Output hidden; open in https://colab.research.google.com to view.

[79]:
```

```