Indices and Logarithms

Tutorial

1. Simplify the followings.

   i. $(\frac{16}{81})^{-1/4}$

   ii. $32^{-2/5}$

   iii. $\frac{2m^2n^{-2}}{2^{-3}n^{-4}}$

   iv. $(16a^4)^{1/2}$

   v. $(a^{-1}b^5)^{-1}$

   vi. $(\frac{1}{x^2y^{-4}})^{-1/2}$

   vii. $8^{n+1} \div 4^{2n-1}$

   viii. $\frac{p^{n+2}p^{3-2n}}{p^{5+3n}}$

   ix. $a^3b^{-5}(a^2b^4)^4$

   x. $(p^2 q^{-4})^{1/2}(27p^{-6} q^3)^{-1/3}$

   xi. $\frac{10^n - 4^n}{5^n - 2^n}$

   xii. $\frac{(2x^2y)^3}{(xy^3)^2(4x^2)^2}$

   xiii. $\frac{7x^5y^2 * 3(xy^3)^2}{21x^2y^3}$

   xiv. $\frac{a^{-1}+b^{-1}}{(a+b)}$

2. Solve the equation.

   i. $4^x = 32$

   ii. $9 * 3^{x-1} = \frac{1}{27}$

   iii. $2^x * 4^{x+1} = 64$

   iv. $8^{x^2} - 16^{x+1} = 0$

   v. $\sqrt{27^{x+4}} = \frac{1}{3^x + 3^{9x}}$

   vi. $16^{3x} = 8^{6x+1}$

   vii. $5^{2x} - 5^{2x-1} = 100$

   viii. $\sqrt{8^{x-2}} = \frac{1}{4^{1-x}2^x}$

   ix. $7^{2x} * \frac{1}{7^{5x}} * 7^{x/2} = \frac{1}{49}$

   x. $(\frac{4^{x^2}}{8})^2 = 8$

3. Find the value of the followings.

   i. $\log_4 \frac{1}{64} + \log_9 3^{-1}$

   ii. $\log_3 18 - \log_3 6 + \frac{1}{2}\log_3 9$

   iii. $\log_4 2 + \log_4 32 - \log_4 \frac{1}{8}$

   iv. $\log_{49} 7 - \log_8 \frac{1}{64}$

   v. $2\log_5 10 + 3\log_5 2 - \log_5 32$

   vi. $\frac{\log_6 216 + (\log_7 42 - \log_7 6)}{\log_7 49}$

   vii. $\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}$

4. Solve the equation.
   i. $\log_{10}(5x - 5) = \log_{10}(x - 1) + 2$
   ii. $\log_5(8x - 4) = 2\log_5 3 + \log_5 4$
   iii. $\log_3 4x - \log_3(2x - 1) = 1$
   iv. $2\log_b x = 2\log_b(1 - a) + 2\log_b(1 + a) - \log_b(\frac{1}{a} - a)^2$
   v. $\log_{10} 5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$
   vi. $\log_{\sqrt{8}} x = 3\frac{1}{3}$

5. If $\log_{10} 7 = a$, then find $\log_{10} \frac{1}{70}$.

6. If $\log_{10} 2 = 0.3010$, then find value of $\log_{10} 80$.

7. If $\log_{10} 2 = 0.3010$, then find the value of $\log_5 512$.

8. The level of sound D in decibels is defined as follows
   D = 10 $\log_{10}(I / 10^{-16})$, where I is the sound intensity in watts per centimeters squared.
   i. Determine the level in decibels of a sound with intensity I = 10^{-8} watts/cm^2.
   ii. Two sounds of intensities I_1 and I_2 have decibel levels of 60 and 80 respectively. Determine the ratio of the intensities I_2 / I_1?

9. Early in the century the earthquake in San Francisco registered 8.3 on the Richter scale. In the same year, another earthquake was recorded in South America that was four time stronger.
   i. What was the magnitude of the earthquake in South American?
   ii. A recent earthquake in San Francisco measured 7.1 on the Richter scale. How many times more intense was the San Francisco earthquake?
   M=$\log_{10} \frac{I}{S}$, where M is magnitude, I is intensity and S is intensity of standard earth quack

10. Half-life is the time required for a radioactive element to decay to half its original mass. Radium (Ra) is a radioactive element that decays naturally according to the following equation:
   N=N$_0$ $(\frac{1}{2})^{\frac{t}{d}}$
   Where N is the mass in grams left after t year(s), N$_0$ is the original mass in grams, and d is the half-life of the element.
   In 3000 years, a 100-g sample of radium decays to a mass of 27.04 g. In 1000 years, how much of a 250-g sample of radium will be left?