

## **Section 6: Introduction to Percentages and Interest Rates**

### **6.1 Calculation of Profit and Loss**

When goods are bought for one sum of money and sold for another, there is a profit or loss depending on whether the selling price is greater or less than the cost price. The **cost price** (C.P) is the sum of money paid to buy the goods. The price at which the goods are sold is the **selling price** (S.P). If the selling price is more than the cost price, then the seller makes a **profit** which is equal to the difference between the selling price and the cost price.

**Profit = Selling Price – Cost Price** (if the selling price is greater than the cost price)

However if the selling price is less than the cost price, then the seller suffers a **loss** which is equal to the difference between the cost price and the selling price.

**Loss = Cost Price - Selling Price** (if the cost price is greater than the selling price)

### **6.2 Percentage Profit and Discount**

Example:

Consider the following cost prices and respective selling prices of two items.

- |      |                       |                         |
|------|-----------------------|-------------------------|
| (i)  | Cost Price - Rs. 25;  | Selling Price – Rs. 30  |
| (ii) | Cost Price – Rs. 300; | Selling Price - Rs. 305 |

In each of the above cases, the profit is Rs. 5. But, in the first case the profit is  $\frac{5}{25} = \frac{1}{5}$  of the cost price while in the second case the profit is  $\frac{5}{300} = \frac{1}{60}$  of the cost price.

Expressing these as percentages we obtain,

the profit as a percentage in the first case =  $\frac{1}{5} \times 100\% = 20\%$

the profit as a percentage in the second case =  $\frac{1}{60} \times 100\% = 1.67\%$

Thus despite the profit being the same in both cases, the rate of profit is different, and the better deal for the seller is the first case.

We see that the calculation of the actual gains or losses does not provide us with any useful basis for comparison, but the profit per cent and the loss per cent do.

We note that the profit per cent has been calculated with reference to the cost price, and unless otherwise specified, profit per cent and loss per cent are always to be understood in this sense.

Thus

$$\text{Profit per cent} = \frac{\text{profit}}{\text{cost}} \times 100\%$$

$$\text{Loss per cent} = \frac{\text{loss}}{\text{cost}} \times 100\%$$

Example:

- (i) A salesman buys a wristwatch for Rs. 1100, spends Rs. 250 to repair it and sells it for Rs. 1500. What is his profit per cent?
- (ii) Renuka sells a pair of shoes for Rs. 736 by making a profit of 15%. How much did she spend to buy the pair of shoes?
- (iii) Ruvan sells an old TV for Rs. 5000 by incurring a loss of 20%. What was the price that he initially bought it for? To have gained a profit of 5%, how much should he have sold it for?

Solution:

- (i) Cost price = Rs. 1100 + Rs. 250 = Rs. 1350  
Profit = Rs. 1500 – Rs. 1350 = Rs. 150  
Thus profit per cent =  $\frac{150}{1350} \times 100\% = 11\frac{1}{9}\%$
- (ii) If the cost of the pair of shoes was Rs.  $x$ , then  $x + \frac{15}{100}x = 736$ .  
Therefore  $x(1 + \frac{3}{20}) = 736$ ; i.e.,  $\frac{23}{20}x = 736$ .  
Therefore  $x = \frac{736 \times 20}{23} = 32 \times 20 = 640$ .  
i.e., Renuka bought the pair of shoes for Rs. 640.
- (iii) If the price that Ruvan bought the TV for is Rs.  $x$ , then since  
“Selling Price = Cost Price – Loss, we obtain  $5000 = x - \frac{20}{100} \times x$ .  
i.e.,  $5000 = \frac{80}{100} \times x$ . Therefore  $x = 5000 \times \frac{100}{80} = 6250$ . i.e., the price he initially bought it for is Rs. 6250. Since “Selling Price = Cost Price + Profit”, to have gained a profit of 5%, he should have sold it for  
 $\text{Rs. } 6250 + 6250 \times \frac{5}{100} = \text{Rs. } 6250 \times \frac{105}{100} = \text{Rs. } 6562.50$ .

## Discounts

Shop keepers at times have sales to get rid of old stocks. At such times goods are sold at prices that are lower than the price on the 'tag' which is the **marked price**. The shop keeper usually still makes a profit, though less than what he would have made if he sold the item for the marked price.

### Example (Challenging Problems):

- (i) A shop keeper gives a discount of 10% on the marked price of a shirt and still makes a profit of 25%. If the marked price is Rs. 500, what was the actual cost of the shirt?
- (ii) Kamal buys a pair of shoes on sale at a discount of 20%. He sells it for Rs. 880, making a profit of 10%. What are the marked price and the discount price?

### Solution:

- (i) The marked price of the shirt is Rs. 500.  
Since "Selling Price = Marked Price – Discount", the selling price of the shirt with the discount is Rs.  $(500 - 500 \times \frac{10}{100}) = \text{Rs. } 500 \times \frac{90}{100} = \text{Rs. } 450$ .  
If the actual cost of the shirt is Rs.  $x$ , then since "Selling Price = Cost Price + Profit",  $450 = x + x \times \frac{25}{100} = x \times \frac{125}{100}$ . Thus  $x = 450 \times \frac{100}{125} = 360$ ; i.e., the actual cost of the shirt was Rs. 360.
- (ii) Suppose Kamal buys the pair of shoes for Rs.  $x$ .  
Selling Price = Cost Price + Profit. Since Kamal makes a profit of 10% by selling the pair of shoes for Rs. 880 we obtain,  $880 = x + x \times \frac{10}{100} = x \times \frac{110}{100}$ .  
Thus  $x = 880 \times \frac{100}{110} = 800$ ; i.e., the discount price is Rs. 800.  
Suppose the marked price is  $y$ .  
Then since, "Selling Price = Marked Price – Discount",  
 $800 = y - \frac{20}{100} \times y = \frac{80}{100} \times y$ .  
Thus the marked price  $y = \text{Rs. } \frac{800 \times 100}{80} = \text{Rs. } 1000$ .

### 6.3 Working with Tax

#### Income Tax

**Income Tax** is used by governments to finance public services offered by the government such as the armed services, civil services, national health services etc. Any person who has an income above a certain minimum amount has to pay income tax to the Government.

Income Tax in Sri Lanka is based on the amount a person earns in a tax year which begins on the 1<sup>st</sup> of April of a year and ends on the 31<sup>st</sup> of March of the next year. Certain amounts of each person's income are not taxed. These amounts are called **tax allowances**. The residue of the income left after the tax allowances are deducted is called the **taxable income**.

For Sri Lanka, for the year of assessment 2006/2007 the tax payable on the taxable income was computed using the following table.

<b>Taxable Income Range (Rs.)</b>	<b>Tax on the lower amount of the range (Rs)</b>	<b>Rate on the excess taxable income over the lower amount</b>
0 – 300,000	-	5%
300,000 – 500,000	15,000	10%
500,000 – 700,000	35,000	15%
700,000 – 900,000	65,000	20%
900,000 – 1,100,000	105,000	25%
1,100,000-1,600,000	155,000	30%
1,600,000 and over	305,000	35%

#### Example:

- (i) Suppose Sonali's taxable income is Rs. 550 000. What is her total income tax payable?
- (ii) Suppose Ruwan's taxable income is Rs 1 800 000. What is his total income tax payable?

#### Solution:

- (i) By considering the above given table we see that the tax payable on Rs. 500 000 is Rs. 35 000 and the tax payable on the excess taxable income of

Rs 50 000 is  $\text{Rs. } 50\,000 \times \frac{15}{100} = \text{Rs. } 7\,500$ . Thus the total tax payable is  $\text{Rs. } 35\,000 + \text{Rs. } 7\,500 = \text{Rs. } 42\,500$ .

- (ii) The tax payable by Ruvan on Rs 1 600 000 is Rs. 305 000. The tax payable on the excess taxable income of Rs. 200 000 is  $\text{Rs. } 200\,000 \times \frac{35}{100} = \text{Rs. } 70\,000$ . Thus Ruvan's total income tax payable is Rs. 375 000.

## **VAT**

Apart from income tax, the Government also raises money by taxes on goods and services. These taxes are called indirect taxes as they are included in the price to the consumer. The main indirect tax is the value-added tax or VAT. This is charged on good that we buy and also on services such as meals in restaurants. VAT is usually not charged on essential items such as most food items. Prices are sometimes quoted exclusive of VAT. The price you would then pay would be the price quoted plus VAT. The standard rate of VAT is 15% and the rate of VAT on luxury items is 20%.

### **Example:**

- (i) VAT of 20% is charged on luxury items. If a refrigerator which is considered a luxury item costs Rs. 32 000 exclusive of VAT, what is the price inclusive of VAT?
- (ii) A luxury item costs Rs. 33 600 inclusive of 20% VAT. What is its price excluding VAT?
- (iii) An item costs Rs. 2 990 inclusive of 15% VAT. How much is the VAT?

### **Solution:**

- (i) The price of the refrigerator exclusive of VAT is Rs. 32 000. The VAT on the refrigerator is  $\text{Rs. } 32\,000 \times \frac{20}{100} = \text{Rs. } 6\,400$ . Thus the price of the refrigerator inclusive of VAT is Rs. 38 400.
- (ii) Suppose the luxury item costs Rs.  $x$  without VAT.  
Then  $33\,600 = x + \frac{20}{100}x = \frac{120}{100}x$ .  
Therefore  $x = \frac{33600 \times 100}{120} = 28\,000$
- (iii) The cost of the item inclusive of VAT is Rs. 2 990. Suppose the item costs Rs.  $x$  without VAT. Then  $2\,990 = x + \frac{15}{100}x = \frac{115}{100}x$ .

$$\text{Therefore } x = \frac{2990 \times 100}{115} = 2600$$

Therefore the VAT on the item is Rs. 2 990 – Rs. 2 600 = Rs. 390

#### **6.4 Calculation of Simple Interest and Compound Interest**

People borrow money from money lenders, banks and other financial institutions for various requirements. The borrowed money is usually paid back after a certain period, with an additional amount which is the charge for the money lent. Such a charge is called the **interest**, and the sum lent is called the **principal**.

Interest is usually given as a percentage on the principal for each year until the loan is repaid. The sum paid on each Rs. 100 of the loan for each year is called the **rate per cent per annum**.

When the interest is paid on the original principal only, throughout the whole term of the loan, it is called **simple interest**. When interest, as it becomes due is used to increase the principal, the interest is said to be **compound**.

The principal together with its interest for the stated time is called the **amount** for that time.

##### **Simple Interest**

Simple interest depends on

- (i) the amount borrowed or lent
- (ii) the rate per cent
- (iii) the period of the loan

When the principal  $P$ , rate per cent per annum  $r$ , and the period of the loan  $T$  (in years) are given, the simple interest can be calculated by the formula

$$I = \frac{PrT}{100}$$

##### **Example:**

- (i) What is the simple interest on a loan of Rs. 5000, invested for 3 years at a rate of 9%? What is the amount payable after 3 years?
- (ii) If the simple interest on a loan after 2 years is Rs. 2400 calculated at a rate of 12 per cent per annum, is what is the principal amount?
- (iii) How many years would it take a deposit of Rs. 1000 to accumulate an interest of Rs. 300, if simple interest is paid at a rate of 12% per annum?

Solution:

(i)  $I = \frac{5000 \times 9 \times 3}{100} = 1350$ . Thus the simple interest is Rs. 1350 and the amount payable after 3 years is Rs. 5 000 + Rs. 1 350 = Rs. 6 350.

(ii)  $2400 = \frac{P \times 12 \times 2}{100}$ . Thus  $P = \text{Rs. } \frac{2400 \times 100}{24} = \text{Rs. } 10\,000$ .

(iii)  $300 = \frac{1000 \times 12 \times T}{100}$ . Thus  $T = \frac{300 \times 100}{1000 \times 12} = 2.5$  years.

### **Compound Interest**

Money is said to be invested (or loaned out) at **compound interest** when each instalment of interest as it becomes due is added to the principal. In this case the principal is continually being increased and the interest for each period is the interest on the amount at the end of the preceding period.

Example:

Suppose Rs. 1000 is invested at an annual compound interest rate of 10% for 3 years. Then, at the end of the first year, the interest is Rs. 100. Thus the principal for the second year is Rs. 1100. Therefore the interest at the end of the second year is Rs. 110 and the principal amount for the third year is Rs. 1100 + Rs. 110 = Rs. 1210. Thus the interest for the third year is Rs. 121. Therefore the amount at the end of three years is Rs. 1 331 and the total compound interest paid is Rs. 1331 – Rs. 1000 = Rs. 331.

Note: If Rs. 1000 was invested for 3 years at a simple interest rate of 10%, then the interest accrued after 3 years would be Rs. 300. Thus compound interest is beneficial for investors, but not for borrowers.

When the principal  $P$ , rate per cent per annum  $r$ , and the period of the loan or deposit  $t$  (in years) are given, the amount accrued after  $t$  years when interest is compounded can be calculated by the formula

$$A = P \left( 1 + \frac{r}{100} \right)^t$$

Sometimes interest is paid half yearly or quarterly. In such cases, each half year or quarter is considered as a separate period at the corresponding interest rate.

Therefore, if the principal  $P$  is invested (or borrowed) at a rate per cent per annum  $r$  for a period of  $t$  years, and interest is compounded  $n$  times per year, then the amount accrued after  $t$  years can be calculated by the formula

$$A = P \left( 1 + \frac{r}{100n} \right)^{nt}$$

Example:

- (i) If Rs. 2000 is invested in an account paying compound interest at a rate of 12% per annum, what is the value of the investment to the nearest rupee after 3 years?
- (ii) If Rs. 2000 is invested for 2 years at a rate of 10% per annum compounded semi-annually, how much interest would have accrued by the end of the two years?

Solution:

- (i)  $A = 2000(1 + 0.12)^3 = 2809.856$   
Therefore, the value of the investment to the nearest rupee after three years is Rs. 2810.

- (ii) We apply the formula  $A = P \left( 1 + \frac{r}{100n} \right)^{nt}$ .

$P = 2000$ ,  $r = 10$ ,  $n = 2$  and  $t = 2$ . Therefore,  $A = 2000(1 + 0.05)^4 = 2431.0125$ .  
Thus the amount of the investment after two years is Rs. 2431 to the nearest rupee. Thus the interest accrued is Rs. 431.