Time Related Problems

IS 1006 – Discrete Mathematics

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Direct proportion

Two quantities are said to vary directly or be in direct proportion, if they increase or decrease together. If two quantities denoted by x and y respectively are in direct proportion, then $\frac{x}{y}$ is always a positive constant. This constant is called the constant of variation

Note: if k is the constant of variation, then $\frac{x}{y} = k \quad \Leftrightarrow \quad x = ky$

The symbol for direct proportion is ∝.

The statement 't is directly proportional to r' can be written using the proportionality symbol:

$$t \propto r$$

Direct proportion examples

- The cost of items and the number of items purchased vary directly.
- The uniform speed at which a vehicle travels and the distance travelled are in direct proportion.

Direct proportion exercises

1. The weight of a piece of wire is directly proportional to its length. A piece of wire is 25 cm long and has a weight of 6 grams. Another piece of the same wire is 30 cm long. Calculate the weight of the 30 cm piece of wire.
2. A syrup is made by dissolving 2 cups of sugar in 2/3 cups of boiling water. How many cups of sugar should be used for 2 cups of boiling water?
3. A school buys 8 gallons of juice for 100 kids. how many gallons do they need for 175 kids ?
4. On a map, two cities are $2\frac{5}{8}$ inches apart. If $\frac{3}{8}$ inches on the map represents 25 miles, how far apart are the cities (in miles) ?
5. 2 men or 3 women can earn $192 in a day. Find how much 5 men and 7 women will earn in a day?

Inverse Proportion

Two quantities are said to vary inversely or to be inversely proportional if one quantity increases as the other quantity decreases such that the product of the two quantities is always a constant.

Thus if the two quantities are denoted by x and y respectively,
then $xy = k$ where k is the constant of variation.

The statement 'x is inversely proportional to y' is written: $\boldsymbol{x \propto \frac{1}{y}}$

If x_1 and x_2 are two value of x and y_1 and y_2 are the corresponding values of y, then $x_1y_1 = x_2y_2$.

Therefore $\boldsymbol{\frac{x_1}{x_2} = \frac{y_2}{y_1}}$. This also much means that $\boldsymbol{x_1 : x_2 = y_2 : y_1}$.

Inverse Proportion examples

- The number of people working on a job and the time taken to complete the job vary inversely.

- The speed of a vehicle and the time taken to travel a certain distance are inversely proportional.

Inverse Proportion exercises

1. A hostel has enough food for 125 students for 16 days. How long will the food last if75 more students join them?
2. A water tank can be filled in 7 hours by 5 equal sized pumps working together. How much time will 7 pumps take to fill it up?
3. 500 soldiers in a fort had enough food for 30 days. After 6 days, some soldiers were sent to another fort and thus the food lasted for 32 more days. How many soldiers left the fort?
4. 70 patients in a hospital consume 1350 litres of milk in 30 days. At the same rate, how many patients will consume 1710 litres in 28 days?

Work and time

Time and work related problems are direct and inverse variation problems.

I. More men will do more work and less men will do less work (Direct Proportion)
II. More time will result in more work (Direct Proportion)
III. More men will take less time to finish a piece of work (Inverse Proportion)

When we have to compare the work done by several persons it is necessary to first determine the work each does during a fixed time period, usually during a day, an hour or a minute, according to the conditions of the problem.

Work and time examples

1. If 35 men can reap a field in 8 days; in how many days can 20 men reap the same field?

Solution:

35 men can reap the field in 8 days

1 man can reap the field in (35×8) days [less men, more days]

20 men can reap the field in $(35 \times 8)/20$ days [more men, more days]

$= 14$ days

Hence, 20 men can reap the field in 14 days.

Work and time examples

2. 6 oxen or 8 cows can graze a field in 28 days. How long would 9 oxen and 2 cows take to graze the same field?

Solution:

6 oxen = 8 cows $\Rightarrow$ 1 ox = 8/6 cows $\Rightarrow$ 9 oxen $\equiv$ $(8/6 \times 9)$ cows = 12 cows

$\Rightarrow$ (9 oxen + 2 cows) $\equiv$ (12 cows + 2 cows) = 14 cows

Now, 8 cows can graze the field in 28 days

1 cow can graze the field in (28×8) days

14 cows can graze the field in $(28 \times 8)/14$ days = 16 days

Hence, 9 oxen and 2 cows can graze the field in 16 days.

Work and time examples

3. 6 typists working 5 hours a day can type the manuscript of a book in 16 days. How many days will 4 typists take to do the same job, each working 6 hours a day?

Solution:

6 typists working 5 hours a day can finish the job in 16 days

6 typists working 1 hour a day can finish it in (16×5) days

1 typist working 1 hour a day can finish it in $(16 \times 5 \times 6)/6$ days

1 typist working 6 hours a day can finish it in 6 days

4 typists working 6 hours a day can finish it $(16 \times 5 \times 6)/(6 \times 4)$ days

$= 20$ days.

Hence, 4 typists working 6 hours a day can finish the job in 20 days.

Work and time examples

4. A can do a work in 15 days and B in 20 days. If they work on it together for 4 days, then find the fraction of the work that is left.

Solution:

A's 1 day's work =

B's 1 day's work =

(A + B)'s 1 day's work =

(A + B)'s 4 day's work =

Therefore, Remaining work =

Work and time examples

4. A can do a work in 15 days and B in 20 days. If they work on it together for 4 days, then find the fraction of the work that is left.

Solution:

A's 1 day's work = $\frac{1}{15}$

B's 1 day's work = $\frac{1}{20}$

(A + B)'s 1 day's work = $\left(\frac{1}{15}+\frac{1}{20}\right)=\frac{7}{60}$

(A + B)'s 4 day's work = $\frac{7}{60}\times 4=\frac{7}{15}$

Therefore, Remaining work = $1-\frac{7}{15}=\frac{8}{15}$

Work and time exercises

1. Suppose 8 men working on a job complete it in 5 days. How many days will 10 men take to complete the same job?
2. If 30 labourers working 7 hours a day can finish a piece of work in 18 days, how many labourers working 6 hours a day can finish it in 30 days?
3. Sonali takes 5 hours to paint a room. Amal is able to complete the same task in 4 hours. If they work together, how long will it take them to complete the task?
4. Sonali takes 5 hours to paint a room. Amal is able to complete the same task in 4 hours. Suppose they work together for 1 hour and then Amal leaves. How long more will it take Sonali to complete the task?

Distance and Time

If a vehicle travels x kilometres in an hour then we say that the speed of the vehicle is x km/h (x kilometres per hour); i.e., the uniform speed of a moving body is measured by the distance it travels in a unit of time.

Thus we have that,

$$\boldsymbol{Speed = \frac{Distance}{Time}}$$

If a vehicle travels at a constant speed, then $\frac{Distance}{Time}$ = Constant and hence distance varies directly with time.

Distance and Time example

1. A person crosses a 600 m long street in 5 minutes. What is his speed in km per hour?

Solution:

$$Speed = \left(\frac{600\ m}{5 \times 60\ \ s}\right) = 2\ ms^{-1}$$

Converting ms^{-1} to kmh^{-1}

$$Speed = \left(2 \times \frac{60 \times 60}{1000}\right) = \left(2 \times \frac{18}{5}\right) = 7.2\ kmh^{-1}$$

Distance and Time example

2. An aeroplane covers a certain distance at a speed of 240 kmh^{-1} in 5 hours. What should be the speed of aeroplane to cover the same distance in $1\frac{2}{3}$ hours ?

Solution:

$$Distance = Speed \times Time = 240 \times 5 = 1200\ km$$

$$\therefore Required\ \ Speed = \left(\frac{1200\ km}{5/3\ \ h}\right) = 720\ kmh^{-1}$$

Distance and Time example

3. A train can travel 50% faster than a car. Both start from point A at the same time and reach point B, 75 km away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. Find the speed of the car .

Solution:

Let the speed of the car be x kmh^{-1}

Then speed of the train $= \frac{150}{100}x = {}^{3x}/_{2}\ kmh^{-1}$

$$\therefore \frac{75}{x} - \frac{75}{\frac{3x}{2}} = \frac{125}{10 \times 60} \quad \Longrightarrow \quad \frac{75}{x} - \frac{50}{x} = \frac{5}{24} \quad \Longrightarrow \quad x = \left(\frac{25 \times 24}{5}\right) = 120\ kmh^{-1}$$

Volume and Time

A water tank (cistern) has two types of pipes, one that fills the tank and one that empties it. The pipe that fills the tank is known as an inlet and the one that empties a tank is known as an outlet.

If a pipe can fill a tank in x hours, then: **part filled in 1 hour** $= \frac{1}{x}$

If a pipe can empty a tank in y hours, then: **part emptied in 1 hour** $= \frac{1}{y}$

If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where y > x), when both pipes are opened;

the net part filled in 1 hour $= \left(\frac{1}{x} - \frac{1}{y}\right)$

Volume and time example

1. Two pipes can fill a tank in 4 and 6 hours respectively. How long will it take to fill the tank if both pipes are opened together?

Solution

The first pipe fills $\frac{1}{4}$ th of the tank in 1 hour.

The second pipe fills $\frac{1}{6}$ th of the tank in 1 hour.

Thus $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$ th of the tank will be filled in 1 hour if both taps are open.

Hence the amount of time taken to fill the tank if both

pipes are opened together is $\frac{12}{5}h = 2\frac{2}{5}h = 2\ hours\ and\ 24\ minutes$

Volume and time example

2. A pipe fills a tank in 8 hours. Another pipe empties the tank in 12 hours. If both pipes are opened together, how long will it take to fill the tank?

Solution

The first pipe fills $\frac{1}{8}$ th of the tank in 1 hour.

The second pipe empties $\frac{1}{12}$ th of the tank in 1 hour.

Thus $\frac{1}{8} - \frac{1}{12} = \frac{1}{24}$ th of the tank will be filled in 1 hour if both taps are open.

Hence if both pipes are opened together, it would take **24 hours** for the tank to be filled.

Volume and time example

3. Three pipes A, B and C can fill a tank from empty to full in 30 minutes, 20 minutes, and 10 minutes respectively. When the tank is empty, all the three pipes are opened. A, B and C discharge chemical solutions P,Q and R respectively. What is the proportion of the solution R in the liquid in the tank after 3 minutes?

Solution:

Part filled by (A+B+C) in 3 minutes = $3\left(\frac{1}{30}+\frac{1}{20}+\frac{1}{10}\right) = 3 \times \frac{11}{60} = \frac{11}{20}$

Part filled by C in 3 minutes = $\frac{3}{10}$

Thus required ratio = $\frac{\textit{Chemical solution R in tank}}{\textit{Total chemical Solution in tank}} = \left(\frac{3}{10} \times \frac{20}{11}\right) = \frac{6}{11}$

Volume and time exercises

1. 8 taps having the same rate of flow, fill a tank in 27 minutes. If two taps go out of order, how long will the remaining taps take to fill the tank?
2. Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, how long will it take to fill the tank?
3. A pump can fill a tank with water in 2 hours. Because of a leak, it took $2\frac{1}{3}$ hours to fill the tank. Find the time taken to the leak to drain all the water of the tank.
4. Two pipes A and B can fill a tank in 15 minutes and 20 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank?