CSCE 221 Cover Page
Homework Assignment #3
Due April 23 at 23:59 pm to eCampus

First Name Dilanka Last Name Weerasinghe UIN 126007816

User Name dweerasinghe E-mail address dweerasinghe@tamu.edu

Please list all sources in the table below including web pages which you used to solve or implement the current homework. If you fail to cite sources you can get a lower number of points or even zero, read more on Aggie Honor System Oce website: http://aggiehonor.tamu.edu/

Type of sources				
People				
Web pages (provide URL)				
Printed material				
Other Sources				

I certify that I have listed all the sources that I used to develop the solutions/codes to the submitted work. On my honor as an Aggie, I have neither given nor received any unauthorized help on this academic work.

Your Name Dilanka Weerasinghe Date 4/23/2020

Homework 3 (100 points)

due April 24 at 11:59 pm to eCampus.

Write clearly and give full explanations to solutions for all the problems. Show all steps of your work.

Reading assignment:

- Balanced Binary Search Trees
- Skip Lists
- Hash Tables
- Heap and Priority Queue
- Graphs

Problems.

1. (10 points) For the following statements about red-black trees, provide a justication for each true statement and a counterexample for each false one.

   (a) A subtree of a red-black tree is itself a red-black tree.

      i. False because a subtree with a red root would not be a red black tree

   (b) The sibling of an external node is either external or red.

      i. True because every leaf must have the same black depth so if a black had another black it would fail to be a red black tree.

   (c) There is a unique 2-4 tree associated with a given red-black tree.

      i. True because all the nodes in a 2-4 tree can be converted to red and black nodesEach node with zero red is a two node, one red will be a 3 node form and the ones with 2 children will be a 4 node.

   (d) There is a unique red-black tree associated with a given 2-4 tree.

      i. False because the 3 node can be represented in dierent ways.

2. (10 points) Modify this skip list after performing the following series of operations: erase(38), insert(48,x), insert(24,y), erase(42). Provided the recorded coin ips for x and y.

(a)

$-\infty$ $+\infty$

$-\infty$ 17 $+\infty$

$-\infty$ 17 42 $+\infty$

$-\infty$ 17 42 $+\infty$

$-\infty$ 12 17 38 42 $+\infty$

$-\infty$ 12 17 20 38 42 $+\infty$

(b) x = HHHHT; y = HT

(c)

$-\infty$ - $+\infty$

$-\infty$ 17 48 $+\infty$

$-\infty$ 17 - 48 $+\infty$

$-\infty$ 17 - 48 $+\infty$

$-\infty$ 12 17 24 48 $+\infty$

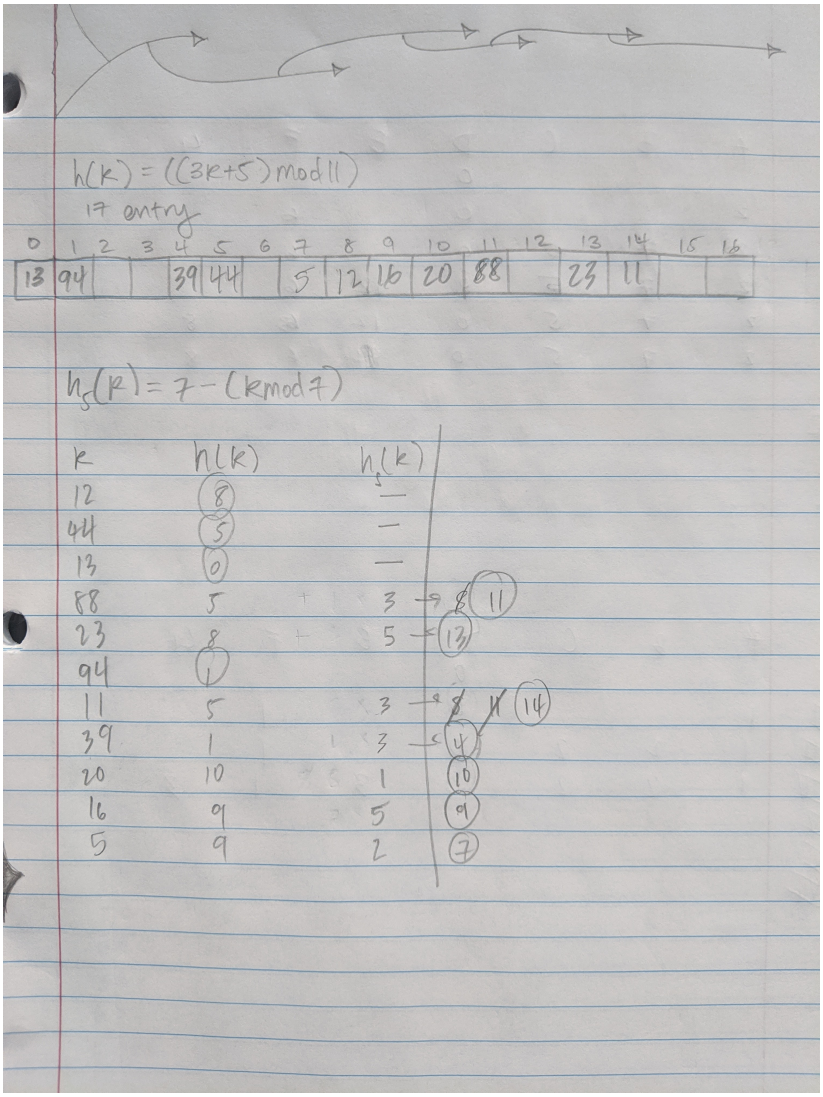
$-\infty$ 12 17 20 24 48 $+\infty$

3. (10 points) Draw the 17-entry hash table that results from using the has function: $h(k) = ((3k + 5) \bmod 11)$ to hash the keys: 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, 5, assuming collisions are handled by double hashing using the secondary hash function: $h_s(k) = (7 - (k \bmod 7))$

(a)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
13	94			39	44		5	12	16	20	88		23	11		

(b)


h(k) = ((3k+5) mod 11)
17 entry
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
13 94 39 44 5 12 16 20 88 23 11
h2(k) = 7 − (k mod 7)
k h(k) h2(k)
12 8 —
44 5 —
13 0 —
88 5 3 → 8 11
23 8 5 → 13
94 1
11 5 3 → 8 14
39 1 3 → 4
20 10 1 10
16 9 5 9
5 9 2 7

4. (10 points) An airport is developing a computer simulation of air-trac control that handles events such as landings and takeos. Each event has a time-stamp that denotes the time when the event occurs. The simulation program needs to eciently perform the following two fundamental operations:

   - Insert an event with a given time-stamp (that is, add a future event)
   - Extract the event with a smallest time-stamp (that is, determine the next event to process)

   Which data structure should be used for the above operations? Why? Provide big-O asymptotic complexity for each operation.

   (a) A binary heap would work very well because it would probably be the best for nding the smallest time stamp. With any sort of other tree the tree is not sorted to nd the min as well as a binary heap would be. Because the tree is perfectly balanced the search time for this algorithym will be O(log(n)). While placing in a node will take longer than other storing algorithyms because nding the minimum time stamp is vital to this operation you cannot use other algorityms like a hash or skip because you do not know what specic value you are trying to grab.

5. (15 points) Find the shortest path from D to all other vertices for the graph below.

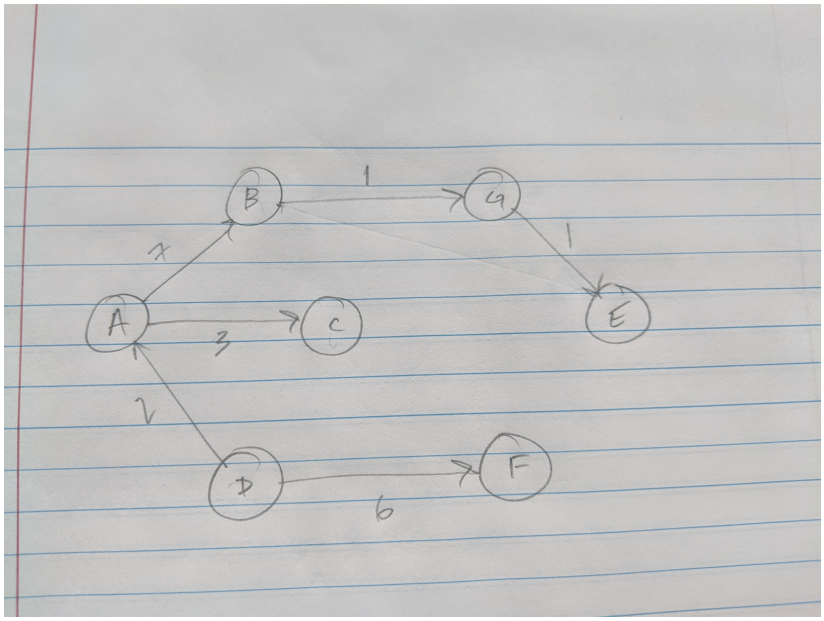
(a) Illustrate the minimum priority queue at each iteration Dijkstra's algorithm.

i.

	A	B	C	D	E	F	G
i				0			
ii	2			0		6	
iii	2	7	5	0		6	
iv	2	7	5	0	10	6	8
v	2	7	5	0	9	6	8
vi	2	7	5	0	9	6	8
vii							

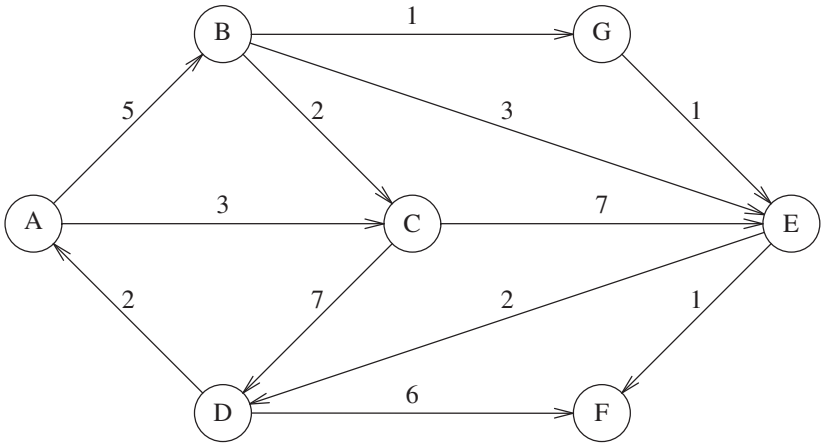
(b) Draw the Shortest Path Tree.

i.



(c) What is the running time of the Dijkstra's algorithm under the assumption that the graph is implemented based on an adjacency list and the minimum priority queue is implemented based on a binary heap?

i. O(elog(v))



6. (15 points) Find the shortest unweighted path from D to all other vertices for the graph below. You can measure the distance from D by number of edges.

(a) Which graph algorithm can solve the problem?

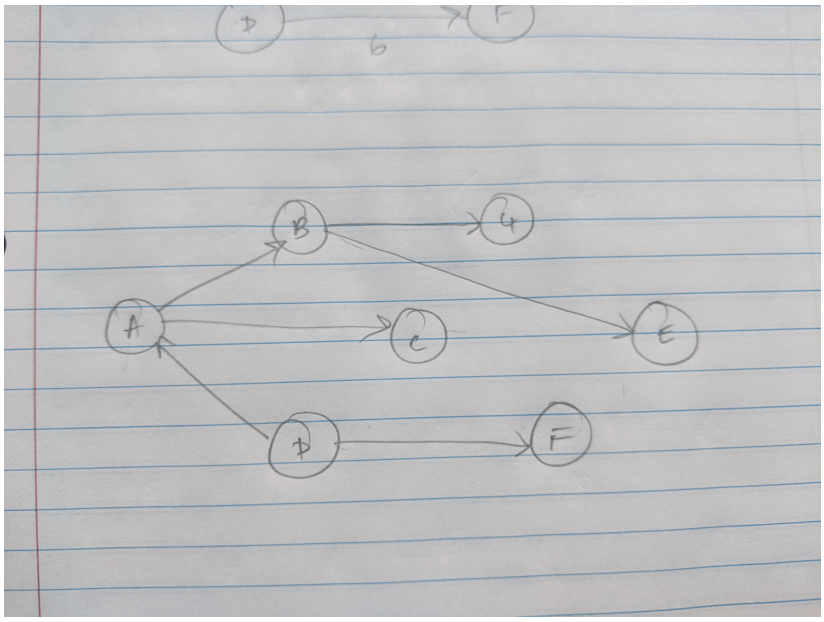
i.

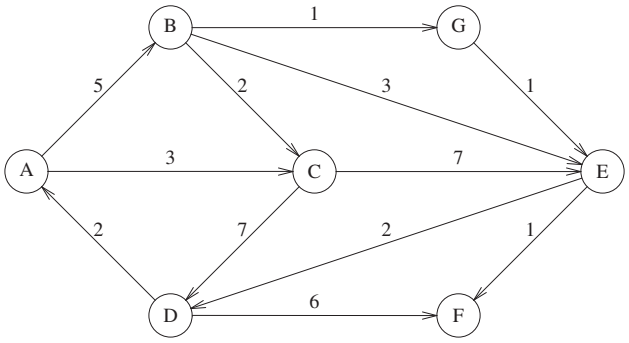
	A	B	C	D	E	F	G
i				0			
ii	1			0		1	
iii	1	2	2	0		1	
iv	1	2	2	0	3	1	3
v							
vi							
vii							

ii. BFS is the algoritym to nd the shortedst path in an unweighted graph.

(b) Draw the Shortest Path Tree.

i.





7. (10 points) Apply the Dijkstra's algorithm to nd the shortest path from the vertex A to all the vertices in the graph below. Does the algorithm return a correct output? Justify your answer using the Dijkstra's Theorem.

(a) The nal result of Dijkstras algoritym on the negative weight problem is that it is not sopposed to be used on negative edges. Because in this specic problem the C value carries in a -2 and out a +3 the dierence to the updating value will not be replaced. There is not a cycle for it to follow.

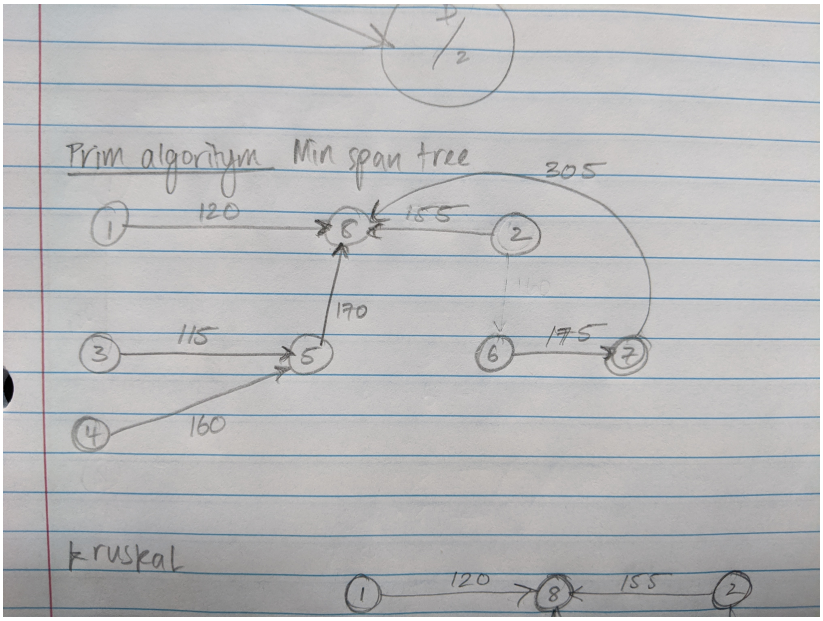
(b)

A	B	C	D
0	4	2	2

8. (20 points) There are eight small island in a lake, and the state wants to build seven bridges to connect them so that each island can be reached from any other one via one or more bridges. The cost of bridge construction is proportional to its length. The distance between pairs of islands are given in the following table.

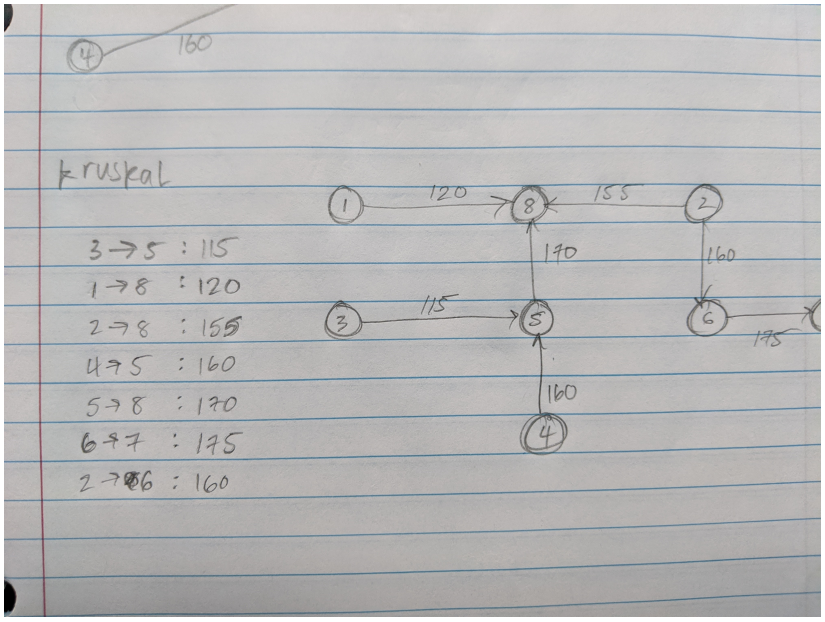
(a) Illustrate the Prim's algorithm using the graph below. Draw the Minimum Spanning Tree. What is the length of the bridges?

	1	2	3	4	5	6	7	8
1	-	240	210	340	280	200	345	120
2	-	-	265	175	215	180	185	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	360	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-



(b) Illustrate the Kruskal's algorithm using the graph below. Draw the Minimum Spanning Tree. What is the length of the bridges?

	1	2	3	4	5	6	7	8
1	-	240	210	340	280	200	345	120
2	-	-	265	175	215	180	185	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	360	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-


4
160
kruskal
3→5 : 115
1→8 : 120
2→8 : 155
4→5 : 160
5→8 : 170
6→7 : 175
2→6 : 160
1
120
8
155
2
170
160
3
115
5
6
175
160
4