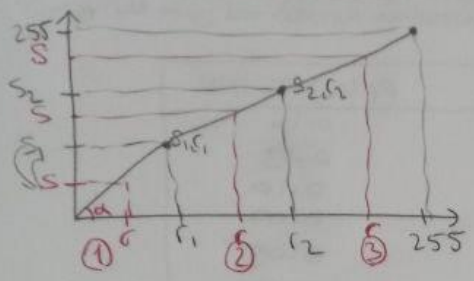
Example: Power-Law (Gamma) transformations

$S = cr^{\gamma}$



① $S = \frac{S_1}{r_1} \cdot r \Rightarrow \frac{S}{S+1} = \frac{r}{r+r_1}$

$= S = \frac{S_1}{r_1} \cdot r$

② $\frac{S_1+S}{S_1+S+S_2} = \frac{r_1+r}{r_1+r+r_2}$

$S_1r_1 + Sr + S_1r_2 + Sr_1 + Sr + Sr_2$

$=$

$S_1r_1 + S_1r + Sr_1 + Sr + S_2r_1 + S_2r$

$\Rightarrow S_1r_2 + Sr_2 = S_2r_1 + S_2r$

$\Rightarrow r_2(S_1+S) = S_2(r_1+r)$

$\Rightarrow S_1+S = \frac{S_2(r_1+r)}{r_2}$

$S = \frac{S_2(r_1+r)}{r_2} - S_1$

③ $\frac{S_2+S}{S_2+S+255} = \frac{r_2+r}{r_2+r+255}$

$S_2r_2 + Sr_2 + 255S_2 + Sr_2 + Sr + 255S = S_2r_2 + S_2r + Sr_2 + Sr + 255r_2 + 255r$

$255(S_2+S) = 255(r_2+r)$

$S = r_2 + (-S_2)$

Example : Histogram Equalization

→ Suppose that a 3-bit image (L=8) of size 64×64 pixels (MN=4096) has the intensity distribution shown in following table. Get the histogram equalization transformation function and give the $p_S(s_k)$ for each s_k.

r_k	n_k	$p_r(r_k)=n_k/MN$
$r_0=0$	680	0.16
$r_1=1$	570	0.13
$r_2=2$	430	0.10
$r_3=3$	322	0.07
$r_4=4$	237	0.05
$r_5=5$	716	0.17
$r_6=6$	874	0.21
$r_7=7$	1073	0.26

$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7 \times 0.16 = 1.12 \rightarrow (1)$

$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7 \times (0.16+0.13) = 2.03 \rightarrow (2)$

$s_2 = T(r_2) = 7\sum_{j=0}^{2} p_r(r_j) = 7 \times (0.16+0.13+0.10) = 2.73 \rightarrow (3)$

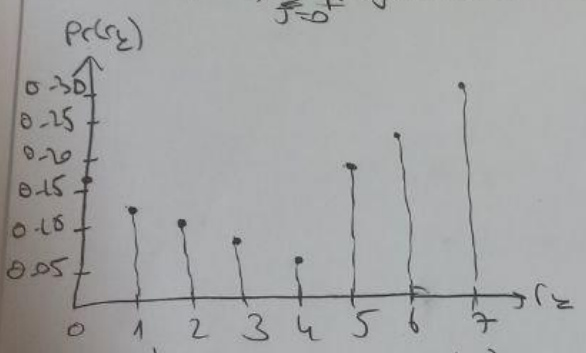
$s_3 = T(r_3) = 7\sum_{j=0}^{3} p_r(r_j) = 7 \times (0.16+0.13+0.10+0.07) = 3.22 \rightarrow (3)$

$s_4 = T(r_4) = 7\sum_{j=0}^{4} p_r(r_j) = 7 \times (0.16+0.13+0.10+0.07+0.05) = 3.57 \rightarrow (4)$

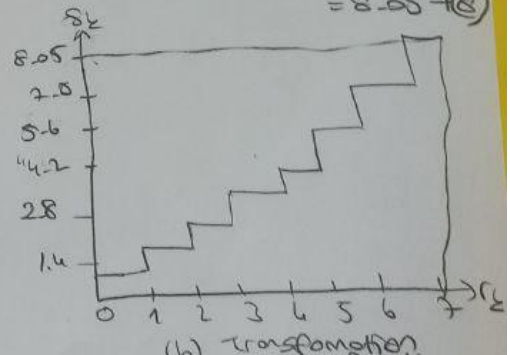
$s_5 = T(r_5) = 7\sum_{j=0}^{5} p_r(r_j) = 7 \times (0.16+0.13+0.10+0.07+0.05+0.17) = 4.76 \rightarrow (5)$

$s_6 = T(r_6) = 7\sum_{j=0}^{6} p_r(r_j) = 7 \times (0.16+0.13+0.10+0.07+0.05+0.17+0.21) = 6.23 \rightarrow (6)$

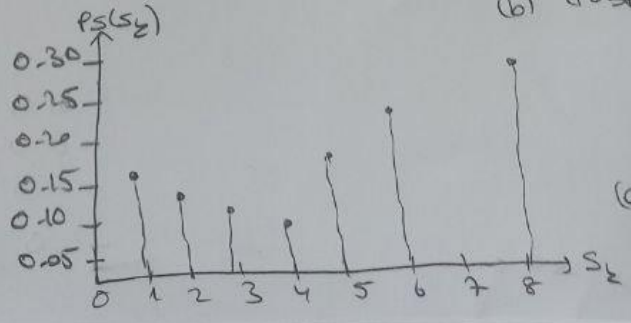
$s_7 = T(r_7) = 7\sum_{j=0}^{7} p_r(r_j) = 7 \times (0.16+0.13+0.10+0.07+0.05+0.17+0.21+0.26) = 8.05 \rightarrow (8)$



(a) original histogram



(b) transformation function



(c) Equalized histogram