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# Statistical Essentials for Health Data Science


**Dileepa Ediriweera**

MBBS, MSc(BioStat), MSc(Biomed Info), PhD

# Outline

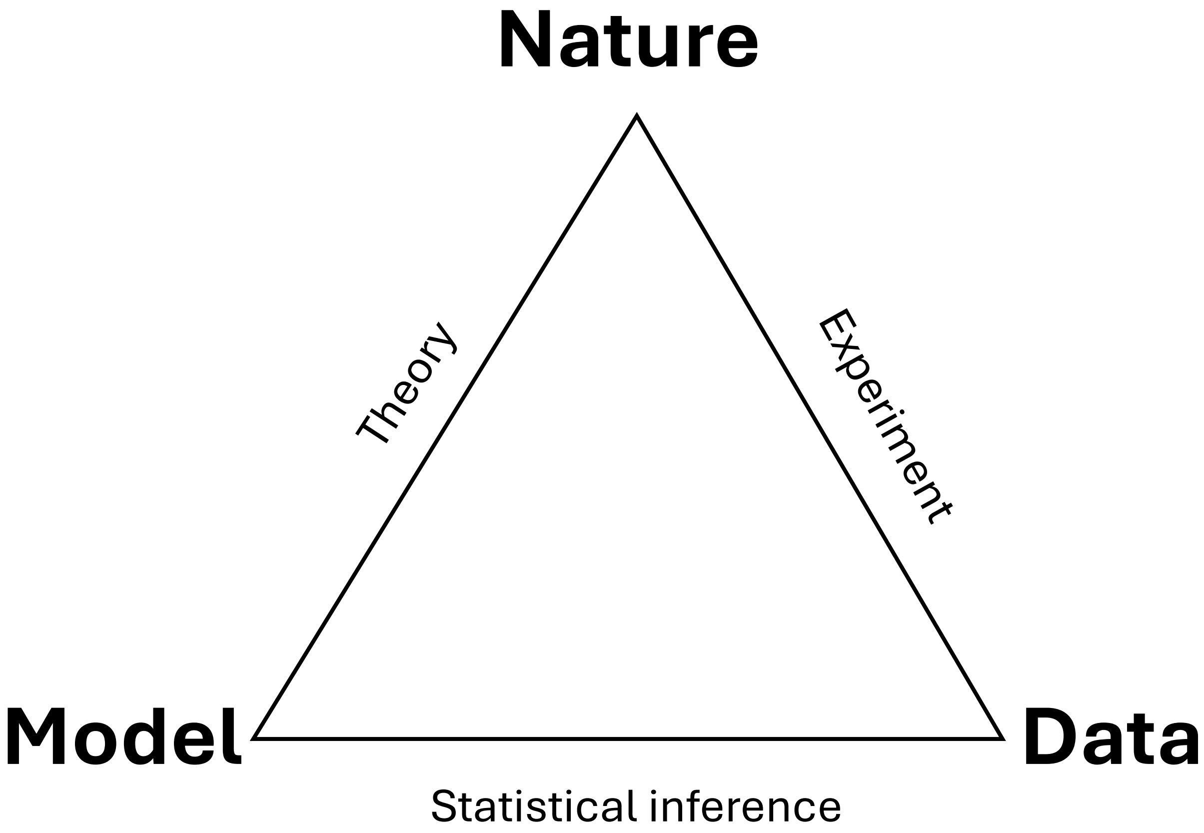
- Session 1: Describing the Health Data
- Session 2: Making Informed Decisions
- Session 3: Exploring Relationships and Prediction

# Why we need science?

# Science

- Goal is to understand nature
- Two pillars of the scientific method
  - Theory
  - Observations
- Theory – predicts how a natural process should behave
- Observations (controlled experiment or direct observation of the natural world) – tells us whether the theory is correct

# Role of statistics within scientific method



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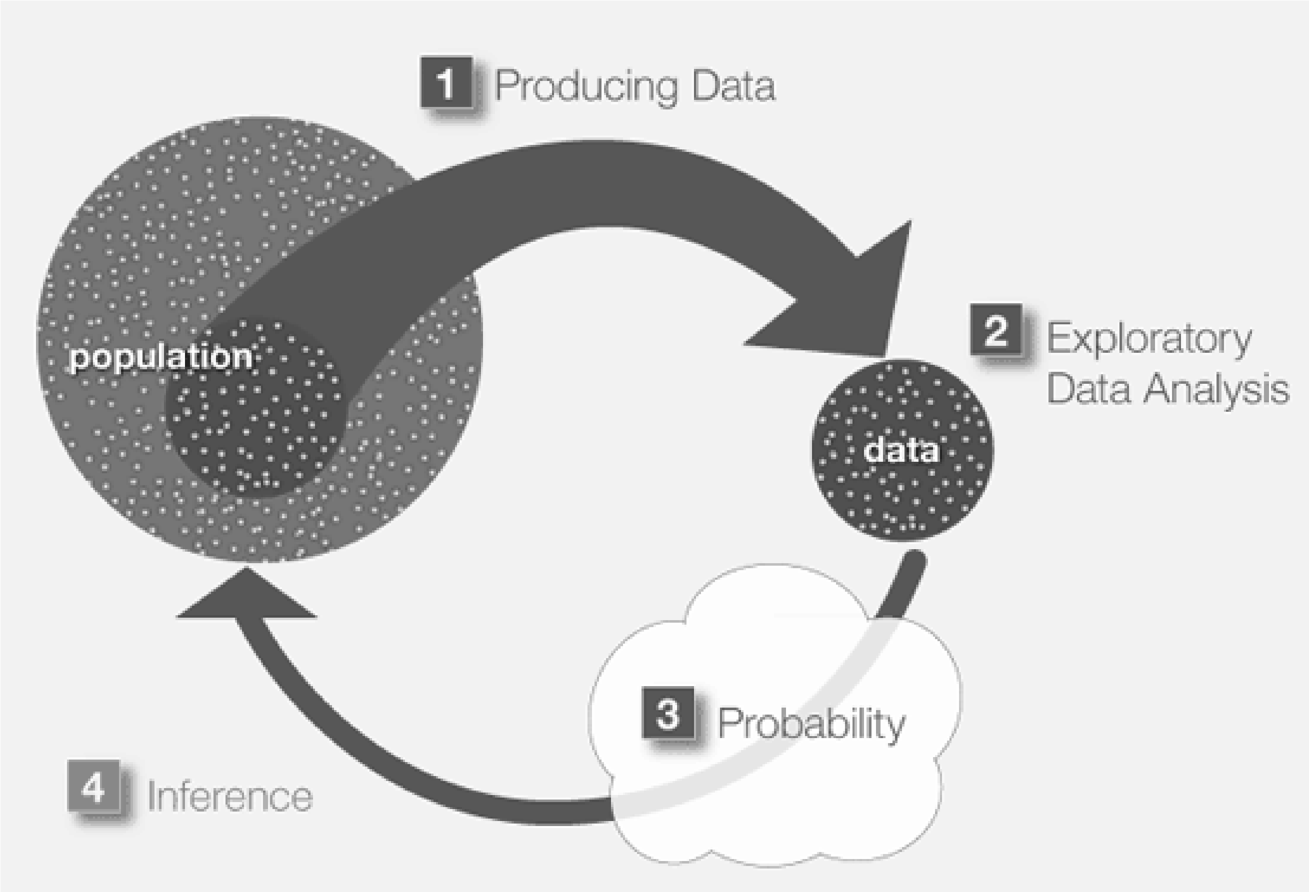
# Session 1: Describing the Health Data

# Statistics

- Collection of data
- Analysis
- Making inference



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# Areas of statistics

- Descriptive statistics
  - Used to summarize, organize and present data in a convenient and communicable form
    - Average science marks of a class
    - Number of students for a given age groups

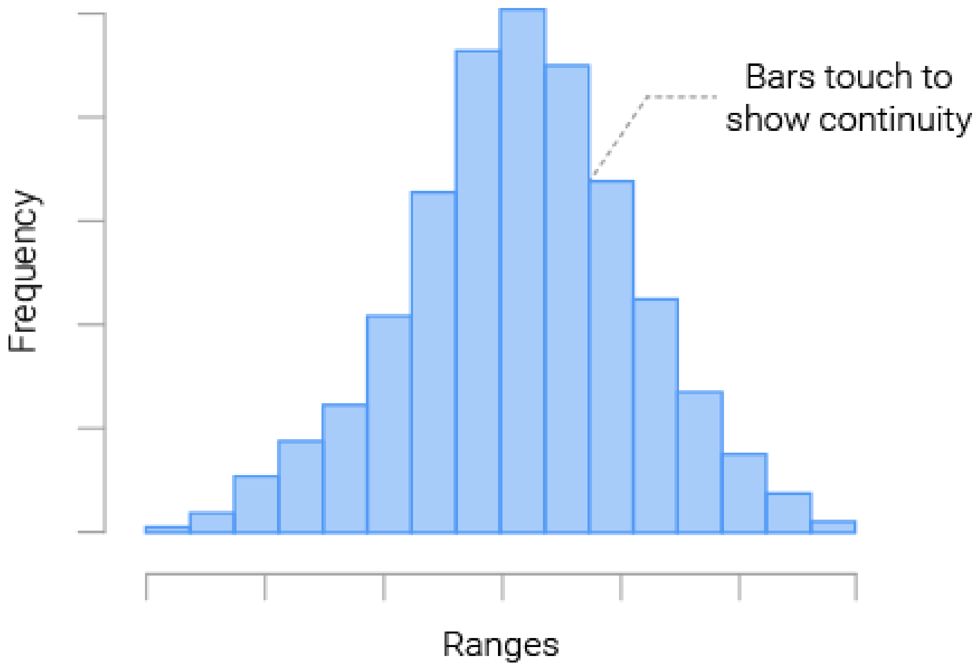
- Inferential statistics
  - Techniques that allow us to make inferences or conclusions about a population based on data that are gathered from a sample
  - This is done either;
    - Estimate parameters (e.g. population mean)
    - Hypothesis testing (e.g. effectiveness of a new drug)

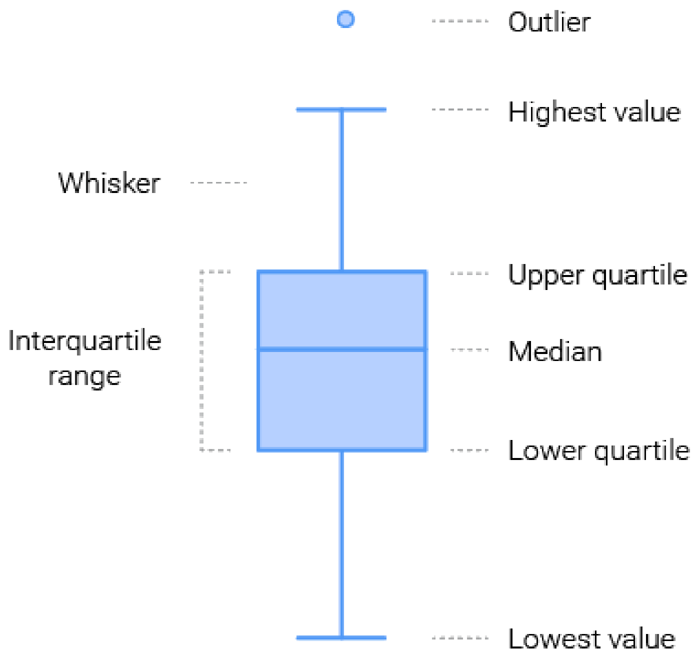
# Descriptive statistics

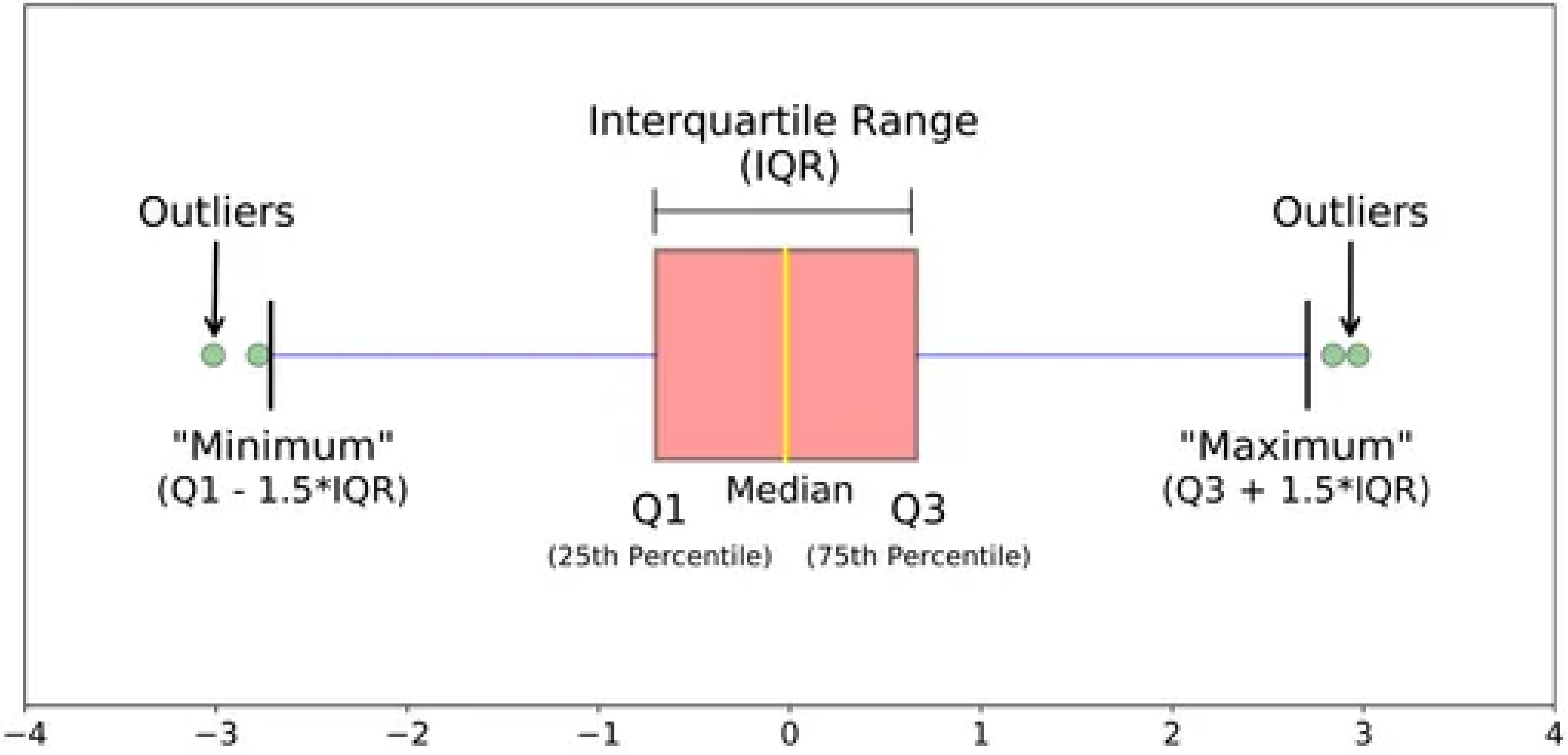
- To describe the distribution of sample data (what data show)
- Graphically
  - Histogram, boxplot, bar charts, pie charts
- Numerically
  - Distribution (frequency distribution table)
  - Central tendency (centre of location)
    - Mean, median, mode
  - Dispersion
    - Standard deviation, range, interquartile range

# Graphical illustrations – single variable

- Continuous data?




Interquartile Range
(IQR)
Outliers
Outliers
"Minimum"
(Q1 - 1.5*IQR)
Q1
(25th Percentile)
Median
Q3
(75th Percentile)
"Maximum"
(Q3 + 1.5*IQR)
−4
−3
−2
−1
0
1
2
3
4

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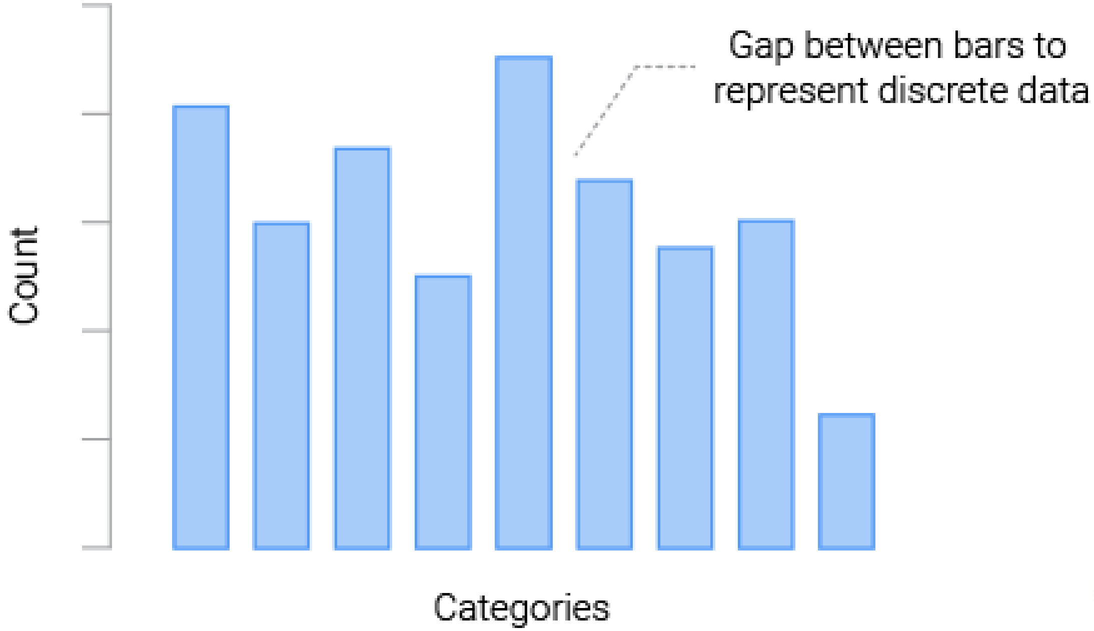
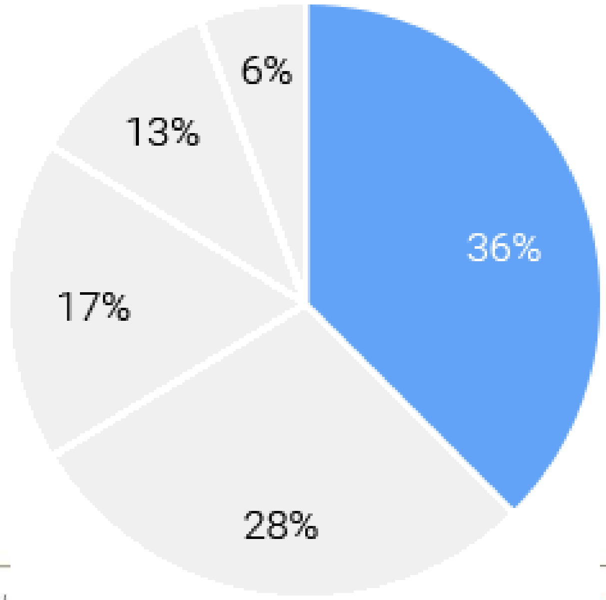
# Graphical illustrations – single variable

- Categorical data ?



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# Frequency distribution table

| Degree | Frequency |
|---|---|
| High School | 2 |
| Bachelor's | 7 |
| MBA | 20 |
| Master's | 3 |
| Law | 4 |
| PhD | 4 |

| Range | Frequency |
|---|---|
| 0-39 | 12 |
| 40-79 | 6 |
| 80-119 | 2 |
| 120-159 | 3 |
| 160-199 | 5 |
| 200-240 | 2 |

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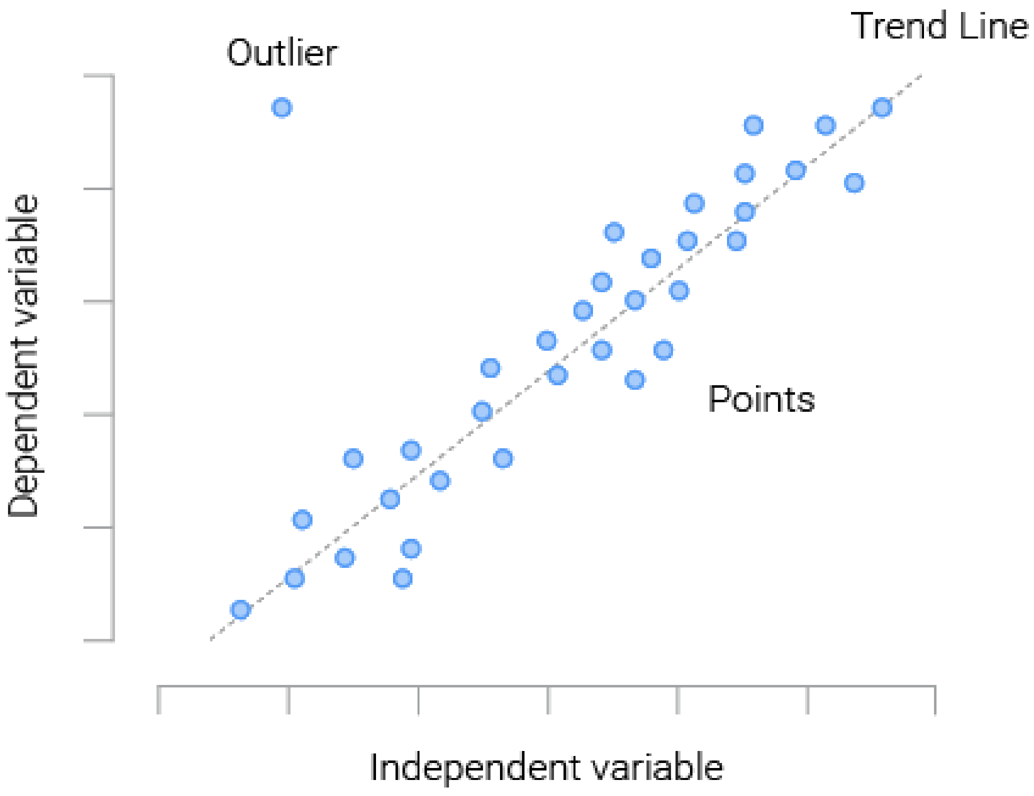
# Graphical illustrations – two variables

- How are two variables correlated?



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# Data distributions

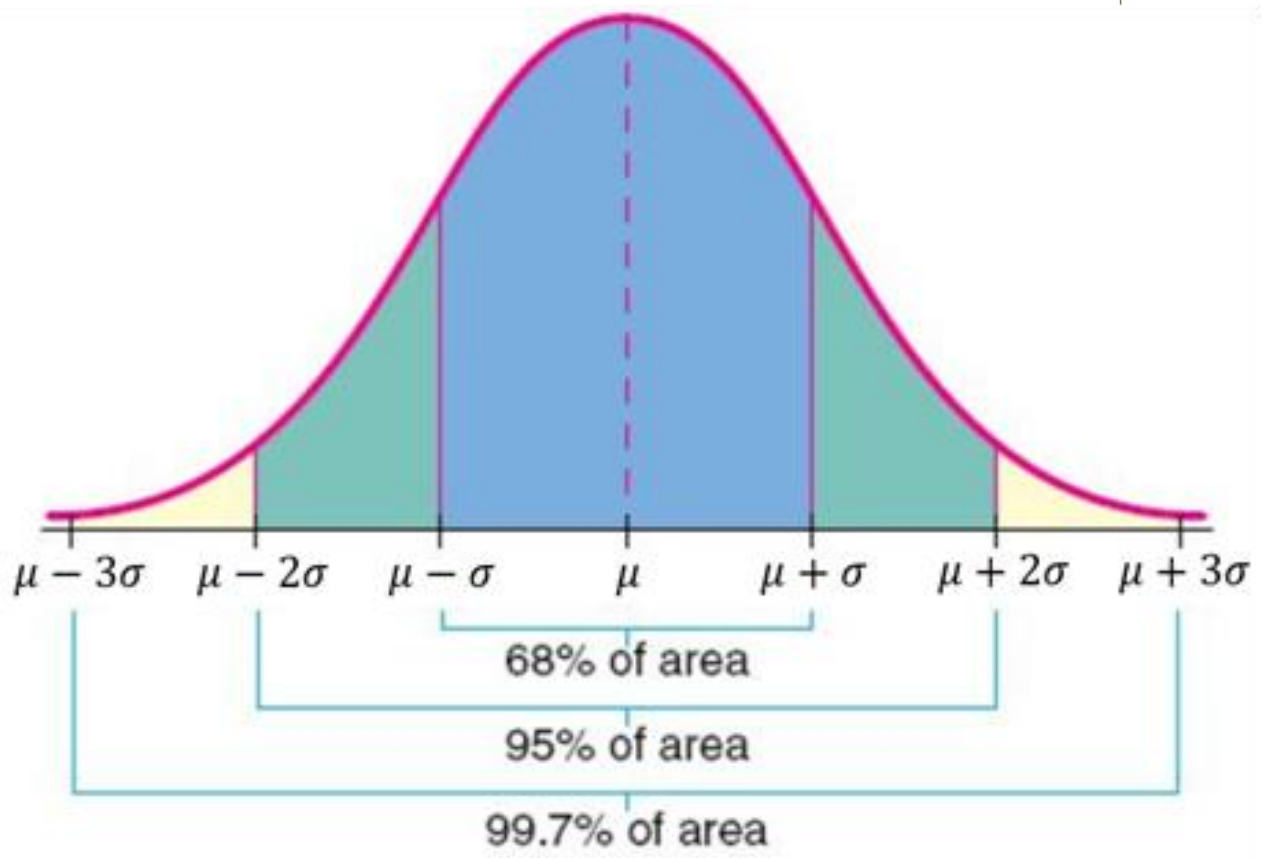
- We need to identify the distribution that best fit the data (and to specify the parameters)
  - Theoretical framework

- Continuous distributions
  - Normal, Lognormal, Exponential, Uniform, Cauchy, Weibull

- Discrete distributions
  - Binomial, Poisson, Negative Binomial, Discrete uniform, Geometric

# Normal Distribution (theoretical)

- Bell shape ("Bell Curve")
- Goes from **-∞** to +∞
- Mean = Median = Mode
- Symmetry around the centre
  - 50% of values less than the mean
  - 50% of values greater than the mean
  - 68% of values are within 1 standard deviation of the mean
  - 95% of values are within 2 standard deviations of the mean
  - 99.7% of values are within 3 standard deviations of the mean



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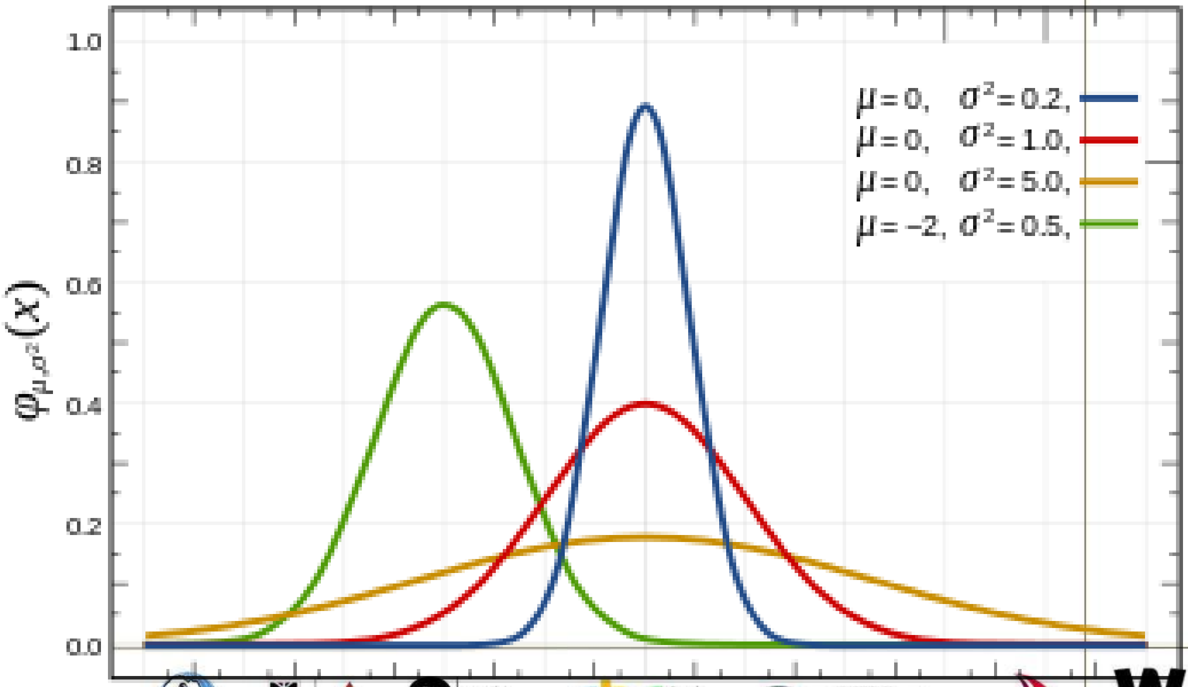
# Statistical distribution has parameters

- Parameters describe the shape of a distribution
  - E.g. Normal distribution
  - $X \sim N(\mu, \sigma)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$



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# github.com/DileepaE

eswatini antivenom foundation

Rwanda Biomedical Centre

University of Global Health Equity

LSTM

wellcome

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# Measures of Central Tendency

- Mean
  - Add up all the numbers and divide by number of observations
- Median
  - The middle number (order the numbers and find the actual middle  number or average of the two numbers if not)
- Mode
  - Most commonly occurring number

- Activity
  - 17, 18, 20, 21, 21, 24, 23, 21, 15, 19
  - 17, 18, 23, 20, 21, 24, 23, 20, 20, 15, 19, 20
  - 17,18, 23, 20, 21, 24, 23, 20, 20, 15, 19, 20,60

$$\mu = \frac{1}{N} \sum_{i=1}^{N} x_i$$

# Measures of Dispersion

- Range
  - Maximum – minimum (this is different to reporting the max and min)

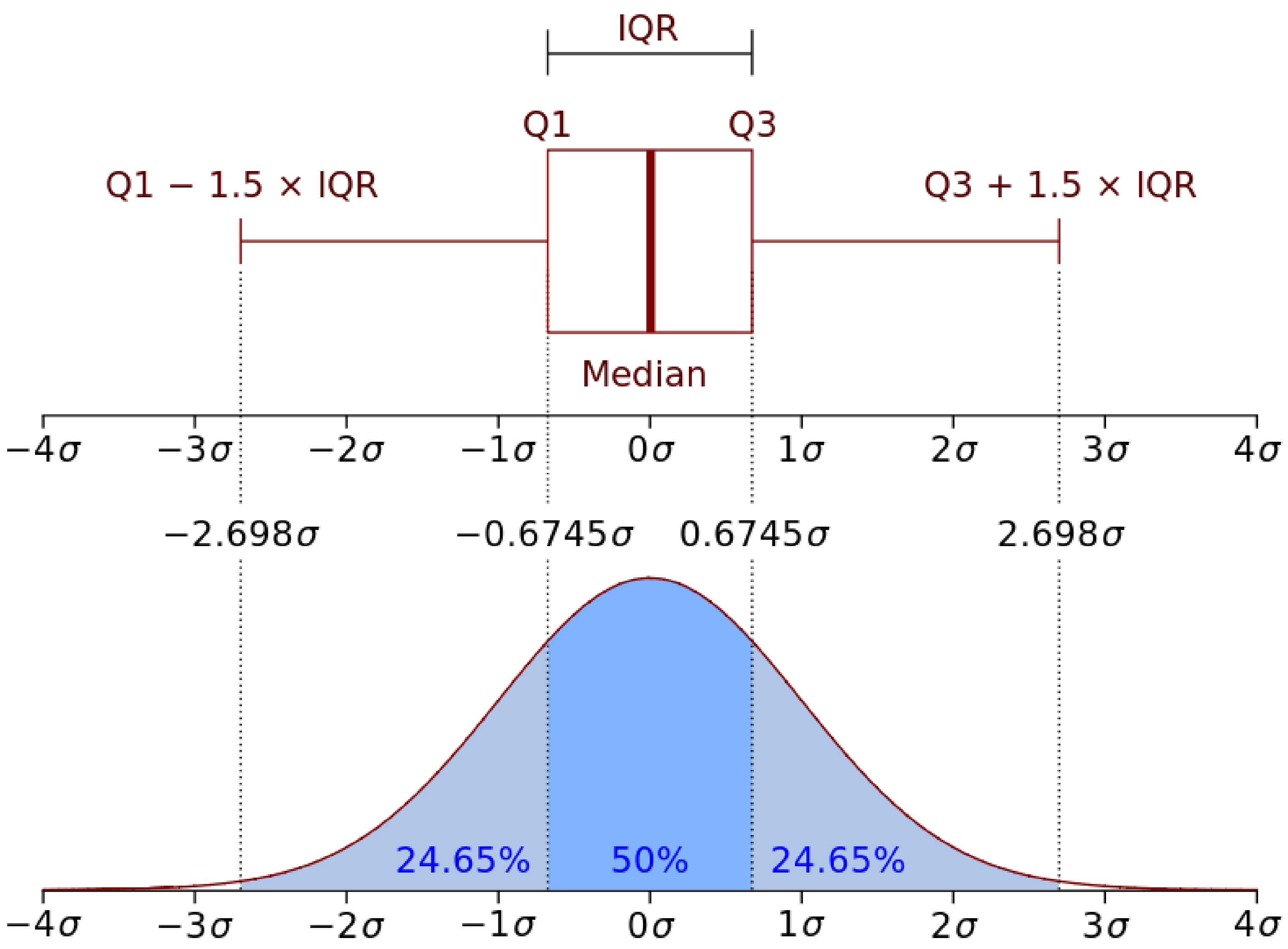
- Variance ($\sigma^2$)
  - Average squared deviation from the mean

- Standard Deviation ( $\sigma$ )
  - Square root of variance

$$\sigma^2 = \frac{\sum(\chi - \mu)^2}{N}$$

# Measures of Dispersion

- Quartile :
  - A division of observations into four (quarters) of equal size
- Interquartile rage
  - First quartile (Q1) : middle number between the smallest and median
  - Second quartile (Q2) : median (50% of the data lies below this point)
  - Third quartile (Q3) : middle value between the median and the highest
  - Q1 – Q3 (this is reporting Q1 and Q3)


IQR
Q1
Q3
Q1 − 1.5 × IQR
Q3 + 1.5 × IQR
Median
−4σ −3σ −2σ −1σ 0σ 1σ 2σ 3σ 4σ
−2.698σ
−0.6745σ
0.6745σ
2.698σ
24.65%
50%
24.65%
−4σ −3σ −2σ −1σ 0σ 1σ 2σ 3σ 4σ

# Selecting most suitable statistic

- If data has a normal distribution
  - Mean (SD)
  - Sensitive to outliers

- If data does not have a normal distribution?
  - Median (IQR)
  - Not sensitive to outliers

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# Introduction to data simulation

- Assume mean (SD) of FBS in general population is 100 (20) mg/dl
- Simulate 20 FBS values from this distribution

```
rnorm(n, mean = 0, sd = 1)
```

| | |
|---|---|
| `n` | number of observations. If `length(n) > 1`, the length is taken to be the number required. |
| `mean` | vector of means. |
| `sd` | vector of standard deviations. |

```
y <- rnorm(20,100,20)
y
```

```
> y
 [1] 124.33053  74.75877  69.40745 149.12174 143.60878 107.27091 106.09180  93.16760  65.99009 110.09239
[11]  92.44645 124.89783  93.32005  74.17525  67.33445 120.53812 101.79306 114.33832  99.01170 103.23623
```

# Testing for normality

- To determine if a data set is well-modeled by a normal distribution

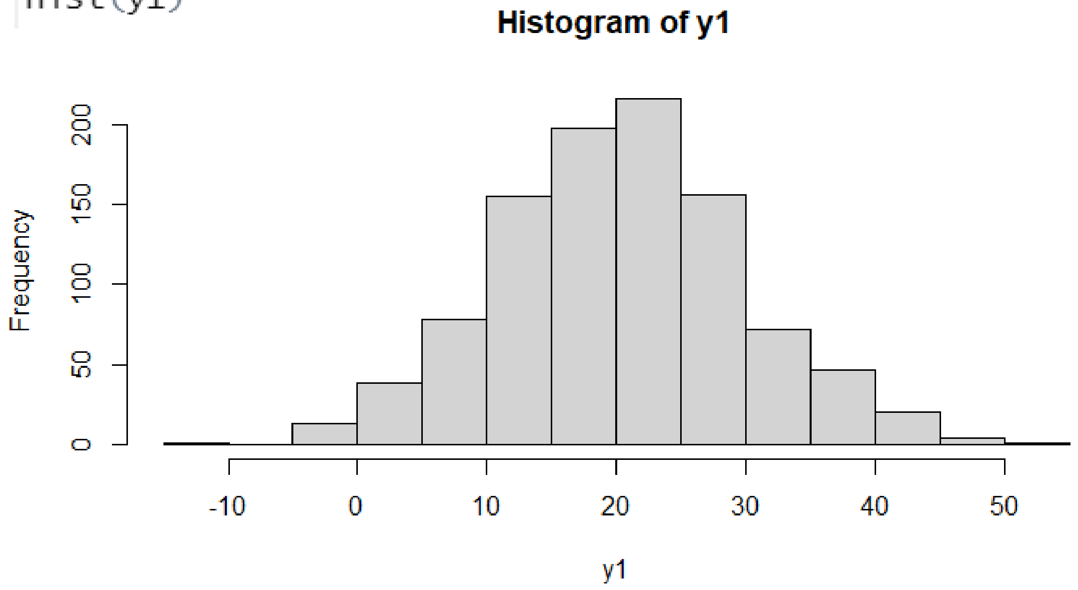
- Graphical methods
  - Histogram
  - Quantile-quantile (QQ) plot of the standardized data against the standard normal distribution

- Statistical tests
  - Shapiro–Wilk test

# Testing for normality

```
y1 <- rnorm(1000,20,10)
```

```
hist(y1)
```

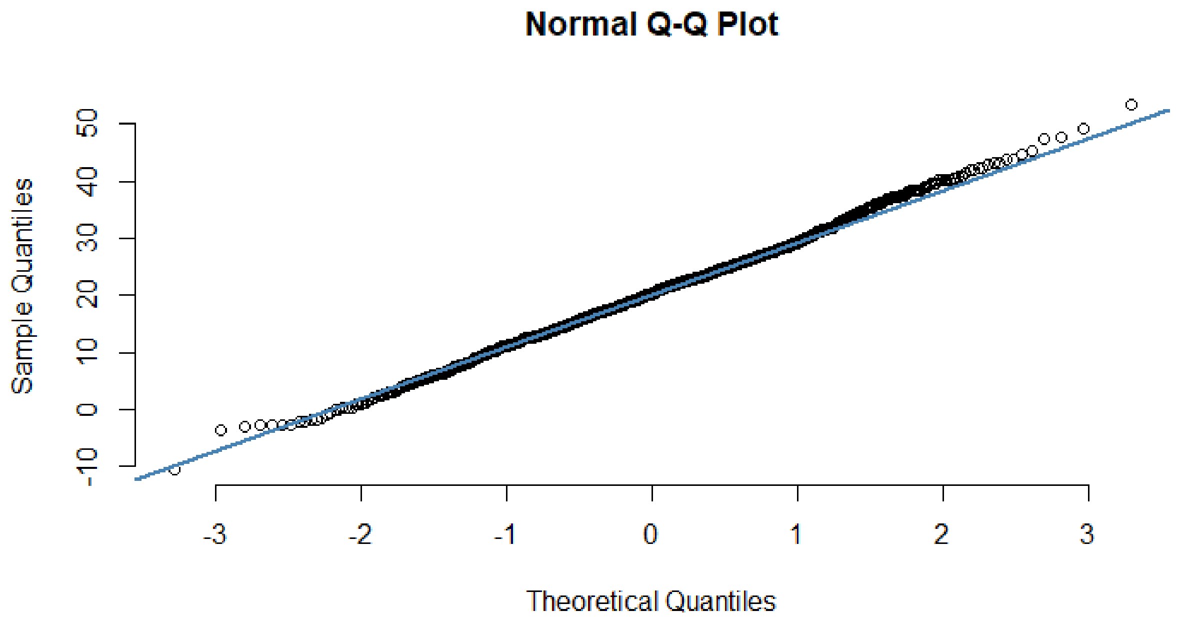



```
> shapiro.test(y1)

        Shapiro-Wilk normality test

data:  y1
W = 0.98454, p-value = 0.2939
```

```
qqnorm(y1, pch = 1, frame = FALSE)
qqline(y1, col = "steelblue", lwd = 2)
```



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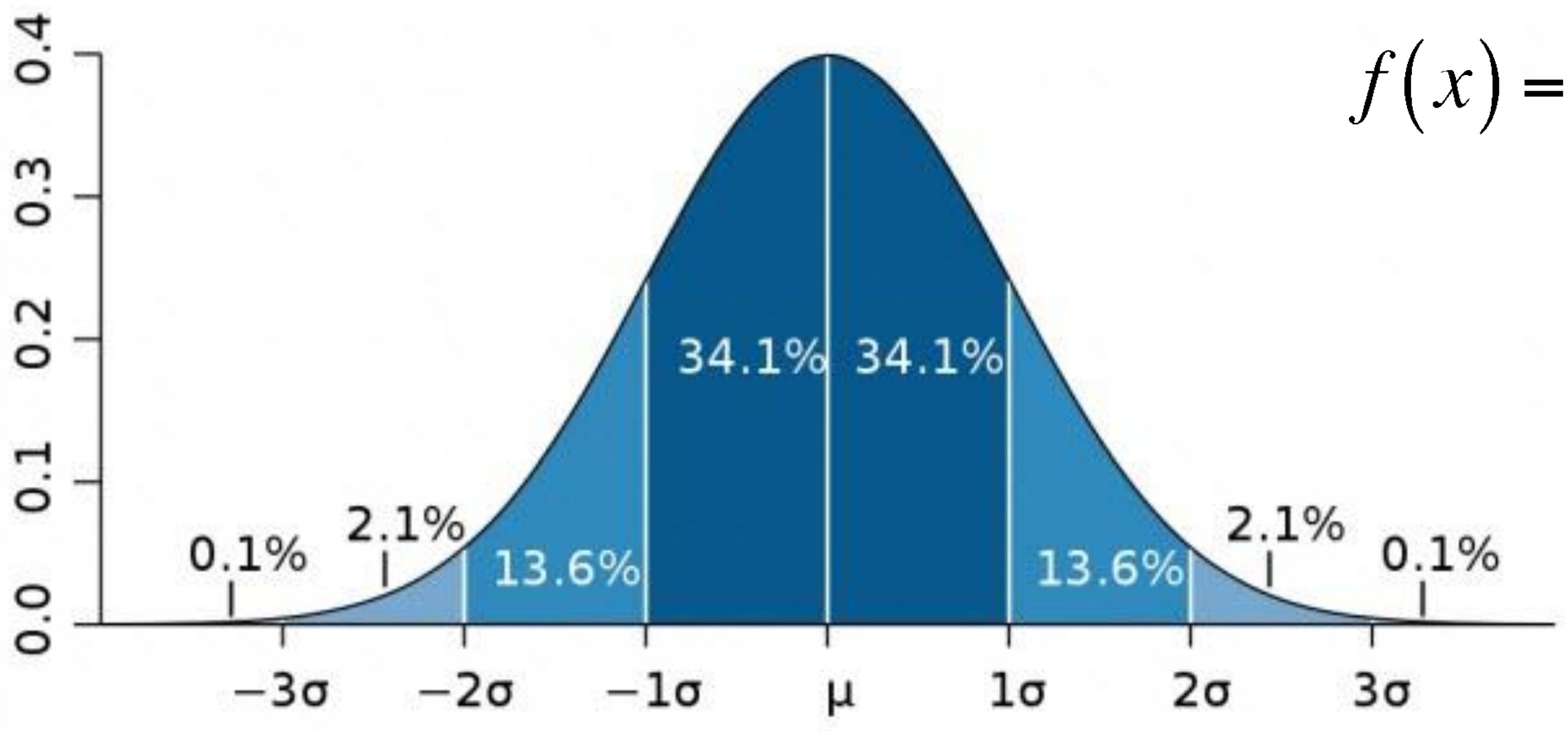
# Normal distribution




$$f(x) = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\delta}\right)}$$

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# Standard normal distribution

- Is a normal distribution
- Bell shape
- Total area under curve  = 1
- Area:
  - -1 < Z < +1 → 68%
  - -2 < Z < +2 → 95%
- Probability
  - P(Z < 0 ) = 0.5

# Why SND?

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty \le x \le \infty$$

$$= \frac{1}{1\times\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-0}{1}\right)^2} \qquad -\infty \le x \le \infty$$

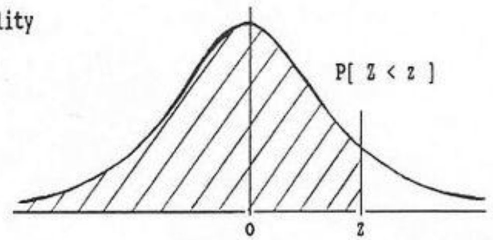
$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z)^2} \qquad -\infty \le z \le \infty \qquad \left(\frac{x-0}{1} = z\right)$$

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\, Z < z \,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$




| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7854 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8804 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |

| z | 3.00 | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| P | 0.9986 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |

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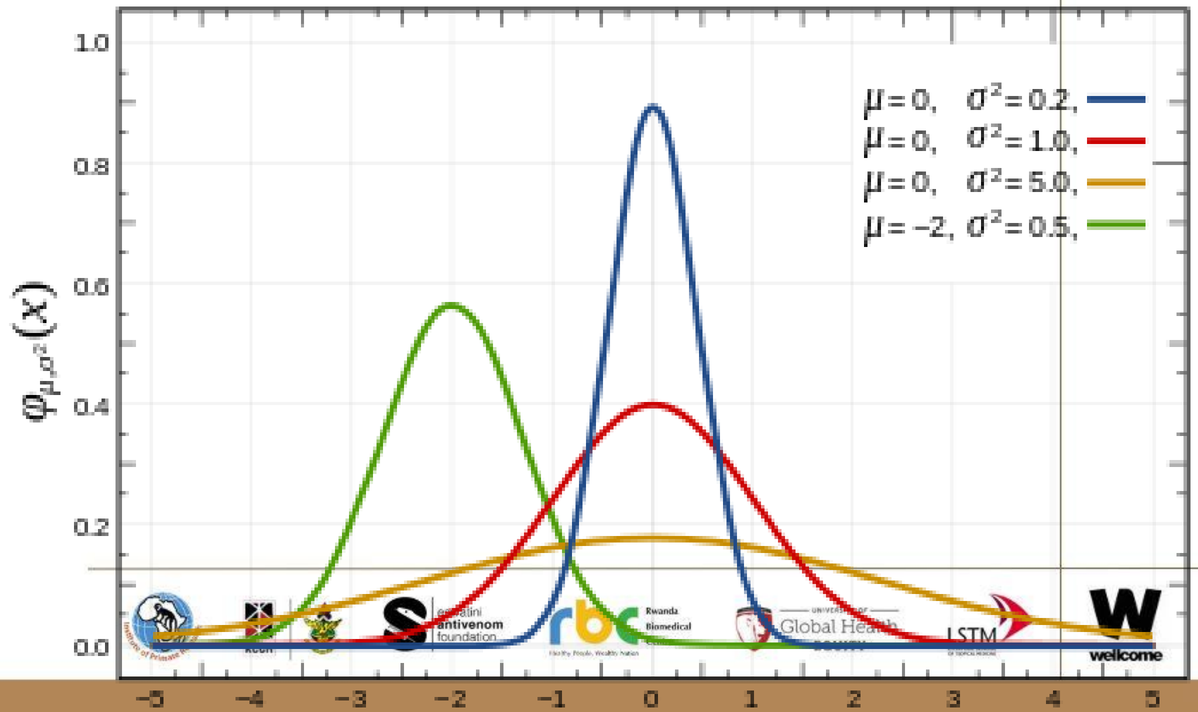
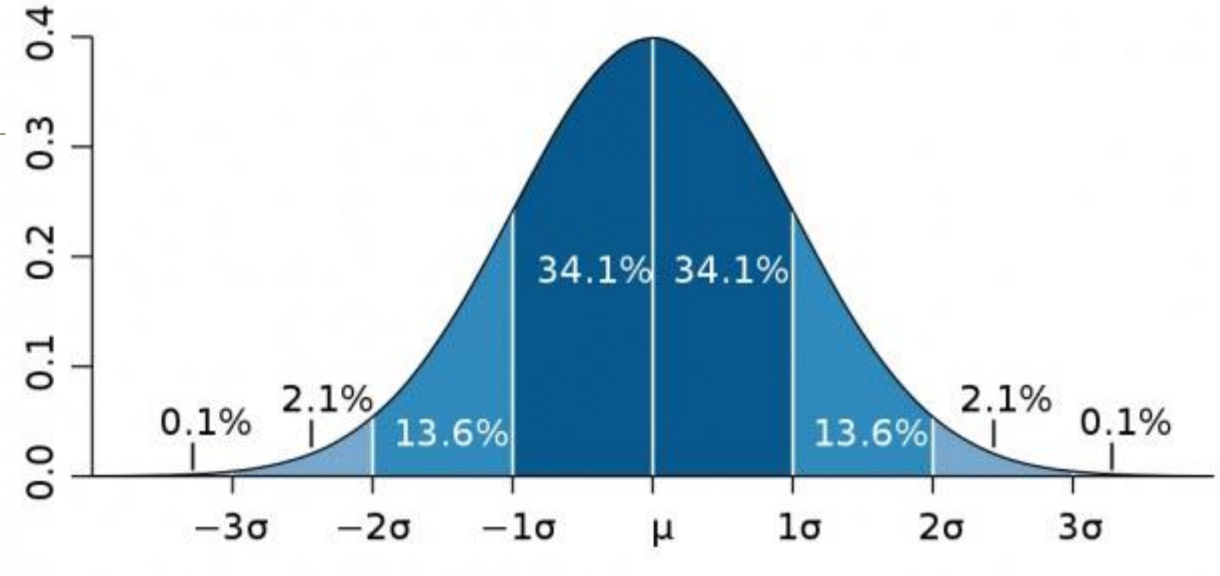
# Example

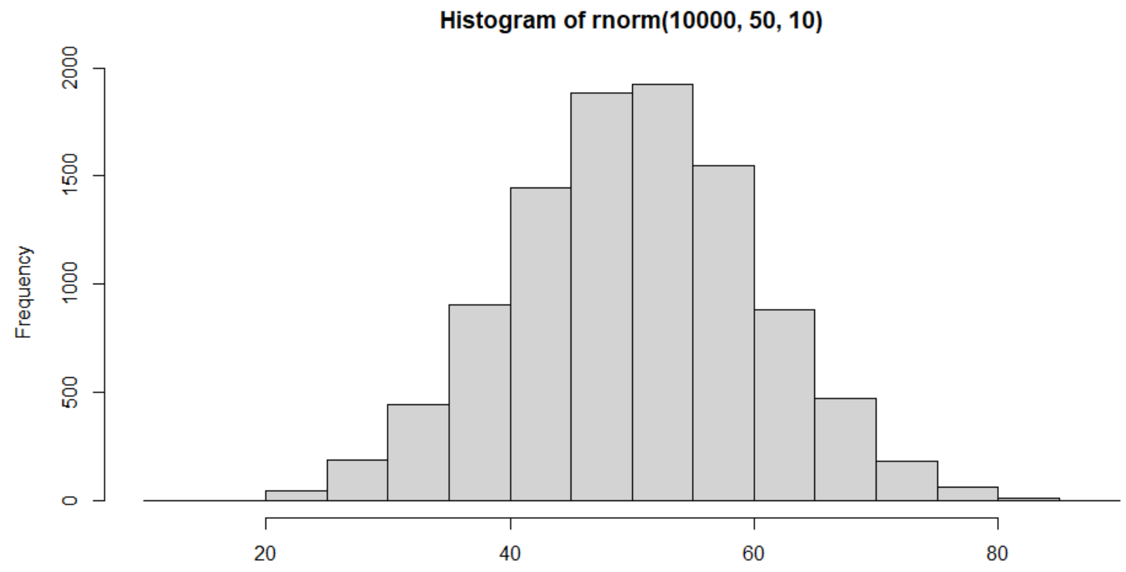
- Blood pressure X~ (120, 10)
  - P ( X > 140) ?

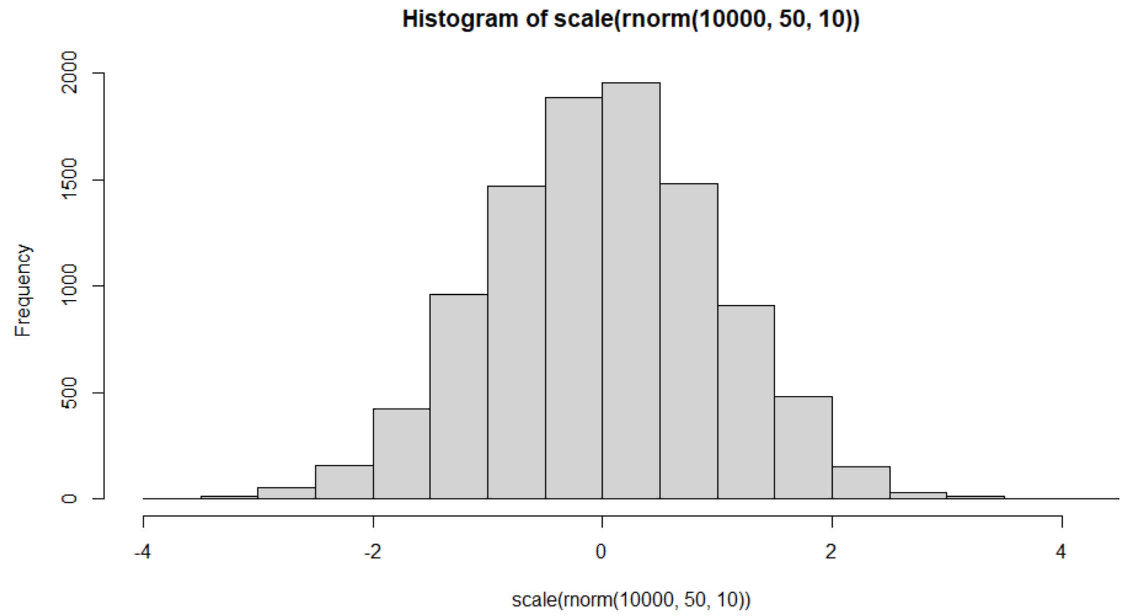
- Blood pressure X~ (150, 12)
  - P ( X > 140) ?

$$f(x) = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\delta}\right)}$$




0.4
0.3
0.2
0.1
0.0
34.1%
34.1%
13.6%
13.6%
2.1%
2.1%
0.1%
0.1%
−3σ
−2σ
−1σ
μ
1σ
2σ
3σ


Histogram of rnorm(10000, 50, 10)
Frequency
0
500
1000
1500
2000
20
40
60
80


Histogram of scale(rnorm(10000, 50, 10))
Frequency
0
500
1000
1500
2000
-4
-2
0
2
4
scale(rnorm(10000, 50, 10))

# Exercise

- P (Z < 0.5)
- P (Z > 0.5)
- P ( Z < 0.4)
- P ( Z < 1.35)
- P (0.4 < Z < 1.35)
- P ( Z < -1)

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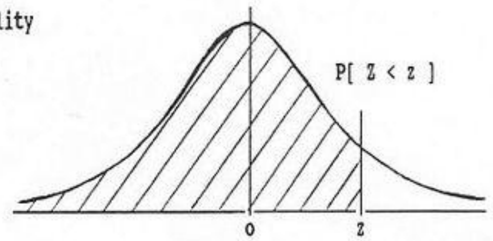


| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
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| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
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| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
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| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7854 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8804 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
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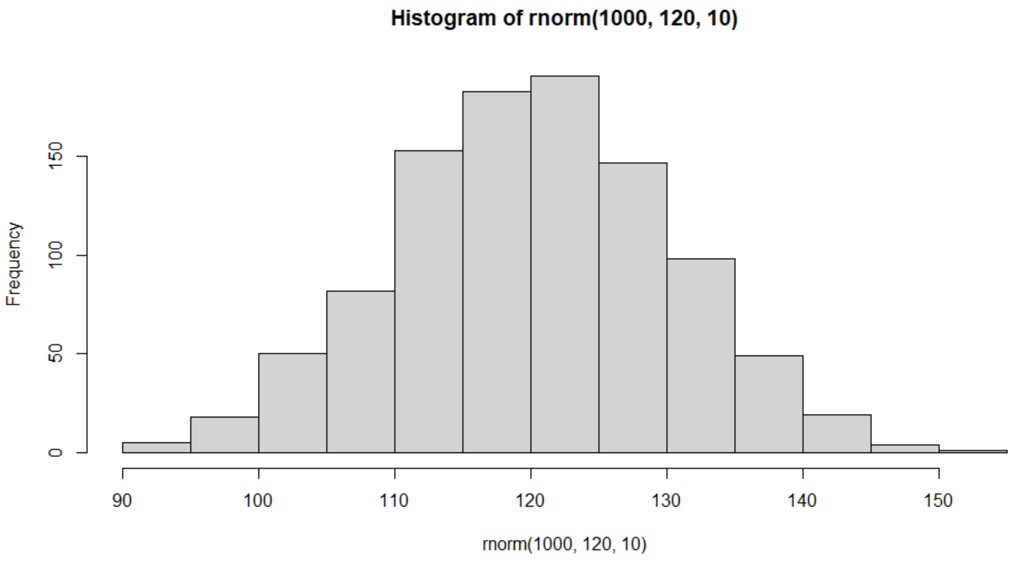
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |

| z | 3.00 | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| P | 0.9986 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |

# Example

- Blood pressure X~ N(120, 10)
    - P ( X > 140) ?

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The table gives the cumulative probability up to the standardised normal value z i.e.

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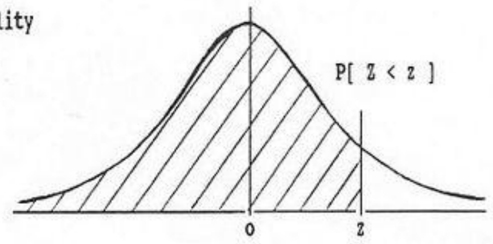


| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7854 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8804 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |

| z | 3.00 | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| P | 0.9986 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |

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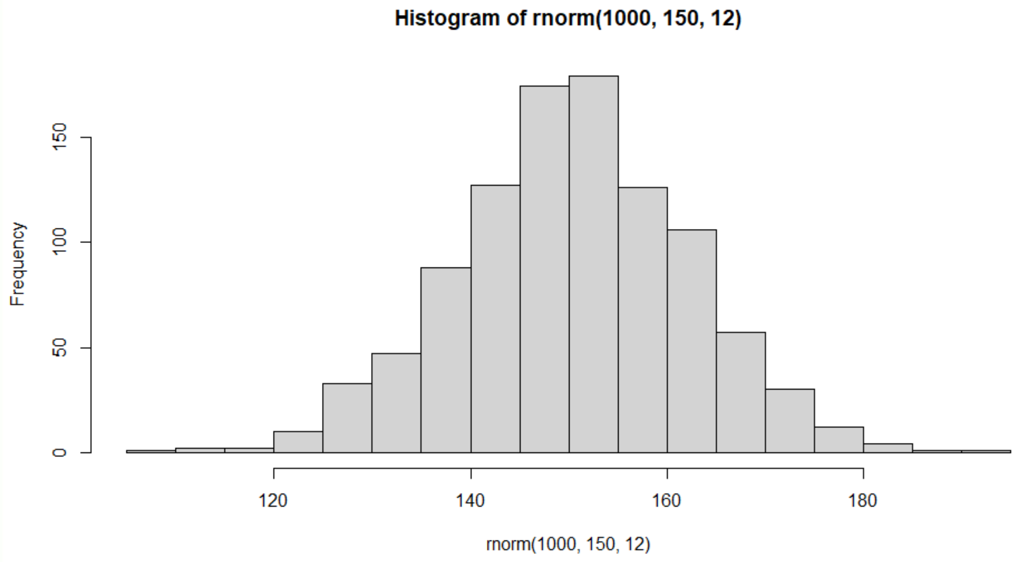
# Example

- Blood pressure X~ N(150, 12)
    - P ( X > 140) ?

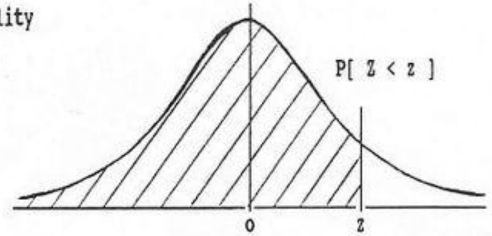



## STANDARD STATISTICAL TABLES

### 1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z
i.e.

$$P[\, Z < z \,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$




| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
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| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
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| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7854 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
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| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
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| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
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| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |

| z | 3.00 | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| P | 0.9986 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |

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# Data types

# Activity

- Identifying data types from common health datasets

# Scale of measurement (types) of data

1. Nominal/categorical
2. Ordinal
3. Interval
4. Ratio

| Offers: | Nominal | Ordinal | Interval | Ratio |
|---|---|---|---|---|
| The sequence of variables is established | – | Yes | Yes | Yes |
| Mode | Yes | Yes | Yes | Yes |
| Median | – | Yes | Yes | Yes |
| Mean | – | – | Yes | Yes |
| Difference between variables can be evaluated | – | – | Yes | Yes |
| Addition and Subtraction of variables | – | – | Yes | Yes |
| Multiplication and Division of variables | – | – | – | Yes |
| Absolute zero | – | – | – | Yes |

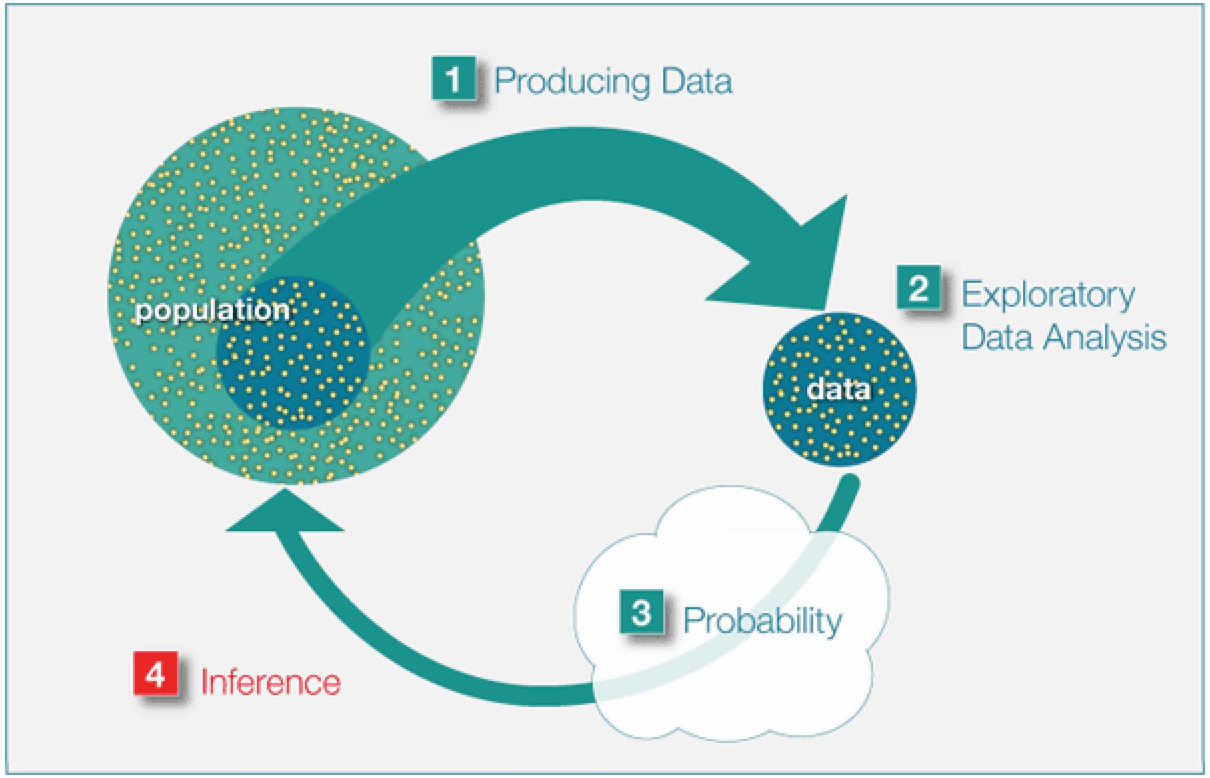
# Session 2: Making Informed Decisions

Inferential statistics : Part I

# Inferential statistics

# Statistics

- Collection of data
- Analysis
- Making inference



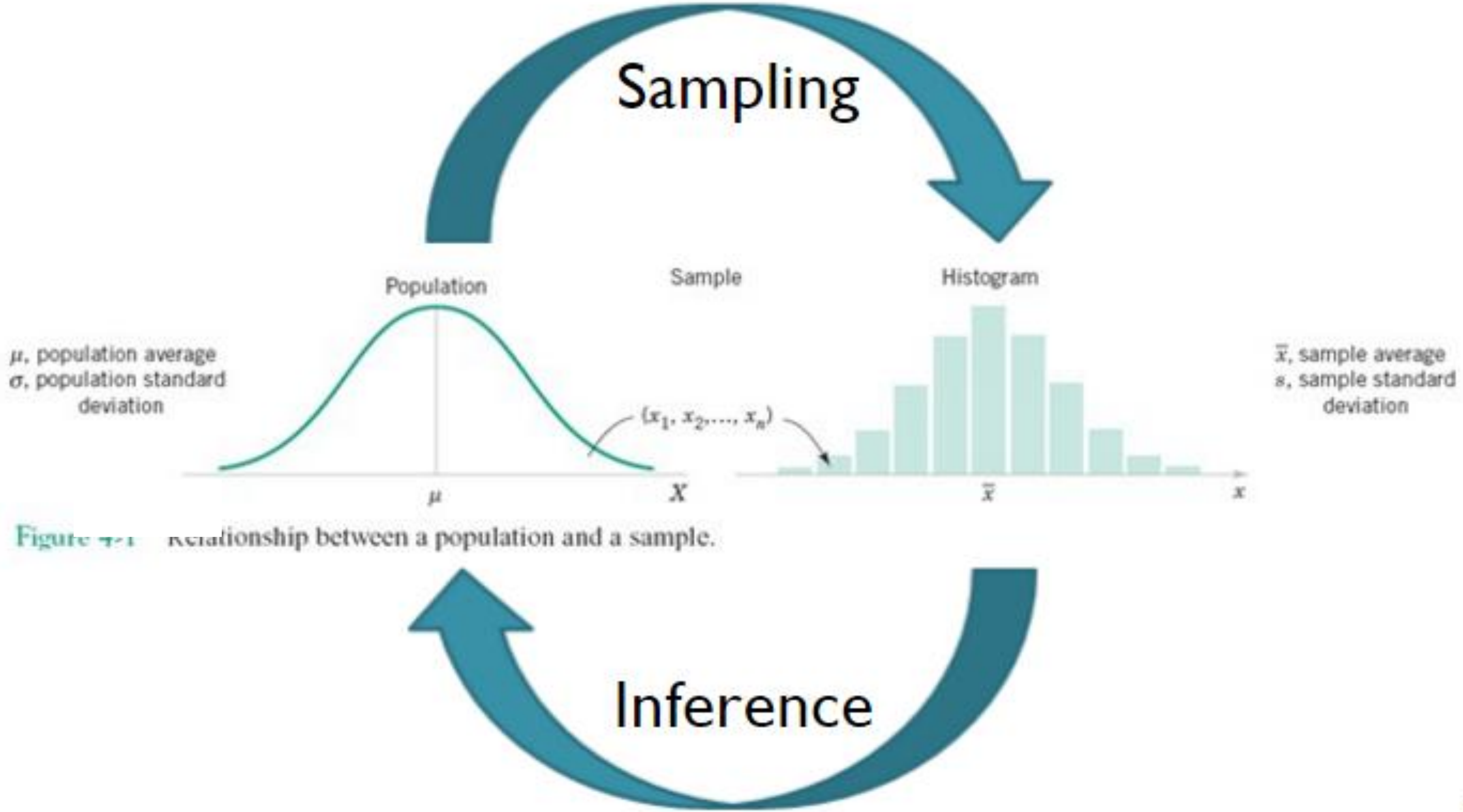
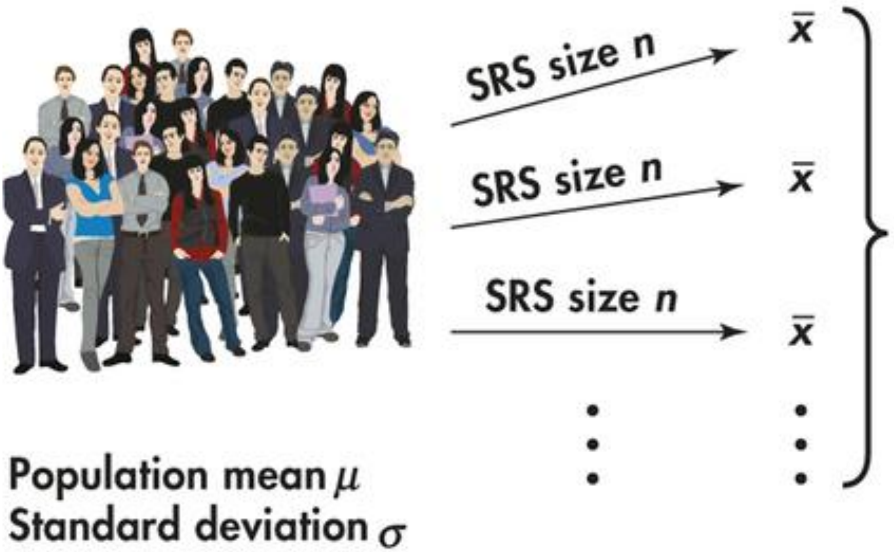



Figure 4-1 Relationship between a population and a sample.

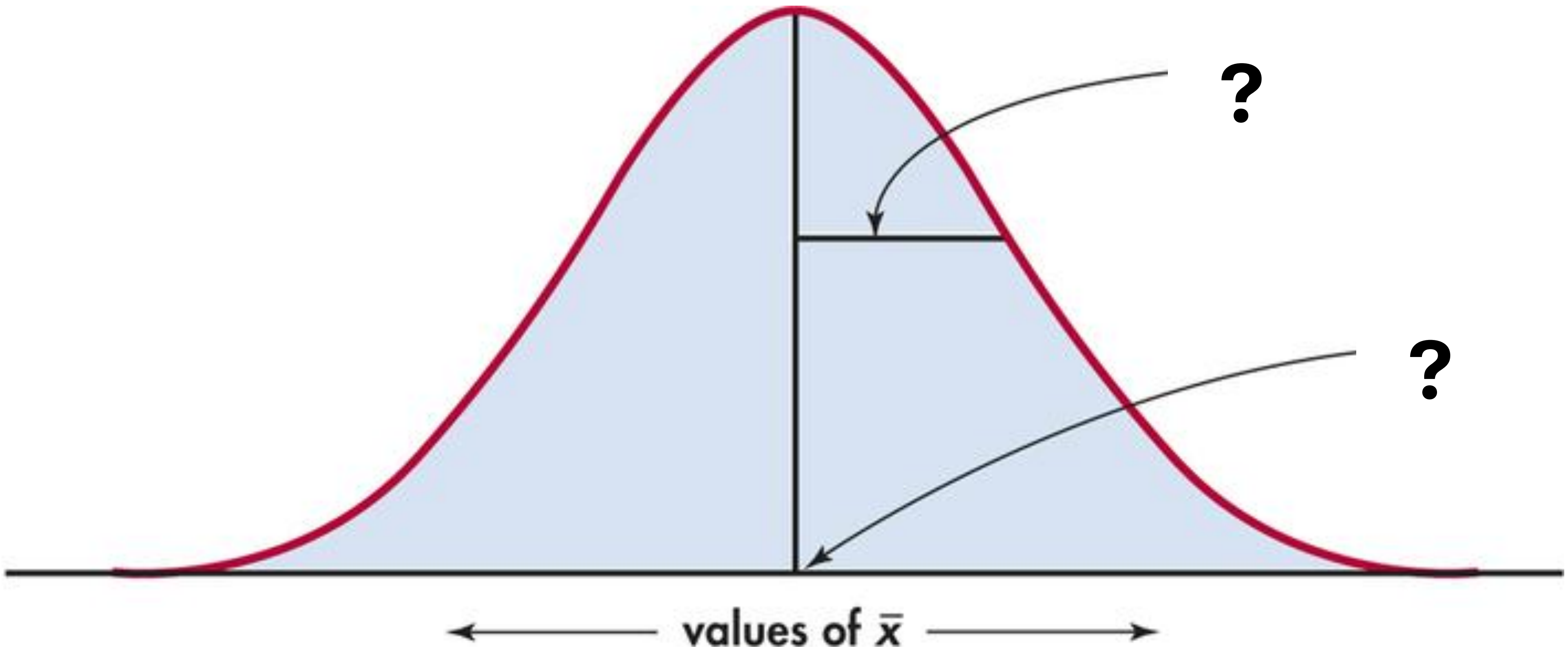
# Population vs Sample

- Population value vs Sample value
  - Parameter vs statistic

- Notations
  - Greek vs English


SRS size n
SRS size n
SRS size n
x̄
x̄
x̄
Population mean μ
Standard deviation σ

# Sampling distribution



# Variables (experimental designs)

- Independent
  - ?
- Dependent
  - ?

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# Variables (experimental designs)

- Independent
  - Cause (independent of other variables in the study)
  - Assumed to have a direct effect on the dependent variable
  - Experimenter has control (we can manipulate)

- Dependent
  - Effect/outcome (depends on changes in the independent variable)
  - Experimenter does not have control (we observe)

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# Outcome data (dependent variable)

- Continuous
  - Infinite number of possibilities within finite interval
  - E.g.
    - Height/weight/time

- Discrete
  - Fixed number of possibilities
  - E.g.
    - Outcome of a coin/disease status

- Basis for the statistical tests

# Learning outcomes

- By the end of this session, you should be able to explain the methods of hypothesis testing

- By the end of this session, you should be able to test hypothesis in below situations
  - Categorical independent variable and continuous dependent variable
  - Categorical independent variable and categorical dependent variable
  - Continuous independent variable and continuous dependent variable
  - Continuous independent variable and categorical dependent variable

# Hypothesis testing

- Start with an idea/imaginary value
- Do a study and test it

- Two ways
  - Draw confidence intervals
  - Perform a test

# Inferential statistics

- Part 1: Independent variable : Categorical
- Part 2: Independent variable : Continuous

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# Part 1: Independent variable : Categorical

- What are the examples for categorical type independent variable?

- What are the examples for
  - Continuous type dependent (outcome) variables related to a categorical type independent variable?
  - Categorical type dependent (outcome) variables related to a categorical type independent variable?

# Independent variable : Categorical (2 groups)

- Continuous outcomes (e.g. FBS)
  - To estimate a mean (one sample)
  - To compare a mean with a given value (one sample)
  - To compare two or more means (two or more samples)

- Categorical outcomes (e.g. DM)
  - To estimate a proportion (one sample)
  - To compare a proportion with a given value (one sample)
  - To compare two or more proportion (two or more samples)

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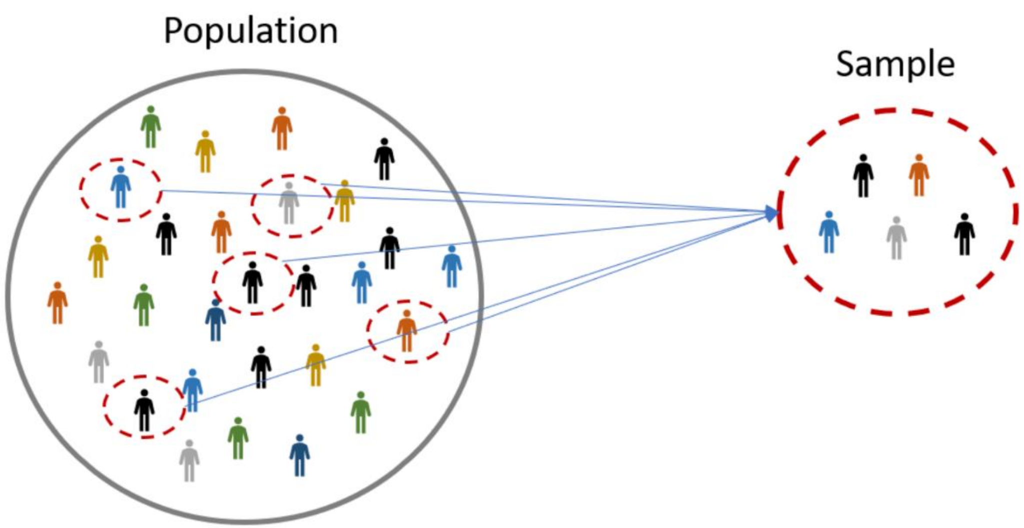
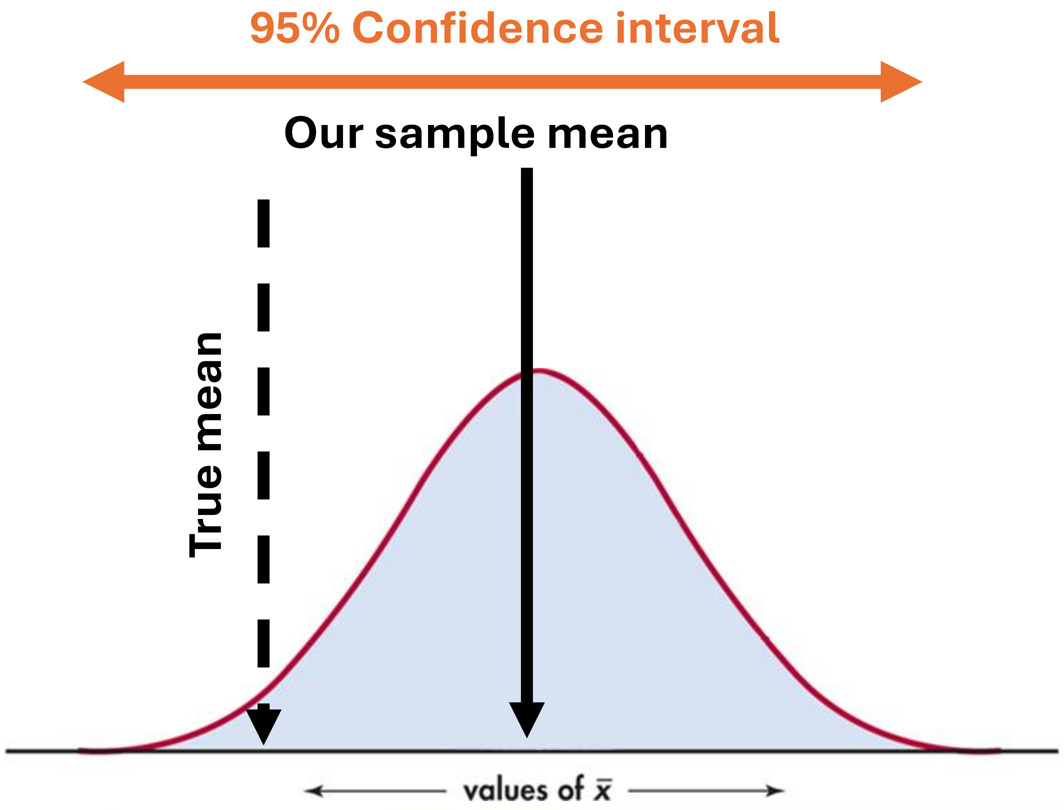
# One sample scenario



# One sample – Estimate population parameter

- Continuous outcomes (e.g. estimate mean FBS level in a population)

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$$

$$\bar{x} \pm z\frac{s}{\sqrt{n}}$$

Task:
Imagine that you have randomly sampled 10000 people. Assume the mean FBS of them is 115 and standard deviation is 20. Estimate the mean FBS value of the above population with 95% confidence. Hint: z = 1.96

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# One sample – Hypothesis testing

## Continuous outcomes (e.g. Mean FBS in a population is 112 mg/dl?)

- 95% CI:

$$\overline{x} \pm z\frac{s}{\sqrt{n}}$$

- One sample t test :
  - Null hypothesis $H_0$ : μ = m0
  - Alternative Hypothesis $H_1$: μ ≠ m0

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Task:
Does mean FBS in the previous example (i.e.in the population where the 10000 people were selected) equal to 112 mg/dl?

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t.test {stats}

# Student's t-Test

## Description

Performs one and two sample t-tests on vectors of data.

## Usage

```
t.test(x, ...)

## Default S3 method:
t.test(x, y = NULL,
       alternative = c("two.sided", "less", "greater"),
       mu = 0, paired = FALSE, var.equal = FALSE,
       conf.level = 0.95, ...)

## S3 method for class 'formula'
t.test(formula, data, subset, na.action, ...)
```

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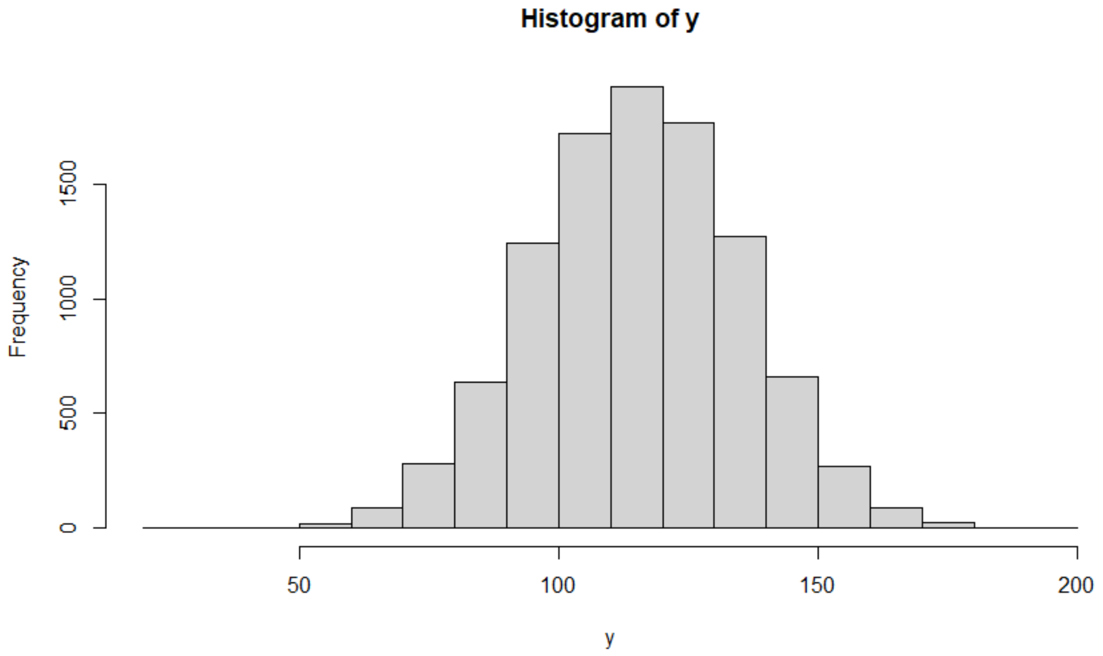
# Simulation example

```
y <- rnorm(10000,115,20)
```




```
> summary(y)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
   27.6   101.5   115.2   115.2   128.9   194.0 
```

```
> t.test(y,mu=112)

	One Sample t-test

data:  y
t = 15.373, df = 9999, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 112
95 percent confidence interval:
 114.6744 115.4561
sample estimates:
mean of x 
 115.0653 
```

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# What happens when sample size changes?

- Imagine this time you randomly sampled 100 people from SL.
- Assume the mean FBS is 115 and standard deviation is 20.
- Estimate the mean FBS value of the above population with 95% confidence.
- Does mean FBS in the previous example (i.e.in the population where the 100 people were selected) equal to 112 mg/dl?
- Hint: z = 1.96

```
> y <- rnorm(100,115,20)
> t.test(y,mu=112)

        One Sample t-test

data:  y
t = 0.33569, df = 99, p-value = 0.7378
alternative hypothesis: true mean is not equal to 112
95 percent confidence interval:
 108.8652 116.4114
sample estimates:
mean of x
 112.6383
```

# One sample – Estimate population parameter

- Binary outcomes (e.g. estimate the prevalence of DM in SL)

$$\hat{p} = \frac{x}{n}$$

$$SE = \sqrt{\frac{p(1-p)}{n}}$$

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$$

Task:
Imagine that you have randomly sampled 10000 people. Assume there are 3000 patients with diabetes. Estimate the prevalence of diabetes in the above population with 95% confidence. Hint: z = 1.96

# One sample – Hypothesis testing

Binary outcomes: (e.g. DM prevalence in a population is 25%?)

- 95% CI:

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$$

One sample proportion test

- Null hypothesis $H_0$: $p = p_0$
- Alternative Hypothesis $H_1$: $p \neq p_0$

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

Task:
Does diabetes prevalence in the previous example (i.e., in the population where the 10000 people were selected) equal 25%?

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# Simulation example

```
> prop.test(x=3000,n=10000,p=0.25)

        1-sample proportions test with continuity correction

data:  3000 out of 10000, null probability 0.25
X-squared = 133.07, df = 1, p-value < 2.2e-16
alternative hypothesis: true p is not equal to 0.25
95 percent confidence interval:
 0.2910470 0.3091075
sample estimates:
  p
0.3
```

```
> prop.test(x=30,n=100,p=0.25)

        1-sample proportions test with continuity correction

data:  30 out of 100, null probability 0.25
X-squared = 1.08, df = 1, p-value = 0.2987
alternative hypothesis: true p is not equal to 0.25
95 percent confidence interval:
 0.2145426 0.4010604
sample estimates:
  p
0.3
```

prop.test {stats} R Documentation

## Test of Equal or Given Proportions

### Description

`prop.test` can be used for testing the null that the proportions (probabilities of success) in several groups are the same, or that they equal certain given values.

### Usage

```
prop.test(x, n, p = NULL,
          alternative = c("two.sided", "less", "greater"),
          conf.level = 0.95, correct = TRUE)
```

Hint:
n = number of trials
x = number of successes
p = $H_0$

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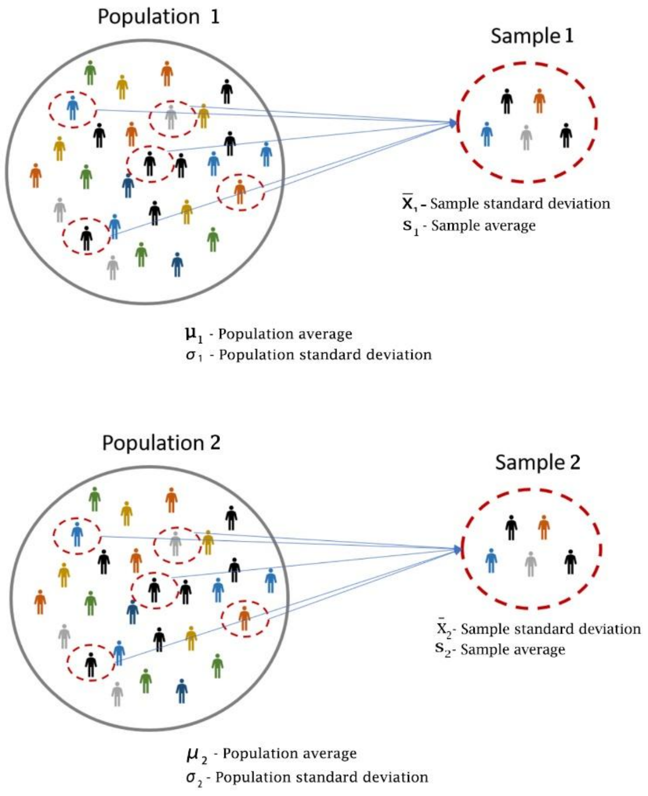
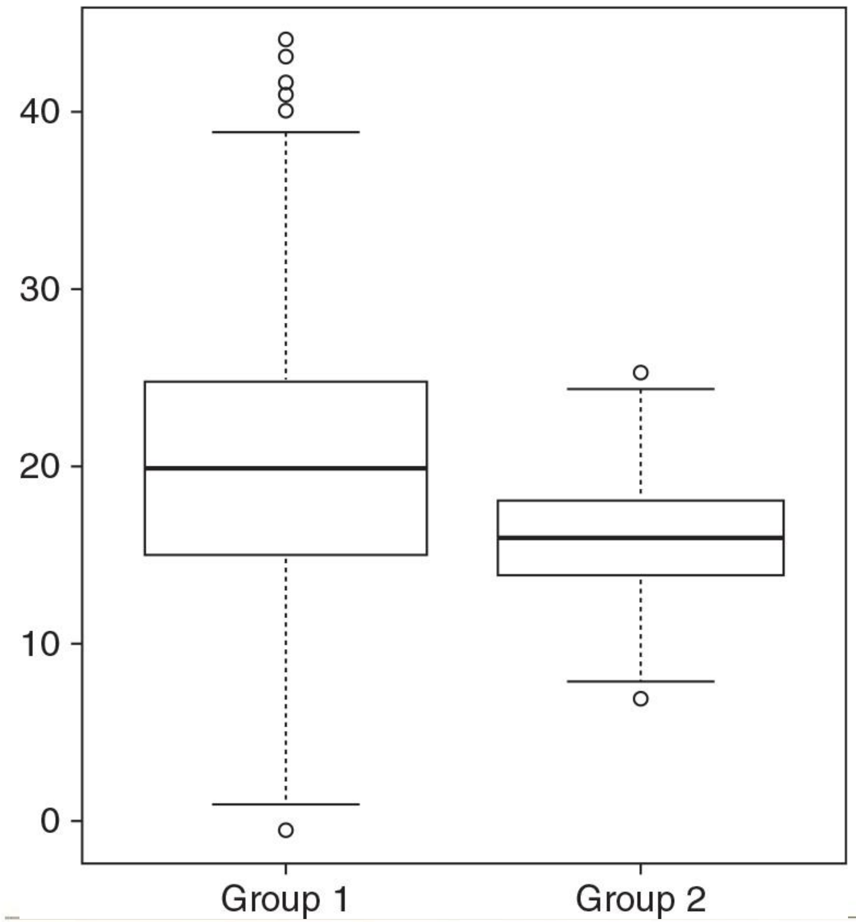
# Two samples



Population 1
Sample 1
$\bar{X}_1$ - Sample standard deviation
$S_1$ - Sample average
$\mu_1$ - Population average
$\sigma_1$ - Population standard deviation
Population 2
Sample 2
$\bar{X}_2$- Sample standard deviation
$S_2$- Sample average
$\mu_2$ - Population average
$\sigma_2$ - Population standard deviation

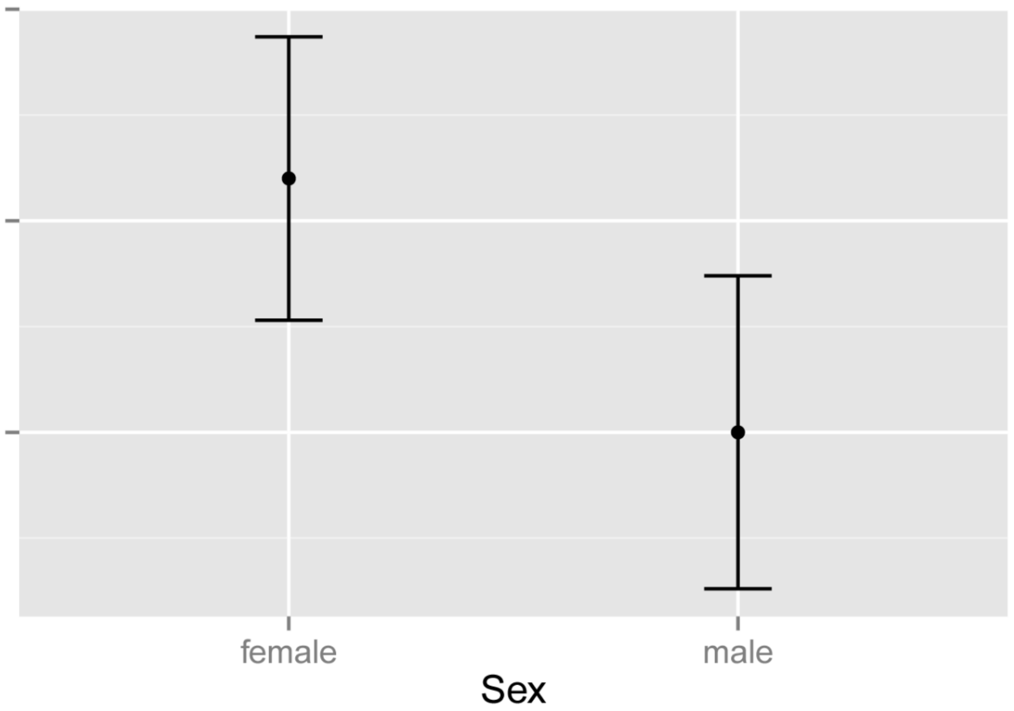
0
10
20
30
40
Group 1
Group 2

# Two sample – Hypothesis testing by 95%CI

- Continuous variables:
  - Example?

- How?
  - Construct 95% CI for both groups and see for overlaps

$$\bar{x} \pm z\frac{s}{\sqrt{n}}$$




Task:
Imagine that you have randomly sampled 100 people from two places. Assume the mean FBS readings are 115 (SD = 20) and 100 (SD = 17) in sample 1 and 2 respectively. Are the mean values different? Hint: z = 1.96

# Two samples – Hypothesis testing by test

## Continuous variables

- Null hypothesis $H_0 : \mu_1 = \mu_2$
- Alternative Hypothesis $H_1 : \mu_1 \neq \mu_2$

- Independent or related (paired) samples
  - Independent sample t test (different samples)
  - Paired t test (one group - pre and post data)

- Equal or unequal variances between groups
  - If equal : Pooled t test (exact t distribution)
  - If unequal : Welch's t test

# Simulation example

```
> y1 <- rnorm(100,25,10)
> y2 <- rnorm(100,30,10)

> summary(y1)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 -0.385  17.832  25.423  25.193  32.750  51.133

> summary(y2)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  4.646  23.948  30.129  29.228  36.211  47.921

> var.test(y1,y2)

        F test to compare two variances

data:  y1 and y2
F = 1.4029, num df = 99, denom df = 99, p-value = 0.09373
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.9439417 2.0850641
sample estimates:
ratio of variances
          1.402918
```

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t.test {stats}

# Student's t-Test

## Description

Performs one and two sample t-tests on vectors of data.

## Usage

```
t.test(x, ...)

## Default S3 method:
t.test(x, y = NULL,
       alternative = c("two.sided", "less", "greater"),
       mu = 0, paired = FALSE, var.equal = FALSE,
       conf.level = 0.95, ...)

## S3 method for class 'formula'
t.test(formula, data, subset, na.action, ...)
```

```
> t.test(y1,y2)

        Welch Two Sample t-test

data:  y1 and y2
t = -2.7886, df = 192.59, p-value = 0.005825
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -6.888075 -1.180903
sample estimates:
mean of x mean of y 
 25.19331  29.22780 

> t.test(y1,y2,var.equal = T)

        Two Sample t-test

data:  y1 and y2
t = -2.7886, df = 198, p-value = 0.00581
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -6.887581 -1.181396
sample estimates:
mean of x mean of y 
 25.19331  29.22780 

> t.test(y1,y2, paired = T)

        Paired t-test

data:  y1 and y2
t = -2.8289, df = 99, p-value = 0.005656
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -6.864361 -1.204617
sample estimates:
mean of the differences 
              -4.034489 
```

t.test {stats}

# Student's t-Test

## Description

Performs one and two sample t-tests on vectors of data.

## Usage

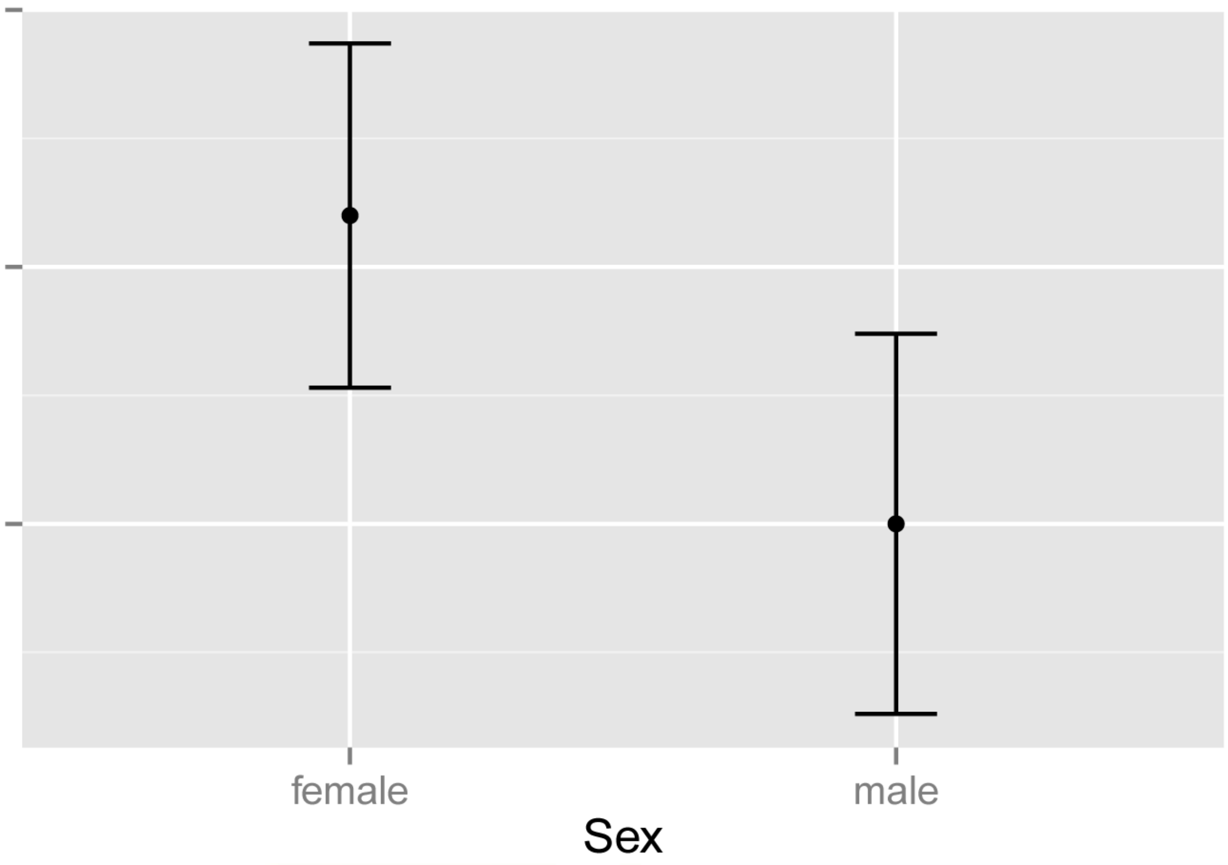
```
t.test(x, ...)

## Default S3 method:
t.test(x, y = NULL,
       alternative = c("two.sided", "less", "greater"),
       mu = 0, paired = FALSE, var.equal = FALSE,
       conf.level = 0.95, ...)

## S3 method for class 'formula'
t.test(formula, data, subset, na.action, ...)
```

# Two sample – Hypothesis testing by 95%CI

- Binary outcomes:
  - Example?

- How?
  - 95% CI for both groups and see for overlaps

$$\hat{p} \pm z\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$$



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# Two samples – Hypothesis testing by test

## Binary outcomes

Two (Independent) sample proportion test

- Null hypothesis $H_0$: $P_1 = P_2$
- Alternative Hypothesis $H_1$: $P_1 \neq P_2$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

McNemar's test (paired)

- Null hypothesis $H_0$: $P_1 = P_2$
- Alternative Hypothesis $H_1$: $P_1 \neq P_2$

$$\chi^2 = \frac{(b-c)^2}{b+c}$$

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# Simulation example

```
> prop.test(x = c(45,50), n = c(100,100))

	2-sample test for equality of proportions with continuity correction

data:  c(45, 50) out of c(100, 100)
X-squared = 0.3208, df = 1, p-value = 0.5711
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.19824347  0.09824347
sample estimates:
prop 1 prop 2 
  0.45   0.50 
```

```
> prop.test(x = c(30,50), n = c(100,100))

	2-sample test for equality of proportions with continuity correction

data:  c(30, 50) out of c(100, 100)
X-squared = 7.5208, df = 1, p-value = 0.006099
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.34293122 -0.05706878
sample estimates:
prop 1 prop 2 
   0.3    0.5 
```

prop.test {stats} R Documentation

## Test of Equal or Given Proportions

### Description

`prop.test` can be used for testing the null that the proportions (probabilities of success) in several groups are the same, or that they equal certain given values.

### Usage

```
prop.test(x, n, p = NULL,
          alternative = c("two.sided", "less", "greater"),
          conf.level = 0.95, correct = TRUE)
```

Hint:

x = number of successes (2 groups)

n = number of trials (2 groups)

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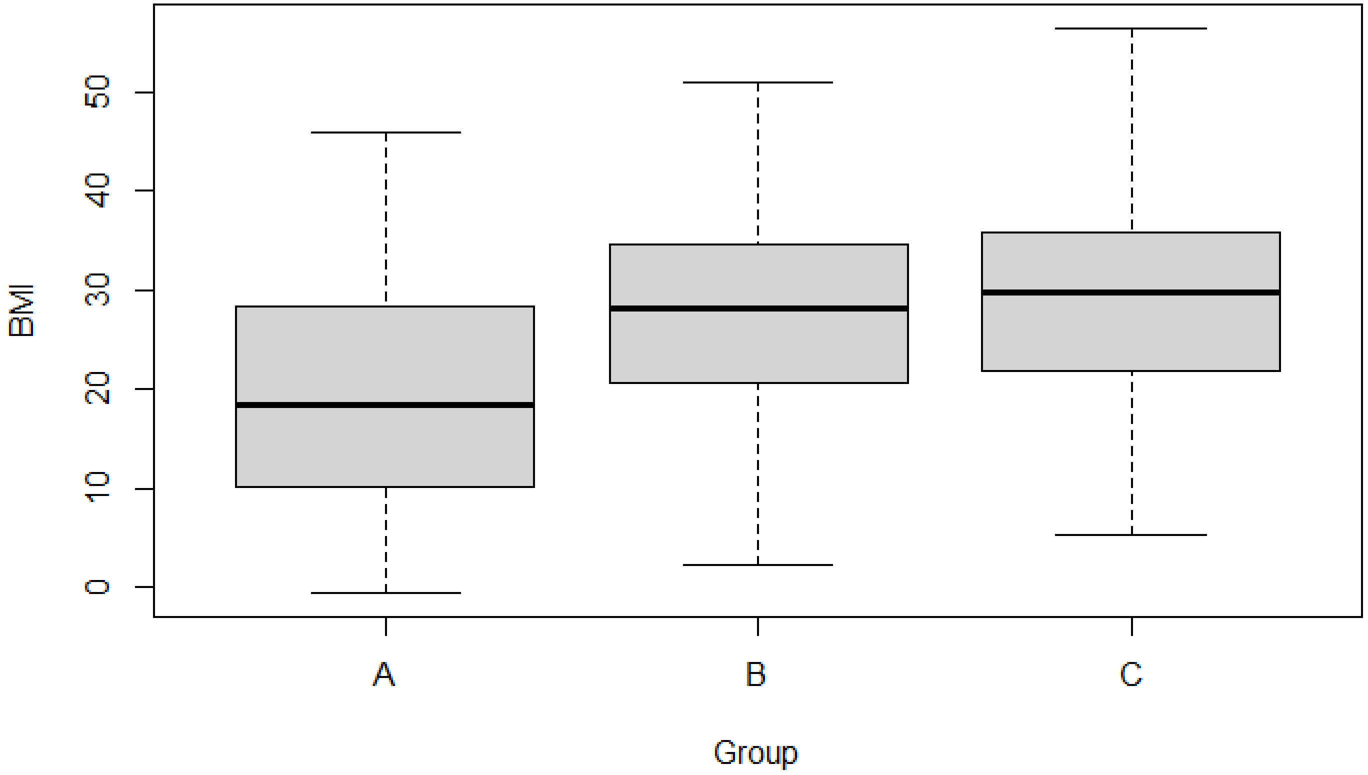
# More than two samples/groups



# More than two samples

- Construct 95% CI and see for overlaps

- Do a test
  - Independent samples : ANOVA and Post Hoc test
  - Dependent samples : Repeated measures ANOVA (Longitudinal data analysis)

# Analysis of variability

- Total variability = Variability due to treatment + Natural variability

- How to calculate?

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# ANOVA table

| Source | df | SS | MS | F |
|---|---|---|---|---|
| Model | p | $\sum(\hat{y}_i - \bar{y})^2$ | $SS_{Model}/df_{Model}$ | $MS_{Model}/MS_{Error}$ |
| Error | N – p - 1 | $\sum(y_i - \hat{y}_i)^2$ | $SS_{Error}/df_{Error}$ | |
| Total | N - 1 | $\sum(y_i - \bar{y})^2$ | | |

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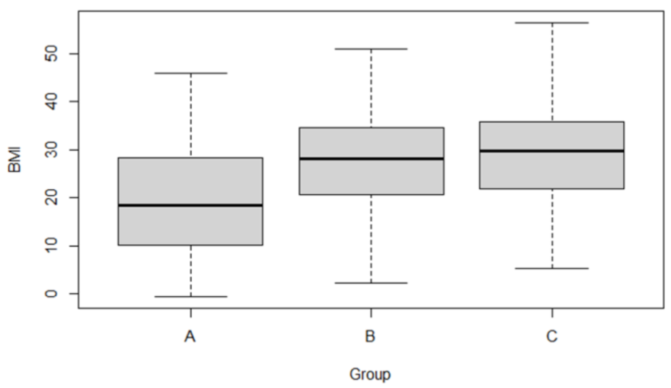
# Simulation example

```
y1 <- rnorm(100,20,10)
y2 <- rnorm(100,28,10)
y3 <- rnorm(100,30,10)

data <- data.frame(group = c(rep("A",100), rep("B",100),rep("C",100)),
                   BMI = c(y1,y2,y3))
```

```
> aov.out <- aov(data$BMI ~ data$group)
> summary(aov.out)
             Df Sum Sq Mean Sq F value   Pr(>F)    
data$group    2   5995  2997.5   28.12 6.56e-12 ***
Residuals   297  31663   106.6                     
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```




```
> TukeyHSD(aov.out)
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = data$BMI ~ data$group)

$`data$group`
         diff       lwr       upr     p adj
B-A  8.732144  5.292587 12.171702 0.0000000
C-A 10.087824  6.648266 13.527381 0.0000000
C-B  1.355679 -2.083878  4.795237 0.6227532
```

- Categorical outcomes
  - Give an example?
  - Pearson's chi-square test

| Exposure | Disease status | | |
|---|---|---|---|
| | Disease | No Disease | |
| Exposed | **12** | **12** | 24 |
| Unexposed | **11** | **32** | 43 |
| | 25 | 36 | 67 |

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$\chi^2$ = the test statistic $\sum$ = the sum of

O = Observed frequencies E = Expected frequencies

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# Simulation example

chisq.test {stats} R Documentation

## Pearson's Chi-squared Test for Count Data

### Description

`chisq.test` performs chi-squared contingency table tests and goodness-of-fit tests.

### Usage

```
chisq.test(x, y = NULL, correct = TRUE,
           p = rep(1/length(x), length(x)), rescale.p = FALSE,
           simulate.p.value = FALSE, B = 2000)
```

```
Smoking <- sample(c("Yes","No"),100,replace = T)
Cancer  <- sample(c("Yes","No"),100,replace = T)
```

```
> table(Smoking ,Cancer)
       Cancer
Smoking No Yes
    No  18  16
    Yes 32  34
```

```
> prop.table(table(Smoking ,Cancer),1)*100
       Cancer
Smoking       No      Yes
    No  52.94118 47.05882
    Yes 48.48485 51.51515
```

```
> chisq.test(table(Smoking ,Cancer))

	Pearson's Chi-squared test with Yates' continuity correction

data:  table(Smoking, Cancer)
X-squared = 0.044563, df = 1, p-value = 0.8328
```

- Categorical outcomes (If sample size is very small in any cell : eg <5)
    - Fisher` s exact test

```
Smoking <- sample(c("Yes","No"),100,replace = T)
Cancer  <- sample(c("Yes","No"),100,prob = c(0.1,0.9),replace = T)
table(Smoking ,Cancer)
```

```
> table(Smoking ,Cancer)
       Cancer
Smoking No Yes
    No  32   2
    Yes 55  11
```

```
> prop.table(table(Smoking ,Cancer),1)*100
       Cancer
Smoking        No       Yes
    No  94.117647  5.882353
    Yes 83.333333 16.666667
```

```
> chisq.test(table(Smoking ,Cancer))

        Pearson's Chi-squared test with Yates'

data:  table(Smoking, Cancer)
X-squared = 1.4525, df = 1, p-value = 0.2281

Warning message:
In chisq.test(table(Smoking, Cancer)) :
  Chi-squared approximation may be incorrect
> fisher.test(table(Smoking ,Cancer))

        Fisher's Exact Test for Count Data

data:  table(Smoking, Cancer)
p-value = 0.209
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
  0.631179 31.207926
sample estimates:
odds ratio
  3.168927
```

# More than 2 groups

| Surgical Apgar Score | No morbidity | Minor morbidity | Major morbidity or mortality |
|---|---|---|---|
| 0-4 | 21 | 20 | 16 |
| 5-6 | 135 | 71 | 35 |
| 7-10 | 158 | 62 | 35 |

- Still can do a Pearson's chi-square test
  - This is a global test (like ANOVA)
  - If there is no difference – then ok
  - If there is a difference – need further analysis (loglinear models)

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# Session 3: Exploring Relationships and Prediction

Inferential statistics : Part II

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# Independent variable : Continuous

- What are the examples for continuous type independent variable?
    - What are the examples for continuous type dependent (outcome) variables related to a continuous type independent variable?
    - What are the examples for categorical type dependent (outcome) variables related to a continuous type independent variable?

# Independent variable : Continuous

- Continuous outcomes (e.g. FBS with age)
  - Correlation analysis
  - Linear regression analysis

- Categorical outcomes (e.g. DM prevalence with age)
  - Logistic regression
  - Multinomial regression

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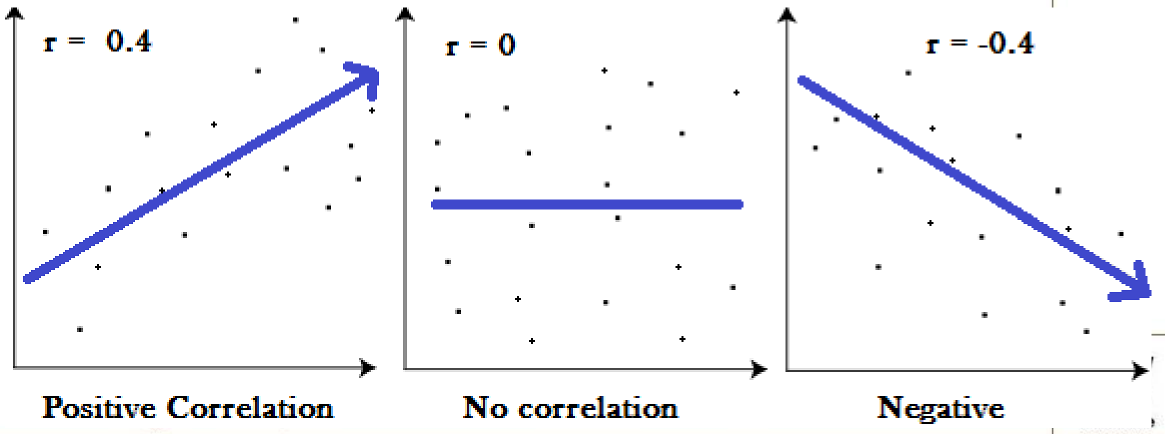
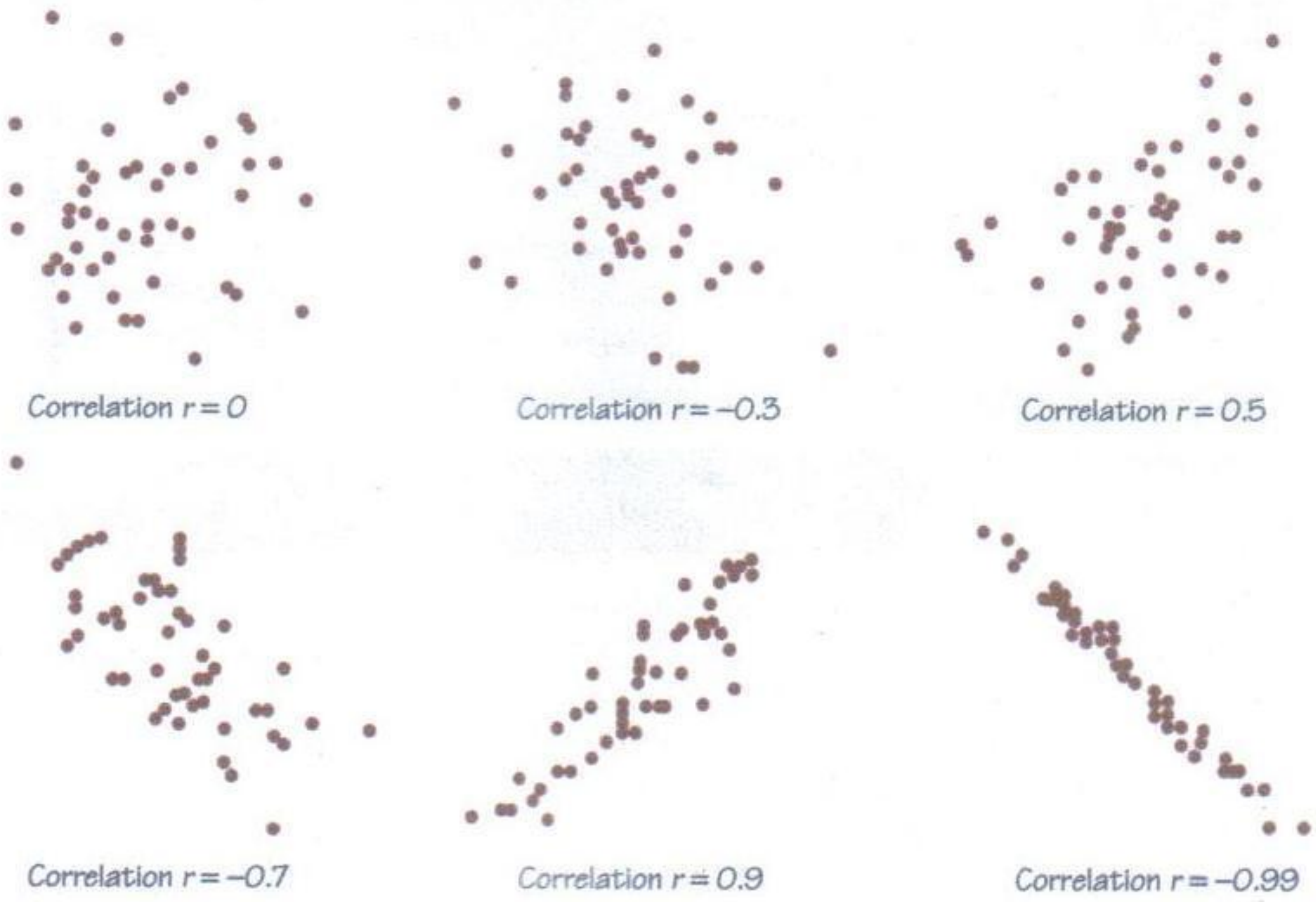
# Pearson's correlation (Pearson's R)

- Strength of relationship (How strong a relationship is ?)
- Not to find the relationship (by regression)
- Correlation coefficient

$$r_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$$



# Correlation



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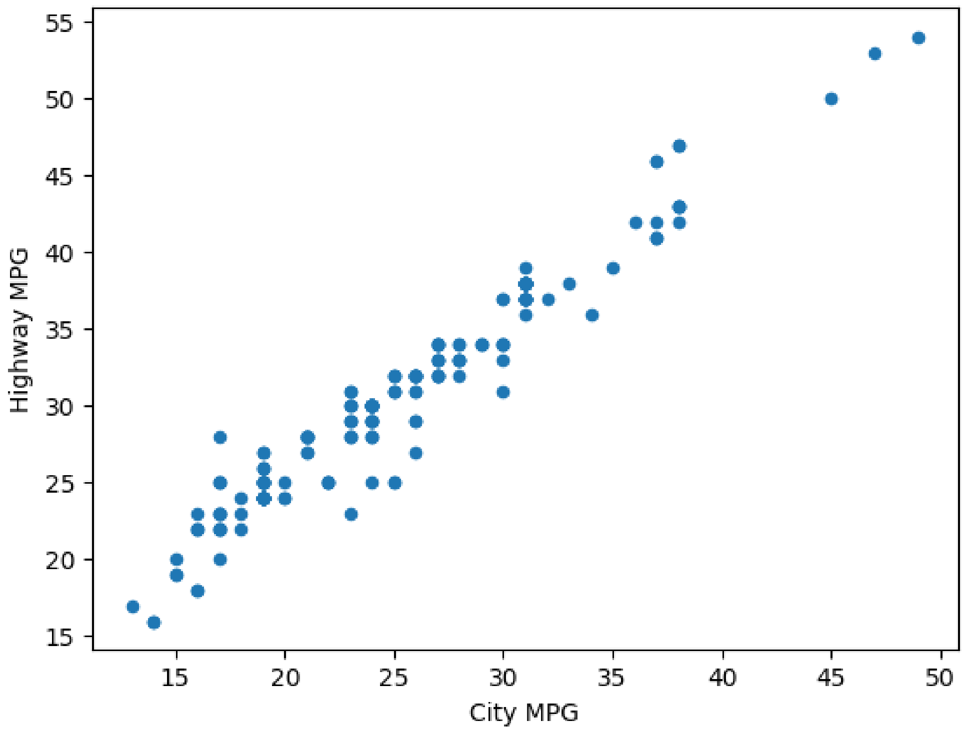
# Regression analysis

- Finding the relationship between variables
- Why we need to know?
    - To predict



# Types

- Linear regression
- Binary logistic regression
- Poisson regression (loglinear models)
- Cox proportional hazard models

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# Regression analysis

- Linear regression makes several key assumptions:
  - Linear relationship
  - Multivariate normality
  - No or little multicollinearity (ie. independent)
  - No auto-correlation (value of y(x+1) is not dependent on the value of y(x))
  - Homoscedasticity (that is the error terms along the regression are equal)

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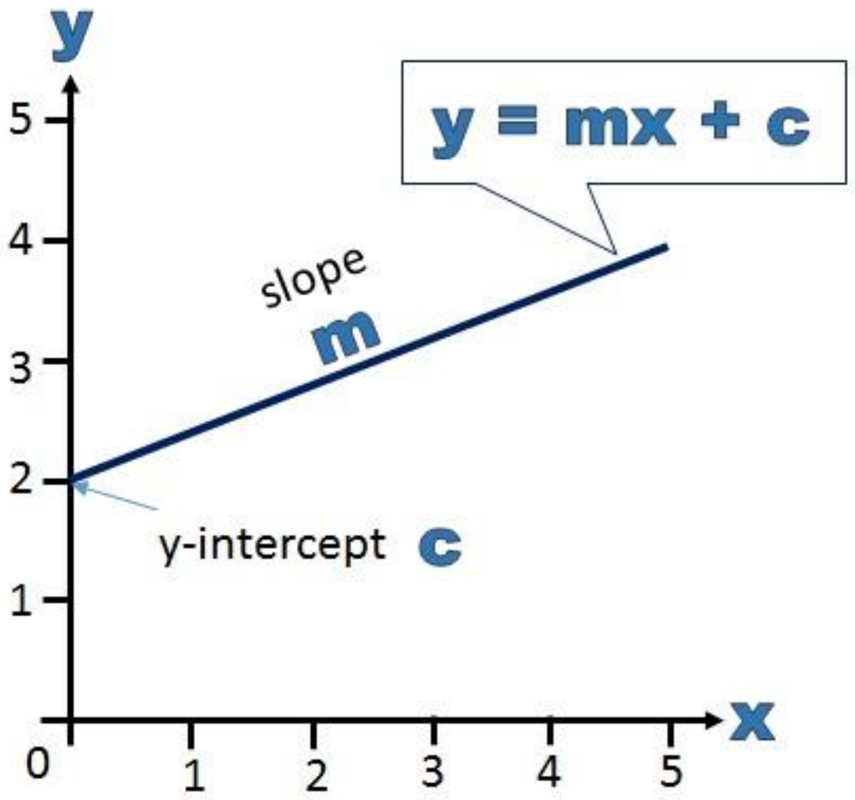
# Mathematics

- How to write this mathematically?
  - $y_i = m\, x_i + c$



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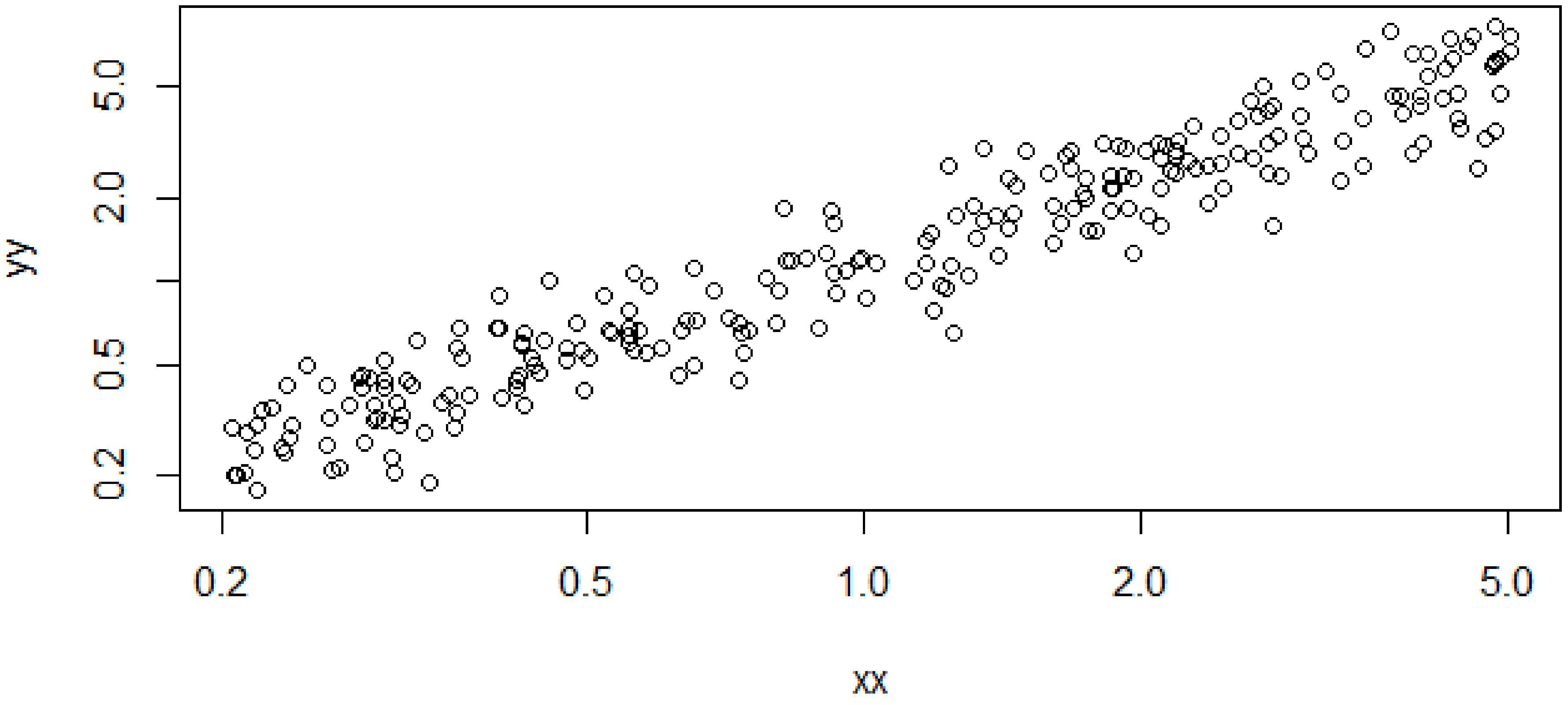
# Real world example



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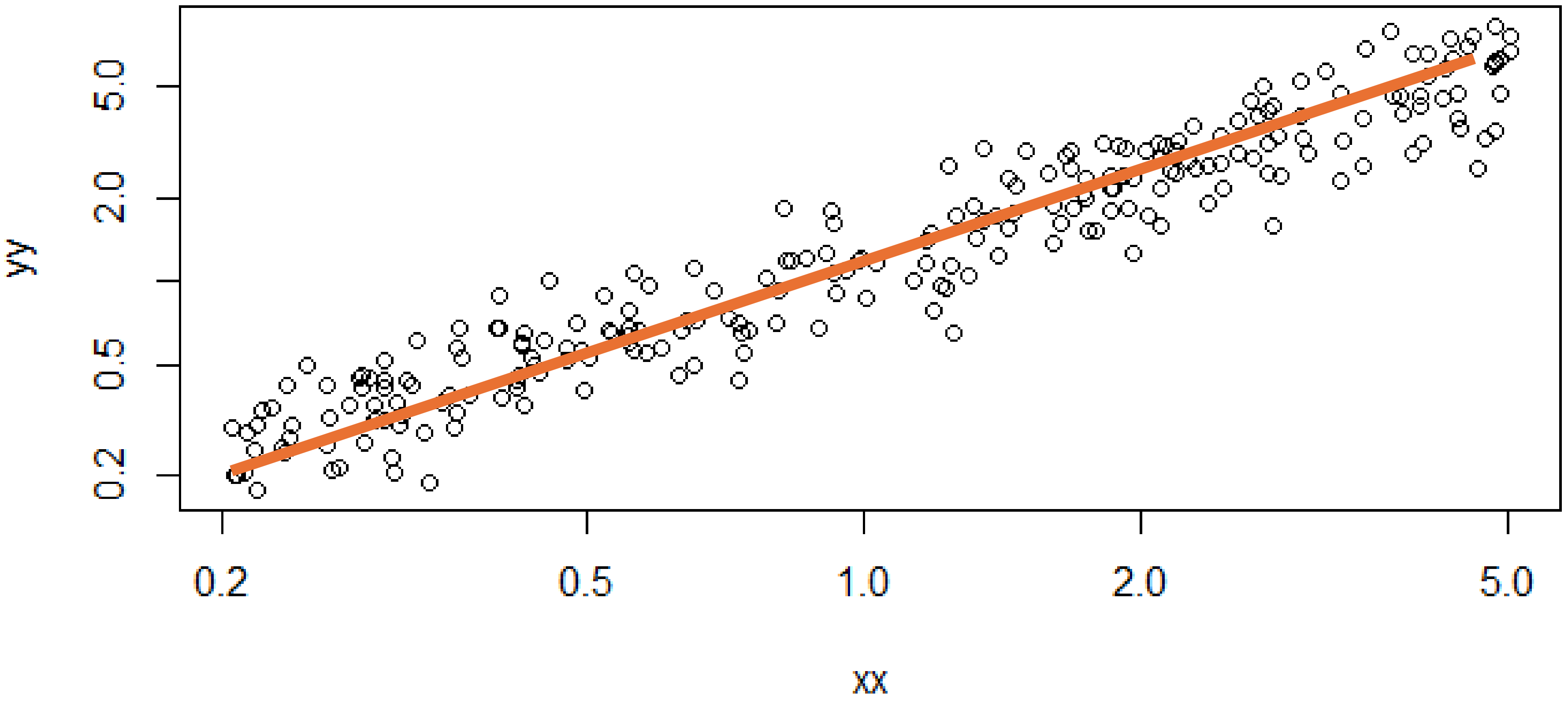
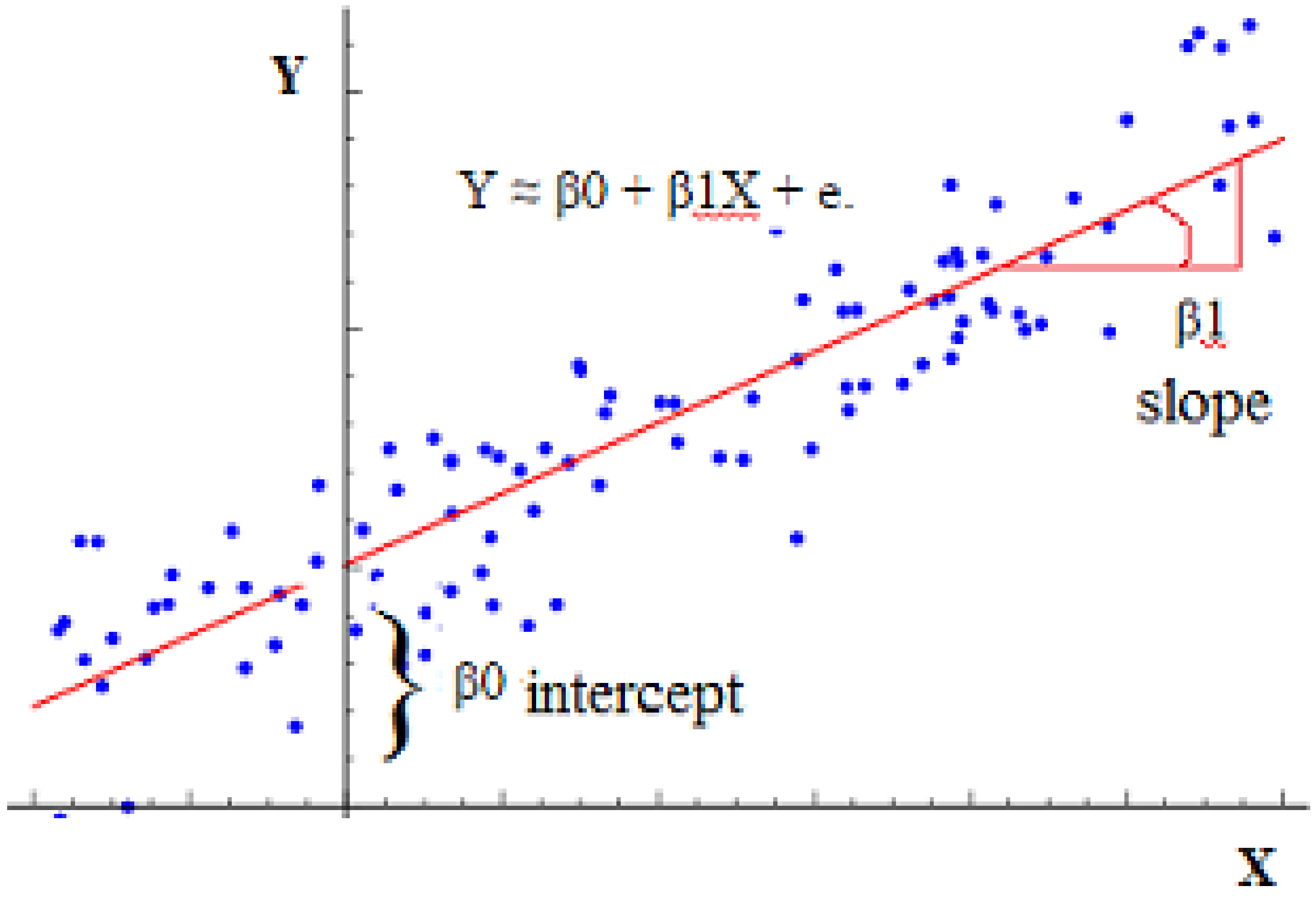
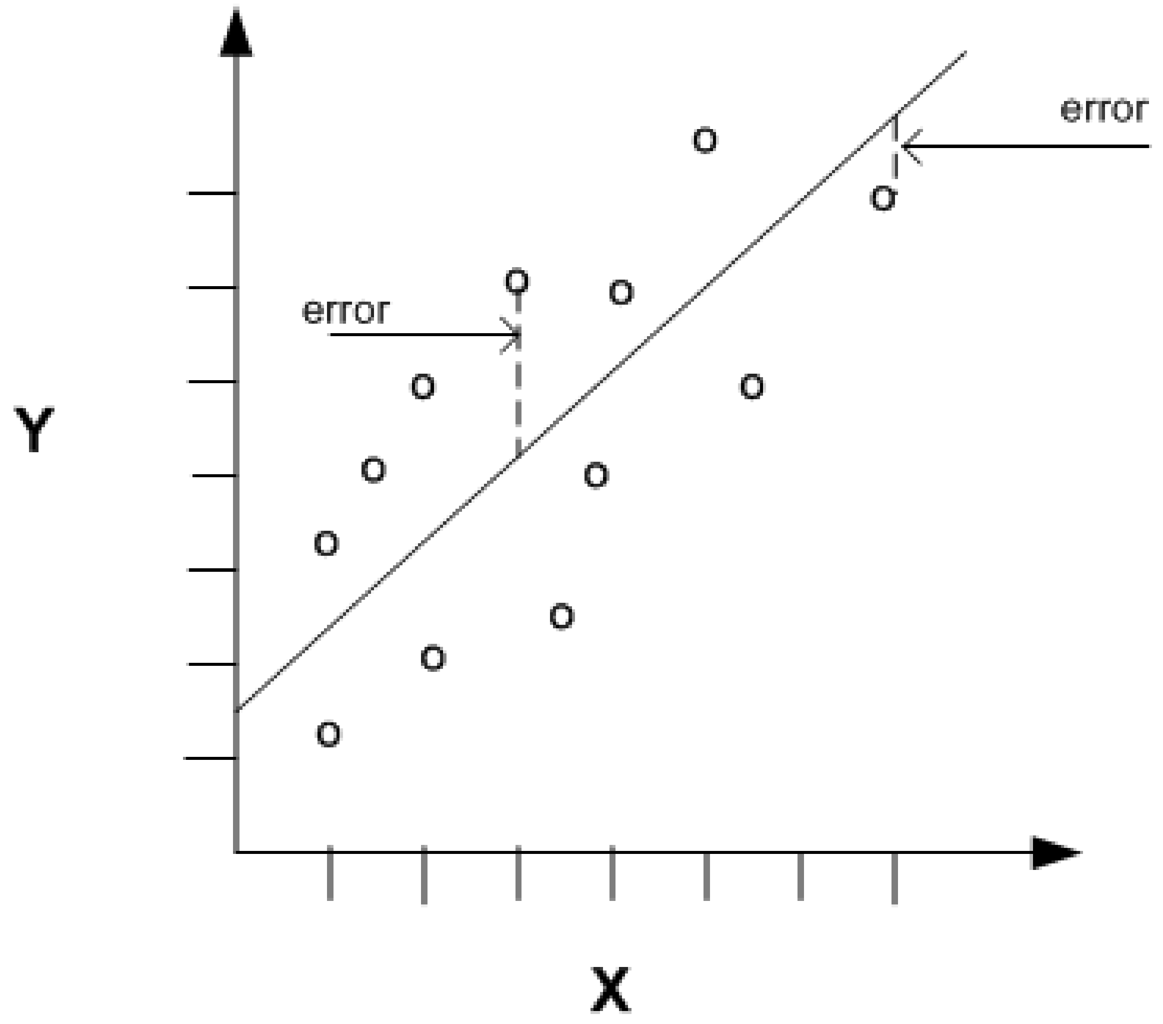
# Real world example




Y
Y ≈ β0 + β1X + e.
β1
slope
β0 intercept
X


error
error
Y
X

# Conceptual model:

- Population regression model

Dependent Variable

Population Y intercept

Population Slope Coefficient

Independent Variable

Random Error term

$$Y_i = \underbrace{\beta_0 + \beta_1 X_i}_{\text{Linear component}} + \underbrace{\varepsilon_i}_{\text{Random Error component}}$$

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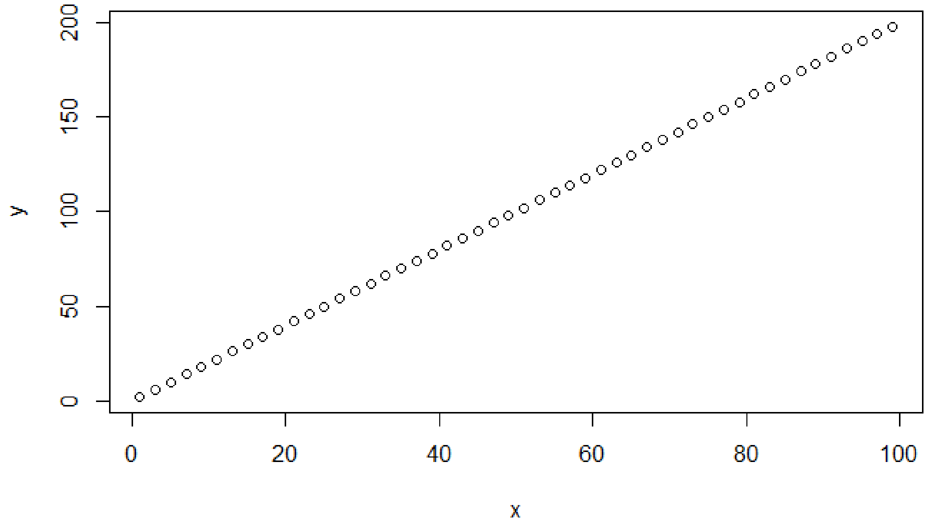
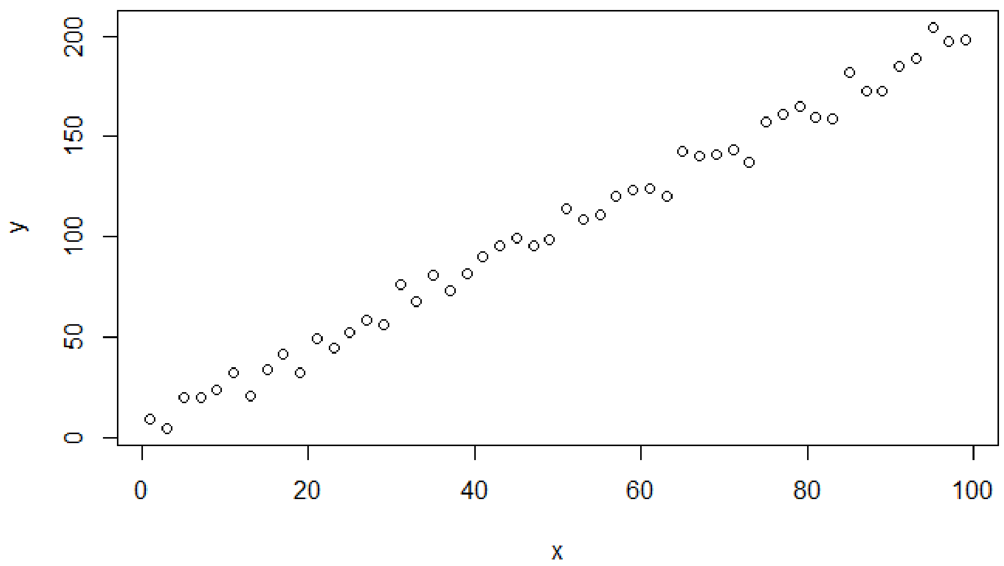
# Simulation example

```
x <- seq(1,100,by=2)
y <- 2*x
plot(x,y)
```

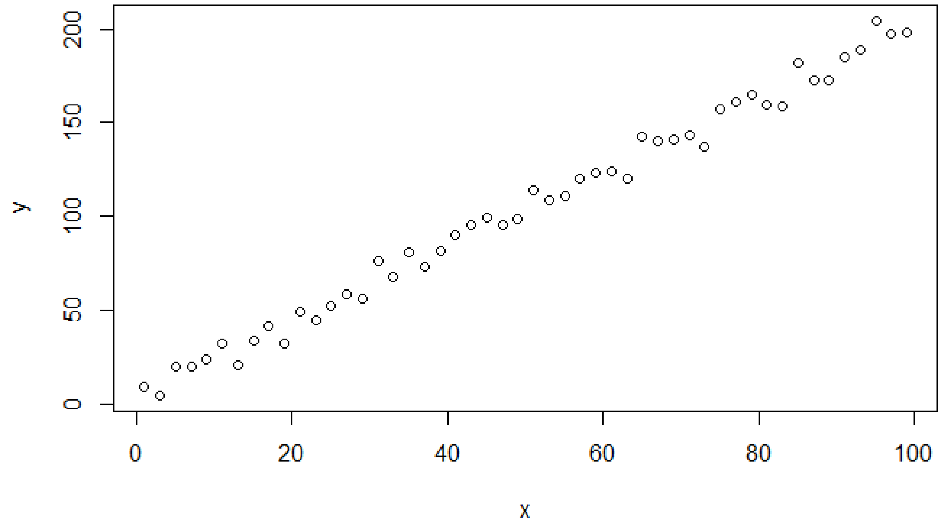
```
x <- seq(1,100,by=2)
y <- 2*x + rnorm(50,5,5)
plot(x,y)
```



# Simulation example

```r
x <- seq(1,100,by=2)
y <- 2*x + rnorm(50,5,5)
plot(x,y)
```




```
> summary(lm(y~x))

Call:
lm(formula = y ~ x)

Residuals:
    Min      1Q  Median      3Q     Max 
-7.8008 -2.9175 -0.0975  2.9962  9.9960 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4.18140    1.29732   3.223  0.00228 ** 
x            2.01687    0.02247  89.753  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.586 on 48 degrees of freedom
Multiple R-squared:  0.9941,	Adjusted R-squared:  0.994 
F-statistic:  8056 on 1 and 48 DF,  p-value: < 2.2e-16
```

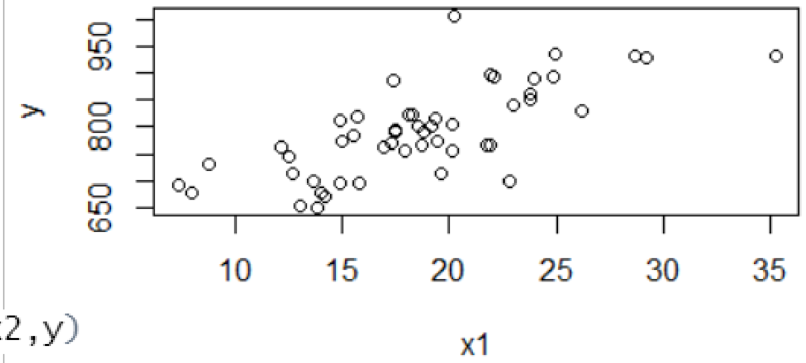
# Expanding simple linear regression

| | |
|---|---|
| Simple Linear Regression | $y = b_0 + b_1 x_1$ |
| Multiple Linear Regression | $y = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_n x_n$ |
| Polynomial Linear Regression | $y = b_0 + b_1 x_1 + b_2 x_1^2 + \ldots + b_n x_1^n$ |

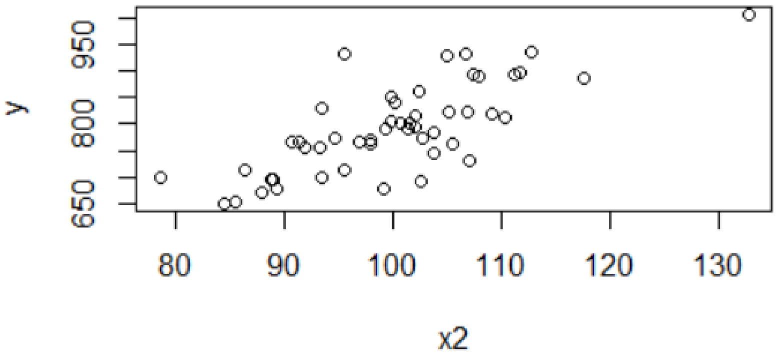
# Example - multiple linear regression

```
x1 <- rnorm(50,20,5)
x2 <- rnorm(50,100,10)
y <- 10*x1 + 6*x2 + rnorm(50,5,5)
```

```
plot(x1,y)
```



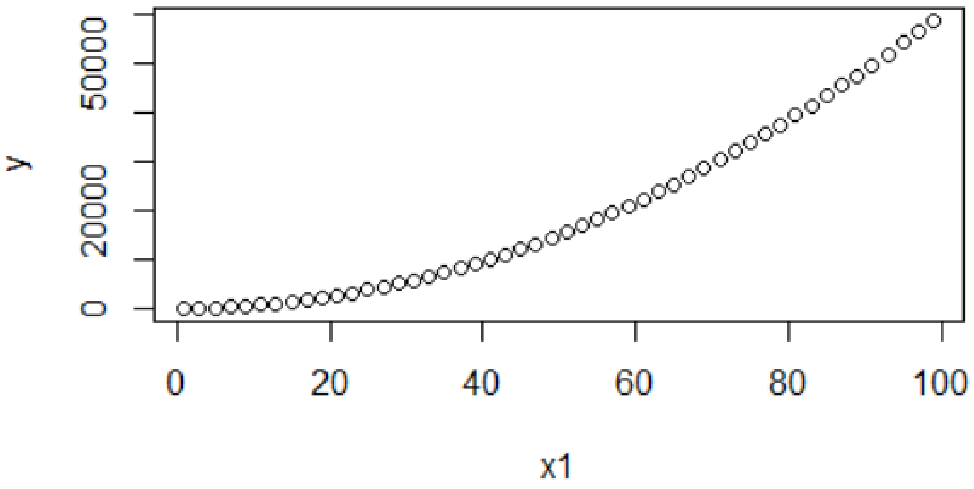

```
plot(x2,y)
```




```
> summary(lm(y~x1+x2))

Call:
lm(formula = y ~ x1 + x2)

Residuals:
    Min      1Q  Median      3Q     Max 
-7.9808 -3.0874 -0.0921  2.3243 11.4903 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.70147    6.81788    0.25    0.804    
x1          10.07570    0.11660   86.41   <2e-16 ***
x2           6.02172    0.06662   90.38   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.412 on 47 degrees of freedom
Multiple R-squared:  0.9973,	Adjusted R-squared:  0.9972 
F-statistic:  8728 on 2 and 47 DF,  p-value: < 2.2e-16
```

# Example – polynomial linear regression

```
x1 <- seq(1,100,by=2)
x2 <- x1^2
y <- 2*x1 + 6*x2 + rnorm(50,5,5)
plot(x1,y)
```

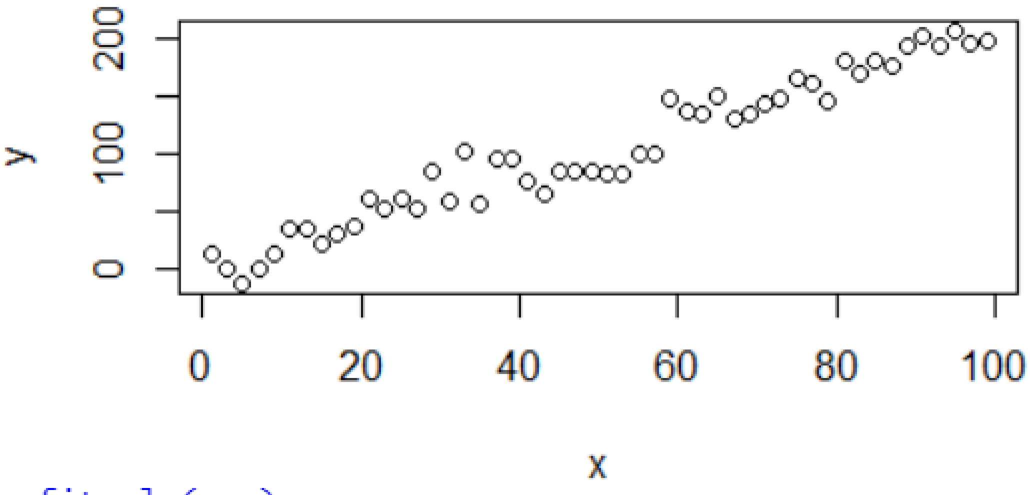



```
> summary(lm(y~x1+x2))

Call:
lm(formula = y ~ x1 + x2)

Residuals:
     Min       1Q   Median       3Q      Max 
-15.1871  -3.3959   0.3643   3.7373  11.6549 

Coefficients:
            Estimate Std. Error  t value Pr(>|t|)    
(Intercept) 8.266053   2.256018    3.664  0.00063 ***
x1          1.799061   0.104224   17.262  < 2e-16 ***
x2          6.001591   0.001009 5946.932  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 5.314 on 47 degrees of freedom
Multiple R-squared:      1,     Adjusted R-squared:      1 
F-statistic: 2.849e+08 on 2 and 47 DF,  p-value: < 2.2e-16
```

# Goodness of a fit

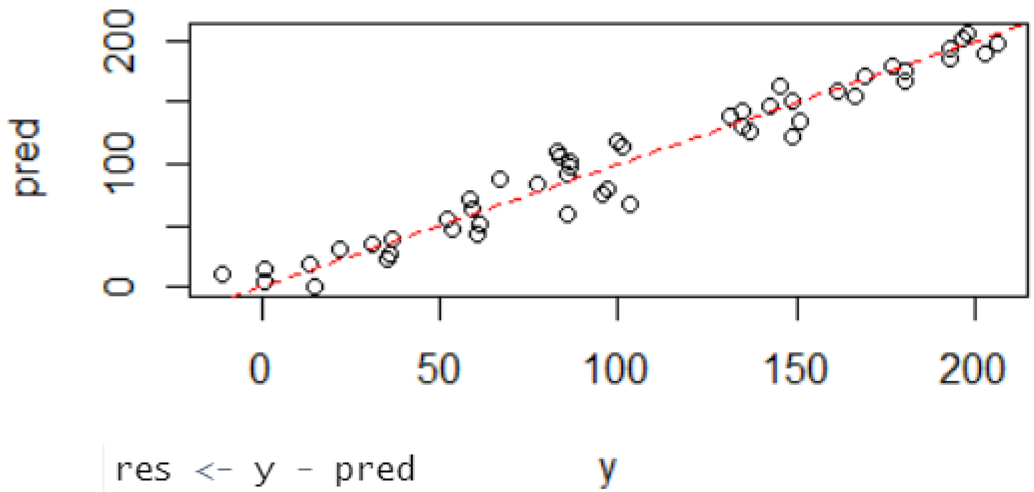
- Outcome vs predictors (independent variables)
- Outcome vs fitted value plot
- Residual vs fitted value plot
- Coefficient of determination ($R^2$ )
- Deviance statistic
- Autocorrelation plot

```
x <- seq(1,100,by=2)
y <- 2*x + rnorm(50,5,15)
plot(x,y)
```

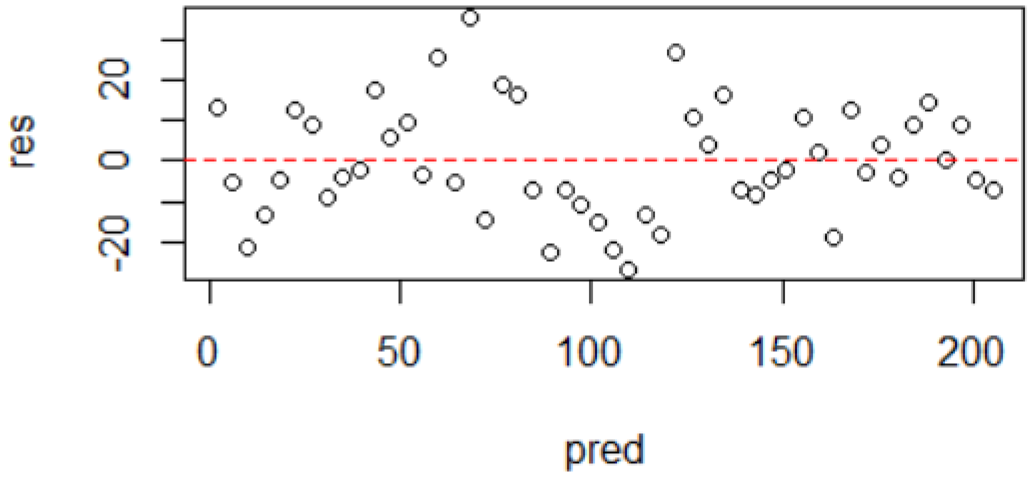


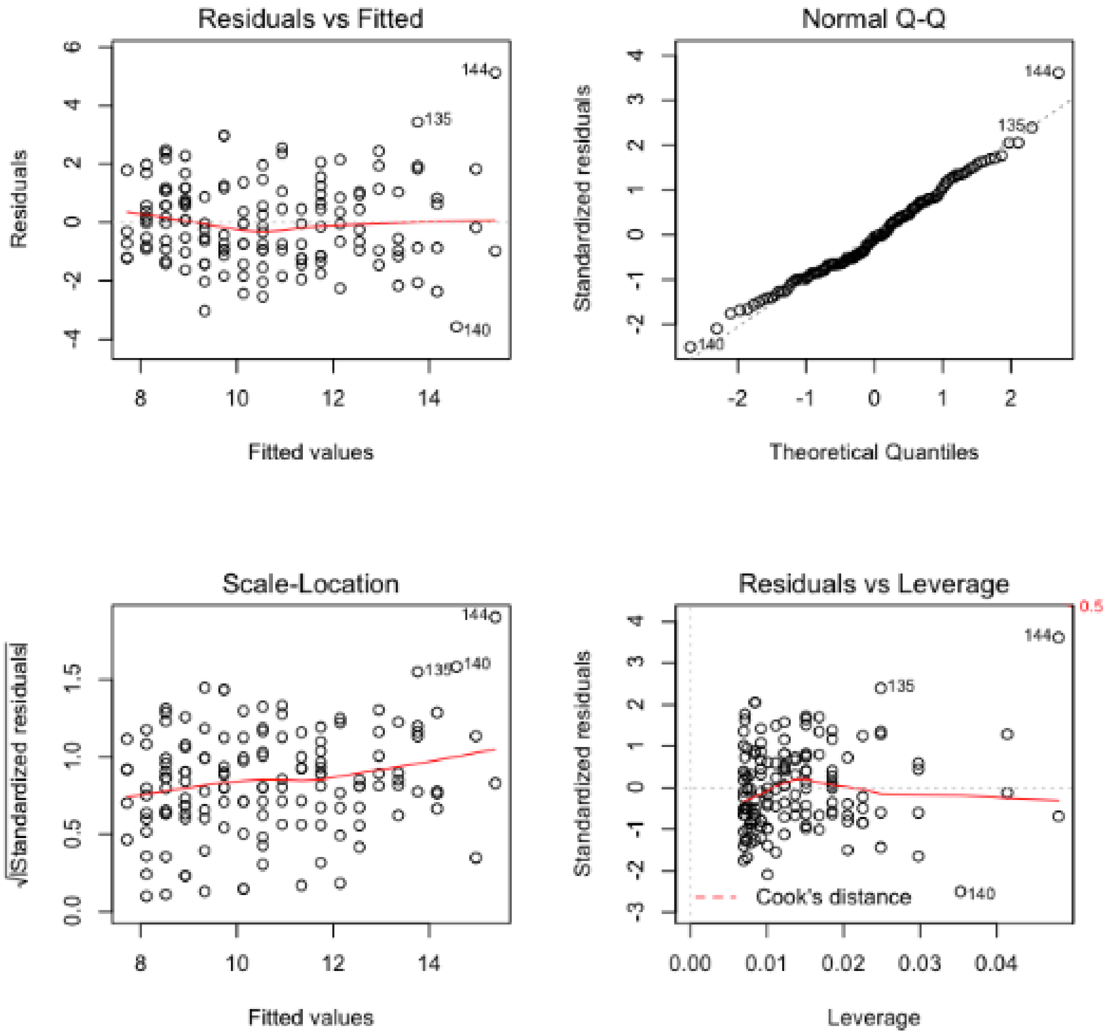

```
> fit<-lm(y~x)
> summary(fit)

Call:
lm(formula = y ~ x)

Residuals:
    Min      1Q  Median      3Q     Max 
-26.541  -8.061  -3.050  10.570  35.286 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.51530    3.95677   -0.13    0.897    
x            2.07919    0.06854   30.34   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 13.99 on 48 degrees of freedom
Multiple R-squared:  0.9504,	Adjusted R-squared:  0.9494 
F-statistic: 920.3 on 1 and 48 DF,  p-value: < 2.2e-16
```

```
pred <- -0.51530+2.07919*x
plot(y, pred)
abline(0,1,col="Red",lty="dashed")
```




```
res <- y - pred
plot(pred,res)
```




Residuals vs Fitted
Residuals
Fitted values
Normal Q-Q
Standardized residuals
Theoretical Quantiles
Scale-Location
√|Standardized residuals|
Fitted values
Residuals vs Leverage
Standardized residuals
Cook's distance
Leverage
0.5
144
135
140

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# Logistic regression

- Outcome – binary (1 or 0)
- Need to transform the outcome variable (continuous variable)

```
> log(0)
[1] -Inf
> 1/0
[1] Inf
> log(Inf)
[1] Inf
```

$$\text{Logit Function} = \log\left(\frac{p}{1-p}\right)$$

# Logistic regression

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X$$

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k$$

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# Example : obtaining predictions

**Table 5** Fitted linear logistic regression model to predict the probability of having OV on screening UGIE

| Variable | Estimate | Std error | Z value | *P* value |
|---|---|---|---|---|
| Intercept | –0.189 | 0.652 | –0.290 | 0.771 |
| Small platelet count | –0.046 | 0.015 | –0.310 | 0.002 |
| CTP class B (compered to Class A) | 2.852 | 0.944 | 3.021 | 0.003 |
| CTP class C (compered to Class A) | 3.695 | 1.229 | 3.005 | 0.003 |

The prediction formula;

Log odds (presence of OV) = -0.189 -0.046*%SP +2.9 [if CTP class B, otherwise zero] +3.7 [if CTP class C, otherwise zero]

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The prediction formula;

Log odds (presence of OV) = -0.189 -0.046*%SP +2.9 [if CTP class B, otherwise zero] +3.7 [if CTP class C, otherwise zero]

$$\ln\left(\frac{P}{1-P}\right) = a + bX$$

$$\frac{P}{1-P} = e^{a+bX}$$

$$P = \frac{e^{a+bX}}{1+e^{a+bX}}$$

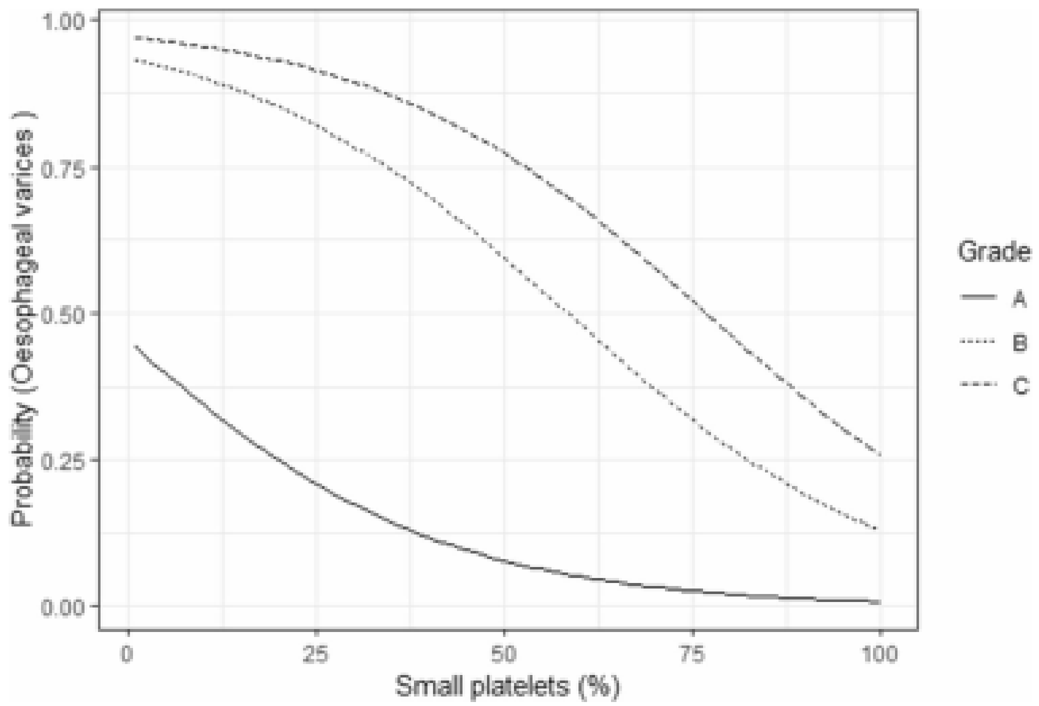
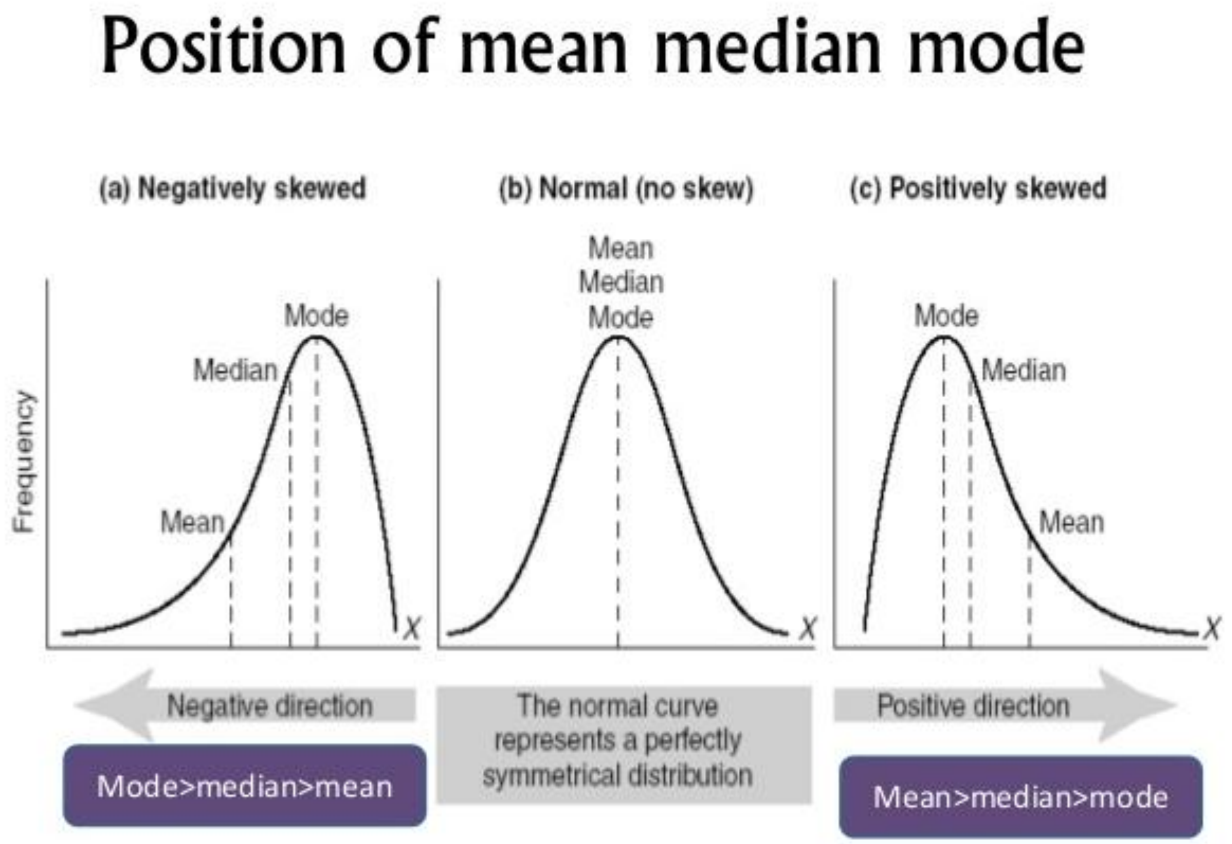



**Fig. 1** Probability of having OV along with the number of small platelets for each CTP class

# When normality is not there?



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# Parametric vs Non-parametric tests

- **Parametric** statistical test is one that makes assumptions about the parameters (defining properties) of the population distribution(s) from which one's data are drawn

- **Non-parametric** test is one that makes no such assumptions.
  - In this strict sense, "non-parametric" is essentially a null category, since virtually all statistical tests assume one thing or another about the properties of the source population(s).

| Parametric tests | Non-parametric tests |
|---|---|
| One sample | |
| One sample *t*-test | Sign test |
| | Wilcoxon's signed rank test |
| Two-sample | |
| Paired *t*-test | Sign test |
| | Wilcoxon's signed rank test |
| Unpaired *t*-test | Mann-Whitney U-test |
| | Kolmogorov-Smirnov test |
| K-sample | |
| ANOVA | Kruskal-Wallis test |
| | Jonckheere test |
| Two-way ANOVA (repeated measure ANOVA) | Friedman test |
| Pearson correlation coefficient ($r$) | Spearman rank order ($\rho$) |

ANOVA – Analysis of variance

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Contact: dileepa@kln.ac.lk

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- bit.ly/stat_workshop_feedback

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# Feedback

1. what did you like about this session?
2. what didn't you like about this session?
3. what did you learn from this session?

dileepa@kln.ac.lk

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# Thank You