1. show that the set of all strictly increasing infinite sequences of positive integers is uncountable.

2. Prove that the collection of all finite subsets of $\mathbb{N}$ is countable.

3. Let $S = \mathbb{Z} \times (\mathbb{Z} - \{0\})$. Define the relation $\sim$ on S where ~~(a,b)~~ $(a,b) \sim (c,d)$ iff $ad = bc$. Prove that $\sim$ is an equivalence relation.

4. Prove that for the set of positive integers, the relation " m is multiple of n " is an order relation.

5. Let S is a set and for $A, B \in P(S)$, we define $A \leq B$ to mean $|A| \leq |B|$. Is this relation a partial order on $P(S)$? Explain.

$P(S) \rightarrow$ power set of S

$|A| \rightarrow$ cardinality of A

$|B| \rightarrow$ cardinality of B

6. consider the set $S = \mathbb{R}$ where $x \sim y$ iff $x^2 = y^2$.

i) Find all the numbers that are related to $x = 1$. Repeat this exercise for $x = \sqrt{2}$ and $x = 0$

ii) prove that $\sim$ is an equivalence relation on S.