Countable & Uncountable Sets

→ regarding "size" of a set.

Def": Two sets A & B are said to be equivalent ($A \approx B$) if $\exists$ a f" $f: A \to B$ that is one-to-one and onto.

Notice that for sets A, B, C following holds:

1. $A \approx A$
2. If $A \approx B$, then $B \approx A$
3. If $A \approx B$ and $B \approx C$, then $A \approx C$.

The set of natural nos. $\mathbb{N} = \{1, 2, \dots\}$.
Any subset of $\mathbb{N}$ of the form $\{1, \dots, n\}$ is called a segment of $\mathbb{N}$, and $n \in \mathbb{N}$ is called no. of elements of the segment.
Two segments $\{1, \dots, m\}$ and $\{1, \dots, n\}$ are equiv. iff $m = n$.

A set that is equivalent to a segment is called a finite set. The empty set is also considered to be finite with zero elements. A set that is not finite is called an infinite set.

Def^{n}: A set A is called countable if it is equivalent to $\mathbb{N}$, i.e., if there is a one-to-one correspondence of $\mathbb{N}$ with the elements of A.

Countable set A usually denoted as $A = \{a_i\}_{i=1}^{n}$ for $n \in \mathbb{N}$.

enumeration of the set A.

An infinite set that is not countable is called uncountable set.

Thm: Every infinite set contains a countable subset.