Asg 4

Examples: Assignment (4)

(a) $\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

(b) $\frac{1}{2} + 1 + \frac{1}{8} + \frac{1}{4} + \frac{1}{32} + \frac{1}{16} \cdots$

} Ratio test & Root test

* Thm

For a metric space (X, d) the following statements hold:

1) X & ϕ are closed sets.

2) Arbitrary intersection of closed sets are closed.

3) Finite union of closed sets is closed sets.

Assignment: Proof of these

Thm. For a f^n $f: (X,d) \longrightarrow (Y,\rho)$ bet^n two metric spaces, the following statements are equivalent:

i) f is continuous on X.

ii) $f^{-1}(O)$ is an open subset of X whenever O is an open subset of Y.

iii) If $\lim x_n = x$ holds in X, then $\lim f(x_n) = f(x)$ holds in Y.

iv) $f(\overline{A}) \subseteq \overline{f(A)}$ holds for every subset A of X.

v) $f^{-1}(C)$ is a closed subset of X whenever C is closed subset of Y.