Thm. Every subset of a countable set is either finite or else countable.

Thm: For an infinite set A, the following statements are equiv.

i) A is countable.

ii) $\exists$ a subset B of $\mathbb{N}$ and a f^n: $f: B \to A$ that is onto.

iii) $\exists$ a f^n $g: A \to \mathbb{N}$ that is one-one.

Thm: Let $\{A_1, A_2, A_3, A_4, \ldots\}$ be a countable family of sets s.t. each A_i is a countable set. Then $A = \bigcup_{n=1}^{\infty} A_n$ is a countable set.

Proof: Let $A_n = \{a_1^n, a_2^n, \ldots\}$ for all $n \in \mathbb{N}$.

$A = \bigcup_{n=1}^{\infty} A_n$. Consider $B = \{2^k \cdot 3^n : k, n \in \mathbb{N}\}$.

Let $f: B \to A$ s.t. $f(2^k \cdot 3^n) = a_k^n$. Since f maps B onto A, A is countable set.

Thm. Let $\{A_1, A_2, \ldots, A_n\}$ be a finite collection of sets s.t. each A_i is countable. Then $A_1 \times A_2 \times \cdots \times A_n$ is countable.

→ The Cartesian product of a finite collection of countable sets is always countable.

Let $A \preceq B$ denote existence of one-one f^n $f: A \to B$. Then "B has at least as many elements as A".

Relation $\preceq$ satisfies following properties:

1. $A \preceq A$ for all sets A.
2. If $A \preceq B$ and $B \preceq C$, then $A \preceq C$.
3. If $A \preceq B$ and $B \preceq A$, then $A \approx B$.

Thm. (Cantor). If A is a set, then $A \preceq P(A)$ and $A \not\approx P(A)$ hold, where $P(A)$ is the power set of A.