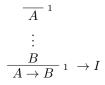
Rules for Natural Deduction (Propositional Logic)

Implication



$$\begin{array}{cc} A & A \to B \\ \hline B & \end{array} \to E$$

Conjunction

$$\frac{\mathbf{A} \quad \mathbf{B}}{A \wedge B} \wedge I$$

$$\frac{A \wedge B}{A} \wedge E_1$$

$$\frac{A \wedge B}{B} \wedge E_2$$

Disjunction

$$\frac{A}{A \vee B} \vee I_1$$

$$\frac{B}{A \vee B} \vee I_2$$

$$\begin{array}{cccc} & \overline{A}^{1} & \overline{B}^{1} \\ & \vdots & \vdots \\ \underline{A \lor B} & \underline{C} & \underline{C} \\ & \underline{C} & \end{array} 1 \lor \underline{E}$$

Truth and Falsity

$$\frac{\perp}{A} \perp E$$

$$\overline{\ \ \, } \top I$$

Negation

$$\begin{array}{c}
\overline{A}^{1} \\
\vdots \\
\overline{A}^{1} & \neg I
\end{array}$$

$$A \qquad A \qquad T$$

Conjunction

Quantifiers

$$\frac{\forall x \ A(x)}{A(t)} \ \forall E$$

$$\begin{array}{c} -1 \\ \vdots \\ A(y) \\ \hline \forall x \ A(x) \end{array} \ _1 \ \forall I$$

$$\frac{A(y)}{A(y)} \stackrel{1}{=} \frac{\exists x \ A(x)}{B} \stackrel{1}{=} \exists E$$

$$\frac{A(t)}{\exists x \ A(x)} \ \exists I$$

Equality

$$t = t$$
 refl

$$\frac{s=t}{t=s}$$
 symm

$$\frac{r=s}{r=t} \frac{s=t}{t}$$
 trans

$$\frac{s=t}{r(s)=r(t)}$$
 subst

$$\frac{s=t}{P(t)} P(s)$$
 subst