International Institute of Information Technology Hyderabad

Discrete Structures (MA5.101)

Assignment 4
Deadline: January 25, 2022 (Wednesday) 23:55 PM
Total Marks: 100
Instructions: Submit ONLY handwritten scanned pdf file in the moodle under Assignments directory.

1. Let $G = \left\{ \begin{pmatrix} a & b \\ a & b \end{pmatrix} : a, b \in \mathcal{R}, a + b \neq 0 \right\}$, where $\mathcal{R}$ is the set of real numbers. Show that
   (a) G is a semigroup under matrix multiplication.
   (b) G has a left identity.
   (c) Each element of G has a right inverse.
   (d) G is a group.

   [10]

2. Let $(G, \circ)$ be a semigroup such that for every $a \in G$, there exists a unique $b \in G$ such that $a \circ b \circ a = a$. Prove that G is a group.

   [10]

3. If G is a finite cycle group with more than one element, then prove that G must have an element of prime order.

   [10]

4. Let H be a subgroup of a group $(G, .)$, and let $N = \cap_{x \in G}\, x.H.x^{-1}$. Prove that N is a normal subgroup of G.

   [10]

5. Show that the intersection of two normal subgroups is a normal subgroup.

   [10]

6. Prove that a subgroup H of a group $\langle G, \cdot \rangle$ is "normal" if and only if $g^{-1} \cdot H \cdot g \subseteq H, \forall g \in G$.

   [10]

7. Let H be a normal subgroup in $(G, .)$. Prove that G/H is abelian if and only if $g_1.g_2.g_1^{-1}.g_2^{-1}$ is in H for all g_1 and g_2 in G.

[10]

8. Prove that a cyclic group is necessarily abelian. But, the converse is not true.

[10]

9. Let H and K be two subgroups of a group G. The *product set* of H and K, written by $H.K$, is the set of all products of the form $h.k$, for h in H and k in K. Show that the product set is a normal subgroup of G, if H and K are normal in G.

[10]

10. The converse of the Lagrange's theorem is NOT true. For this purpose, show that there is a group of order 12, which has no subgroup of order 6.

[10]

All the best!!!