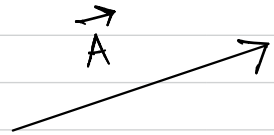
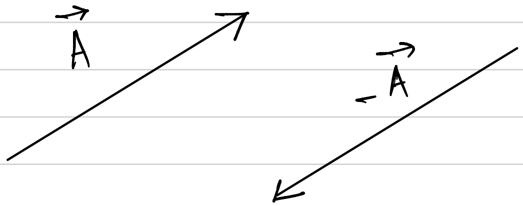
Vector: Has direction & magnitude. Class 1. 22/12/22

Example: Displacement

Written as: Denoted as $\vec{A}$. The magnitude is $|\vec{A}|$

In diagrams: Shown by an arrow. Length of the arrow is proportional to the magnitude & the arrow indicates direction

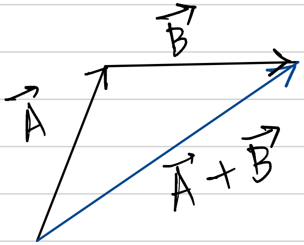




* A vector that is opposite to $\vec{A}$ is $-\vec{A}$

Location is not important

* Addition:



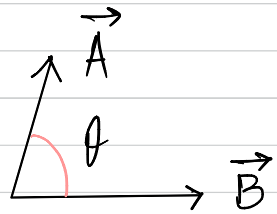
Addition is commutative: $\vec{A}+\vec{B} = \vec{B}+\vec{A}$

* Multiplication by scalar: If multiplied by scalar a then the magnitude increased by factor a, direction unchanged.

Multiplication is distributive : $a(\vec{A}+\vec{B}) = a\vec{A} + a\vec{B}$

* Dot product of two Vectors $\vec{A}$ & $\vec{B}$:

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}| \cos\theta$$



Geometrically: $|\vec{A}|$ times the projection of $\vec{B}$ along $\vec{A}$

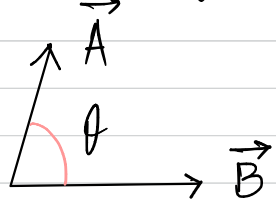
This is a scalar. Also called scalar product

Dot/Scalar product is commutative : $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

and distributive : $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$ Prove this Exercise!

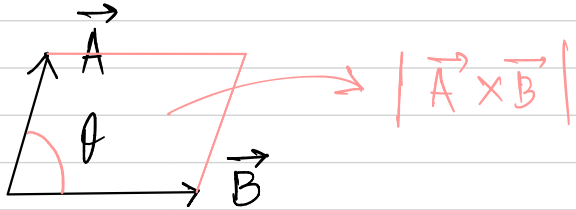
* Cross Product of two vectors $\vec{A}$ & $\vec{B}$: Defined as $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}| \sin\theta \, \hat{n}$

$\hat{n}$ is a unit vector the direction of which is determined by the righ-hand rule.



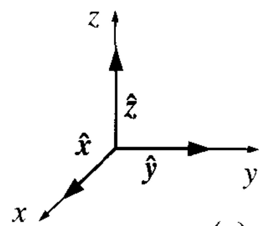
In this figure $\hat{n}$ points onto the page, & $\vec{B}\times\vec{A}$ points out of the page.

Geometrically $|\vec{A} \times \vec{B}|$ is the area of parallelogram generated by $\vec{A}$ & $\vec{B}$



* **Component Form:** Cartesian coordinate system with unit/basis vectors

$\hat{x}, \hat{y}, \hat{z}$

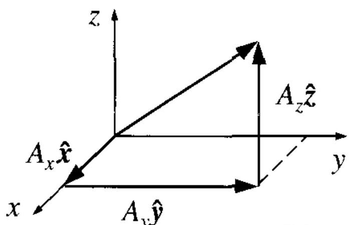


(a)

$\hat{x}\cdot\hat{x} = 1$, $\hat{y}\cdot\hat{y} = 1$ & $\hat{z}\cdot\hat{z} = 1$

$\hat{x}\cdot\hat{y} = 0$, $\hat{x}\cdot\hat{z} = 0$ & $\hat{y}\cdot\hat{z} = 0$

Any vector $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$



(b)

A_x, A_y & A_z are the components of $\vec{A}$ – projection of $\vec{A}$ on to the coordinate axes.

* **Addition:** $\vec{A} + \vec{B} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} + (A_z + B_z)\hat{z}$
* **Multiplication by scalar:** $a\vec{A} = (aA_x)\hat{x} + (aA_y)\hat{y} + (aA_z)\hat{z}$

Dot/Scalar Product: $\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z . B_z$

$$\vec{A}.\vec{A} = A_x^2 + A_y^2 + A_z^2$$

$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$ is the magnitude of the vector $\vec{A}$

Cross Product: From the definition of cross product we find for the unit vectors

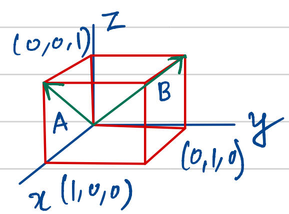
$$\begin{aligned}
\hat{\mathbf{x}} \times \hat{\mathbf{x}} = \quad \hat{\mathbf{y}} \times \hat{\mathbf{y}} &= \hat{\mathbf{z}} \times \hat{\mathbf{z}} = 0, \\
\hat{\mathbf{x}} \times \hat{\mathbf{y}} = -\hat{\mathbf{y}} \times \hat{\mathbf{x}} &= \hat{\mathbf{z}}, \\
\hat{\mathbf{y}} \times \hat{\mathbf{z}} = -\hat{\mathbf{z}} \times \hat{\mathbf{y}} &= \hat{\mathbf{x}}, \\
\hat{\mathbf{z}} \times \hat{\mathbf{x}} = -\hat{\mathbf{x}} \times \hat{\mathbf{z}} &= \hat{\mathbf{y}}.
\end{aligned}$$

Hence $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{x} + (A_z B_x - A_x B_z)\hat{y} + (A_x B_y - A_y B_x)\hat{z}$

The following is easy to remember

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}.$$

Find the angle between face diagonal of a cube

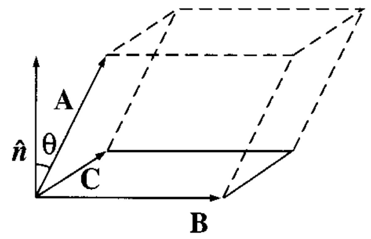


$\vec{A} = 1\hat{x} + 0\hat{y} + 1\hat{z}$; $\vec{B} = 0\hat{x} + 1\hat{y} + 1\hat{z}$

$\Rightarrow \vec{A}.\vec{B} = 1 = |A||B|\cos\theta$

$= \sqrt{2}\sqrt{2}\cos\theta \quad \Rightarrow \theta = \cos^{-1} 1/2 = 60^\circ$

Scalar triple Product: Geometrically $\vec{A}\cdot(\vec{B}\times\vec{C})$ is the volume of a parallelepiped formed by $\vec{A}$, $\vec{B}$ & $\vec{C}$



$$\vec{A}\cdot(\vec{B}\times\vec{C}) = |\vec{A}||\vec{B}\times\vec{C}|\cos\theta$$

$|\vec{B}\times\vec{C}|$ is the area, $|\vec{A}|\cos\theta$ is the height.

It is evident that $\vec{A}\cdot\vec{B}\times\vec{C} = \vec{B}\cdot\vec{C}\times\vec{A} = \vec{C}\cdot\vec{A}\times\vec{B}$

Easy to remember expression for the triple product.

$$\vec{\mathbf{A}}\cdot(\vec{\mathbf{B}}\times\vec{\mathbf{C}}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}.$$

Vector triple Product: $\vec{A}\times\vec{B}\times\vec{C} = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$ this is BAC−CAB rule.