# Lecture 4 – Binary Representation

Chapter 1

### Representation of negative numbers

- In ordinary arithmetic, a negative number is indicated by a minus sign and a positive number by a plus sign
- Computers must represent everything with binary digits
- It is customary to represent the sign with a bit placed in the leftmost position of the number
- The convention is to make the sign bit 0 for positive and 1 for negative
- This can be done using:
  - 1. Signed magnitude representation
  - 2. Signed complement representation
    - 1. Signed 1's complement representation
    - 2. Signed 2's complement representation

### Signed-magnitude representation

- In this notation, the number consists of a magnitude and a symbol ( + or ) or a bit (0 or 1) indicating the sign
- This is similar to the representation of signed numbers used in ordinary arithmetic

- Eg: <u>0</u>1001 represents +9, and <u>1</u>1001 represents -9 in signed magnitude representation
  - -9 is obtained from +9 by changing only the sign bit from 0 to 1
- Weird: +0 is represented as 0000 and minus 0 is represented as 1000. So, two representations for zero – inefficient and may cause errors

### Signed complement representation

- In digital hardware, it is more convenient to use *signed complement* system, for representing negative numbers
- In this system, a negative number is indicated by its complement
  - signed-complement system negates a number by taking its complement
- Since positive numbers always start with 0 (plus) in the leftmost position (in all representations), it follows that the complement will always start with a 1, indicating a negative number
- In signed-1's-complement, -9 is obtained by taking the 1's complement of all the bits of +9 (01001), including the sign bit, ie, 10110
- The signed-2's-complement representation of -9 is obtained by taking the 2's complement of +9, including the sign bit, ie 10111

## Signed complement representation

• Find signed 2's complement representation in 4 bits:

```
• +3
0011
• -7
(7)_{10} = 0111 \implies -7 = 1001
• 0
0000
```

- -39 (00100111) in 8 bit:
  - 10100111 (signed magnitude)
  - 11011000 (signed 1's complement)
  - 11011001 (signed 2's complement)

# Signed complement representation

Find the decimal numbers for the following signed 2's complement representation in 4 bits:

- (1100)<sub>2</sub>
- (1111)<sub>2</sub>
- (0000)<sub>2</sub>
- (1000)<sub>2</sub>

# Interpretations for 4 bit binary numbers

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	_	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100
-5	1011	1010	1101
-6	1010	1001	1110
-7	1001	1000	1111
-8	1000	_	_

# Signed addition

- If the numbers are represented in memory in 2's complement form, the sign of the sum takes care of itself! The result is also in 2's complement representation
- The sign bit is to be included in the addition and if there is a carry, it is discarded
- Examples in 4-bit signed 2's complement representation:

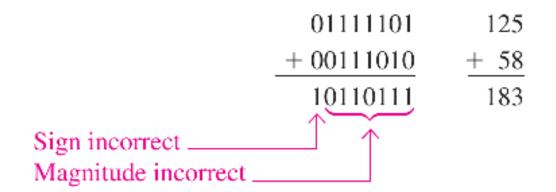
+ 6	00000110	- 6	11111010
+13	00001101	+13	00001101
+19	00010011	+ 7	00000111
+ 6	00000110	- 6	11111010
-13	11110011	<u>-13</u>	11110011
	11111001	-19	11101101

111111001

# Signed addition

 In order to obtain a correct answer, we must ensure that the result has a sufficient number of bits to accommodate the sum

• If we start with two *n*-bit numbers and the sum occupies *n* + 1 bits, we say that an overflow occurs



### Signed subtraction

- Subtraction of two signed binary numbers when negative numbers are in 2's-complement form is simple and can be stated as follows:
  - Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including the sign bit)
  - A carry out is discarded
- This works because: M N = M + (-N)
- Example : 8 3 = 8 + (-3)

```
00001000 \qquad \text{Minuend (+8)}
+ 11111101 \qquad \text{2's complement of subtrahend (-3)}
\frac{1 \ 00000101}{1 \ 00000101} \qquad \text{Difference (+5)}
```