Lecture 2 – Binary numbers and representations

Chapter 1

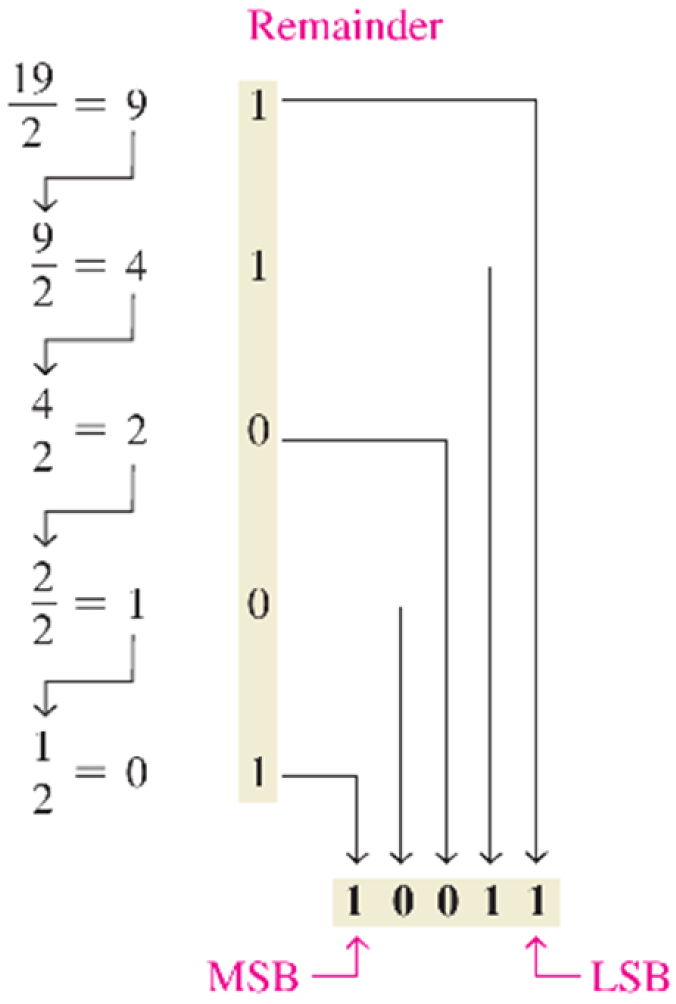
Recap

- Number Systems
  - Decimal
  - Octal
  - Hex
  - Binary
- Conversion from one base to another
- Need for various number systems
  - $(111111111111)_2 = (FFF)_{16}$

Recap: Conversions from decimal

- Algorithm:
  - Divide by radix
  - Save the remainder
  - Repeat till quotient '0'
  - Arrange remainders in reverse order

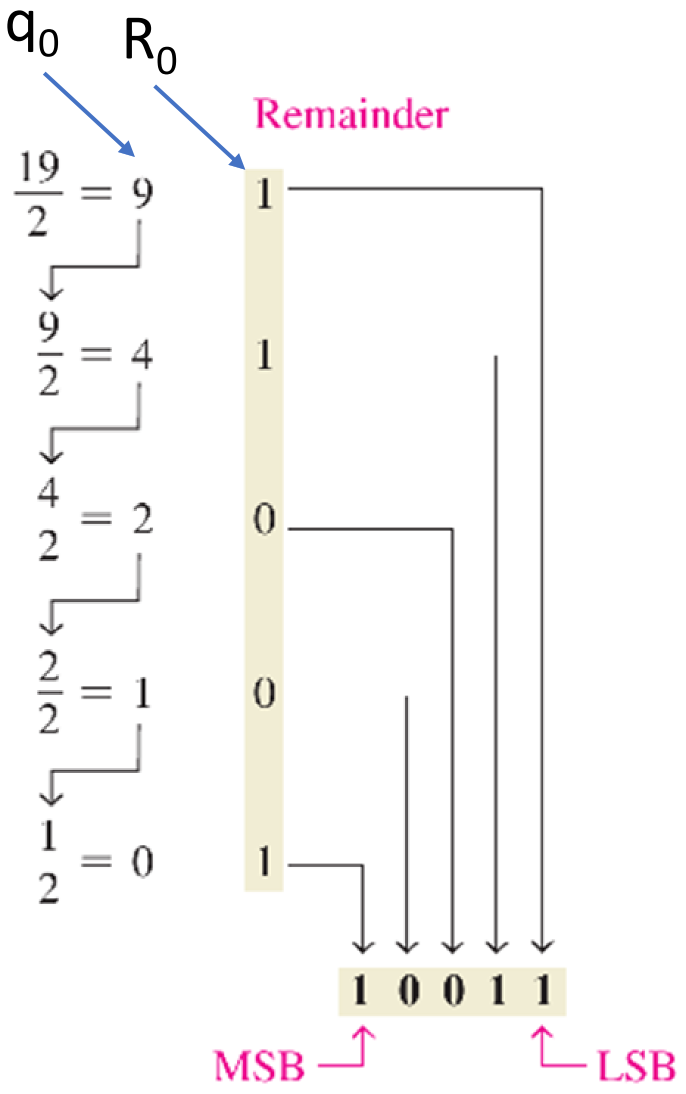
Eg: Convert $(19)_{10}$ to binary



Recap

Base conversion by repeated division – position of MSB and LSB

$$
\begin{aligned}
N &= q_0 r + R_0 \\
&= (q_1 r + R_1) r + R_0 \\
&= q_1 r^2 + R_1 r + R_0 \\
&= (q_2 r + R_2) r^2 + R_1 r + R_0 \\
&= q_2 r^3 + R_2 r^2 + R_1 r + R_0 \\
&= \ldots\ldots \\
&\quad \ldots\ldots \\
&= 0 * r^{n+1} + R_n r^n + R_{n-1} r^{n-1} + \ldots + R_0 r^0
\end{aligned}
$$



The “decimal” point

$$
\begin{aligned}
568.23 &= (5 \times 10^2) + (6 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (3 \times 10^{-2}) \\
&= (5 \times 100) + (6 \times 10) + (8 \times 1) + (2 \times 0.1) + (3 \times 0.01) \\
&= \mathbf{500} + \mathbf{60} + \mathbf{8} + \mathbf{0.2} + \mathbf{0.03}
\end{aligned}
$$

In general: $a_2a_1a_0.a_{-1}a_{-2}$ can be expressed as $a_2r^2+a_1r^1+a_0r^0+a_{-1}r^{-1}+a_{-2}r^{-2}$

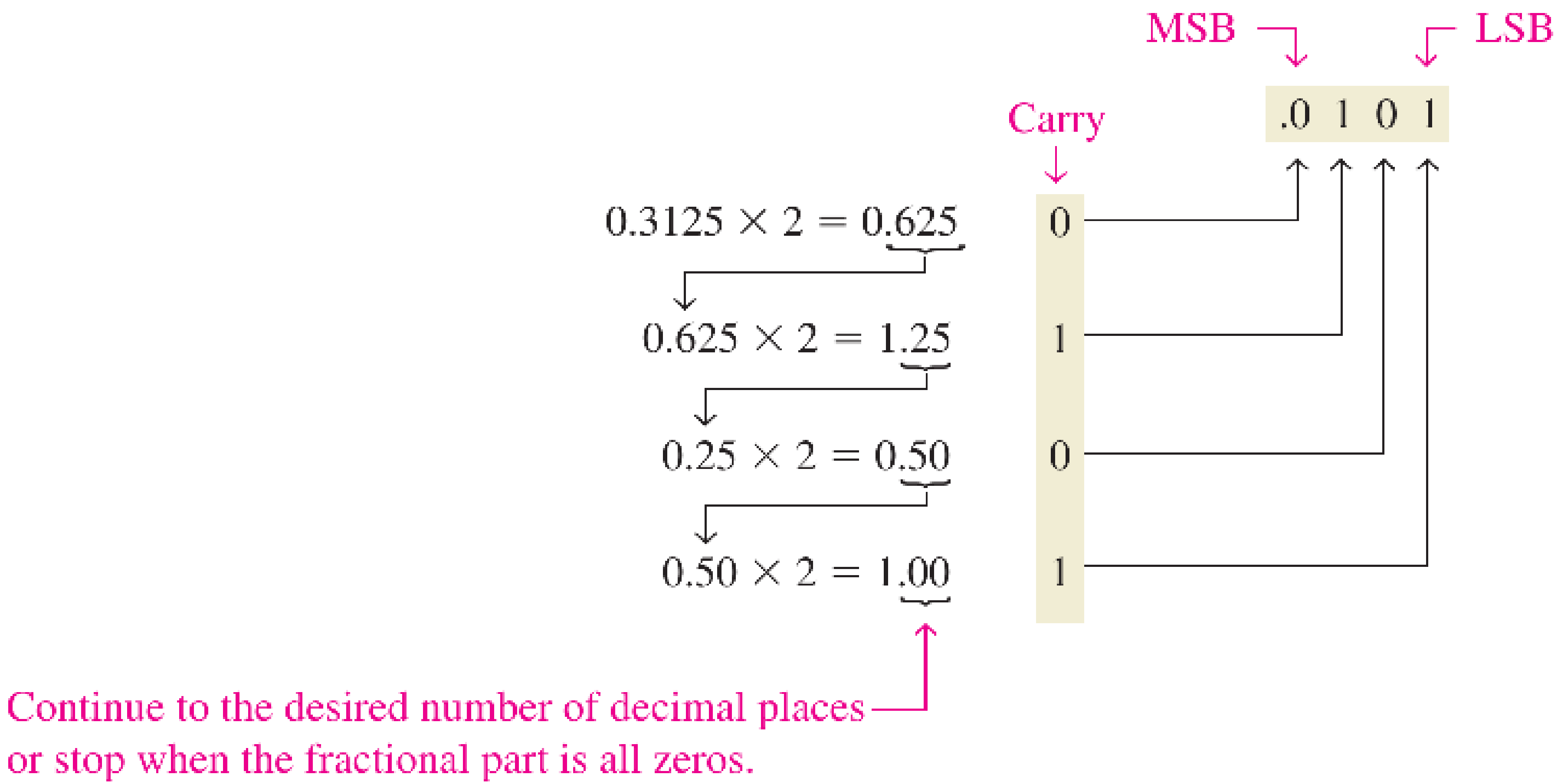
- Binary to decimal:
  - $(1.011)_2 = 1*2^0+0*2^{-1}+1*2^{-2}+1*2^{-3}$

    $= 1+0.25+0.125$

    $= (1.375)_{10}$

The “decimal” point

Convert 0.3125 into binary:



The “decimal” point

Covert $(0.65626)_{10}$ to binary:

0.65626 *2	1.31250	1
0.31250*2	0.62500	0
0.62500*2	1.25000	1
0.25000*2	0.50000	0
0.50000*2	1.00000	1

Binary to Hex conversion:

$$(\underbrace{10}_{2}\ \underbrace{1100}_{C}\ \underbrace{0110}_{6}\ \underbrace{1011}_{B}\ .\ \underbrace{1111}_{F}\ \underbrace{0010}_{2})_2 = (2C6B.F2)_{16}$$

The “decimal” point

Convert $(0.510)_{10}$ to octal:

$$0.513 * 8 = 4.104$$
$$0.104 * 8 = 0.832$$
$$0.832 * 8 = 6.656$$
$$0.656 * 8 = 5.248$$
$$0.248 * 8 = 1.984$$
$$0.984 * 8 = 7.872$$

$(0.510)_{10} = (0.406517\ldots)_8$

Addition in various number systems

- Octal number system
  - $(167)_8 + (765)_8$

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ } \\ 1\ 6\ 7 \\ +7\ 6\ 5 \\ \hline 1\ 1\ 5\ 4 \end{array}$$

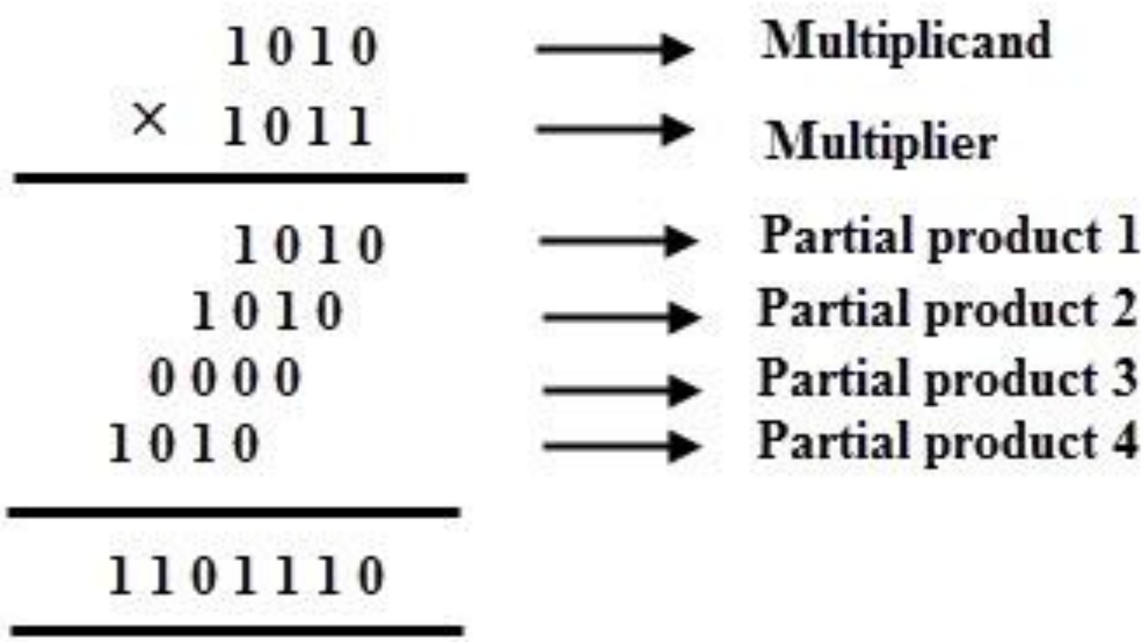
- Hexadecimal number system
  - $(BA3)_{16} + (5DE)_{16}$

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ } \\ B\ A\ 3 \\ +5\ D\ E \\ \hline 1\ 1\ 8\ 1 \end{array}$$

- Binary number system
  - $(1101)_2 + (111)_2 = (10100)_2$

Multiplication

- Binary number system



- Example:
  - $(111)_2 * (110)_2 = (101010)_2$

Complements of numbers

- Complements are used in digital computers to simplify the subtraction operation and for logical manipulation
- Simplifying operations leads to simpler, less expensive circuits to implement the operations
- There are two types of complements for each base-*r* system:

1. The radix complement [r’s complement] – called the 10’s complement in decimal, 2’s complement in binary and so on
2. The diminished radix complement [(r-1)’s complement] – called the 9’s complement in decimal, 1’s complement in binary and so on

Diminished radix complement

- Given an n-digit number *N* in base *r*, the (*r* - 1)'s complement of *N* , i.e., its diminished radix complement, is defined as $(r^n - 1) - N$

- For decimal numbers, the 9's complement of *N* is $(10^n - 1) - N$
- In this case, $10^n \; - \; 1$ is a number represented by *n* 9s
  - Eg: if *n* = 4, we have 10^4 = 10,000 and r^n-1 = 10^4 - 1 = 9999
  - If n=2, we have 10^2 = 100 and r^n – 1 = 10^2 - 1 = 99

- It follows that the 9's complement of a decimal number is obtained by subtracting each digit from 9
  - Eg: 9's complement of 76 = 99 – 76 = 23
  - 9's complement of 1242 = 9999 – 1242 = 8757
  - 9's complement of 99981 is 99999 – 99981 = 18

Diminished radix complement

- For n-bit binary numbers, the 1's complement of *N* is $(2^n - 1) - N$.

- Again, $(2^n - 1)$ is a binary number represented by *n* 1s
  - For example, if $n = 4$, we have $2^4 = (10000)_2$ and $2^4 - 1 = (1111)_2$.
  - If n=2, we have $2^2 = (100)_2$, and $2^2 - 1 = 11$,
- 2's complement of a binary number can be obtained by subtracting each bit from 1
- However, when subtracting binary digits from 1, we can have either 1 - 0 = 1 or 1 - 1 = 0, which causes the bit to change from 0 to 1 or from 1 to 0, respectively
- Therefore, **the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.**
- Examples of 1's complement:

  1's complement of 1011000 = 1111111 – 1011000 = 0100111
  1's complement of 100 = 111 – 100 = 011