



Questions for 17/12/2022

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Q1 - Show that the sequence $a_n = (1 + 1/n)^n$ is a monotone increasing sequence and is bounded above. We call its limit as e.

Q2 - Repeat the same for sequence $x_n = 1 + 1/1! + 1/2! + 1/3! \dots + 1/n!$. And show that limit = e (same as above)

Q3 - Show that the sequence $a_n = (1 + \frac{1}{n})^{n+1}$ is a monotone decreasing. Also find its limit. (limit = e)

Q4 - $u_n = 1 + 1/2 \dots + 1/n - \log n$; $v_n = 1 + 1/2 \dots + 1/(n-1) - \log n$, $n \geq 2$. Show that u_n is a monotone decreasing sequence and $(v_n)_{n=2}^{\infty}$ is an monotone increasing one and they converge to same limit.

Q5 - If $u_1 > 0$ and $u_{n+1} = 1/2(u_n + 9/u_n)$ for $n \geq 1$. Prove that the sequence converges to 3.

Q6 - For $a, b \in \mathcal{N}$, consider the sequence (shown below), for $n > a, b$. Which of the following statements are true? As $n \rightarrow \infty$

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$$d_n = \frac{\binom{n}{a}}{\binom{n}{b}}$$

- a. d_n converges for all values of a and b
- b. converges if $a < b$
- c. converges if $a = b$
- d. converges if $a > b$

Q7 - A function $f : \mathcal{R}$ is defined by $f(x) = x$ if $x \in \mathcal{Q}$ or $= 0$ if $x \in \mathcal{R} - \mathcal{Q}$. Show that f is continuous at 0 and f has a discontinuity of the second kind at every other point in \mathcal{R} . (\mathcal{Q} is Rational set)

Q8 - A function $f : [0, 1] \rightarrow \mathcal{R}$ is continuous on $[0, 1]$ and assume it takes rational values. If $f(1/2) = 1/2$, prove that $f(x) = 1/2$ for all $x \in [0, 1]$.