Points: 40 | MA5.101
Due: 4th January 23:59 | Monsoon 2022

Assignment 3

Instructions

1. The given problems are to be solved **only** using fitch-style proof of natural deduction. Strictly adhere to standard convention and write clear and concise proofs.
2. A rulesheet for natural deduction has been attached here for reference. Refrain from using rules/theorems outside the rulesheet unless explicitly asked otherwise.
3. Naming of submissions should strictly follow the format **rollnumber_A3**. Penalty will be imposed for not following the same.
4. Plagiarism will attract penalty.

Problem 1

Prove the following implications (conditionals).

a) $(\forall x\ A(x) \land \exists x\ (A(x) \to B(x))) \to \exists x\ (A(x) \land B(x))$

b) $\forall x\ (A(x) \to B(x)) \to ((\exists x\ \neg B(x)) \to (\exists x\ \neg A(x)))$

[10 points]

Problem 2

Show that the formulae $\forall x\ \forall y\ (P(x) \to Q(y))$ and $(\exists x\ P(x)) \to (\forall y\ Q(y))$ are logically equivalent.

[10 points]

Problem 3

Prove the following double implication (biconditional). (**Hint:** Show the back implication using a proof of contradiction).

$$\exists x\ \exists y\ (P(x) \to Q(y)) \longleftrightarrow (\forall x\ P(x)) \to (\exists y\ Q(y))$$

[15 points]

Problem 4

Consider the domain of real numbers $\mathbb{R}$ and the function $f : \mathbb{R} \to \mathbb{R}$ such that $\forall x\ f(2 \cdot x) = f(x)$ where $\cdot$ denotes the multiplication operator over real numbers. Assume standard interpretation of multiplication and constant symbols $2, 4, 8$, etc. Prove that $\forall x\ f(x) = f(8 \cdot x)$.

[5 points]