Assignment 3

*Don't try googling, come to office hours. These are basic problems and similar problem will come in midsem

1. **Sup Sup**

   Show that if A and B are bounded subsets of $\mathcal{R}$, then $A \cup B$ is a bounded set. Show also that sup($A \cup B$) = sup{sup(A), sup(B)}.

2. **Finite set contain sup**

   Show that a nonempty finite set $S \subset \mathcal{R}$ contains its supremum.

   Hint - Use Mathematical Induction

3. **Some properties of sup and inf**

   Let S be a non-empty bounded set in $\mathcal{R}$.

   a. Let a > 0, and let aS := {as : s $\in$ S}. Prove that inf(aS) = a inf(S) and sup(aS) = a sup(S)

   b. Let b < 0, and let bS := {bs : s $\in$ S}. Prove that sup(bS) = b inf(S) and sup(bS) = b inf(S)

4. **Inverse of element in a Group**

   Let G be a group, and show that $(xy)^{-1} = y^{-1}x^{-1}$ for any two element x and y in G. Show also that $(x^{-1})^{-1} = x$ for any element x in G.

5. **Singular and non-singular elements**

   Let R be a Ring with identity (i.e contains an object denoted by 1 that when multiplied by any other element of ring returns the same object). Now if an element x in R has an inverse (that is $x \times x^{-1} = 1$), then x is said to be non-singular or invertible. And if inverse doesn't exist, it is called singular. With this defination in mind, show the following -

   a. A ring with idenity whose all non-zero element are invertible and is commutative w.r.t to multiplication is a field. (Just follow the axiom, it is clear as sea water)

b. The set of all $N \times N$ sized matrices, whose element are in $\mathcal{R}$ is not a field. Also check whether it has some singular matrices other than zero.

c. Show that set of even intergers is a ring. Also tell whether 2 is singular or non-singular.

6. **Modulo rings**

Let m be a positive integer, and I_m the set of all non-negative intergers less than m: $I_m = \{0, 1, \ldots, m-1\}$. Let a and b are two numbers in I_m, we define there sum and product as follows: $a + b = (a + b)\text{mod}(m)$ and $ab = (ab)\text{mod}(m)$. That is remainder after division by m of “ordinary” sum and product. Show that I_m with these operation is a ring.

a. (Not graded) Also show that I_m is a field $\iff$ m is a prime number.

7. **Limit of a sequence**

Consider the ever green sequence $(a_n)_{n=0}^{\infty}$ where $a_n = \frac{1}{n}$.

a. What is the limit of this sequence?

b. We all know it is zero, prove it!

Note - While proving please use ideas of proof, especially how to write proof of double quantifiers (you have done quantifer in discrete structures 😄). No marks will be awarded otherwise!

Hint - Use epsilon defination of limit

8. **Properties of Limits**

This is just one, prove all other relevant property too. Say $(a_n)_{n=0}^{\infty}$ and $(b_n)_{n=0}^{\infty}$ are 2 sequences which converge, i.e limit exist. Show that limit of $(a_n + b_n)_{n=0}^{\infty}$ = limit of $(a_n)_{n=0}^{\infty}$ + limit of $(b_n)_{n=0}^{\infty}$. Use ideas of proof and quantifier, and again no marks otherwise.