Rules for Natural Deduction (Propositional Logic)

Implication

$$\dfrac{\dfrac{}{A}\,{\scriptstyle 1}\quad\vdots\quad B}{A \to B}\,{\scriptstyle 1}\ \to I$$

$$\dfrac{A \qquad A \to B}{B} \to E$$

Conjunction

$$\dfrac{\mathrm{A} \qquad \mathrm{B}}{A \wedge B} \wedge I \qquad \dfrac{A \wedge B}{A} \wedge E_1 \qquad \dfrac{A \wedge B}{B} \wedge E_2$$

Disjunction

$$\dfrac{A}{A \vee B} \vee I_1 \qquad \dfrac{B}{A \vee B} \vee I_2$$

$$\dfrac{A \vee B \qquad \begin{array}{c}\overline{A}\,{\scriptstyle 1}\\ \vdots \\ C\end{array} \qquad \begin{array}{c}\overline{B}\,{\scriptstyle 1}\\ \vdots \\ C\end{array}}{C}\,{\scriptstyle 1} \vee E$$

Truth and Falsity

$$\dfrac{\bot}{A} \bot E \qquad \dfrac{}{\top} \top I$$

Negation

$$\dfrac{\dfrac{}{A}\,{\scriptstyle 1}\quad\vdots\quad \bot}{\neg A}\,{\scriptstyle 1}\ \neg I$$

$$\dfrac{\neg A \qquad A}{\bot} \neg E$$

Conjunction

$$\dfrac{\dfrac{}{\neg A}\,{\scriptstyle 1}\quad\vdots\quad \bot}{A}\,{\scriptstyle 1}\ PbC$$

Quantifiers

$$\frac{\forall x\ A(x)}{A(t)}\ \forall E$$

$$\frac{\begin{array}{c}\overline{\quad}\ {\scriptstyle 1}\\ \vdots\\ A(y)\end{array}}{\forall x\ A(x)}\ {\scriptstyle 1}\ \forall I$$

$$\frac{\exists x\ A(x) \qquad \begin{array}{c}\overline{A(y)}\ {\scriptstyle 1}\\ \vdots\\ B\end{array}}{B}\ {\scriptstyle 1}\ \exists E$$

$$\frac{A(t)}{\exists x\ A(x)}\ \exists I$$

Equality

$$\frac{}{t = t}\ \text{refl}$$

$$\frac{s = t}{t = s}\ \text{symm}$$

$$\frac{r = s \qquad s = t}{r = t}\ \text{trans}$$

$$\frac{s = t}{r(s) = r(t)}\ \text{subst}$$

$$\frac{s = t \qquad P(s)}{P(t)}\ \text{subst}$$