

EC 2.101 - Digital Systems and Microcontrollers

Practice Sheet 2 (Lec 1 – Lec 12)

Q1. Three Variable Maps

a. $F(x, y, z) = \sum(0, 2, 4, 5, 6)$

		y			
		00	01	11	10
x	0	m_0	m_1	m_3	m_2
		1			1
x	1	m_4	m_5	m_7	m_6
		1	1		1
		z			

$$F = z' + xy'$$

b. $F(x, y, z) = \sum(3, 4, 5, 6, 7)$

		yz			
		00	01	11	10
x	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6
		z			

$$F = x + yz$$

c. $F(x, y, z) = x'yz + xy'z' + xyz'$

A 2x4 Karnaugh map for variables x, y, and z. The vertical axis is labeled x with values 0 and 1. The horizontal axis is labeled yz with values 00, 01, 11, and 10. The map contains 1s in the following cells: (x=0, yz=11), (x=1, yz=00), (x=1, yz=01), and (x=1, yz=10). The 1s in the bottom row (x=1) are circled together, representing the term xz'. The 1 in the top row (x=0) at yz=11 is circled, representing the term x'yz.

$$F = x'yz + xz'$$

Q2. Four Variable Maps

a. $F(w, x, y, z) = \sum(1, 3, 4, 5, 6, 7, 9, 11, 13, 15)$

A 4-variable Karnaugh map for variables w, x, y, and z. The vertical axis is labeled wx with values 00, 01, 11, and 10. The horizontal axis is labeled yz with values 00, 01, 11, and 10. The map is divided into four 2x2 quadrants by a vertical line labeled y and a horizontal line labeled x. The quadrants are labeled m0-m3 (top-left), m4-m7 (top-right), m8-m11 (bottom-left), and m12-m15 (bottom-right). The 1s are located in the following cells: (wx=00, yz=01), (wx=00, yz=11), (wx=01, yz=00), (wx=01, yz=01), (wx=01, yz=11), (wx=01, yz=10), (wx=11, yz=01), (wx=11, yz=11), (wx=10, yz=01), and (wx=10, yz=11). The 1s in the top-right quadrant (m3, m7) are shaded dark gray. The 1s in the bottom-left quadrant (m9, m11) are shaded dark gray. The 1s in the top-left quadrant (m1, m5) are shaded light gray. The 1s in the bottom-right quadrant (m13, m15) are shaded light gray.

$$F = z + xw'$$

b. $F(A, B, C, D) = \sum(3, 7, 11, 13, 14, 15)$

A 4-variable Karnaugh map for variables A, B, C, and D. The vertical axis is labeled AB with values 00, 01, 11, and 10. The horizontal axis is labeled CD with values 00, 01, 11, and 10. The map is divided into four 2x2 quadrants by a vertical line labeled C and a horizontal line labeled B. The quadrants are labeled m0-m3 (top-left), m4-m7 (top-right), m8-m11 (bottom-left), and m12-m15 (bottom-right). The 1s are located in the following cells: (AB=00, CD=11), (AB=01, CD=11), (AB=11, CD=01), (AB=11, CD=11), (AB=11, CD=10), (AB=10, CD=11), and (AB=10, CD=10). The 1s in the top-right quadrant (m3, m7) are shaded dark gray. The 1s in the bottom-left quadrant (m13, m15) are shaded dark gray. The 1s in the top-left quadrant (m1, m5) are shaded light gray. The 1s in the bottom-right quadrant (m14, m15) are shaded light gray.

$$F = CD + ABD + ABC$$

c. $F(w, x, y, z) = x'z + w'xy' + w(x'y + xy')$

		yz		y	
wx		00	01	11	10
w	00	m_0	m_1 1	m_3 1	m_2
	01	m_4 1	m_5 1	m_7	m_6
	11	m_{12} 1	m_{13} 1	m_{15}	m_{14}
	10	m_8	m_9 1	m_{11} 1	m_{10} 1
				z	

x

$$F = xy' + x'z + wx'y$$

d. $F(A,B,C,D) = AD' + B'C'D + BCD' + BC'D$

		CD		C	
AB		00	01	11	10
A	00	m_0	m_1 1	m_3	m_2
	01	m_4	m_5 1	m_7	m_6 1
	11	m_{12} 1	m_{13} 1	m_{15}	m_{14} 1
	10	m_8 1	m_9 1	m_{11}	m_{10} 1
				D	

B

$$F = AD' + C'D + BCD'$$

For product of maxterms representation, we need to optimize the zeros and write F' as a sum of minterms and take its complement.

		CD	
AB		00	01
	00	0	1
	01	0	1
	11	1	0
	10	1	0

$$F' = A'C'D' + A'B'C + CD$$

$$\Rightarrow F = (A'C'D')' \cdot (A'B'C)' \cdot (CD)' = (A + C + D)(A + B + C')(C' + D')$$

Q3. Multi-Level Circuits

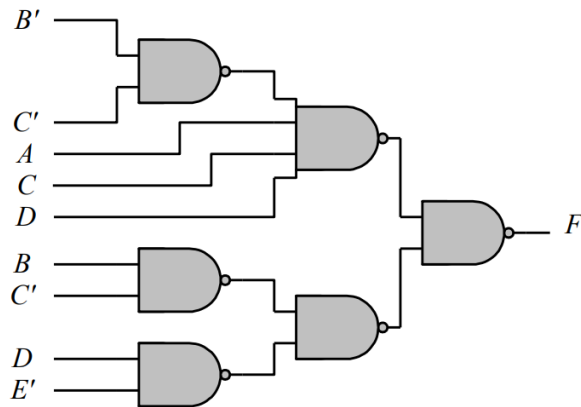
- a. $CD(B + C)A + (BC' + DE')$ using only NAND gates

$$F = ACD(B + C) + (BC' + DE')$$

$$F' = [ACD(B + C)]' [BC' + DE']'$$

$$F' = [ACD(B'C')]' [BC' + DE']'$$

$$F' = [CD(B'C')'A]' [[(BC')' (DE')]']'$$



- b. $CD(B + C)A + (BC' + DE')$ using only NOR gates

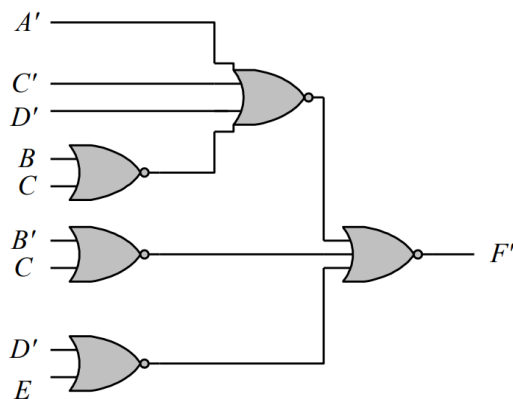
$$F = ACD(B + C) + (BC' + DE')$$

$$F' = [ACD(B + C) + (BC' + DE')]'$$

$$F' = [(A' + C' + D')(B + C) + (B' + C)' + (D' + E)']'$$

$$F' = [((A' + C' + D') + (B + C)')' + (B' + C)' + (D' + E)']'$$

$$F' = [(A' + C' + D' + (B + C)')' + (B' + C)' + (D' + E)']'$$

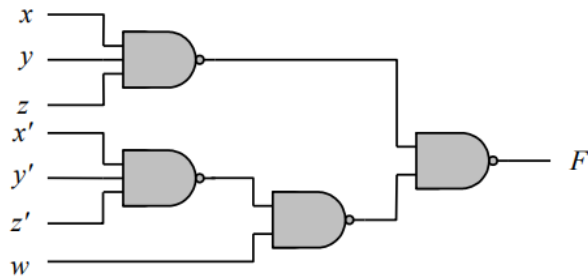


- c. $w(x + y + z) + xyz$ using only NAND gates

$$F = w(x + y + z) + xyz$$

$$F' = [w(x + y + z)]' [xyz]'$$

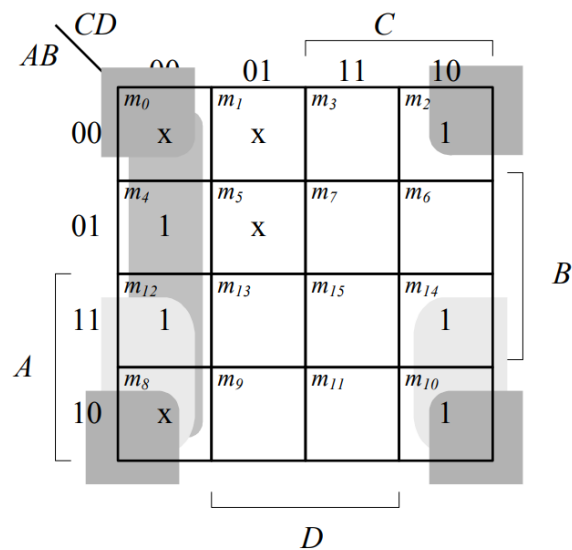
$$F' = [w(x'y'z')']'(xyz)'$$



Q4. Implement the following Boolean function F , together with the don't-care conditions d , using no more than two NOR gates:

$$F(A, B, C, D) = \sum(2, 4, 10, 12, 14)$$

$$d(A, B, C, D) = \sum(0, 1, 5, 8)$$

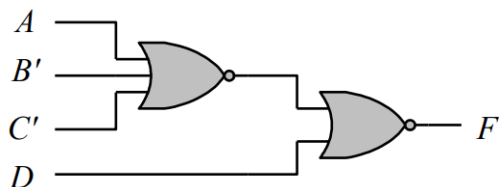


$$F = B'D' + AD' + C'D'$$

$$F' = D + A'BC$$

$$F = [D + A'BC]'$$

$$F = [D + (A + B' + C')']'$$



Q5. Design a half subtractor with 2 input variables, X(minuend) and Y(subtrahend) and 2 output variables, D(Difference) and B(Borrow)

<i>Input variables</i>		<i>Output variables</i>	
<i>X</i>	<i>Y</i>	<i>D</i>	<i>B</i>
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

From the minterms of the above table, it is clear that the Boolean expressions for the D and B are as follows:

$$D = X'Y + XY'$$

$$B = X'Y$$

