Points: 30 | Due: 17th November 23:59 | **Assignment 1** | MA5.101 | Monsoon 2022

Problem 1

Consider the following statements:

i The door is open or it didn't rain yesterday.

ii If it rained yesterday, it's sunny today.

State whether they are true or false with justification, given that it did not rain yesterday and that the door is closed. [3 points]

Solution

i True because $False \lor True$ is True.

ii True because $False \implies ?$ is True.

Grading

0.5 points for the correct answer and 1 point for the correct justification for both statements.

Problem 2

Identify the three atomic propositions present in the following statements. Use small case English letters to represent the identified propositions. Finally, write the formulas for the compound statements using connectives for all the statements.

i Oranges are ripe along the path and it is safe to walk along the path.

ii Dogs have not been seen in the area and walking on the path is safe, but oranges are ripe along the path.

iii If oranges are ripe along the path, walking is safe if and only if dogs have not been seen in the area.

iv Walking is not safe on the path whenever dogs have been seen in the area and oranges are ripe along the path.

v For walking on the path to be safe, it is necessary but not sufficient that oranges not be ripe along the path and for dogs not to have been seen in the area.

[7 points]

Solution

- d : Dogs have been in the area.
- o : Oranges are ripe along the path.
- s : Walking is safe on the path.

i $(o \wedge s)$

ii $(\neg d \wedge s \wedge o)$

iii $(o \rightarrow (s \leftrightarrow \neg d))$

iv $((d \wedge o) \rightarrow \neg s)$

v $(s \rightarrow (\neg o \wedge \neg d)) \wedge \neg((\neg o \wedge \neg d) \rightarrow s)$

Grading

2 points for identifying atomic propositions. 1 point for each correct formula.

Problem 3

Determine the contrapositive of the following statements.

i If it rains today or the field is crowded, then we can't play football.

ii Lakers advance to the finals if and only if they win their remaining matches.

[2 points]

Solution

i If we can play football, then it won't rain today and the field won't be crowded.

ii Lakers do not win their remaining matches if and only if they do not advance to the finals.

Grading

1 point each for the correct contrapositive statement.

Problem 4

Write the truth table for following formulas

i $(A \to B) \to \neg B$

ii $(X \oplus Y) \to Z$

iii $(P \to Q) \land (Q \to R)$

[3 points]

Solution

A	B	$A \implies B$	$\neg B$	$(A \to B) \to \neg B$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

X	Y	Z	$X \oplus Y$	$(X \oplus Y) \to Z$
T	T	T	F	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \land (Q \to R)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Grading

1 point for each correct truth table.

Problem 5

Determine which of the following are logically equivalent to each other:

i $(X \land Y) \lor (\neg X \land \neg Y)$

ii $\neg X \lor Y$

iii $(Y \land X) \lor \neg X$

[4 points]

Solution

X	Y	$\neg X$	$\neg Y$	$X \land Y$	$Y \land X$	$\neg X \lor Y$	$\neg X \land \neg Y$	$(X \land Y) \lor (\neg X \land \neg Y)$	$(Y \land X) \lor \neg X$
T	T	F	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F	F	F
F	T	T	F	F	F	T	F	F	T
F	F	T	T	F	F	T	T	T	T

From the table, one can see (ii) and (iii) are logically equivalent.

Grading

1 point for the correct truth table for all three formulas. 1 point for correct final answer.

Problem 6

Consider the following formulas:

1. $(X \land Y) \lor Y \lor Z \to X \oplus Y \oplus Z$
2. $A \to A \oplus B$

Fix the truth value of any one of the variables in the first statement such that both statements are logically equivalent. [3 points]

Solution

Setting $Y = f$, we get.

$$\begin{aligned}(X \land f) \lor f \lor Z \to X \oplus f \oplus Z &= (X \land f) \lor Z \to X \oplus f \oplus Z \\ &= f \lor Z \to X \oplus f \oplus Z \\ &= Z \to X \oplus f \oplus Z \\ &= Z \to X \oplus Z \\ &= A \to A \oplus B \quad \text{[Variable Renaming]}\end{aligned} \tag{1}$$

One can alternatively show the same using the truth table.

Grading

1 point for identifying that the variable Y needs to be set to F. 2 points for proving logical equivalence.

Problem 7

Simplify the boolean expression $(p \lor \neg q \lor \neg r) \land (p \lor \neg q \lor r) \land (p \lor q \lor \neg r)$. Annotate each step with the name of the law used in that step. [4 points]

Solution

$p \lor (\neg q \land \neg r)$

Grading

1 point for the correct answer. 1.5 points for annotation. 1.5 points for steps.

Problem 8

Convert the boolean expression $\neg((\neg a \implies \neg b) \wedge \neg c)$ to Disjunctive Normal Form. Annotate each step with the name of the law used in that step.

Solution

$$\begin{aligned}
\neg((\neg a \implies \neg b) \wedge \neg c) &= \neg((\neg\neg a \vee \neg b) \wedge \neg c) \quad \text{[definition]} \\
&= \neg((a \vee \neg b) \wedge \neg c) \quad \text{[double negation]} \\
&= \neg(a \vee \neg b) \vee \neg\neg c \quad \text{[de morgan's law]} \\
&= \neg(a \vee \neg b) \vee c \quad \text{[double negation]} \\
&= (\neg a \wedge b) \vee c \quad \text{[de morgan's law]}
\end{aligned} \tag{2}$$

[4 points]

Grading

1 point for the correct answer. 1.5 points for annotation. 1.5 points for steps.