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Author(s): Thomas Ricketts and James Levine

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LOGIC AND TRUTH IN FREGE

Thomas Ricketts and James Levine

I—Thomas Ricketts

Throughout his career, Frege battles against a naturalistic empiricist view of cognition and the confused admixture of psychology with logic that he blames on it.¹ In particular, Frege rejects the identification of judgments with combinations of mental representations (*Vorstellungen*) that makes judgments—like sense impressions, imaginings, feelings, and moods—into states of consciousness, the contents of judgment into contents of consciousness. Frege holds that judgment is intersubjective and objective: two individuals may make the same judgment; and what individuals hold true is true or false independently of their cognition. He urges that no view that identifies the contents of judgment with contents of consciousness can do justice to the intersubjective objectivity of judgment.

Frege's critique is commonly taken to turn straightforwardly on an alleged subjectivity of states of consciousness. For example, because two individuals cannot share numerically identical mental representations, the empiricist viewpoint cannot do justice to the intersubjectivity of judgment. Frege's conception of a *Gedanke*, a thought, appears to be a platonist antidote to an overly mentalistic view of cognition. Frege's thoughts are the contents of judgment and the bearers of truth and falsity. They are not a kind of mental state; nor are they part of the spatio-temporal order of causally interacting things. Rather, thoughts are causally inert (*unwirklich*) objects that subsist in a third realm, as Frege puts it, independent of us. This ontological status of thoughts in turn is held to explain or to explicate or to ground the intersubjective objectivity of judgment.²

1. In emphasizing Frege's opposition to a naturalistic empiricism, I follow Hans Sluga, *Gottlob Frege*, (London: Routledge and Kegan Paul, 1980), chap. 1.

2. For a recent expression of this interpretation, see Tyler Burge, 'Frege on Knowing the Third Realm', *Mind*, 101 (1992), p. 645. See also p. 646f. However, in note 16 on p. 644 Burge denies that Frege's platonism provides a 'general theory of linguistic representation or even of intentionality in judgment.'

I contest this interpretation of Frege. It misses what is distinctive and deep about the view of judgment and truth that prompts Frege's antipathy to empiricism and underlies his conception of logic. At the core of his polemic against empiricism is a rejection of any representation-theoretic view of truth. He believes that for an empiricist view of cognition to sustain the intersubjective objectivity of judgment, it must have recourse to some such view of truth, to a conception of truth as correspondence. This is where the subjectivity of the mental comes into play. For if judgments were combinations of ideas, then the objective truth of a judgment '... could consist only in a relation to something that is not inner, not mental.'³ Frege's view of judgment, I shall argue, rules out any conception of truth as correspondence, indeed any conception of truth as a property of statements or judgments. This interpretative approach prompts reexamination of the point of Frege's classifying thoughts as unwirklich. I shall argue that this classification is not intended to provide any substantive explanation of or grounding for the intersubjective objectivity of judgment.

Frege says, 'The word "true" points the direction for logic...', and he goes on immediately to call the laws of logic the laws of truth.⁴ As it is Frege's view of truth that is at issue, I propose, reversing Frege's exposition, to approach matters by a consideration of his conception of logic. How must Frege think of truth, for the label 'laws of truth' to fit the logical axioms Frege actually states? We shall see that the role the notion of truth plays in Frege's conception of logic is different from its role in more contemporary conceptions.

3. 'Logic' (1897), *Nachgelassene Schriften* (Hereafter cited as *NS*), edited by Hans Hermes, et al. (Hamburg: Felix Meiner, 1983), p. 145 (p. 134). A statement of Frege's regress argument, which I discuss in §II, immediately follows this remark. The translations from Frege's writings are my own. Following references to *NS*, I give in parentheses the parallel reference to the English translation, *Posthumous Writings*, translated by Peter Long and Roger White (Chicago: The University of Chicago Press, 1979).

4. 'Thoughts,' p. 58 and p. 59. See also, 'Logic' (before 1891), *NS*, p. 3 (p. 3) and 'Logic' (1897), *NS*, p. 139 (p. 128). Page references to Frege's papers and books are to the original publications. This pagination is either reproduced in German reprints of the books or is indicated marginally in most German and English editions. Except where otherwise indicated by parenthetical references, the cited Frege articles are translated in *Collected Papers on Mathematics, Logic, and Philosophy* edited by Brian McGuinness (Oxford: Basil Blackwell, 1984).

I

Michael Dummett asserts, 'Reality cannot be said to obey a law of logic; it is our thinking about reality that obeys such a law or flouts it.'⁵ However correct this precept may be for some contemporary views of logic, it is false of Frege's. It has been long established that Frege has a universalist conception of logic.⁶ On this view, logical laws are maximally general truths, substantive generalizations that are 'about reality' in the same fashion that the laws of geometry, physics, and chemistry are.

There are two central features of the universalist conception. First, quantification is permitted into predicate as well as subject positions; and the quantifiers in logical laws, indeed in any statement, are taken to be unrestricted over logical types. Quantifiers binding variables in proper name positions generalize over all objects; those binding variables in unary predicate positions generalize over all concepts under which objects fall; etc. The second feature is more subtle: logical laws are maximally general in that the only vocabulary required for their expression is the topic-universal vocabulary required for statements on any topic whatsoever, vocabulary that includes, for example, expressions for negation, for identity, for subsumption of concepts. This vocabulary is the proprietary vocabulary of the science of logic.⁷ Purely logical laws then set forth generalizations that, not mentioning any objects or concepts investigated by the special sciences, do not distinguish among these. It is in this sense that logic, on the universalist conception, is the maximally general science. The maximal generality of logical laws secures their universal applicability in demonstrative reasoning in any subject matter. Suppose the universalist logician wants to apply her logical laws to prove one particular scientific statement *q* from another *p*. Typically, she might first from general logical laws

5. Michael Dummett, *The Logical Basis of Metaphysics* (Cambridge, MA: Harvard University Press, 1991), p. 2.

6. Jean van Heijenoort made this point in his classic paper 'Logic as Calculus and Logic as Language', *Synthese* 17 (1967), pp. 324–30. See also Warren Goldfarb, 'Logic in the Twenties: the Nature of the Quantifier,' *Journal of Symbolic Logic* 44 (1979), pp. 351–68. Goldfarb observes that Dummett anachronistically assimilates Frege's conception of logic to a post-Gödelian one.

7. See 'On the Foundations of Geometry' (1906), p. 428. I take topic-universality to be what Frege has in mind when he speaks of the 'formal part' of everyday language in 'Boole's Calculating Logic and the Begriffsschrift', *NS*, p. 14 (p. 13).

prove the universal generalization of ‘if p then q ’: in this generalization any unstructured names apart from the logical vocabulary are replaced by variables over objects, concepts, etc. Then, instantiating variables in this generalization by names, she can obtain ‘if p then q ’, from which the given premise yields the desired conclusion by modus ponens.

Frege’s conception of logic thus differs markedly from more contemporary views. His logical laws do not describe valid forms of argument; they are about neither sentences nor the thoughts sentences express; they do not use a truth-*predicate*.⁸ Furthermore, in his controversy with Hilbert, Frege scorns talk of varying interpretations of sentences as a confused way of expressing what is properly said by the use of quantification, including quantification into predicate positions.⁹ He has then no semantic conception of logical consequence, in the post-Tarskian sense of ‘semantic’.

More than this, Frege lacks any general conception of logical consequence, any overarching conception of logic.¹⁰ Frege has only a retail conception of logic, not a wholesale one. He tells us what logic is by identifying specific laws and inferences as logical. The universalist conception of logic specifies the sort of content logical laws have; it does not state a defining criterion of the logical. So, while purely logical laws are maximally general truths, Frege is not committed to the claim that every truth couched in topic-universal vocabulary is a logical law. Here Frege’s approach to logic sharply contrasts with Wittgenstein’s. Wittgenstein’s notion of a tautology gives him a general conception of the logical. For this reason, many early readers of the *Tractatus* in Britain and Vienna viewed it as a decisive advance over a universalist conception of logic.¹¹

8. Frege’s horizontal is **not** a truth-predicate. It is, rather, a predicate that means a concept under which only the True falls. It yields a truth, only when its argument place is filled by a name of the True. Filling its argument place with any name either of a sentence or of a thought yields a falsehood. Hence, it cannot then be used for semantic ascent to formulate, for example, the soundness of inference rules.

9. See ‘On the Foundations of Geometry’ (1906), pp. 384–86.

10. The closest he comes, in a very tentative discussion in part 3 of ‘On the Foundations of Geometry’ (1906), p. 423, is a characterization of a notion of logical dependence: one truth is logically dependent on another, if the first can be obtained from the second and logical laws by logical inferences. Neither in this paper nor elsewhere does Frege give a general characterization of logical laws and inferences.

11. See Burton Dreben and Juliet Floyd, “‘Tautology’—How not to use a Word,” *Synthese* 87 (1991).

If Frege has no general conception of the logical, how does he approach the task of providing an explicit formulation of logical principles? Frege seeks to state the principles of logic, the principles of demonstrative reasoning, in order to be able to make explicit in any proof the premises from which the conclusion has been inferred. In this enterprise he is hindered, he tells us, by the irregularity and ambiguity in the expression everyday language gives to topic-universal notions, above all generality.¹² Frege devises his *Begriffsschrift* to overcome these drawbacks, and recognizes that his goal of fully explicit and rigorous proofs requires, in addition to the identification of evident truths of the formalism as axioms, the provision of inference rules. These cannot be stated within the formalism so as to serve as premises for inferences. Frege accordingly specifies in German, outside the system, evidently truth-preserving rules permitting the inference of one type of sentence from another. The types of sentences are described in notational terms, and the only specific expressions—names, in Frege's technical sense—mentioned in these rules are those expressing topic-universal notions.

Frege aims to arrive at a surveyable group of logical principles that, as he says, in *Foundations of Arithmetic* §91, 'suffice for all cases'. Frege is clear that the comprehensiveness of a group of logical principles is subject only to 'experimental' test. He aims to state logical principles that suffice for the reconstruction inside the *Begriffsschrift* of accepted, apparently correct reasoning from plausibly implicit premises. No overarching conception of logical truth or consequence plays a role here. Accordingly, following the 1879 publication of *Begriffsschrift*, Frege publicizes its significance for logic by showing how various mathematical and geometrical claims can be expressed in it with the addition of the requisite vocabulary.¹³ He also points to the derivations of the second and third sections of *Begriffsschrift*, above all his development of a general theory of series, as giving proof (*Nachweis*)

12. See *Begriffsschrift* (Halle: L. Nebert, 1879), p. iv and *Die Grundlagen der Arithmetik* (Breslau: W. Koebner, 1884) §91, p. 103.

13. See 'Applications of the *Begriffsschrift*'—an 1879 lecture Frege gave to the Jena Society for Medicine and Natural Science—in *Conceptual Notation and Other Articles* edited and translated by Terrell Ward Bynum (Oxford: Oxford University Press, 1972), and 'Boole's Calculating Logic', *NS*, especially pp. 23–36 (pp. 21–32).

...that I can manage everywhere with my primitive laws (Urgesetze). Here, of course, we attain only a probability based on the fact that I can manage in many cases.¹⁴

None of this is to say that a principle is constituted as a logical principle by its use in formalizing accepted, intuitively correct argumentation. Rather, Frege calls attention to the use and application of the *Begriffsschrift* to motivate calling it logic.

In his polemic against psychologism, Frege warns against any attempt to ground logic:

I hold it for a sure sign of a mistake, if logic has need of metaphysics and psychology, sciences that themselves require fundamental logical principles (logischen Grundsätze). Where here is then the bedrock (Urboden) on which everything rests?¹⁵

The basic idea here is clear enough: as every science draws on logic, no science can provide a foundation for logic. Frege's universalist conception of logic moulds his understanding of this theme. Basic logical principles are maximally general truths. These principles cannot be established on the basis of the more specific claims of the special sciences. More than a language for the maximally general science of logic, Frege conceives his *Begriffsschrift* as a framework for universal science. He envisions that, simply with the addition of the requisite specialized vocabulary, any fact or law uncovered by the special sciences can be expressed in it.¹⁶ Explanations in the mature sciences are provided by proofs in this framework of one truth from more basic truths. In this framework, there will be no proofs, no explanations, for basic logical laws. The very attempt would, from Frege's viewpoint, be vitiated by vicious circularity. Nor is there some metalinguistic, or extra-scientific, or transcendental grounding for logic. For Frege, truth is scientific truth. There is no realm of truths not subject to logic, not expressible within the framework of the *Begriffsschrift*.

14. 'Boole's Calculating Logic,' *NS*, p. 42 (p. 38). I believe that this passage is the closest Frege comes to claiming comprehensiveness for a formulation of logic. Perhaps as a result of the reflections that led him to embrace Basic Law V, Frege makes no claim of comprehensiveness for the system of *Grundgesetze*, and seems to be open to discovering further basic logical principles. See 'On the *Begriffsschrift* of Mr. Peano and My Own', p. 221.

15. *Grundgesetze der Arithmetik*, v. 1 (Jena: H. Pohle, 1893), p. xix. See also *Grundlagen*, §26, the final two sentences, p. 36.

16. See *Begriffsschrift*, p. vi; 'On the Scientific Justification for a *Begriffsschrift*', pp. 55–56 (pp. 88–89 in *Conceptual Notation*).

Frege voices his universalist conception of logic by calling logical laws ‘the laws of truth’ (Gesetze des Wahrseins). Logical laws are not laws of thought, not generalizations about human cognition. Instead, they prescribe how we should think. Nevertheless, Frege denies that logical laws are, as regards their content, general normative prescriptions. Instead, they are laws in the sense in which laws of nature are laws—generalizations that set forth what is.¹⁷ Any general assertion can be viewed, Frege holds, as prescribing that one ought to think in accordance with it, the laws of geometry and physics no less than the laws of logic. The laws of geometry set forth generalizations concerning spatial concepts and relations—they are, so to speak, the laws of truth concerning these concepts and relations. Similarly, the fundamental laws of any special science set forth true generalizations, laws of truth, for some range of concepts proprietary to that science. By their topic-universality, the laws of logic are the laws of truth for every object, concept, and relation—they are the laws of truth simpliciter. In the same way that the assertion of the laws of geometry prescribe how one should think about geometrical matters, the laws of logic prescribe how one should think about every subject matter. Just as any geometrical investigation or application draws on the Euclidean axioms, any inquiry draws on the basic laws of truth, draws on maximally general truths. Frege says in the foreword to *Grundgesetze*:

Each law that states what is can be conceived as prescribing that one ought to think in accord with it, and so, in this sense, is a law of thought. This holds no less for geometrical and physical laws than for logical ones. The latter have better title to the name ‘laws of thought’ only if one wishes to say that they are the most general laws that prescribe how one should think about anything at all that one thinks about.¹⁸

The universalist conception of logic thus carries with it a monolithic view of truth. In addition to scientific statements formalizable in the begriffsschrift, there are what Frege calls elucidations. These are the things one might say to convey some notion that cannot be defined in other more basic terms. Some elucidatory remarks, having facilitated communication, may

17. ‘Thoughts’, p. 58.

18. *Grundgesetze der Arithmetik*, v. 1, p. xv. See also, ‘Logic’ (1897), *NS*, p. 139 (p. 128).

themselves be straightforwardly stated in the *begriffsschrift*. Other elucidations, like those employing analogies, will not be thus formulable. Frege's logical innovations and notational novelties require extensive elucidation. Some of this rhetoric—including, I believe, much of what we tend to think of as Frege's semantics—is not statable within the framework of the *begriffsschrift*. Frege's universalist conception of logic gives it an anomalous status.¹⁹

II

I have stressed Frege's conception of logic as a science alongside others, distinguished by the maximal generality of its laws. This emphasis slights logic's defining concern with inference,²⁰ a concern that surfaced in my talk of the applicability of general logical laws and in my mention of the need for inference rules in addition to axioms in Frege's formulation of logic. Here we encounter a further, deeper way in which the word 'true' points the way for logic.

For Frege, our capacity to make judgments, to recognize truths, is an unquestioned given. One facet of our capacity for knowledge is the ability to draw demonstrative inferences, to recognize one truth on the basis of the recognition of other truths. How is the notion of truth operative in Frege's descriptions of judgment, inference, and assertion to be understood? Frege observes that the word 'true' is a grammatical predicate, and in the usage he wants to highlight, commonly predicated of sentences. But sentences, considered only as series of marks or noises, are not true or false. A sentence is true or false in virtue of its content, its sense. Frege's empiricist *bête noir* thinks of the judgments that sentences express as combinations of mental representations. Accordingly, the truth

19. Frege's elucidations of the function–object distinction are the clearest examples of elucidatory remarks that cannot be captured in the *begriffsschrift*. In several places in his writing quite apart from the function–object distinction, Frege shows an awareness of the need to use variously inaccurate, vague, or figurative statements to convey difficult ideas. See *Grundgesetze*, p. 4; 'Logic' (Before 1891), *NS*, p. 7 (p. 7); 'Boole's Calculating Logic', *NS*, p. 30 (p. 27); 'Logic' (1897), *NS*, p. 158 (p. 146); Frege to Hilbert 27.12.99, *Wissenschaftlicher Briefwechsel*, edited by Hans Hermes et al. (Hamburg: Felix Meiner, 1976) p. 63; 'On the Foundations of Geometry' (1906), p. 301; 'Logic in Mathematics', *NS*, p. 224 (p. 207). Here I am very much indebted to Joan Weiner, who emphasizes the importance of Frege's view of elucidations to understanding his viewpoint in her book *Frege in Perspective* (Ithaca: Cornell University Press, 1990).

20. 'Logic' (Before 1891) *NS*, p. 3 (p. 3) and '17 Key Sentences on Logic', *NS*, p. 190 (p. 175).

of judgments about material bodies must, it seems, consist in some agreement between the mental representations and the bodies they represent. Frege's regress argument against the definability of truth is directed against this view.

Suppose that truth can be defined as a certain correspondence between mental representations and reality.²¹ Relying on the elucidatory slogan that judgment is the recognition of truth, Frege deploys an 'open question' style argument to generate a vicious regress, urging that any such characterization of truth would render judgment impossible. Let us use the phrase 'corresponds with Reality' as a placeholder for the alleged definition of truth. To judge that Socrates is mortal is to recognize the truth that Socrates is mortal. On the correspondence definition of truth, one is in a position to recognize this truth only by applying the definition to determine that the idea representing that Socrates is mortal corresponds with Reality. But to determine anything to be the case is to make a judgment, in this case the judgment that a certain idea has a certain property. To be in a position to make this judgment requires a second application of the definition, this time to the idea representing the correspondence of the idea of Socrates' being mortal with Reality. We have thus embarked on a regress in which cognizers are in a position to judge that *p* only if they have already judged that the idea of *p* corresponds with Reality. Cognizers are then never then in a position to make any judgments at all, a manifest absurdity.

As Dummett notes, the regress argument has the appearance of sophistry.²² To overcome this appearance, we should use the argument to unearth Frege's guiding assumptions about judgment and truth. How must Frege view judgment in order that the regress just noted be genuine and vicious? To begin with, the argument assumes first that successive applications of a characterization of truth as correspondence will generate a series of distinct contents of judgment. Surely, this assumption is plausible with respect to a definition of truth for mental representations, the express target of the argument. Even so, the regress would not be vicious, either if

21. Frege states the regress argument in 'Thoughts', p. 60 and 'Logic' (1897), *NS*, p. 140 (pp. 128–29) and p. 146 (p. 134).

22. Michael Dummett *Frege: Philosophy of Language*, second edition (Cambridge, MA: Harvard University Press, 1984), p. 443. Indeed, Dummett argues that Frege's argument, as stated, is specious.

we could over a finite time span make infinitely many judgments or if each judgment had as its content an infinite conjunction. Frege explicitly rejects the first option²³ and would, no doubt, reject the second as well. After all, Frege thinks each distinct content of judgment can be affirmed or denied by itself. Just this view underlies his test that two sentences express different senses, if it is possible to hold true what one says, while holding false what the other says.

More importantly, the regress argument assumes that if truth is characterized as correspondence, then the judgment that **p** requires the prior judgment that **p** corresponds to Reality, i.e., that it is true that **p**. Here is the tendentious linchpin of the argument. To many contemporary ears, this assumption will sound gratuitous. Surely, a person can use mental representations to arrive at knowledge without knowing how those representations represent, without knowing anything about those representations. This view in turn rests on a sharp separation of the notion of truth from that of judgment, a separation familiar from many historical and contemporary views. Judgment is identified with (the formation of) belief, and truth and falsehood are properties of beliefs or of what is believed. G. E. Moore in 'The Nature of Judgment', and Russell following him, adopt such a view. Judging is a binary relation between minds and propositions. Propositions are non-linguistic, nonmental complexes. Each proposition is either true or false, truth and falsity being simple, indefinable properties of propositions. As Russell observes, on this approach it is not at all platitudinous that individuals do or ought to strive to make true judgments.²⁴

Although Frege urges that what is true is so independent of anyone's judging it so, he does not share the Moore–Russell approach to judgment. For Frege, the conceptions of judgment and truth are intertwined, as enunciated in his dictum that judgment is

23. See Frege's discussion of Dedekind's notorious proof that there are infinitely many objects in 'Logic' (1897), *NS*, pp. 148–49 (p. 136).

24. See G. E. Moore, 'The Nature of Judgment', *Mind* n.s. 8 (1899) and his entry 'Truth and Falsity' for *Dictionary of Philosophy and Psychology*, v. 2, edited by James Mark Baldwin (New York: Macmillan, 1901), p. 717. Russell puzzles over the relationship between judgment and truth in 'Meinong's Theory of Complexes and Assumptions', *Mind* n.s. 13 (1904), especially p. 524. I think that Russell's worries here partly motivate his shift some years later to a multiple relation analysis of judgment and a correspondence conception of truth.

the recognition (anerkennen) of the truth of a thought. This dictum is equally elucidatory of judgment and truth.²⁵ Truth is thus the goal of judging, and judging is the recognition of truth. We have no grasp on the one apart from a grasp of the other. Frege's use here of a quasi-factive verb with jurisprudential associations is deliberate.²⁶ he aligns judgment with knowledge, not belief. To make a judgment is to acquire a piece of knowledge; our capacity for judgment is a capacity to arrive at knowledge. The capacity for judgment is defeasible in that not every exercise of it results in knowledge: not every holding-true (fürwahrhalten) is a recognition-as-true. Still, it is recognition-as-true, not holding-true, that is the prior notion. A holding-true is a putative recognition-as-true, an exercise, perhaps flawed, of the capacity for knowledge.

In the context of this view of judgment, Frege considers the proposal to identify truth with some particular property of mental states. This proposal, in giving us a characterization of truth independent from our purchase on judging, foists a particular construal on Frege's elucidatory dictum: to recognize a thought to be true must be to recognize a mental representation to have a particular property. The genus judgment is thus assimilated to one of its species, the discrimination whether things of some restricted range have a particular property. In the context of the assumptions noted three paragraphs back, this equation of a species with its genus generates a vicious regress.

The regress argument explains why Frege thinks that an empiricist view of cognition collapses into subjective idealism, and accordingly restricts his critique of the identification of thoughts with mental representations to very crude views. For deprived of a correspondence view of truth, the empiricist must, Frege believes, hold that combinations of ideas in individual minds, apart from any relation to anything extra-mental, are in

25. The dictum is prominent in Frege's late essay, 'Thoughts', p. 62. It is stated in the four earlier drafts of this material that survive in Frege's Nachlaß. See 'Logic' (Before 1891), p. 2 (p. 2); 'Logic' (1897), p. 150 (p. 139); 'Introduction to Logic,' p. 201 (p. 185); 'A Brief Survey of my Logical Doctrines,' p. 213 (p. 197). It also appears in *Grundgesetze* §5, p. 9. I take it to express a stable element in Frege's conception of judgment and truth.

26. Moriz Heyne in *Deutsches Wörterbuch* (Leipzig: 1890) defines 'anerkennen' by: 'stärkeres erkennen, mit dem Beisinn des Würdigens und der Gesetzlichkeit.' Theodor Hensius gives a definition in *Volkthümliches Wörterbuch der deutschen Sprache*, Bd. 1 (Hannover: 1818) that also occurs in several later dictionaries: 'etwas für das erkennen und erklären, was es wirklich ist.'

themselves what are true or false. In Frege's eyes, such a conception of judgment lacks motivation altogether:

Standing by a river, swirls of water are often observed. Wouldn't it be absurd to claim that such a swirl was valid or true, or even false? And even if the atoms or molecules in my brain danced around each other a thousand times more spirited and frenzied than gnats on a nice summer night, would it not be just as absurd to maintain that this dance was valid or true? And if explanations were such dances, could they be said to be true? And finally, is it any different, if the explanations were jumbles of ideas?²⁷

Nothing in Frege's regress argument turns on any special features of mental representations or on the particular property truth is identified with. Frege accordingly takes the argument to rule out any definition of truth, concluding that 'the content [Inhalt] of the word "true" is unique [einzigartig] and indefinable.'²⁸ But the regress argument, as I have presented it, seems to support the stronger conclusion that truth is not a property at all. Frege does, of course, frequently talk as if truth were a primitive, indefinable property of some thoughts, just as identity is an indefinably primitive relation. However, in several places Frege resists this treatment of truth, albeit without providing a definitive discussion of the predicate 'true'. Indeed, in the passage from the paper 'Thoughts' just quoted, he signals this resistance by calling the content of the word 'true,' not only indefinable, but also unique, indicating that its content does not comfortably fit in any logical category. On the next page, Frege brings out this singularity with the observation that anyone who recognizes that some thing has a property thereby finds the thought that the thing has the property to be true; and he urges that no content is added to a sentence by prefixing the words 'it is true that' to it. He suggests that 'what we are dealing with here is certainly not in the otherwise ordinary sense a property,' but says that he nevertheless will speak 'as if truth were a property until some more suitable way of talking is found.'²⁹

27. 'Logic', (1897), *NS*, p. 156 (p. 144). This remark concludes a passage in which Frege argues that once judgments are identified with mental states, no psychophysical account of the origins of these states can qualify any of them as objective truths. I believe that the entire line of reflection here presupposes the regress argument, which Frege has already stated twice in this draft essay.

28. 'Thoughts', p. 60. See 'Logic' (1897), *NS*, pp. 140 (p. 129).

A short piece in Frege's *Nachlaß* entitled 'My Basic [grundlegende] Logical Insights' illuminates these enigmatic remarks. Noting the superfluity in some contexts of the phrase 'it is true that', Frege claims, 'The word "true" has a sense that contributes nothing to the sense of a sentence in which it occurs as a predicate.' He continues:

But precisely in this way, this word shows itself suited to point to the essence of logic....For in this way, the word 'true' seems to make the impossible possible, namely to make what corresponds to the asserting force appear as a contribution to the thought. Although this attempt fails—or rather precisely by means of its failure—it points to the distinguishing feature (*Eigenart*) of logic.... 'True' makes an unsuccessful attempt to point to the essence of logic in that what really pertains to logic lies not in the word 'true' but in the asserting force with which a sentence is spoken.³⁰

Frege says in conclusion that the need to use the seemingly contentless (*inhaltsleer*) word 'true' in the establishment (*Grundlegung*) of logic arises from the imperfection of language.

What guiding insight does Frege present here? The recognition-as-true of a thought is what an assertion of a sentence that expresses the thought publicly manifests. The word 'true' is an attempt to formulate as a predicate the import of asserting something, by splitting off 'true' from 'recognize-as-true' and treating 'true' as simply a predicate of what is judged. Were this possible, given Frege's characterization of judgment and assertion, every assertion would then *ipso facto* implicitly predicate truth of a thought. The contentless character of the word 'true' shows the failure of this attempt to formulate as a predicate, as a condition, the goal of judgment. For example, since the sentence 'Sea water is salty', expresses the same sense as 'It is true that sea water is salty', use of a truth-predicate here makes nothing explicit that was previously implicit. In the use of the sentence 'Sea water is salty' to assert that sea water is salty, nothing is predicated of what is asserted. In general, in an assertion nothing is predicated of the thought expressed by the asserted sentence.

29. 'Thoughts', p. 61f. 'Trotz diesem Zweifel will ich mich zunächst nach dem Sprachgebrauche folgend so ausdrücken, als ob die Wahrheit eine Eigenschaft wäre, bis etwas Zutreffenderes gefunden sein wird.' Note the use of the subjunctive.

30. 'My Basic Logical Insights', *NS*, p. 272 (p. 252). According to Scholz, this piece dates from 1915 and so shortly precedes 'Thoughts.'

Hence, the attempt to express the goal of judgment by a regular predicate is futile.

Frege's view of truth here is not a version of the redundancy theory. Frege does not use the equivalence of sentences of the form 'p' and 'it is true that p' to argue that the truth predicate is eliminable, and to conclude from this eliminability that truth is not a property. Instead, I read Frege, in the context of his conception of judgment, to argue that were truth a property, then a truth-predicate would be required to make a predication implicit in every assertion explicit. The superfluity of certain uses of the word 'true' shows that the word 'true' does not function in this way. The term 'property' is not part of Frege's own favoured ontological vocabulary; and his remarks on the way the truth-predicate fits into his logical grammar are hesitant. I suggest that the best way to accommodate Frege's remarks about 'true' is to deny that this predicate means a concept. Frege says that 'true' must have a sense, albeit an 'empty' one, in order for sentences in which it occurs to have a sense.³¹ The unique sense of 'true' insures that its lack of meaning does not deprive some of the sentences in which it occurs of a meaning.

Frege makes much the same point about the truth-predicate in his paper 'On Sense and Meaning', when he discusses his identification of the truth-values, the True and the False, as the meanings of sentences. Frege observes the superfluity of some singular predicative uses of 'true', and concludes that the relationship of a thought to the True is not that of subject to predicate, but that of sense to meaning. Frege is not here analyzing the simple unary predicate 'is true' by the compound predicate 'presents the True', as though being true were a matter of bearing a relation to a certain object. Having distinguished the sense of a (non-sentential) proper name from its meaning, Frege observes that we care whether names have a meaning only in the context of science, of inquiry into the truth of thoughts expressed by sentences containing the name: 'The striving after truth is what everywhere drives us from sense to meaning.'³² Frege emphasizes that judgment 'is something entirely singular and incomparable (ganz Eigenartiges und Unvergleichliches)',³³ an advance from a

31. 'My Basic Logical Insights', *NS*, p. 272 (p. 252).

32. 'On Sense and Meaning', p. 33.

33. 'On Sense and Meaning', p. 35.

thought to a truth-value, from the level of sense to the level of meaning, of what is objective.³⁴ The relationship between a thought and its truth-value is not describable in a sentence—it is not a matter of a thought's falling under a concept or of a relation's holding between two objects. Rather, the relationship between the thought that Socrates is mortal and the True is linguistically expressed by an indication of the asserting force with which a sentence expressing the thought is uttered by someone who has recognized-the-truth of the thought.³⁵

How, though, does the essence of logic concern the asserting force with which statements may be uttered? The task of logic is the erection of principles of inference, principles for the recognition as true of one thought on the basis of the recognition as true of other thoughts.³⁶ Such inferences are what the assertion of subconclusions in proofs publicly manifest. In this way, logic concerns asserting force. I noted that in Frege's notationally exacting formulation of logic, some principles must take the form of inference rules. It is tempting to use a truth-predicate in generalizations to explain these rules, to get a novice in the use of *begriffsschrift* to appreciate its soundness. Indeed, use of a truth-predicate is all but a pedagogical necessity to explain to a novice the significance of various notational distinctions and manipulations in *begriffsschrift*. Frege thinks such explanations would be unnecessary, if everyday language gave uniform and perspicuous expression to the topic-universal notions of logic. Then either there would be no need to formulate logical principles—because no difficulty in readily identifying the premises on which any conclusion depends—or logical principles could without ado be read off from language.³⁷ Within logic, no

34. 'On Sense and Meaning', p. 35 and p. 34.

35. See 'On Sense and Meaning', p. 34; 'Introduction to Logic', *NS*, p. 211 (p. 194); and 'Logic in Mathematics', *NS*, p. 251–52 (pp. 233–34).

36. Frege characterizes inference as the recognition of one thought as true on the basis of the previous recognition of another thought as true in accordance with logical laws. See 'On the Foundations of Geometry' (1906), p. 387 and Frege to Dingler 31.I.17, *Wissenschaftlicher Briefwechsel*, p. 30. I take the mention of logical laws to be an amplification of the notion of basis (*Grund*) at work in this elucidation of inference. For Frege all inference is logical (demonstrative) inference, and non-logical principles of inference are, properly speaking, non-logical premises for inference. Frege's view of inferring as a kind of judging makes unproblematic his otherwise puzzling and repeated remark that nothing can be inferred from false premises. See, for example, 'Negation', p. 56, 'Compound Thought', p. 47, and Frege to Jourdain (undated), *Wissenschaftlicher Briefwechsel*, p. 118.

37. See 'My Basic Logical Insights', *NS*, p. 272 (p. 252), the penultimate paragraph.

truth-predicate is required, only an indication of asserting force. Here, I believe, is the more suitable way of speaking to which Frege alludes in ‘Thoughts’.

Some will urge that this interpretation must be incorrect, for this view of the truth-predicate would debar Frege from proving the soundness of the *begriffsschrift*, and thus exhibiting its basis. This objection gets Frege’s priorities backward. Even apart from its use of a truth-predicate, Frege would find the attempt to prove his formalism sound to be pointless. Such a proof could achieve scientific status only via formalization inside the framework provided by the formulation of logic it proves sound. The resulting circularity would, in Frege’s eyes, vitiate the proof as any sort of justification for the formalism. It is striking how Frege avoids even informal soundness arguments in his exposition of inference rules in *Grundgesetze* §§14–25. Frege explains his inference rules by arguing informally in a mixture of German and *begriffsschrift* for the truth of conditionals corresponding to representative applications of the rules.³⁸

I have been trying to explain why, on Frege’s view of judgment, truth, the goal of judgment, is not to be conceived as a property, a concept, a condition on thoughts. It is a consequence of this view that there can be, in a sense, no genuine theorizing about logic. There is only theorizing within logic—the proof of derived logical laws from basic logical laws and the application of logic in formal proofs within the framework of the *begriffsschrift* to the laws and facts uncovered by the special sciences. Here Frege’s view of judgment meshes with and motivates his universalist conception of logic. Our grasp of the goal of judgment, our occupancy of the status of cognizers, is exhibited, not in our applications of the predicate ‘true’, but in our manifestations of judgments in assertions. To be more precise, an individual’s assertions exhibit that individual’s mastery of what truth concerning some particular subject matter comes to. A grasp of the notion of truth per se is displayed in an individual’s ability to reason, to draw logical inferences, to recognize one truth on the basis of the recognition

38. Frege’s phrase ‘is the True’ is not a truth-predicate; it is the translation into German of Frege’s horizontal. See footnote 8. The one place where Frege has recourse to the use of a truth-predicate in generalizations is in his very tentative discussion of independence proofs in the third part of ‘On the Foundations of Geometry’, (1906), pp. 426–27. He opines that these generalizations would be the laws of a new science.

of others. Frege aims to give notationally explicit, rigorous formulation to the principles whose application is implicit in the exercise of this ability via the logical axioms and inference rules of his *Begriffsschrift*. In this way, 'In the laws of truth the significance (*Bedeutung*) of the word "true" is unfolded.'³⁹

III

In summarizing his life's work, Frege says:

What is distinctive about my conception of logic comes out first in that I give top priority to the content of the word 'true' and then that I immediately introduce thoughts as that concerning which the question of truth arises.⁴⁰

We have seen that Frege's notion of a thought does not figure directly in his view of the subject matter of logic. Logical laws are not about thoughts, except insofar as they generalize over all objects. Nor do logical laws describe, reflect, or depend on relations among thoughts. The notion of a thought, instead, figures in Frege's conception of judgment. He explicates the notion by indicating its application in the systematic description of inquiry, judgment, and assertion. So, when I ask whether the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides, I grasp a thought and am confronted with a demand to recognize it as true or reject it as false. To make a judgment is to recognize the truth of a thought, which may then be communicated to others by the assertion of a sentence expressing the thought.

Frege's understanding of judging shapes his conception of what is judged. He holds, 'What I recognize as true, I judge to be true completely independent of my recognition of its truth, and also independent of whether I think of it.'⁴¹ The independence that Frege calls attention to here pertains to judging, to what it is to recognize something to be true. He says in the unpublished 1897 'Logic' manuscript:

If someone wanted to contradict that what is true is true independent of our recognition, he would by his very assertion

39. 'Thoughts', p. 59. Cf. 'Logic' (Before 1891), *NS*, p. 3 (p. 3).

40. 'Notes for Ludwig Darmstaedter', *NS*, p. 273 (p. 253).

41. 'Thoughts', p. 74.

contradict what he asserted, similar to the way in which a Cretan who said that all Cretans lie would.⁴²

The contradiction here is not formal, but, as we might say, pragmatic. It would be self-defeating to try to assert, to put forward as true, the claim that what is true is not true independent of cognition, because truth, according to Frege, is truth-independent-of-cognition. As I have argued, this notion, so to speak, of objective truth cannot be split off from recognition-of-the-objective-truth-of, from asserting force, to be the content of a predicate. The basic elements in Frege's conception of thoughts articulate aspects of the objective truth cognized in acts of judging.

First, Frege believes that while an act of recognizing the truth of a thought occurs at a particular time, the truth recognized is tenseless. That is, thoughts are not recognized as true for some times, but not others; nor are thoughts affirmed of some things, but not others. Any restriction or condition on the judgment of a thought properly pertains to the thought, to the content judged. This feature of judging is evident, Frege believes, in the grammatical predicate 'is true', which Frege takes to be untensed and to contain no hidden argument places. Similarly, on this basis Frege maintains that tensed sentences as well as sentences containing personal pronouns and demonstratives do not by themselves express thoughts. Rather, it is only the sentence together with relevant features of the circumstances of a particular utterance that expresses a thought.⁴³

Second, being true is independent from being held true. In the foreword to *Grundgesetze*, Frege says:

Being true (Wahrsein) is something other than being held true (Fürwahrgehalten werden), whether by one, by many, or by all, and is in no way reducible to it. It is not a contradiction that something is true which is held by everyone to be to be false....If it is true that I am writing this on July 13, 1893 in my room, while outside the wind howls, it remains true, even if every human should later hold it false.⁴⁴

42. 'Logic' (1897), *NS*, p. 144 (p. 132). The editors of the *Nachgelassene Schriften* observe that earlier scholars working on the Frege Nachlaß noted that this paragraph and the preceding one were crossed out in the manuscript. This remark is the closest Frege ever comes to mentioning the semantical paradoxes.

43. See 'Thoughts', pp. 76 and 64; *Grundgesetze*, p. xvi; 'Logic' (1897), *NS*, p. 146f (p. 134); 'Logic' (Before 1891), *NS*, pp. 4–5 (pp. 4–5).

44. *Grundgesetze*, p. xvf.

The notion of independence invoked here is, in fact, an informal notion of irrefutability. Frege's point is that the thought that Socrates is mortal is consistent, for example, with the thought that everyone denies that Socrates is mortal. That is, there is no fundamental characterization of belief, of holding-true, from which, for example, the conjunction

Socrates is mortal and everyone believes Socrates is not mortal,
is refutable.

Third, the cognition-independence of truth carries with it the possibility of agreement in judgment. Two individuals may both judge that the square of the hypotenuse of a right triangle is equal to the sum of the square of the other two sides. And if they both so judge, then there is something they both know. Frege takes the surface grammar here seriously.⁴⁵ Construing the clauses of indirect discourse to be proper names, he takes judging to be a relation that holds between cognizers and thoughts. When two individuals agree, there is a numerically identical object to which they bear a certain relation. This view does not explain the possibility of agreement, as though two people can make the same judgment because there is some thing they both judge. Rather, against the backdrop of the logical grammar Frege finds in descriptions of cognition, it sets this possibility forth.

Fourth, the timeless, cognition-independent character of the truth recognized in acts of judging also makes it appropriate to speak of understanding the thought expressed in a question as grasping something already there, something not produced by thinking. What sort of objects are these? We have seen how Frege uses the regress argument to reject the identification of thoughts with states of consciousness. With scant argument, he dismisses any identification of thoughts with some range of objects in the spatio-temporal causal order. Thoughts are then causally inert, unwirklich. As its etymology suggests, the *unwirklich* is Frege's default category. He emphasizes that thoughts are unwirklich in order to underscore the inadequacy of an empiricist conception of judgment which, apart from ideas, allows at best only for the recognition of things that via their causal interactions with us are represented by ideas. In the essay 'Thoughts', at the end of Frege's

45. See 'Logic' (1897), *NS*, p. 148f (p. 137).

career, after the paradox, thoughts then take up the role in his polemic against empiricism played by numbers in *Foundations of Arithmetic* and the foreword to *Grundgesetze*.

I said that these features of Frege's view of thoughts articulate aspects of his notion of objective truth. They should not be viewed as giving an account or explanation in ontological terms that secures objective truth. Frege's view of judgment precludes any explanation here. The judgment of the existential generalization that there are Fregean thoughts has the objective status of all our judgments. There is no judgment that voices an explanation within science—the only sort of explanation there is—of this status. In the context of Frege's view of judgment, the recognition of mind-independent objects with features that bar their identification with spatial, causally active objects is unproblematic, but also unexplanatory of objectivity.

Indeed, on Frege's view of judgment, there is no general notion of something's making a truth's true. Such a notion has figured prominently in contemporary characterizations of Frege as a realist who holds that the thoughts expressed by our sentences are their truth-conditions, which are satisfied or not by the way that things objectively are, independent of our knowledge. Frege's understanding of truth, in precluding a genuine truth-predicate—one usable in generalizations—also rules out truth-conditional semantics, rules out, that is, a theory of how (the thoughts expressed by) sentences are determined to be true or false by the items referred to in them. The sentence 'Sea water is salty' expresses the thought that sea water is salty. The attempt to state 'truth-conditions' here yields, on Frege's view, only the tautology that sea water is salty if and only if sea water is salty. As Frege lacks any truth-conditional semantics, the term 'realist', as Dummett explains it, is not applicable to his position. But Frege is no idealist either. He rather begins philosophizing from a conception of objective truth that is immanent within judgment-making. There is no explanation or account of this objectivity. There is only the fully explicit manifestation using the begriffsschrift of the (objective) judgments we have made.⁴⁶

46. Tyler Burge commented on an earlier version of this paper at an American Philosophical Association symposium; I am grateful for his criticisms. I have also benefited from criticisms and comments from Cora Diamond, Burton Dreben, Gary Ebbs, Warren Goldfarb, Peter Hylton, Gary Kemp, Jason Stanley, Markus Stepanians, and especially Jamie Tappenden.

LOGIC AND TRUTH IN FREGE

Thomas Ricketts and James Levine

II—James Levine

In 'Logic and Truth in Frege', Thomas Ricketts brings together a number of apparently disparate elements of Frege's philosophy: his view of thoughts as belonging to a third realm which is neither mental nor physical; his rejection of any correspondence theory of truth; and his views of judgment and of logic.

Ricketts' central claim is that Frege's view of thoughts as subsisting in a third realm is best viewed, not as an attempt 'to explicate or ground the intersubjective objectivity of judgment', but rather as following from his most basic views of judgment and truth, views which are embedded in his conception of logic.¹ As Ricketts makes clear, once Frege introduces 'the thought' as that 'for which the question of truth can arise at all',² then—without any consideration of grounding communication or intersubjective agreement in judgment—Frege is already committed to denying that thoughts are either physical or mental.

For suppose one grants with Frege that physical entities—such as sentences considered merely as marks or noises—can be neither true nor false; and that, instead, only the contents expressed by such physical entities may be either true or false.³ Then one might wish to hold that such contents, or thoughts, expressed are mental entities; and, on this view, truth would have to consist in some kind of correspondence between such mental entities and external reality.⁴ But, according to Ricketts, Frege's most basic views of judgment, views which reflect themselves in his conception of logic, force him to reject any correspondence theory of truth. And, for Ricketts, it is in his regress argument against the definability of

1. See Ricketts, pp.121–122.

2. 'Thoughts' p. 60. (I follow Ricketts in giving references for papers appearing in *CP* to the page numbers of the original publications, given in the margins of *CP*.) See also, for example, 'Notes for Ludwig Darmstaedter', *PW*, p. 253/*NS*, p. 273.

3. See, for example, 'Logic II', *PW*, p. 129/*NS*, pp. 140–141.

4. *Ibid.*, *PW*, p. 134/*NS*, p. 145, cited by Ricketts in his note 3.

truth that Frege brings these views of judgment to bear against correspondence theories. So, for Ricketts, clarifying the regress argument alone—without any consideration of Frege's views of communication or intersubjective agreement—will show why Frege is forced to hold that thoughts cannot be mental entities; and, given his view that they cannot be physical entities either, it thereby forces him to hold that thoughts can be neither mental nor physical.

My purpose here is not to challenge the general interpretative strategy of locating Frege's discussions of thoughts in the context of his most basic logical commitments; rather, my purpose is to examine more closely the key element in the above argument against regarding thoughts as mental entities—namely, Frege's rejection of any correspondence conception of truth. I will argue that, strictly speaking, Frege's regress argument does not preclude a correspondence conception of truth: relying as it does on the details of Frege's views of definability, the regress argument rules out a Fregean *definition* of truth but does not, by itself, rule out a correspondence '*elucidation*' of truth. And I will argue further that what *does* preclude Frege from giving even a correspondence '*elucidation*' of truth is his view of logical truth as maximally general truth. Ricketts emphasizes this feature of Frege's position, but does not relate it to Frege's rejection of a correspondence view of truth in the same way I will.

By distinguishing the regress argument against the definability of truth from views of logic which force Frege to rule out even a correspondence '*elucidation*' of truth, I hope to clarify some differences between Frege and the early Wittgenstein. For I will argue that while there is a sense in which Wittgenstein agrees with Frege that truth can only be '*elucidated*' but not defined, he adheres, as opposed to Frege, to a correspondence '*elucidation*' of truth. And I will attempt to make clear that it is only by rejecting Frege's conception of logic that Wittgenstein is able to '*elucidate*' a correspondence conception of truth.

I

Ricketts suggests that in order to clarify Frege's argument against the definability of truth, we should ask how must Frege view judgment in order for the argument to be valid.⁵ I agree, but with

5. Ricketts, p. 129.

the proviso that in order to understand fully what Frege's argument does, and does not, show, we must also consider Frege's views of definition. Frege's argument, I will attempt to show, rests solely on his views of definability together with his view that the act of judgment or assertion presupposes the notion of truth.

Frege emphasizes throughout his writings that, within a system of science, not every term can be defined. For Frege, in a definition, a sign new to the system (the definiendum) is stipulated as having the same sense and meaning as a group of signs already in the system (the definiens).⁶ While the signs in the definiens may themselves have been introduced by definition, if we trace the chain of definitions back far enough 'we shall always come upon something which being a simple, is indefinable, and must be admitted to be incapable of further analysis'.⁷ While Frege emphasizes that the 'simple' or 'primitive' elements of a system—the 'building blocks' of that system—cannot be defined within that system, he allows that these primitive elements can be 'elucidated' outside of that system.

Thus in he writes:

We must admit logically primitive elements that are indefinable.... Since definitions are not possible for primitive elements, something else must enter in. I call it elucidation [Erläuterung].⁸

And a few pages later, in arguing that 'no definition is possible' of the word 'between', Frege writes:

... we shall simply have to recognize a primitive element and be satisfied with an elucidation. The latter, however, cannot appear in the system but rather must precede it. Within the system, one will just have to presuppose the word 'between' as known; just as one can never circumvent the necessity of assuming some words to be known.⁹

When Frege discusses elucidations, he stresses their imprecise nature. In formulating elucidations we are accorded a certain 'leniency of judgment';¹⁰ in communicating elucidations to

6. See, for example, 'Logic in Mathematics' *PW*, pp. 207–208/*NS*, p. 224 and *BLA*, pp. 82–83.

7. 'On Formal Theories of Arithmetic', p. 96.

8. 'Geometry II', p. 301.

9. *Ibid.*, pp. 305–306.

10. *Ibid.*, p. 302.

others, 'we must be able to count on a little goodwill and cooperative understanding, even guessing'.¹¹ Elucidation will often proceed by means of 'hints' and 'metaphors'¹² or 'figurative modes of expression'.¹³

As opposed to elucidations, definitions have a role to play within a system of science. In particular—and this will be central to his argument against the definability of truth—Frege has a specific view as to what is involved in the application (*Anwendung*) of a definition within a system of science. While a definition is first introduced as a mere stipulation, it can be used later as an assertion of identity, in which case it will function 'as a premise of inferences'.¹⁴ In particular, for Frege, a definition will be thus used or applied whenever we prove a theorem in which the defined sign appears. Thus he writes:

... if a word or sign which has been introduced by definition is used in a theorem, the only way in which it can make its appearance there is through the application [*Anwendung*] of the definition or the identity which immediately follows from it. If such an application [*Anwendung*] is never made, then there must be a mistake somewhere....¹⁵

For Frege, then, in order to establish a theorem in which a defined sign appears, we must apply (*anwenden*) that definition at some point in the proof. More specifically, Frege holds that in order to establish any result in which the definiendum appears, we must apply the definition—as an asserted identity, relating the definiens and the definiendum—to results in which only the definiens appear.

Consider, for example, how the Gaussian definition of numerical congruence modulo 3—a definition Frege often cites as an example of a legitimate definition¹⁶—may be applied. This definition may be introduced into a system by the generalized identity:

11. *Ibid.*, p. 301.

12. See, for example, 'What is a Function?', p. 665.

13. 'Geometry II', p. 301.

14. *Ibid.*, p. 303. See also 'Geometry I', p. 320 and *BLA*, pp. 82–83.

15. 'Logic in Mathematics', *PW*, p. 212/*NS*, p. 229.

16. See 'Geometry I', p. 369; 'Geometry II', p. 305; 'Logic in Mathematics', *PW*, pp. 230–234/*NS*, pp. 248–252; *COR*, p. 45/ *WB*, p. 72.

3 goes evenly into $(b-a)=a$ is congruent to b modulo 3, with the definiens to the left, and the definiendum to the right, of the identity-sign. Regarding this definition, Frege writes:

... [it] fulfils its purpose because it reduces the meaning of the word 'congruent' to the meanings of the expressions 'different' and 'a number divides evenly into a number', which are already known: It constructs the meaning out of known building blocks.¹⁷

And shortly after this passage, Frege indicates, in effect, how this definition can be applied:

... if we posit the Gaussian definition of number-congruence, then in order to recognize that 2 is congruent to 8 modulo 3, we need only the propositions

$$8-2=3+3$$

and

$$3 \text{ goes evenly into } 3+3$$

which neither contain the sign for congruence nor presuppose knowledge of it.¹⁸

That is to say, in order to prove that 2 is congruent to 8 modulo 3 (a theorem containing the definiendum), we must apply the definition of numerical congruence to results involving only the definiens (namely, that 3 goes evenly into $(8-2)$).

In Frege's Begriffsschrift notation, a sentence which is asserted within the system, is preceded by the sign '⊢', so that in this example sketched by Frege, one may first write:

$$\vdash (8-2)=(3+3)$$

and:

$$\vdash 3 \text{ goes evenly into } (3+3).$$

Then, applying the definition of numerical congruence, which now appears as the asserted identity:

$$\vdash 3 \text{ goes evenly into } (8-2)=2 \text{ is congruent to } 8 \text{ modulo } 3,$$

we are able to conclude, from these three assertions:

$$\vdash 2 \text{ is congruent to } 8 \text{ modulo } 3.$$

17. 'Geometry I', p. 305.

18. Ibid., p. 306.

What is crucial to Frege is that the judgments to which we apply the definition in the proof (here the first two judgments) must make no use of the sign of congruence nor ‘presuppose any knowledge of it’. For if the definition succeeds in ‘construct[ing] the meaning logically out of known building blocks’, then the judgments to which we apply the definition must appeal only to the ‘known building blocks’ (the *definiens*), not to the defined notion (the *definiendum*).

Frege’s argument that truth is indefinable is that there is no similar way to apply any proposed definition of truth without already presupposing the notion of truth. Thus in the 1897 paper ‘Logic’, Frege writes

Now it would be futile to employ a definition in order to make it clearer what is to be understood by ‘true’. If, for example, we wished to say ‘an idea is true if it corresponds to reality’ nothing would have been achieved, since in order to *apply* this definition [um dies anzuwenden] we should have to decide whether some idea or other did correspond to reality—in other words, whether it is true that the idea corresponds to reality. Thus we should have to presuppose the very thing that is being defined.¹⁹ [My emphasis]

Likewise, in ‘Thoughts’, after similarly criticizing the definition of truth as ‘the correspondence of an idea with something real’, Frege continues:

And any other attempt to define truth also breaks down. For in a definition certain characteristics would have to be specified. And in *application* [Anwendung—my emphasis] to any particular case the question would always arise whether it were *true* that the characteristics were present. So we should be going round in a circle.²⁰

In both these passages, Frege indicates that the reason truth cannot be defined is that such a definition could not be legitimately applied. Whereas legitimate definitions (such as the Gaussian definition of numerical congruence) can be applied without presupposing the notion defined, no definition of truth could be applied without presupposing the notion of truth.

19. ‘Logic II’, *PW*, p. 128/NS, pp. 139–140. The English translation in *PW* eliminates the last main clause of the penultimate sentence.

20. ‘Thoughts’, p. 60. Unless noted otherwise, emphasis is in the original.

Consider, as Frege suggests in ‘Logic’, any proposed definition of truth of the form ‘A is true if and only if it has such-and-such properties or stands in such-and-such a relation to such-and-such a thing’.²¹ This definition could take the form of the generalized identity:

$\emptyset A = A$ is true,

with variable ‘A’ and where the constant ‘ \emptyset ’ consists only of previously introduced terms, knowable without appeal to the notion of truth. Thus in applying the definition to establish that some specific A_1 is true, we would have to establish that $\emptyset A_1$. In Frege’s terminology, we would have to judge or assert that $\emptyset A_1$; in Begriffsschrift notation, we would have to be able to write:

$\vdash \emptyset A_1$.

Then, applying the definition as the asserted identity:

$\vdash \emptyset A_1 = A_1$ is true,

we could thereby conclude:

$\vdash A_1$ is true.

Frege’s point, however, is that even when we have written ‘ $\vdash \emptyset A_1$ ’—that is, even when we have expressed the judgment to which we were supposed to apply the proposed definition of truth—we are already presupposing the notion of truth. For even though the sentence ‘ $\emptyset A_1$ ’ does not contain the word ‘true’ and makes no mention of truth, in judging or asserting that $\emptyset A_1$, we are, according to Frege, already presupposing the notion of truth.

It is here, then, that Frege’s view of judgment comes into play. Frege characterizes judgment as the recognition or acknowledgment of the truth of a thought.²² Ricketts stresses that in doing so, Frege not only relates the notion of judgment to the notion of truth; he also seems to adopt a ‘quasi-factive’ account of judgment, whereby judgment is aligned with knowledge rather than belief.²³ For Frege, to judge that $\emptyset A_1$ is to recognize that it is true that $\emptyset A_1$. However, for the purposes of Frege’s argument, it is not necessary that judgment be ‘quasi-factive’; one need only hold that the notions of judgment and truth are conceptually linked. Thus, for

21. ‘Logic II’, *PW*, pp. 128–129/NS, p. 140.

22. See, for example, ‘Thoughts’, p. 62.

23. Ricketts, pp. 131–132.

example, if one holds simply that truth is the aim of judgment or belief, without aligning judgment (or belief) with knowledge, one would still hold that judging that $\emptyset A_1$ involves the notion of truth.²⁴ And this is all that Frege needs for his argument. For so long as judging that $\emptyset A_1$ involves the notion of truth in any way, the notion of truth has appeared in the judgment to which we were to apply the proposed definition. And this shows for Frege that we have not succeeded in constructing the notion of truth out of more primitive ‘building blocks’, for judgments to which we apply definitions are not supposed to presuppose the defined notions.

As I have presented it, Frege’s argument here is not, in essence, an infinite regress argument.²⁵ The regress which Dummett,²⁶ and, following him, Ricketts discuss is generated if one accepts, as Frege does, that judging that $\emptyset A_1$ is the same as judging that it is true that $\emptyset A_1$. For now, if we use the proposed definition of truth to eliminate this use of the word ‘true’ which has appeared in merely judging that $\emptyset A_1$, we will conclude that judging that $\emptyset A_1$ requires judging that $\emptyset \emptyset A_1$. But, again, this will be the same as judging that it is true that $\emptyset \emptyset A_1$, which, by the definition, will require judging that $\emptyset \emptyset \emptyset A_1$, and so on ad infinitum. I believe, however, that it is misleading to present Frege’s argument in this way.

While Frege holds that if a notion is definable, that notion is, in principle, eliminable from the system of science, he also holds that, strictly speaking, we *apply* a definition in order to *introduce* the definiendum in the course of proving a result in which it appears (as when we prove that 2 is congruent to 8 modulo 3) not to *eliminate* a word previously occurring in a system. For, on Frege’s view, it is only through the application of definitions that defined terms are introduced to the system in the first place. So, strictly

24. Ricketts suggests, p. 130, that Frege’s argument requires a stronger view of the connection between judgment and truth than that which Russell accepts. I do not believe that this is so. In his 1905 paper ‘The Nature of Truth’, pp. 493–494, Russell presents an argument very similar to, if not identical with, Frege’s for the indefinability of truth. Later in that paper (p. 504), Russell refers to that argument as showing ‘the impossibility of defining truth without presupposing it’. In §II, I discuss how strongly one would have to separate the notions of judgment and truth in order to avoid Frege’s argument; in §IV, I compare features of Russell’s position with Frege’s.

25. And in the passages I have cited in notes 19 and 20, Frege does not state his argument in terms of an infinite regress.

26. See Dummett, pp. 442–443. Blackburn, pp. 226–229 and Carruthers, pp. 17–32 also follow Dummett in presenting Frege’s argument in terms of an infinite regress.

speaking, the infinite regress is generated by a *misapplication* of the definition, an attempt, not to introduce the word 'true' for the first time, but rather (as when we go from 'It is true that $\emptyset A_1$ ' to ' $\emptyset \emptyset A_1$ ') to eliminate an already occurring use of the word. For Frege, the problem with the proposed definition is established once it is shown that the notion of truth has appeared in the judgment to which we were to apply the definition. For, by itself, this already establishes that the proposed definition has not succeeded in 'constructing' truth out of more primitive 'building blocks'. There is no need to argue further that attempts to eliminate the word 'true' which can appear in the judgments to which we are to apply a proposed definition of truth will lead (through a *misapplication* of the definition) to an infinite regress.

More generally, Frege's point is not that the proposed theory of truth would require us to perform an infinite number of tasks before we could establish that any thought is true. Rather, his point is that no proposed definition is capable of eliminating the notion of truth in favour of more primitive 'building blocks'. For, by Frege's view of definition, a definition can be applied only to a judgment which does not presuppose the defined notion; and, by Frege's view of judgment, the act of judgment presupposes the notion of truth. Hence by his view of definition together with his view of judgment, Frege is committed to holding that no definition is possible of truth. For applying a definition of truth would require making a judgment which does not presuppose the notion of truth; and, for Frege, this is impossible since any judgment presupposes the notion of truth.

II

The question arises, then, as to what Frege's argument against the definability of truth shows regarding his view of truth.

First, the argument shows why Frege holds that truth is not only indefinable, but also '*sui generis*'.²⁷ For the reason Frege holds that truth is indefinable is not because truth is presupposed in the *content* of judgments preceding the application of a proposed definition but rather because it is presupposed in the *act* of judging any content preceding the application of the definition. Thus, for

27. 'Thoughts', pp. 60, 61. The German is 'einzigartig', translated by Ricketts as 'unique'.

Frege, truth is reflected, not in our use of the word 'true', but rather in our acts of assertion. Accordingly, in the paragraph following his indefinability argument, Frege writes:

... it is really by using the form of an assertoric sentence that we assert truth, and to do this we do not need the word 'true'. Indeed we can say that even where we use the form of expression 'it is true that ...' the essential thing is really the assertoric form of the sentence.²⁸

For Frege, truth is a primitive undefinable element in any system of science, not because it will be an irreducible 'building block' that makes its appearance in the content of the primitive truths of any science, but rather because it makes its appearance in the act of judgment, which is needed for asserting any primitive truth of any science. Whereas the other undefinables of the *Begriffsschrift*, such as negation or identity, are symbolized by signs which contribute to the content of sentences, truth is not symbolized by any sign which contributes to any content.²⁹ Instead, in the *Begriffsschrift* notation, truth makes its appearance in the vertical judgment-stroke; and as Frege remarks, the judgment-stroke is not a name (of either an object or a function) but is rather 'a sign of its own special kind'³⁰ which 'contains the act of assertion'.³¹

Second, Frege's argument against the definability of truth does not turn on any features of a correspondence theory of truth that would distinguish it, say, from a pragmatic, or coherence theory of truth. For by Frege's argument, so long as we formulate a definition whereby sentences of the form 'A is true' may be replaced by sentences of the form ' $\emptyset A$ ', then whatever 'substantial property' we represent by ' \emptyset '—be it 'corresponds with reality' or 'coheres with our other beliefs' or 'is pragmatically useful'—we will face the same problem that such a definition cannot be applied without already presupposing the notion of truth. For Frege, 'a definition is a constituent of the system of a science',³² hence, to

28. 'Logic II', *PW*, p. 129/*NS*, p. 140. See also 'Logic in Mathematics', *PW*, p. 233/*NS* pp. 251–252, and 'My Basic Logical Insights', *PW*, p. 252/*NS*, p. 272.

29. Russell also treats truth as a unique undefinable. In the first paragraph of Chapter I of *PoM*, p. 3, Russell indicates that truth is an undefinable of mathematics (logic) even though 'it is not a constituent of the propositions' of mathematics (logic).

30. *BLA*, p. 82.

31. *Ibid.*, p. 39.

32. 'Geometry II', p. 303.

argue that truth is indefinable is to argue that there can be no explanation (Erklärung) of what truth is from within a system of science.³³ Frege's argument, then, is not directed against any specific feature of the notion of *correspondence* in the correspondence theory of truth; instead, it is directed against the notion of *theory* in any theory of truth, where the purpose of a theory is to give an explanation or definition of its subject-matter from within a system of science.

Third, that Frege's argument thus rules out any theory of truth where truth is defined by a 'substantial property' does not show, as some have suggested,³⁴ that Frege's position is in accord with contemporary 'minimalist' or 'deflationary' views of truth. For Frege's argument applies not only to definitions in which 'A is true' is analyzed as ' $\emptyset A$ ', where ' \emptyset ' represents some substantial property; it also applies to the 'deflationary' definition:

A=A is true,

in which the definiendum 'is true' is eliminated in paraphrase not in favour of some substantial predicate, but is rather simply eliminated.³⁵ For Frege, even this definition cannot be applied without already presupposing the notion of truth. Suppose we wish to use this definition to establish that A_1 is true. Here, we would have to establish A_1 —that is, we would have to be able to write:

$\vdash A_1$.

Then, applying the definition, as the asserted identity:

$\vdash A_1 = A_1$ is true

we would be able to conclude:

$\vdash A_1$ is true.

33. Frege often associates explanation with definition (see, for example, 'Geometry II', pp. 304–305), and he also does so in his argument against the definability of truth (see 'Thoughts', p. 60). Occasionally Frege uses 'Erklärung' as he would use 'Erläuterung' (see, for example, 'On Concept and Object', p. 193). I will follow what seems to be Frege's typical practice and associate explanation only with definition.

34. See, for example, Horwich, p. 6, note 2.

35. This is how Ayer, pp. 87–89, defines truth (on his 'redundancy' version of deflationism). Contemporary minimalists hold that the truth-predicate cannot be eliminated in this fashion, since this definition will not cover all cases of our use of the truth-predicate. In particular, it will not cover cases (such as 'Everything she says is true') where the sentences (or propositions) of which truth is being predicated are not explicitly expressed. See, for example, Quine, pp. 80–82.

But again, for Frege, even in the first step—even in writing ‘ $\vdash A_1$ ’—we are already presupposing the notion of truth in the assertoric force with which we utter ‘ A_1 ’. Once again, we are presupposing the notion of truth in the judgment to which we were to apply the definition. For Frege, that is, the act of judging that A_1 presupposes the notion of truth as much as the act of judging that $\emptyset A_1$, where ‘ \emptyset ’ is some ‘substantial property’. Hence, for Frege, applying the ‘deflationary’ definition of truth presupposes the notion of truth no less than applying a ‘substantial’ definition of truth does.

Frege agrees with the minimalist in accepting the equivalence of ‘It is true that p ’ and ‘ p ’; in fact, he holds that these have the same sense.³⁶ However, Frege takes this equivalence in a different way from the minimalist. The minimalist holds, first, that all that there is to the ‘nature’ of truth will be explained by accounting for our use of the word ‘true’ and, second, that everything regarding our use of the word ‘true’ can be explained by the equivalence. Thus the minimalist concludes that since everything regarding our use of the word ‘true’ can be explained without analyzing it in terms of some ‘substantial property’, there is nothing which is the ‘essence’ or ‘nature’ of truth.³⁷ While Frege agrees that asserting that p is true is equivalent to asserting that p , he holds further that asserting that p presupposes the notion of truth. Thus he holds, in opposition to the minimalist, that the nature of truth is reflected precisely *not* in our use of the word ‘true’, but rather in the ‘assertoric force with which a sentence is spoken’.³⁸ While Frege agrees with the minimalist that the nature of truth is not reflected by any predicate which could contribute to the content of a sentence, he does not conclude from this that truth has no nature. Instead, he holds that the nature of truth is reflected in the ‘assertoric force’ with which we affirm a content, not in any element of the content affirmed.

In recent years, Hilary Putnam has argued that the consistent minimalist must supply an account of assertion which presupposes no notion of truth.³⁹ For if the minimalist wishes to deflate truth

36. See, for example, ‘My Basic Logical Insights’, *PW*, p. 251/NS, p. 271.

37. See, for example, Horwich, pp. 4–6.

38. See the passages cited in note 28.

39. See, for example, his ‘On Truth’.

by emphasizing that asserting that *p* is true is equivalent to asserting that *p*, then the minimalist owes us an account of assertion which does not itself presuppose a notion of truth. Since Putnam holds that no such account of assertion is forthcoming, he concludes that truth is a 'substantial notion'. Understood properly, Frege's argument against the definability of truth is similarly directed against minimalism. For Frege holds that truth is indefinable because he holds, with Putnam, that the act of judgment or assertion presupposes the notion of truth. Putnam argues that to sever the connection between judgment and truth, a minimalist (such as Quine) might be led to view our assertions solely in terms of a cause and effect account of how we come to make certain noises in certain contexts.⁴⁰ Frege similarly suggests that psychological logicians who fail to recognize the intrinsic connection between judgment and truth are led to view our judgments simply as a 'dance of the atoms and molecules in [the] brain' or as 'jumbles of ideas' connected by mere associative 'processes'.⁴¹ And for Putnam and Frege, these positions are reductions to absurdity of the minimalist attempt to account for judgment without presupposing the notion of truth.

But now the question as to what Frege's view of truth is becomes even more pressing. For Putnam argues that assertion presupposes truth only in order to argue against minimalism and in favour of the view that truth is a 'substantial notion'. In Frege, on the other hand, the view that assertion presupposes the notion of truth becomes directed not only against minimalist accounts of truth but also, as I have discussed, against any substantial theory of truth. So, again, the question remains as to what Frege's view of truth is. To begin to address this question, I consider some features of Frege's view of elucidation and compare his position with that of the early Wittgenstein.

First of all, as I have emphasized, in arguing that truth is indefinable, Frege has concluded that no explanation or definition is possible of truth from *within* any system of science. But, as I have also mentioned, Frege holds that while primitive elements of

40. Ibid., pp. 320–321.

41. 'Logic II', *PW*, p. 144/NS, p. 156, cited by Ricketts in his note 27. I take it that the view Frege is here criticizing involves a sharper separation of judgment and truth than any separation Russell would accept (see note 24).

a system are incapable of being explained within that system, they are capable of being elucidated outside of it. So, taken by itself, the argument does not preclude Frege from holding that truth is a 'substantial' notion, albeit one which is capable only of elucidation, not definition. While the argument precludes any substantial *theory* of truth, where theories are explanations or definitions expressible within a system of science, it does not, by itself, rule out a substantial *elucidation* of truth outside any system of science.

In fact, Frege's argument seems to commit him to the view that truth *is* a substantial notion which can only be elucidated, not defined. For in so far as the issue between minimalism and substantial views of truth just *is* the issue as to whether judgment presupposes the notion of truth, then Frege's argument is directly opposed to minimalism and in favour of a substantial view. But given Frege's view of *definability*, any substantial notion of truth will be undefinable (since if the act of judgment presupposes the notion of truth, no definition of truth will be applicable without presupposing the notion to be defined). But if Frege's argument thus commits him to the view that truth is a substantial notion which is incapable of definition, the argument, by itself, gives no indication as to how the notion of truth is to be elucidated. As far as the argument is concerned, Frege could adhere to a 'correspondence' or 'coherence' or 'pragmatic' elucidation of truth.

It is not my purpose here to try and find in Frege a fully coherent or worked-out account of the status of elucidations. In general, Frege relegates elucidations to sentences which lie outside of systems of science: they 'cannot appear in the system but rather must precede it'.⁴² But he also indicates that sentences which express thoughts which are either true or false belong within 'the realm of science',⁴³ thus suggesting that any such sentence can fall within a system of science. But then the question arises as to whether elucidations express thoughts which have a truth-value.⁴⁴ Frege acknowledges that some of his elucidations—as of function and concept and object—are if 'taken literally', misleading and 'inaccurate'.⁴⁵ But he also presents his basic characterizations of

42. 'Geometry II', p. 306.

43. See, for example, 'Logic in Mathematics', *PW*, p. 232/NS, p. 250.

44. This issue is forcefully raised by Weiner, especially Chapter 6.

45. 'On Concept and Object', p. 204. See also, for example, 'Introduction to Logic', *PW*, p. 193/NS, p. 210.

judgment and of thought as elucidations, not definitions;⁴⁶ and he does not indicate that these elucidations are inevitably misleading. Nor does he indicate that his elucidations of his primitive logical signs—for example, his horizontal-stroke and conditional-stroke or the sign of identity—are inevitably misleading.

Without considering, then, whether Frege has, or can have, a coherent view of elucidations, my only point in this context is that if Frege has reasons for opposing even a correspondence elucidation of truth (as I believe he does), the source of that opposition will have to be located outside his argument against the definability of truth. For, by itself, that argument yields no conclusion as to how truth is to be elucidated. This point becomes strengthened when we consider the position of the early Wittgenstein. For while Wittgenstein agrees with the conclusion of Frege's argument that the nature of truth cannot be explained or defined within a system of science, there is, nevertheless, a sense in which he 'elucidates' a correspondence view of truth.

Unlike Frege, Wittgenstein is clear in not allowing any genuine sentence to fall outside the system of science: for Wittgenstein, the sentences with sense just *are* the sentences of natural science.⁴⁷ Whereas Frege grants elucidations a status (however tenuous) of genuine sentences which do not belong to a system of science, Wittgenstein's position precludes there being any elucidations which are both genuine sentences and also lie outside of science. Accordingly, when Wittgenstein discusses elucidations (Erläuterungen) in the *Tractatus*, he indicates that they are (genuine) sentences,⁴⁸ and denies that they are 'philosophical sentences' which lie outside of science.⁴⁹ For Wittgenstein, elucidations are (genuine) sentences which are clarifications of other (genuine) sentences. Whereas Frege holds that an elucidation is a sentence of ordinary language which *cannot* be translated into a Begriffsschrift notation, for Wittgenstein, an elucidation of a sentence of ordinary language may be a translation of that sentence into a logically perspicuous Begriffsschrift notation.

46. See 'On Sense and Meaning', p. 35; 'Thoughts', p. 60.

47. See *Tractatus* 6.53, where 'what can be said' is identified with 'the sentences of natural science'.

48. See *Tractatus* 3.263.

49. See *Tractatus* 4.112.

Wittgenstein's account of elucidations, however, does not address the status of the remarks of the *Tractatus* themselves. For the remarks of the *Tractatus* are not themselves in Begriffsschrift notation, nor are they translatable into it. In fact, the remarks of the *Tractatus* seem to have exactly the sort of status Wittgenstein denies that any (genuine) sentence can have. For, like Frege's pre-systematic elucidations, the remarks of the *Tractatus* seem to be 'philosophical sentences' which lie outside the system of science. Thus Wittgenstein cannot consistently regard the remarks of the *Tractatus* as genuine elucidations, nor can he even regard them as genuine sentences. Accordingly, in 6.54 Wittgenstein writes:

My sentences are elucidatory in this way [erläutern dadurch]: he who understands me finally recognizes them as nonsensical [unsinnig] when he has used them—as steps—to climb up beyond them....

He must surmount these sentences; then he sees the world aright.⁵⁰

Here, then, Wittgenstein indicates that while the remarks of the *Tractatus* can be elucidatory (or can elucidate), he does not indicate that they are elucidations. Elucidations are genuine sentences; specifically, they are logically perspicuous translations of other genuine sentences. Whatever they are, the remarks of the *Tractatus* are not that. Wittgenstein's remarks can be elucidatory, not because they give logically perspicuous translations of genuine sentences, but only if one recognizes that they are nonsensical—that they are not genuine sentences at all. In particular, the remarks of the *Tractatus* can be elucidatory only once one recognizes that they are nonsensical attempts to say what can only be shown. In themselves, the remarks of the *Tractatus* neither say nor show anything—they are nonsense. But what they seem, or attempt, to say is shown by the genuine sentences of language.

Just as I am not concerned here to examine the status of Frege's elucidations, neither am I concerned here to examine Wittgenstein's show/say distinction.⁵¹ Rather, I am only concerned to point out that with his show/say distinction in place, Wittgenstein

50. See in this connection Wittgenstein's *Letters to C.K. Ogden*, p. 51, where he discusses the translation of 6.54.

51. I recognize that Diamond argues against reading 6.54 in connection with the show/say distinction and seems to suggest that once 'the ladder has been thrown away' there will be no show/say distinction left. Here is not the place to examine this issue. In any case, Diamond is willing to contrast Wittgenstein's views of 'the truths of logic' with Frege's (see, for example, p. 203); and this is also my focus in what follows.

can agree with Frege that there can be no *theory* of truth, where a theory has the status of statements within science, while still embracing a correspondence view of truth. For in the *Tractatus* are remarks which present a correspondence view of truth.⁵² For Wittgenstein, these remarks are not genuine statements at all, but are rather nonsensical attempts to say what can only be shown. For Wittgenstein, that is, no genuine sentence may *say* what the nature of truth consists in; rather, genuine sentences *show* that truth consists in correspondence with reality.

I agree with Ricketts that Frege does not accept a correspondence view of truth in any sense whatsoever. That is, Frege not only denies that truth can be defined in terms of correspondence; he would also reject the view that truth should be elucidated in terms of correspondence. My only point so far is that, properly understood, his indefinability argument does not establish this strong conclusion. As Wittgenstein's position suggests, one can deny that the nature of truth can be stated within a system of science while still maintaining that truth can be 'elucidated' (in Frege's terminology) or 'shown' (in Wittgenstein's) to consist in correspondence with reality. My concern in what follows, then, is to examine what it is in Frege's position, if not his indefinability argument, that precludes him from holding, with Wittgenstein, that truth can be 'elucidated' or 'shown' to consist in correspondence, even if it cannot be so defined within a system of science.⁵³

III

By 1910, Russell comes to accept a correspondence account of truth. In the course of introducing his position in *The Problems of Philosophy*, Russell mentions 'three requisites which any theory [of truth] must fulfil', and he indicates that it is the third of his three 'requisites' which leads him to adopt a correspondence view.⁵⁴ My purpose in this section is to show how the early

52. See, for example, 2.222.

53. I recognize that Frege indicates that his indefinability argument, by itself, rules out any correspondence view of truth. Thus he nowhere considers and rules out any correspondence 'elucidation' of truth as distinct from a correspondence 'definition'. What I have argued, then, is that the indefinability argument is not as strong as Frege himself takes it to be. But I am also arguing that, given his view of logic, Frege has good reasons available to him, which he does not state, for rejecting even a correspondence 'elucidation' of truth.

54. *PoP*, pp. 120–121. I am only taking Russell's third 'requisite' as necessary, not sufficient, for accepting a correspondence view.

Wittgenstein's consistent application of Russell's third 'requisite' leads him to accept a view of logic which is incompatible with Frege's.

Russell presents his third 'requisite' by writing:

... the truth or falsehood of a belief always depends upon something which lies outside the belief itself. If I believe that Charles I died on the scaffold, I believe truly, not because of any intrinsic quality of my belief ... but because of an historical event which happened two and a half centuries ago.... Hence, although truth and falsehood are properties of beliefs, they are properties dependent upon the relations of the beliefs to other things, not upon any internal quality of the beliefs.⁵⁵

As Russell indicates later, in accepting this third 'requisite', he holds that truth and falsehood are 'extrinsic properties' of beliefs.⁵⁶

In stating this requirement, Russell is, I take it, articulating a feature common to what Ricketts calls 'representation-theoretic' views of truth. On these sorts of views, the bearer of a truth-value represents what would have to obtain in order for it to be true, but it does not determine its own truth-value. Russell holds (by 1910) that the bearer of a truth-value is a belief; but this is not essential to Russell's 'requisite'. Rather, what is crucial to this 'requisite' is that whatever the truth-bearer of a truth-value may be (be it a sentence or belief or abstract proposition), what determines its truth-value is not 'internal' or 'intrinsic' to it; instead, what determines its truth-value is the presence or absence of a 'truth-maker' which is 'extrinsic' to that truth-bearer.

Russell's requirement thus imposes a metaphysical constraint on anyone who would accept a correspondence view of truth. For if its truth-value is not 'intrinsic' to the bearer, then, metaphysically speaking, that bearer would be the same entity, would have the same 'intrinsic' nature, regardless of what truth-value it has. And if its truth-value is 'extrinsic' to the bearer, then, metaphysically speaking, what determines its truth-value—the 'truth-maker'—is independent of that bearer: whether that 'truth-maker' is present or absent is not metaphysically determined by the 'intrinsic' nature of the bearer. That is to say, the truth-bearer

55. *Ibid.*, p. 121.

56. *Ibid.*, p. 129.

could be what it is and the truth-maker fail to be present (in which case the truth-bearer will be false), or the truth-bearer could be what it is and the truth-maker be present (in which case the truth-bearer will be true). By Russell's requirement, both cases will be metaphysically possible; and thus, by Russell's requirement, in order for a truth-bearer to be either correspondence-true or correspondence-false, its truth-value will be metaphysically contingent.

In the *Tractatus*, Wittgenstein consistently applies Russell's requirement, and, in doing so, he ends up with the result that the sentences of logic are not correspondence-true or -false. That is to say, by applying Russell's requirement, Wittgenstein is led to hold that the notion of truth which applies to the sentences of logic is not the same (correspondence) notion of truth which applies to every other sentence.

According to Wittgenstein, a sentence with sense is a 'picture of reality', and, in conformity with Russell's requirement, it has its truth-value contingently. Consider, for example, how Wittgenstein accounts for the sentence 'aRb' as a picture, where 'a' and 'b' are names and 'R' is a relation expression. The names 'a' and 'b' are 'elements' of the picture: they are representatives of objects. The relational expression 'R' is not a further 'element' in the picture: it does not serve to stand for some further entity. Rather, it contributes to the 'structure' of the picture and by doing so it shows how the objects corresponding to the names would have to be arranged for the sentence to be true.⁵⁷ For Wittgenstein, then, the sentence (picture) determines which objects it is about (the names in that sentence 'rigidly designate' those objects), and it determines how those objects would have to be arranged in order for it to be true. But what determines the truth-value of the sentence is how those objects are actually arranged, and the sentence does not determine that. For Wittgenstein holds that all that is 'intrinsic' to the 'nature' of an object are its possible combinations with other objects.⁵⁸ Thus it is metaphysically possible for 'aRb' to secure reference to the objects it is about and still be either true or false: since it is an 'external' property of those

57. For Wittgenstein's distinction between 'elements' and 'structure' of a picture, see 2.13–2.15.

58. See *Tractatus* 2.0123–2.01231.

objects whether they are arranged as the sentence represents them, what determines the truth-value of the sentence has not been determined merely by determining the objects it is about. In accord with Russell's 'requisite', what determines the truth-value of that sentence (namely, how those objects are actually arranged), is not 'intrinsic' to the sentence itself.⁵⁹ As Wittgenstein writes in his *Notebooks*: 'The sentential sign guarantees the possibility of the fact which it presents (not, that this fact is actually the case)....'⁶⁰ And again: 'The sentence must *contain* (and in this way show) the *possibility of its truth*. But not more than the *possibility*.'⁶¹

Wittgenstein's consistent application of Russell's correspondence 'requisite' thus leads him to deny that there is any 'real necessity' in nature—to deny, that is, that the 'nature' or 'essence' of any object determines its occurrence in a given state of affairs. For if there were any such 'real necessity' in nature, then any sentence representing that state of affairs would not be 'correspondence' true. Once the sentence had determined the object it was about, the sentence would have thereby determined (metaphysically) its own truth-value. Since the 'nature' of that object requires that it occur in that state of affairs, it would not be possible for that sentence to be about *that* object and not be true. And in so far as it is an intrinsic property of that sentence to be about that object, then, contrary to Russell's requisite, that sentence would have determined its own truth-value from its own intrinsic properties.

Furthermore, Wittgenstein's consistent application of Russell's correspondence requirement leads him to hold that the sentences of logic are not correspondence-true. For suppose 'p' is a sentence which is correspondence-true or correspondence-false. Such a sentence, then, does not determine its truth-value from its own 'intrinsic nature'. But if we form an instance of what Frege would regard as a logical law using that sentence—if, for example, we

59. Note that Wittgenstein's 'truth-makers' are not themselves entities (objects); rather, they are arrangements of objects. That is, what determines whether a sentence is true is not whether there is a certain entity, but is rather how the entities that there are (the objects) are actually arranged. Thus Wittgenstein's version of the correspondence view of truth does not involve reifying facts (or states of affairs).

60. *NB*, p. 27.

61. *NB*, p. 16.

form the sentence 'p or not-p'—then the sentence we have thus formed is no longer correspondence-true or correspondence-false. For given our understanding of the signs 'or' and 'not', it will not be possible for the sentence 'p or not-p' to be false. Whether 'p' is true or false, 'p or not-p' will still be true. Whereas the truth-value of 'p' is not intrinsic to the sentence itself and is rather determined by something 'external' to it, the truth-value of 'p or not-p' is intrinsic to the sentence itself.

Hence, Wittgenstein cannot hold that 'p or not-p' has its truth-value in virtue of the presence or absence of any extrinsic 'truth-maker': the way in which 'p or not-p' is true is not in accord with Russell's correspondence 'requisite'. Thus by holding that the bearers of correspondence truth-values are sentences which are 'pictures of reality', or are sentences with sense, or are sentences which represent possible situations, Wittgenstein has to deny that 'p or not-p' or any other instance of what Frege would recognize as a logical law is a 'picture of reality' or represents a possible situation or has sense.⁶² For Wittgenstein, the sentences of logic are tautologies which 'say nothing' and have no sense. Whereas their truth is determined by the 'symbol alone', the truth-value of any non-logical sentence is not determined by the symbol alone, but rather by something external to that symbol.⁶³ And hence for Wittgenstein the sentences of logic are true (and their negations are false) not in virtue of any correspondence notion of truth or falsity. Given his consistent application of Russell's correspondence requirement, that is, Wittgenstein is forced to deny that the notion of truth and falsity which applies to sentences of logic and their negations is the same notion of truth and falsity—a correspondence notion of truth and falsity—which applies to every other sentence.

It is at this point that the application of Russell's correspondence 'requisite' conflicts with Frege's view of logic. For, as Ricketts stresses, Frege holds that the truths of logic are true in the exact same way as the truths of chemistry or geology or of any other science. What distinguishes the truths of logic from other truths is not the notion of truth which applies to them, but rather their generality.

62. See *Tractatus* 4.461–4.462.

63. See *Tractatus* 6.1113.

Thus, for example, in ‘Logic’ Frege writes:

... we do not demand that [logic] should go into what is peculiar to each branch of knowledge and its subject-matter. On the contrary, the task we assign logic is only that of saying what holds with the utmost generality for all thinking, whatever its subject-matter.... Consequently we can also say: logic is the science of the most general laws of truth.⁶⁴

For Frege, that is, sciences other than logic have a specific subject-matter. They refer to specific objects or employ specific concepts or relations. What is characteristic of logic, on the other hand, is that—aside from what Ricketts calls the ‘topic-universal’ vocabulary of negation, identity, generality, and so forth—logic makes no reference to any specific object or concept or relation. The truths of logic, then, are maximally general truths which quantify over all objects, concepts, and relations.⁶⁵

On Frege’s view, the world includes a totality of entities—objects, concepts, relations, and so on. What distinguishes one science from another—or one truth from another—is which of these entities it applies to. Facts (at least on Frege’s early use of this term) are truths which are not general, that is, are truths about particular objects.⁶⁶ Laws, on the other hand are general truths. Laws of the special sciences apply to one domain within the totality of entities. Thus, for example, the laws of geometry apply only to what is spatial. And to pick out their ‘peculiar’ subject-matter, the laws of geometry will employ concepts and relations specific to its domain. Logic, alone, applies to every entity: hence it makes no reference to any particular object; and it makes no use

64. ‘Logic II’, *PW*, p. 128/*NS*, p. 139.

65. Ricketts claims, p. 124, that Frege is committed only to the view that maximally general truth is necessary for logical truth, not that it is sufficient, but he cites no passage where Frege allows that a maximally general truth need not be a logical truth. In *FA*, p. 83, Frege seems to indicate that any fully generalized relational proposition, like any fully generalized object-concept proposition, will be a proposition of logic. And this suggests that Frege regards maximally general truth as not merely necessary, but also sufficient, for logical truth. The point is not crucial for my discussion here, since all that is needed for a conflict between Frege and Wittgenstein is that Frege hold that the same notion of truth applies to the truths of logic as to other truths, not that he hold that every maximally general truth is a logical truth. As I discuss in §IV, Russell clearly identifies logical truth with maximally general truth, and it is on this point that Wittgenstein criticizes Russell.

66. See *FA*, pp. 4, 90, note 1. In ‘Logic II’, *PW*, p. 131/*NS*, p. 142, Frege similarly seems to use ‘fact’ for non-general truth. By ‘Thoughts’, p. 74, Frege seems to have identified any true thought with a fact.

of any specific concept or relation in order to limit its domain of applicability.⁶⁷

For Frege, then, while truths are distinguished by their level of generality, they are not distinguished modally. Frege has no notion of worlds different from this world; he only has the notion of this world (the world) and the entities in it.⁶⁸ Accordingly, in the *Begriffsschrift*, Frege indicates that we call a proposition necessary or possible depending on the state of our knowledge.⁶⁹ Thus he does not distinguish truths into those which are metaphysically contingent from those which are metaphysically necessary; for lacking any notion of worlds other than this world, Frege has no way to distinguish a thought which is true in this world and false in another from a thought which is true in every world. All he can distinguish is a thought which applies to a limited domain of entities within this (the) world from a thought which applies to every entity in this (the) world.

By thus distinguishing truths in terms of their generality, and by rejecting any distinction of truths in terms of their (metaphysical) modality, Frege is able to regard the sentences of logic as having content and as being true in the same way as every other true sentence. For, according to Frege, the content of a sentence determines what entities it applies to. Thus the sentences of logic will have content—in fact, will have maximal content—since they apply to every entity. And like other true sentences, the sentences of logic will be true of the entities they apply to. What distinguishes the sentences of logic from other true sentences is not the sense in which they are true, but rather that they are truths regarding every entity.

Accepting a correspondence theory of truth, on the other hand, leads to the conclusion that the sentences of logic are not true in the sense in which other sentences are true. For by Russell's 'requisite', a (correspondence) truth-bearer has its truth-value only contingently. And once the notion of metaphysically

67. Thus when Frege argues that arithmetic, unlike geometry, should be associated with logic, he emphasizes that arithmetic, unlike geometry, has maximal applicability. See Tappenden, especially section III.

68. As van Heijenoort writes: 'For Frege it cannot be a question of changing universes. One could not even say that he restricts himself to *one* universe. His universe is *the* universe.' (p. 325)

69. *Conceptual Notation*, p. 114.

contingent truth has been introduced, it will then follow, as I have discussed, that an instance of a logical law will have its truth-value necessarily, in which case, by Russell's 'requisite', it cannot be correspondence-true. And, in which case, it cannot have 'content' in the same way as sentences which are correspondence-true or -false.⁷⁰ By introducing modality into the conception of truth, the correspondence view, as consistently applied by Wittgenstein, leads to the conclusion that what distinguishes the truths of logic from other truths is not their maximal generality, as Frege holds, but rather their necessity and (hence) their lack of content and lack of correspondence-truth, as Wittgenstein holds.

It is this conflict between Frege's view of logic and a correspondence conception of truth that precludes him from accepting any correspondence view, not anything that is implicit in his argument against the definability of truth. For, as I have argued, Wittgenstein can accept the conclusion of the indefinability argument (namely, that no definition or explanation of truth is possible within a system of science) and still accept a correspondence view of truth (as shown but not said), because he is willing to reject Frege's view of logic as the maximally general science. This relation between accepting a correspondence view of truth and rejecting Frege's conception of logic becomes more clear—as does Frege's conception of truth—if we consider the relation between Frege's position and Russell's as well as the relation between Russell's position and Wittgenstein's.

IV

Prior to 1910, Russell's position is similar in many respects to Frege's. Like Frege (both before and after 1891), Russell regards logical truth as maximally general truth.⁷¹ Like Frege, he rejects any fundamental metaphysical distinction between necessary and

70. In 'Compound Thoughts', p. 50, Frege argues that instances of logical laws (and their negations) have content and express thoughts. His argument is that they must express thoughts, since only thoughts have truth-values. Again, he is assuming that the notion of truth which applies to logical truth is the same notion of truth which applies to any other truth. He is perhaps here responding to Wittgenstein, as by this time, Frege had corresponded with Wittgenstein regarding the *Tractatus*.

71. Thus, for example, in his 1901 draft of the first chapter of *PoM*, Russell writes: '...logic may be defined as (1) the study of what can be said of *everything*, i.e. of the propositions which hold of all entities, together with (2) the study of the constants which occur in true propositions concerning everything.' (p. 187)

contingent truth. And—what I have argued is related—like Frege, Russell rejects any correspondence view of truth.

Thus, in holding that ‘the subject of modality ought to be banished from logic, since propositions are simply true or false’,⁷² Russell emphasizes:

... there seems to be no true proposition of which there is any sense in saying that it might have been false. One might as well say that redness might have been a taste and not a colour. What is true, is true; what is false, is false; and concerning fundamentals, there is nothing more to be said.⁷³

Like Frege, then, the early Russell lacks the modal resources to sustain a correspondence view of truth. Lacking any notion of contingency, Russell cannot hold that one and the same proposition could be true and could be false depending on the presence or absence of a metaphysically independent ‘truth-maker’. By rejecting modal distinctions as fundamental, Russell, like Frege, is in no position to hold, with Wittgenstein, that whereas non-logical truth is contingent (and correspondence-true), logical truth is necessary (and is not correspondence-true). Rather, he can hold, with Frege, that what distinguishes logical truths from other truths is not their modality, but rather their maximal generality.

Furthermore, like Frege prior to 1891, Russell holds prior to 1910, that the entities composing a propositional content will be the very entities—the objects, concepts, and relations—which that content is about and which are relevant to determining the truth-value of that content.⁷⁴ And this feature of Russell’s early position is also clearly opposed to any correspondence view of truth. For if everything relevant to determining its truth-value is internal to the propositional content itself, then there is no room for any metaphysically independent ‘truth-maker’ to determine the truth-value of that content. As Moore writes in defending the position he shares with the early Russell: ‘It is the impossibility of finding any ... difference between a truth [that is, a true proposition] and the

72. ‘Necessity and Possibility’, p. 520.

73. *PoM*, p. 454. See also ‘The Nature of Truth’, pp. 503–504, and *PoL*, pp. 23–34.

74. For Frege’s pre-1891 view that objects and concepts are constituents of propositional contents, see, for example, ‘Boole’s Logical Calculus and the Concept-Script’, *PW*, pp. 16–17/*NS*, pp. 17–19, and *FA*, p. 82.

reality to which it is supposed to correspond which refutes the [correspondence] theory.’⁷⁵ For Moore and Russell, that is, there can be no difference between the true proposition and the reality or ‘fact’ to which it is supposed to correspond, since that proposition contains the same entities related in the same way as that ‘fact’. Hence, for Moore and Russell, there is nothing extrinsic to the proposition which determines its truth-value.⁷⁶

Like their view of logic, then, this view of the constituents of propositional contents mitigates against a correspondence view of truth. But for both Frege and Russell, their view of logic as maximally general truth is more fundamental than this view of the constituents of propositional contents. For Frege (after 1891) and Russell (after 1910) both reject this view of the constituents of propositional contents while still maintaining that logical truth is maximally general truth. However, whereas Frege abandons his early view of the constituents of propositional contents while still maintaining a view of truth consistent with his view of logic, Russell abandons his early view of propositional contents by introducing a view of truth which he would realize, through the influence of Wittgenstein, undermines his view of logic.

One reason for rejecting the view of the constituents of propositional contents which Frege (prior to 1891) and Russell (prior to 1910) both accept is that it creates a certain epistemological difficulty. For if the constituents of a propositional content include all the entities relevant to determining the truth-value of that content, and if apprehending that content requires knowing (or ‘grasping’ or being acquainted with) each of its constituents, then it is hard to see how one could apprehend that content without thereby knowing its truth-value. And both Frege and Russell wish to hold, in general at least, that we can apprehend a content without knowing its truth-value.⁷⁷

Frege avoids this epistemological difficulty after 1891 through his distinction between sense and meaning. For after introducing

75. Moore, p. 717.

76. Cartwright discusses this early view of Moore and Russell.

77. See, for example, Russell, ‘The Nature of Truth’, p. 504, where he emphasizes that we cannot determine the truth-value of a proposition by ‘inspection’. In ‘Logic I’, *PW*, p. 7/NS, p. 8, Frege writes: ‘We grasp the content of a truth before we recognize it is true....’ I am not suggesting (especially in the case of Frege) that this epistemological difficulty is the sole reason why Russell and Frege reject their early views of propositional contents.

this distinction, Frege holds that while the propositional content (the thought) consists of the senses, not the meanings, of words, the entities which that content are about are the meanings, not the senses, of words.⁷⁸ While Frege holds that sense ‘determines’ meaning, he also holds that we can grasp a sense without thereby knowing what meaning it has ‘determined’.⁷⁹ For Frege, when we make a judgment, we are taking a ‘step from the level of thoughts to the level of meaning’.⁸⁰ In particular, for Frege, ‘a thought is related to its truth-value as the sense of its sign to its meaning’.⁸¹ Thus, for Frege, the reason we can apprehend a thought without thereby knowing its truth-value is because we can apprehend sense without thereby knowing the meaning it has determined.

Because Frege no longer holds that the entities which a propositional content is about are constituents of that content, it has seemed natural to some to attribute a ‘representation-theoretic’ view of truth to Frege.⁸² It may seem, that is, that what determines the truth-value of a (post-1891) Fregean thought, as opposed, say, to a (pre-1910) Russellian proposition, is not anything ‘internal’ or ‘intrinsic’ to the thought itself, but are rather entities ‘extrinsic’ to that thought. Furthermore, on a view of the relation of sense to meaning which is commonly attributed to Frege, he would, indeed, adhere to a position consistent with Russell’s correspondence ‘requisite’. For suppose that sense is a function from possible worlds to meanings;⁸³ suppose, that is, that sense does not determine meaning *simpliciter*, but rather that sense together with contingent features of the world determines meaning. In this case, Frege would hold that the same sense will have a different meaning—and the same thought will have a different truth-value—in different possible worlds. And thus he would hold, in accord with Russell’s ‘requisite’, that what determines the truth-value of a truth-bearer is not wholly ‘intrinsic’ to that truth-bearer itself, but is rather something metaphysically independent of it.

78. See, for example, Frege’s letter to Russell, *COR*, p. 149/WB, p. 231.

79. See, for example, ‘On Sense and Meaning’, pp. 27–28.

80. *Ibid.*, p. 34.

81. ‘Logic in Mathematics’, *PW*, p. 234/NS, p. 252.

82. This view of Frege is suggested in Dummett, pp. 464f and in Baker and Hacker, pp. 344f.

83. This understanding of Fregean sense is attributed to Carnap by Kaplan, pp. 90–91. In accounting for sense in this way, Carnap (in *Meaning and Necessity*) is, in effect, combining Fregean sense with a Wittgensteinian view of logic.

It should be clear, however, that Frege is in no position to accept this account of sense. For Frege has no notion of metaphysically possible worlds distinct from this world; and it is his rejection of metaphysical modality that enables him to hold that the same notion of truth applies to the truths of logic as to any other truths. Thus although Frege's (post-1891) thoughts differ from Russell's (pre-1910) propositions, these differences do not bear on the nature of truth. While Frege's thoughts, unlike Russell's propositions, do not contain the entities they are about among their constituents, Frege's thoughts are as 'intrinsically' related to the entities they are about and to their truth-values as are Russellian propositions. With the introduction of sense, Frege is able to hold that we can apprehend a thought without thereby knowing what entity it is about and without thereby knowing its truth-value; but he does not thereby hold that that thought (metaphysically) could have been about different entities or (metaphysically) could have had a different truth-value. For Frege, that is, it is epistemologically, but not metaphysically, possible that a thought have a different truth-value or that a sense determine a different meaning.⁸⁴

Thus the way Frege resolves the epistemological difficulty as to how we can apprehend a propositional content without thereby knowing its truth-value does not introduce any metaphysical notions inconsistent with his view of logic. Russell, on the other hand, resolves the same epistemological difficulty by introducing his correspondence view of truth.⁸⁵ By doing so, he not only holds, with Frege, that we can apprehend a content without thereby knowing its truth-value; he also holds, as opposed to Frege, that the truth-value of that content is determined by the presence or absence of a metaphysically independent 'truth-maker'.⁸⁶ When Russell introduces this view of truth, he does not

84. Baker and Hacker recognize that Frege is in no position to hold that the same sense could (metaphysically) determine different meanings (see pp. 312–313, note 9), but they do not apply this point in considering Frege's views of truth (pp. 344f).

85. By 1910, Russell is in no position to accept Frege's resolution of this epistemological problem, because by 1905 in 'On Denoting', he had already rejected Frege's sense/meaning distinction.

86. Specifically, for Russell, understanding 'aRb' requires being acquainted with the individuals a and b and the relation R 'in itself'; but what determines whether 'aRb' is true is whether R 'actually relates' from a to b. To sustain his correspondence requirement, Russell thus has to hold that the individuals a and b and the relation R 'in itself' do not (metaphysically) determine whether R actually relates from a to b. See, for example, *PoP*, Chapter XII.

recognize any conflict between it and his view of logic as maximally general truth.⁸⁷ However, through the influence of Wittgenstein, Russell comes to recognize the conflict and abandons his view of logic.

Russell comes to question his characterization of logical truth as maximally general truth in the context of considering certain axioms—the axioms of infinity and reducibility, and the multiplicative axiom—which he was led to accept in order to derive arithmetic from logic. These axioms are maximally general; so, by Russell's view, if they are true, they should be logical truths. Even prior to the influence of Wittgenstein, Russell recognizes that these axioms have a different status from other logical truths: they 'cannot be demonstrated' from other logical axioms, but neither are they 'self-evident'.⁸⁸ However, before the influence of Wittgenstein, Russell is only concerned with the epistemological status of these axioms. That is, he is concerned with the reasons we might have for accepting or rejecting them; but he does not seem to have concluded that the anomalous status of these axioms undermines his characterization of logical truth as maximally general truth.⁸⁹

In a letter to Russell written in November of 1913, Wittgenstein introduces a decision procedure (equivalent to the truth-table test for logical truth) for propositional logic, which he calls his 'ab-notation'. He indicates, in effect, that if that notation shows a proposition as coming out true in all possible cases (as in a truth-table where a proposition comes out true in all lines), the proposition 'is a true, logical prop[osition]'. If the test shows a

87. Thus, for example, in 'The Philosophical Importance of Mathematical Logic', p. 35, Russell still identifies a 'proposition of pure logic' with 'a proposition which does not contain any other constant than logical constants'. One reason Russell may have seen no problem in adopting a correspondence view of truth is that to avoid the semantic paradoxes, he now holds that 'the words "true" and "false" have many different meanings, according to the kind of proposition to which they are applied'. ('The Theory of Logical Types', p. 9. This paper became Chapters II and III of the Introduction to *Principia Mathematica*.) Thus Russell no longer holds that the same notion of truth applies to general judgments as to atomic judgments (see 'The Theory of Logical Types', p. 12), and, on account of this, he is perhaps able to maintain his view that logical truth is maximally general truth while holding that atomic judgments are correspondence-true or -false.

88. See 'On the Axioms of the Infinite and the Transfinite', p. 43 and 'The Theory of Logical Types', pp. 30–31.

89. Even though Russell holds that the axiom of infinity, as opposed to the other two axioms, can be established only by empirical considerations (see 'On the Axioms of the Infinite and the Transfinite', p. 52), he does not thereby reach the conclusion that some maximally general propositions are not logical propositions.

proposition as coming out false in all possible cases, then ‘the prop[osition] is false and logical’. Wittgenstein then writes:

If finally neither is the case the prop[osition] may be true or false but is in no case logical.... [Y]our axiom of reducibility... *is not a logical prop[osition] at all* and the same applies to the axioms of infinity and the mult[iplicative] ax[iom]. IF *these are true prop[osition]s* they are *what I shall call ‘accidentally’ true and not ‘essentially’ true*. Whether a prop[osition] is accidentally or essentially true can be seen by writing it down in the ab-Notation and applying the above rule. What I—in stating this rule—called ‘logical’ prop[osition] is a prop[osition] which is either essentially true or essentially false. This distinction of accid[entally] and essent[ially] true prop[osition]s explains—by the way—the feeling one always had about the infin[ity] ax[iom] and the axiom of reducibility, the feeling that if they were true they would be so only by a lucky accident.⁹⁰

Wittgenstein is here attacking Russell’s conception of logic. He is arguing that what distinguishes logical propositions—true or false—from other propositions is not their maximal generality, as Russell holds, but rather their modal status. For Wittgenstein, that is, logical propositions are ‘essentially’ true or ‘essentially’ false. Non-logical propositions have their truth-value only ‘accidentally’. And, for Wittgenstein, since some maximally general propositions—including the axioms of infinity and reducibility and the axiom of multiplicity—have their truth-value only ‘accidentally’, then Russell is wrong to hold that every maximally general proposition is either a logical truth or a logical falsehood.

Furthermore, Wittgenstein’s argument follows from his accepting Russell’s correspondence ‘requisite’ on truth for non-logical propositions. By Russell’s ‘requisite’, if a truth-bearer has a correspondence truth-value, it will have that truth-value ‘extrinsically’—that is, (metaphysically) ‘accidentally’. That is, it will have the character Wittgenstein here attributes to non-logical propositions. Likewise, by Russell’s ‘requisite’, if a truth-bearer has its truth-value ‘intrinsically’—that is, ‘essentially’—then it will not be either correspondence-true or -false; and in this letter, Wittgenstein indicates that logical truths are ‘tautologous’ and

90. *LRKM*, pp. 36–37. Dreben and Floyd, pp. 31–33, also discuss the impact on Russell of this letter and Wittgenstein’s next letter to Russell.

logical falsehoods are 'self-contradictory'.⁹¹ Thus in this letter, Wittgenstein applies Russell's correspondence 'requisite' on truth to conclude, as against Russell (if not Frege⁹²), that some maximally general truths are not logical truths and to conclude, as against Frege (if not Russell by this point⁹³), that the notions of truth and falsity which apply to logical propositions are not the same (correspondence) notions of truth and falsity that apply to other propositions.

We do not have Russell's response to Wittgenstein. But it seems that he resisted Wittgenstein's position, for in his next letter to Russell, Wittgenstein reiterates his arguments against Russell's view of logic. He emphasizes again that the characteristic mark of logical propositions is not that they are maximally general, but rather that they are tautologies.⁹⁴ He argues that since we can 'imagine worlds' in which some maximally general propositions are false, then those propositions are not propositions of logic. And he emphasizes that 'the propositions of logic—and only they—have the property that their truth or falsity, as the case may be, finds its expression in the very sign for the proposition'.⁹⁵ Thus again, Wittgenstein indicates that what characterizes logical truth and falsity is not its maximal generality, but rather that, as opposed to non-logical (correspondence) truth and falsity, it is intrinsic to its truth-bearer.

While Russell seems to have resisted Wittgenstein's objections to his view of logic when Wittgenstein first posed them in these letters, in later years Russell clearly accepts them. In particular, in 'The Philosophy of Logical Atomism' and *Introduction to Mathematical Philosophy* (both written in 1918⁹⁶), Russell indicates that he was wrong to identify logical truth with maximally general truth.⁹⁷ While he still holds that every logical

91. *LRKM*, p. 37.

92. Depending on whether Frege holds that all maximally general truth are logical truths. See note 65.

93. Given that Russell already holds by this point (for unWittgensteinian reasons) that different notions of truth and falsity apply to different kinds of propositions. See note 87.

94. *LRKM*, p. 41. At this point Wittgenstein holds that logical propositions are generalizations of tautologies. By the *Tractatus*, he holds that ungeneralized propositions can also be logical. See 6.1231.

95. *LRKM*, p. 42.

96. See the 'Chronology' in *BR8*, pp. xxxviii–xxxix.

97. For a similar discussion, see the 'Introduction to the Second Edition' of *PoM*, pp. vii–ix.

truth is maximally general, he no longer holds that every maximally general truth is logical. Furthermore, he uses the modal arguments of Wittgenstein's 1913 letters to show this. Thus he argues in 'The Philosophy of Logical Atomism' that

The proposition that there are exactly 30,000 things in the world can also be expressed in purely logical terms, and is certainly not a proposition of logic but an empirical proposition (true or false), because a world containing more than 30,000 things and a world containing fewer than 30,000 things are both possible, so that if it happens that there are exactly 30,000 things, that is what one might call an accident and is not a proposition of logic.⁹⁸

And in *Introduction to Mathematical Philosophy*, he indicates similarly that the axiom of infinity is not a proposition of logic, even though it is maximally general, because there are "possible worlds" in the Leibnizian sense' in which it would be false.⁹⁹ Further, Russell holds that a logical proposition 'has got to be in some sense or other like a tautology',¹⁰⁰ but admits (as does Wittgenstein in 1913¹⁰¹) that he does not know how to define 'tautology'. In a footnote, Russell indicates that while Wittgenstein had been 'working on the problem' of defining 'tautology', Russell does not know 'whether he has solved it, or even whether he is alive or dead'.¹⁰²

By August of 1919, Russell not only knew that Wittgenstein was alive but had also read the *Tractatus*. And in a letter to Wittgenstein in that month, Russell writes:

I am convinced you are right in your main contention, that logical prop[osition]s are tautologies, which are not true in the sense that substantial prop[osition]s are true.¹⁰³

98. 'PLA', p. 210.

99. *IMP*, p. 203.

100. 'PLA', p. 211; see also *IMP*, p. 203.

101. *LRKM*, p. 42: 'As to what tautologies really are, however, I myself am not yet able to say quite clearly....'

102. *IMP*, p. 205.

103. 'Unpublished Correspondence between Russell and Wittgenstein', p. 107. In connection with the issue raised in note 51, note that Wittgenstein replies to Russell by writing: 'Now I'm afraid you haven't really got hold of my main contention, to which the whole business of logical prop[osition]s is only a corollary. The main point is the theory of what can be expressed (gesagt) by prop[osition]s—i.e. by language—and, which comes to the same, what can be *thought* and what can not be expressed by prop[osition]s, but only shown (gezeigt); which, I believe, is the cardinal problem of philosophy.' (*LRKM*, p. 71)

What I have attempted to make clear, in effect, is that it is by holding that ‘the sense in which substantial prop[osition]s are true’ is a ‘representation-theoretic’ sense of truth—one which accords with Russell’s correspondence ‘requisite’ on truth—that Wittgenstein is led to hold that logical propositions not true in that ‘substantial sense’. By applying Russell’s correspondence ‘requisite’ on truth, according to which truth-value is an ‘extrinsic’, and hence metaphysically contingent, property of a truth-bearer, Wittgenstein is led to conclude that a sentence of logic has its truth-value ‘intrinsically’ and necessarily, and so is not ‘substantially’ true or false. Wittgenstein recognizes that if one accepts Russell’s correspondence ‘requisite’ on truth, then one will have to deny that the same notion of truth applies to both logical and non-logical truth. Wittgenstein accepts Russell’s ‘requisite’ and accepts the conclusion which follows. Frege, in contrast, always held that the same notion of truth applies to logical and non-logical truth alike; and, in doing so, he consistently rejected any correspondence view. Russell began by accepting Frege’s position; but, by introducing the correspondence view of truth to Wittgenstein, he inadvertently provided the means by which he was eventually led to reject his early view of logic.¹⁰⁴

REFERENCES

Works of Frege

(Date of publication (composition for unpublished writings) follows the references.)

[BLA] *The Basic Laws of Arithmetic*, translated by Montgomery Furth, University of California Press, Berkeley, 1964 (1893)

‘Boole’s Logical Calculus and the Concept-Script’ in *PW/NS* (1880–81)

‘Compound Thoughts’ in *CP* (1923–26)

Conceptual Notation in Conceptual Notation and Related Articles, translated and edited by Terrell Ward Bynum, Clarendon Press, Oxford, 1972 (1879)

[FA] *The Foundations of Arithmetic*, translated by J.L. Austin, second revised edition, Northwestern University Press, Evanston, IL, 1978 (1884)

‘Introduction to Logic’ in *PW/NS* (1906)

[‘Logic I’] ‘Logic’ (first essay of that title) in *PW/NS* (before 1891)

[‘Logic II’] ‘Logic’ (second essay of that title) in *PW/NS* (1897)

‘Logic in Mathematics’ in *PW/NS* (1914)

‘My Basic Logical Insights’ in *PW/NS* (1915)

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- 'Notes for Ludwig Darmstaedter' in *PW/NS* (1919)
 'On Concept and Object' in *CP* (1892)
 'On Formal Theories of Arithmetic' in *CP* (1885)
 ['Geometry I'] 'On the Foundations of Geometry: First Series' in *CP* (1903)
 ['Geometry II'] 'On the Foundations of Geometry: Second Series' in *CP* (1906)
 'On Sense and Meaning' in *CP* (1892)
 [COR] *Philosophical and Mathematical Correspondence*, translated by Hans Kaal, The University of Chicago Press, Chicago, 1980
 'Thoughts' in *CP* (1918–19)
 'What is a Function?' in *CP* (1904)
 [WB] *Wissenschaftlicher Briefwechsel*, edited by Gottfried Gabriel et al., Felix Meiner, Hamburg, 1976

Collections referred to above:

- [CP] *Collected Papers on Mathematics, Logic, and Philosophy*, edited by Brian McGuinness, Basil Blackwell, Oxford, 1984
 [NS] *Nachgelassene Schriften*, edited by Hans Hermes, et al., Felix Meiner, Hamburg, 1983
 [PW] *Posthumous Writings*, translated by Peter Long and Roger White, The University of Chicago Press, Chicago, 1979

Works of Russell

- [IMP] *Introduction to Mathematical Philosophy*, George Allen & Unwin, London, 1919
 'On the Axioms of the Infinite and the Transfinite' in *BR6* (1911)
 'On Denoting' in *BR4* (1905)
 'The Nature of Truth' in *BR4* (1905)
 'Necessity and Possibility' in *BR4* (1905)
 'Part I of the *Principles*, Draft of 1901' in *BR3* (1901)
 'The Philosophical Importance of Mathematical Logic' in *BR6* (1911)
 [PoL] *The Philosophy of Leibniz*, George Allen & Unwin, London, 1900
 ['PLA'] 'The Philosophy of Logical Atomism' in *BR8* (1918)
 [PoM] *The Principles of Mathematics*, second edition, Cambridge University Press, Cambridge, 1937 (first edition 1903)
 [PoP] *The Problems of Philosophy*, Oxford University Press paperback, Oxford, 1959 (first published 1912)
 'The Theory of Logical Types' in *BR6* (1910)
 'Unpublished Correspondence between Russell and Wittgenstein', edited by B.F. McGuinness and G.H. von Wright, *Russell: The Journal of the Bertrand Russell Archives*, New Series Vol.10, No. 2, Winter 1990–91

Collections referred to above

- [BR3] *The Collected Papers of Bertrand Russell*, Volume 3, edited by Gregory H. Moore, Routledge, London, 1993
 [BR4] *The Collected Papers of Bertrand Russell*, Volume 4, edited by Alasdair Urquhart, Routledge, London, 1994
 [BR6] *The Collected Papers of Bertrand Russell*, Volume 6, edited by John G. Slater, Routledge, London, 1992
 [BR8] *The Collected Papers of Bertrand Russell*, Volume 8, edited by John G. Slater, Routledge, London, 1986

Works of Wittgenstein

- Letters to C.K. Ogden*, edited by G.H. von Wright, Basil Blackwell, Oxford, 1973
 [LRKM] *Letters to Russell, Keynes and Moore*, edited by G.H. von Wright, Cornell University Press, Ithaca, 1974
 [NB] *Notebooks 1914–1916*, second edition, edited by G.H. von Wright and G.E.M. Anscombe, The University of Chicago Press, Chicago, 1979
Tractatus Logico-Philosophicus, translated by C.K. Ogden, Routledge & Kegan Paul, London, 1922; translated by Brian McGuinness and David Pears, Routledge & Kegan Paul, London, 1961

Other References

- Ayer, A.J. *Language, Truth and Logic*, Dover Publications, New York, 1952 (first edition 1935)
 Baker, Gordon and Hacker, Peter, *Frege: Logical Excavations*, Basil Blackwell, Oxford, 1984
 Blackburn, Simon, *Spreading the Word*, Clarendon Press, Oxford, 1984
 Carnap, Rudolf, *Meaning and Necessity*, The University of Chicago Press, Chicago, 1947
 Carruthers, Peter, 'Frege's Regress', *Proceedings of the Aristotelian Society*, lxxxii (1981)
 Cartwright, Richard, 'A Neglected Theory of Truth' in his *Philosophical Essays*, The MIT Press, Cambridge, MA, 1987
 Diamond, Cora, 'Throwing Away the Ladder' in her *The Realistic Spirit*, The MIT Press, Cambridge, MA, 1991
 Dreben, B. and Floyd, J., 'Tautology: How not to use a Word', *Synthese* 87, 1991
 Dummett, Michael, *Frege: Philosophy of Language*, second edition, Harvard University Press, Cambridge, MA, 1981
 Horwich, Paul, *Truth*, Basil Blackwell, Oxford, 1990
 Kaplan, David, 'Transworld Heir Lines' in *The Possible and the Actual*, edited by Michael Loux, Cornell University Press, 1979
 Moore, G.E., 'Truth' in *Dictionary for Philosophy and Psychology*, v.2, edited by James Mark Baldwin, Macmillan, New York, 1901
 Quine, W.V., *Pursuit of Truth*, revised edition, Harvard University Press, Cambridge, MA, 1992
 Putnam, Hilary, 'On Truth' in his *Words and Life*, edited by James Conant, Harvard University Press, Cambridge, MA, 1994
 Ricketts, Thomas, 'Logic and Truth in Frege', *Proceedings of the Aristotelian Society, Supplementary Volume*, 1996
 Tappenden, Jamie, 'Geometry and Generality in Frege's Philosophy of Arithmetic', *Synthese*, 102, 1995
 van Heijenoort, Jean 'Logic as Calculus and Logic as Language', *Synthese* 17, 1967
 Weiner, Joan, *Frege in Perspective*, Cornell University Press, Ithaca, 1990