Introduction to PL
Rules of Inference

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Email Subject：CS104/108 + *ID* + *Name*: *TOPIC*

Sakai: CS104/CS108-数理逻辑导论/(H)

Introduction to Mathematical Logic (I2ML)

Content

- Exercise 02 Assignment
- Basics of Logic
- Rules of Inference (Basics of Natural Deduction)
- Informal Definition of Propositional Logic (PL)
- (Some topics unfinished in the 1st lecture)

Next Lecture:

- Natural Deduction
- PL as a formal language
- Semantics of PL

Exercise 02: Reading and Trial Assignment

Record your time spent (in 0.1 hours) with brief tasks and durations in your learning log by hand writing!

Reading for lecture 2:

- textD_ch02-PropositionalLogic.pdf (in 1 week)

  (textD: "Logic and Proof" is an open book online: https://avigad.github.io/logic_and_proof/)
- textB_ch01-1.2-NaturalDeduction.pdf (in 2 weeks)
- textA_ch02-AnalyzingArguments.pdf (in 2 weeks)

Trial Assignment:

Solve 3 questions in 2.5 Exercises on page 16 of textD. Send your answer in pdf file to the corresponding course email in 2 weeks:

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2.5 Exercises

1. Here is another (gruesome) logic puzzle by George J. Summers, called "Murder in the Family."

   Murder occurred one evening in the home of a father and mother and their son and daughter. One member of the family murdered another member, the third member witnessed the crime, and the fourth member was an accessory after the fact.

   1. The accessory and the witness were of opposite sex.
   2. The oldest member and the witness were of opposite sex.
   3. The youngest member and the victim were of opposite sex.
   4. The accessory was older than the victim.
   5. The father was the oldest member.
   6. The murderer was not the youngest member.

   Which of the four—father, mother, son, or daughter—was the murderer?

   Solve this puzzle, and *write a clear argument* to establish that your answer is correct.

2. Using the mnemonic F (Father), M (Mother), D (Daughter), S (Son), Mu (Murderer), V (Victim), W (Witness), A (Accessory), O (Oldest), Y (Youngest), we can define propositional variables like FM (Father is the Murderer), DV (Daughter is the Victim), FO (Father is Oldest), VY (Victim is Youngest), etc. Notice that only the son or daughter can be the youngest, and only the mother or father can be the oldest.

   With these conventions, the first clue can be represented as

$$((FA \vee SA) \to (MW \vee DW)) \wedge ((MA \vee DA) \to (FW \vee SW)),$$

   in other words, if the father or son was the accessory, then the mother or daughter was the witness, and vice-versa. Represent the other five clues in a similar manner.

   Representing the fourth clue is tricky. Try to write down a formula that describes all the possibilities that are not ruled out by the information.

3. Consider the following three hypotheses:

   - Alan likes kangaroos, and either Betty likes frogs or Carl likes hamsters.
   - If Betty likes frogs, then Alan doesn't like kangaroos.
   - If Carl likes hamsters, then Betty likes frogs.

   Write a clear argument to show that these three hypotheses are contradictory.

Course Structure

- **Lectures**：1-16周
  - CS104，星期五上午　3-4节，商学院101教室(课室变更!!!)
  - CS108，星期四晚上　9-10节，三教205

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What is Logic?

premisses (postulates, axioms)

reasoning (studied in logic)

conclusion (theorem)

The study of logic is the study of reasoning.

The basic question is what conclusions can be drawn with absolute certainty from a particular set of premises.

What entails what?

What follows from what?

Why? What are the evaluation criteria?

How to establish/define the evaluation criteria?

How to evaluate arguments/reasoning?

It is LOGIC to answer these fundamental questions.

Conclusion Indicators

A word or phrase (such as "therefore" or "thus") appearing in an argument and usually indicating that what follows it is the conclusion of that argument.

Here is a partial list of **conclusion indicators**:

therefore	for these reasons
hence	it follows that
so	I conclude that
accordingly	which shows that
in consequence	which means that
consequently	which entails that
proves that	which implies that
as a result	which allows us to infer that
for this reason	which points to the conclusion that
thus	we may infer

Premise Indicators

In an argument, a word or phrase (like “because” and “since”) that normally signals that what follows it are statements serving as premises.

Here is a partial list of premise indicators:

since	as indicated by
because	the reason is that
for	for the reason that
as	may be inferred from
follows from	may be derived from
as shown by	may be deduced from
inasmuch as	in view of the fact that

The words and phrases we have listed may help to indicate the presence of an argument or identify its premises or conclusion, but such indicators do not necessarily appear.

The Three “Law of Thought”

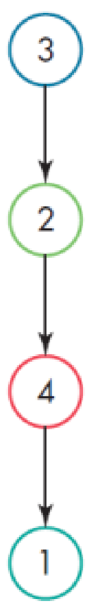
Some early thinkers, after having defined logic as “the science of the laws of thought,” went on to assert that there are exactly three *basic* laws of thought, laws so fundamental that obedience to them is both the necessary and the sufficient condition of correct thinking. These three have traditionally been called:

- The **principle of identity**. This principle asserts that *if any statement is true, then it is true*. Using our notation we may rephrase it by saying that the principle of identity asserts that every statement of the form $p \supset p$ must be true, that every such statement is a tautology.
- The **principle of noncontradiction**. This principle asserts that *no statement can be both true and false*. Using our notation we may rephrase it by saying that the principle of noncontradiction asserts that every statement of the form $p \cdot \sim p$ must be false, that every such statement is self-contradictory.
- The **principle of excluded middle**. This principle asserts that *every statement is either true or false*. Using our notation we may rephrase it by saying that the principle of excluded middle asserts that every statement of the form $p \vee \sim p$ must be true, that every such statement is a tautology.

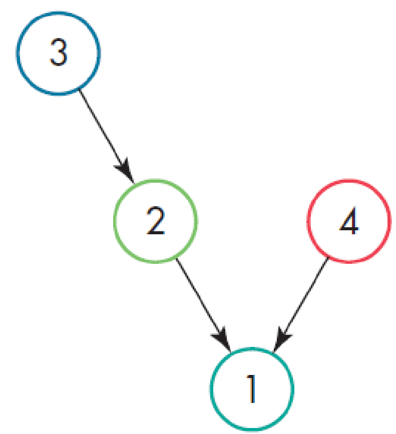
Analyzing Arguments

① Football analysis is trickier than the baseball kind because ② Football really is a team sport. ③ Unlike in baseball, all eleven guys on the field are involved in every play. ④ Who deserves the credit or blame is harder to know than it looks.[4]

The diagram looks like this:



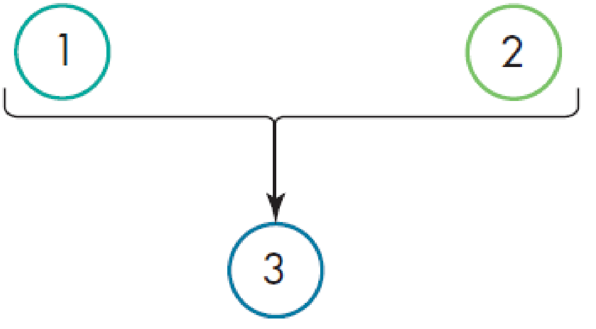
An alternative plausible interpretation of this argument can be represented by a different diagram:



Analyzing Arguments

① General Motors makes money (when it does) on new cars and on the financing of loans. ② Car dealers, by contrast, make most of their money on servicing old cars and selling used ones. ③ So car dealers can thrive even when the automaker languishes.

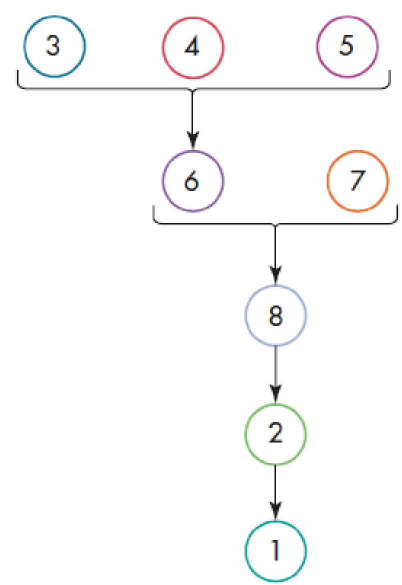
By bracketing the premises in the diagram of this argument, we show that its premises give support only because they are joined, thus:



Analyzing Arguments

① Solar-powered cars can never be anything but experimental devices. ② Solar power is too weak to power even a mini-car for daily use. ③ The solar power entering the atmosphere is about 1 kilowatt per square yard. Because of ④ the scattering in the atmosphere, and because ⑤ the sun shines half a day on the average at any place on earth, ⑥ average solar power received is 1/6 kilowatt, or 4 kilowatt hours a day. . . . Tests on full-size cars indicate that ⑦ 300,000 watt hours are required in a battery for an electric car to perform marginally satisfactorily. So, ⑧ 40 square yards of cells would be needed to charge the car batteries, about the size of the roof of a tractor-trailer. ① It is not undeveloped technologies that put solar power out of the running to be anything but a magnificently designed experimental car. It is cosmology.[17]

The first proposition in this passage, asserting that "solar powered cars can never be more than experimental," is the final conclusion. It is repeated in more elaborate form at the end of the passage, as a diagram of the passage shows:



A Puzzle

The following puzzle, titled "Malice and Alice," is from George J. Summers' *Logical Deduction Puzzles.*

Alice, Alice's husband, their son, their daughter, and Alice's brother were involved in a murder. One of the five killed one of the other four. The following facts refer to the five people mentioned:

1. A man and a woman were together in a bar at the time of the murder.
2. The victim and the killer were together on a beach at the time of the murder.
3. One of Alice's two children was alone at the time of the murder.
4. Alice and her husband were not together at the time of the murder.
5. The victim's twin was not the killer.
6. The killer was younger than the victim.

Which one of the five was the victim?

Summers' book offers the following hint: "First find the locations of two pairs of people at the time of the murder, and then determine who the killer and the victim were so that no condition is contradicted."

A Solution

Things to be noticed:

It is helpful to draw a diagram, and to be systematic about searching for an answer.

The question seems to presuppose that there is a unique answer to the question.

(see textD for more details)

A Solution

From (1), (2), and (3), the roles of the five people were as follows: Man and Woman in the bar, Killer and Victim on the beach, and Child alone.

Then, from (4), either Alice's husband was in the bar and Alice was on the beach, or Alice was in the bar and Alice's husband was on the beach.

If Alice's husband was in the bar, the woman he was with was his daughter, the child who was alone was his son, and Alice and her brother were on the beach. Then either Alice or her brother was the victim; so the other was the killer. But, from (5), the victim had a twin, and this twin was innocent. Since Alice and her brother could only be twins to each other, this situation is impossible. Therefore Alice's husband was not in the bar.

So Alice was in the bar. If Alice was in the bar, she was with her brother or her son.

A Solution

If Alice was with her brother, her husband was on the beach with one of the two children. From (5), the victim could not be her husband, because none of the others could be his twin; so the killer was her husband and the victim was the child he was with. But this situation is impossible, because it contradicts (6). Therefore, Alice was not with her brother in the bar.

So Alice was with her son in the bar. Then the child who was alone was her daughter. Therefore, Alice's husband was with Alice's brother on the beach. From previous reasoning, the victim could not be Alice's husband. But the victim could be Alice's brother because Alice could be his twin.

So *Alice's brother was the victim and Alice's husband was the killer.*

A Solution

This argument relies on some "extralogical" elements, e.g., that a father cannot be younger than his child, and that a parent and his or her child cannot be twins. But the argument also involves a number of common logical terms and associated patterns of inference. Next, we will focus on some of the key logical terms occurring in the argument above, words like "and," "or," "not," and "if … then."

Our goal is to give an account of the patterns of inference that govern the use of those terms. To that end, using the methods of symbolic logic, we will introduce variables *A, B, C, … to stand for fundamental statements, or propositions*,and symbols $\wedge$, $\vee$, $\neg$, *and* $\to$ *to stand for "and," "or," "not," and "if … then … ," respectively. Doing so will let us focus* on the way that compound statements are built up from basic ones using the logical terms, while abstracting away from the specific content. We will also adopt a stylized notation for representing inferences as *rules: the inscription*

$$\frac{A \qquad B}{C}$$

indicates that statement C is a logical consequence of A and B.

Rules of Inference: Implication

First pattern of inference: “if … then …”

If Alice was in the bar, Alice was with her brother or her son.

Alice was in the bar.

Alice was with her brother or son.

Modus Ponens, or “Implication Elimination”

$$\frac{A \to B \qquad A}{B} \to\mathrm{E}$$

If you have a proof of *A → B, possibly from some hypotheses, and a proof of A, possibly from* hypotheses, then combining these yields a proof of *B, from the hypotheses in both subproofs.*

Rules of Inference: Implication

The rule for deriving an "if … then" statement is more subtle. Consider the beginning of the third paragraph, which argues that if Alice's husband was in the bar, then Alice or her brother was the victim. Abstracting away some of the details, the argument has the following form:

Suppose Alice's husband was in the bar.

Then …

Then …

Then Alice or her brother was the victim.

Thus, if Alice's husband was in the bar, then Alice or her brother was the victim.

Rules of Inference: Implication

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Rules of Inference: Implication

This is a form of *hypothetical reasoning (Implication Introduction)*.

On the supposition that A holds, we argue that B holds as well. If we are successful, we have shown that *A implies B, without supposing A. In other words, the temporary assumption that A* holds is "canceled" by making it explicit in the conclusion.

$$\dfrac{\dfrac{\quad}{A}^{\,1} \\ \vdots \\ B}{A \to B}\,1 \;\; \to\mathrm{I}$$

The hypothesis is given the label 1; when the introduction rule is applied, the label 1 indicates the relevant hypothesis.

The line over the hypothesis indicates that the assumption has been "canceled" by the introduction rule.

Rules of Inference: Conjunction

Conjunction, that is, the word "and", simply combines the two assertions into one.

Alice's brother was the victim.

Alice's husband was the killer.

Therefore Alice's brother was the victim and Alice's husband was the killer.

Conjunction Introduction :

$$\frac{A \qquad B}{A \wedge B} \wedge \mathrm{I}$$

Rules of Inference: Conjunction

Two Conjunction Elimination rules to extract the two components:

The l and r stand for "left" and "right".

Alice's husband was in the bar and Alice was on the beach.

So Alice's husband was in the bar.

$$\frac{A \wedge B}{A}\ \wedge E_l$$

Or:

Alice's husband was in the bar and Alice was on the beach.

So Alice was on the beach.

$$\frac{A \wedge B}{B}\ \wedge E_r$$

Rules of Inference: Negation and Falsity

In logical terms, showing "not A" amounts to showing that A leads to a contradiction.

Suppose Alice's husband was in the bar.

...

This situation is impossible.

Therefore Alice's husband was not in the bar.

This is another form of hypothetical reasoning, similar to that used in establishing an "if … then" statement: we temporarily assume A, show that leads to a contradiction, and conclude that "not A" holds.

In symbols, the rule reads as follows:

$$\dfrac{\dfrac{}{A}{}^{1} \\ \vdots \\ \bot}{\neg A}\ 1\ \neg\mathrm{I}$$

We have introduced a new symbol, $\bot$. *It corresponds to natural language* phrases like "this is a contradiction" or "this is impossible."

Rules of Inference: Negation and Falsity

The elimination rule is that if we have both “A” and “not A,” then we have a contradiction.

The killer was Alice’s husband and the victim was the child he was with.

So the killer was not younger than his victim.

But according to (6), the killer was younger than his victim.

This situation is impossible.

In symbolic logic, the rule of inference is expressed as follows:

$$\frac{\neg A \qquad A}{\bot}\ \neg\mathrm{E}$$

Rules of Inference: Negation and Falsity

What are the rules governing $\perp$? *In the proof system we will introduce in the next chapter, there is no introduction rule;* "false" is false, and there should be no way to prove it, other than extract it from contradictory hypotheses.

On the other hand, the system provides a rule that allows us to conclude anything from a contradiction:

$$\frac{\perp}{A}\ \perp\mathrm{E}$$

The elimination rule also has the fancy Latin name, *ex falso sequitur quodlibet, which means "anything you want follows* from falsity."

This elimination rule is harder to motivate from a natural language perspective, but, nonetheless, it is needed to capture common patterns of inference.

Rules of Inference: Negation and Falsity

Anything you want follows from falsity.

Assume 1 = 0; (or 1 = 2; or a = b for any a and b such that a != b)

Then for any x and y, we have:

x = 0 { proof: x = 1*x = 0*x = 0 ➔ x = 0}

and

x = y { proof: x-y = 1*(x-y) = 0*(x-y) = 0 ➔ x = y }

Rules of Inference: Negation and Falsity

Notice that if we define $\neg A$ *to be* $A \rightarrow \perp$*, then the rules for negation introduction and elimination are nothing more than* implication introduction and elimination, respectively.

We can think of $\neg A$ *expressed colorfully by saying "if A is true,* then pigs have wings," where "pigs have wings" stands for $\perp$.

Having introduced a symbol for "false," it is only fair to introduce a symbol for "true." In contrast to "false," "true" has no elimination rule, only an introduction rule:

$$\frac{}{\top}$$

Put simply, "true" is true.

Rules of Inference: Disjunction

The introduction rules for disjunction, otherwise known as "or":

Alice's daughter was alone at the time of the murder.

Therefore, either Alice's daughter was alone at the time of the murder, or Alice's son was alone at the time of the murder.

In symbolic terms, the two introduction rules are as follows:

$$\frac{A}{A \vee B}\,\vee\mathrm{I_l} \qquad \frac{B}{A \vee B}\,\vee\mathrm{I_r}$$

Here, again, the l and r stand for "left" and "right".

Rules of Inference: Disjunction

The disjunction elimination rule is trickier, but it represents a natural form of case-based hypothetical reasoning.

Either Alice's brother was in the bar, or Alice's son was in the bar.

Suppose, in the first case, that her brother was in the bar. Then … Therefore, her husband was on the beach.

On the other hand, suppose her son was in the bar. In that case, … Therefore, in this case also, her husband was on the beach.

Either way, we have established that her husband was on the beach.

In symbols, this pattern is expressed as:

$$\dfrac{A \vee B \qquad \begin{array}{c} \overline{A}^{\,1} \\ \vdots \\ C \end{array} \qquad \begin{array}{c} \overline{B}^{\,1} \\ \vdots \\ C \end{array}}{C}\ 1\ \vee\mathrm{E}$$

Rules of Inference: Disjunction

Another pattern of reasoning that is commonly used with "or":

Either Alice's husband was in the bar, or Alice was in the bar.

Alice's husband was not in the bar.

So Alice was in the bar.

In symbols, we would render this rule as follows:

$$\frac{A \vee B \qquad \neg A}{B}$$

We will see in the next chapter that it is possible to *derive this rule from the others. As a result, we will not take this to* be a fundamental rule of inference in our system.

Rules of Inference: If and only if, iff

In mathematical arguments, it is common to say of two statements, *A and B, that "A holds if and only if B holds."*

This assertion is sometimes abbreviated *"A iff B," and means simply that A implies B and B implies A.*

I claim that Alice is in the bar if and only if Alice's husband is on the beach.

To see this, first suppose that Alice is in the bar.

Then …

Hence Alice's husband is on the beach.

Conversely, suppose Alice's husband is on the beach.

Then …

Hence Alice is in the bar.

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Conversely, suppose Alice's husband is on the beach.

Then …

Hence Alice is in the bar.

$$\dfrac{\begin{array}{c}\overline{A}^{\,1}\\ \vdots \\ B\end{array}\qquad\qquad \begin{array}{c}\overline{B}^{\,1}\\ \vdots \\ A\end{array}}{A \leftrightarrow B}\;1\;\;{\leftrightarrow}\mathrm{I}$$

Rules of Inference: If and only if, iff

The elimination rules for iff are unexciting. In informal language, here are the “left” rule and the “right” rule:

Alice is in the bar if and only if Alice’s husband is on the beach.
Alice is in the bar.
Hence, Alice’s husband is on the beach.

Alice is in the bar if and only if Alice’s husband is on the beach.
Alice’s husband is on the beach.
Hence, Alice is in the bar.

Rendered in natural deduction, the rules are as follows:

$$\frac{A \leftrightarrow B \qquad A}{B} \leftrightarrow\mathrm{E}_l \qquad \frac{A \leftrightarrow B \qquad B}{A} \leftrightarrow\mathrm{E}_r$$

Rules of Inference: Proof by Contradiction

Looks at two informal arguments:

Suppose Alice's husband was in the bar.

...

This situation is impossible.

Therefore Alice's husband was not in the bar.

Suppose Alice's husband was not on the beach.

...

This situation is impossible.

Therefore Alice's husband was on the beach.

In the first argument, a negation is *introduced into the conclusion*, *whereas in the second, it is eliminated from* the hypothesis.

Using negation introduction to close the second argument would yield the conclusion "It is not the case that Alice's husband was not on the beach."

Rules of Inference: Proof by Contradiction

Indeed, the rule is equivalent to adding an axiom that says that for every statement A, "not not A" is equivalent to A.

There is a style of doing mathematics known as "constructive mathematics" that denies the equivalence of "not not A" and A.

Constructive arguments tend to have much better computational interpretations; a proof that something is true should provide explicit evidence that the statement is true, rather than evidence that it can't possibly be false.

In natural deduction, proof by contradiction is expressed by the following pattern:

$$\begin{array}{c} \overline{\neg A}^{\;1} \\ \vdots \\ \dfrac{\perp}{A}\;\mathrm{RAA},1 \end{array}$$

It also has a fancy name, *reductio ad absurdum,* "reduction to an absurdity."

The assumption $\neg$*A is canceled at the final inference.*

Informal Intro to the Language of PL

The language of propositional logic starts with symbols *A, B, C, … which are intended to range over basic assertions,* or propositions, which can be true or false.

Compound expressions are built up using parentheses and the logical symbols introduced in the last section.

For example,

$$((A \wedge (\neg B)) \to \neg(C \vee D))$$

is an example of a propositional formula.

Informal Intro to the Language of PL

When writing expressions in symbolic logic, we will adopt an order of operations which allows us to drop superfluous parentheses.

When parsing an expression:

- Negation binds most tightly.
- Then, conjunctions and disjunctions bind from right to left.
- Finally, implications and bi-implications (iff) bind from right to left.

So, for example, the expression $\neg A \vee B \to C \wedge D$ is

$$\text{understood as } ((\neg A) \vee B) \to (C \wedge D).$$

Translating statements in arguments into symbols and formulas (expressions) is left for your reading and exercises.

We will introduce formal definition of PL in later weeks.

Example 1.2 (some valid patters of reasoning).

1. if P_1, then P_2
not P_2
therefore: not P_1

$$\begin{array}{r} P_1 \to P_2 \\ \neg P_2 \\ \hline \neg P_1 \end{array} \qquad \textit{Modus Tollens}$$

2. if P_1, then P_2
P_1
therefore: P_2

$$\begin{array}{l} P_1 \to P_2 \\ P_1 \\ \hline P_2 \end{array} \qquad \textit{Modus Ponens (MP)}$$

3. P_1 if and only if (iff) P_2
not P_1
therefore: not P_2

$$\begin{array}{l} P_1 \rightleftarrows P_2 \\ \neg P_1 \\ \hline \neg P_2 \end{array}$$

4. not (P_1 and P_2)
P_1
therefore: not P_2

$$\begin{array}{l} \neg(P_1 \wedge P_2) \\ P_1 \\ \hline \neg P_2 \end{array}$$

5. P_1 or P_2
not P_2
therefore: P_1

$$\begin{array}{l} P_1 \vee P_2 \\ \neg P_2 \\ \hline P_1 \end{array}$$

We have introduced above $\rightleftarrows$ for 'if and only if (iff)', $\wedge$ for 'and', $\vee$ for the inclusive 'or', i.e., $P_1 \vee P_2$ stands for 'P_1 or P_2 or both P_1 and P_2'. The reader should verify that all patterns in Example 1.2 are valid.

The following two patterns of reasoning are frequently used in practice, although they are invalid:

if P_1, then P_2 not P_1 therefore: not P_2	$$\frac{\begin{array}{l} P_1 \rightarrow P_2 \\ \neg P_1 \end{array}}{\neg P_2}$$
if P_1, then P_2 P_2 therefore: P_1	$$\frac{\begin{array}{l} P_1 \rightarrow P_2 \\ P_2 \end{array}}{P_1}$$

So, the following concrete arguments are not correct:

If it rains, then the street becomes wet.
It does not rain.
Therefore: The street does not become wet.

If it is raining, then the street becomes wet.
The street becomes wet.
Therefore: It is raining.

Propositional Logic (PL)

We have introduced a new language for representing patterns of reasoning, the alphabet of which consists of the symbols:

$P_1,\ P_2,\ P_3, \ldots$	called atomic formulas
$\rightleftarrows, \rightarrow, \wedge, \vee, \neg$	called connectives
(,)	called parentheses.

REMARK: May have multiple different symbols to represent same connectives in different books:

$\rightleftarrows,\ \leftrightarrow,\ \equiv,\ \Leftrightarrow$

$\rightarrow,\ \supset,\ \Rightarrow$

$\wedge,\ \bullet$

$\vee,\ +$

$\neg,\ \sim$

Formulas:

1. $P_1, P_2, P_3, \ldots$ are formulas. In other words, if P is an atomic formula, then P is a formula.
2. If A and B are formulas, then $(A \rightleftarrows B)$, $(A \to B)$, $(A \wedge B)$ and $(A \vee B)$ are formulas.
3. If A is a formula, then $(\neg A)$ is a formula too.

Example 1.3. P_1, P_3 and P_5 are formulas.

$(\neg P_1)$ and $(P_3 \to P_5)$ are formulas.

$((\neg P_1) \vee (P_3 \to P_5))$ is a formula.

Tautology, Contradiction, Satisfiable?

Predicate Logic (First-Order Logic, FOL)

All men are mortal.
Socrates is a man.
Therefore, Socrates is mortal.

The structure of the argument above is the following pattern:

For all objects x, if x is a person, then x is mortal.	$\forall x[P(x) \rightarrow M(x)]$
Socrates is a person.	$P(c)$
Therefore: Socrates is mortal.	$M(c)$

Using $\forall x$ for 'for all x', $P(x)$ for 'x has the property P (to be a Person)', $M(x)$ for 'x has the property M (to be Mortal)' and c for 'Socrates'.

Next consider the following elementary argument:

John is ill
Therefore: someone is ill.

$$\frac{I(c)}{\exists x[I(x)]}$$

we need one more symbol: $\exists x$, for
'there is at least one x such that ...'.

For instance, the following correct argument can be built up from the elementary steps just specified.

> John loves Jane and John is getting married.
> If John is getting married, then he is looking for another job.
> Hence: John is looking for another job or he does not love Jane.

The underlying pattern of reasoning is:

$$\frac{\begin{array}{c} P_1 \wedge P_2 \\ P_2 \to P_3 \end{array}}{P_3 \vee \neg P_1}$$

And indeed, this pattern can be built up from the elementary steps specified above as follows:

$$\dfrac{\dfrac{\dfrac{\text{premiss}}{P_1 \wedge P_2}}{P_2} \qquad \dfrac{\text{premiss}}{P_2 \to P_3}}{\dfrac{P_3}{P_3 \vee \neg P_1}}$$

And the four elementary steps of reasoning specified above can be supplemented by a few more elementary steps to form what is called Gentzen's [8] system of *Natural Deduction*

Example 1.23 Finally, we return to the argument of Examples 1.1 and 1.2, which can be coded up by the sequent $p \wedge \neg q \rightarrow r,\ \neg r,\ p \vdash q$ whose validity we now prove:

1	$p \wedge \neg q \rightarrow r$	premise
2	$\neg r$	premise
3	p	premise
4	$\neg q$	assumption
5	$p \wedge \neg q$	$\wedge$i 3, 4
6	r	$\rightarrow$e 1, 5
7	$\bot$	$\neg$e 6, 2
8	$\neg\neg q$	$\neg$i 4–7
9	q	$\neg\neg$e 8

(From page 23 of textB.)

The basic rules of natural deduction:

	introduction	*elimination*
$\wedge$	$\dfrac{\phi \qquad \psi}{\phi \wedge \psi}\ \wedge\mathrm{i}$	$\dfrac{\phi \wedge \psi}{\phi}\ \wedge\mathrm{e}_1 \qquad \dfrac{\phi \wedge \psi}{\psi}\ \wedge\mathrm{e}_2$
$\vee$	$\dfrac{\phi}{\phi \vee \psi}\ \vee\mathrm{i}_1 \qquad \dfrac{\psi}{\phi \vee \psi}\ \vee\mathrm{i}_2$	$\dfrac{\phi \vee \psi \qquad \boxed{\begin{array}{c}\phi\\ \vdots\\ \chi\end{array}} \qquad \boxed{\begin{array}{c}\psi\\ \vdots\\ \chi\end{array}}}{\chi}\ \vee\mathrm{e}$
$\rightarrow$	$\dfrac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \psi\end{array}}}{\phi \rightarrow \psi}\ \rightarrow\mathrm{i}$	$\dfrac{\phi \qquad \phi \rightarrow \psi}{\psi}\ \rightarrow\mathrm{e}$

The basic rules of natural deduction:

	introduction	*elimination*
$\neg$	$$\dfrac{\boxed{\begin{matrix}\phi \\ \vdots \\ \bot\end{matrix}}}{\neg\phi}\ \neg\mathrm{i}$$	$$\dfrac{\phi \quad \neg\phi}{\bot}\ \neg\mathrm{e}$$
$\bot$	(no introduction rule for $\bot$)	$$\dfrac{\bot}{\phi}\ \bot\mathrm{e}$$
$\neg\neg$		$$\dfrac{\neg\neg\phi}{\phi}\ \neg\neg\mathrm{e}$$

Some useful derived rules:

$$\frac{\phi \to \psi \quad \neg\psi}{\neg\phi}\ \mathrm{MT} \qquad\qquad \frac{\phi}{\neg\neg\phi}\ \neg\neg\mathrm{i}$$

$$\frac{\boxed{\begin{array}{c}\neg\phi\\ \vdots\\ \bot\end{array}}}{\phi}\ \mathrm{PBC} \qquad\qquad \frac{}{\phi \lor \neg\phi}\ \mathrm{LEM}$$

Figure 1.2. Natural deduction rules for propositional logic.

Program verification

$$\frac{(\!|\phi|\!)\ C_1\ (\!|\eta|\!) \qquad (\!|\eta|\!)\ C_2\ (\!|\psi|\!)}{(\!|\phi|\!)\ \ C_1; C_2\ \ (\!|\psi|\!)}\ \text{Composition}$$

$$\frac{}{(\!|\psi[E/x]|\!)\ x = E\ (\!|\psi|\!)}\ \text{Assignment}$$

$$\frac{(\!|\phi \wedge B|\!)\ C_1\ (\!|\psi|\!) \qquad (\!|\phi \wedge \neg B|\!)\ C_2\ (\!|\psi|\!)}{(\!|\phi|\!)\ \texttt{if}\ B\ \{C_1\}\ \texttt{else}\ \{C_2\}\ (\!|\psi|\!)}\ \text{If-statement}$$

$$\frac{(\!|\psi \wedge B|\!)\ C\ (\!|\psi|\!)}{(\!|\psi|\!)\ \texttt{while}\ B\ \{C\}\ (\!|\psi \wedge \neg B|\!)}\ \text{Partial-while}$$

$$\frac{\vdash_{\mathrm{AR}} \phi' \to \phi \qquad (\!|\phi|\!)\ C\ (\!|\psi|\!) \qquad \vdash_{\mathrm{AR}} \psi \to \psi'}{(\!|\phi'|\!)\ C\ (\!|\psi'|\!)}\ \text{Implied}$$

Figure 4.1. Proof rules for partial correctness of Hoare triples.

TextBooks

textA_IntroductionToLogic-14ed--Copi-Cohen-2011.pdf

textB_LogicInComputerScience--2ed-Huth-Ryan-2004.pdf

textC_PhilosophicalAndMathematicalLogic--Swart-2018.pdf

textD_LogicAndProof-R3.18.4--Avigad-Lewis-Doorn-2021.pdf
(open book online: https://avigad.github.io/logic_and_proof/)

textE_MathematicalLogic-Preprint--Slaman-Woodin-2019.pdf

textF_MathematicalLogicForComputerScience-3ed--Ben-Ari-2012.pdf

textG_LogicForComputerScientists-Reprint--Schoning-2008.pdf

textH_AMathematicalIntroductionToLogic-2ed--Enderton-2001.pdf

Besides logic, I2ML will try to help

- Thinking and communicating precisely
- Problem solving
- Creative thinking
- Critical thinking

BUT this is only one side (teaching) of the course. Another side (studying) is your participation and efforts on the course...

Course Evaluation:

- Class participation/Quizes/Exercises 25%
- Assignments 25% (on sakai course site with exact due date-time)
- Final exam 50%

Important !!!

- Most ideas will be covered in class but some details might be skipped. These details are covered in the necessary reading.
- Assignments, Quizes and Exercises should be done individually although mini-group discussions are allowed for Assignments and Exercises
- Any dishonest behaviour and cheating in assignments will be dealt with severely
- If you get an idea for a solution from others or online you must acknowledge the source in your submission

南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

计算机科学与工程系
Department of Computer Science and Engineering

Undergraduate Students Declaration Form

This is______________ (student ID: ______________, who has enrolled in _________ course of the Department of Computer Science and Engineering. I have read and understood the regulations on courses according to “Regulations on Academic Misconduct in courses for Undergraduate Students in the SUSTech Department of Computer Science and Engineering”. I promise that I will follow these regulations during the study of this course.

Signature：

Date:

Exercise 02: Reading and Trial Assignment

Record your time spent in 0.1 hours with brief tasks and durations in your learning log by hand writing!

Reading for lecture 2:

- textD_ch02-PropositionalLogic.pdf (in 1 week)

  (textD: "Logic and Proof" is an open book online: https://avigad.github.io/logic_and_proof/)
- textB_ch01-1.2-NaturalDeduction.pdf (in 2 weeks)
- textA_ch02-AnalyzingArguments.pdf (in 2 weeks)

Trial Assignment:

Solve 3 questions in 2.5 Exercises on page 16 of textD. Send your answer in pdf file to the corresponding course email in 2 weeks:

CS104: program07 @ yeah.net **CS108: mxhe1 @ yeah.net**

Summary

- Exercise 02 Assignment
- Basics of Logic
- Rules of Inference (Basics of Natural Deduction)
- Informal Definition of Propositional Logic (PL)
- (Some unfinished topics in the 1st lecture)

Next Lecture:

- Natural Deduction
- PL as a formal language
- Semantics of PL