Adversarial Search

Minimax Algorithm

- **Minimax Algorithm** is a basic method to solve game-tree problems.

MINIMAX-DECISION and EXPECTIMINIMAX

function MINIMAX-DECISION(*state*) **returns** *an action*
  return arg max $_{a \in \text{ACTIONS}(s)}$ MIN-VALUE(RESULT(*state*, *a*))

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function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for each *a* **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(*state*, *a*)))
  return v

---

function MIN-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow \infty$
  for each *a* **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(*state*, *a*)))
  return v

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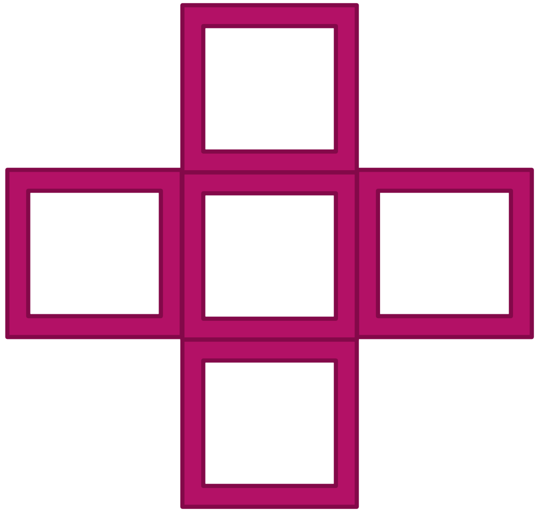
function EXPECTIMINIMAX(s) =
  UTILITY(s) **if** TERMINAL-TEST(s)
  $\max_a$ EXPECTIMINIMAX(RESULT(s, a)) **if** PLAYER(s)= MAX
  $\min_a$ EXPECTIMINIMAX(RESULT(s, a)) **if** PLAYER(s)= MIN
  $\sum_r P(r)$ EXPECTIMINIMAX(RESULT(s, r)) **if** PLAYER(s)= CHANCE

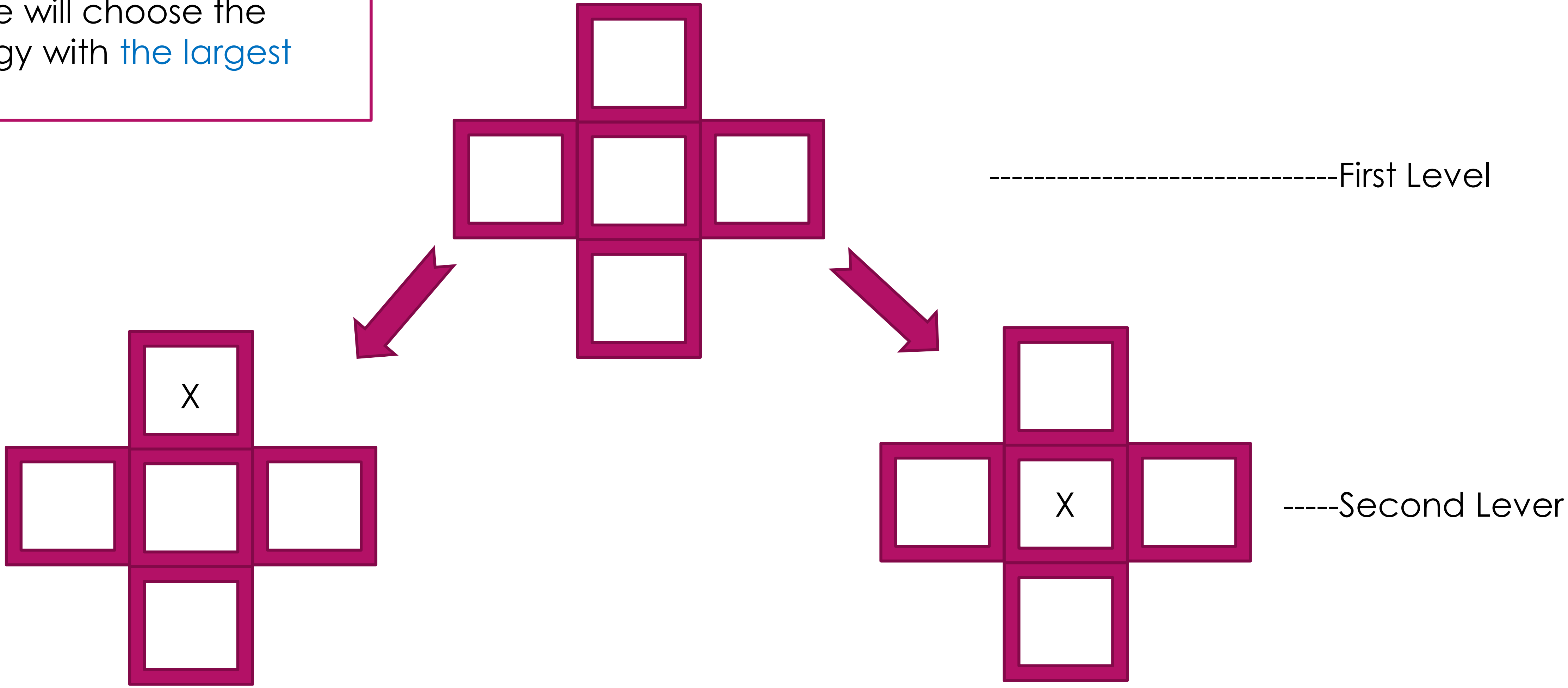
A Basic Example

- Fill X or O in the box
- X plays first
- Termination:
  - If there are two consecutive X or O, then it wins
- Utility function:
  - If X wins, score=1
  - If O wins, score=-1
  - If tie, score=0



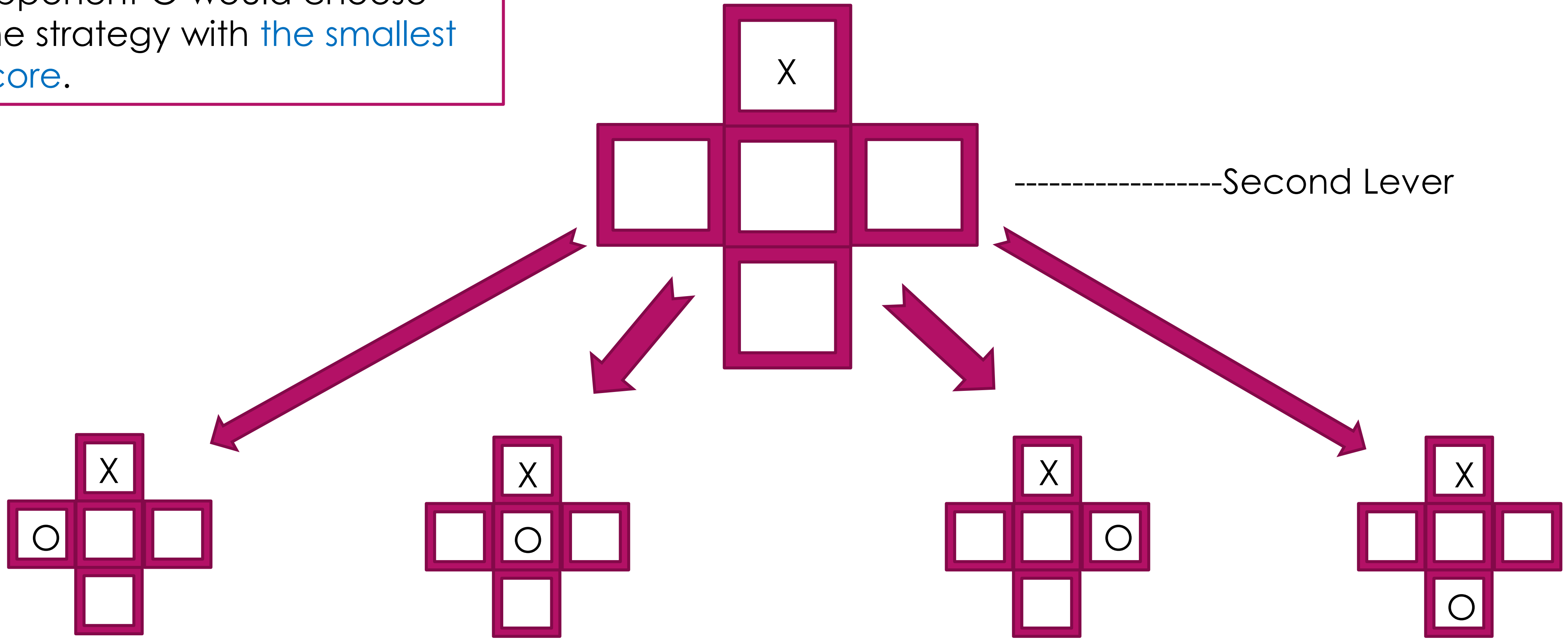
A Basic Example: The First Level with Max

MAX(): If X wants to win, he/she will choose the strategy with the largest score

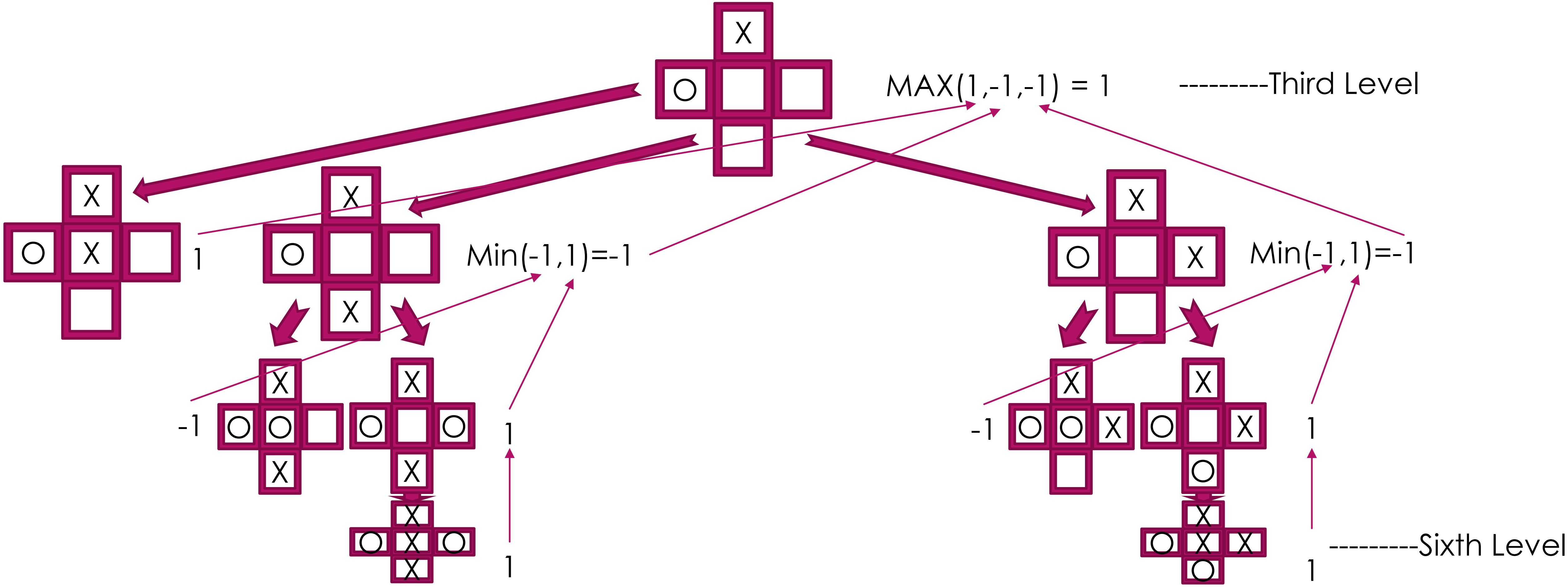


A Basic Example: The Second Level with Min

MIN(): If X has chosen the center-above position, the opponent O would choose the strategy with the smallest score.

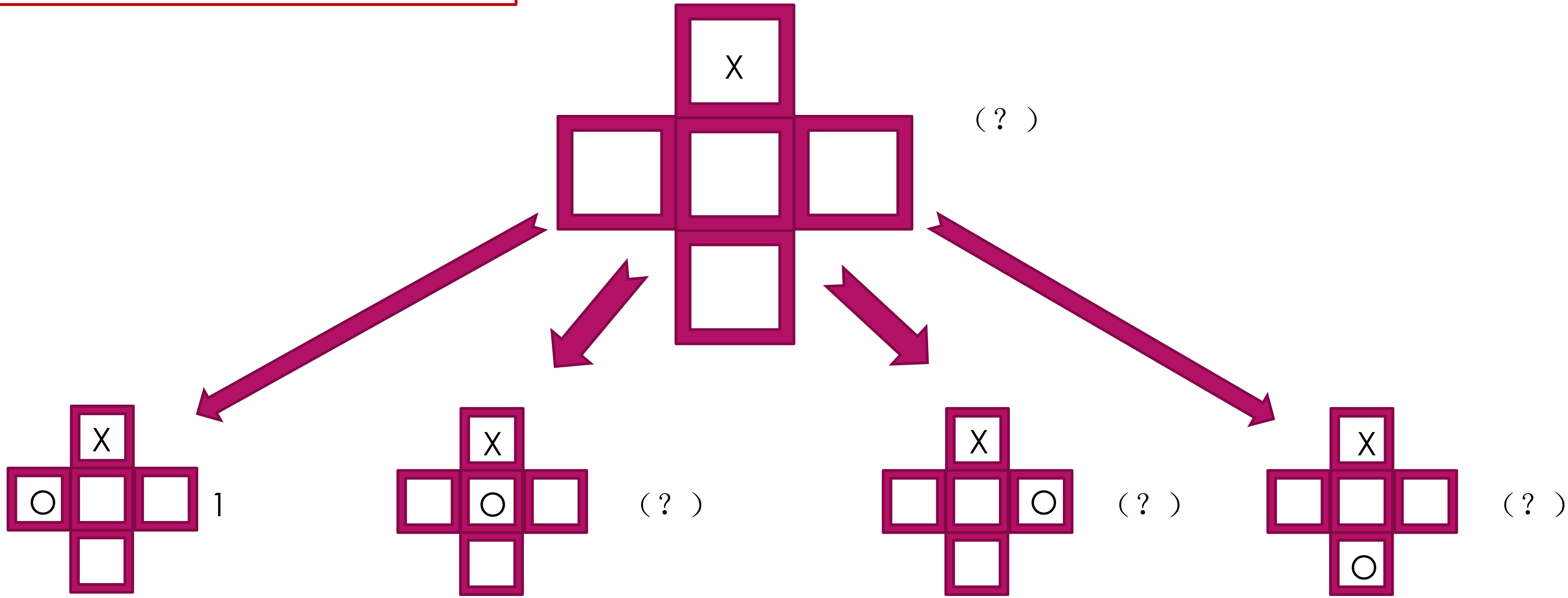


A Basic Example: Third Level to Sixth Level

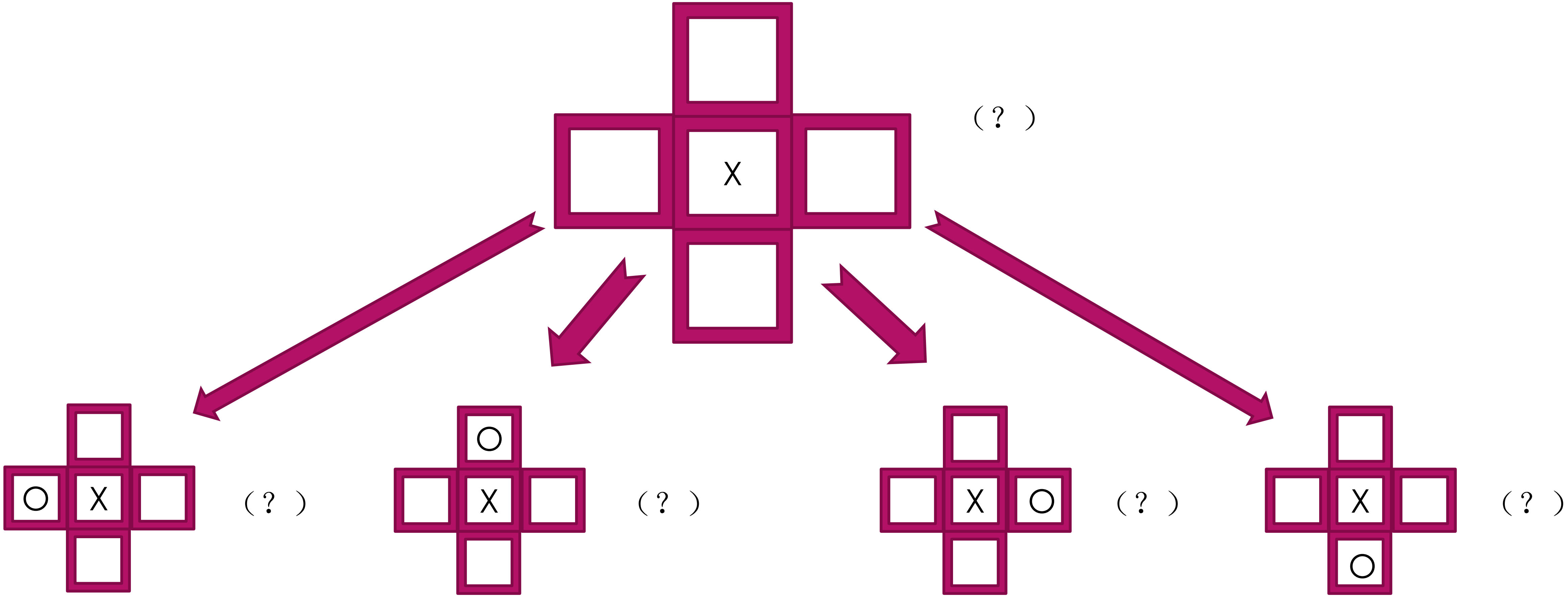


Fill in the Blank

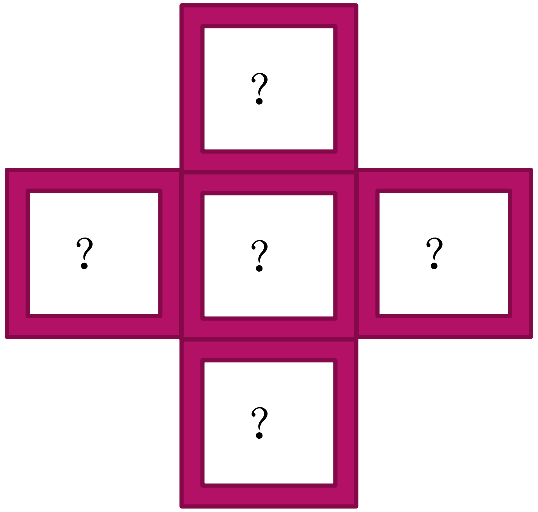
Please complete the scores in the remaining boxes according to the above procedure



Fill in the Blank



Which position would X choose in the first step?

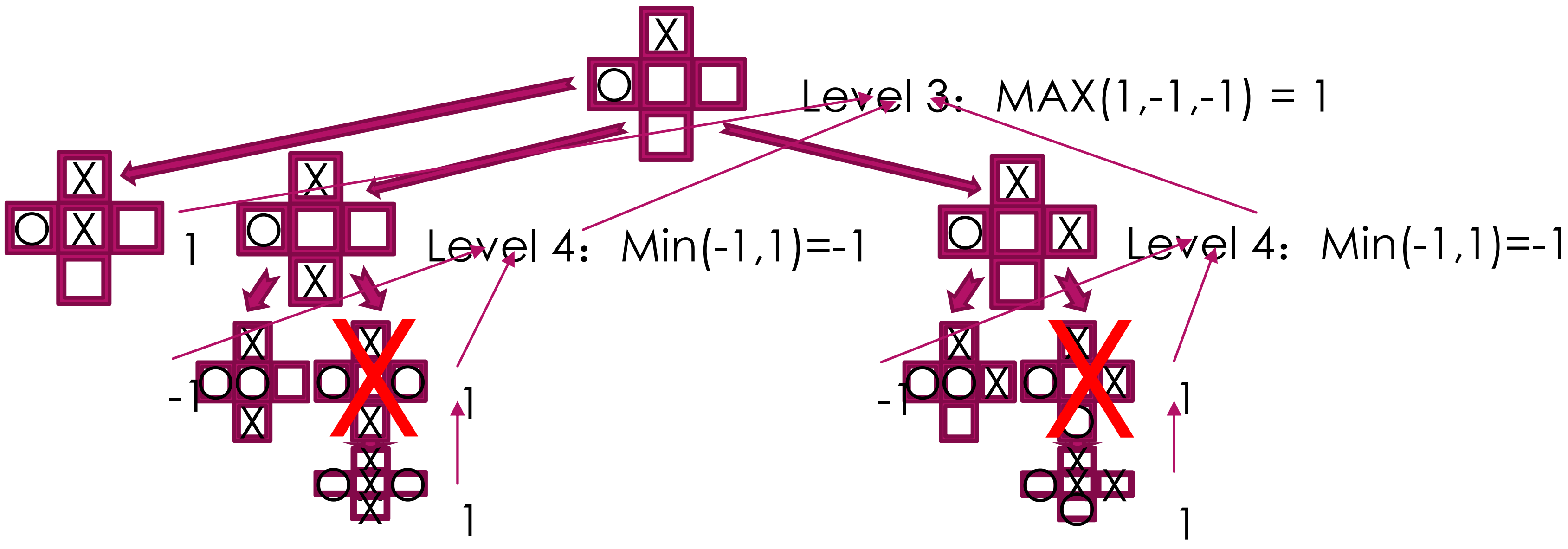


In the implementation of the Minimax algorithm:

- If we arrive at a symmetric case, is it necessary to search twice?
- In the process, can you summarize which searches are redundant?
- If you were to design a suitable evaluation function, how would you design it?

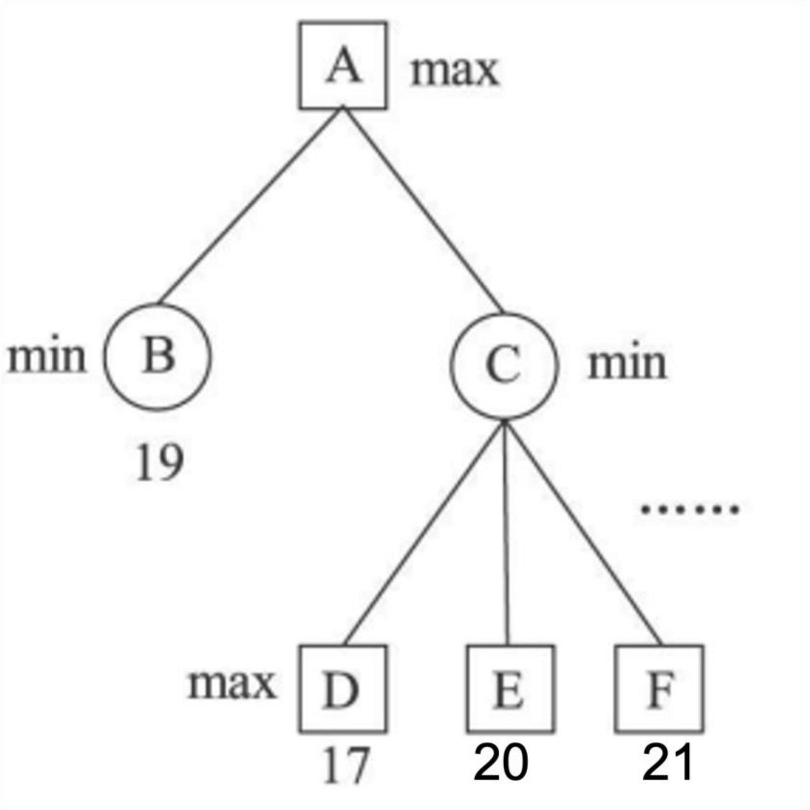
Pruning

- The Level-3 needs to get the maximum value of the Level-4. As the search proceed, the leftmost node in Level-4 already gets value 1 and return it to Level-3. Then, Level-3 continue to call the second node in Level-4, which already gets value −1. Note −1 < 1.
- Then, does the second node in Level-4 need to continue to search its right branch?
- If the right branch gets value larger than -1, it is obvious that Min still gets the value −1 when the search is finished. If the right branch gets value smaller than -1, it is clear that Max in Level-3 still gets the value 1. So, the result of Level-3 will remain the same no matter which value the right branch of the second node in Level-4 returns.
- Conclusion: The second right branch of the fourth layer can be cut off.



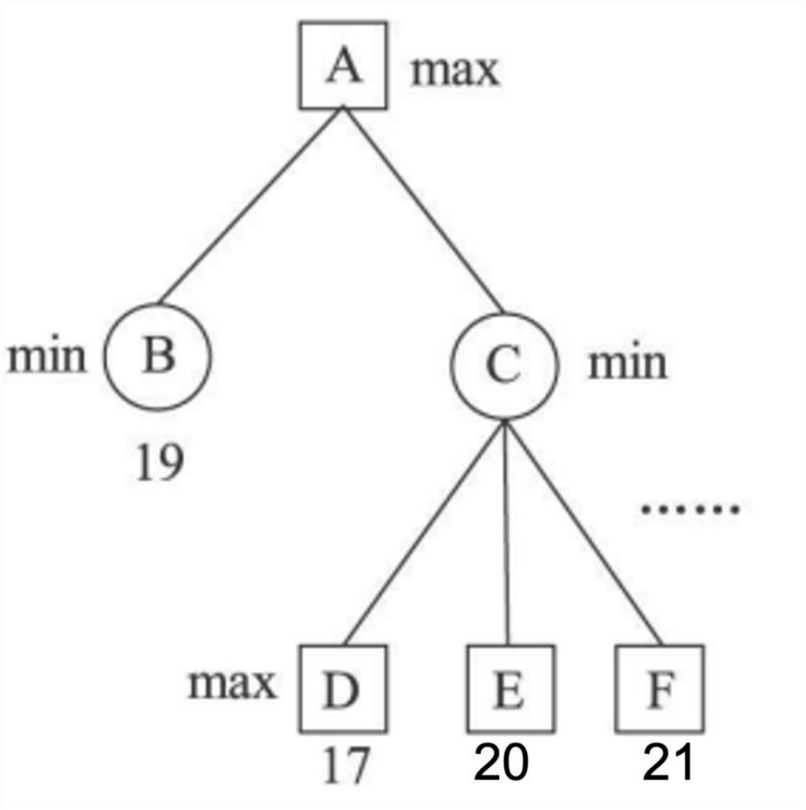
Alpha Pruning

- The value of **node A** should be the greater of the values of **node B** and **node C**. **Node B** is now known to have a value greater than the value of **node D**. Since the value of **node C** should be the **smallest** of the values of its children, this minimum value must be no larger than the value of **node D**, and therefore must be less than the value of **node B**, indicating the meaningless of the search of other children of **node C**, e.g., **node E and F**. Now, we can cut off the subtree rooted at **node C**. This optimization is called **Alpha pruning**.
- Question: What happens if searching **branches E and F** are in front of **D**?



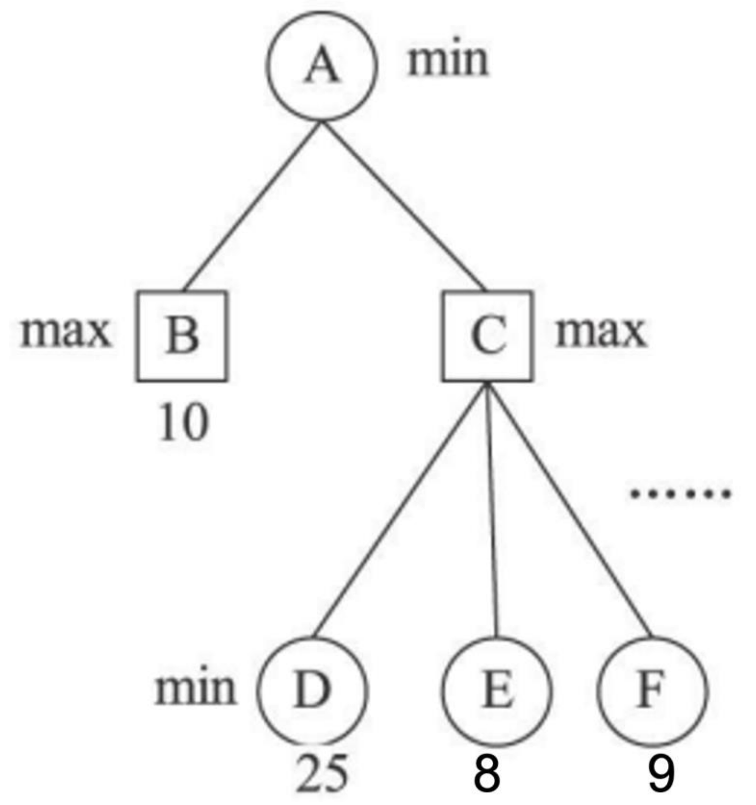
Alpha Pruning: Explanation

- **MAX Level**: At node A, the maximum value found in the child node is saved in **alpha**. This **alpha** value is passed to the next level along with the function call.
- The next level is **MIN level**. The minimum value currently found by the node of the **MIN level** is no larger than **alpha** value, so there is no need to continue searching.
- A sub-node needs to keep updating its own **beta** value. If the node branch does not terminate, and the currently found minimum value < **beta**, we need to update the **beta** value and pass it to the next level of the node.



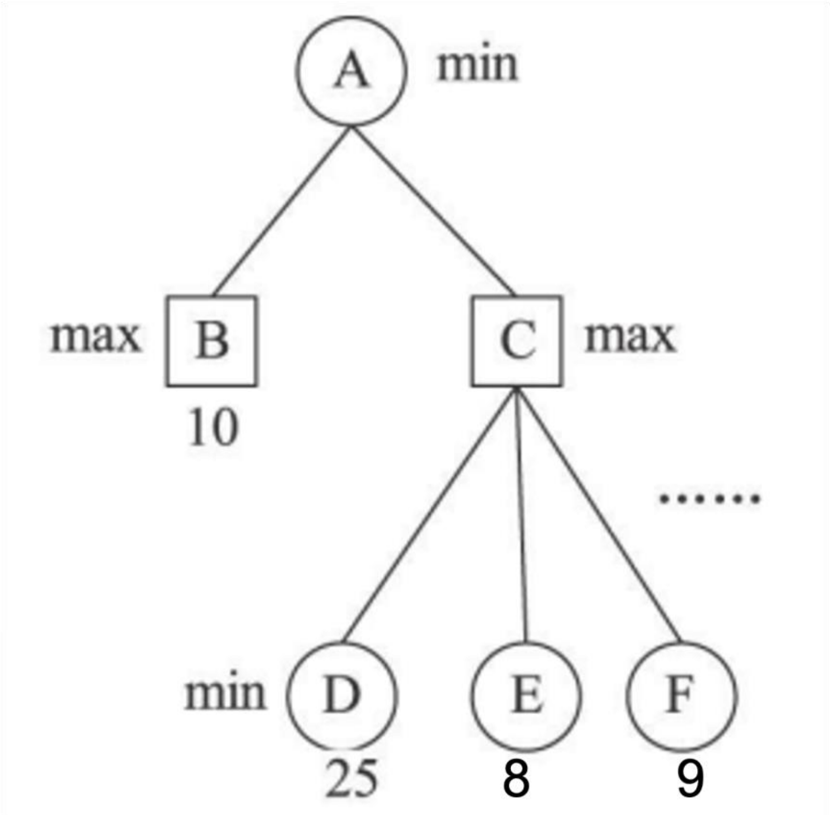
Beta Pruning

- The value of **node A** should be the lesser of the values of **node B** and **node C**. **Node B** is known to have a value less than the value of **node D**. Since the value of **node C** should be the **largest** of its sub-node values, this maximum value must be no less than the value of **node D**, and therefore greater than the value of **node B**, indicating that continuing to search for other children of **node C** have no meaning, and all subtrees rooted at **node C** can be **cut off**. This optimization is called **Beta pruning**.
- Question: What would happen if the branches of **E and F** are in front of **D**?



Beta Pruning: Explanation

- **MIN Level**: At **node A**, the **minimum value** found in the child node is saved in **beta**. This **beta** value is passed to the next level along with the function call.
- The next level is **MAX level**. The maximum value currently found by the node of the **MAX level** is no less than **beta** value, so there is no need to continue searching.
- A sub-node (max) needs to keep updating its own **alpha** value. If the node branch does not terminate, and the currently found maximum value >**alpha**, we need to **update** the **alpha** value and pass it to the next level of the node.



Alpha-Beta Pruning

- Applying Alpha-Beta pruning to the Minimax algorithm, we derive the Alpha-Beta search algorithm.
- Its optimization uses properties of Minimax and does not change the result of Minimax.
- The optimization depends on the order of nodes.

function ALPHA-BETA-SEARCH(*state*) **returns** an action
  $v \leftarrow$ MAX-VALUE(*state*, $-\infty$, $+\infty$)
  return the *action* in ACTIONS(*state*) with value v

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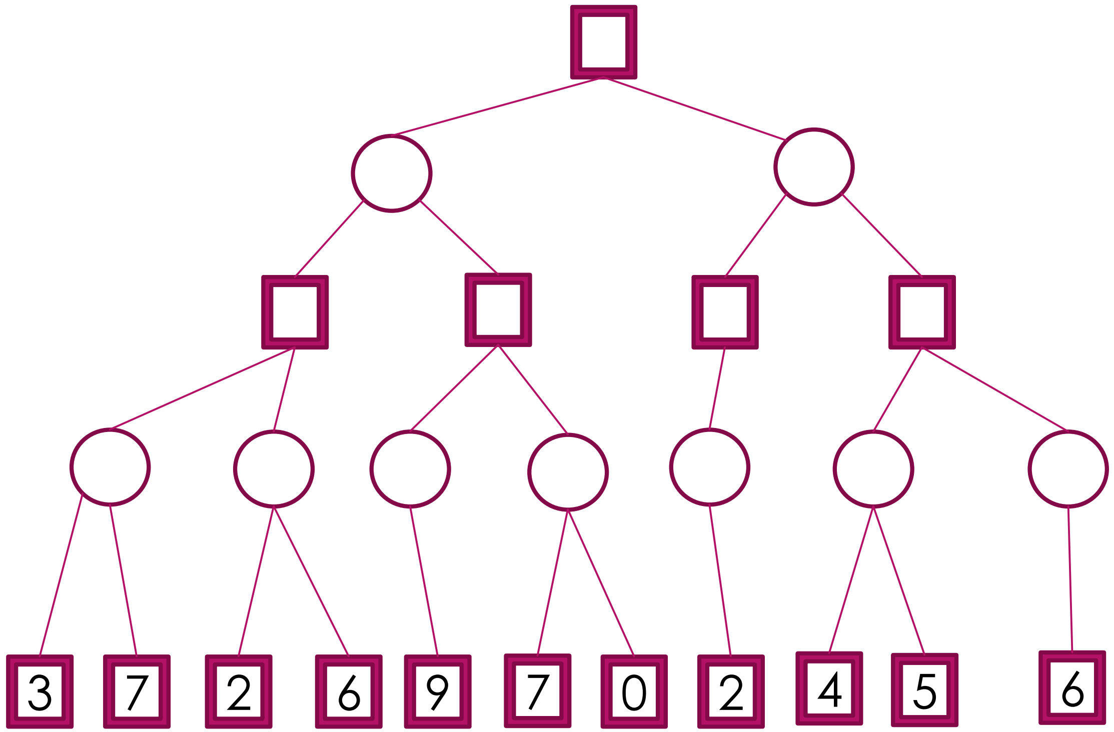
function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for each *a* **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(*state*, *a*), α, β))
    if $v \geq \beta$ **then return** v Beta Pruning
    $\alpha \leftarrow$ MAX(α, v) update alpha value
  return v

---

function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for each *a* **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(*state*, *a*), α, β))
    if $v \leq \alpha$ **then return** v Alpha Pruning
    $\beta \leftarrow$ MIN(β, v) Update beta value
  return v

Application of Alpha-Beta Pruning

- The execution result of a MINIMAX algorithm is shown in the right figure.
- Using the Alpha-Beta pruning algorithm to prune the right figure.



Tic-Tac-Toe

- http://aimacode.github.io/aima-javascript/5-Adversarial-Search/