Assignment 03 & 04 of I2ML-s23

Q1 Show that the following sequents are not valid by finding a valuation (model) in which the truth values of the formulas to the left of *are 1s (Ts) and the truth value of* the formula to the right of *is 0 (F).* *(6 * 10pts = 60 pts)*

(a) $\neg p \lor (q \to p) \vdash \neg p \land q$

(b) $\neg r \to (p \lor q),\ r \land \neg q \vdash r \to q$

(c) $\neg p,\ p \lor q \vdash \neg q$

(d) $p \to (\neg q \lor r),\ \neg r \vdash \neg q \to \neg p$

(e) $p \to q \vdash p \lor q$

(f) $p \to (q \lor r) \vdash (p \to q) \land (p \to r)$

Q2 Prove the validity of the following sequents (study all of them first, and then choose 4 of the hardest sequents for you to prove and do not duplicate with formulas or sequents you have proved in the assignment 1 or 2) by formal proof rules and format learnt in Lecture Notes 05 & 06. (Pay attention that F1 $\vdash$ F2 is a shorthand of { F1 } $\vdash$ F2 in formal proof.)
(*4 * 10pts = 40pts*)

(a) $\phi_1 \wedge \neg\phi_2 \vdash \neg(\phi_1 \rightarrow \phi_2)$
(b) $\neg\phi_1 \wedge \neg\phi_2 \vdash \phi_1 \rightarrow \phi_2$
(c) $\neg\phi_1 \wedge \phi_2 \vdash \phi_1 \rightarrow \phi_2$
(d) $\phi_1 \wedge \phi_2 \vdash \phi_1 \rightarrow \phi_2$
(e) $\neg\phi_1 \wedge \phi_2 \vdash \neg(\phi_1 \wedge \phi_2)$
(f) $\neg\phi_1 \wedge \neg\phi_2 \vdash \neg(\phi_1 \wedge \phi_2)$
(g) $\phi_1 \wedge \neg\phi_2 \vdash \neg(\phi_1 \wedge \phi_2)$
(h) $\neg\phi_1 \wedge \neg\phi_2 \vdash \neg(\phi_1 \vee \phi_2)$
(i) $\phi_1 \wedge \phi_2 \vdash \phi_1 \vee \phi_2$
(j) $\neg\phi_1 \wedge \phi_2 \vdash \phi_1 \vee \phi_2$
(k) $\phi_1 \wedge \neg\phi_2 \vdash \phi_1 \vee \phi_2$.

Q3 Use mathematical induction on *n to prove the following equivalence: (20 pts)*

$$((\phi_1 \wedge (\phi_2 \wedge (\cdots \wedge \phi_n) \dots) \to \psi) \equiv (\phi_1 \to (\phi_2 \to (\dots (\phi_n \to \psi) \dots)))).$$

Q4 Find a formula of propositional logic φ which contains only the atoms *p, q* and *r* and which is true only when *p* and *q* are false, or when $\neg q \wedge (p \vee r)$ is true. *(10pts)*

Q5 Derive the following statements by formal proof learnt in Lecture Notes 05 & 06. (*2 * 15 pts = 30 pts*)

5a) $\emptyset \vdash (p \Rightarrow (q \vee r)) \vee (r \Rightarrow \neg p)$

5b) $\{r \vee (s \wedge \neg t), (r \vee s) \Rightarrow (u \vee \neg t)\} \vdash t \Rightarrow u$

Q6 Prove the following equivalences: *(2 * 15 pts = 30 pts)*

6a) $p \Rightarrow q \;\equiv\; \neg p \vee q \;\equiv\; \neg(p \wedge \neg q) \;\equiv\; \neg q \Rightarrow \neg p$

6b) $(p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r) \;\equiv\; p \Leftrightarrow q \Leftrightarrow r$

Q7 Simplify *(10 pts)*

$$\underbrace{p \oplus \ldots \oplus p}_{n} \oplus \underbrace{\neg p \oplus \ldots \oplus \neg p}_{k} \equiv ?$$

Q8 For the formula: $\neg(\neg((s \Rightarrow \neg(p \Leftrightarrow q))) \oplus (\neg q \vee r))$

8a) Convert the above formula into NNF;

8b) Remove $\Rightarrow$, $\Leftrightarrow$, *and* $\oplus$ before turning the above into NNF, redo 8a).

(*2 * 10pts = 20pts*)