Assignment 02 of I2ML-s23

Q1 Use truth tables to prove the validity of the 3 hardest sequents you thought from the following list. Pay attention that $A, B, C \vdash D$ is equivalent to $A \wedge B \wedge C \rightarrow D$. *(3 * 10 = 30 pts)*

1. $p \vee q \vdash q \vee p$
2. $(p \wedge q) \wedge r \vdash p \wedge (q \wedge r)$
3. $p \rightarrow q \vdash \neg q \rightarrow \neg p$
4. $(p \wedge q) \vee (p \wedge r) \vdash p \wedge (q \vee r)$
5. $\vdash \neg p \vee q \rightarrow (p \rightarrow q)$
6. $\vdash (p \rightarrow q) \rightarrow \neg p \vee q$
7. $\neg(p \rightarrow q) \vdash q \rightarrow p$
8. $W \rightarrow X, Y \rightarrow Z \vdash W \vee Y \rightarrow X \vee Z$
9. $\vdash \neg(A \wedge B) \rightarrow (A \rightarrow \neg B)$
10. $p \wedge q \rightarrow r \vdash (p \rightarrow r) \vee (q \rightarrow r)$
11. $\vdash \neg(A \leftrightarrow \neg A)$
12. $A \leftrightarrow B \vdash \neg A \leftrightarrow \neg B$

Q2 Use truth tables to prove the validity of the 3 hardest formulas you thought from the following list. Pay attention that do not repeat with Q1. *(3 * 10 = 30 pts)*

1. Commutativity of $\wedge$: $A \wedge B \leftrightarrow B \wedge A$
2. Commutativity of $\vee$: $A \vee B \leftrightarrow B \vee A$
3. Associativity of $\wedge$: $(A \wedge B) \wedge C \leftrightarrow A \wedge (B \wedge C)$
4. Associativity of $\vee$ $(A \vee B) \vee C \leftrightarrow A \vee (B \vee C)$
5. Distributivity of $\wedge$ over $\vee$: $A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C)$
6. Distributivity of $\vee$ over $\wedge$: $A \vee (B \wedge C) \leftrightarrow (A \vee B) \wedge (A \vee C)$
7. $(A \rightarrow (B \rightarrow C)) \leftrightarrow (A \wedge B \rightarrow C)$.
8. $(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$
9. $((A \vee B) \rightarrow C) \leftrightarrow (A \rightarrow C) \wedge (B \rightarrow C)$
10. $\neg(A \vee B) \leftrightarrow \neg A \wedge \neg B$
11. $\neg(A \wedge B) \leftrightarrow \neg A \vee \neg B$
12. $\neg(A \wedge \neg A)$
13. $\neg(A \rightarrow B) \leftrightarrow A \wedge \neg B$
14. $\neg A \rightarrow (A \rightarrow B)$
15. $(\neg A \vee B) \leftrightarrow (A \rightarrow B)$
16. $A \vee \bot \leftrightarrow A$
17. $A \wedge \bot \leftrightarrow \bot$
18. $A \vee \neg A$
19. $\neg(A \leftrightarrow \neg A)$
20. $(A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$
21. $(A \rightarrow C \vee D) \rightarrow ((A \rightarrow C) \vee (A \rightarrow D))$
22. $(((A \rightarrow B) \rightarrow A) \rightarrow A)$

Q3 For the given formula below, draw its parse tree and list the set of its all subformulas. *(10 + 10 = 20 pts)*

$((p \rightarrow \sim q) \vee (p \wedge r) \rightarrow s) \vee \sim r$

Q4 Finish 3 tasks for each parse true in Figure A and Figure B: **1)** List the corresponding formula with fully dressed parentheses; **2)** List the shortest formula with proper precedence order and right associative discussed in lectures; **3)** Given a valuation/model and a parse tree of a formula, compute the truth value of the formula for that valuation/model (as done in a bottom-up fashion in Figure 1.7 on page 40 of textB). (*2 * (5+5+5) = 30 pts*)

4.1 **Figure A** in which *q evaluates to T and p and r evaluate to F;*

4.2 **Figure B** where we let *p be T, q be F and r be T.*

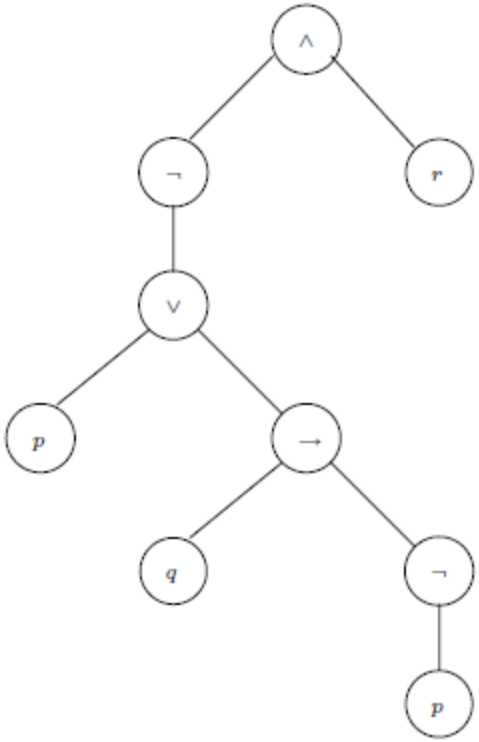

Figure A. In-order linear representation of the tree is a logical formula

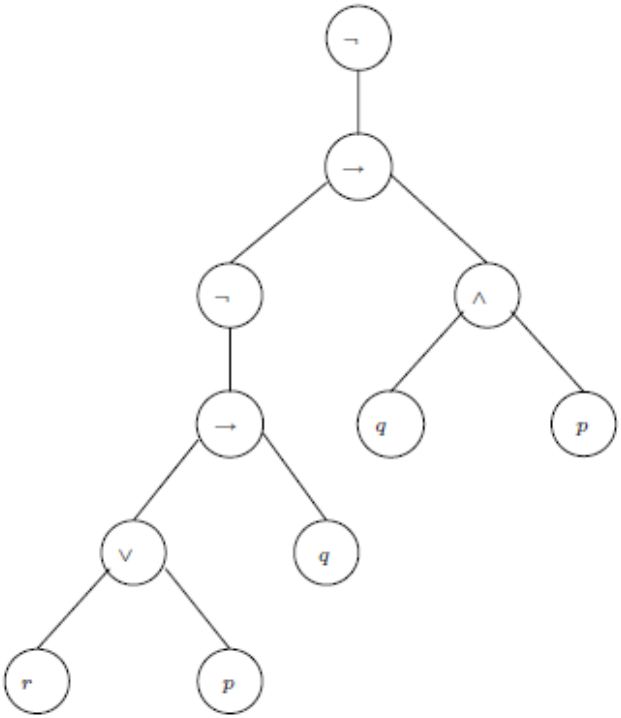

Figure B. A parse tree of a negated implication

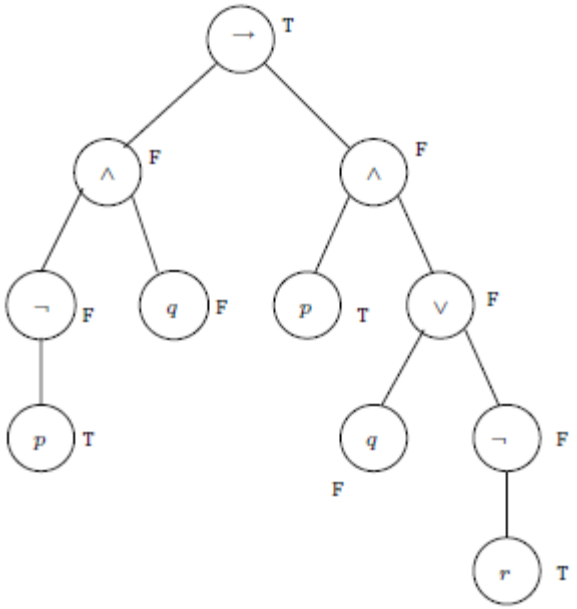

Figure 1.7. The evaluation of a logical formula under a given valuation *(On page 40 of textB)*