Propositional Logic I

An argument is a pattern of inference. It is *valid* if there is no interpretation under which all the argument's premises are true but the conclusion is false. Arguments are valid if they do not produce falsehoods from true premises.

Propositions

A proposition is an entity that has a truth value. It is either true (T) or false (F). Usually, a proposition is expressed in a sentence of a natural language:

(1) 'Jim likes Wagner' is an English sentence that expresses the proposition *that* Jim likes Wagner; or

(2) 'Descartes aimait le vin' is a French sentence that expresses the proposition *that* Descartes liked wine.

Preliminaries

(a) Greek letters (ϕ and ψ) are metavariables, which stand for *any* sentence of the language of propositional logic (PL). They are metalinguistic variables that we use to think about the object language, which is here PL.[1] Occasionally, *A*, *B*, and *C* are used as metavariables.

(b) There is a distinction between *use* and *mention*. Quotation marks allow us to mention a word or sentence: (i) Socrates has a beard; (ii)⋆ 'Socrates' has a beard; (iii) 'Der Hund bellt' is a German sentence; (iv) The formula '$P \vee Q$' is a sentence in PL; (v)⋆ *P* means that Socrates is bearded.

Approaching PL

(3) Jim plays the piano or Jackie sings. Jackie does not sing. So Jim plays the piano.

(4) (Jim plays the piano) *or* (Jackie sings). *Not* (Jackie sings). *So* (Jim plays the piano).

(5) Lexicon or interpretation: P = (Jim plays the piano), Q = (Jackie sings)

(6) *P* or Q. Not Q. So *P*.

(7) $P \vee Q, \sim Q \therefore P$, where '$\therefore$' is an inference marker that means 'therefore' in PL.[2] This is sometimes also put in a vertical way, including a line as the inference marker:

$$\begin{array}{l} P \vee Q \\ \sim Q \\ \hline P \end{array}$$

The aim of PL is to represent or express how propositions are connected. Translation into PL can reveal the logical structure of natural languages.

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1 Similarly, '$\models$' is a metalinguistic symbol too; it does not belong to PL.

2 No quotation marks in the object language; in metalanguage: '$(P \vee Q)$', '$\sim Q$' $\models$ 'P' says that the PL inference which concludes from '$(P \vee Q)$' and '$\sim Q$' to 'P' is valid.

Syntax of PL

The grammar of PL defines which symbols are meaningful sentences in PL. The syntax sets out what counts as a well-formed formula (wff) in that language.

(i) Basic sentence letters or atomic formulae: P, Q, R, S, etc. (with or without indices for more sentences)[3]

(ii) Connectives or logical constants: &, ∨, ~, ⊃, ≡ (connectives generate complex formulae)

(iii) Brackets for scope: (,)

Definition of a wff in PL:

- Any atomic formula is a wff.
- If ϕ and ψ are wff, then $(\phi \,\&\, \psi)$, $(\phi \vee \psi)$, $\sim\phi$, are also wffs.[4]
- Nothing else is a wff in PL.

Semantics of PL

PL needs an interpretation, which makes its sentences meaningful and available to a truth evaluation. (Remember: propositions have truth values.)

Connective	*English*	*Symbol*
Negation[5]	'it is not the case that'	~ or ¬
Conjunction	'and', 'both'	& or ∧ or ·
Disjunction	'or', 'at least one'	∨
Material Implication, Conditional	'if … then'	⊃ or →
Material Equivalence, Biconditional	'if and only if'	≡ or ↔

Truth Functionality: the truth of a complex formula is a function of the truth of its constituent formulae. The connectives are defined by a specific distribution of truth values (T, F, or 1, 0). There are 16 such distributions for dyadic connectives.

ϕ	$\sim\phi$
T	F
F	T

ϕ	ψ	$\phi \,\&\, \psi$
T	T	T
T	F	F
F	T	F
F	F	F

&	T	F
T	T	F
F	F	F

ϕ	ψ	$\phi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

∨	T	F
T	T	T
F	T	F

The *conjunction* of ϕ and ψ is true if and only if ϕ and ψ are both true, and false otherwise. The (inclusive) *disjunction* of ϕ and ψ is true if and only if at least one of ϕ and ψ is true, and false otherwise.

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3 In older English and current German texts, small letters p, q, etc. are often used.

4 The sentences $(\phi \supset \psi)$ and $(\phi \equiv \psi)$ are also wffs. But they can be reduced to those mentioned above. Note too that more precisely, we should say: if ϕ and ψ are wff, so is $\ulcorner(\phi \,\&\, \psi)\urcorner$. The new quotation marks ('Quine corners') make clearer that the symbols of the object language are mentioned while the metasymbols are used. So, $\ulcorner(\phi \,\&\, \psi)\urcorner$ means "the result of writing '(' followed by ϕ (any atomic formula of PL) followed by '&' followed by ψ followed by ')'."

5 The negation is a *monadic* 'connective', which only takes one wff. The other connectives are dyadic, and take two wffs.

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