Assignment#3 CS207 Fall 2023

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PROBLEM 1. Analyze the sequential circuit with two JK flip-flops A and B with two inputs x and y and one output z. The input equation and output equation is given.

SOLUTION. The state equation for the circuit is given by

$$\begin{aligned}A(t+1) &= J_A(t)A' + K_A'(t)A = A'Bx + A'B'y' + A(B'xy')' \\ &= A'Bx + A'B'y' + A(B + x' + y) \\ &= A'Bx + A'B'y' + AB + Ax' + Ay \\ &= \Sigma(0, 2, 6, 7, 8, 9, 10, 12, 13, 14, 15)\end{aligned}$$

$$\begin{aligned}B(t+1) &= J_B(t)B' + K_B'(t)B = A'B'x + B(A + xy')' \\ &= A'B'x + A'Bx'y \\ &= \Sigma(2, 3, 4, 5, 7)\end{aligned}$$

and the output equation is given by

$$z(t) = Ax'y' + Bx'y' = \Sigma(4, 8, 12)$$

So the state table is

Table 1: State Table

Present State		**Input**		**Next State**		**Output**	**JKFF Input**			
A	B	x	y	A	B	z	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	1	0	1	1	1	1
0	0	1	1	0	1	0	0	0	1	0
0	1	0	0	0	1	1	0	0	0	0
0	1	0	1	0	1	0	0	0	0	0
0	1	1	0	1	0	0	1	0	1	1
0	1	1	1	1	1	0	1	0	1	0
1	0	0	0	1	0	1	1	0	0	1
1	0	0	1	1	0	0	0	0	0	1
1	0	1	0	1	0	0	1	1	0	1
1	0	1	1	0	0	0	0	0	0	1
1	1	0	0	1	0	1	0	0	0	1
1	1	0	1	1	0	0	0	0	0	1
1	1	1	0	1	0	0	1	0	0	1
1	1	1	1	1	0	0	1	0	0	1

and the state diagram and timing diagram is

Figure 1: State Diagram

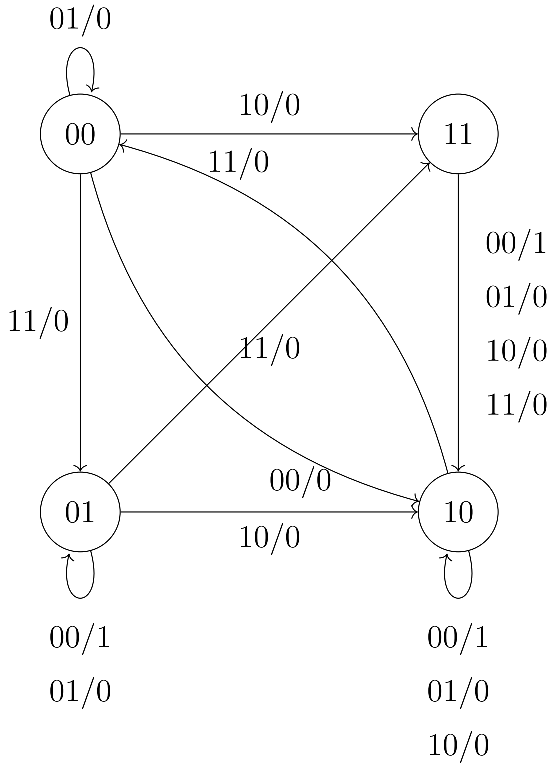
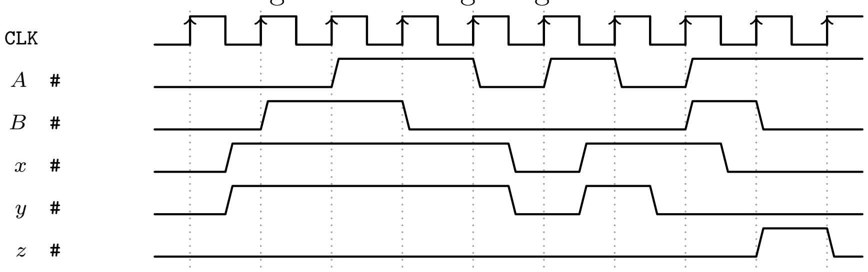


Figure 2: Timing Diagram



PROBLEM 2. Analyze the sequential circuit with two TFFs A and B.

Solution. From the diagram, we can derive the input equations

$$T_A = A + B$$
$$T_B = A' + B$$

and there's no output equation. So the state equation would be

$$A(t+1) = T_A \oplus Q_A = (A + B) \oplus A$$
$$B(t+1) = T_B \oplus Q_B = (A' + B) \oplus B$$

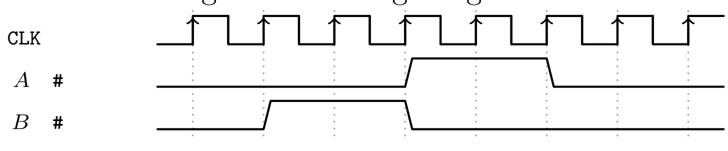
and the state table is

Table 2: State Table

Present State		**Next**	
A	B	A	B
0	0	0	1
0	1	1	0
1	0	0	0
1	1	0	0

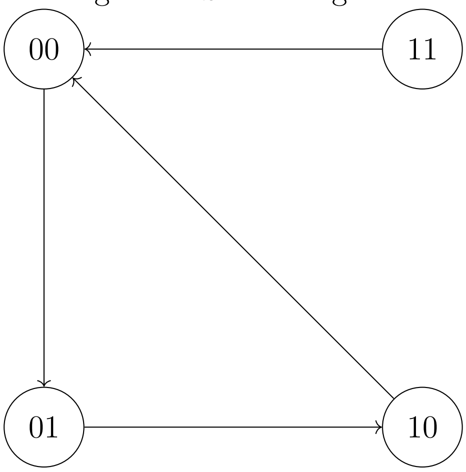
the fourth state is useless. And the timing diagram and state diagram is

Figure 3: Timing Diagram



Problem 3. For the block diagram, find the state table and state diagram.

Figure 4: State Diagram



Solution. **a)** The input functions are

$$\begin{aligned} J_1 &= X \\ K_1 &= (Q_2'X)' \\ J_2 &= X \\ K_2 &= (Q_1X)' \end{aligned}$$

and the state equation is

$$\begin{aligned} Q_1(t+1) &= J_1Q_1' + K_1'Q_1 = Q_1'X + Q_1Q_2'X \\ Q_2(t+1) &= J_2Q_2' + K_2'Q_2 = Q_2'X + Q_1Q_2X \\ Q_2'(t+1) &= (X' + Q_2)(Q_1' + Q_2' + X') \\ &= X' + Q_2(Q_1' + Q_2') = X' + Q_1'Q_2 \end{aligned}$$

and the output function is

$$F = X \oplus Q_2(t+1)' = X \oplus (X' + Q_1'Q_2)$$

so the state table is

b) It's a Mealy Machine since the output F is determined by the input X and the state.

Table 3: State Table

Present State		**Input**	**Next State**		**Output**	**JKFF Input**			
$Q_1(t)$	$Q_2(t)$	X	$Q_1(t+1)$	$Q_2(t+1)$	F	J_1	K_1	J_1	K_1
0	0	0	0	0	1	0	1	0	1
0	0	1	1	1	1	1	0	1	1
0	1	0	1	0	1	0	1	0	1
0	1	1	0	0	0	1	1	1	1
1	0	0	0	0	1	0	1	0	1
1	0	1	1	1	1	1	0	1	0
1	1	0	0	0	1	0	1	0	1
1	1	1	0	1	1	1	1	1	0

Figure 5: State Diagram

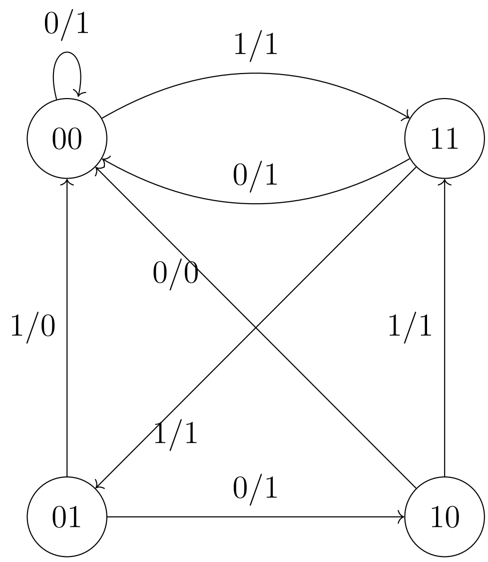
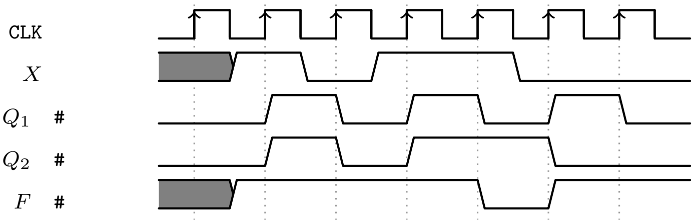


Figure 6: Timing Diagram



c) The timming diagram based on the state table is

Problem 4. Obtain the simplified input equations for a sequential circuit with TFF and the state diagram is given.

Solution. From the state diagram, we can derive the state table so the

Table 4: State Table

Present State		**Input**	**Next State**	
$Q_1(t)$	$Q_2(t)$	X	$Q_1(t+1)$	$Q_2(t+1)$
0	0	0	0	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	1

state equations are

$$Q_1(t+1) = T_1 \oplus Q_1(t) = \Sigma(2,3,4,5,7)$$
$$Q_2(t+1) = T_2 \oplus Q_2(t) = \Sigma(0,2,4,7)$$

and the K-Maps for each TFF are

$Q_1 \backslash Q_2X$	00	01	11	10
0	0	0	1	1
1	1	1	1	0

$Q_1 \backslash Q_2X$	00	01	11	10
0	1	0	0	1
1	1	0	1	0

So the simplified state equations are

$$\begin{aligned} Q_1(t+1) &= Q_1'Q_2 + Q_2X + Q_1Q_2' \\ &= Q_1 \oplus (Q_1 \oplus (Q_1'Q_2 + Q_2X + Q_1Q_2')) \\ &= Q_1 \oplus (Q_1'Q_2 + Q_1Q_2X') \end{aligned}$$

$$\begin{aligned} Q_2(t+1) &= Q_2'X' + Q_1'X' + Q_1Q_2X \\ &= Q_2 \oplus (Q_2 \oplus (Q_2'X' + Q_1'X' + Q_1Q_2X)) \\ &= Q_2 \oplus (Q_1Q_2X' + Q_1'Q_2X) \end{aligned}$$

So the input equations are

$$T_1 = Q_1'Q_2 + Q_1Q_2X'$$
$$T_2 = Q_1Q_2X' + Q_1'Q_2X$$

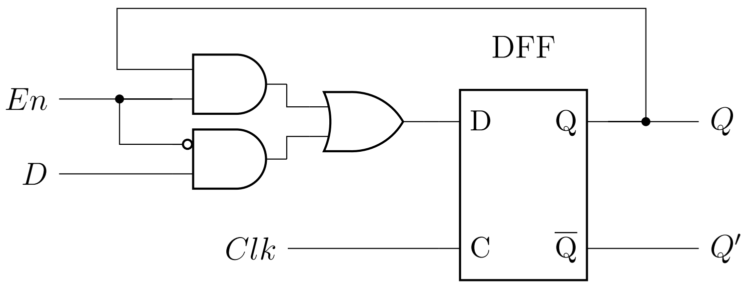
Problem 5. Design a DFF with enable input whose function table is given.

Solution. From the characteristic table, we can derive the input equation

$En \backslash DQ$	00	01	11	10
0	0	1	1	0
1	0	0	1	1

$$Q(t) = Q(t+1)En' + DEn$$

so the circuit is



Problem 6. Design a certain filp-flop according the following description.

Solution. **a)** The characteristic table is

A	B	$Q(t+1)$	$Q(t+1)'$	
0	0	0	1	clear to 0
0	1	$Q(t)$	$Q'(t)$	no change
1	0	$Q'(t)$	$Q(t)$	complement
1	1	1	0	set to 1

b) From the characteristic table, we can derive the K-maps first

$Q(t) \backslash AB$	00	01	11	10
0	0	0	1	1
1	0	1	1	0

so the simplified equation is

$$Q(t+1) = AQ'(t) + BQ(t)$$

c) The excitation table is

$Q(t)$	$Q(t+1)$	A	B	Operation
0	0	0	X	no change
0	1	1	X	set to 1
1	0	X	0	clear to 1
1	1	X	1	no change

d) From the characteristic equation, we have

$$Q(t+1) = AQ'(t) + (B')'Q(t)$$

that is, we can substitude the inputs with JKFF's inputs

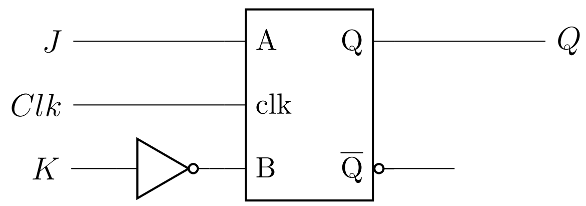
$$Q(t+1) = JQ'(t) + K'Q(t) \Rightarrow J = A \qquad K = B'$$

which is indicates the input equations of our flip-flop

$$A = J$$
$$B = K'$$

so the block diagram is

JKFF



PROBLEM 7. For the following state table, simplify the state table and draw the state diagram. Then design the sequential circuit using JKFF.

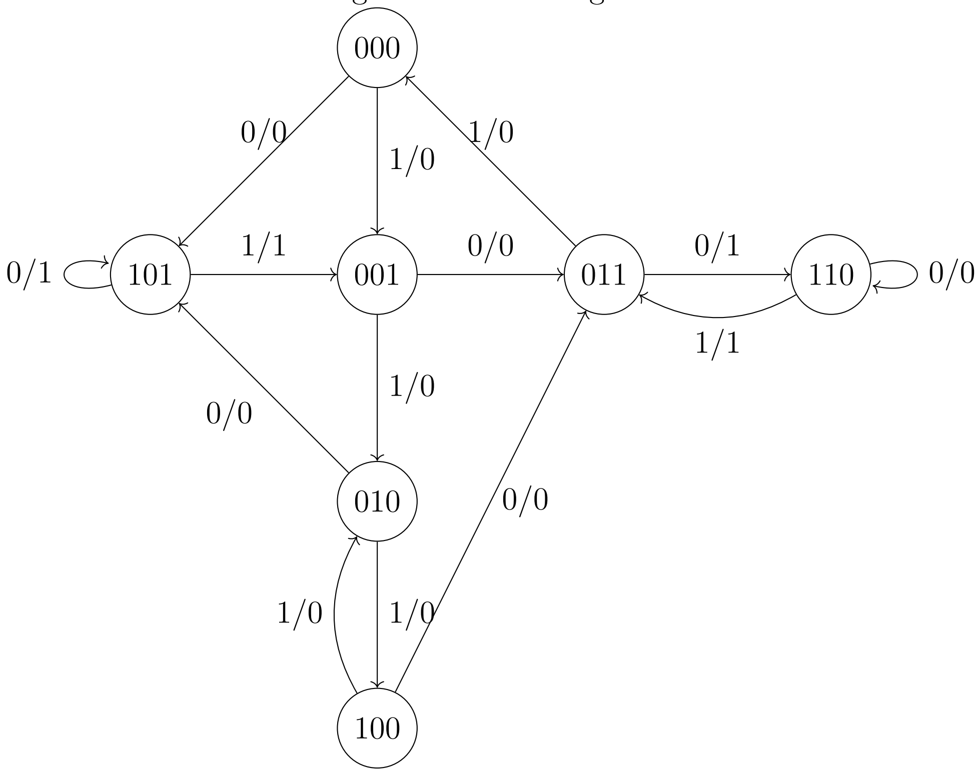
SOLUTION. To simplify the state, we first notice that the state of h is the same as state d, so we can remove the state h and rewrite the transition to h into d. Then we assign codes to the states and obtain the state table

Table 5: State Table

Present State	**Next State**		**Output**	
ABC	$x = 0$	$x = 1$	$x = 0$	$x = 1$
000	101	001	0	0
001	011	010	0	0
010	101	100	0	0
011	110	000	1	0
100	011	010	0	0
101	101	010	1	1
110	110	011	0	1
111	X	X	X	X

and the coresponding state diagram is

Figure 7: State Diagram



Next we obtain the state equations and output equation using the state table and K-map

$AB \backslash Cx$	00	01	11	10
00	1	0	0	0
01	1	1	0	1
11	X	X	X	X
10	X	X	X	X

$AB \backslash Cx$	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	0	1	X	X
10	1	1	1	0

AB \ Cx	00	01	11	10
00	0	0	1	1
01	X	X	X	X
11	X	X	X	X
10	1	1	1	0

AB \ Cx	00	01	11	10
00	X	X	X	X
01	1	1	1	0
11	0	0	X	X
10	X	X	X	X

AB \ Cx	00	01	11	10
00	1	1	X	X
01	1	0	X	X
11	0	1	X	X
10	1	0	X	X

AB \ Cx	00	01	11	10
00	X	X	1	0
01	X	X	1	1
11	X	X	X	X
10	X	X	1	0

Figure 8: Output K-Map

AB \ Cx	00	01	11	10
00	0	0	0	0
01	0	0	0	1
11	0	1	X	X
10	0	0	1	1

Table 6: State Table

Present State			Input	Next State			JKFF Input					
A	B	C	x	A	B	C	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	1	0	1	1	X	0	X	1	X
0	0	0	1	0	0	1	0	X	0	X	1	X
0	0	1	0	0	1	1	0	X	1	X	X	0
0	0	1	1	0	1	0	0	X	1	X	X	1
0	1	0	0	1	0	1	1	X	X	1	1	X
0	1	0	1	1	0	0	1	X	X	1	0	X
0	1	1	0	1	1	0	1	X	X	0	X	1
0	1	1	1	0	0	0	0	X	X	1	X	1
1	0	0	0	0	1	1	X	1	1	X	1	X
1	0	0	1	0	1	0	X	1	1	X	0	X
1	0	1	0	1	0	1	X	0	0	X	X	0
1	0	1	1	0	1	0	X	1	1	X	X	1
1	1	0	0	1	1	0	X	0	X	0	0	X
1	1	0	1	0	1	1	X	1	X	0	1	X
1	1	1	X	X	X	X	X	X	X	X	X	X

so the input equations are

$$J_A = C'x' + BC' + BCx'$$

$$K_A = Ax + AB'C'$$

$$J_B = A'C + AC' + Ax$$

$$K_B = A'C'Cx$$

$$
\begin{aligned}
J_C &= A'B' + A'C'x' + ABx + AB'x' \\
K_C &= Cx + BC
\end{aligned}
$$

and the output equation is

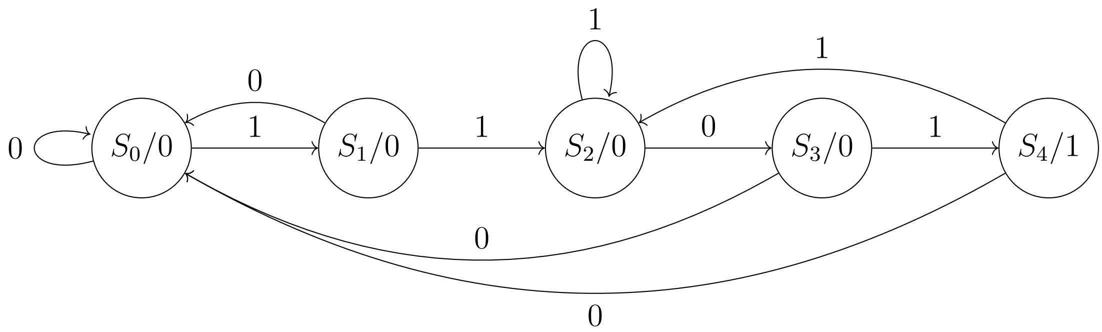
$$y = AC + ABx + BCx'$$

and the state equations are

$$
\begin{aligned}
A(t+1) &= J_A A' + K'_A A \\
&= A'(C'x' + BC' + BCx') + A(Ax + AB'C')' \\
&= A'C'x' + A'BC' + A'BCx' + ABx' + ACx' \\
B(t+1) &= J_B B' + K'_B B \\
&= B'(A'C + AC' + Ax) + A(A'C'Cx)' \\
&= A + A'B'C \\
C(t+1) &= J_C C' + K'_C C \\
&= C'(A'B' + A'C'x' + ABx + AB'x') + C(Cx + BC)' \\
&= A'B'C' + A'C'x' + ABC'x + AB'C'x' + BCx
\end{aligned}
$$

Problem 8. Design a sequence detector using DFF to recognize the occurrence of bits 1101 with Moore machine in overlapping mode.

Solution. According to the description, we can draw the state diagram



and the state table is

Table 7: State Table

Present State	**Next State**		Output
ABC	$x = 0$	$x = 1$	y
(S_0) 000	000	001	0
(S_1) 001	000	010	0
(S_2) 010	011	010	0
(S_3) 011	000	100	0
(S_4) 100	000	010	1
101	X	X	X
110	X	X	X
111	X	X	X

So we can obtain the input and state equations from the K-maps

$$A(t+1) = D_A(t) = BCx$$
$$B(t+1) = D_B(t) = BC' + Ax + B'Cx$$
$$C(t+1) = D_C(t) = BC'x' + A'B'C'x$$

and the output equation is

$$y = A$$

And the K-maps are

$AB \backslash Cx$	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	X	X	X	X
10	0	0	X	X

$AB \backslash Cx$	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	X	X	X	X
10	0	1	X	X

$AB \backslash Cx$	00	01	11	10
00	0	1	0	0
01	1	0	0	0
11	X	X	X	X
10	0	0	X	X

$AB \backslash Cx$	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	X	X	X	X
10	1	1	X	X