Logic & Bayesian Network Questions

Propositional Logic

- The best combination of the six important players in the volleyball team (1, 3, 4, 6, 9, 12) should follow the following rules:
  - Players 4 and 6 need to play together.
  - Player 3 does not play the game if and only if player 1 does not play.
  - Either player 3 or player 6 should appear on the court, and they can not appear at the same time.
  - If players 9 and 12 are on the court, then player 4 must be on the court.

- **If players 1 and 12 need to play at the same time in one game, who of the other 4 players should play?**

(1) **Please use propositional logic to represent the above statement.**

(2) **Convert them into a CNF.**

(3) **Apply the DPLL algorithm to derive which players should be on the court.**

First-order Logic

- Anyone who passes the AI exam and wins a prize is happy.
- Anyone willing to learn or lucky can pass all the exams.
- Zhansan is not willing to learn but he is lucky.
- Any lucky person can win a prize.

- Show $ZhangSan$ is Happy using resolution.

Bayesian Network

- Suppose the ratio of boys and girls in SUSTech is 7:3, the ratio of long hair and short hair among boys is 1:9, and the ratio of long hair and short hair among girls is 8:2, please do:
  - Draw the Bayesian network that represents the above relationship.
  - If you meet a student with short hair on the campus, what is the probability that it is a girl?

Approximate Inference with Bayesian Networks

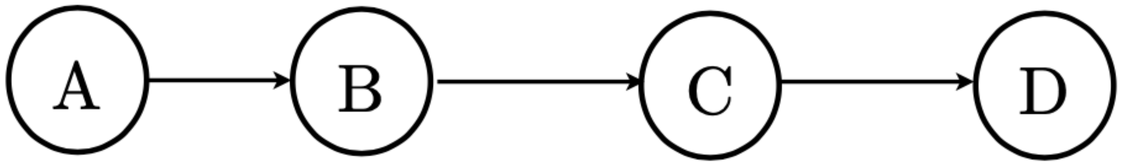
- Assume the right Bayes' net, and the corresponding distributions over the variables in the Bayes' net:
  - You are given the following samples:

  (A, B, ¬C, ¬D), (A, ¬B, C, ¬D), (¬A, B, C, ¬D), (¬A, ¬B, C, ¬D)

  (A, ¬B, ¬C, D), (A, B, C, ¬D), (¬A, B, ¬C, D), (¬A, ¬B, C, ¬D)

  (i) If these samples came from doing Prior Sampling, calculate our sample estimate of P (C).

  (ii) Now we will estimate P (C| A, ¬D). Cross out the samples that would not be used when doing Rejection Sampling for this task and write down the sample estimate of P (C| A, ¬D).



P(A)
1/4

P(B | A)	
A	1/5
¬A	3/4

P(C | B)	
B	1/2
¬B	1/3

P(D | C)	
C	1/6
¬C	7/8

- Using Likelihood Weighting Sampling to estimate P(¬A | B, ¬D), the following samples were obtained. What is the weight of each sample?

Sample	Weight
¬A B C ¬D	________
A B C ¬D	________
A B ¬C ¬D	________
¬A B ¬C ¬D	________

- From the weighted samples, estimate P(¬A | B, ¬D).

- Recall that during the MCMC-Ask function (Gibbs Sampling), samples are generated through an iterative process. Assume that the only evidence that is available is A ==TRUE. Which sequence(s) below could have been generated by Gibbs Sampling?

"State" of network = current assignment to all variables.

Generate next state by sampling one variable given Markov blanket
Sample each variable in turn, keeping evidence fixed

function MCMC-ASK(X, $\mathbf{e}$, bn, N) **returns** an estimate of $P(X|\mathbf{e})$
  local variables: $\mathbf{N}[X]$, a vector of counts over X, initially zero
    $\mathbf{Z}$, the nonevidence variables in bn
    $\mathbf{x}$, the current state of the network, initially copied from $\mathbf{e}$

  initialize $\mathbf{x}$ with random values for the variables in $\mathbf{Y}$
  for $j = 1$ to N **do**
    for each Z_i in $\mathbf{Z}$ **do**
      sample the value of Z_i in $\mathbf{x}$ from $\mathbf{P}(Z_i|mb(Z_i))$
        given the values of $MB(Z_i)$ in $\mathbf{x}$
      $\mathbf{N}[x] \leftarrow \mathbf{N}[x] + 1$ where x is the value of X in $\mathbf{x}$
  return NORMALIZE($\mathbf{N}[X]$)

Markov Blanket

Can also choose a variable to sample at random each time

Sequence 1	
1:	A ¬B ¬C D
2:	A ¬B ¬C D
3:	A ¬B C D

Sequence 2	
1:	A ¬B ¬C D
2:	A ¬B ¬C ¬D
3:	¬A ¬B ¬C D

Sequence 3	
1:	A ¬B ¬C D
2:	A ¬B ¬C ¬D
3:	A B ¬C ¬D

Sequence 4	
1:	A ¬B ¬C D
2:	A ¬B ¬C ¬D
3:	A B ¬C D