Assignment#1 CS207 Fall 2023

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PROBLEM 1. Convert the following numbers with different bases.

SOLUTION. **a)** $(22200.11)_3$ **b)** $(176.6)_{12}$ **c)** $(167)_{10}$ **d)** $(26.24)_8$

Table 1: Process of (a)

Integer	**Remainder**	**Decimal**	**Remainder**
234		.5	
78	0	.5	1
26	0	.5	1
8	2	$\vdots$	$\vdots$
2	2	$\vdots$	$\vdots$
0	2	answer	$(22200.11)_3$

Table 2: Process of (b)

Integer	**Remainder**	**Decimal**	**Remainder**
234		.5	
19	6	0	6
1	7		
0	1	answer	$(176.6)_{12}$

Process of (c): $5 \times 6^0 + 3 \times 6^1 + 4 \times 6^2 = (167)_{10}$
Process of (d):

$$(26.24)_8 = (\underset{2}{\underline{010}}\ \underset{6}{\underline{110}}\ .\ \underset{2}{\underline{010}}\ \underset{4}{\underline{100}})_2$$

Problem 2. Determine the possible radices of the numbers in each operation.

Solution.
a) Suppose the radix to be x, and we have,

$$\begin{aligned} &1 \cdot x^3 + 2 \cdot x^2 + 3 \cdot x^1 + 4 \cdot x^0 \\ &+5 \cdot x^3 + 4 \cdot x^2 + 3 \cdot x^1 + 2 \cdot x^0 = 6 \cdot x^3 + 6 \cdot x^2 + 6 \cdot x^1 + 6 \cdot x^0 \\ &\qquad\qquad\qquad\qquad\qquad\qquad\quad = 6 \cdot x^3 + 6 \cdot x^2 + 6 \cdot x^1 + 6 \cdot x^0 \end{aligned}$$

without any carry operation. Therefore, the radix barely need to be over than the biggest number, say 6.
Thus, $x > 6$, and radices could be $7, 8, 9, 10, \cdots$.
b) Same as (a) and we have

$$\begin{aligned} 3 \cdot x^2 + 2 \cdot x^0 &= (1 \cdot x^1 + 2 \cdot x^0 + 1 \cdot x^{-1})(2 \cdot x^1) \\ &= (2 \cdot x^2 + 4 \cdot x^1 + 2 \cdot x^0 \\ \Rightarrow x^2 - 4x = 0 &\quad \text{Solution: } x = 4 \text{ or } 0 \end{aligned}$$

Radix cannot be zero or one, so the radix is 4.

Problem 3. Simplify the following Boolean expressions to the **indicated** number of literals algebraically.

Solution. **a)** The simplification is as follow

$$\begin{aligned}
&(a' + c)(a' + c')(a + b + c'd) \\
&= (a' + c \cdot c')(a + b + c'd) \\
&= a' \cdot (a + b + c'd) \\
&= a \cdot a' + ab' + a'c'd \\
&= ab' + a'c'd \\
&= a' \cdot (b + c'd) \qquad \text{4 literals}
\end{aligned}$$

b) Simplify the expression and got

$$\begin{aligned}
&abc'd + a'bd + abcd \\
&= abd \cdot (c + c') + a'bd \\
&= abd + a'bd \\
&= bd \cdot (a + a') \\
&= bd \qquad \text{2 literals}
\end{aligned}$$

Problem 4. Simplify the following Boolean expressions to a **minimum** number of literals algebraically.

Solution. **a)** Conbining terms using *Distributive Law*

$$\begin{aligned}
&(a + c)(a' + b + c)(a' + b' + c) \\
&= (a + c)(a' + c + bb') \\
&= (a + c)(a' + c) \\
&= c + aa' \\
&= c \qquad \text{1 literals}
\end{aligned}$$

b) Conbining the terms

$$
\begin{aligned}
F(a,b,c) &= a'b'c' + a'b'c + a'bc' + a'bc + ab'c \\
&= a'b' \cdot (c + c') + a'b(c + c') + (a + a')b'c \\
&= a'b' + a'b + b'c \\
&= a'(b + b') + b'c \\
&= a' + b'c \qquad \text{3 literals}
\end{aligned}
$$

PROBLEM 5. Convert the expressions into sum of minterms and product of maxterms.

SOLUTION. **a)** Transfer the expression into SOP terms with 4 literals each

$$
\begin{aligned}
F(a,b,c,d) &= acd' + ab'c + bd' + a'c' \\
&= acd'(b + b') + ab'c(d + d') + (a + a')b(c + c')d' + a'(b + b')c'(d + d') \\
&= abcd' + ab'cd' + ab'cd + abc'd' + a'bcd' + a'bc'd' + a'b'c'd + a'b'c'd' + a'bc'd
\end{aligned}
$$

Thus, Minterms: $F(a,b,c,d) = \Sigma(0,1,4,5,6,10,11,12,14)$
Maxterms: $F(a,b,c,d) = \Pi(2,3,7,8,9,13,15)$

b) Same procedure as (a)

$$
\begin{aligned}
F(x,y,z) &= (x' + y)(x' + z) \\
&= x' + yz \\
&= x'(y + y')(z + z') + (x + x')yz \\
&= xyz + x'yz + x'yz' + x'y'z + x'y'z'
\end{aligned}
$$

Thus, Minterms: $F(a,b,c,d) = \Sigma(0,1,2,3,7)$
Maxterms: $F(a,b,c,d) = \Pi(4,5,6)$

PROBLEM 6. Simplify the functions $F_1(A, B, C)$ and $F_2(A, B, C)$ to
a) expressions with 3 literals(F_1) and 2 literals(F_2) using algebraic method
b) by K map in sum of product form.

SOLUTION. **a)** Got the expression using minterms and simplify them algebraically

$$\begin{aligned} F_1(A, B, C) &= \Sigma(2, 3, 7) \\ &= A'BC' + A'BC + ABC \\ &= A'B + BC \\ &= B(A' + C) \qquad \text{3 literals} \end{aligned}$$

$$\begin{aligned} F_2(A, B, C) &= \Sigma(0, 2, 5, 7) \\ &= A'B'C' + A'BC' + AB'C + ABC \\ &= A'C' + AC \\ &= (A \oplus C)' \qquad \text{2 literals} \end{aligned}$$

b) F_1

A \ BC	00	01	11	10
0	0	0	1	1
1	0	0	1	0

F_2

A \ BC	00	01	11	10
0	1	0	0	1
1	0	1	1	0

$$\begin{aligned} F_1(A, B, C) &= A'B + BC \\ &= B(A' + C) \end{aligned}$$

$$\begin{aligned} F_2(A, B, C) &= AC + A'C' \\ &= (A \oplus C)' \end{aligned}$$

PROBLEM 7. Using K maps to find a simplest sum-of-products expression for the following Boolean functions.

SOLUTION. **a)** Draw the K map and box the cells

$WX \backslash YZ$	00	01	11	10
00	1	0	1	1
01	0	0	1	1
11	1	1	1	0
10	0	0	1	1

Answer: $\boxed{F(W,X,Y,Z) = WXY' + WX'Y + W'X'Z' + YZ + W'Y}$

b) Same as (a)

$AB \backslash CD$	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	1	0
10	1	0	1	1

Answer: $\boxed{F(A,B,C,D) = B'D' + ACD}$

Problem 8. With K maps, find the simplest sum-of-products form of the function $F = fg$, where $f = abd' + c'd + a'cd' + b'cd'$ and $g = (a+b+d')(b'+c'+d)(a'+c+d')$.

Solution. From the expression $F = fg$ we can derive that

$$\text{maxterms of } F = \text{maxterms of } f + \text{maxterms of } g$$

So we got

$$\begin{aligned} f(a,b,c,d) &= \Sigma(1,2,5,6,9,10,12,13,14) \\ &= \Pi(0,3,4,7,8,11,15) \end{aligned}$$

$$g(a,b,c,d) = \Pi(1,3,6,9,13,14)$$

$$\begin{aligned} \Rightarrow F(a,b,c,d) &= \Pi(0,1,3,4,6,7,8,9,11,13,14,15) \\ &= \Sigma(2,5,10,12) \end{aligned}$$

ab \ cd	00	01	11	10
00	0	0	0	1
01	0	1	0	0
11	1	0	0	0
10	0	0	0	1

Answer: $\boxed{F = b'cd' + a'bc'd + abc'd'}$

Problem 9. Obtain the simplest sum-of-products expression for $F(A, B, C, D) = \Sigma(1, 2, 4, 7, 8, 9, 11) + d(0, 3, 5)$ and implement it with **a)** NAND gates only, **b)** AND NOR gates only and then draw the two logic diagrams.

Solution. **a)** Simplify the expression into SOP form

AB \ CD	00	01	11	10
00	X	1	X	1
01	1	X	1	0
11	0	0	0	0
10	1	1	1	0

$$\begin{aligned} F(A, B, C, D) &= A'B' + A'C' + A'D + B'C' + B'D \\ &= \big((A'B')' \cdot (A'C')' \cdot (A'D)' \cdot (B'C')' \cdot (B'D)'\big)' \end{aligned}$$

So, the expression is implemented by NAND-NAND form.

b) Got the simplified expression of F from (a)

$$\begin{aligned} F(A, B, C, D) &= A'B' + A'C' + A'D + B'C' + B'D \\ F'(A, B, C, D) &= \big[A'(B' + C' + D) + B'(C' + D)\big]' \\ &= \big[A'(B' + C' + D)\big]' \cdot \big[B'(C' + D)\big]' \\ &= (A + (BCD') \cdot (B + CD') \\ &= AB + ACD' + BCD' \\ \Rightarrow F(A, B, C, D) &= (AB + ACD' + BCD')' \end{aligned}$$

Thus, the function is implemented by AND-NOR form.

c) The diagrams are drawn below

Figure 1: NAND only diagram

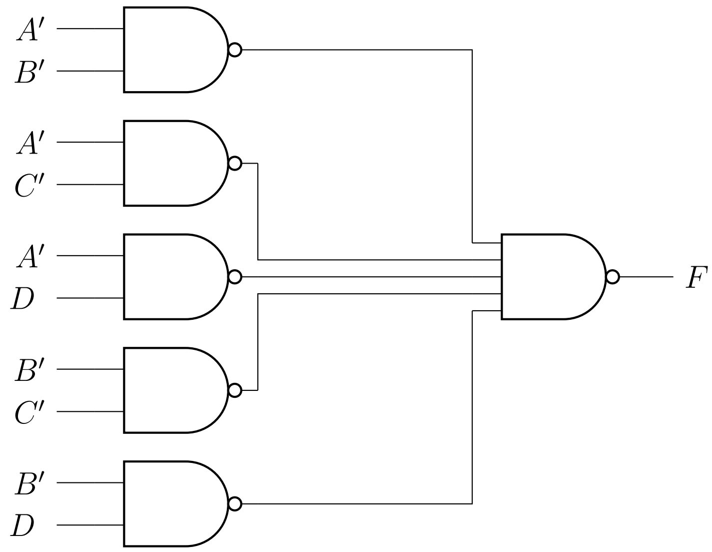

Figure 2: AND NOR only diagram