


Department of
Computer Science and Engineering
计算机科学与工程系

CSE5014 CRYPTOGRAPHY AND NETWORK SECURITY

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PRGs

- A *PRG* is an efficient, deterministic algorithm that expands a *short*, *uniform seed* into a *longer*, *pseudorandom* output

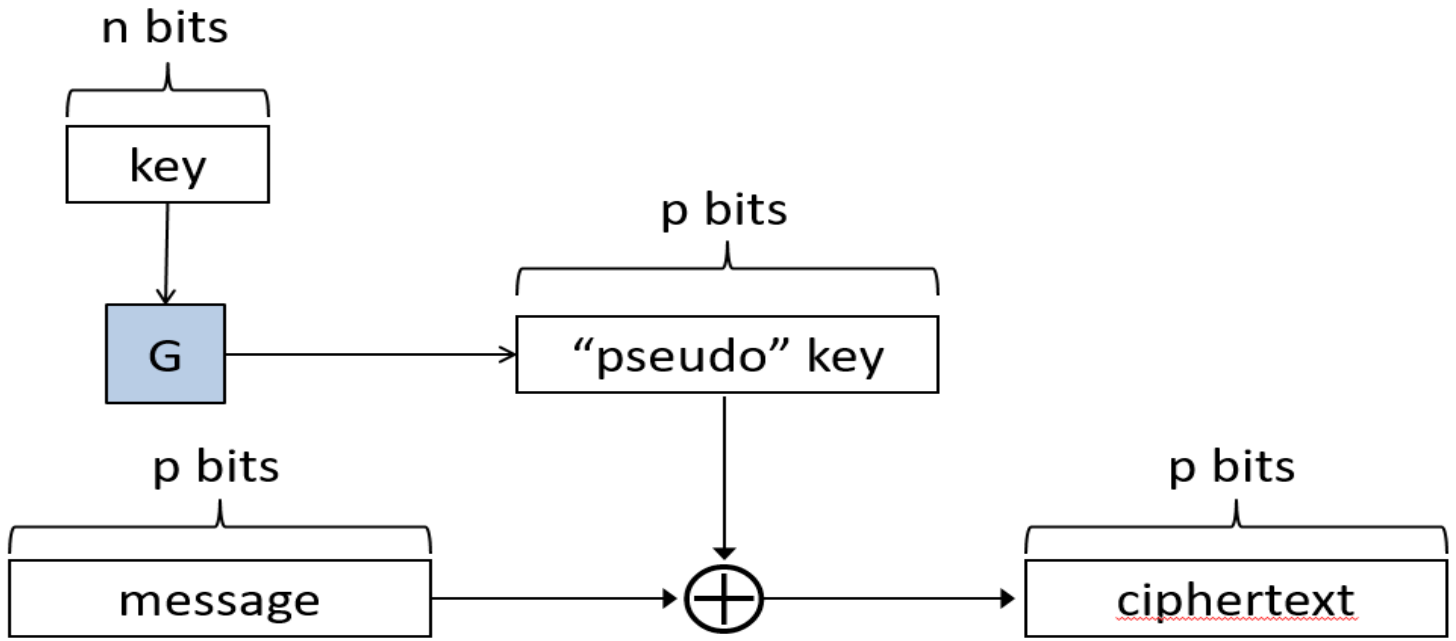
  Let G be a deterministic, poly-time algorithm that is *expanding*, i.e., $|G(x)| = p(|x|) > |x|$.

- For all efficient distinguishers A, there is a negligible function ϵ such that

  $$|\Pr_{x \leftarrow U_n}[A(G(x)) = 1] - \Pr_{y \leftarrow U_{p(n)}}[A(y) = 1]| \leq \epsilon(n)$$

  No efficient A can distinguish whether it is given $G(x)$ (for uniform x) or a uniform string y!

Pseudo one-time pad



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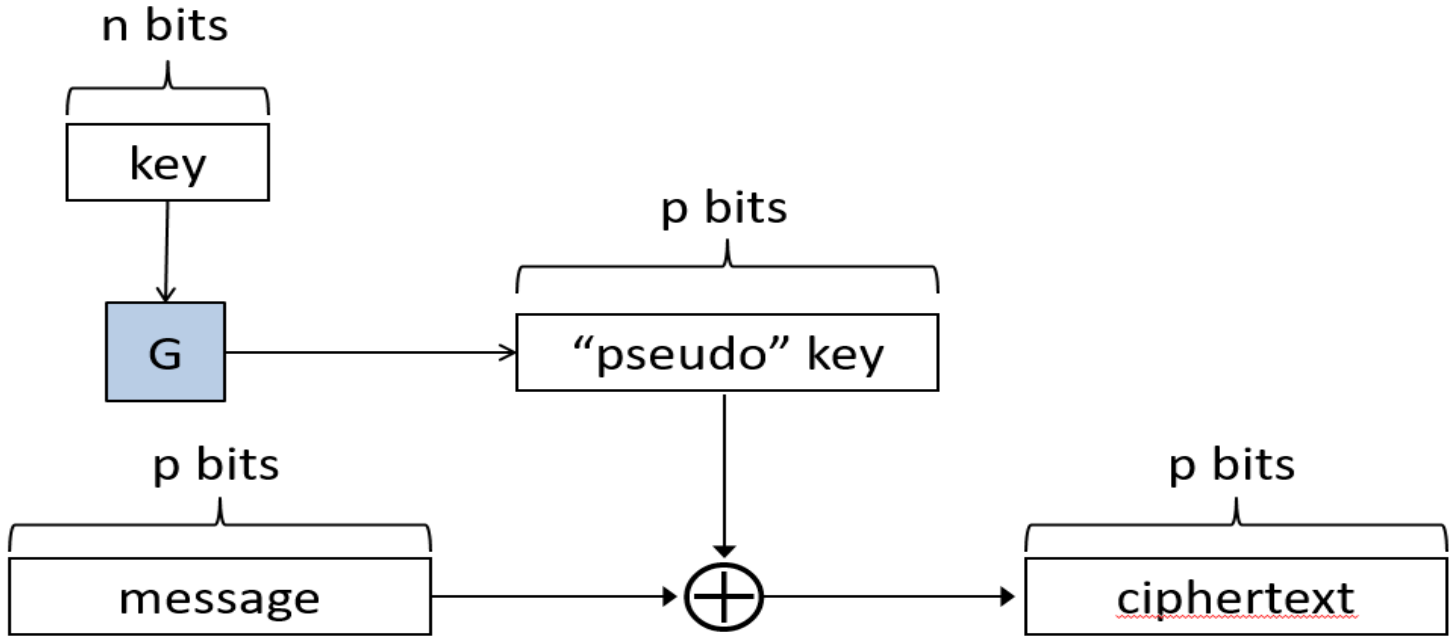

- **Theorem 3.3** If G is a pseudorandom generator (PRG), then the pseudo one-time pad (pseudo-OTP) Π is *EAV-secure* (i.e., *computationally secure*)

CPA-security

- Fix Π, A

  Define a randomized experiment $PrivKCPA_{A,\Pi}(n)$:

  1. $k \leftarrow Gen(1^n)$
  2. $A(1^n)$ interacts with an *encryption oracle* $Enc_k(\cdot)$, and then outputs m_0, m_1 of the same length
  3. $b \leftarrow \{0,1\}$, $c \leftarrow Enc_k(m_b)$, give c to A
  4. A can continue to interact with $Enc_k(\cdot)$
  5. A outputs b'; A succeeds if $b = b'$, and experiment evaluates to 1 in this case

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  Definition 4.1 Π is *secure against chosen-plaintext attacks (CPA-secure)* if for all PPT attackers A, there is a *negligible* function ϵ such that

$$\Pr[PrivKCPA_{A,\Pi}(n) = 1] \leq 1/2 + \epsilon(n)$$

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Definition 4.2 F is a *pseudorandom function* if F_k, for uniform $k \in \{0,1\}^n$ is indistinguishable from a uniform function $f \in Func_n$

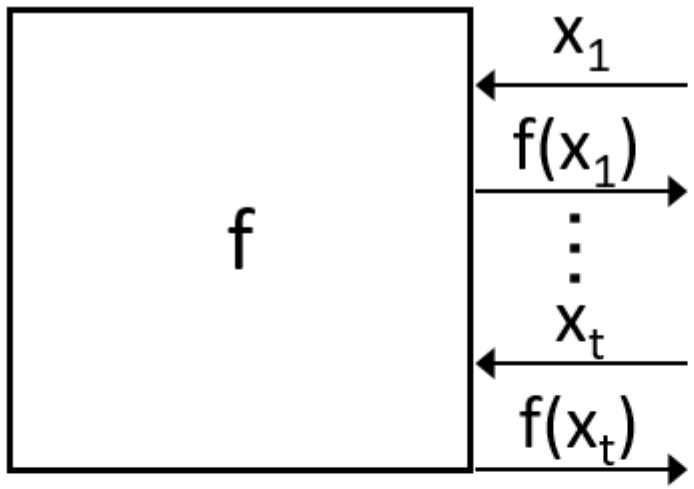
Formally, for all poly-time distinguishers D:

$$\left|\Pr_{k \leftarrow \{0,1\}^n}[D^{F_k(\cdot)}(1^n) = 1] - \Pr_{f \leftarrow Func_n}[D^{f(\cdot)}(1^n) = 1]\right| \leq \epsilon(n)$$

PRFs

$f \in Func_n$ chosen uniformly at random

World 0



??

(poly-time)

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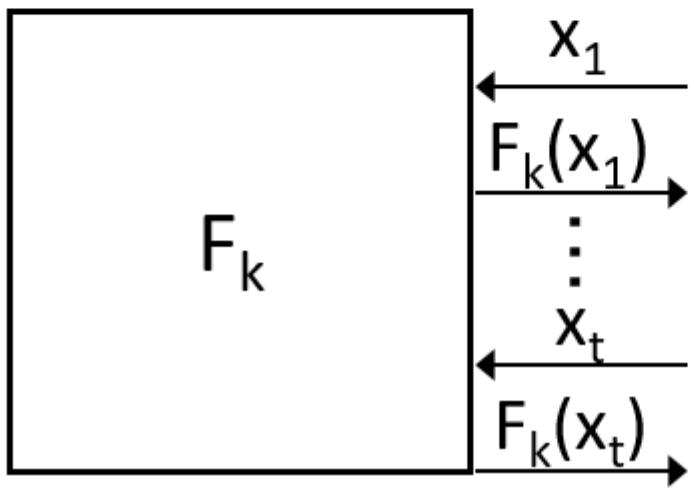

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- For large enough n, a random permutation is indistinguishable from a random function.
  - In practice, PRPs are also good PRFs

PRFs vs. PRGs

- PRF F immediately implies a PRG G:
  - Define $G(k) = F_k(0 \ldots 0) | F_k(0 \ldots 1)$
  - I.e., $G(k) = F_k(\langle 0 \rangle) | F_k(\langle 1 \rangle) | F_k(\langle 2 \rangle) | \ldots$, where $\langle i \rangle$ denotes the n-bit encoding of i

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- PRF can be viewed as a PRG with random access to exponentially long output
  - The function F_k can be viewed as the $n2^n$-bit string $F_k(0\ldots0)|\ldots|F_k(1\ldots1)$

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  Hard to distinguish F_k from uniform $f \in Perm_m$

AES

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- Will discuss details later in the course
  - Rijndael named after Vincent Rijmen and Joan Daemen

History of Block Cipher

- 1972: NIST (then NBS) called for encryption standard proposals

  1976: IBM responsed: "Lucifer"

  NSA tweaked Lucifer to get *Data Encryption Standard* (DES) and approved it

  The key length is "short": 56 bits

  By late 1990's, most commercial applications used 3DES: three applications of DES with independent keys

  It had been used as a standard for encryption until 2000. DES was subject to exhaustive key search attacks.

History of Block Cipher

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  1998: 15 candidates accepted in June

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  Block length: 128 bits, key length: 128/192/256 bits

  Stronger and faster than 3DES

  Efficient in both software and hardware

  Simple in design, suitable for smard cards (memory requirement)

CPA-secure encryption

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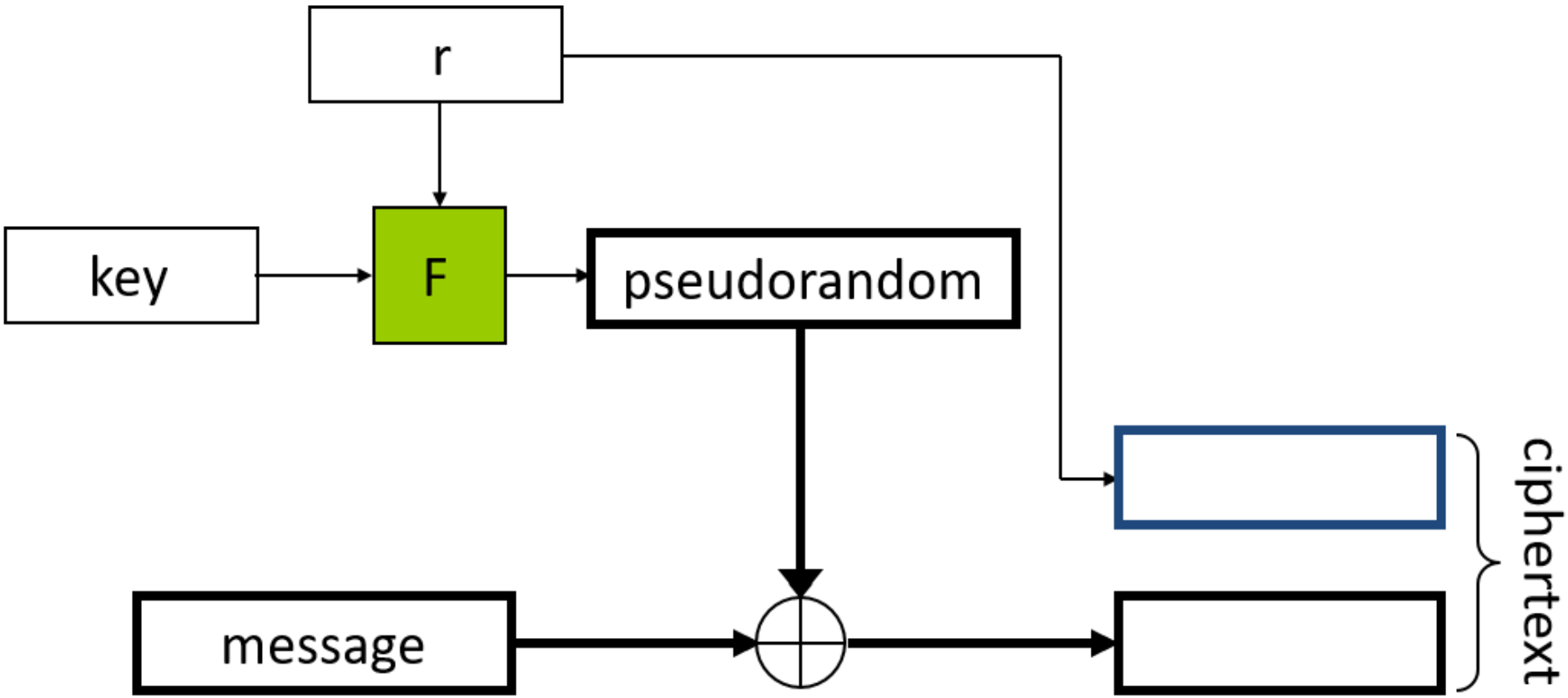

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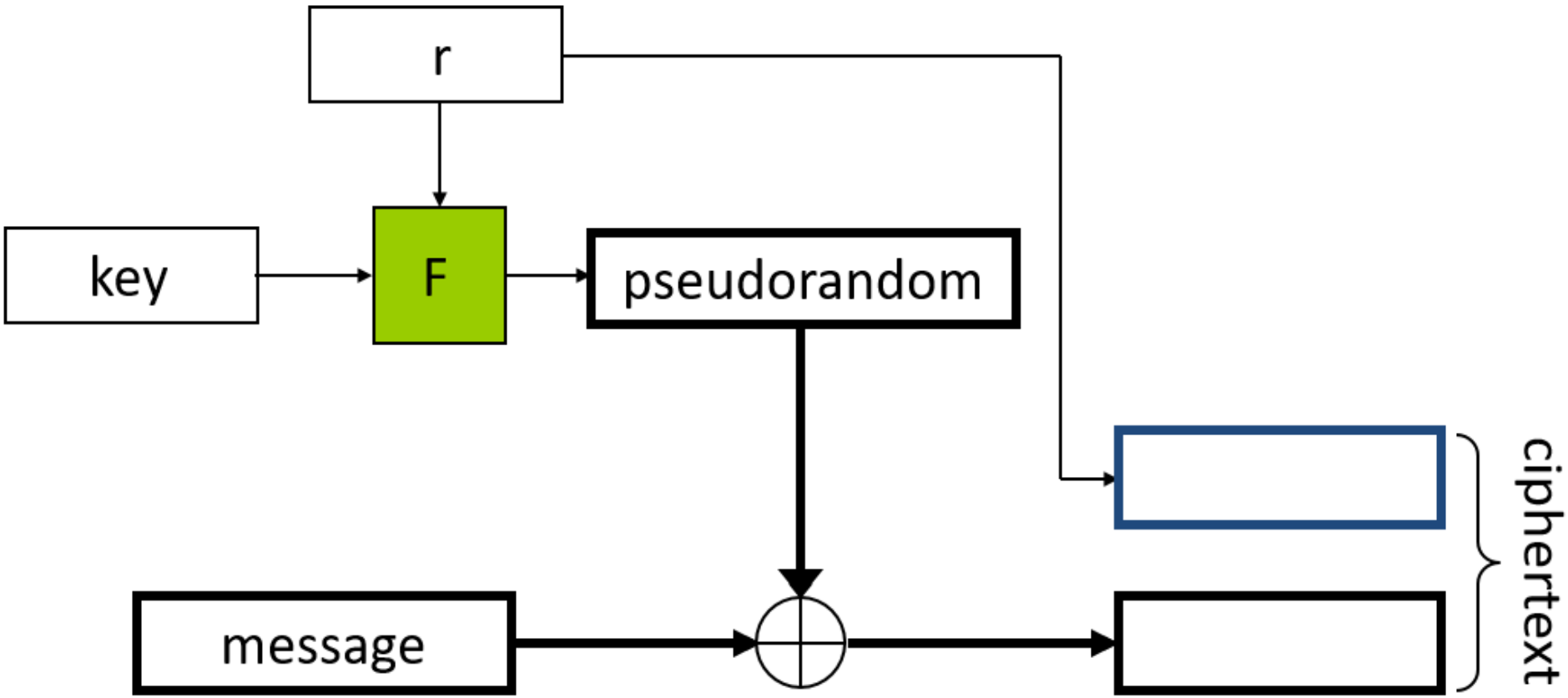

Theorem 5.1 If F is a pseudorandom function, then this scheme is *CPA-secure*.

Note

- The key may be as long as the message

- But the same key can be used to safely encrypt *multiple* messages

Security?

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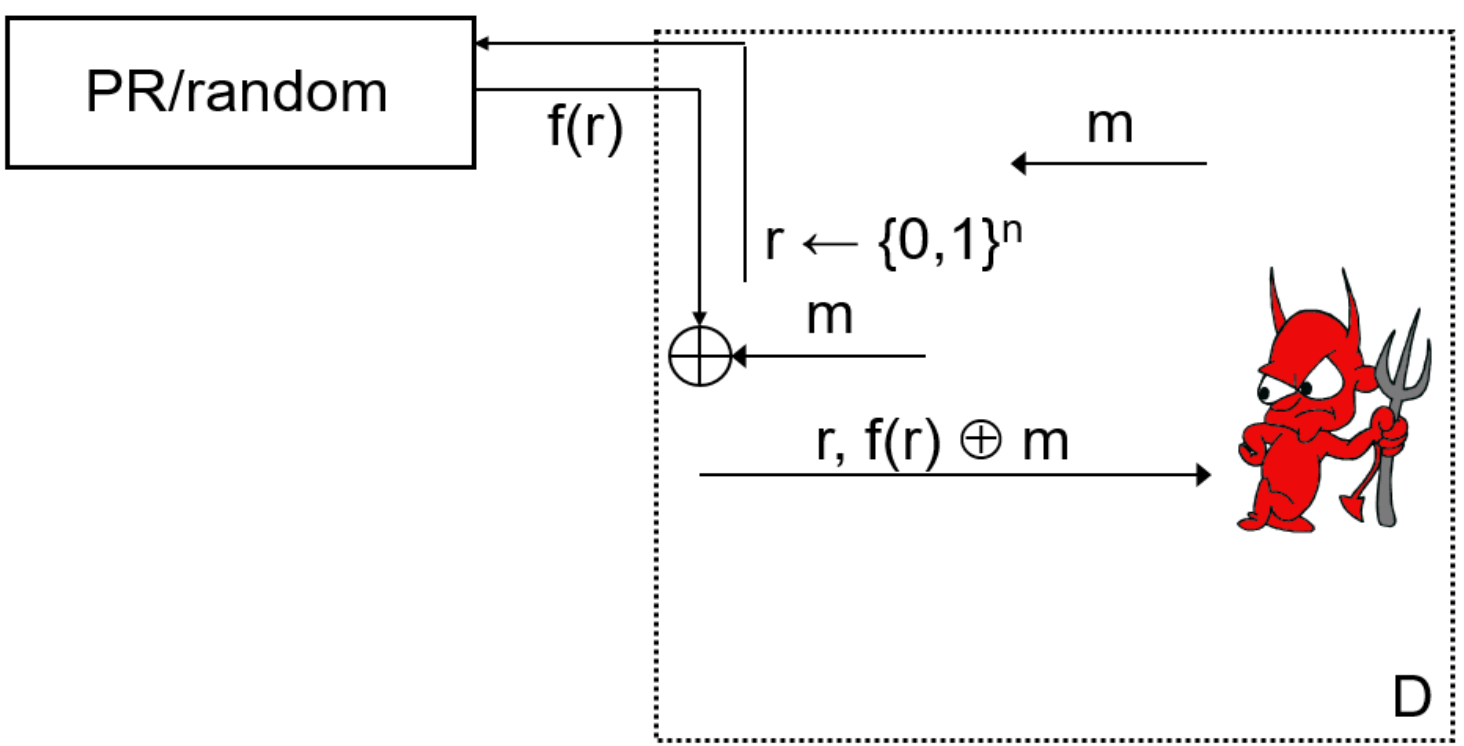

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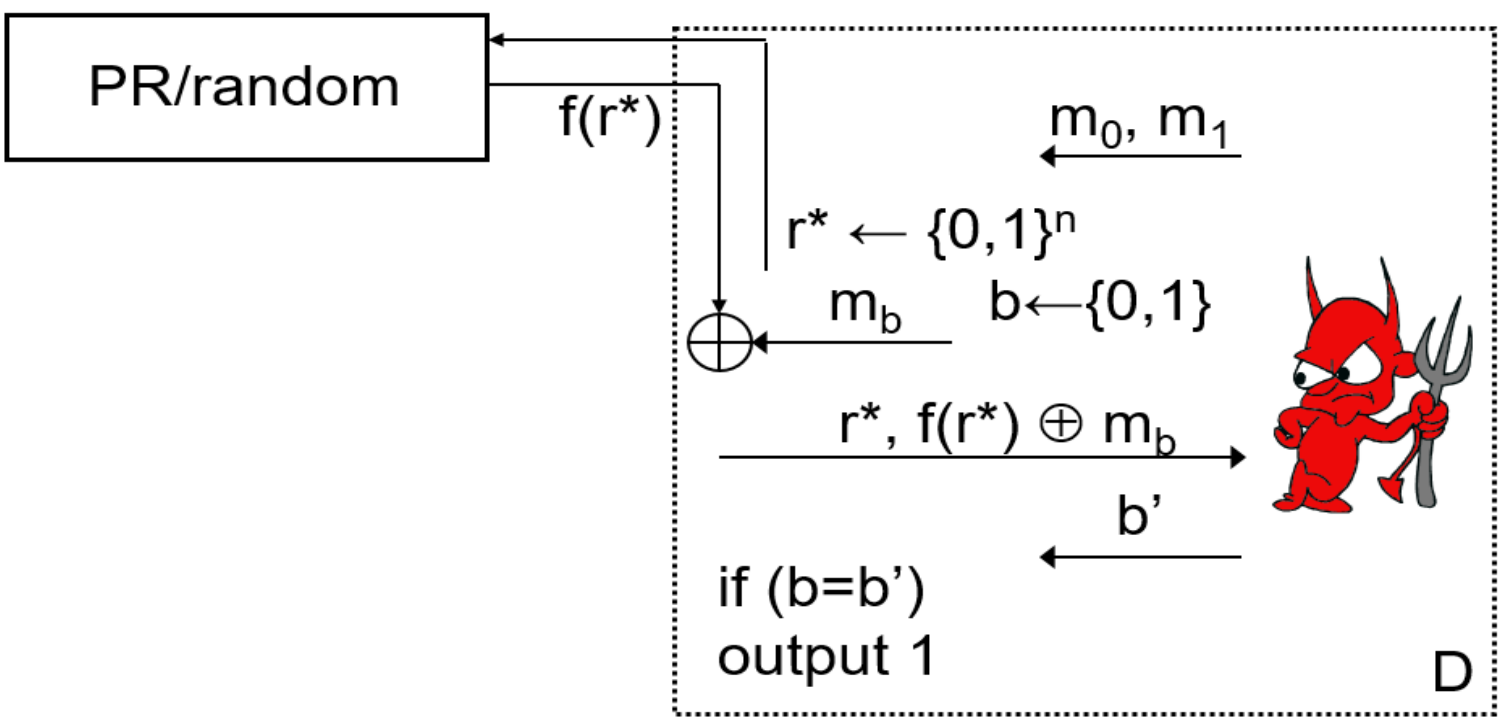

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  If $f = F_k$ for uniform k, then the view of Adv is exactly as in $PrivKCPA_{Adv,\Pi}(n)$

  $\Rightarrow$

  $\Pr_{k \leftarrow \{0,1\}^n}[D^{F_k(\cdot)} = 1] = \Pr[PrivKCPA_{Adv,\Pi}(n) = 1] = \mu(n)$

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  Note: $q(n)/2^n + \epsilon(n) = \epsilon'(n)$ is negligible

Real-world security?

- The security bound we proved is *tight*

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- What happens if a nonce r is ever reused?
- What is the probability that the nonce used in some challenge ciphertext is also used for some other ciphertext?
- What happens to the bound if the nonce is chosen *non-uniformly*?

CPA-secure encryption

- We have shown a *CPA-secure* encryption scheme based on any block cipher/ PRF
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- Drawbacks?
  - A 1-block plaintext results in a 2-block ciphertext
  - Only defined for encryption of n-bit messages

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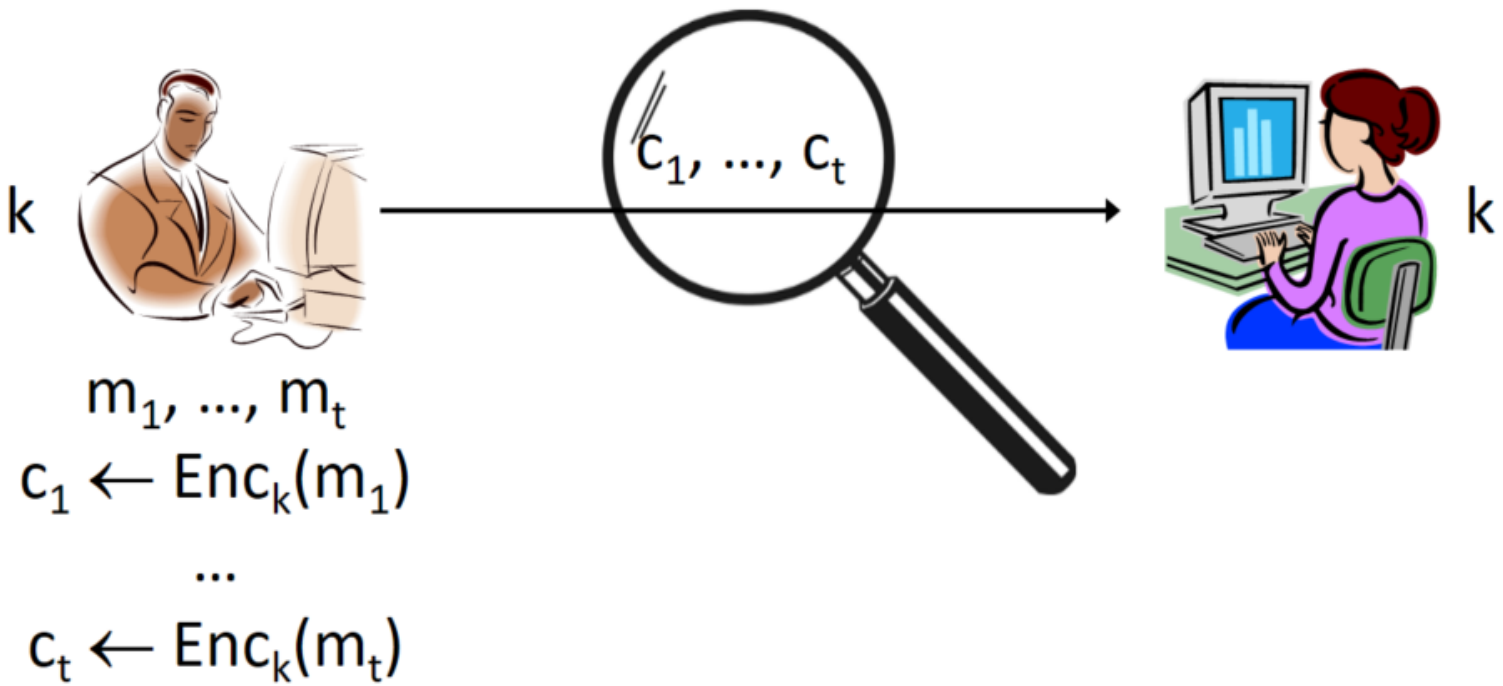

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- Can we do better?
- Modes of operation
  - *Block-cipher* modes of operation
  - *Stream-cipher* modes of operation

CTR (Counter) mode

- $Enc_k(m_1, \ldots, m_t)$ // note: t is arbitrary
  - Choose $ctr \leftarrow_R \{0,1\}^n$, set $c_0 = ctr$
  - For $i = 1$ to t:
    - $c_i = m_i \oplus F_k(ctr + i)$
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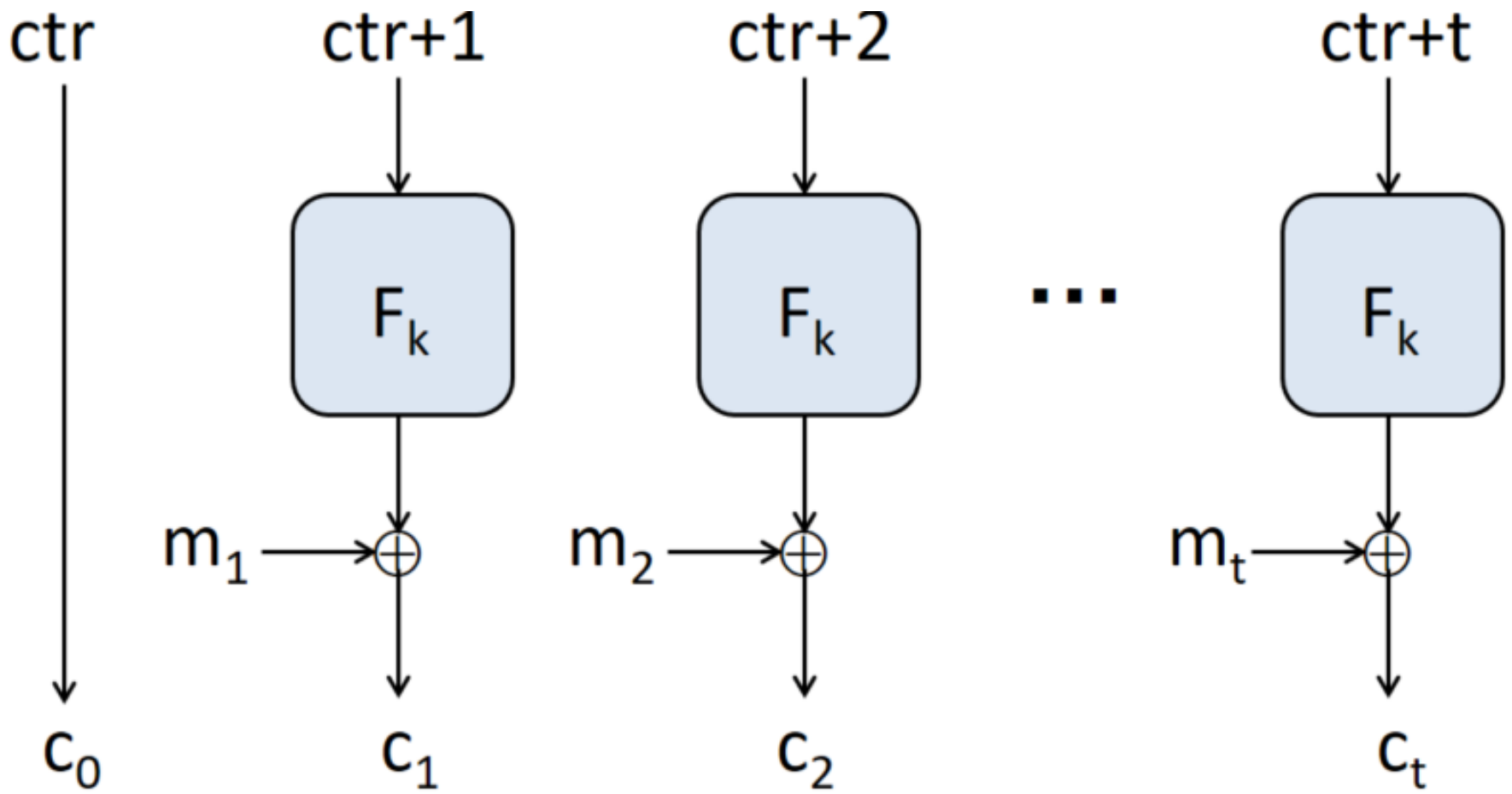

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  - Moreover, it is independent of every other such sequence unless $ctr_i + j = ctr_{i'} + j'$ for some i, j, i', j'
  - Just need to bound the probability of that event

CBC (Cipher Block Chaining) mode

- $Enc_k(m_1, \ldots, m_t)$ // note: t is arbitrary
  - Choose $c_0 \leftarrow_R \{0,1\}^n$ (also called the IV)
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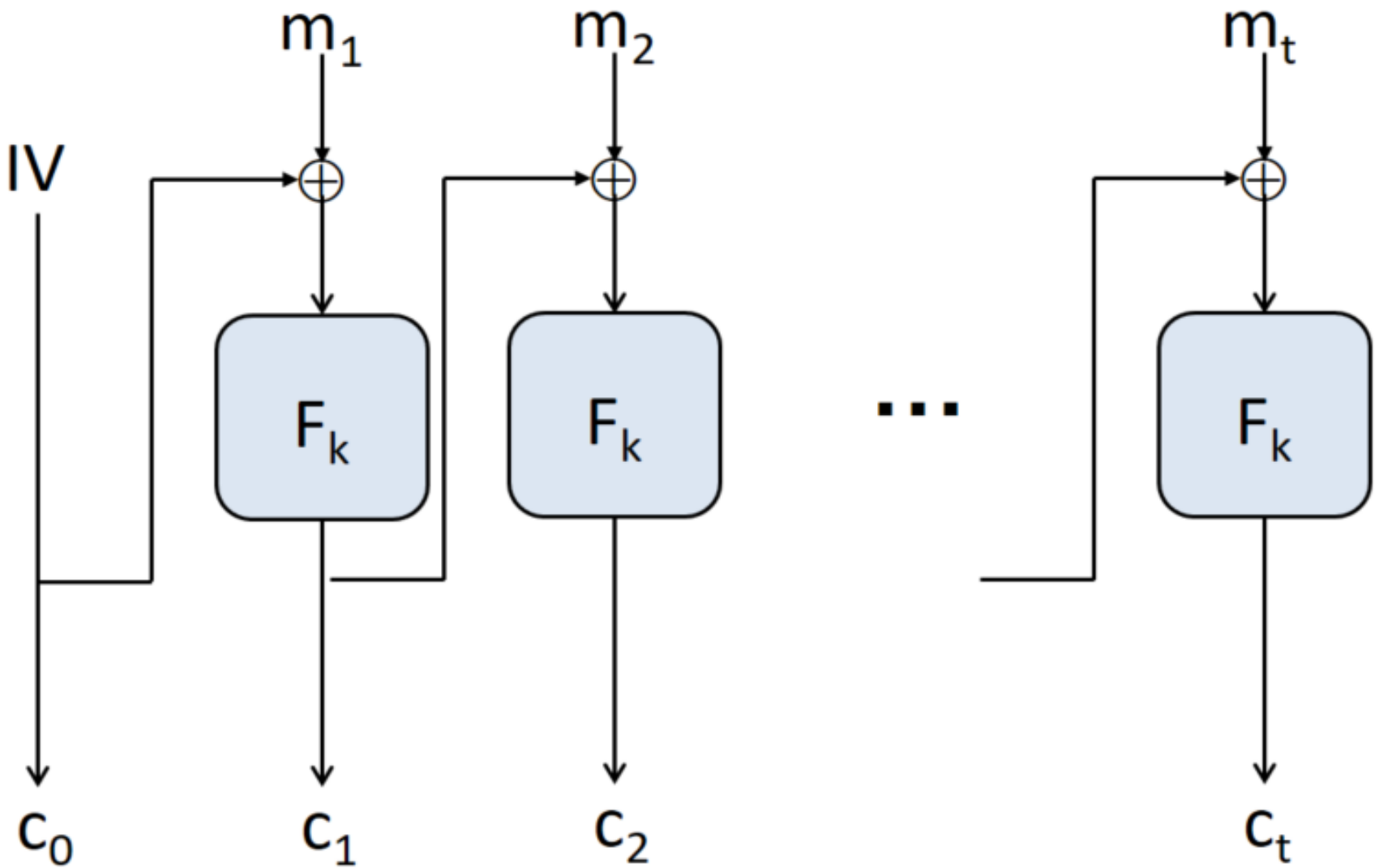

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- **Proof** is more complicated than for CTR mode

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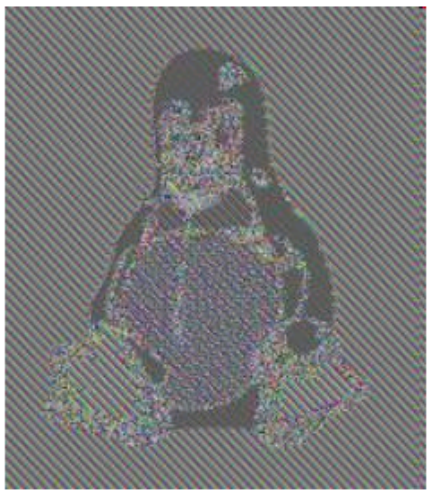

encrypted using ECB mode

Stream ciphers

- As we defined, PRGs are *limited*
  - They have fixed-length output
  - They produce output in "one shot"

- In practice, PRGs are based on *stream ciphers*
  - Can be viewed as producing an "infinite" stream of pseudorandom bits, on demand
  - More flexible, more efficient

Stream ciphers

- Pair of efficient, deterministic algorithms (Init, GetBits)

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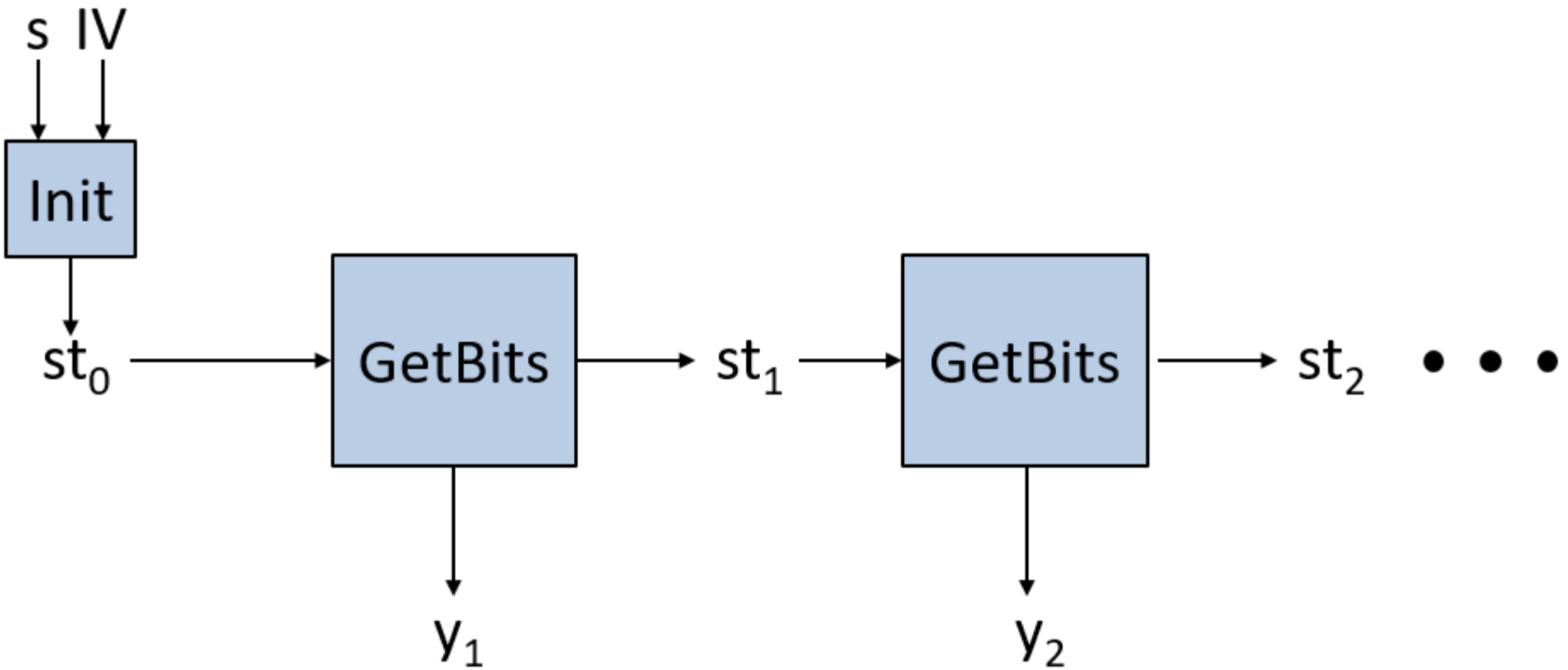
  - Init takes a seed s_0 (and optional IV), and outputs initial state st_0
  - GetBits takes the current state st and outputs a bit y along with updated state st'

Stream ciphers

- Pair of efficient, deterministic algorithms (Init, GetBits)
  - Init takes a seed s_0 (and optional IV), and outputs initial state st_0
  - GetBits takes the current state st and outputs a bit y along with updated state st'
    - In practice, y would be a block rather than a bit

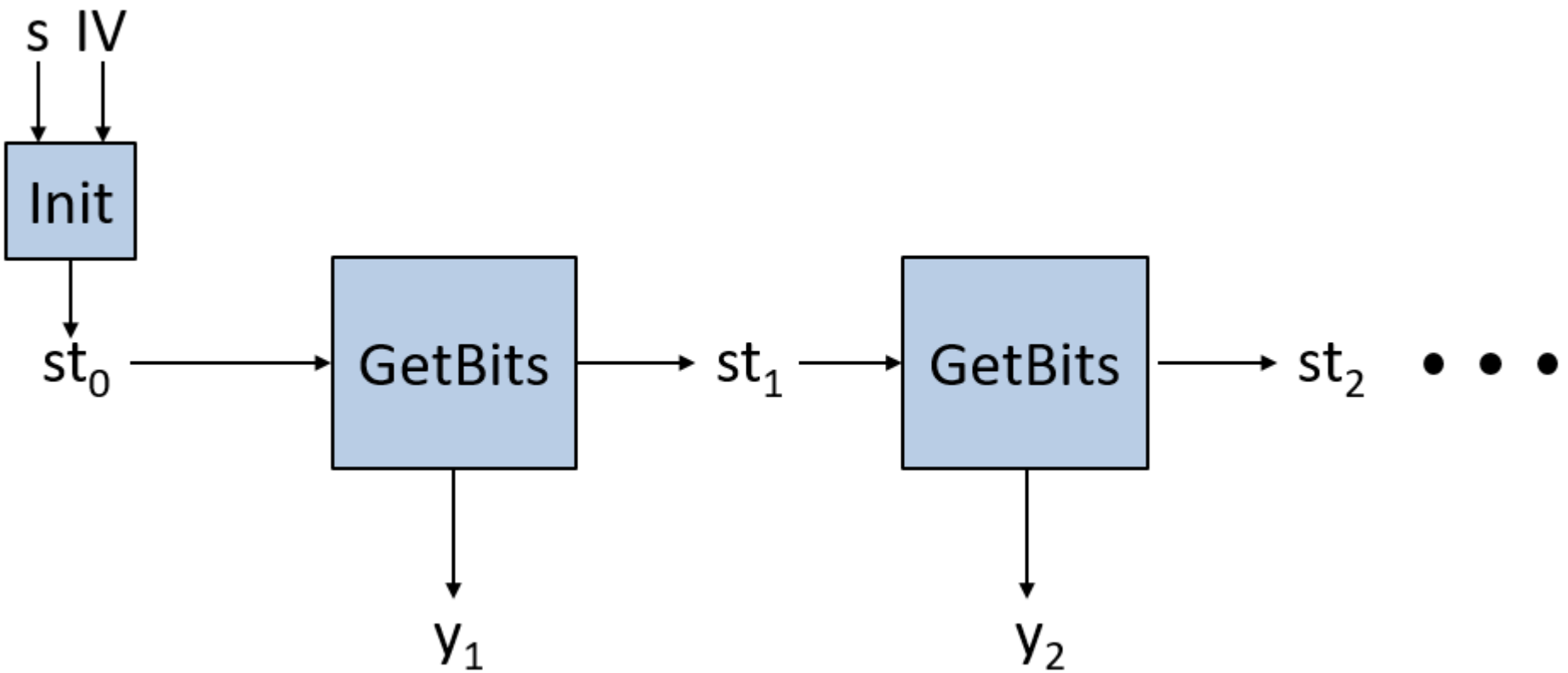
Stream ciphers

- Can use (Init, GetBits) to generate any desired number of output bits from an initial seed



Stream ciphers

- A *stream cipher* is *secure* (informally) if the output stream generated from a uniform seed is pseudorandom
  - I.e., regardless of how long the output stream is (so long as it is polynomial)



Next Lecture

- stream cipher, CCA security ...