GOTTFRIED WILHELM LEIBNIZ

PHILOSOPHICAL PAPERS AND LETTERS

A Selection Translated and Edited, with an Introduction by

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TENTAMEN ANAGOGICUM: AN ANAGOGICAL ESSAY IN THE INVESTIGATION OF CAUSES

Ca. 1696

Leibniz agreed with the Cartesians that natural events are to be explained mechanistically, but he insisted that the consideration of final causes was of significance in the derivation of mechanical laws themselves. As example he submitted a demonstration of the laws of refraction and reflection as a special case, maximally determined, of an infinite number of possible laws. His argument, historically important as the beginning of the famous dispute on the principle of least action *in the 18th century, and the revived interest in a principle of the* extremum *today, was first published in the* Acta eruditorum *in June, 1682.*[1] *This clear and concise formulation is obviously later. The form and content suggest that it may be a continuation of the* Specimen dynamicum *(No. 46). The mention of the celebrated brachistochrone problem dates it after June, 1696, however, when John Bernoulli first proposed this problem. The essay remained unpublished until Gerhardt included it in his edition.*

[G., VII, 270–79]

I[2] have shown on several occasions that the final analysis of the laws of nature leads us to the most sublime principles of order and perfection, which indicate that the universe is the effect of a universal intelligent power. As the ancients already held, this truth is the chief fruit of our investigations; without mentioning Pythagoras and Plato, whose primary aim was such an analysis, even Aristotle sought to demonstrate a prime mover in his works, particularly in his *Metaphysics*. It is true that these ancient thinkers were not informed about the laws of nature as are we, since they lacked many of the methods which we have and of which we ought to take advantage. The knowledge of nature gives birth to the arts, it gives us many means of conserving life, and it even provides us with conveniences; but the satisfaction of spirit which comes from wisdom and virtue, in addition to being the greatest ornament of life, raises us to what is eternal, whereas this life, in contrast, is most brief. As a result, whatever serves to establish maxims which locate happiness in virtue and show that everything follows the principle of perfection is infinitely more useful to man, and even to the state, than all that serves the arts. Discoveries useful to life, moreover, are very often merely the corollaries of more important insights; it is true here too that those who seek the kingdom of God find the rest on their way.

The inquiry into final causes in physics is precisely the application of the method which I think ought to be used, and those who have sought to banish it from their philosophy have not adequately considered its usefulness. For I do not wish to do them the injury of thinking that they have evil designs in doing this. Others followed them, however, who have abused their position, and who, not content with excluding

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final causes from physics but restoring them elsewhere, have tried to destroy them entirely and to show that the Creator of the universe is most powerful, indeed, but without any intelligence. There have been still others who have not admitted any universal cause, like the ancients who recognized nothing in the universe but a concourse of corpuscles. This seems plausible to those minds in whom the imaginative faculty predominates[3], because they believe that they need to use only mathematical principles, without having any need either for metaphysical principles, which they treat as illusory, or for principles of the good, which they reduce to human morals; as if perfection and the good were only a particular result of our thinking and not to be found in universal nature.

I recognize that it is rather easy to fall into this error, especially when one's thinking stops at what imagination alone can supply, namely, at magnitudes and figures and their modifications. But when one pushes forward his inquiry after reasons, it is found that the laws of motion cannot be explained through purely geometric principles or by imagination alone. This is also why some very able philosophers of our day have held that the laws of motion are purely arbitrary. They are right in this if they take *arbitrary* to mean coming from choice and not from geometric necessity, but it is wrong to extend this concept to mean that laws are entirely indifferent, since it can be shown that they originate in the wisdom of their Author or in the principle of greatest perfection, which has led to their choice.

This consideration gives us the true middle term that is needed for satisfying truth as well as piety. We know that while there have been, on the one hand, able philosophers who recognized nothing except what is material in the universe, there are, on the other hand, learned and zealous theologians who, shocked at the corpuscular philosophy and not content with checking its misuse, have felt obliged to maintain that there are phenomena in nature which cannot be explained by mechanical principles; as for example, light, weight, and elastic force. But since they do not reason with exactness in this matter, and it is easy for the corpuscular philosophers to reply to them, they injure religion in trying to render it a service, for they merely confirm those in their error who recognize only material principles. The true middle term for satisfying both truth and piety is this: all natural phenomena could be explained mechanically if we understood them well enough, but the principles of mechanics themselves cannot be explained geometrically, since they depend on more sublime principles which show the wisdom of the Author in the order and perfection of his work.

The most beautiful thing about this view seems to me to be that the principle of perfection is not limited to the general but descends also to the particulars of things and of phenomena and that in this respect it closely resembles the method of *optimal forms*, that is to say, of forms *which provide a maximum or minimum*, as the case may be – a method which I have introduced into geometry in addition to the ancient method of *maximal and minimal quantities*. For in these forms or figures the *optimum* is found not only in the whole but also in each part, and it would not even suffice in the whole without this. For example, if in the case of the curve of shortest descent between two given points, we choose any two points on this curve at will, the part of the line intercepted between them is also necessarily the line of shortest descent with regard to them.[4] It is in this way that the smallest parts of the universe are ruled in accordance with the order of greatest perfection; otherwise the whole would not be so ruled.

It is for this reason that I usually say that there are, so to speak, two kingdoms even in corporeal nature, which interpenetrate without confusing or interfering with each other – the realm of power, according to which everything can be explained *mechanically* by efficient causes when we have sufficiently penetrated into its interior, and the realm of wisdom, according to which everything can be explained architectonically[5], so to speak, or by final causes when we understand its ways sufficiently. In this sense one can say with Lucretius not only that animals see because they have eyes but also that eyes have been given them in order to see, though I know that some people, in order the better to pass as free thinkers, admit only the former. Those who enter into the details of natural machines, however, must have need of a strong bias to resist the attractions of their beauty. Even Galen, after learning something about the function of the parts of animals, was so stirred with admiration that he held that to explain them was essentially to sing hymns to the honor of divinity. I have often wished that an able physicist would undertake to prepare a special work whose title – or whose aim at least – would be *The Hymn of Galen.*

What is more, our thinking sometimes furnishes us with considerations revealing the value of final causes, not merely in increasing our admiration for the supreme Author, but also in making discoveries among his works. Some day I shall show this in a special case in which I shall propose as a general principle of optics that a ray of light moves from one point to another by the path which is found to be easiest in relation to the plane surfaces which must serve as the rule for other surfaces. For it must be kept in mind that if we claimed to use this principle as an efficient cause, and as if the easiest path would prevail among all the possible competing rays, it would be necessary to consider the whole surface as it is, without considering the plane tangent to it, and then the principle would not always work out successfully, as I shall show presently.[6] But far from concealing that there is a certain final cause involved in this principle – an objection which was once made against Mr. Fermat, who had used it in his *Dioptrics* – I have found it more beautiful and more important than that of mechanism for a more sublime application. And an able author who has published a work on optics in England has expressed his indebtedness to my view.[7] Order demands that curved lines and surfaces be treated as composed of straight lines and planes, and a ray is determined by the plane on which it falls, which is considered as forming the curved surface at that point. But the same order demands that the effect of the greatest ease be obtained in relation to the planes, at least those which serve as elements to other surfaces, since it cannot be obtained with regard to these surfaces also. This is all the more true since it thus satisfies, with respect to these curves, another principle which now supersedes the preceding one, and which holds that in the absence of a minimum it is necessary to hold to the *most determined*, which can be the *simplest* even when it is a *maximum.*

Now we find that the ancients, and among others Ptolemy, already used this hypothesis of the easiest path of a ray which falls on a plane, to account for the equality of the angles of incidence and reflection, the principle at the basis of catoptrics. It is by this same hypothesis that Mr. Fermat provided a reason for the law of refraction according to the sines, or to formulate it in other terms as Snell did, according to the secants. But what is more, I have no doubt whatever that this law was first discovered by this method. It is known that Willebrord Snell, one of the greatest geometricians of his time and well versed in the methods of the ancients, invented it, having even written a work which was not published because of its author's death. But since he had

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taught it to his disciples, all appearances point to the conclusion that Descartes, who had come to Holland a little later and who was most interested in this problem, learned it there. For the way in which Descartes has tried to explain the law of refraction by efficient causes or by the composition of directions in imitation of the reflection of bullets is extremely forced and not intelligible enough. To say no more about it here, it shows clearly that it is an afterthought adjusted somehow to the conclusion and was not discovered by the method he gives. So we may well believe that we should not have had this beautiful discovery so soon without the method of final causes.

I recall that capable writers have frequently objected that this principle does not seem to work in reflection itself when applied to curved surfaces and that in concave mirrors the path of reflection happens sometimes to be the longest. But in addition to what I have already said, that according to architectonic principles, curved surfaces must be ruled by the planes tangent to them, I shall now explain how it remains always universally true that the ray is directed in the most determined or unique path, even in relation to curves. It is also worth noting that in the method of analysis by maxima and minima, the same operation suffices for the problems of the greatest and the smallest, without distinguishing between them except in applying the method to different cases, since we seek the most determined magnitude in both cases, which is sometimes the greatest and sometimes the smallest in its order, the analysis being based solely on the disappearance of a difference or on the unique result of reuniting twins, and not at all on a comparison of the greatest and smallest with all other magnitudes. For given a curve *AB*, concave or convex, and an axis *ST* to which the ordinates of the curve are referred; then it is seen that to each ordinate, like *Q* or *R*, there corresponds another one equal to it, its twin, *q* or *r* (Figure 33). But there is one particular ordinate

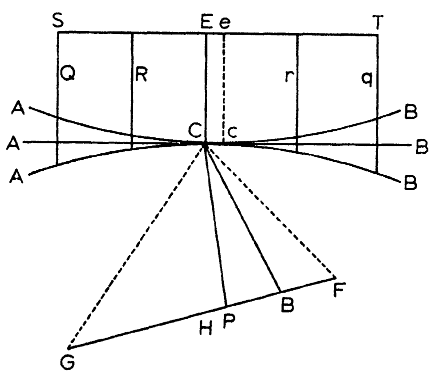

Fig. 33.

EC which is unique, or the only determinate one of its magnitude, and has no twin, since the two twins *EC* and *ec* coincide in it and make but one. And this *EC* is the greatest ordinate of the concave curve and the smallest of the convex curve. So instead of two infinitely near ordinates in all other cases, having a difference of *dm* if the ordinate is called *m*, whose ratio to *Ee*, a correspondingly small part of the axis, would give the angle of the curve or of its tangent to the axis *ST*, the infinitely close ordinates or twins become coincident in this case at C and have no difference; dm becomes 0, and the tangent at C is parallel to the axis. Thus the basis of the analysis is this uniqueness caused by the union of the twins, without any concern as to whether the ordinate is the greatest or smallest. The calculus shows us this in particular, in this very matter. Let ACB be any mirror whatever, plane, concave, or convex; and let two points F and G be given (Figure 34). To find the point of reflection C, such that the path FCG

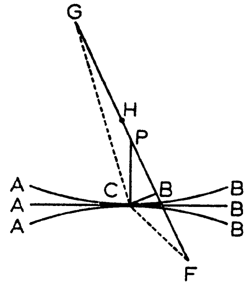

Fig. 34.

is the unique, singular, or most determined path in size, or what the ancients once called the *μοναχὸν*, either the greatest or the smallest, that is, whichever it happens to be. For those which are not unique in this sense are doubles or twins, having another of the same length corresponding to them. Draw FG, whose middle point is H, and between C and FG draw CB perpendicular to FG, and CP perpendicular to the mirror. Call HF or HG a, HB x, CB y; then BP will be $-ydy$, dx being taken negatively. Then CF will be $\sqrt{y^2 +}$ shortest, though it is in fact the shortest when that which should provide the rule is $\overline{x^2 - 2ax + a^2}$, and CG will be $\sqrt{y^2 + x^2 + 2ax + a^2}$. Now $CF + CG = m$. Differentiating, one gets $d \cdot CF + d \cdot CG = 0$; that is,

$$\frac{ydy + xdx - adx}{CF} + \frac{ydy + xdx + adx}{CG} = 0,$$

or

$$\frac{CF}{CG} = \frac{(a - x - ydy)\,dx}{(a + x + ydy)\,dx}.$$

But $a - x$ is BF and $a + x$ is GB. Therefore

$$\frac{CF}{CG} = \frac{BF + BP}{GB - BP},$$

or

$$\frac{CF}{CG} = \frac{PF}{PG}.$$

This shows that the angle of direction FCG is bisected by CP, the perpendicular to the curve[8], or that the angles of incidence and of reflection are equal, whatever may be the reflecting surface.

The same truth holds also with regard to refraction; that is, whether the surface of

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separation be plane or curved, provided only that it be uniformly determined, the broken ray always passes from a point in one medium to a point in the other medium by the most determined or unique path, which has, so to speak, no twin brother in length of time. This is something I do not remember having seen commented on before. It is easy to prove it by an analysis similar to that given above. Let a figure be set up just like that above, except that instead of a mirror there is a surface ACB, flat, concave, or convex, which separates two media penetrated by the ray, and which changes its direction. The ratio of the resistance of the medium $ACBF$ to that of the medium $AGCB$ shall be as f to g; then $f \cdot CF + g \cdot CG = m$.
Differentiating, we get

$$\frac{f\,(ydy + xdx - adx)}{CF} + \frac{g\,(ydy + xdx + adx)}{CG} = 0,$$

and as a result (calculating as above),

$$\frac{CF}{CG} = \frac{f \cdot PF}{g \cdot PG}.$$

Now it is easy to derive from this theorem the proportionality of sines. For let the

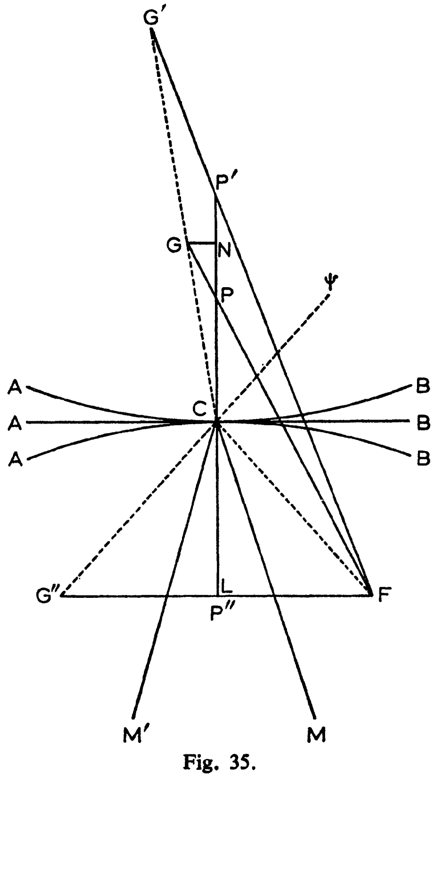

Fig. 35.

ray FC strike the refracting surface ACB at C, and let the refracted ray, CG, be taken equal to the incident ray FC. Draw FG cutting the straight line CP perpendicular to the surface, at P. From points F and G draw normals FL and GN to CP (Figure 35). Now since CG and CF are taken equal, it follows by the equation of the preceding paragraph that PF is to PG as g to f. Then because of the similar triangles PLF and PNG the sine FL will be to the sine GN as g to f[9], that is, reciprocally as the resistances of the media. And the sines of the angles of refraction will be proportional to the sines of the angles of incidence.[10]

This makes us see, finally, that the rule of the unique path, or the path most determined in length of time, applies generally to the direct and the broken ray, whether reflected or refracted, whether by plane or by curved surfaces, whether convex or concave, without distinguishing in the process whether the time is the longest or the shortest, though it is in fact the shortest when that which should provide the rule is taken into consideration, that is, the tangent plane; nature being governed, as it is, by sovereign wisdom, shows the general design throughout of controlling curves by straight lines or planes tangent to them, as if the curves were composed of these, although this is not strictly true.

One also comes in this way to understand some general theorems common to catoptrics and dioptrics. For the rectangle constructed by multiplying the ray on one side of the surface by the opposite segment of the base line (namely, the rectangle $CF \cdot PG$) is always proportional to the corresponding rectangle on the opposite side, made by multiplying the ray of the other side by the segment of the base opposite it (that is, the rectangle $CG \cdot PF$); or the ratio of the product of one side of the broken ray times the opposite segment of the base, to the product of the other side of the ray by its opposite segment is equal to the ratio of the resistances of the opposite media. In the case of simple reflection, where the media have the same nature, this becomes the ratio of equals, and then this theorem gives

$$\frac{CF \cdot P''G''}{CG'' \cdot P''F} = 1^{11}$$

or

$$CF \cdot P''G'' = CG'' \cdot P''F,$$

or, as above,

$$\frac{CF}{CG''} = \frac{P''F}{P''G''};$$

that is, the angles of incidence and reflection are equal.

But there could still be a case of reflection mixed with refraction, of which the solution is easy; for that proposed by Descartes does not seem consistent with the nature of light. The ray FC could at the same time meet the mirror ACB and the new medium MCA or $M'CA$ at C, in which case it would be reflected backward, but the angle of reflection would not be equal to the angle of incidence. It will not be difficult to determine this, since one has only to think, in place of the ray FC, of the ray ψC which would pass, if continued directly, into CG'', and it will be found that the ray FC, falling upon the mirror CB and into the new medium CM at the same time, will then be turned, by reflection and refraction together, to go as the ray ψC would go if turned by the refraction of the medium CM alone.

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This principle still deserves to be studied empirically, however, not to determine the quantity, but to see whether it can perhaps give us something specific, especially with regard to colors. Thus I should also like to have someone investigate empirically another transition from refraction to reflection, which occurs when the ray which strikes the medium has taken on an obliqueness too great to penetrate it; I should like to have this case, too, applied to colors as well as to a crystal of double reflection.[12] This also deserves application, furthermore, to the experiences of color arising from refraction. But this is said in passing.

This principle of nature, that it acts in the most determined ways which we may use, is purely architectonic in fact, yet it never fails to be observed. Assume the case that nature were obliged in general to construct a triangle and that for this purpose only the perimeter or the sum of the sides were given, and nothing else; then nature would construct an equilateral triangle. This example shows the difference between architectonic and geometric determinations. Geometric determinations introduce an absolute necessity, the contrary of which implies a contradiction, but architectonic determinations introduce only a necessity of choice whose contrary means imperfection – a little like the saying in jurisprudence: *Quae contra bonos mores sunt, ea nec facere nos posse credendum est.*[13] So there is even in the algebraic calculus what I call the law of justice, which greatly aids us in finding good solutions. If nature were brutish, so to speak, that is, purely material or geometrical, the above case would be impossible, and unless something more determinative were given than merely the perimeter, nature would not produce a triangle. But since nature is governed architectonically, the half-determinations of geometry are sufficient for it to achieve its work; otherwise it would most often have been stopped. And this is particularly true with regard to the laws of nature. Perhaps someone will deny that what I have said above applies to the laws of motion and will maintain that an entirely geometric demonstration can be given of them. I reserve the proof of the contrary for another discourse, where I shall show that they cannot be derived from their sources without assuming architectonic grounds. One of the most important of these, which I believe I am the first to have introduced into physics, is the law of continuity, which I discussed many years ago in the *Nouvelles de la république des lettres*, where I showed with examples how it serves as the touchstone of theories.[14] It serves not merely to test, however, but also as a very fruitful principle of discovery, as I plan to show some day. But I have also found other very beautiful and extended laws of nature, quite different, however, from those usually employed, yet always depending on architectonic principles. Nothing seems to me to be more effectual in proving and admiring the sovereign wisdom of the Author of things as shown in the very principles of things themselves.

REFERENCES

[1] See Introduction, Sec. V and p, 61, note 36.

[2] The text as given in G. begins with what is really a note added later by Leibniz (Cout. OF., p. 587). This note is as follows: "Whatever leads us to the supreme cause is called anagogical by philosophers as well as theologians. So we begin here to show that no other reason can be given for the laws of nature than the assumption of an intelligent cause. Or we show also that in the investigation of final causes there are cases in which it is necessary to consider the simplest or most determinate, without distinguishing whether this is a maximum or a minimum; that

the same thing is to be seen in the differential calculus; that the general laws for the direction of a ray, derived from final causes, give a beautiful example of this, without distinguishing whether the ray is reflected or refracted or whether the surface is curved or a plane. Certain new general theorems can be derived which apply equally to refraction and reflection. We show that the analysis of the laws of nature and the investigation of causes lead us to God; and how, in the method of final causes as in the differential calculus, we do not consider maxima and minima only, but the most determined and most simple in general." Leibniz also notes: "See if some of these propositions are already to be found in Barrow or elsewhere."

3 That is, the faculty of interpretation within the limits of sensory perception (see p. 41, note 12, and p. 553, note 3).

4 This is a clear allusion to the brachistochrone problem propounded by John Bernoulli in 1696; Leibniz restates the principle used by James Bernoulli in his solution: "If a curve has a certain property of maximum or minimum, every portion or element in the curve also has the same property" (see Mach, *The Science of Mechanics*, 5th English ed., pp. 521–29).

5 Leibniz follows the Aristotelian meaning of the term (*Nic. Ethics* i. 1) which applies to final causes or ends as explanatory of subordinate ends. Leibniz's architectonic therefore differs from Kant's in being metaphysical as well as methodological, though both rest upon a harmony of forms or possibilities.

6 Leibniz's meaning in this difficult paragraph is made clearer by his actual analyses of reflection and refraction which follow, in which he uses the plane tangent at the point of incidence in place of the curved reflecting or refracting surface itself. His argument rests upon the old discovery that maxima or minima involve in their determination the reduction of two equal values to a single one. This principle he views as an instance of the metaphysical principle of maximal determination or of the optimum.

7 Leibniz notes, "Mr. Molyneux" (see p. 452, note 16). On Fermat's interpretation of the problem see Mach, *The Principles of Physical Optics*, New York 1925, p. 34.

8 A line drawn from a vertex of a triangle dividing the opposite side into segments proportional to the adjacent sides bisects the angle at the vertex.

9 Leibniz's marginal note: "From this another theorem can also be derived which is common to catoptrics and dioptrics, and which seems most elegant to me. It as as follows. If two points are taken in a broken ray in such a way that the line which joins them is divided equally by the perpendicular to the surface of separation, the ratio of the two rays is always constant and proportional to the resistances of the media. For example, if F and G' are taken so that CP' cuts FG' in two equal parts FP' and $G'P'$, the ratio of ray FC to ray CG' will always be constant, namely, as f to g. For this reason they are equal in the same medium, which is the case in reflection."

10 That is, the sines of angles FCL and GCP, since $CF = CG$.

11 The equation of G. is obviously incorrectly copied.

12 Iceland spar, whose property of double refraction had been discovered by Huygens and was discussed in the correspondence of Leibniz with him in 1694.

13 "Things which are contrary to moral principles, we ought also to believe we are unable to do."

14 See No. 37.