# Artificial Intelligence (CS303)
# Practice 4

## CSP EXAMPLE: N-QUEEN

# Outline

- Problem description for the N-Queen problem
- Backtracking Search (BTS) for the N-Queen problem
- Improving BTS for the N-Queen problem
- Local search for the N-Queen problem
- Summary

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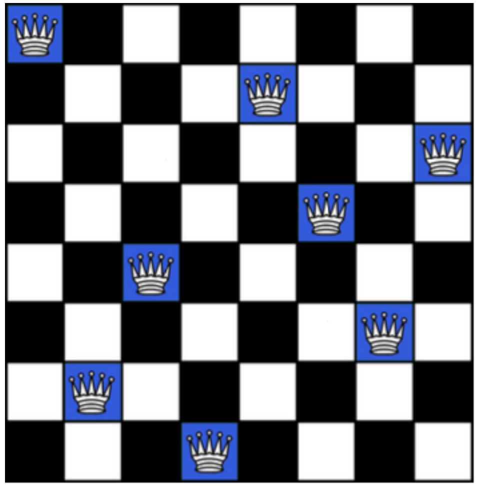
# A brief Review of the CSP

- A Constraint Satisfaction Problem (CSP) consists of three elements:
  - A set of variables: X
  - A set of domains for each variable: D
  - A set of constraints C that specify allowable combinations of values

- Solving the CSP: finding the assignment(s) that satisfy all constraints.

# The $N$-Queen Problem

- The $N$-Queen is the problem of placing $N$ chess queens on an $N \times N$ chessboard so that no two queens attack each other.



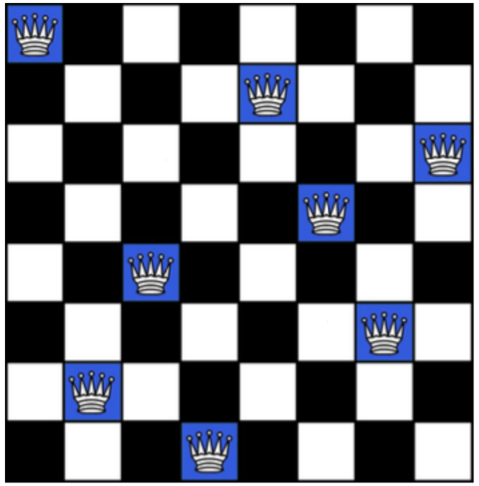
Variables: $X = \{(x_1, y_1), (x_2, y_2), \dots (x_N, y_N)\}$

Domain: $D = \{1,2, \dots N\}$

Constraints: $x_i \neq x_j, y_i \neq y_j, |x_i - x_j| \neq |y_i - y_j|, \forall i \neq j$

# Solution Representation (1)

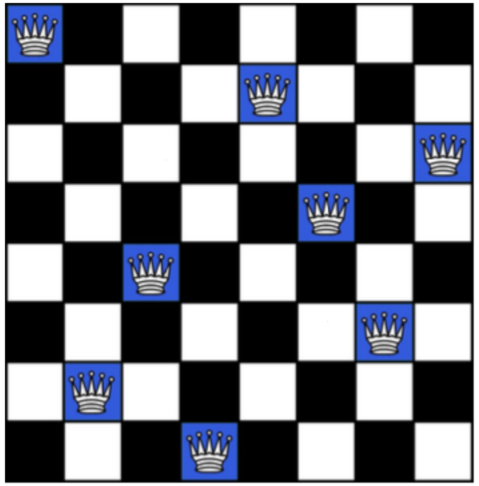
- One variable per queen, $Q_1, Q_2, \dots, Q_n$.
- Each variable could be a tuple $(x, y)$, $\quad x, y \in [1, n]$.



$$\begin{aligned}
Q_1 &= (1,1)\\
Q_2 &= (2,5)\\
Q_3 &= (3,8)\\
Q_4 &= (4,6)\\
Q_5 &= (5,3)\\
Q_6 &= (6,7)\\
Q_7 &= (7,2)\\
Q_8 &= (8,4)
\end{aligned}$$

# Solution Representation (2)

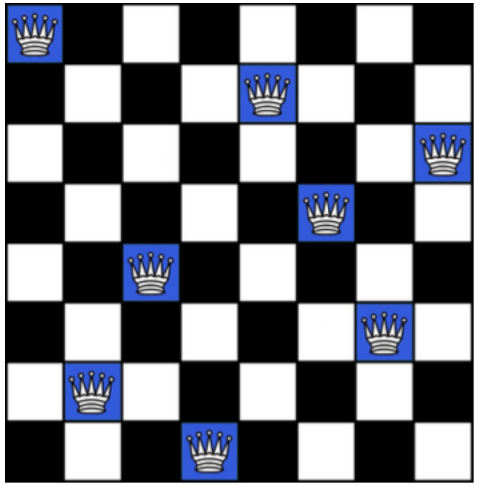
- One variable per queen, $Q_1, Q_2, \ldots, Q_n$.
- Each variable could have a value ∈ $[1, n^2]$.



$$
\begin{aligned}
Q_1 &= 1\\
Q_2 &= 13\\
Q_3 &= 24\\
Q_4 &= 30\\
Q_5 &= 35\\
Q_6 &= 47\\
Q_7 &= 50\\
Q_8 &= 60
\end{aligned}
$$

# Solution Representation (3)

- One variable per queen, $Q_1, Q_2, \ldots, Q_n$.
- Each variable could have a value $\in [1, n]$.



$$
\begin{aligned}
Q_1 &= 1\\
Q_2 &= 5\\
Q_3 &= 8\\
Q_4 &= 6\\
Q_5 &= 3\\
Q_6 &= 7\\
Q_7 &= 2\\
Q_8 &= 4
\end{aligned}
$$

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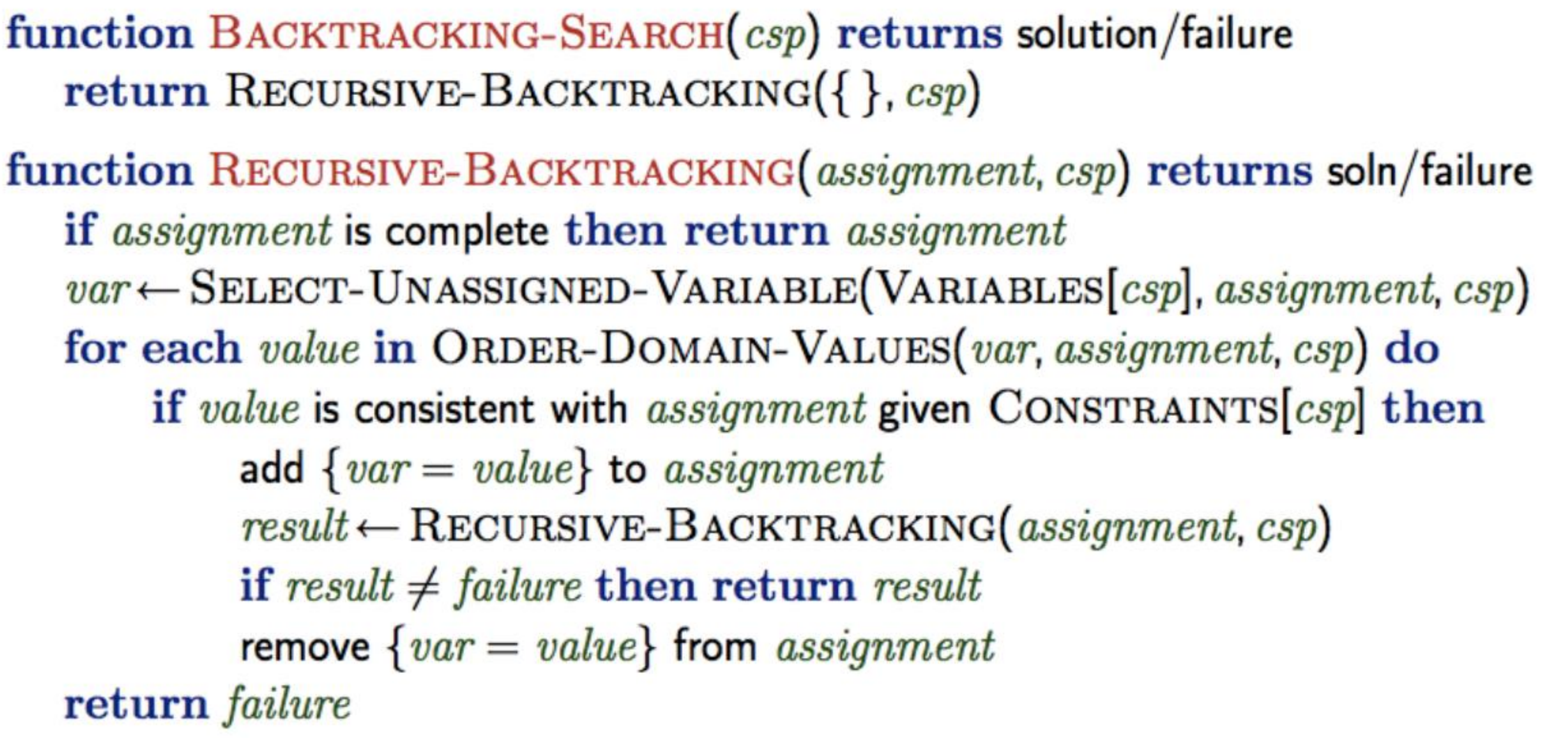
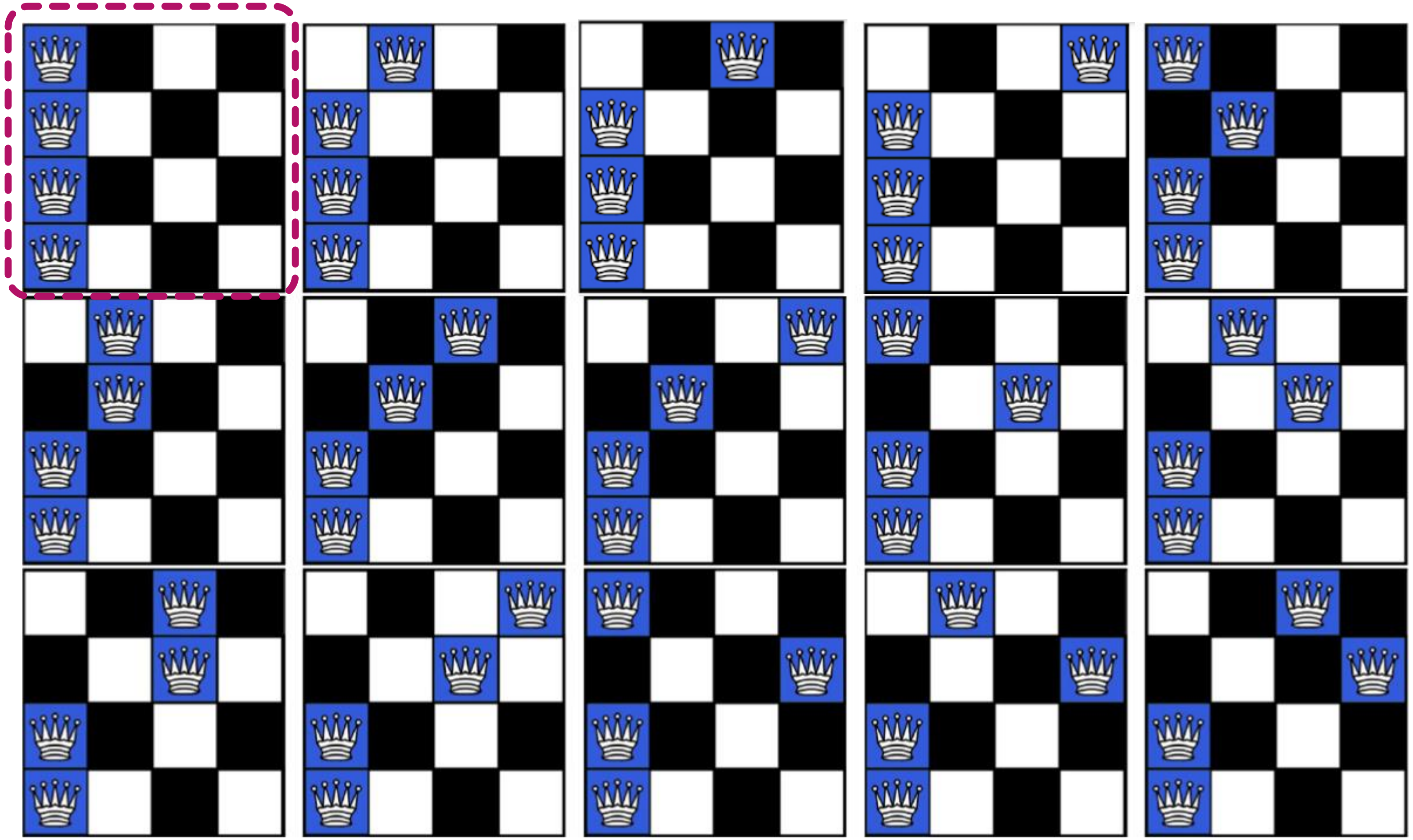
# Backtracking Search(BTS) for CSP



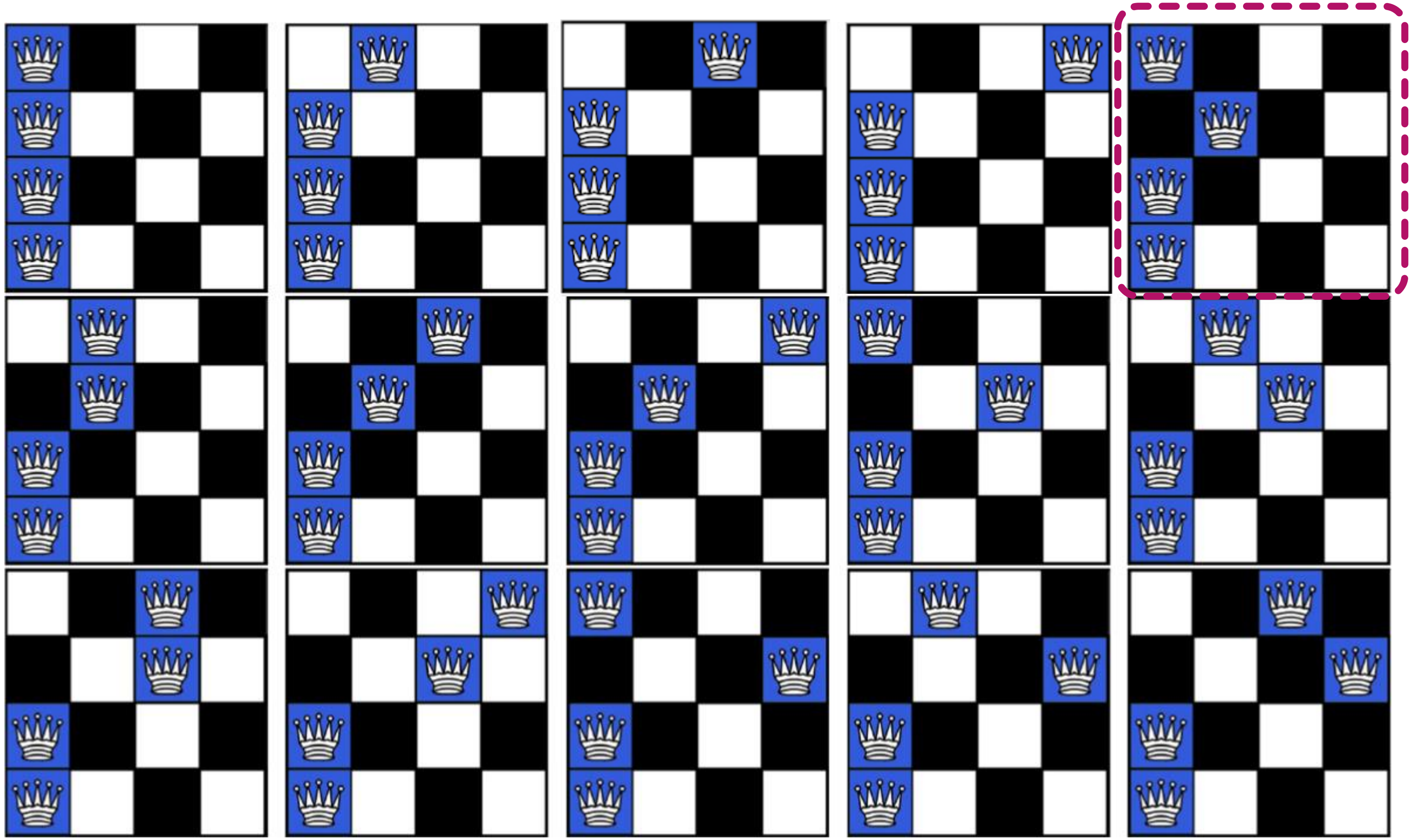
```
function BACKTRACKING-SEARCH(csp) returns solution/failure
    return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment given CONSTRAINTS[csp] then
            add $\{var = value\}$ to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if $result \neq failure$ then return result
            remove $\{var = value\}$ from assignment
    return failure
```

# Backtracking Search(BTS) for N-Queen



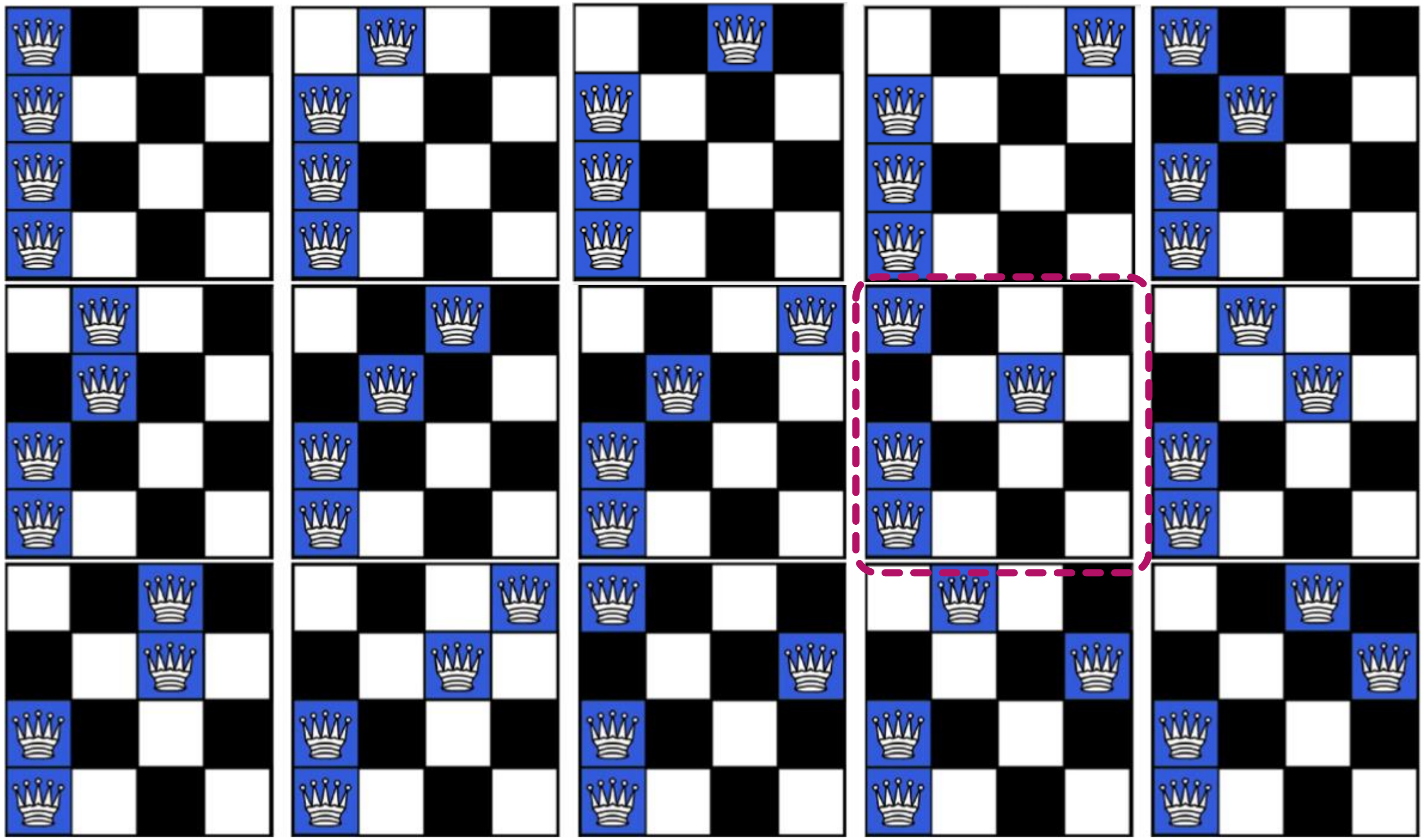
$N = 4$

# Backtracking Search(BTS) for N-Queen



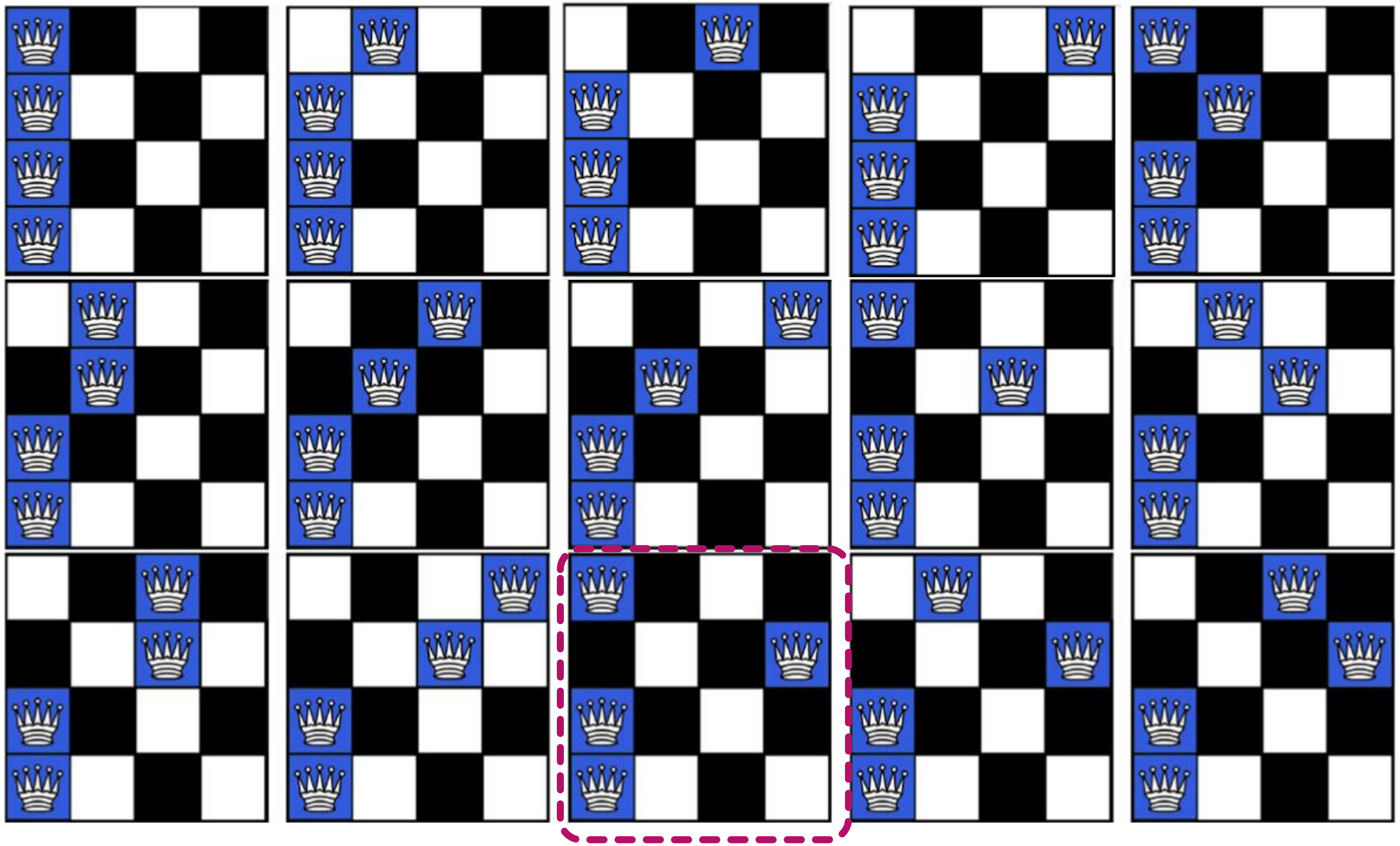
$N = 4$

# Backtracking Search(BTS) for N-Queen



$N = 4$

# Backtracking Search(BTS) for N-Queen



$N = 4$

continue...

# Outline

- Problem description for the N-Queen problem
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# How to improve BTS

- Which variable should be assigned next?
- In what order should its values be tried?
- Can we detect inevitable failure early?

# Improve Backtracking Efficiency

- **Which variable should be assigned next?**

  **Minimum Remaining Value:** Choose the variable with the fewest legal values in its domain.

- **In what order should its values be tried?**

  **Least Constraining Value:** Given a variable, choose the least constraining value, i.e., the one that rules out the fewest values in the remaining variables

- **Can we detect inevitable failure early?**

  **Inference:** Forward checking and using constraint propagation, e.g., arc consistency test.

# An Example for solving N-Queen

- **Minimum Remaining Value:**

Choose the variable with the fewest legal values in its domain.

- **Inference**:

forward checking, keep track of remaining legal values for the unassigned variables. Terminate when any variable has no legal values.

# An Example for solving 8-Queen

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Q1 | Q | X | X | X | X | X | X | X | |
| Q2 | X | X | | | | | | | 6 |
| Q3 | X | | X | | | | | | 6 |
| Q4 | X | | | X | | | | | 6 |
| Q5 | X | | | | X | | | | 6 |
| Q6 | X | | | | | X | | | 6 |
| Q7 | X | | | | | | X | | 6 |
| Q8 | X | | | | | | | X | 6 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Q1 | Q | X | X | X | X | X | X | X | |
| Q2 | X | X | Q | X | X | X | X | X | |
| Q3 | X | X | X | X | | | | | 4 |
| Q4 | X | | X | X | X | | | | 4 |
| Q5 | X | | X | | X | X | | | 4 |
| Q6 | X | | X | | | X | X | | 4 |
| Q7 | X | | X | | | | X | X | 4 |
| Q8 | X | | X | | | | | X | 5 |

- Start: All queens have 8 possible choices
- choose **Q1**, let Q1 = 1
- remove all assignments inconsistent with Q1 = 1

- All queens have 6 possible choices
- choose **Q2**, let Q2 = 3
- Continue to remove all assignments inconsistent with Q2 = 3
- Then: Q3:4 Q4:4 Q5:4 Q6:4 Q7:4 Q8:5

# An Example for solving 8-Queen

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Q1 | Q | X | X | X | X | X | X | X | |
| Q2 | X | X | Q | X | X | X | X | X | |
| Q3 | X | X | X | X | Q | X | X | X | |
| Q4 | X | | X | X | X | X | | | 3 |
| Q5 | X | | X | | X | X | X | | 3 |
| Q6 | X | X | X | | X | X | X | X | 1 |
| Q7 | X | | X | | X | | X | X | 3 |
| Q8 | X | | X | | X | | | X | 4 |

- choose **Q3**, let Q3 = 5
- Continue to remove all assignments inconsistent with Q3 = 5
- Then: Q4:3 Q5:3 Q6:1 Q7:3 Q8:4

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Q1 | Q | X | X | X | X | X | X | X | |
| Q2 | X | X | Q | X | X | X | X | X | |
| Q3 | X | X | X | X | Q | X | X | X | |
| Q4 | X | X | X | X | X | X | | | 2 |
| Q5 | X | | X | X | X | X | X | | 2 |
| Q6 | X | X | X | Q | X | X | X | X | |
| Q7 | X | | X | X | X | | X | X | 2 |
| Q8 | X | X | X | X | X | X | | X | 1 |

- choose **Q6**, let Q6 = 4
- Continue to remove all assignments inconsistent with Q6 = 4
- Then: Q4:2 Q5:2 Q7:2 Q8:1

# An Example for solving 8-Queen

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Q1 | Q | X | X | X | X | X | X | X | |
| Q2 | X | X | Q | X | X | X | X | X | |
| Q3 | X | X | X | X | Q | X | X | X | |
| Q4 | X | X | X | X | X | X | | | 2 |
| Q5 | X | | X | X | X | X | X | | 2 |
| Q6 | X | X | X | Q | X | X | X | X | |
| Q7 | X | | X | X | X | X | X | X | 1 |
| Q8 | X | X | X | X | X | X | Q | X | |

- choose **Q8**, let Q8 = 7
- Continue to remove all assignments inconsistent with Q8 = 7
- Then: Q4:2  Q5:2   Q7:1

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Q1 | Q | X | X | X | X | X | X | X | |
| Q2 | X | X | Q | X | X | X | X | X | |
| Q3 | X | X | X | X | Q | X | X | X | |
| Q4 | X | X | X | X | X | X | | | 2 |
| Q5 | X | X | X | X | X | X | X | | 1 |
| Q6 | X | X | X | Q | X | X | X | X | |
| Q7 | X | Q | X | X | X | X | X | X | |
| Q8 | X | X | X | X | X | X | Q | X | |

- choose **Q7**, let Q7 = 2
- Continue to remove all assignments inconsistent with Q7 = 2
- Then: Q4:2  Q5:1

# An Example for solving 8-Queen

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Q1 | Q | X | X | X | X | X | X | X | |
| Q2 | X | X | Q | X | X | X | X | X | |
| Q3 | X | X | X | X | Q | X | X | X | |
| Q4 | X | X | X | X | X | X | X | X | 0 |
| Q5 | X | X | X | X | X | X | X | Q | |
| Q6 | X | X | X | Q | X | X | X | X | |
| Q7 | X | Q | X | X | X | X | X | X | |
| Q8 | X | X | X | X | X | X | Q | X | |

- choose **Q5**, let Q5 = 8
- Continue to remove all assignments inconsistent with Q5 = 8
- **Then Q4 has no legal values, go back.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Q1 | Q | X | X | X | X | X | X | X |
| Q2 | X | X | Q | X | X | X | X | X |
| Q3 | X | X | X | X | | Q | | |
| Q4 | X | | X | X | X | | | |
| Q5 | X | | X | | X | X | | |
| Q6 | X | | X | | | X | X | |
| Q7 | X | | X | | | | X | X |
| Q8 | X | | X | | | | | X |

- go back to **Q3**, let Q3=6
- go on with the procedure
- …

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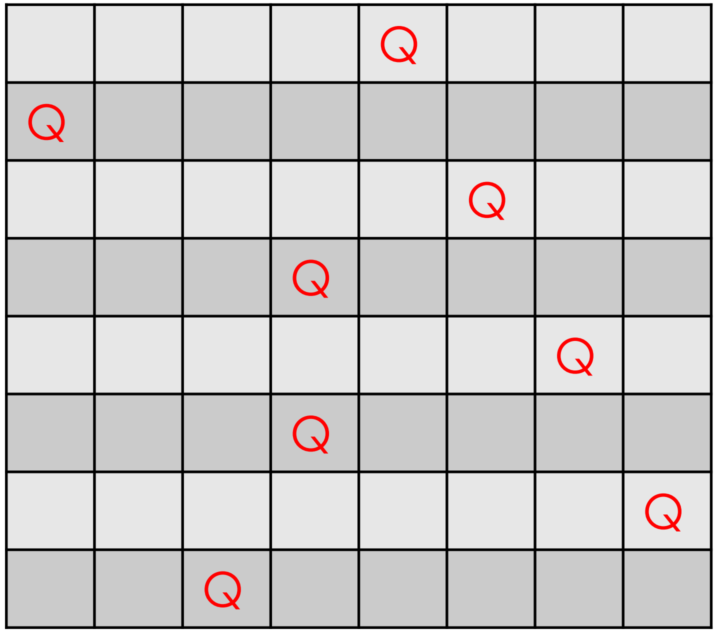
# min_conflicts

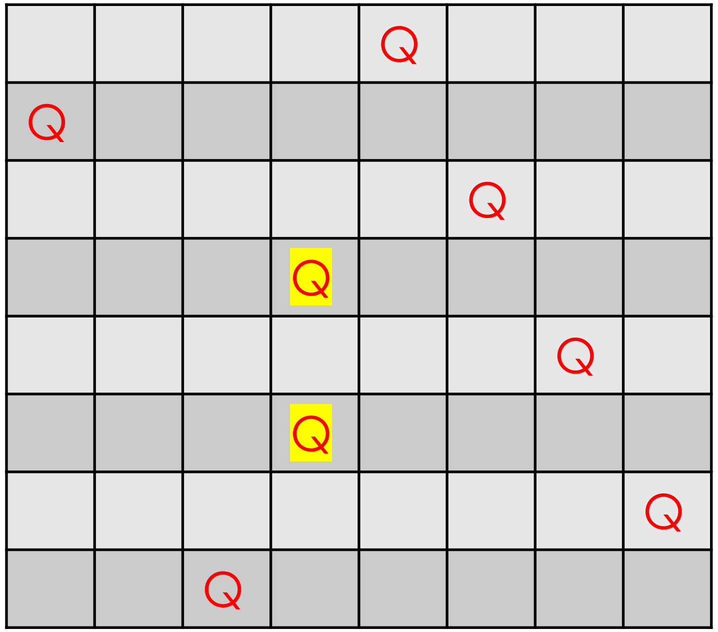
- First generate a complete assignment for all variables (this set of assignments may conflict)
- Repeat the following steps until there are no conflicts:
    - Randomly Select a variable that causes conflicts
    - Reassign the value of this variable to another value that with the least constraint conflicts with other variables

# min_conflicts：8-Queen Problem



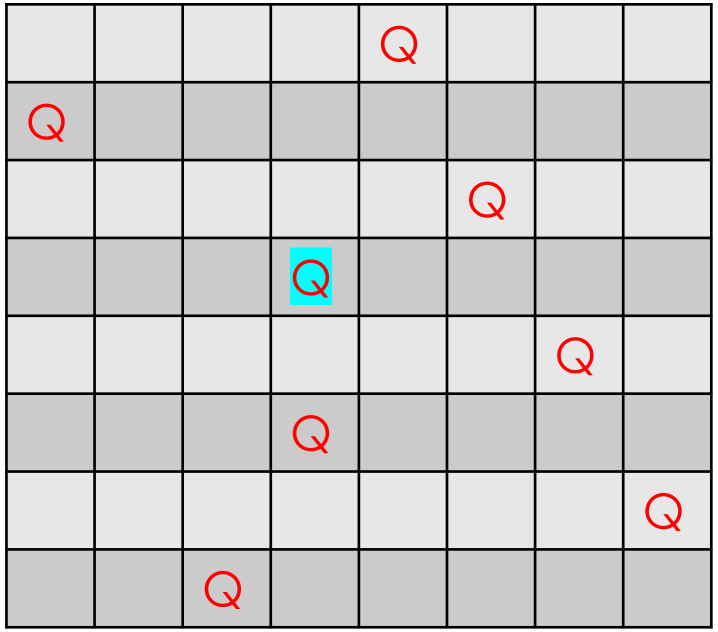
Generate random numbers in the range [1, 8] for each Q

Give an initial complete assignment (in most cases, this assignment will not satisfy all constraints)



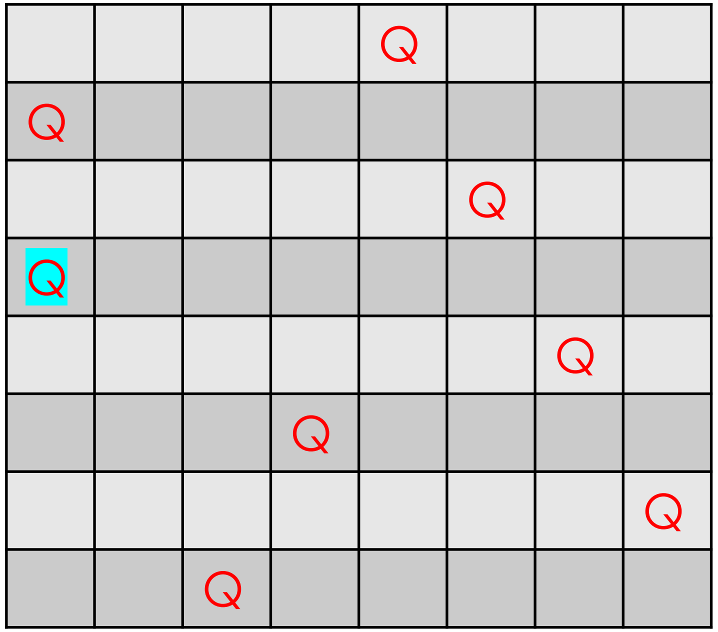
Some conflicted variables

# min_conflicts：8-Queen Problem



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | Q | | | |
| Q | | | | | | | |
| | | | | | Q | | |
| | | | Q | | | | |
| | | | | | | Q | |
| | | | Q | | | | |
| | | | | | | | Q |
| | | Q | | | | | |

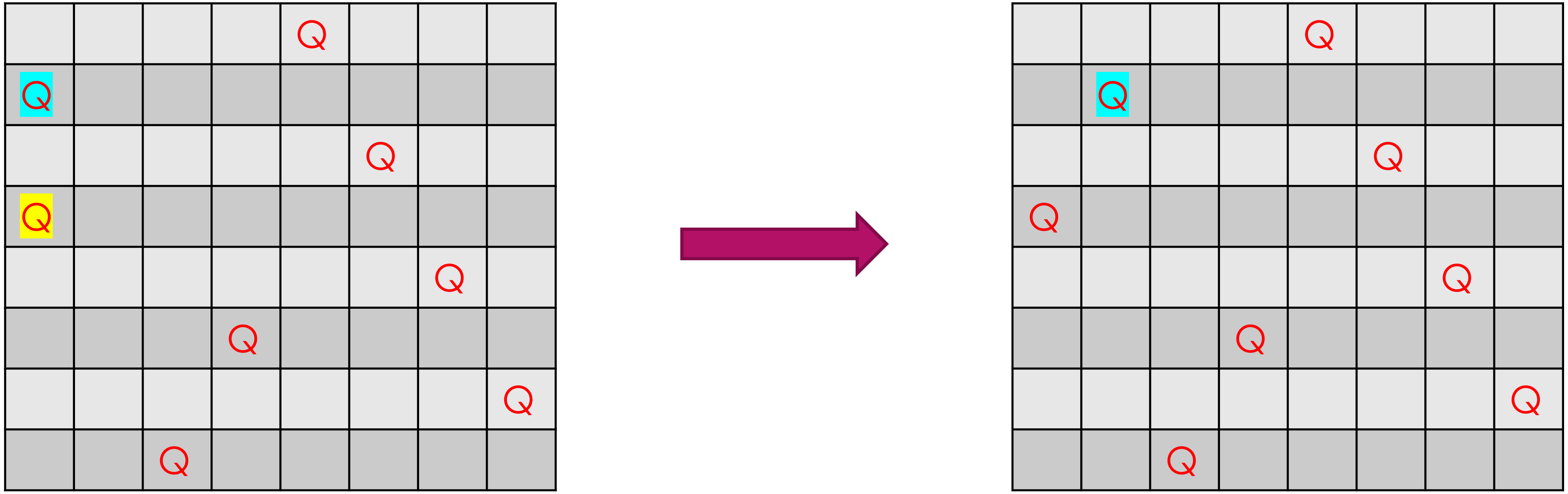
Randomly choose a conflicted variable



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | Q | | | |
| Q | | | | | | | |
| | | | | | Q | | |
| Q | | | | | | | |
| | | | | | | Q | |
| | | | Q | | | | |
| | | | | | | | Q |
| | | Q | | | | | |

Change to the position with the minimum conflict value.

# min_conflicts：8-Queen Problem



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | Q | | | |
| Q | | | | | | | |
| | | | | | Q | | |
| Q | | | | | | | |
| | | | | | | Q | |
| | | | Q | | | | |
| | | | | | | | Q |
| | | Q | | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | Q | | | |
| | Q | | | | | | |
| | | | | | Q | | |
| Q | | | | | | | |
| | | | | | | Q | |
| | | | Q | | | | |
| | | | | | | | Q |
| | | Q | | | | | |

Randomly choose a conflicted variable

Change to the position with the minimum conflict value.

Solved.

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# Summary

- Formulate the N-Queen problem as a CSP
- Solve N-Queen with Backtracking Search
- Solve N-Queen with Local Search
- **CSP Practice DDL: 22:00, Nov.1ST**