Reference Answers for Assignment 02 of I2ML-s23

Q1 Use truth tables to prove the validity of the 3 hardest sequents you thought from the following list. Pay attention that $A, B, C \vdash D$ is equivalent to $A \wedge B \wedge C \rightarrow D$. *(3 * 10 = 30 pts)*

1. $p \vee q \vdash q \vee p$
2. $(p \wedge q) \wedge r \vdash p \wedge (q \wedge r)$
3. $p \rightarrow q \vdash \neg q \rightarrow \neg p$
4. $(p \wedge q) \vee (p \wedge r) \vdash p \wedge (q \vee r)$
5. $\vdash \neg p \vee q \rightarrow (p \rightarrow q)$
6. $\vdash (p \rightarrow q) \rightarrow \neg p \vee q$
7. $\neg(p \rightarrow q) \vdash q \rightarrow p$
8. $W \rightarrow X, Y \rightarrow Z \vdash W \vee Y \rightarrow X \vee Z$
9. $\vdash \neg(A \wedge B) \rightarrow (A \rightarrow \neg B)$
10. $p \wedge q \rightarrow r \vdash (p \rightarrow r) \vee (q \rightarrow r)$
11. $\vdash \neg(A \leftrightarrow \neg A)$
12. $A \leftrightarrow B \vdash \neg A \leftrightarrow \neg B$

8. $W \to X,\ Y \to Z \vdash W \vee Y \to X \vee Z$

Proof： To prove the validity of the above sequent, we may check the validity of the following equivalent sequent:

$$\vdash (W \to X) \wedge (Y \to Z) \to (W \vee Y \to X \vee Z)$$

So we check the values of the formula

$$(W \to X) \wedge (Y \to Z) \to (W \vee Y \to X \vee Z)$$

one by one for all 16 models with variable set { W, X, Y, Z } by the following truth table.

W	X	Y	Z		$(W \rightarrow X) \wedge (Y \rightarrow Z) \rightarrow (W \vee Y \rightarrow X \vee Z)$														
0	0	0	0		0	1	0	1	0	1	0	1	0	0	0	1	0	0	0
0	0	0	1		0	1	0	1	0	1	1	1	0	0	0	1	0	1	1
0	0	1	0		0	1	0	0	1	0	0	1	0	1	1	0	0	0	0
0	0	1	1		0	1	0	1	1	1	1	1	0	1	1	1	0	1	1
0	1	0	0		0	1	1	1	0	1	0	1	0	0	0	1	1	1	0
0	1	0	1		0	1	1	1	0	1	1	1	0	0	0	1	1	1	1
0	1	1	0		0	1	1	0	1	0	0	1	0	1	1	1	1	1	0
0	1	1	1		0	1	1	1	1	1	1	1	0	1	1	1	1	1	1
1	0	0	0		1	0	0	0	0	1	0	1	1	1	0	0	0	0	0
1	0	0	1		1	0	0	0	0	1	1	1	1	1	0	1	0	1	1
1	0	1	0		1	0	0	0	1	0	0	1	1	1	1	0	0	0	0
1	0	1	1		1	0	0	0	1	1	1	1	1	1	1	1	0	1	1
1	1	0	0		1	1	1	1	0	1	0	1	1	1	0	1	1	1	0
1	1	0	1		1	1	1	1	0	1	1	1	1	1	0	1	1	1	1
1	1	1	0		1	1	1	0	1	0	0	1	1	1	1	1	1	1	0
1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

From all the 1's truth values in the column representing the leading connective of the formula in above table, the validity of the original sequent is proved. QED

Q2 Use truth tables to prove the validity of the 3 hardest formulas you thought from the following list. Pay attention that do not repeat with Q1. *(3 * 10 = 30 pts)*

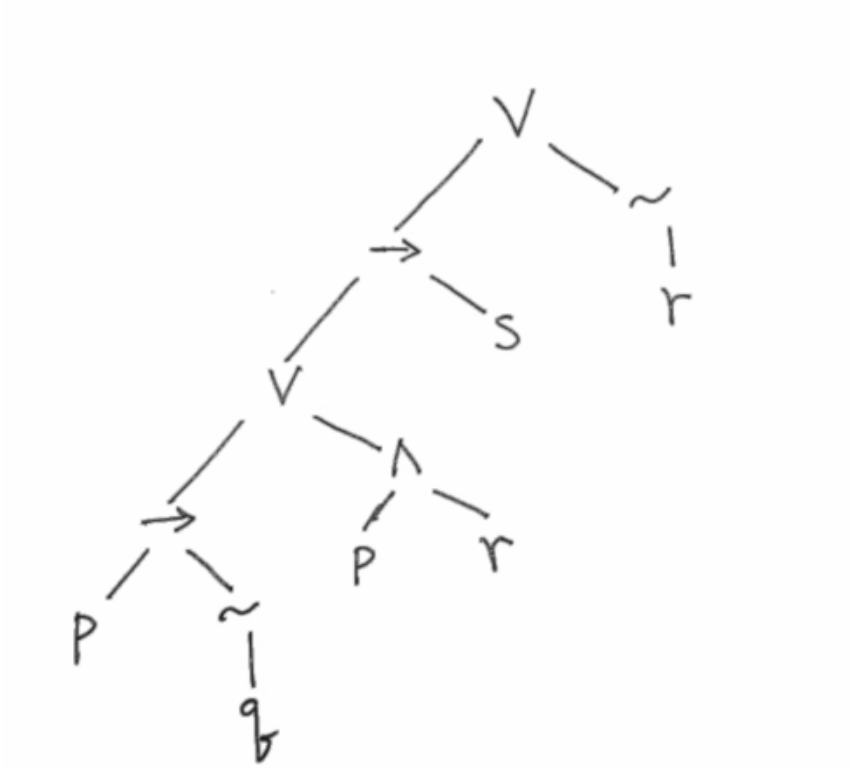
1. Commutativity of $\wedge$: $A \wedge B \leftrightarrow B \wedge A$
2. Commutativity of $\vee$: $A \vee B \leftrightarrow B \vee A$
3. Associativity of $\wedge$: $(A \wedge B) \wedge C \leftrightarrow A \wedge (B \wedge C)$
4. Associativity of $\vee$ $(A \vee B) \vee C \leftrightarrow A \vee (B \vee C)$
5. Distributivity of $\wedge$ over $\vee$: $A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C)$
6. Distributivity of $\vee$ over $\wedge$: $A \vee (B \wedge C) \leftrightarrow (A \vee B) \wedge (A \vee C)$
7. $(A \rightarrow (B \rightarrow C)) \leftrightarrow (A \wedge B \rightarrow C)$.
8. $(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$
9. $((A \vee B) \rightarrow C) \leftrightarrow (A \rightarrow C) \wedge (B \rightarrow C)$
10. $\neg(A \vee B) \leftrightarrow \neg A \wedge \neg B$
11. $\neg(A \wedge B) \leftrightarrow \neg A \vee \neg B$
12. $\neg(A \wedge \neg A)$
13. $\neg(A \rightarrow B) \leftrightarrow A \wedge \neg B$
14. $\neg A \rightarrow (A \rightarrow B)$
15. $(\neg A \vee B) \leftrightarrow (A \rightarrow B)$
16. $A \vee \bot \leftrightarrow A$
17. $A \wedge \bot \leftrightarrow \bot$
18. $A \vee \neg A$
19. $\neg(A \leftrightarrow \neg A)$
20. $(A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$
21. $(A \rightarrow C \vee D) \rightarrow ((A \rightarrow C) \vee (A \rightarrow D))$
22. $(((A \rightarrow B) \rightarrow A) \rightarrow A)$

注： Q1和Q2只给一个相对复杂小题的参考答案。评分标准为，真值表前后的恰当叙述为3分(前后只缺一个根据情况扣1～2分)，完整的真值表7分，真值表要包含完整的models（变量0/1取值组合），每个待求的公式及其中所有子表达式的值(原子命题变量值可以不与模型重复)，每缺一个子公式的值，按比例扣分。证明的结果，只能是全1列(对有效公式)或对应行完全相等的两列(对两个等价的公式) 。

Q3 For the given formula below, draw its parse tree and list the set of its all subformulas. *(10 + 10 = 20 pts)*

$$((p \rightarrow \sim q) \vee (p \wedge r) \rightarrow s) \vee \sim r$$

Ref. Ans. The corresponding parse tree is as follows:



The set of subformulas for the given formula in fully dressed parentheses is

$$
\begin{array}{l}
\{\ ((p \to \sim q) \vee (p \wedge r) \to s) \vee \sim r, \\
\quad ((p \to \sim q) \vee (p \wedge r) \to s), \\
\quad ((p \to \sim q) \vee (p \wedge r)), \\
\quad (p \to \sim q), \\
\quad (p \wedge r), \\
\quad (\sim q),\ (\sim r), \\
\quad p,\ q,\ r,\ s \\
\}
\end{array}
$$

In shorthand form:

$$
\begin{array}{l}
\{\ ((p \to \sim q) \vee (p \wedge r) \to s) \vee \sim r, \\
\quad (p \to \sim q) \vee (p \wedge r) \to s, \\
\quad (p \to \sim q) \vee (p \wedge r), \\
\quad p \to \sim q, \\
\quad p \wedge r, \\
\quad \sim q,\ \sim r, \\
\quad p,\ q,\ r,\ s \\
\}
\end{array}
$$

Q4 Finish 3 tasks for each parse true in Figure A and Figure B: **1)** List the corresponding formula with fully dressed parentheses; **2)** List the shortest formula with proper precedence order and right associative discussed in lectures; **3)** Given a valuation/model and a parse tree of a formula, compute the truth value of the formula for that valuation/model (as done in a bottom-up fashion in Figure 1.7 on page 40 of textB). (*2 * (5+5+5) = 30 pts*)

4.1 **Figure A** in which *q evaluates to T and p and r evaluate to F;*

4.2 **Figure B** where we let *p be T, q be F and r be T.*

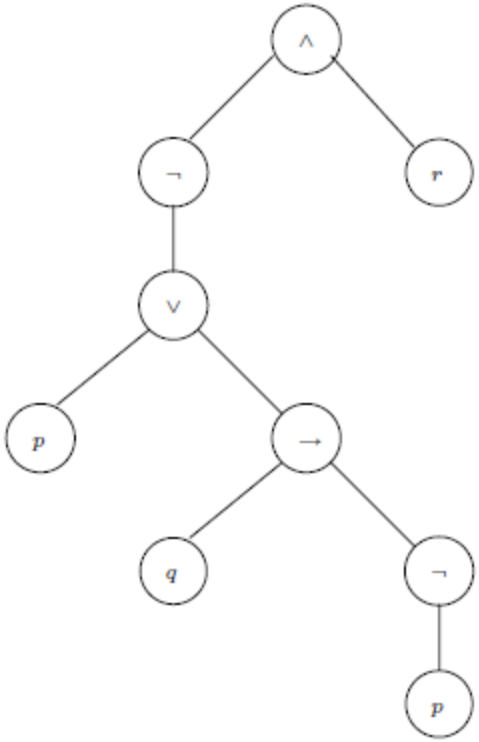


Figure A. In-order linear representation of the tree is a logical formula

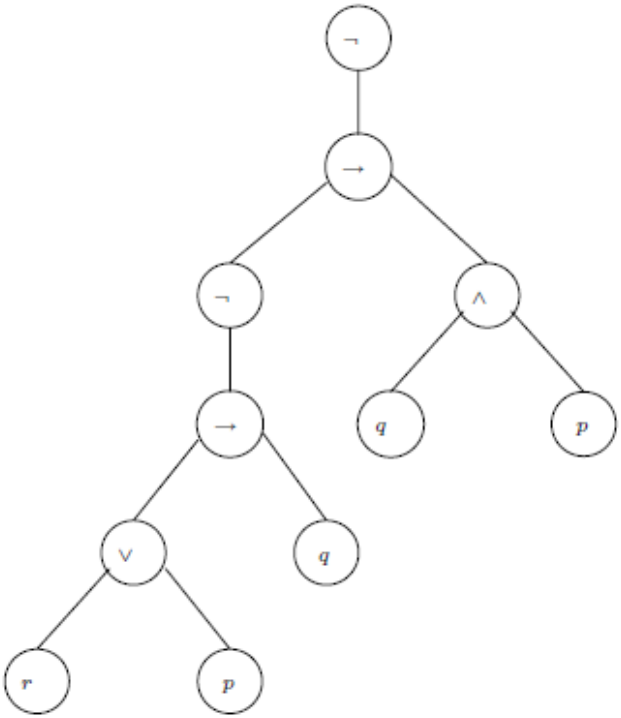


Figure B. A parse tree of a negated implication

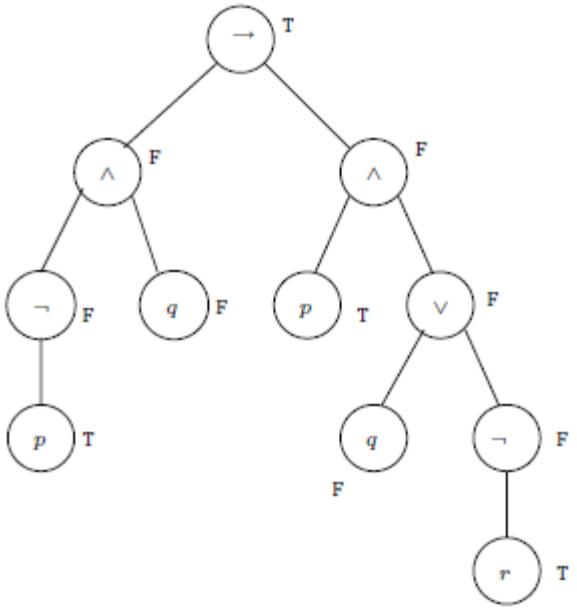


Figure 1.7. The evaluation of a logical formula under a given valuation *(On page 40 of textB)*

Q4 Ref Ans:

For Figure A:

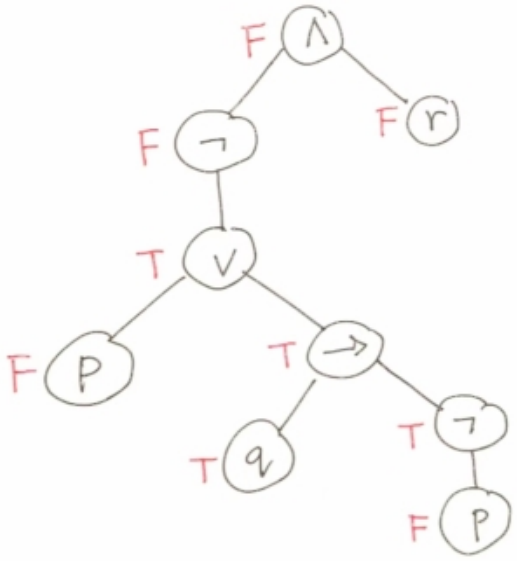
1) The corresponding formula with fully dressed parentheses:

$(\sim(p \vee (q \rightarrow (\sim p))) \wedge r$

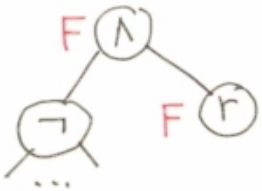
2). The shortest formula

$\sim(p \vee (q \rightarrow \sim p)) \wedge r$

3). The evaluation is as follows:



simplely:



Q4 Ref. Ans:

For Figure B.

1). The fully dressed formula:

$\neg(\neg((r \vee p) \rightarrow q) \rightarrow (q \wedge p))$

2). The shortest formula:

$\neg(\neg(r \vee p \rightarrow q) \rightarrow q \wedge p)$

3). The evaluation is as follows: