# Quantum algorithm for the Vlasov-Maxwell equations

Declan Millar*§, Vadim Elisseev*¶‖, Benjamin Symons†, Dilhan Manawadu†, Michael Garn†, Jason Ledwidge*, Animesh Datta‡, Tom Goffrey‡, Stefano Mensa†

*IBM Research Europe †Hartree Centre, UKRI-STFC ‡University of Warwick, UK
§University of Southampton, UK ¶Wrexham Glyndŵr University, UK ‖University of Oxford, UK

## 1. Motivation: plasma physics simulation

- The Vlasov-Maxwell system models the dynamics of collisionless, classical, non-relativistic plasma.
- Six-dimensional system; classical large-scale simulations intractable.
- This necessitates:
  - Reduced dimensionality models.
  - Approximate, reduced physics models.
- An efficient quantum algorithm may make full simulation feasible.
- Possible basis for simulation of the Vlasov-Fokker-Planck equation.
- May allow accurate calculations of kinetic heat transport towards a multi-scale plasma physics simulation package.

## 2. Vlasov-Maxwell system

- The Vlasov-Maxwell equations are a family of coupled nonlinear partial differential equations,

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} f_s + \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0.$$

- First step: model the more limited Vlasov-Poisson system [1].
- Test case—Landau damping in one phase space dimension with an initial Maxwellian distribution function [2].

## 3. Quantum algorithm

- Assess and validate the state preparation and Hamiltonian simulation detailed in Ref. [1].
- Combine the techniques for efficient implementation detailed in Refs. [3] and [4]:
  - Quantum Carleman linearization.
  - Finite difference algorithm for the Poisson equation.
- Bottleneck: measurement of the electric field:
  - Electric field corresponds to one component of the state vector.
  - Explore using amplitude amplification [2].

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## 4. Empirical validation on quantum hardware

- Understand the end-to-end performance, including:
  - Quantum-state initialization.
  - Hamiltonian simulation.
  - Quantum time evolution.
  - Measurement.
- Compare the performance of our algorithm with:
  - Quantum algorithms in Ref. [1] and [5].
  - State-of-the-art classical Vlasov-Maxwell solvers.
- Demonstrate empirically on an least 127-qubit quantum computer.
- We are writing a software package based on the Qiskit SDK [6].
- Release the source code as part of Qiskit Community.

## 5. Complexity

- Quantify the space and time complexity of the quantum-Carleman-linearization-based Vlasov-Maxwell solver.
- Dependent on the:
  - Nature—absolute or relative.
  - Level of error we demand.
  - Nature of the inputs and outputs.
- Complexity of the quantum algorithm will be a product of these quantities and the algorithm for solving the Poisson equation [4].
- Improve the overall complexity by drawing on Refs. [4] and [7].

## 6. Extensions

- Study the end-to-end complexity of more general cases, such as:
  - Higher phase-space dimensions—may benefit from the tridiagonal nature of the discretized Poisson.
  - Non-Maxwellian distribution functions.
  - Presence of magnetic fields.
  - Model the Vlasov-Maxwell-Fokker-Planck system.

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