GATE 2021 EE 43

EE23BTECH11022 - G DILIP REDDY

Question:
Consider a continuous-time signal $x(t)$ defined by $x(t) = 0$ for $|t| > 1$, and $x(t) = 1 - |t|$ for $|t| \leq 1$. Let the Fourier transform of $x(t)$ be defined as $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$. The maximum magnitude of $X(\omega)$ is ________ . (GATE 2021 EE 43)

Solution:

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t} dt \tag{1}$$

$$X(f) = \int_{-1}^{1} (1 - |t|)\, e^{-j2\pi f t} dt \tag{2}$$

$$X(f) = \int_{-1}^{1} e^{-j2\pi f t} dt - \int_{-1}^{1} |t|\, e^{-j2\pi f t} dt \tag{3}$$

$$X(f) = 2\int_{0}^{1} \cos(2\pi f t)\, dt - 2\int_{0}^{1} t\cos(2\pi f t)\, dt \tag{4}$$

$$X(f) = 2\frac{\sin(2\pi f)}{2\pi f} - 2\left[\frac{\sin(2\pi f)}{2\pi f} + \frac{\cos(2\pi f)}{(2\pi f)^2} - \frac{1}{(2\pi f)^2}\right] \tag{5}$$

$$X(f) = 2\frac{1 - \cos(2\pi f)}{(2\pi f)^2} \tag{6}$$

$$X(f) = 2\frac{2\sin^2\left(\frac{2\pi f}{2}\right)}{(2\pi f)^2} \tag{7}$$

$$X(f) = \frac{\sin^2(\pi f)}{(\pi f)^2} \tag{8}$$

$$f \to 0 \implies X(f) \to 1 \tag{9}$$

Maximum of magnitude of $X(f) = 1$

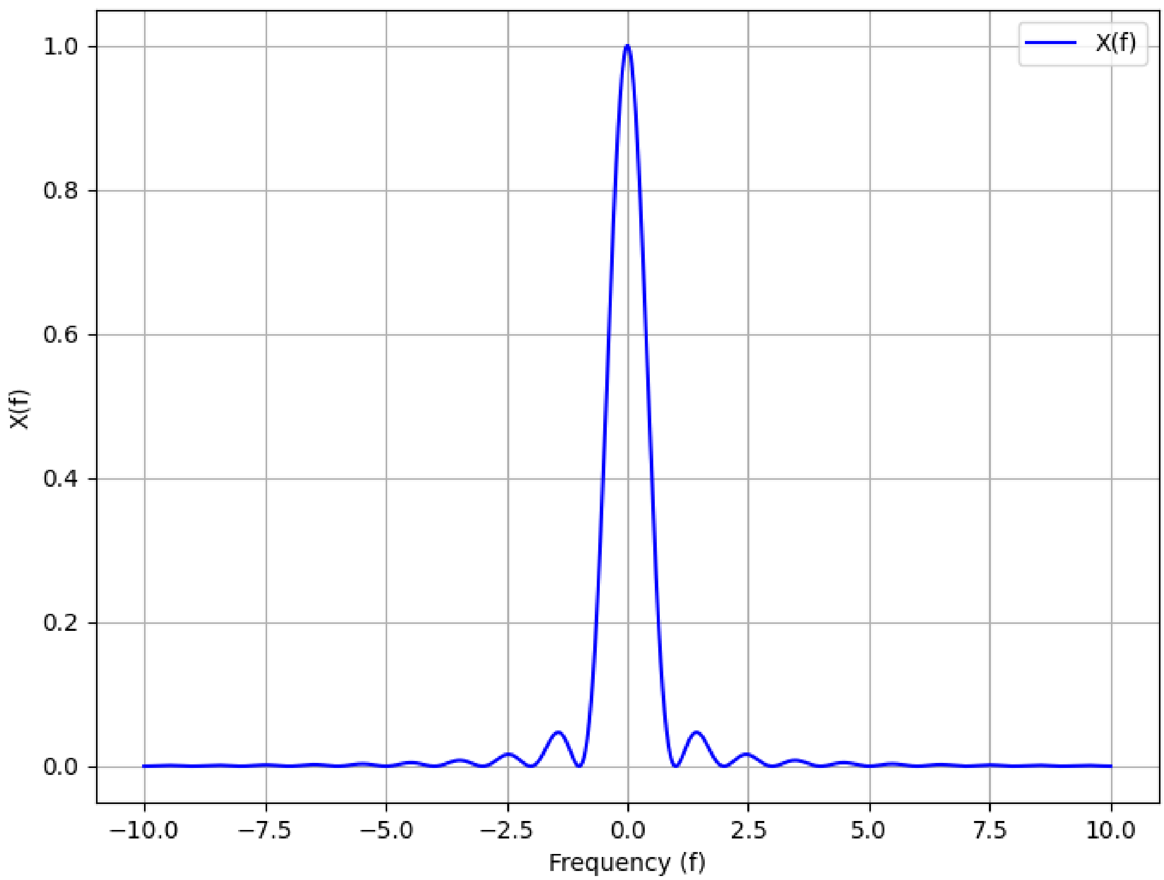

Fig. 1: plot of X(f)