GATE 2021 ee 43

EE23BTECH11022 - G DILIP REDDY

Question:
Consider a continuous-time signal $x(t)$ defined by $x(t) = 0$ for $|t| > 1$, and $x(t) = 1 - |t|$ for $|t| \leq 1$. Let the Fourier transform of $x(t)$ be defined as $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$. The maximum magnitude of $X(\omega)$ is ________. (GATE 2021 EE 43)

Solution:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt \tag{1}$$

$$X(\omega) = \int_{-1}^{1} (1 - |t|)\, e^{-j\omega t} dt \tag{2}$$

$$X(\omega) = \int_{-1}^{1} e^{-j\omega t} dt - \int_{-1}^{1} |t|\, e^{-j\omega t} dt \tag{3}$$

$$X(\omega) = 2\int_{0}^{1} \cos(\omega t)\, dt - 2\int_{0}^{1} t\cos(\omega t)\, dt \tag{4}$$

$$X(\omega) = 2\frac{\sin(\omega)}{\omega} - 2\left[\frac{\sin(\omega)}{\omega} + \frac{\cos(\omega)}{\omega^2} - \frac{1}{\omega^2}\right] \tag{5}$$

$$X(\omega) = 2\frac{1 - \cos(\omega)}{\omega^2} \tag{6}$$

$$X(\omega) = 2\frac{2\sin^2\left(\frac{\omega}{2}\right)}{\omega^2} \tag{7}$$

$$\omega \to 0 \implies X(\omega) \to 1 \tag{8}$$

Maximum of magnitude of $X(\omega) = 1$