

11.9.3.6

EE23BTECH11022 - G DILIP REDDY

Question:

For what values of x , the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P ?

Solution:

Let a, b, c be any three consecutive terms of a GP whose common difference is r then,

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = r \quad (1)$$

$$\Rightarrow \frac{x}{(-\frac{2}{7})} = \frac{(-\frac{7}{2})}{x} \quad (2)$$

$$x^2 = (-\frac{2}{7}) \cdot (-\frac{7}{2}) \quad (3)$$

$$x^2 = 1 \quad (4)$$

$$x = 1, -1 \quad (5)$$

$$r = \frac{x}{(-\frac{2}{7})} \quad (6)$$

$$\Rightarrow r = \frac{7}{2} \text{ or } -\frac{7}{2} \quad (7)$$

$$T_n = -\frac{2}{7} \cdot (-\frac{7}{2})^n \text{ or } -\frac{2}{7} \cdot (\frac{7}{2})^n \quad (8)$$

$$T_n = (-\frac{7}{2})^{n-1} \text{ or } -(\frac{7}{2})^{n-1} \quad (9)$$

The signal corresponding to this is

$$x(n) = (-\frac{7}{2})^{n-1} u(n) \text{ or } -(\frac{7}{2})^{n-1} u(n) \quad (10)$$

Variable	Description	Value
a_0	First term of the GP	$-(2/7)$
r	Common ratio of the GP	$-(7/2) \text{ or } (7/2)$
$x(n)$	General term	$(-7/2)^{n-1} \text{ or } -(7/2)^{n-1}$

TABLE 1: Variables Used

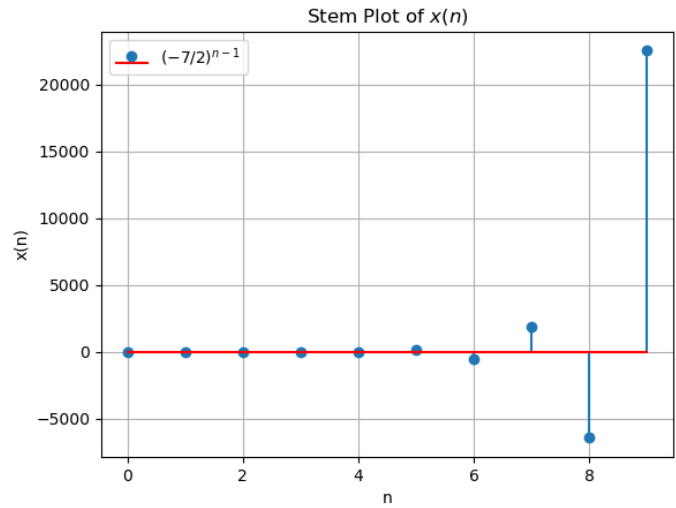


Fig. 1: Stem Plot of $x(n)$

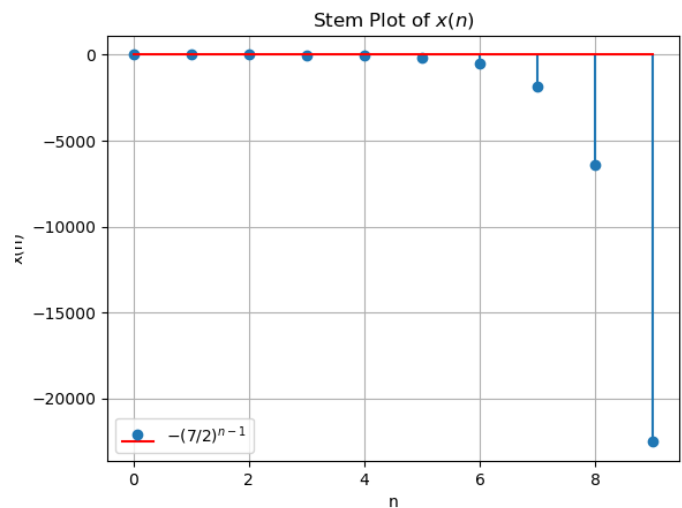


Fig. 1: Stem Plot of $x(n)$

Applying z-Transform:

case -1 :

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\left(-\frac{7}{2} \right)^{n-1} u(n) \right) z^{-n} \quad (11)$$

$$X(z) = 0 + \sum_{n=0}^{\infty} \left(\left(-\frac{7}{2} \right)^{n-1} \right) z^{-n} \quad (12)$$

$$X(z) = \sum_{n=0}^{\infty} \left(\left(-\frac{7}{2} \right)^{n-1} \right) z^{-n} \quad (13)$$

$$\left(\frac{2}{7} \right) X(z) = \sum_{n=0}^{\infty} \left(\left(-\frac{2}{7} \right)^{-n} \right) z^{-n} \quad (14)$$

$$\left(\frac{7}{2} \right) X(z) = \sum_{n=0}^{\infty} \left(\left(-\frac{2z}{7} \right)^{-n} \right) \quad (15)$$

For this to converge $\left| \frac{7}{2z} \right| < 1$

$$\Rightarrow -1 < \frac{7}{2z} < 1 \quad (16)$$

$$z \in \left(-\infty, -\frac{7}{2} \right) \cup \left(\frac{7}{2}, \infty \right) \quad (17)$$

Multiplying $-\frac{7}{2z}$ on both sides of equation 15

$$-\left(\frac{7}{2z} \right) \left(\frac{7}{2} \right) X(z) = \sum_{n=0}^{\infty} \left(\left(-\frac{2z}{7} \right)^{-(n+1)} \right) \quad (18)$$

$$-\left(\frac{7}{2z} \right) \left(\frac{7}{2} \right) X(z) = \sum_{n=1}^{\infty} \left(\left(-\frac{2z}{7} \right)^{-n} \right) \quad (19)$$

$$-\left(\frac{7}{2z} \right) \left(\frac{7}{2} \right) X(z) = -\left(-\frac{2z}{7} \right)^{-0} + \sum_{n=0}^{\infty} \left(\left(-\frac{2z}{7} \right)^{-n} \right) \quad (20)$$

$$-\left(\frac{7}{2z} \right) \left(\frac{7}{2} \right) X(z) = -1 + \sum_{n=0}^{\infty} \left(\left(-\frac{2z}{7} \right)^{-n} \right) \quad (21)$$

Subtracting eqn 15 from eqn 21

$$\left(\frac{7}{2z} + 1 \right) \left(\frac{7}{2} \right) X(z) = 1 \quad (22)$$

$$X(z) = \left(\frac{2}{7} \right) \left(\frac{2z}{2z+7} \right) \quad (23)$$

$$\Rightarrow X(z) = \left(\frac{1}{7} \right) \left(\frac{4z}{2z+7} \right) \quad (24)$$

case -2 :

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\left(-\frac{7}{2} \right)^{n-1} u(n) \right) z^{-n} \quad (25)$$

$$X(z) = 0 + \sum_{n=0}^{\infty} \left(\left(-\frac{7}{2} \right)^{n-1} \right) z^{-n} \quad (26)$$

$$X(z) = \sum_{n=1}^{\infty} \left(\left(-\frac{7}{2} \right)^{n-1} \right) z^{-n} \quad (27)$$

$$\left(\frac{7}{2} \right) X(z) = \sum_{n=0}^{\infty} \left(\left(-\frac{7}{2} \right)^{-n} \right) z^{-n} \quad (28)$$

$$\left(\frac{7}{2} \right) X(z) = \sum_{n=0}^{\infty} \left(\left(-\frac{2z}{7} \right)^{-n} \right) \quad (29)$$

For this to converge $\left| \frac{7}{2z} \right| < 1$

$$\Rightarrow -1 < \frac{7}{2z} < 1 \quad (30)$$

$$z \in \left(-\infty, -\frac{7}{2} \right) \cup \left(\frac{7}{2}, \infty \right) \quad (31)$$

Multiplying $\frac{7}{2z}$ on both sides of equation 29

$$\left(\frac{7}{2z} \right) \left(\frac{7}{2} \right) X(z) = \sum_{n=0}^{\infty} \left(\left(-\frac{2z}{7} \right)^{-(n+1)} \right) \quad (32)$$

$$\left(\frac{7}{2z} \right) \left(\frac{7}{2} \right) X(z) = \sum_{n=1}^{\infty} \left(\left(-\frac{2z}{7} \right)^{-n} \right) \quad (33)$$

$$\left(\frac{7}{2z} \right) \left(\frac{7}{2} \right) X(z) = -(-1)^{-1} + \sum_{n=1}^{\infty} \left(\left(-\frac{2z}{7} \right)^{-n} \right) \quad (34)$$

$$\left(\frac{7}{2z} \right) \left(\frac{7}{2} \right) X(z) = 1 + \sum_{n=1}^{\infty} \left(\left(-\frac{2z}{7} \right)^{-n} \right) \quad (35)$$

Subtracting eqn 29 from eqn 35

$$\left(\frac{7}{2z} - 1 \right) \left(\frac{7}{2} \right) X(z) = 1 \quad (36)$$

$$X(z) = \left(\frac{2}{7} \right) \left(\frac{2z}{7-2z} \right) \quad (37)$$

$$\Rightarrow X(z) = \left(\frac{1}{7} \right) \left(\frac{4z}{7-2z} \right) \quad (38)$$