11.9.3.6

EE23BTECH11022 - G DILIP REDDY

Question:
For what values of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P ?

Solution:
Let r be the common ratio

$$\frac{x}{\left(-\frac{2}{7}\right)} = \frac{\left(-\frac{7}{2}\right)}{x} \tag{1}$$

$$\implies \frac{x}{\left(-\frac{2}{7}\right)} = \frac{\left(-\frac{7}{2}\right)}{x} = r \tag{2}$$

$$x^2 = \left(-\frac{2}{7}\right)\cdot\left(-\frac{7}{2}\right) \tag{3}$$

$$x^2 = 1 \tag{4}$$

$$x = 1, -1 \tag{5}$$

$$r = \frac{x}{\left(-\frac{2}{7}\right)} \tag{6}$$

$$\implies r = \frac{7}{2} \text{ or } -\frac{7}{2} \tag{7}$$

$$T_n = -\frac{2}{7}\cdot\left(-\frac{7}{2}\right)^n \text{ or } -\frac{2}{7}\cdot\left(\frac{7}{2}\right)^n \tag{8}$$

$$T_n = \left(-\frac{7}{2}\right)^{n-1} \text{ or } -\left(\frac{7}{2}\right)^{n-1} \tag{9}$$

The signal corresponding to this is

$$x_1(n) = \left(-\frac{7}{2}\right)^{n-1} u(n) \tag{10}$$

$$x_2(n) = -\left(\frac{7}{2}\right)^{n-1} u(n) \tag{11}$$

Variable	Description	Value
$x(0)$	First term of the GP	$-\left(\frac{2}{7}\right)$
r	Common ratio of the GP	
$x(n)$	General term	

TABLE 1: Variables Used

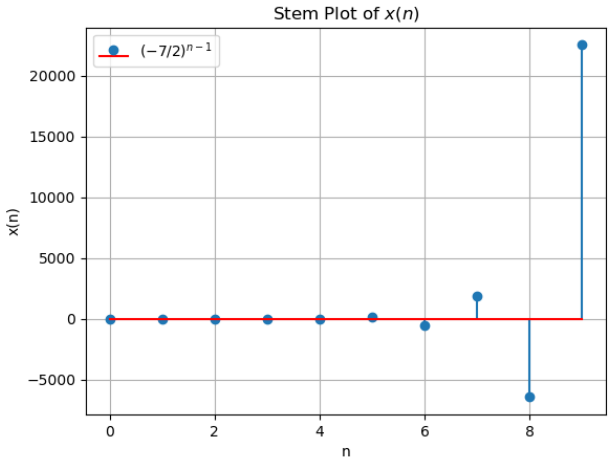

Fig. 1: Stem Plot of x_1(n)

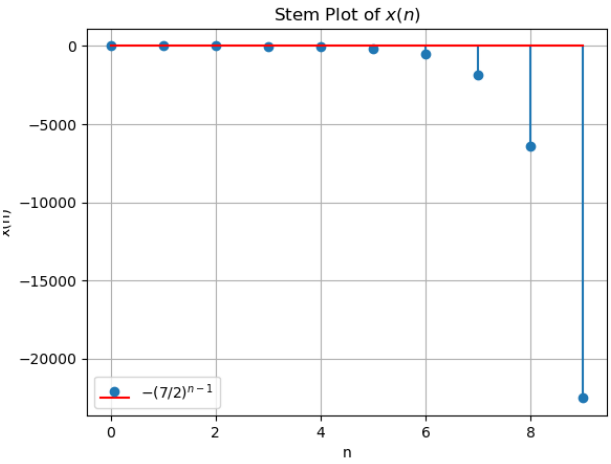

Fig. 2: Stem Plot of $x_2(n)$

Applying z-Transform:

$$\text{ROC}: z \in \left(-\infty, -\frac{7}{2}\right) \cup \left(\frac{7}{2}, \infty\right) \tag{12}$$

$$\implies X_1(z) = \left(\frac{1}{7}\right)\left(\frac{4z}{7+2z}\right) \tag{13}$$

$$\implies X_2(z) = \left(\frac{1}{7}\right)\left(\frac{4z}{7-2z}\right) \tag{14}$$