11.9.3.6

EE23BTECH11022 - G DILIP REDDY

Question:
For what values of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P ?

Solution:
Let a, b, c be any three consecutive terms of a GP whose common difference is r then,

$$\implies \frac{b}{a} = \frac{c}{b} = r \tag{1}$$

$$\implies \frac{x}{(-\frac{2}{7})} = \frac{(-\frac{7}{2})}{x} \tag{2}$$

$$x^2 = (-\frac{2}{7}) \cdot (-\frac{7}{2}) \tag{3}$$

$$x^2 = 1 \tag{4}$$

$$x = 1, -1 \tag{5}$$

$$r = \frac{x}{(-\frac{2}{7})} \tag{6}$$

$$\implies r = \frac{7}{2}\, or - \frac{7}{2} \tag{7}$$

$$T_n = -\frac{2}{7} \cdot (-\frac{7}{2})^n \text{ or } -\frac{2}{7} \cdot (\frac{7}{2})^n \tag{8}$$

$$T_n = (-\frac{7}{2})^{n-1} \text{ or } -(\frac{7}{2})^{n-1} \tag{9}$$

The signal corresponding to this is

$$x(n) = (-\frac{7}{2})^{n-1} u(n) \text{ or } -(\frac{7}{2})^{n-1} u(n) \tag{10}$$

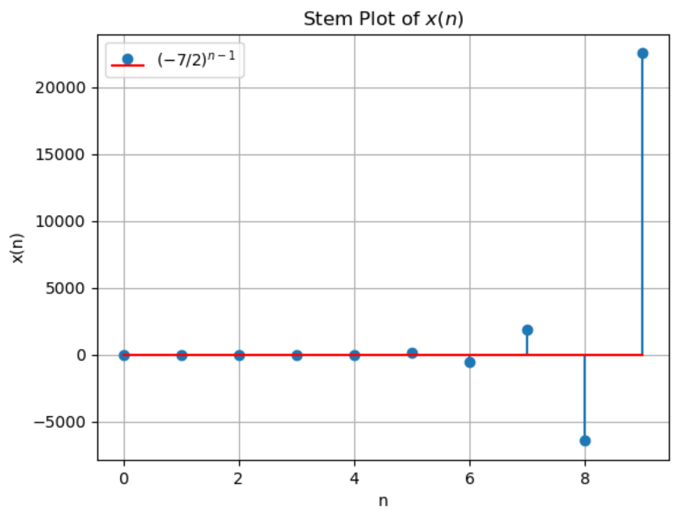

Fig. 1: Stem Plot of x(n)

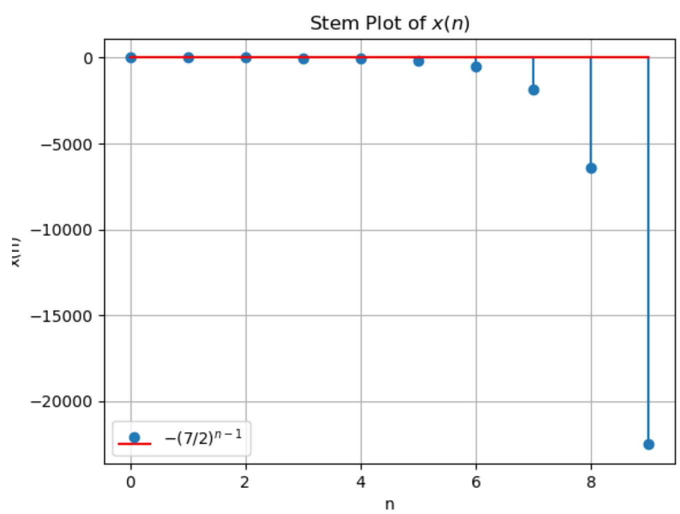

Fig. 1: Stem Plot of x(n)

Applying z-Transform:

case -1 :

$$X(z) = \sum_{n=-\infty}^{\infty}((-\frac{7}{2})^{n-1} u(n))\, z^{-n} \tag{11}$$

$$X(z) = 0 + \sum_{n=0}^{\infty}((-\frac{7}{2})^{n-1})\, z^{-n} \tag{12}$$

$$X(z) = \sum_{n=0}^{\infty}((\ frac27)(-\frac{2}{7})^{-n})\, z^{-n} \tag{13}$$

$$(\frac{2}{7})X(z) = \sum_{n=0}^{\infty}((-\frac{2}{7})^{-n})\, z^{-n} \tag{14}$$

$$(\frac{7}{2})X(z) = \sum_{n=0}^{\infty}((-\frac{2z}{7})^{-n}) \tag{15}$$

For this to converge $|\frac{7}{2z}| < 1$

$$\implies -1 < \frac{7}{2z} < 1 \tag{16}$$

$$z \in (-\infty, -\frac{7}{2}) \cup (\frac{7}{2}, \infty) \tag{17}$$

Multiplying $-\frac{7}{2z}$ on both sides of equation 15

$$-(\frac{7}{2z})(\frac{7}{2})X(z) = \sum_{n=0}^{\infty}((-\frac{2z}{7})^{-(n+1)}) \tag{18}$$

$$-(\frac{7}{2z})(\frac{7}{2})X(z) = \sum_{n=1}^{\infty}((-\frac{2z}{7})^{-n}) \tag{19}$$

$$-(\frac{7}{2z})(\frac{7}{2})X(z) = -(-\frac{2z}{7})^{-0} + \sum_{n=0}^{\infty}((-\frac{2z}{7})^{-n}) \tag{20}$$

$$-(\frac{7}{2z})(\frac{7}{2})X(z) = -1 + \sum_{n=0}^{\infty}((-\frac{2z}{7})^{-n}) \tag{21}$$

Subtracting eqn 15 from eqn 21

$$(\frac{7}{2z} + 1)(\frac{7}{2})X(z) = 1 \tag{22}$$

$$X(z) = (\frac{2}{7})(\frac{2z}{2z+7}) \tag{23}$$

$$\implies X(z) = (\frac{1}{7})(\frac{4z}{2z+7}) \tag{24}$$

case -2 :

$$X(z) = \sum_{n=-\infty}^{\infty}(-(\frac{7}{2})^{n-1} u(n))\, z^{-n} \tag{25}$$

$$X(z) = 0 + \sum_{n=0}^{\infty}-((\frac{7}{2})^{n-1})\, z^{-n} \tag{26}$$

$$X(z) = \sum_{n=1}^{\infty}-((\frac{2}{7})(\frac{2}{7})^{-n})\, z^{-n} \tag{27}$$

$$(\frac{7}{2})X(z) = \sum_{n=0}^{\infty}-((\frac{2}{7})^{-n})\, z^{-n} \tag{28}$$

$$(\frac{7}{2})X(z) = \sum_{n=0}^{\infty}-((\frac{2z}{7})^{-n}) \tag{29}$$

For this to converge $|\frac{7}{2z}| < 1$

$$\implies -1 < \frac{7}{2z} < 1 \tag{30}$$

$$z \in (-\infty, -\frac{7}{2}) \cup (\frac{7}{2}, \infty) \tag{31}$$

Multiplying $\frac{7}{2z}$ on both sides of equation 29

$$(\frac{7}{2z})(\frac{7}{2})X(z) = \sum_{n=0}^{\infty}-((\frac{2z}{7})^{-(n+1)}) \tag{32}$$

$$(\frac{7}{2z})(\frac{7}{2})X(z) = \sum_{n=1}^{\infty}-((\frac{2z}{7})^{-n}) \tag{33}$$

$$(\frac{7}{2z})(\frac{7}{2})X(z) = -(-1)^{-1} + \sum_{n=1}^{\infty}-((\frac{2z}{7})^{-n}) \tag{34}$$

$$(\frac{7}{2z})(\frac{7}{2})X(z) = 1 + \sum_{n=1}^{\infty}-((\frac{2z}{7})^{-n}) \tag{35}$$

Subtracting eqn 29 from eqn 35

$$(\frac{7}{2z} - 1)(\frac{7}{2})X(z) = 1 \tag{36}$$

$$X(z) = (\frac{2}{7})(\frac{2z}{7-2z}) \tag{37}$$

$$\implies X(z) = (\frac{1}{7})(\frac{4z}{7-2z}) \tag{38}$$