11.9.3.6

EE23BTECH11022 - G DILIP REDDY

Question:
For what values of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P ?

Solution:
Let a, b, c be any three consecutive terms of a GP whose common difference is r then,

$$\implies \frac{b}{a} = \frac{c}{b} = r$$
$$\implies \frac{x}{(-\frac{2}{7})} = \frac{(-\frac{7}{2})}{x}$$
$$x^2 = (-\frac{2}{7}) \cdot (-\frac{7}{2})$$
$$x^2 = 1$$
$$x = 1, -1$$
$$r = \frac{x}{(-\frac{2}{7})}$$
$$\implies r = \frac{7}{2} \, or - \frac{7}{2}$$

Assuming $-\frac{2}{7}$ as the first term of the GP, The possible general term of the GP are

$$T_n = -\frac{2}{7} \cdot (-\frac{7}{2})^{n-1} \text{ or } -\frac{2}{7} \cdot (\frac{7}{2})^{n-1}$$
$$T_n = (-\frac{7}{2})^{n-2} \text{ or } -(\frac{7}{2})^{n-2}$$

The signal corresponding to this is

$$x(n) = (-\frac{7}{2})^{n-2} u(n) \text{ or } -(\frac{7}{2})^{n-2} u(n)$$

Applying z-Transform:

case -1 :

$$X(z) = \sum_{n=-\infty}^{\infty} ((-\frac{7}{2})^{n-2} u(n)) z^{-n}$$
$$X(z) = 0 + \sum_{n=1}^{\infty} ((-\frac{7}{2})^{n-2}) z^{-n}$$
$$X(z) = \sum_{n=1}^{\infty} ((\frac{4}{49})(-\frac{2}{7})^{-n}) z^{-n}$$
$$(\frac{49}{4})X(z) = \sum_{n=1}^{\infty} ((-\frac{2}{7})^{-n}) z^{-n}$$
$$(\frac{49}{4})X(z) = \sum_{n=1}^{\infty} ((-\frac{2z}{7})^{-n}) \tag{1}$$

For this to converge $|\frac{7}{2z}| < 1$

$$\implies -1 < \frac{7}{2z} < 1$$
$$z \in (-\infty, -\frac{7}{2}) \cup (\frac{7}{2}, \infty)$$

Multiplying $-\frac{7}{2z}$ on both sides of equation 1

$$-(\frac{7}{2z})(\frac{49}{4})X(z) = \sum_{n=1}^{\infty} ((-\frac{2z}{7})^{-(n+1)})$$
$$-(\frac{7}{2z})(\frac{49}{4})X(z) = \sum_{n=2}^{\infty} ((-\frac{2z}{7})^{-n})$$
$$-(\frac{7}{2z})(\frac{49}{4})X(z) = -(-\frac{2z}{7})^{-1} + \sum_{n=1}^{\infty} ((-\frac{2z}{7})^{-n})$$
$$-(\frac{7}{2z})(\frac{49}{4})X(z) = (\frac{7}{2z}) + \sum_{n=1}^{\infty} ((-\frac{2z}{7})^{-n}) \tag{2}$$

Subtracting eqn 1 from eqn 2

$$-(\frac{7}{2z} + 1)(\frac{49}{4})X(z) = (\frac{7}{2z})$$

$$X(z) = -(\frac{4}{49})(\frac{7}{2z})(\frac{2z}{2z+7})$$
$$\implies X(z) = -(\frac{4}{7})(\frac{1}{2z+7}) \tag{3}$$

case -2 :

$$X(z) = \sum_{n=-\infty}^{\infty} (-(\frac{7}{2})^{n-2}\, u(n)\,)\, z^{-n}$$

$$X(z) = 0 + \sum_{n=1}^{\infty} -((\frac{7}{2})^{n-2})\, z^{-n}$$

$$X(z) = \sum_{n=1}^{\infty} -((\frac{4}{49})(\frac{2}{7})^{-n})\, z^{-n}$$

$$(\frac{49}{4})X(z) = \sum_{n=1}^{\infty} -((\frac{2}{7})^{-n})\, z^{-n}$$

$$(\frac{49}{4})X(z) = \sum_{n=1}^{\infty} -((\frac{2z}{7})^{-n}) \tag{4}$$

For this to converge $|\frac{7}{2z}| < 1$

$$\implies -1 < \frac{7}{2z} < 1$$

$$z \in (-\infty, -\frac{7}{2}) \cup (\frac{7}{2}, \infty)$$

Multiplying $\frac{7}{2z}$ on both sides of equation 4

$$(\frac{7}{2z})(\frac{49}{4})X(z) = \sum_{n=1}^{\infty} -((\frac{2z}{7})^{-(n+1)})$$

$$(\frac{7}{2z})(\frac{49}{4})X(z) = \sum_{n=2}^{\infty} -((\frac{2z}{7})^{-n})$$

$$(\frac{7}{2z})(\frac{49}{4})X(z) = -(-\frac{2z}{7})^{-1} + \sum_{n=1}^{\infty} -((\frac{2z}{7})^{-n})$$

$$(\frac{7}{2z})(\frac{49}{4})X(z) = (\frac{7}{2z}) + \sum_{n=1}^{\infty} -((\frac{2z}{7})^{-n}) \tag{5}$$

Subtracting eqn 4 from eqn 5

$$(\frac{7}{2z} - 1)(\frac{49}{4})X(z) = (\frac{7}{2z})$$

$$X(z) = (\frac{4}{49})(\frac{7}{2z})(\frac{2z}{7-2z})$$

$$\implies X(z) = (\frac{4}{7})(\frac{1}{7-2z}) \tag{6}$$