

Day- 80, Feb 18, 2025 (Falgun 07, 2081)

# Motivation to Derivatives -

$$\text{Velocity} = \frac{\text{distance traveled}}{\text{time}}$$

$$\left. \begin{array}{l} \text{velocity} \Rightarrow 10 - 15 \text{ seconds} \\ \text{between} \end{array} \right\} -$$

$$\text{velocity} = \frac{y(15) - y(10)}{5}$$

$$\Rightarrow \frac{202 - 122}{5}$$

$$\Rightarrow 16 \text{ m/s}$$

These are given values.

Slope =  $\frac{\text{rise}}{\text{run}}$

$\Rightarrow \frac{\text{change in distance } (\Delta x)}{\text{Change in time } (\Delta t)}$

$$\Rightarrow \frac{x(15) - x(10)}{t(15) - t(10)}$$

$$\Rightarrow \frac{202\text{m} - 122\text{m}}{15\text{s} - 10\text{s}}$$

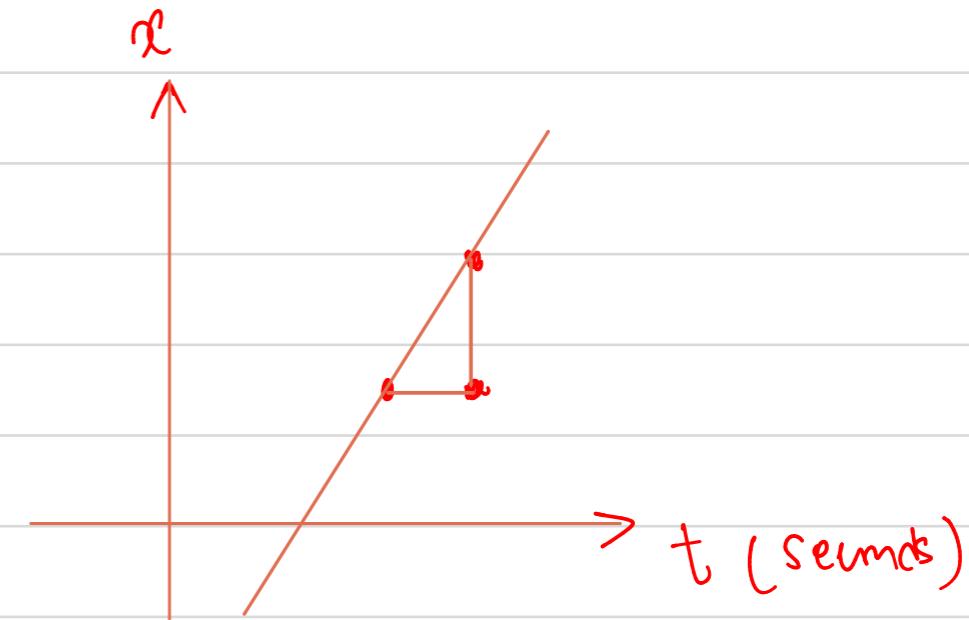
$$= \frac{80\text{m}}{5\text{s}} \quad \boxed{\text{Slope} \approx 16\text{m/s}}$$

$t = 12.5$  seconds

$t = 12$  to  $t = 13$

$$y(13) - y(12) \mid_1$$

$$\Rightarrow 0.5\text{m/s}$$



Q8 [The velocity is  $16\text{m/s}$ ]

$t(\text{s})$	$x(\text{m})$
10	122
11	138
12	155
13	170
14	186
15	202

$$\text{Slope} = \frac{\Delta x}{\Delta y}$$

so,

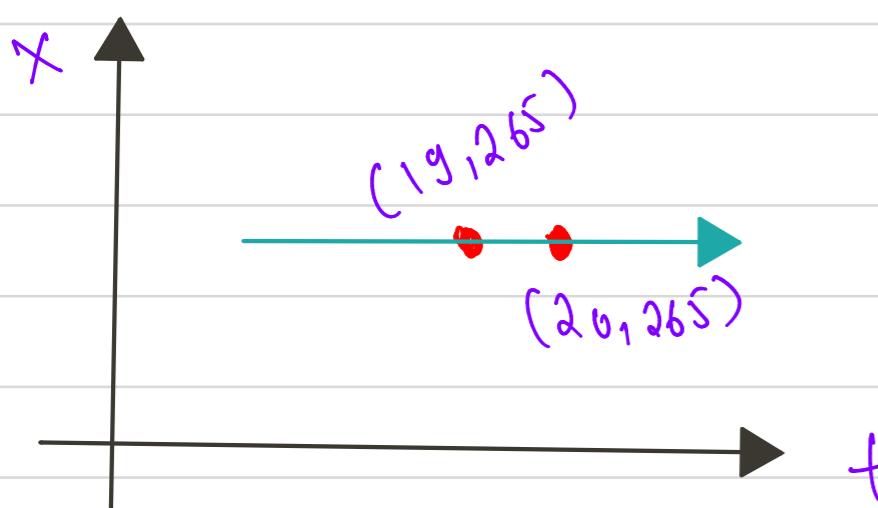
$$\frac{dx}{dt}$$

We call instantaneous Rate of change  $\rightarrow$

moving tiny tiny  $dx$  in tiny tiny time

Derivative =  $\frac{dx}{dt}$  tiny tiny changes at that point.

No Slope



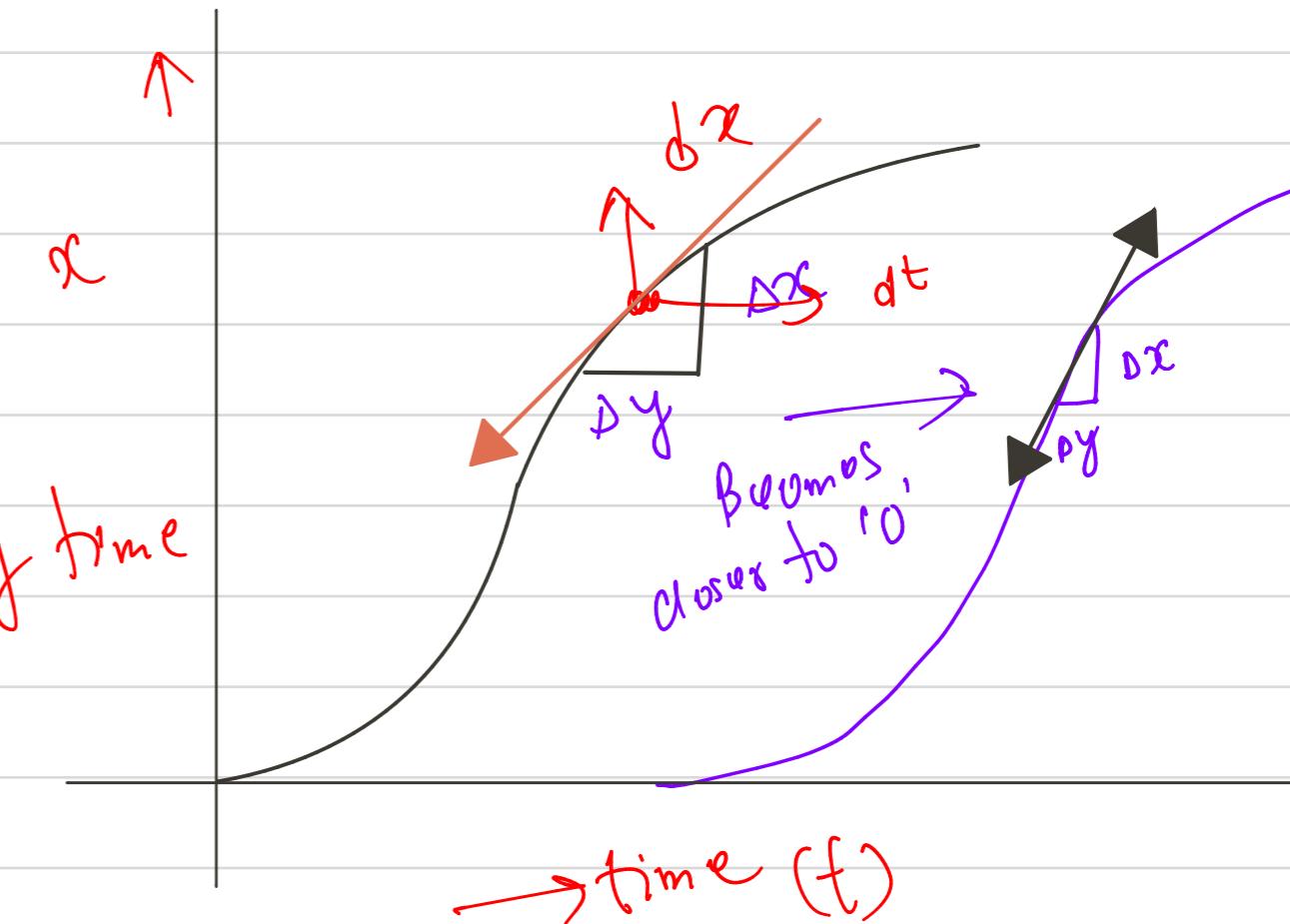
$$\text{Slope} = \frac{\Delta x}{\Delta t}$$

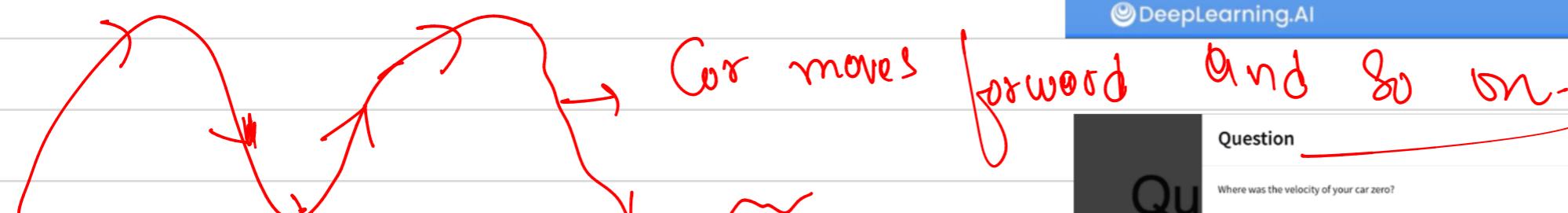
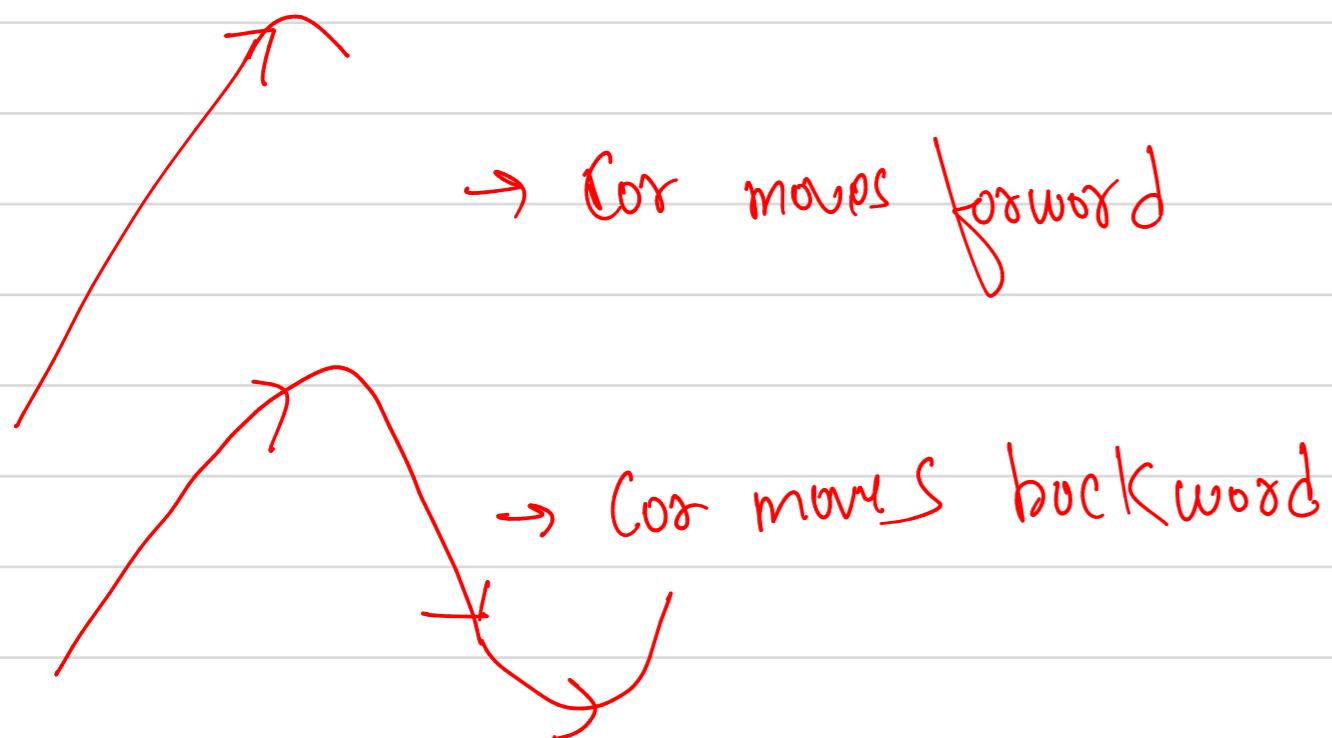
$$\Rightarrow \frac{x(20) - x(19)}{20 - 19}$$

$$= \frac{265 - 265}{1}$$

$$\left[ \text{Slope} = 0\% \right] \rightarrow$$

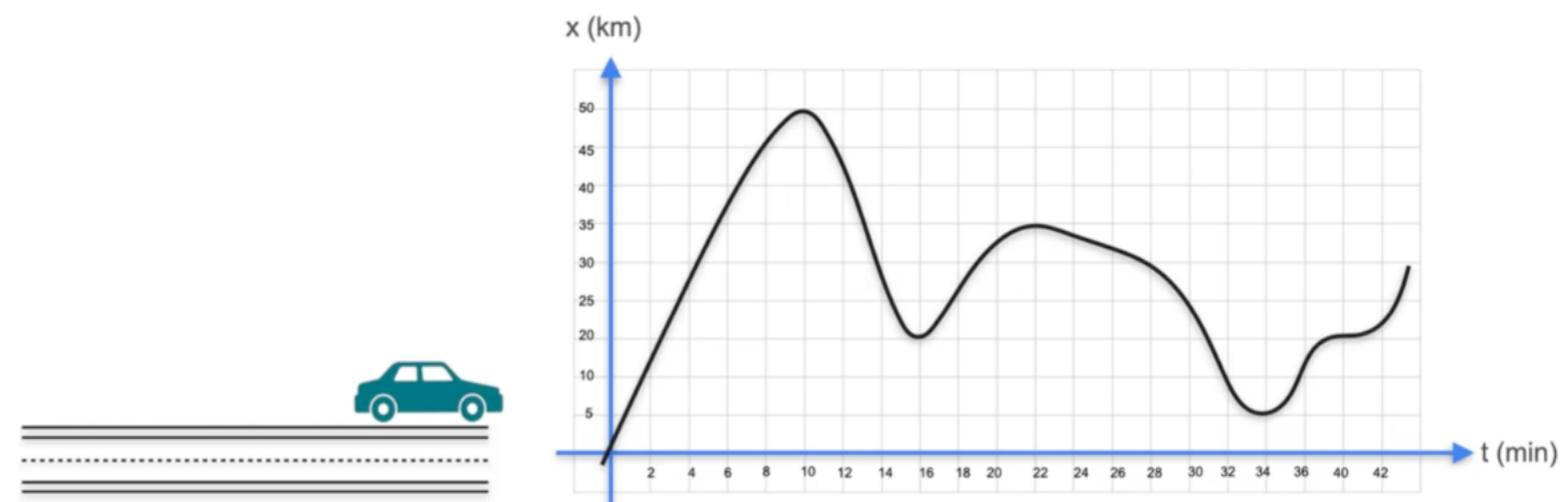
No rise in  $\Delta x$   
Zero slope.





→ Find the Horizontal Slope where  
the velocity is  $\approx 0$  ?

## Minima and Maxima



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Qu

Question

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## Quiz : Concept of Derivatives 3

# Where tangent line is horizontal  
it can be max or min?

# local Maxima

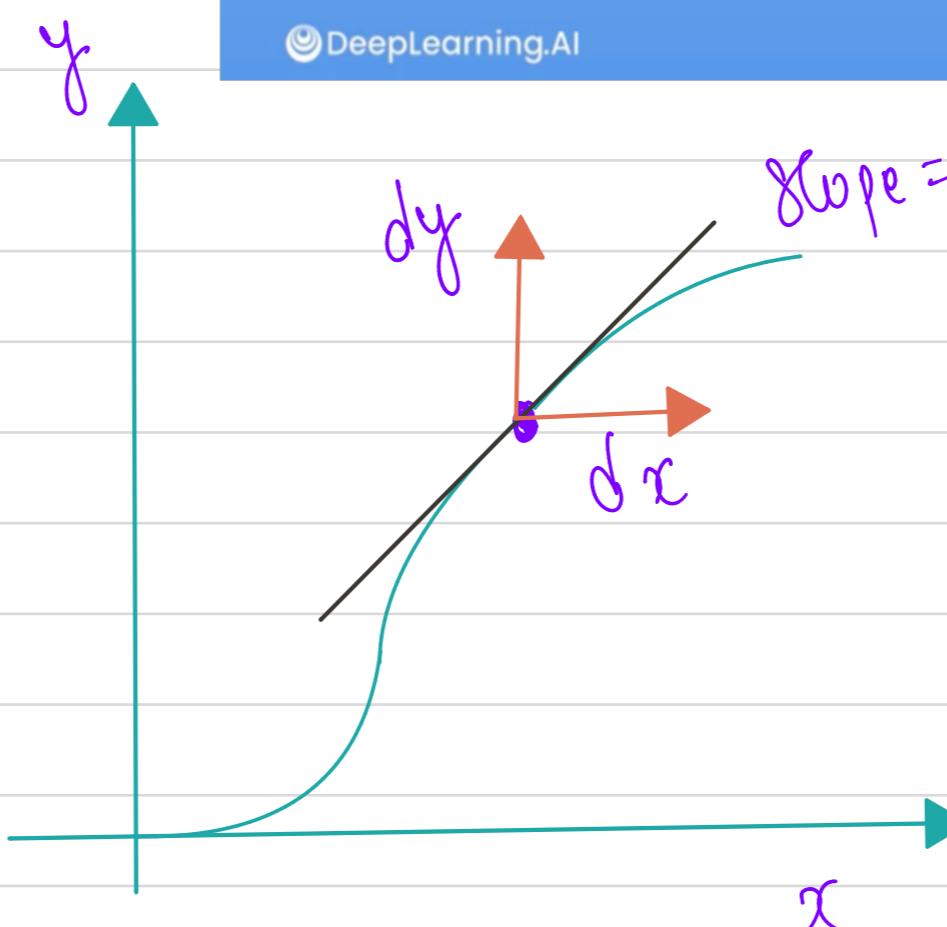
# local Minima

# Derivatives!

$$\text{Slope} = \frac{dx}{dt}$$

$$\text{Slope at a point} \Rightarrow \frac{dx}{dt}$$

$$y = f(x)$$

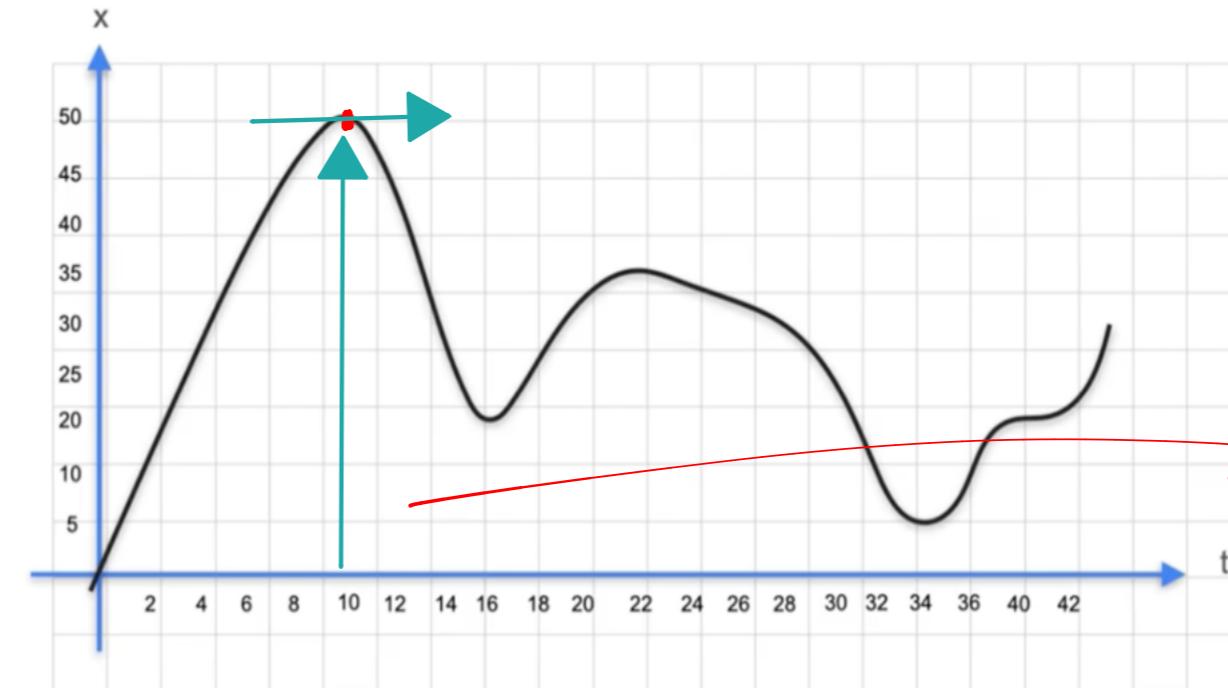


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$\rightarrow$  Lagrange's And Leibniz's Notation:

$f'(x) \rightarrow$  Lagrange's

$\frac{dy}{dx} = \frac{d}{dx} f(x) \rightarrow$  Leibniz's Notation



At what time was the car farthest from its starting point?

$\rightarrow$  Farhest  
 $\rightarrow$  Highest Distance:  
 $\rightarrow$  point where Car stopped:

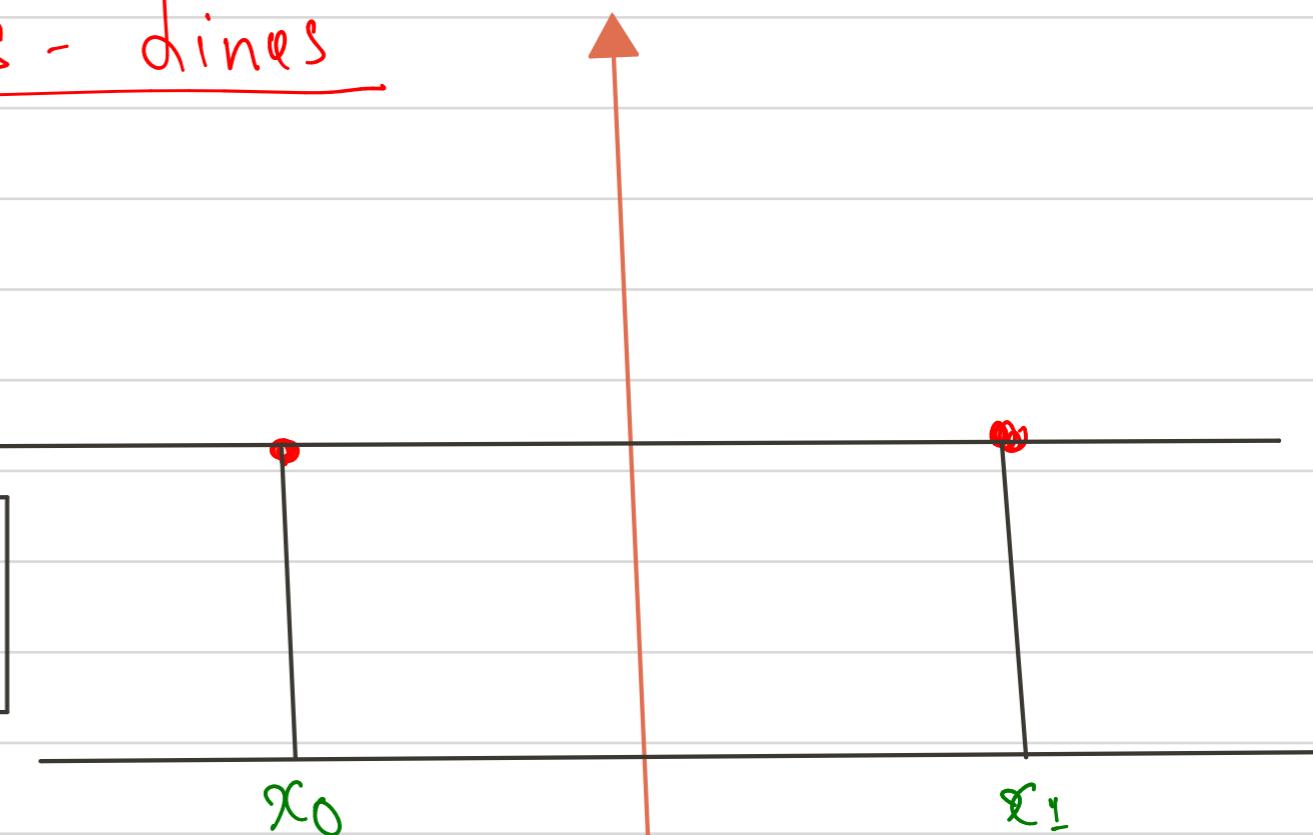
## # Derivatives - Some Common Derivatives - lines

$$y = f(x) = C$$

Slope?

$$\frac{\Delta y}{\Delta x} \Rightarrow \frac{C - C}{x_1 - x_0} \Rightarrow 0$$

$$f'(x) = 0$$

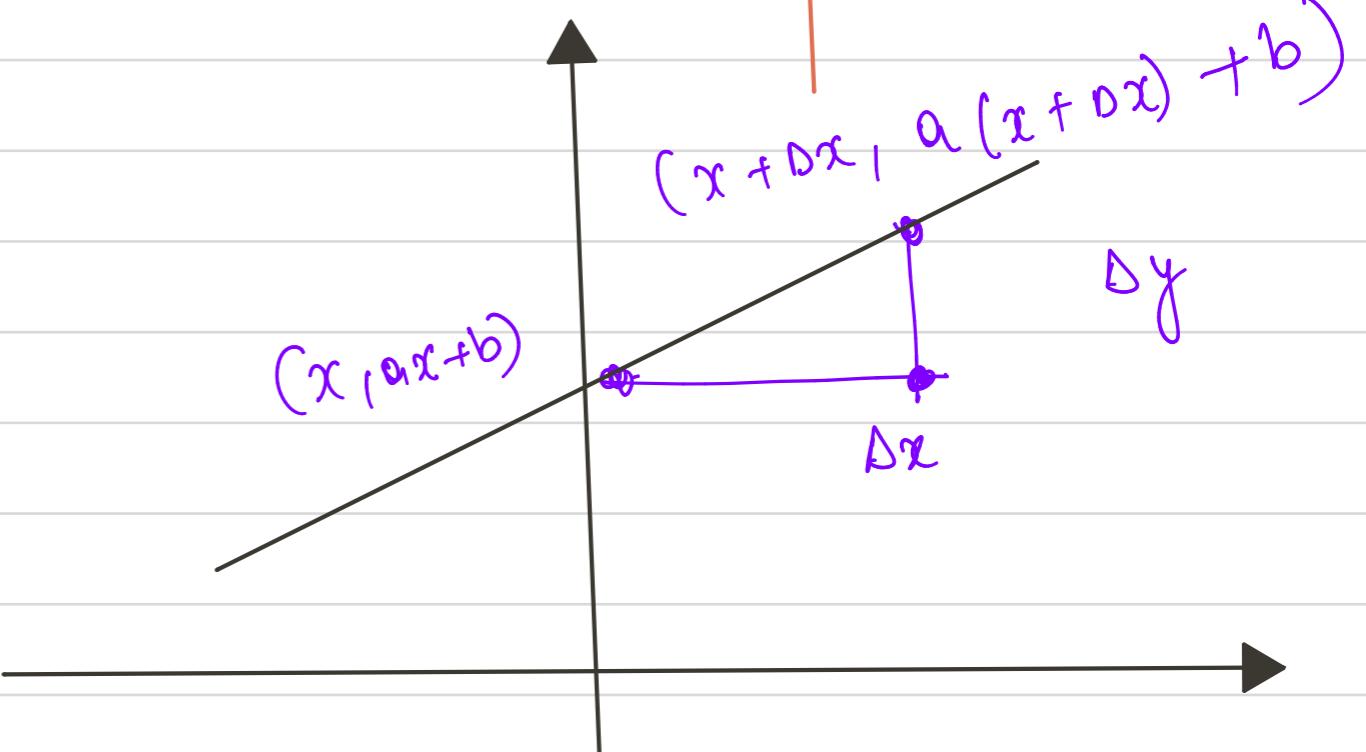


$$f(x) = ax + b$$

$$f'(x) \Rightarrow a$$

$$\frac{\Delta y}{\Delta x} \Rightarrow \frac{\text{rise}}{\text{run}}$$

$$\frac{\Delta y}{\Delta x} = \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x}$$



$$\frac{df}{dx} \cdot \frac{dy}{dx} \Rightarrow \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x}$$

$$\Rightarrow a \cdot \frac{\Delta x}{\Delta x}$$

$$\left[ \frac{df}{dx} \Rightarrow a \right]$$

## # Derivative of Quadratic functions

Quadratics:  $y = f(x) \rightarrow x^2$

Slope:  $\frac{df}{dx} = \frac{(x + \Delta x)^2 - (x)^2}{\Delta x}$

$x = 1$	$\Rightarrow$	$\Delta x$	$+0.1$	$+0.2$	$+0.4$	$+0.8$	$+1.6$	$+3.2$	$\dots$	$+1000$
		$Df$	3	2.25	0.562	0.265	0.128	-	-	0.002
		Slope	3	2.25	2.25	2.125	2.065	-	-	2.001

Moving towards to the 2

$$f'(1) \Rightarrow \frac{d}{dx} f(x)$$

$$\Rightarrow 2$$

$$\Rightarrow 2x^1.$$

$$y = f(x) = x^2$$

$$\frac{df}{dx} \Rightarrow \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= 1 \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$\Rightarrow x^2 + 2 \Delta x \cdot x + (\Delta x)^2 - x^2$$

$$= 2x + \Delta x$$

as  $\Delta x \rightarrow 0$

$$f(x) \Rightarrow x^2 \Rightarrow f'(x) \Rightarrow 2x$$

# dx of Cubic functions:

Cubic:  $y = x^3$

Slope  $\Rightarrow \frac{df}{dx} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

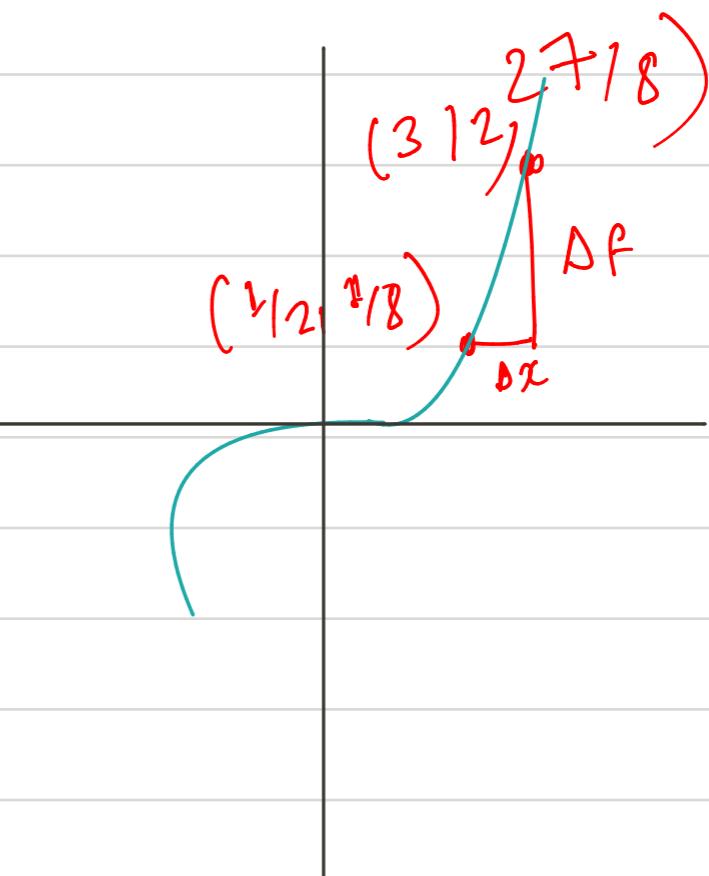
$$\Delta x = 1.0$$

$$df = 3.25$$

$$\left(\frac{1}{2} + 1\right)^3 - \left(1\right)^3 = 27/8 - 1/8$$

$$\Rightarrow \frac{3.25}{1}$$

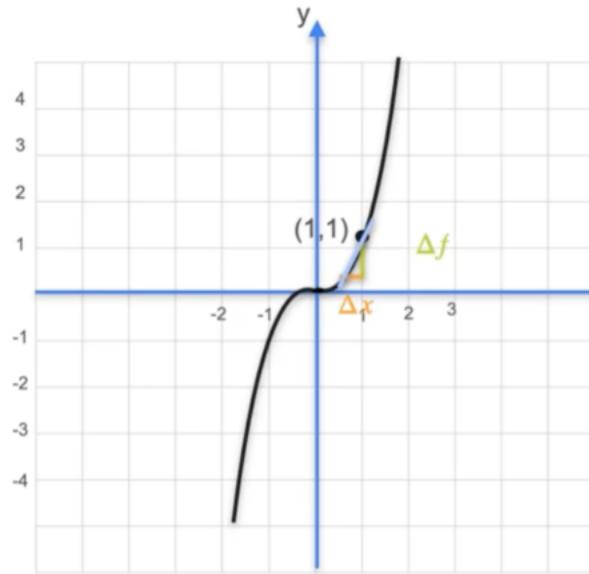
If  $x = 0.5$



$$f'(0.5) = \frac{d}{dx} f(0.5) \Rightarrow (0.75) \\ \Rightarrow 3 \times (0.5)^2$$

## Derivative of Cubic Functions

1

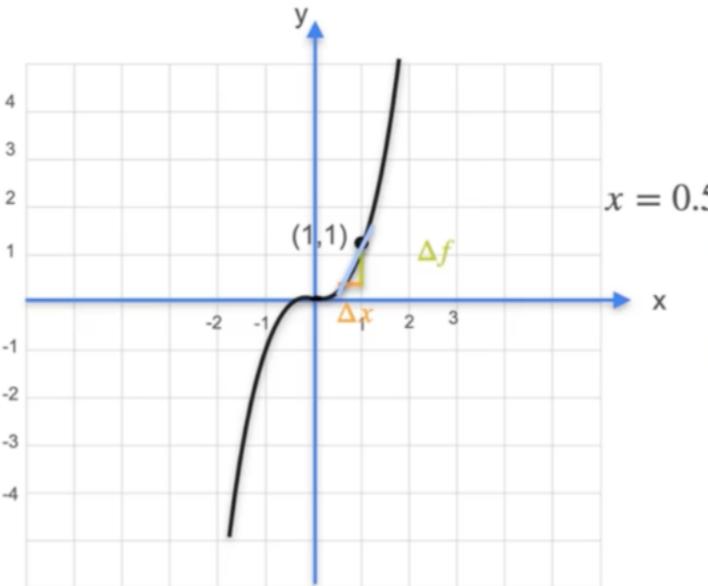


Cubic:  $y = f(x) = x^3$   
 Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	1/2
Δf	3.25	0.86
Slope	3.25	1.75

## Derivative of Cubic Functions

2

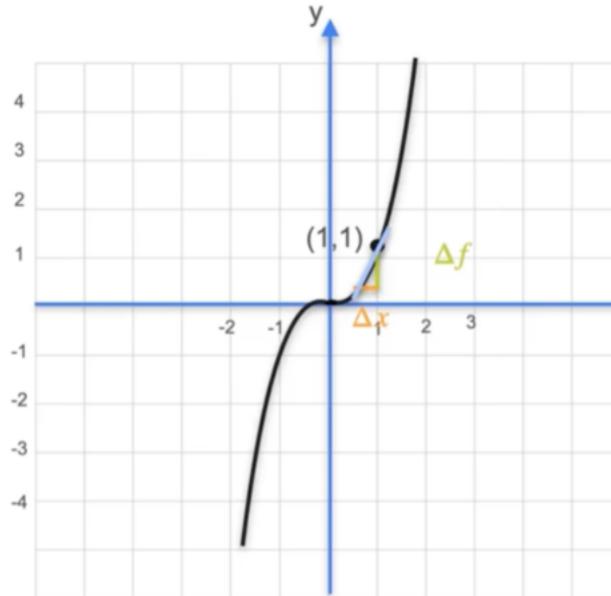


Cubic:  $y = f(x) = x^3$   
 Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	1/2	1/4
Δf	3.25	0.86	0.30
Slope	3.25	1.75	1.188

## Derivative of Cubic Functions

3

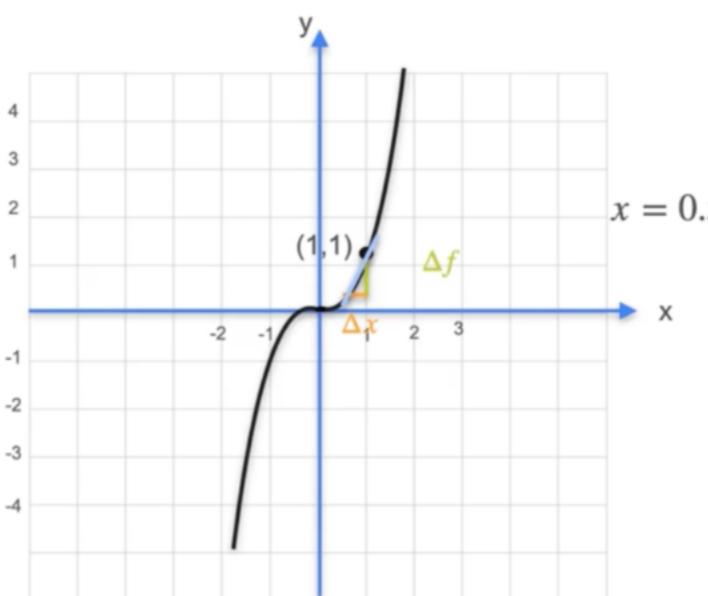


Cubic:  $y = f(x) = x^3$   
 Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16
Δf	3.25	0.86	0.30	0.12	
Slope	3.25	1.75	1.188	0.95	

## Derivative of Cubic Functions

4



Cubic:  $y = f(x) = x^3$   
 Slope:  $\frac{\Delta f}{\Delta x} = \frac{(x + \Delta x)^3 - (x)^3}{\Delta x}$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	3.25	0.86	0.30	0.12	0.05	0.0008
Slope	3.25	1.75	1.188	0.95	0.85	0.752

$$\text{Cubic: } y = f(x) \Rightarrow x^3$$

$$\text{Slope: } \frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\begin{aligned}\frac{\Delta f}{\Delta x} &\approx \frac{(x + \Delta x)^3 - x^3}{\Delta x} \\ &= \frac{x^3 + 3x(\Delta x)^2 + 3x^2 \cancel{\Delta x} + (\Delta x)^3 - x^3}{\Delta x} \\ &\Rightarrow 3x \cancel{\Delta x} + 3x^2 \cancel{\Delta x} \quad \text{As } \Delta x \rightarrow 0\end{aligned}$$

$$f(x) \approx x^3$$

$$f'(x) \Rightarrow 3x^2$$

## Some Common Derivatives - Other Power Functions:

Inverse:  $y = f(x)$

$$\Rightarrow \frac{1}{x} \\ \Rightarrow x^{-1}$$

$$f'(1) = \frac{d}{dx} f(1)$$

$$\Rightarrow -1 \times (1)^{-2}$$

$$f'(1) \Rightarrow -1$$

Slope:  $\frac{\Delta f}{\Delta x} \Rightarrow \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

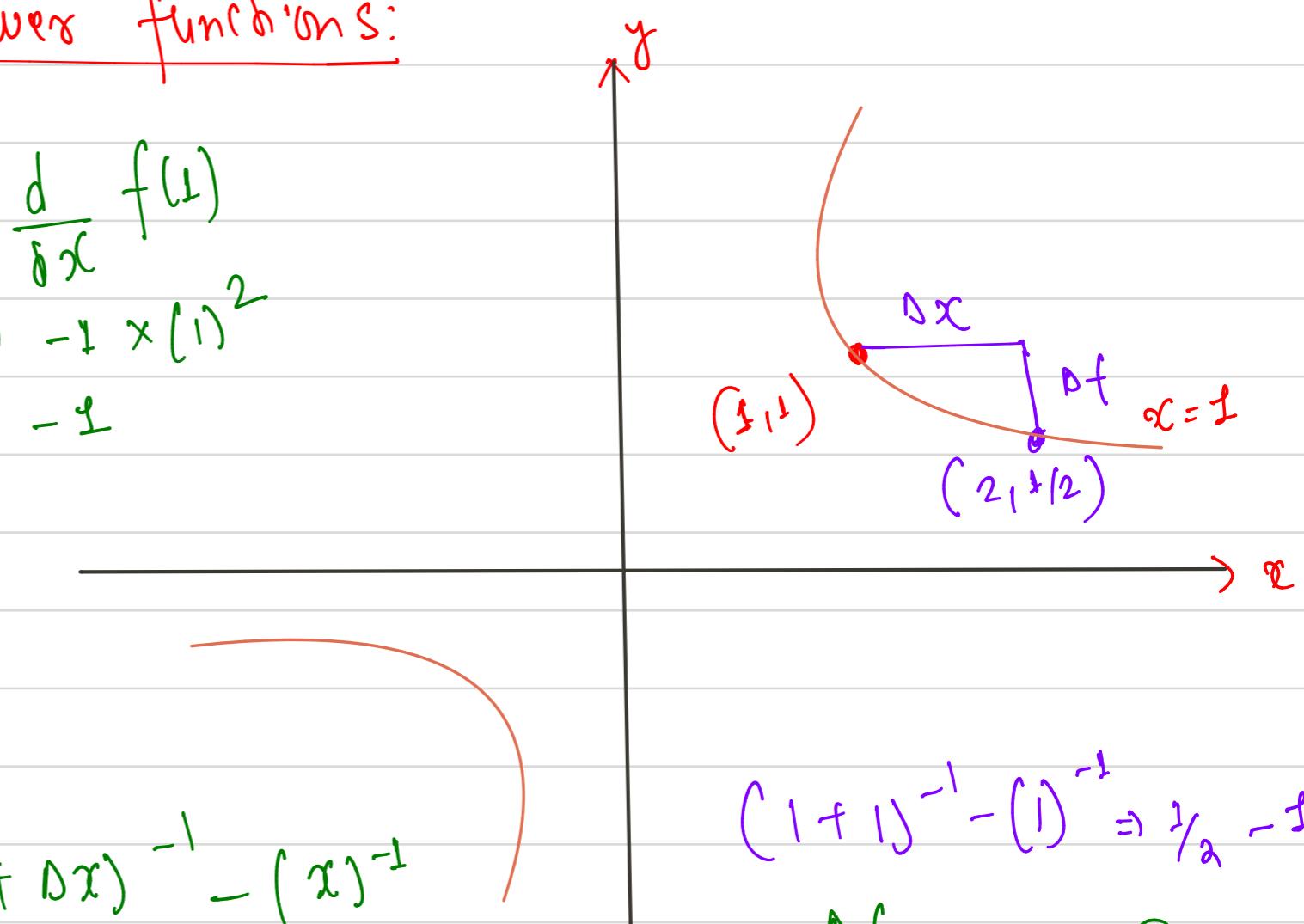
When  $\Delta x = \frac{1}{2}$  then  $\Delta f \Rightarrow (x + \Delta x)^{-1} - (x)^{-1}$

$$\Delta f \Rightarrow (1 + 0.5)^{-1} - (1)^{-1}$$

$$\Rightarrow 2/3^{-1}$$

$$\Delta f \Rightarrow -0.33$$

Then  $\frac{-0.33}{0.5} \Rightarrow -0.67$



$$(1 + 1)^{-1} - (1)^{-1} \Rightarrow \frac{1}{2} - 1$$

$$\frac{\Delta f}{\Delta x} = -0.5$$

$\Delta x$	$\Delta f$	Slope	Converges to
1.0	-0.5	-0.5	-1
1/2	-0.33	-0.33	-0.5
1/4	-0.2	-0.2	-0.33
1/8	-0.125	-0.125	-0.2
1/16	-0.09375	-0.09375	-0.125
1/32	-0.0625	-0.0625	-0.09375
1/64	-0.046875	-0.046875	-0.0625
1/128	-0.03125	-0.03125	-0.046875
1/256	-0.0234375	-0.0234375	-0.03125
1/512	-0.015625	-0.015625	-0.0234375
1/1024	-0.0115234375	-0.0115234375	-0.015625
1/2048	-0.0078125	-0.0078125	-0.0115234375
1/4096	-0.00537109375	-0.00537109375	-0.0078125
1/8192	-0.00390625	-0.00390625	-0.00537109375
1/16384	-0.00265625	-0.00265625	-0.00390625
1/32768	-0.00184375	-0.00184375	-0.00265625
1/65536	-0.0013125	-0.0013125	-0.00184375
1/131072	-0.0009375	-0.0009375	-0.0013125
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$$\text{slope } \frac{df}{dx} = \frac{f(x + dx) - f(x)}{dx}$$

$$\Rightarrow \frac{(x+dx)^{-1} - x^{-1}}{dx}$$

$$\Rightarrow \frac{\frac{1}{x+dx} - \frac{1}{x}}{dx}$$

$$\Rightarrow \frac{x - (x - dx)}{x(x+dx)}$$

$$\Rightarrow -\frac{1}{x^2 + x \cdot dx}$$

$$\Rightarrow -\frac{1}{x^2}$$

$$\Rightarrow -x^{-2}$$

$$\begin{aligned} \frac{d}{dx}(x^n) &= nx^{n-1} \\ \therefore f(x) &= x^{-1} \\ f'(x) &\Rightarrow -x^{-2} \end{aligned}$$

## Differentiation of Power functions

$$f(x) = x^2$$

$$f'(x) = 2x^1$$

$$f(x) = x^3$$

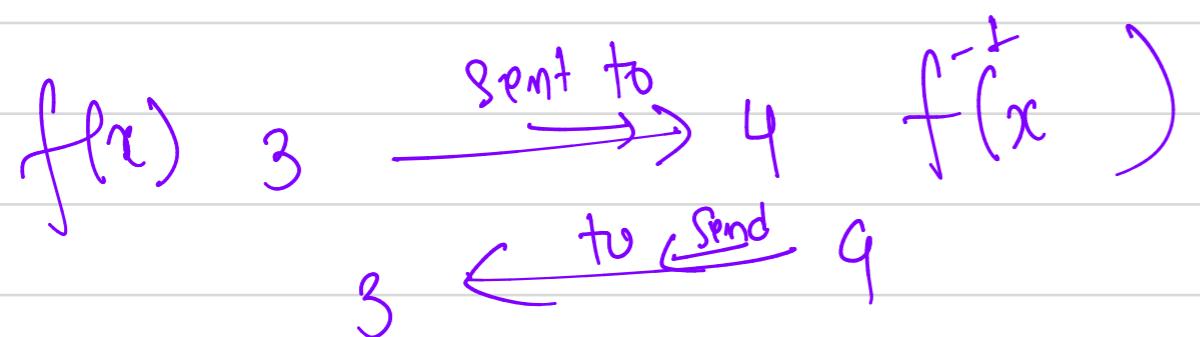
$$f'(x) = 3x^2$$

$$f(x) = x^{-1}$$

$$f'(x) = (-1)x^{-2}$$

True for  
 $\frac{d}{dx}(x^n) = nx^{n-1}$

## # Inverse Function And Its derivative



$$\begin{matrix} x & \xrightarrow{f} & x^2 & \xrightarrow{g} & x \\ & & x^2 & \xrightarrow{g(x)} & \sqrt{x} \end{matrix}$$

$g(x)$  and  $f(x)$  are inverse's

$$g(x) = f^{-1}(x)$$

$$g(f(x)) = x$$

$$\sqrt{x^2} = x \quad \text{for } x \geq 0$$

## # Derivative of Inverse:

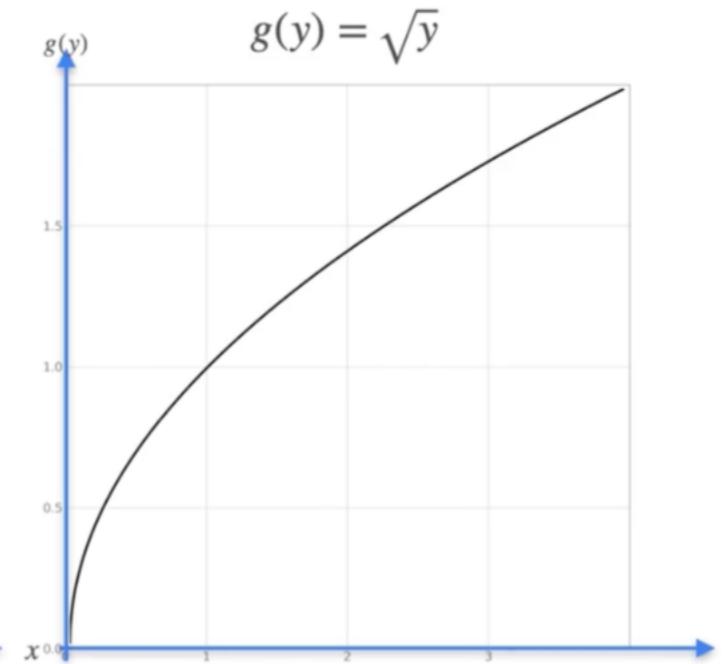
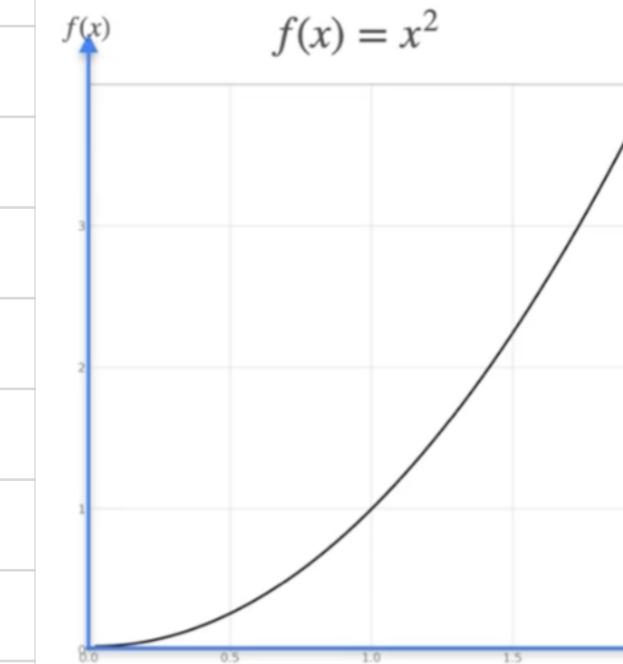
$$f(\frac{1}{2}) \Rightarrow \frac{1}{4}$$

$$g(f(\frac{1}{2})) = g(\frac{1}{4}) \\ \Rightarrow \frac{1}{2}$$

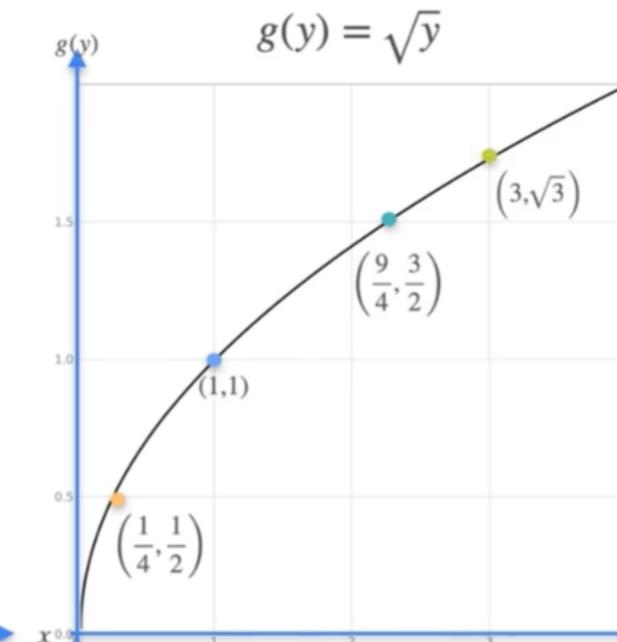
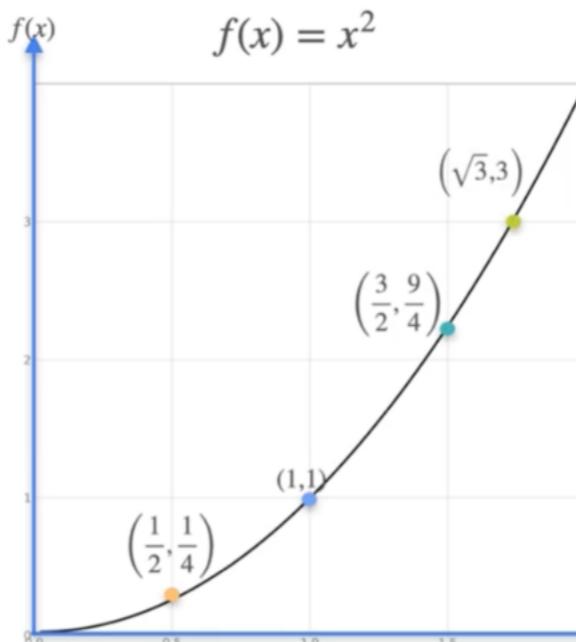
$$f(1) \Rightarrow 1$$

$$g(f(1)) \Rightarrow g(1) \\ \Rightarrow 1$$

## Derivative of the Inverse



## Derivative of the Inverse



$$f(1/2) = 1/4$$

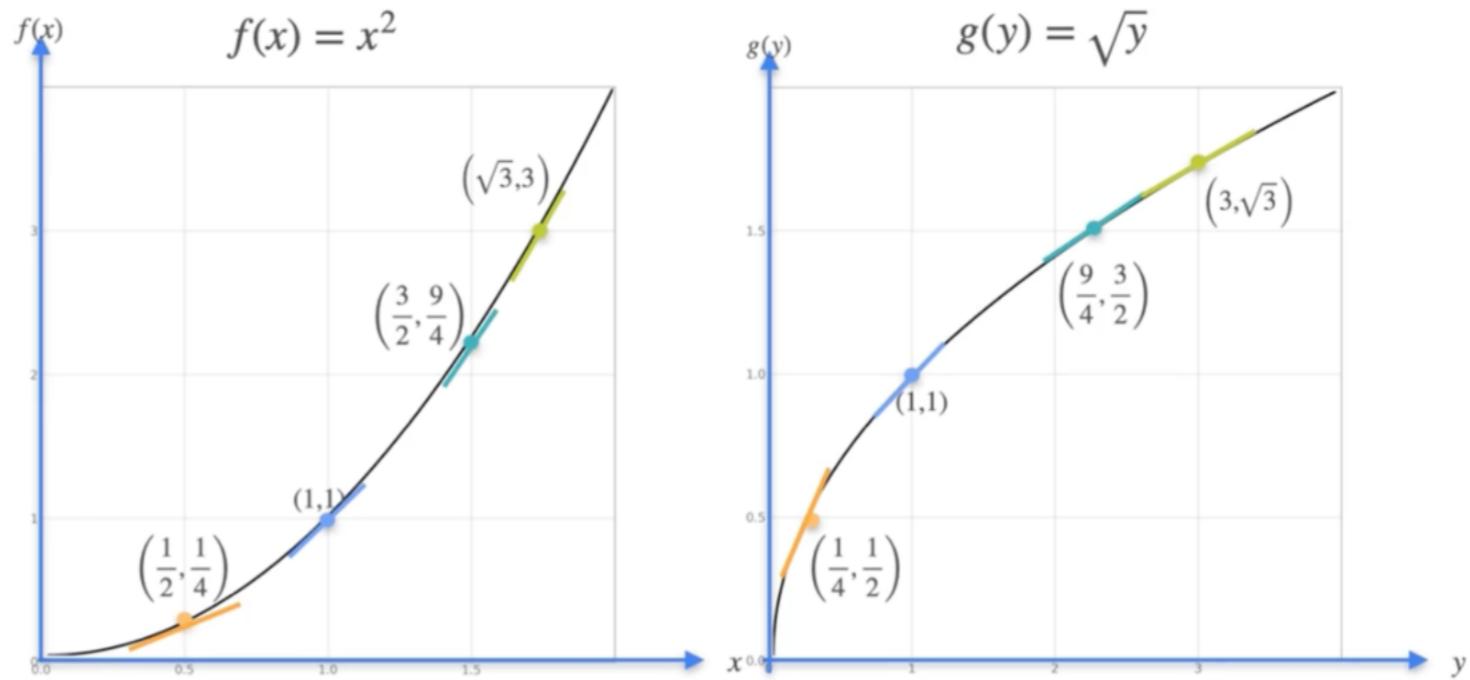
$$g(f(1/2)) = g(1/4) \\ = 1/2$$

$$f(1) = 1$$

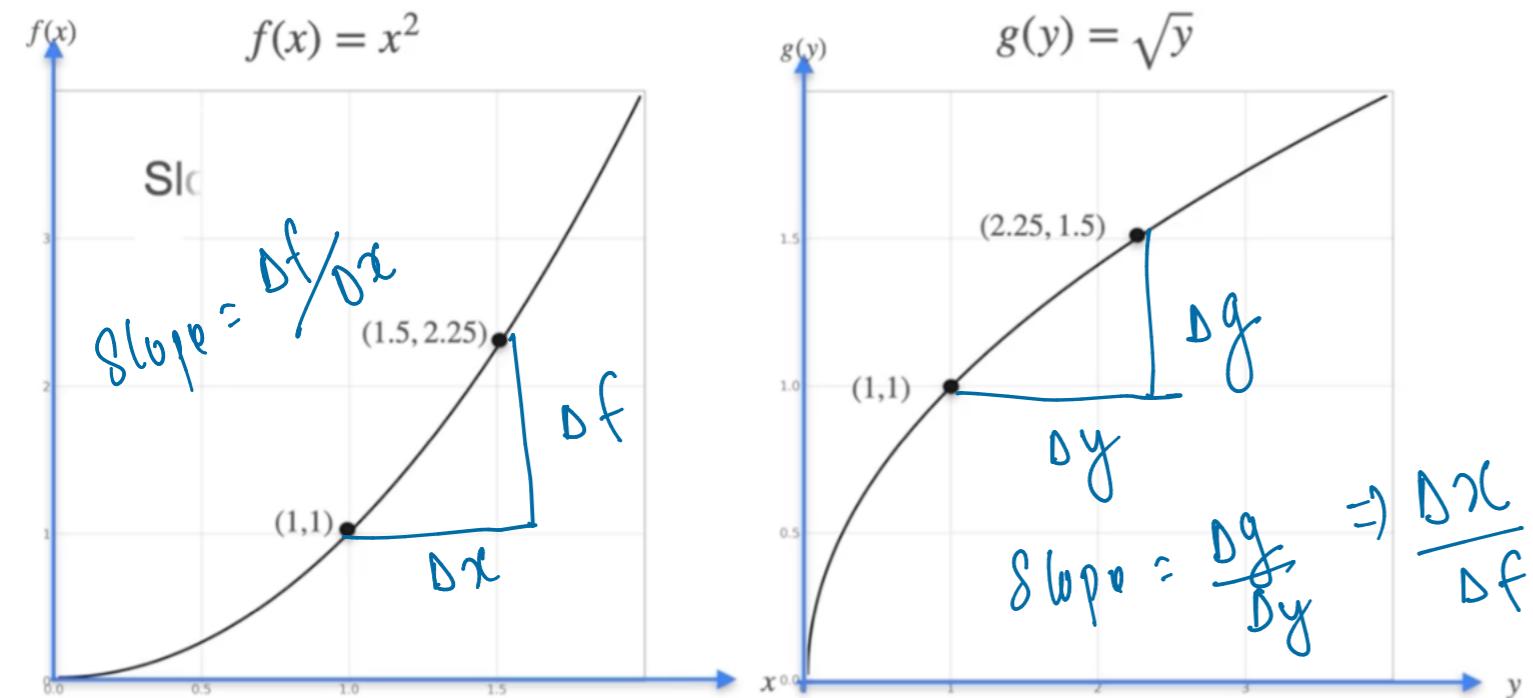
$$g(f(1)) = g(1) \\ = 1$$

## Derivative of the Inverse

$f(x) = x^2$   
 $g(y) = \sqrt{y}$   
 Both <sup>even</sup> slopes ↑ have interesting  
 Potentials if we observe closer.



## Derivative of the Inverse



Since both are reflections of each other

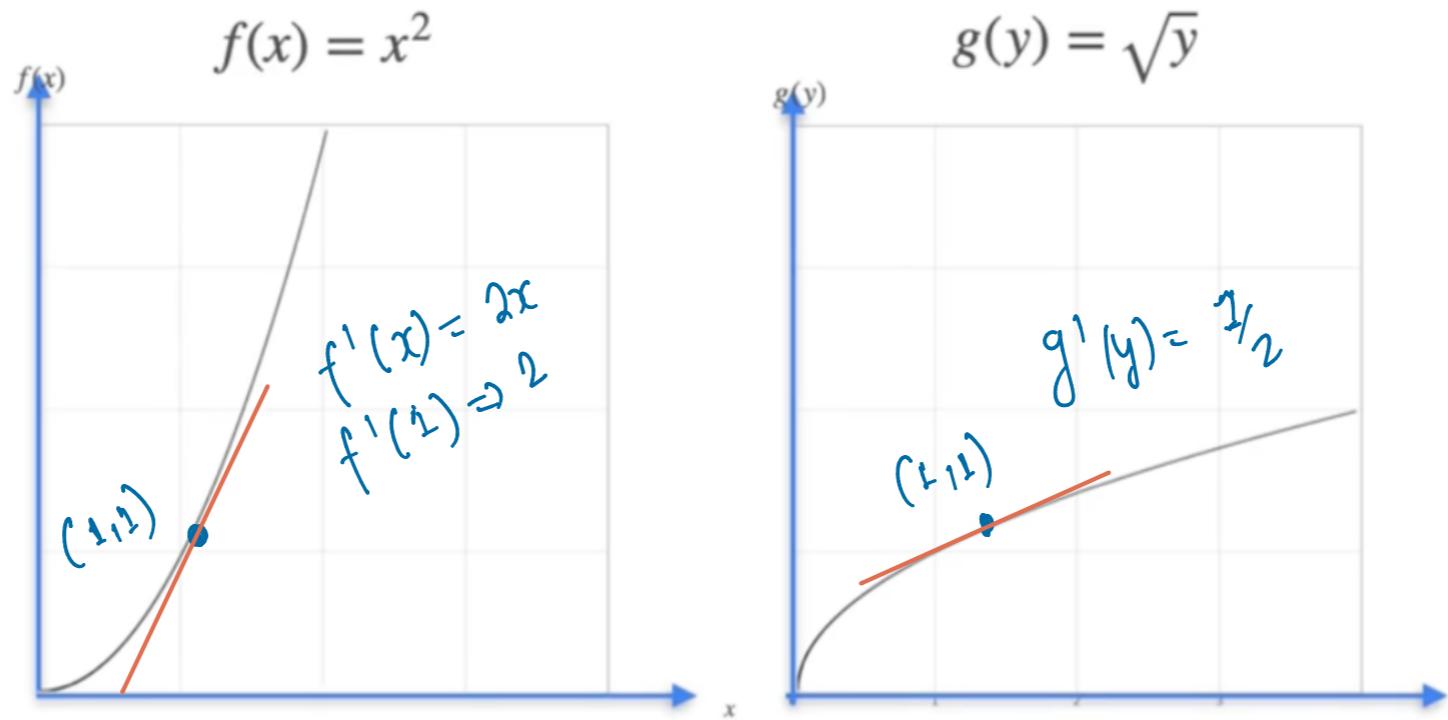
$$\frac{df}{dx} = f'(x)$$

$$\frac{dg}{dy} = g'(y)$$

$$g'(y) = \frac{1}{f'(x)}$$

If  $f$  and  $g$  are inverse functions, then the derivative of  $g$  is  $\frac{1}{f'(x)}$  over the derivative of  $x$ .

## Derivative of the Inverse



$$g'(y) = \frac{1}{f'(x)}$$

at the point  $(1,1)$

$$f(1) = 1$$

$$g(1) = 1$$

$$g'(1) \Rightarrow \frac{1}{f'(1)}$$

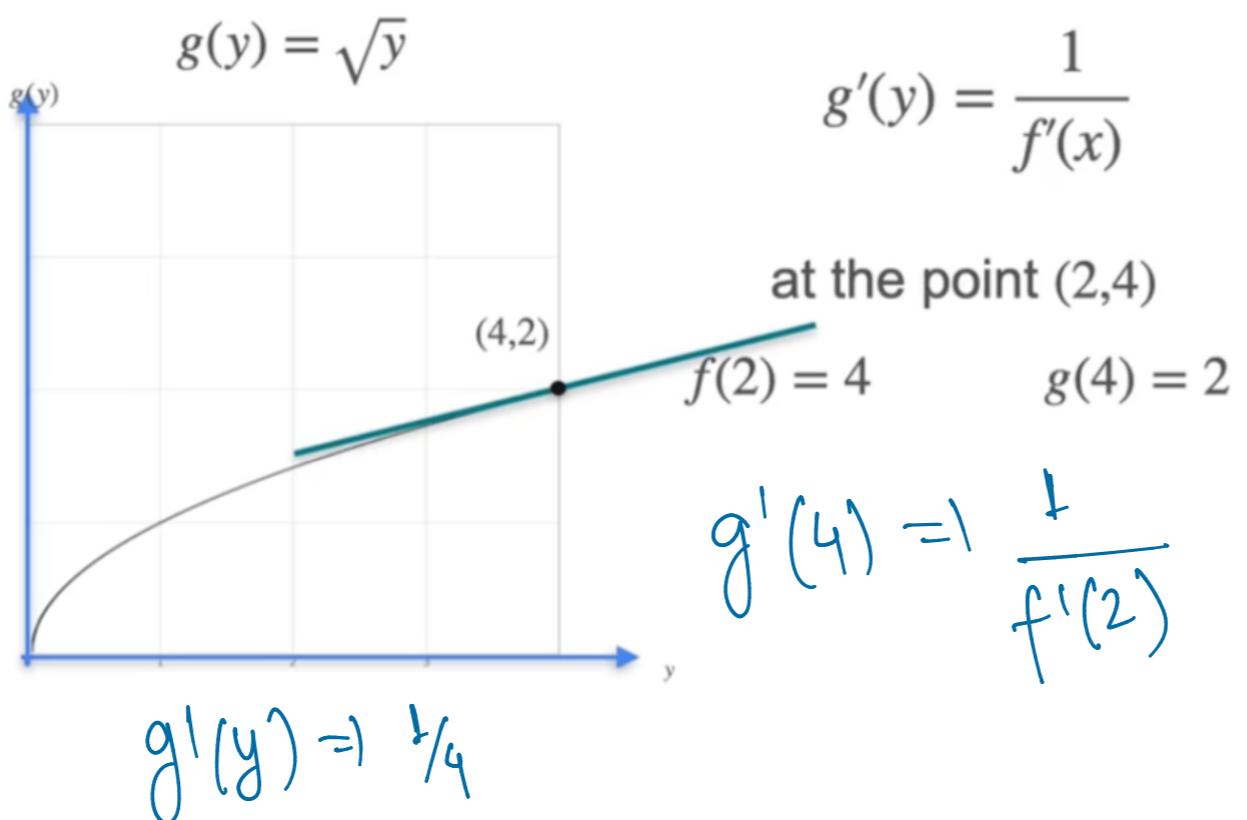
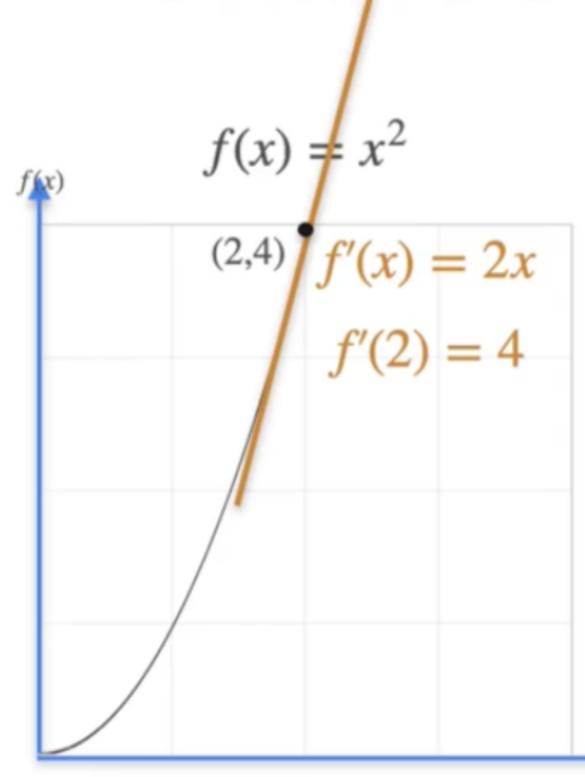
Source: Calculus

DeepLearning.AI

for Machine Learning and Data Science Course on Coursera

offered by Deep learning.

# Derivative of the Inverse



DeepLearning.AI

$\Delta x$ ,  $\Delta y$ ,  $f'(x)$  or  $f(x)$

$\frac{d}{dx} f$ ,  $\frac{dy}{dx}$ ,  $\frac{d^2 f(x)}{dx^2}$

If a function does one  
then the inverse function  
is the one that  
Undo the another.