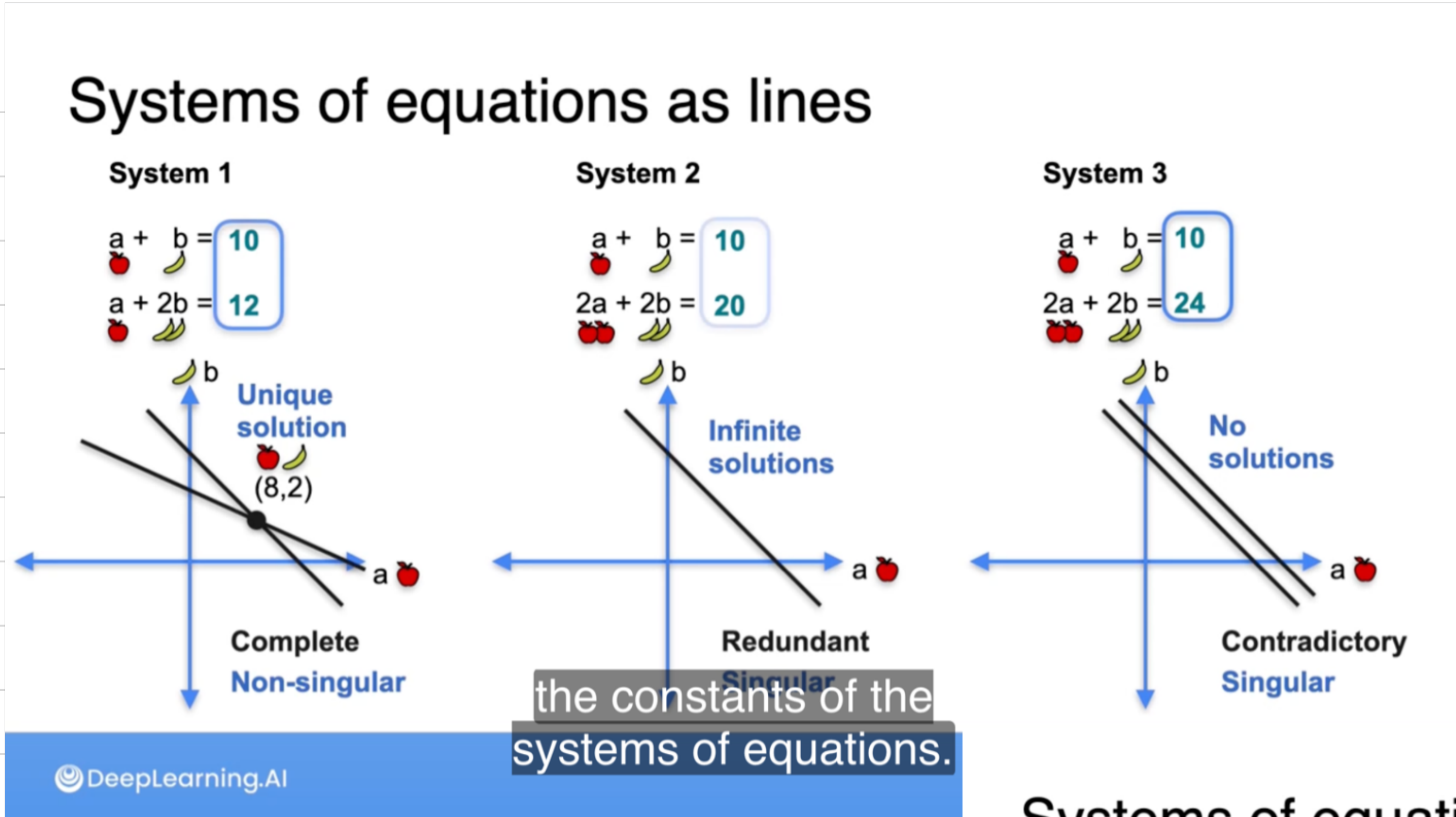
Day - 89, Feb - 27, 2025 (Falgun 15, 2081)

1) Geometric Notion of Singularity
2) Singular vs Non-Singular Matrices
3) Linear Independence and Dependence
4) The Determinant (2x2 and 3x3 Matrix)
5) Quiz Questions and Numerical Examples
6) Interactive Tools (GUI) for Linear Systems

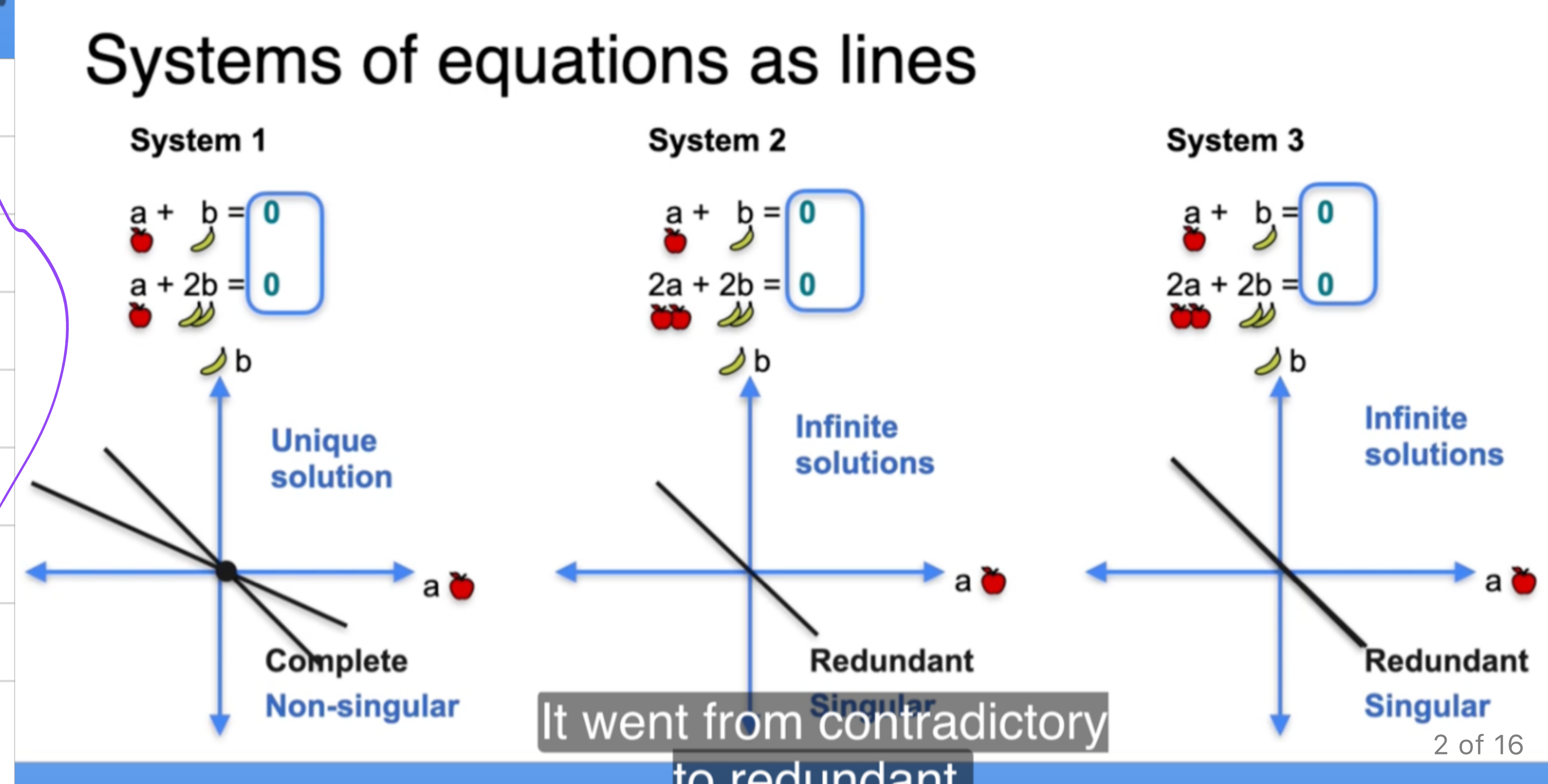
Source: Coursera Linear Algebra for Machine Learning and Data Science offered by DeepLearning.AI.



Changes.

a + b = 13 ← Doesn't contribute to System

They are constant



Singular vs Non-Singular Matrices

Systems of equations as matrices

System 1

$a + b = 0$

$a + 2b = 0$

Non-Singular

a	b
1	1
1	2

Non Singular matrix.

(Unique solution)

System 2

$a + b = 0$

$2a + 2b = 0$

Singular System

a	b
1	1
2	2

Singular Matrix.

(Infinitely many solutions)

into one system when you turn the constants into zero.

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Constants don’t matter for singularity

System 1	System 2	System 3	System 4
$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$
$a + 2b + c = 15$	$a + b + 2c = 15$	$a + b + 2c = 15$	$2a + 2b + 2c = 15$
$a + b + 2c = 12$	$a + b + 3c = 20$	$a + b + 3c = 18$	$3a + 3b + 3c = 20$
Unique Solution	Infinite Solutions	No Solutions	Infinite Solutions
Complete	Redundant	Contradictory	Redundant
Non-Singular	Singular	Singular	Singular

You found that the first
one has a unique solution,

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Turn the constants with 0's

Constants don't matter for singularity

System 1

$a + b + c = 10$
$a + 2b + c = 15$
$a + b + 2c = 12$

↓

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

↓

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

↓

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

↓

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

is singular or
non-singular is to

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Constants don't matter for singularity

System 1

a + b + c = **10**
a + 2b + c = **15**
a + b + 2c = **12**

Unique Solution:
a = 0
b = 0
c = 0
Complete
Non-Singular.

System 2

a + b + c = **10**
a + b + 2c = **15**
a + b + 3c = **20**

Infinite Solutions
c = 0
a + b = 0
i.e. a = -b
Redundant &
Singular

System 3

a + b + c = **10**
a + b + 2c = **15**
a + b + 3c = **18**

System 4

a + b + c = **10**
2a + 2b + 2c = **15**
3a + 3b + 3c = **20**

Infinite Solutions:
a + b + c = 0
i.e. c = - a - b

You found that the first
one has a unique solution,

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Constants don’t matter for singularity

System 1

$$\begin{aligned} a + b + c &= 0 \\ a + 2b + c &= 0 \\ a + b + 2c &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Non-Singular

System 2

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

Singular

System 4

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

Singular

and c. The matrices
are these ones.

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Linear Independence and Dependence

first equation = information

if first equation == second equation
it is singular matrix

else: it is non-singular matrix

Linear dependence between rows

Non-singular

$a + b = 0$

$a + 2b = 0$

a	b
1	1
1	2

No eqn is the multiple of the other one
(Rows are linearly independent)

No row is the multiple of the other one

Singular system

$a + b = 0$

$2a + 2b = 0$

$\times 2$

a	b
1	1
2	2

$\times 2$

Second equation is a multiple of the first one

Second row is a multiple of the first row

⇓

Rows are linearly dependent.

Rows linearly
- Dependent (Singular)
- Independent (Non-Singular)

$a = 1$
$b = 2$
$a + b = 3$

$$\begin{array}{r} a + 0b + 0c = 1 \\ + \; 0a + b + 0c = 2 \\ \hline a + b + 0c = 3 \end{array}$$

$\Rightarrow$ $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Row1 + Row 2 = Row 3

Row 3 depends upon Row 1 & Row 2 so the Rows are Linearly Dependent.

$\therefore$ Matrix & System are Singular.

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

1	1	1
2	2	2
3	3	3

$$\begin{array}{r} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ \hline 3a + 3b + 3c = 0 \end{array}$$

Row 1 + Row 2 = Row 3

Row 3 depends on rows 1 and 2.

Rows are Linearly Dependent

Linear dependence and independence

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

1	1	1
1	1	2
1	1	3

$a + b + c = 0$
$a + b + 3c = 0$

$2a + 2b + 4c = 0 \quad / 2$

$a + b + 2c = 0$

Average of Row 1 and Row 3 is Row 2

→ Row 2 depends on rows 1 and 3

→ Rows are Linearly Dependent

No Relations between equations.

↳ No Relations between rows

↳ Rows are Linearly Dependent

the equations? Well, here's one.

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Quiz: Linear dependence and independence

Problem: Determine if the following matrices have linearly dependent or independent rows

1	0	1
0	1	0
3	2	3

3 Row 1 + 2 Row 2 = Row 3

Dependent (Singular)

1	1	1
1	1	2
0	0	-1

Row 1 - Row 2 = Row 3

Dependent (Singular)

1	1	1
0	2	2
0	0	3

No Relations

Independent (Non-Singular)

1	2	5
0	3	-2
2	4	10

2 Row 1 = Row 3

Dependent (Singular)

dependent or independent rows. The answers are in.

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System of Linear Equations (Determinant)

if (Determinant == 0) {

matrix is Singular

else:

Matrix is Non Singular.

Linear Dependence Between Rows

Non-Singular Matrix

1	1
1	2

$\boxed{1 \mid 1} \times ? = \boxed{ \mid }$

Rows are Linearly Independent.

Singular Matrix

1	1
2	2

$\boxed{1 \mid 1} \times 2 = \boxed{2 \mid 2}$

Rows are Linearly Dependent.

Determinant:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Matrix is Singular if

$[a \mid b] \times K = [c \mid d]$

$aK = c$

$bK = d$

$$\frac{c}{a} = \frac{d}{b} = K$$

$ad = bc$

Determinant $\Rightarrow \boxed{ad - bc = 0}$

$|D| \Rightarrow ad - bc$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$\Rightarrow 2 - 1$

$= 1$

$|D| \neq 0$ so Non-Singular

$$|D| = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$\Rightarrow 2 - 2$

$\Rightarrow 0$. $|D| = 0$ So, Singular.

Solutions: Determinant

Matrix 1: det $= 5 \cdot 3 - 1 \cdot (-1) = 15 + 1 = 16$

5	1
-1	3

Non-singular

Matrix 2: det $= 2 \cdot 3 - (-1) \cdot (-6) = 6 - 6 = 0$

2	-1
-6	3

Singular

which is precisely zer

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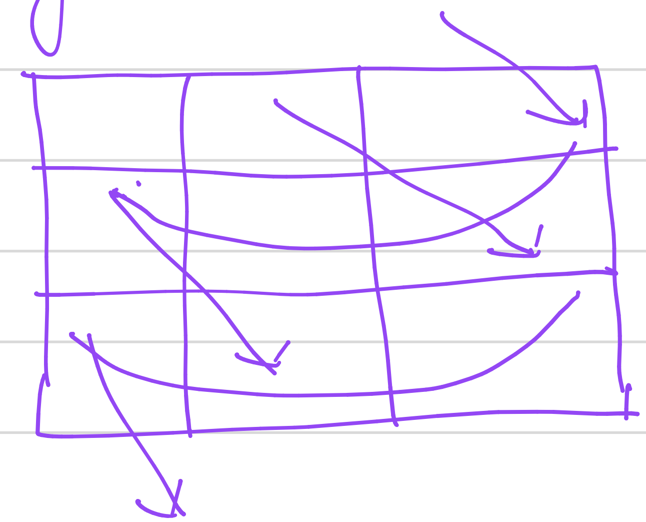
Digonals in 3x3 Matrix .



$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

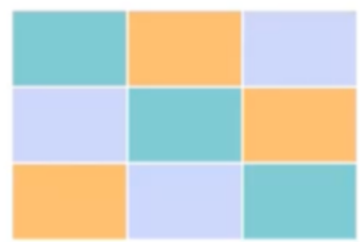
$$\det|M_3| \Rightarrow 1 \cdot \begin{vmatrix} 2 & 2 \\ 0 & 3 \end{vmatrix} - 1 \begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix} + 1 \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix}$$

$$\Rightarrow 1 \cdot (2 \cdot 3 - 0) - 1(0 - 0) + 1(0 - 0)$$

$$\Rightarrow 6$$

Determinant

Add



Subtract

that go the other way around.

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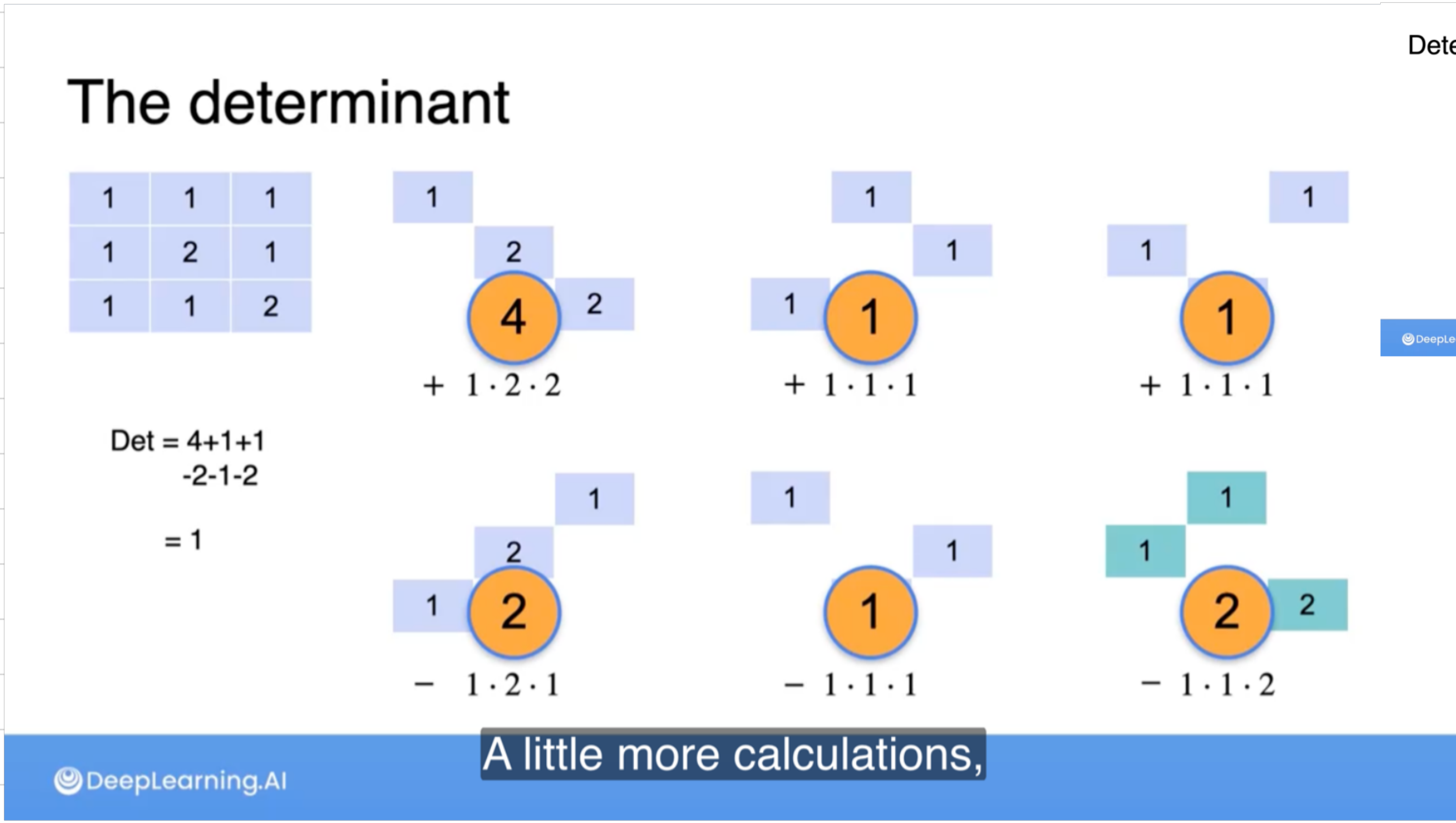

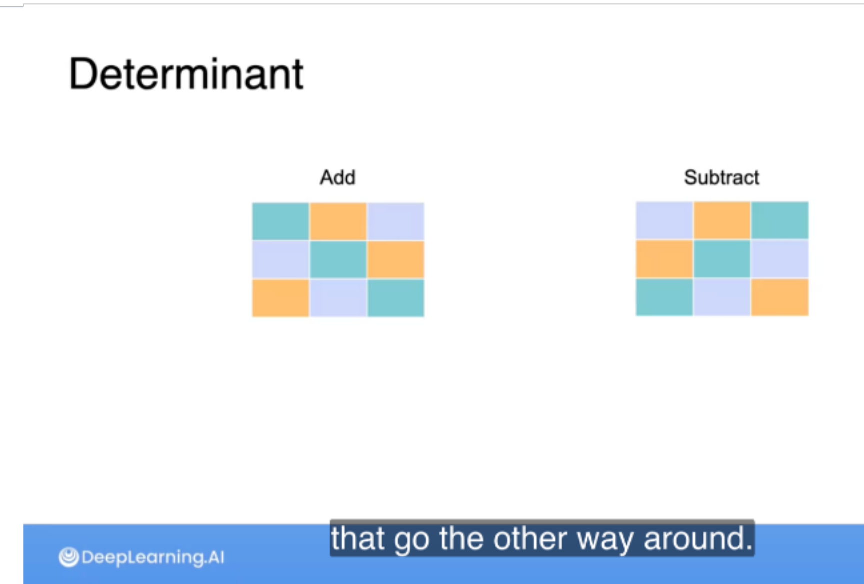

Example:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 3 & 3 \end{bmatrix}$$

(det = 0.)

So, Left to Right

$1 \cdot 1 \cdot 3 + 0 \cdot 0 \cdot 3 + 1 \cdot 3 \cdot 0 \rightarrow 3$

Right to left

$1 \cdot 1 \cdot 3 + 0 \cdot 0 \cdot 3 + 1 \cdot 3 \cdot 0 \rightarrow 3$

$\therefore M_1 \rightarrow 3 - 3 \rightarrow 0$

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

Singular

1	1	1
1	1	2
0	0	-1

Determinant = 0

Singular

1	1	1
0	2	2
0	0	3

Determinant = 6

Non-singular

1	2	5
0	3	-2
2	4	10

Determinant = 0

Singular

Singular matrices have determinant

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$$1\begin{vmatrix} 2 & 2 \\ 4 & 5 \end{vmatrix} - 2\begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} + 3\begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix}$$

$$\Rightarrow 1(10-8) - 2(0-2) + 3(0-2)$$

$$\Rightarrow +2 + 4 + (-6)$$

$\Rightarrow$ - [illegible]

$$1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}$$

$$\Rightarrow 1(2-1) - 2(2+1) + 1(4-1)$$

$$\Rightarrow 1 - 6 + 3$$

$$\Rightarrow -2$$

The determinant

1	1	1
0	2	2
0	0	3

Det = 6+0+0-0-0-0

= 6

$+\ 1\cdot 2\cdot 3$ $\quad +\ 1\cdot 2\cdot 0$ $\quad +\ 1\cdot 0\cdot 0$

$-\ 1\cdot 2\cdot 0$ $\quad -\ 1\cdot 2\cdot 0$ $\quad -\ 1\cdot 0\cdot 3$

all the terms in the determinant are going to

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