Day-88, Feb 26, 2025 (Falgun 14, 2081)

1. Linear Algebra Applied I
2. Linear Algebra Applied II
3. System of Sentences
4. System of Equations
5. System of Equations as Lines and planes.

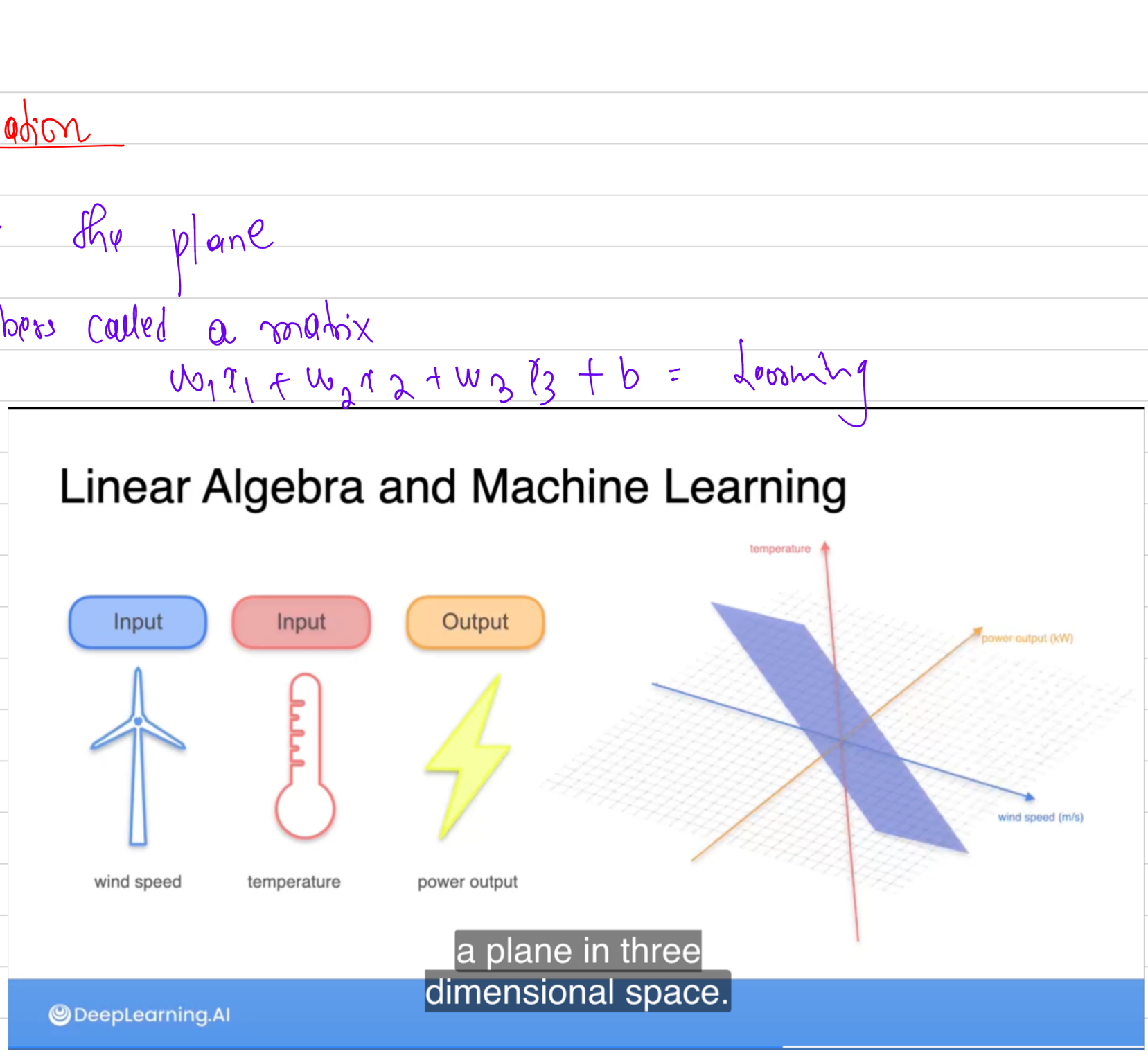
System of Linear Equation

- A set of lines in the plane
- An array of numbers called a matrix

$w_1 x_1 + w_2 x_2 + w_3 x_3 + b =$ Learning

Features (Input)

1. feature 1 - wind speed
2. temperature
3. pressure
4. Humidity
5. Other features
6. Output.



$$\begin{bmatrix} w_1 x_1^{(1)} + w_2 x_2^{(1)} + \ldots + w_n x_n^{(1)} + b = y^{(1)} \\ w_1 x_1^{(2)} + w_2 x_2^{(2)} + \ldots + w_n x_n^{(2)} + b = y^{(2)} \end{bmatrix}$$

Linear Algebra and Machine Learning

System of Linear Equations

$$\begin{aligned} w_1 x_1^{(1)} + w_2 x_2^{(1)} + \ldots + w_n x_n^{(1)} + b &= y^{(1)} \\ w_1 x_1^{(2)} + w_2 x_2^{(2)} + \ldots + w_n x_n^{(2)} + b &= y^{(2)} \\ w_1 x_1^{(3)} + w_2 x_2^{(3)} + \ldots + w_n x_n^{(3)} + b &= y^{(3)} \\ &\vdots \\ w_1 x_1^{(m)} + w_2 x_2^{(m)} + \ldots + w_n x_n^{(m)} + b &= y^{(m)} \end{aligned}$$

before we dive into
the weeks materials.

Linear Algebra and Machine Learning

wind speed	temperature	pressure	humidity	...	feature n		target
$w_1 x_1^{(1)}$	$+ w_2 x_2^{(1)}$	$+ w_3 x_3^{(1)}$	$+ w_4 x_4^{(1)}$	$+$	$w_n x_n^{(1)}$	$+ b$	$= y^{(1)}$
$w_1 x_1^{(2)}$	$+ w_2 x_2^{(2)}$	$+ w_3 x_3^{(2)}$	$+ w_4 x_4^{(2)}$	$+$	$w_n x_n^{(2)}$	$+ b$	$= y^{(2)}$
$\vdots$							
$w_1 x_1^{(m)}$	$+ w_2 x_2^{(m)}$	$+ w_3 x_3^{(m)}$	$+ w_4 x_4^{(m)}$	$+$	$w_n x_n^{(m)}$	$+ b$	$= y^{(m)}$

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denote which row of data a set of features belong to.

Linear Algebra and Machine Learning

$$W \cdot X + b = \hat{y}$$

$$\begin{bmatrix} w_1 & w_2 & w_3 & w_4 & \dots & w_n \end{bmatrix} \cdot \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & x_4^{(1)} & \dots & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & x_4^{(2)} & \dots & x_n^{(2)} \\ \vdots & & & & & \\ x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & x_4^{(m)} & \dots & x_n^{(m)} \end{bmatrix} + b = \begin{bmatrix} y^{(1)} & y^{(2)} & \dots & y^{(m)} \end{bmatrix}$$

Vector

Matrix

Don't worry if you're not already

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linear Algebra is all about vector and Matrix Manipulation.

Linear Algebra and Machine Learning

Linear Regression as Machine Learning.

wind speed | temperature | pressure | humidity | ... | feature n | target

$$w_1 x_1^{(1)} + w_2 x_2^{(1)} + w_3 x_3^{(1)} + w_4 x_4^{(1)} + \ldots + w_n x_n^{(1)} + b = y^{(1)}$$

$$w_1 x_1^{(2)} + w_2 x_2^{(2)} + w_3 x_3^{(2)} + w_4 x_4^{(2)} + \ldots + w_n x_n^{(2)} + b = y^{(2)}$$

$$\vdots$$

$$w_1 x_1^{(m)} + w_2 x_2^{(m)} + w_3 x_3^{(m)} + w_4 x_4^{(m)} + \ldots + w_n x_n^{(m)} + b = y^{(m)}$$

System of Linear Equations

predict y given a set of x features,

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Check your Knowledge

Independent

a — Linear Algebra	Your algebra score added to your calculus score minus your probability score was 6	$c+p+a=6$
c — Calculus	Your algebra score minus your calculus score plus double your probability score was 4.	$a-c+2p=4$
p — Probability & Statistics	Four times your algebra score minus double your calculus score added to your probability score was 10	$4a-2c+p=10$

Represent these statements as a system of linear equations.

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c represent your calculus score,

Check your Knowledge

$$a + c - p = 6$$

What are the weights, w? a, c, p

What are the features, x?

$$a - c + 2p = 4$$

The targets, y? 6, 4, 10

$$4a - 2c + p = 10$$

Is this system Singular or not?

Can Solve this as System of Equations, represented as

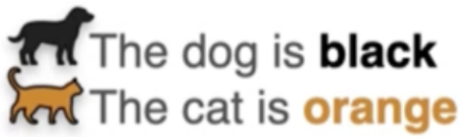
the features x and the target y?

A matrix and vector?

Systems of Sentences. [Sentences gives information about the world].

Systems of sentences

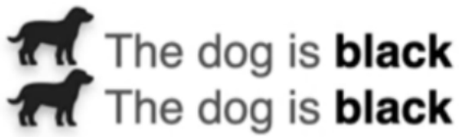
System 1



→ Complete

→ Non-Singular

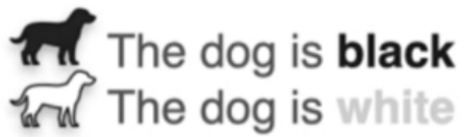
System 2



→ Some Information (Redundant)

→ Singular

System 3



→ Contradictory

→ Cat can be black & white.

→ Singular

contains two sentences,
the sentences he repeat themselves and

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Systems of sentences

System 1

The dog is **black**
The cat is **orange**
The bird is **red**

→ Complete
→ Non-singular

System 2

The dog is **black**
The dog is **black**
The bird is **red**

→ Redundant
→ Singular

System 3

The dog is **black**
The dog is **black**
The dog is **black**

→ Redundant
→ Singular

System 4

The dog is **black**
The dog is **white**
The bird is **red**

→ Contradictory
→ Singular

Rank of Redundant Sys 3 ? Sys 2.

So the first system is complete as it carries three different piece of

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Non-singular System carries as many pieces of information as a sentences (Complete system)

Equations is equivalent to the Sentences.

Sentences → Equations

Sentences

Between the dog and the cat, one is black

⇒

Sentences with numbers

The price of an apple and a banana is $10

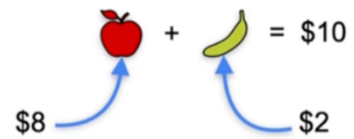
⇒

Equation

$a + b = 10$.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost $10.



- **Day 2:** You bought an apple and two bananas and they cost $12.

- **Solution:** An apple costs $8, a banana costs $2.

Thus, each apple costs $8, and each banana costs $2.

Quiz: Systems of equations 2

Problem 1: You're trying to figure out the price of apples, bananas, and cherries at the store. You go three days in a row, and bring this information.

- **Day 1:** You bought an apple, a banana, and a cherry, and paid $10.
- **Day 2:** You bought an apple, two bananas, and a cherry, and paid $15.
- **Day 3:** You bought an apple, a banana, and two cherries, and paid $12.

How much does each fruit cost?

Now, the question is,
how much does each fruit cost?

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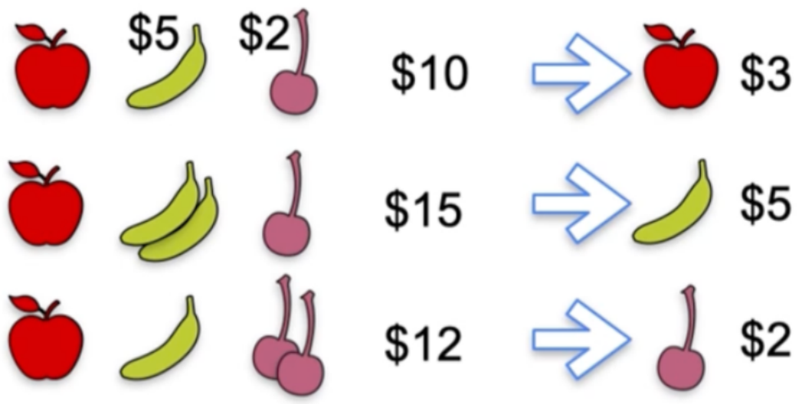

System of equations 1

$$a + b + c = 10$$
$$a + 2b + c = 15$$
$$a + b + 2c = 12$$

a + 2b + c = 15, and a + b + 2c = 12.

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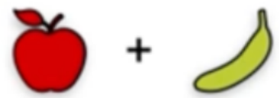
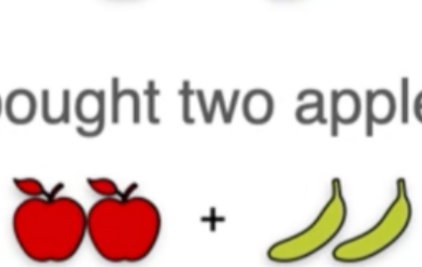
Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost $10.



- **Day 2:** You bought two apples and two bananas and they cost $20.



Same thing!!!

8	2
5	5
8.3	1.7
0	10

Infinitely many solutions!

→ Might have No Information

don't have enough information.

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Solution: Systems of equations 4

- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought two apples and two bananas and they cost $24.

Contradiction!

No solutions!

→ Both equations are conflicting

And this concludes that
the system has no solutions.

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Systems of equations

System 1

a + b = 10

a + 2b = 12

Unique solution:

a = 8

b = 2

Complete

Non-singular.

System 2

a + b = 10

2a + 2b = 20

Infinite solutions

a = 8, 7, 6 ...

b = 2, 3, 4

Redundant

Singular.

System 3

a + b = 10

2a + 2b = 24

No solution

Contradictory

Singular

For this reason,
the system is complete and non singular.

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Quiz: More systems of equations

System 1

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinitely many solutions
$c = 5$.
$a + b = 5$
$5 + 2c = 15$
$c = 5$
$(0, 5, 5)(1, 4, 5)(2, 3, 5)$

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

No solutions
from 1st and 2nd
$c = 5$
from 2nd and 3rd
$c = 3$

System 3

$a + b + c = 10$ → 2x
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$ → 3x.

Infinitely many solutions.
Any number 3 that add to be work
$(0, 0, 10)(2, 7, 1)$

Some of them may have no solution or have an infinite number of solutions.

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What is a linear equation?

We study linear equations in linear algebra

Linear

$a + b = 10$

$2a + 3b = 15$

$3.4a - 48.99b + 2c = 122.5$

Numbers (Constant)

Σ gives Constant.

Non-linear

$a^2 + b^2 = 10$

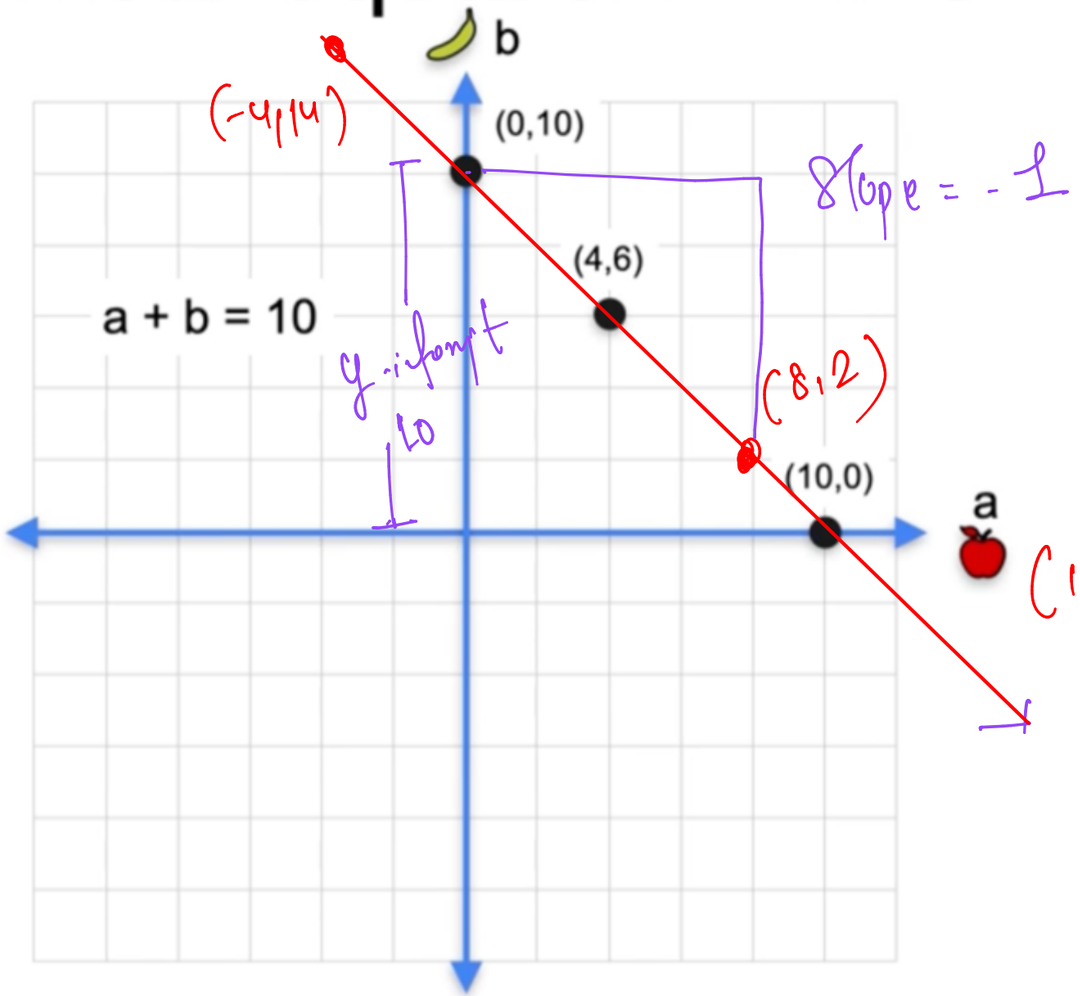
$\sin(a) + b^5 = 15$

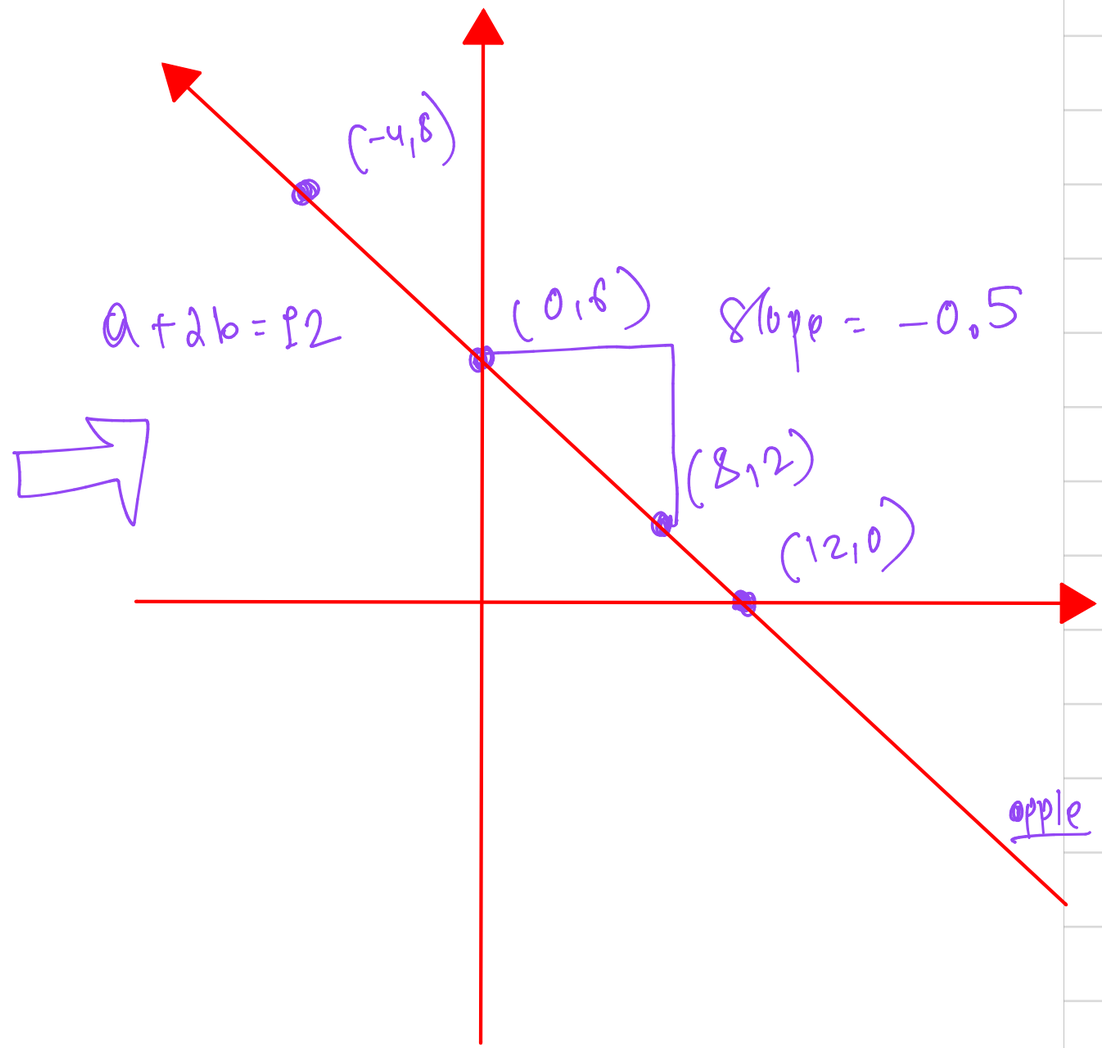
$2^a - 3^b = 0$

What does that mean?

System of Equations as Lines and Planes:

Linear equation → line





So this is a equals 4 and b =6.

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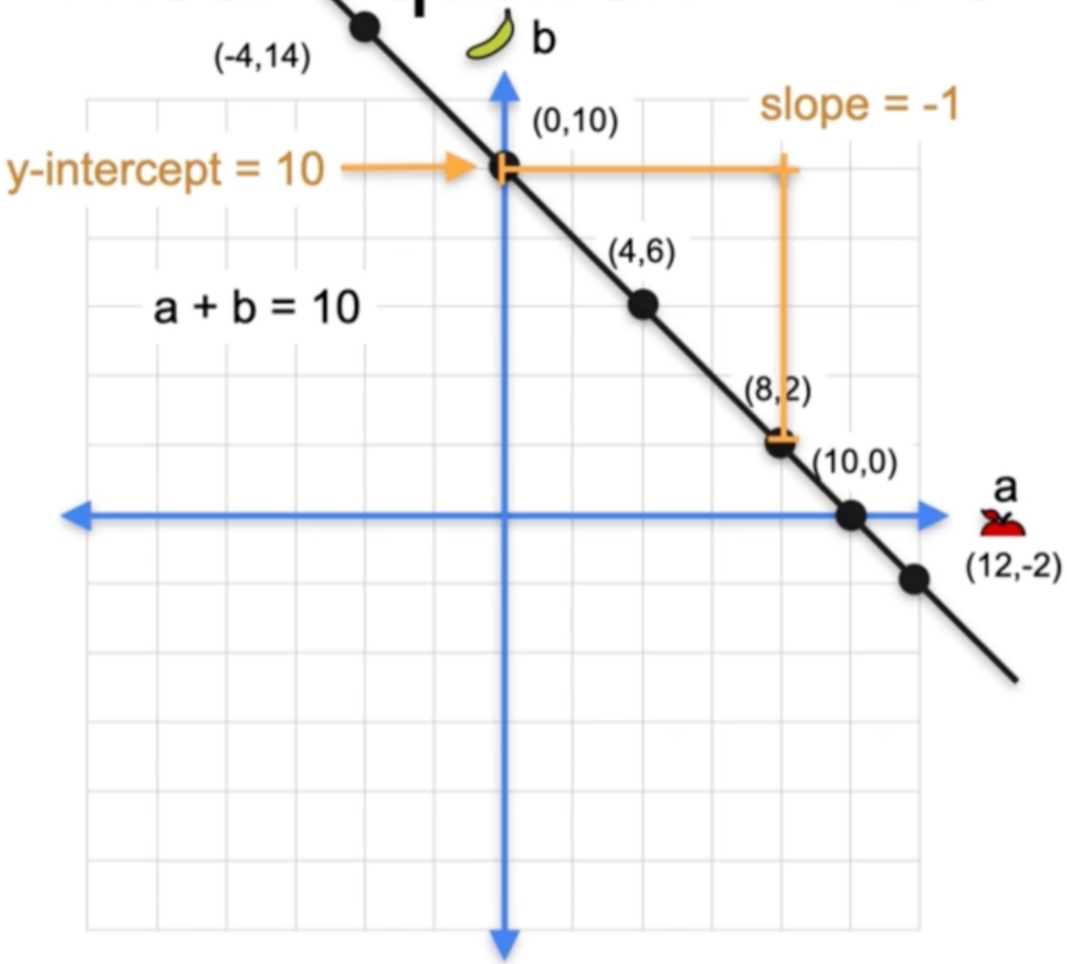
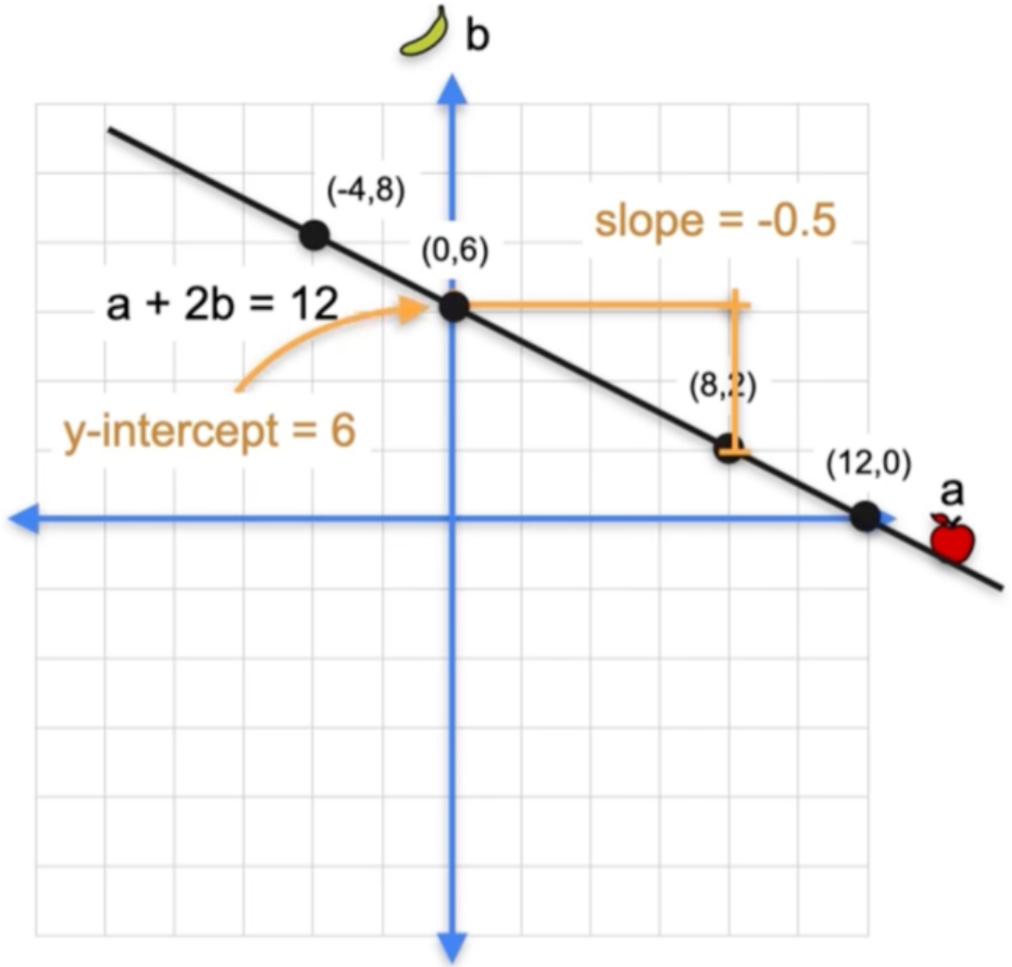
Linear equation → line





And for the line on the right, it is 6.

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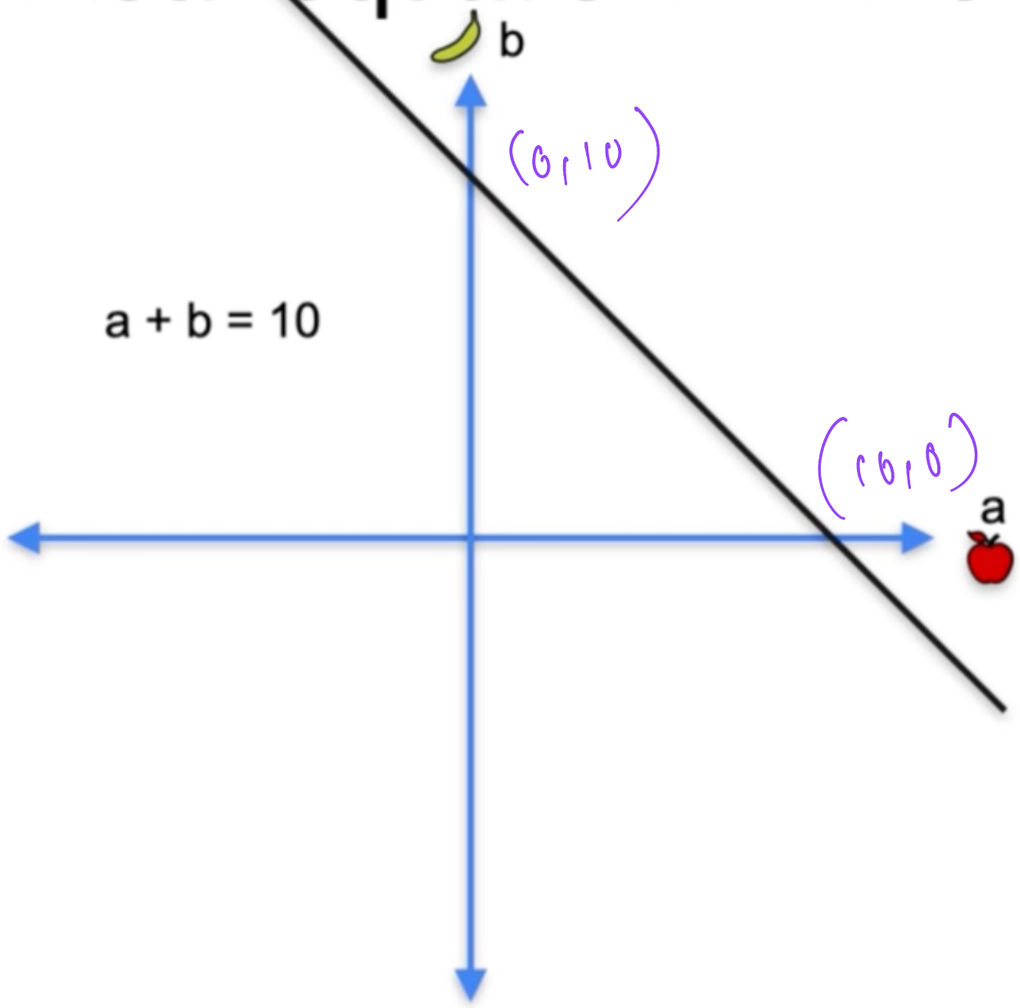
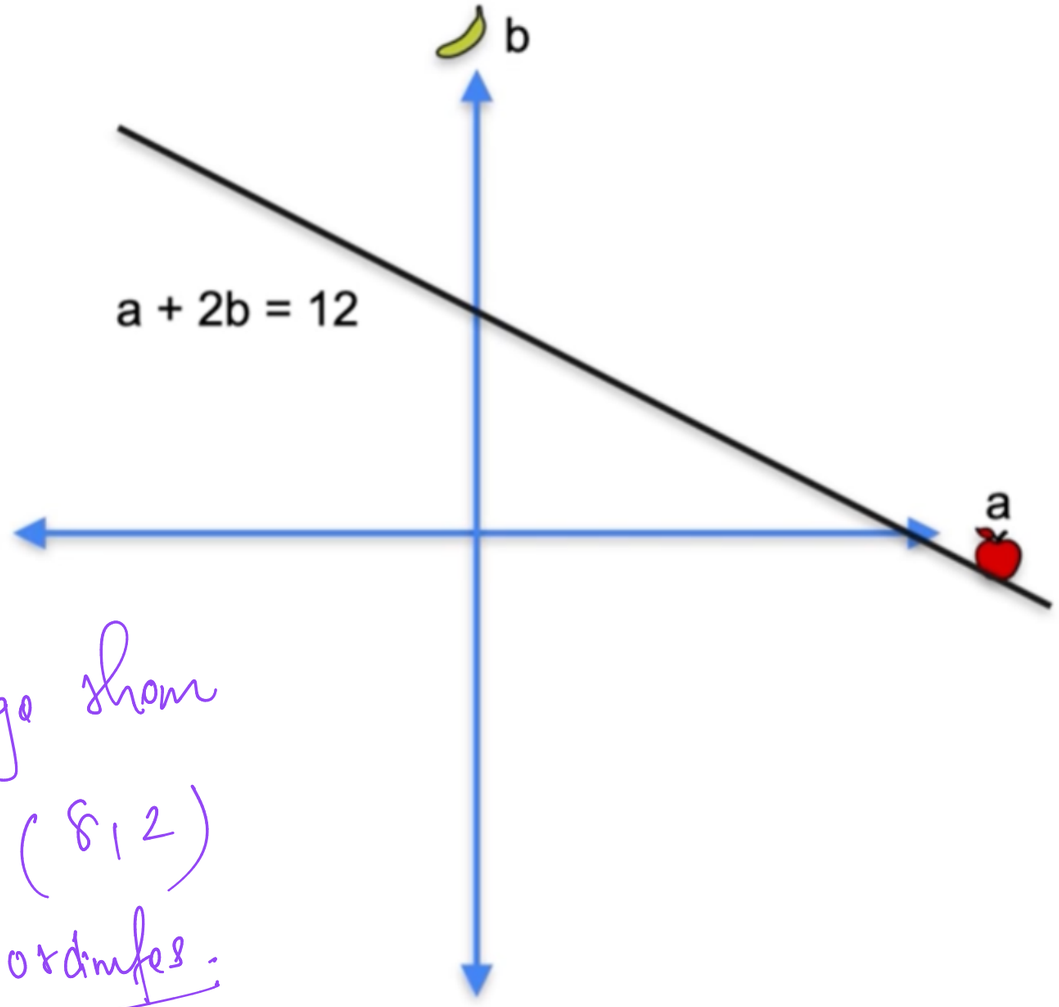
Linear equation → line





Mpoge shom

gives → (8,2)

Co-ordinates:

Each equation is associated to a line.

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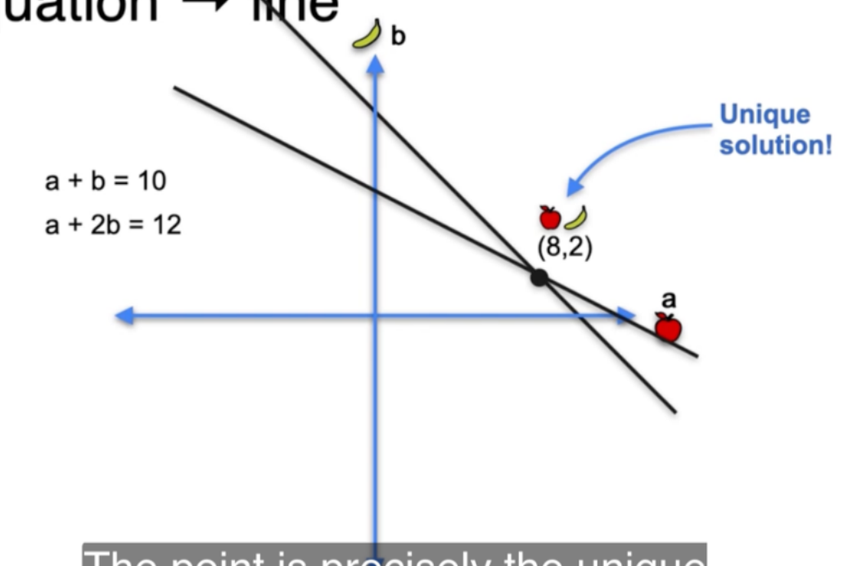
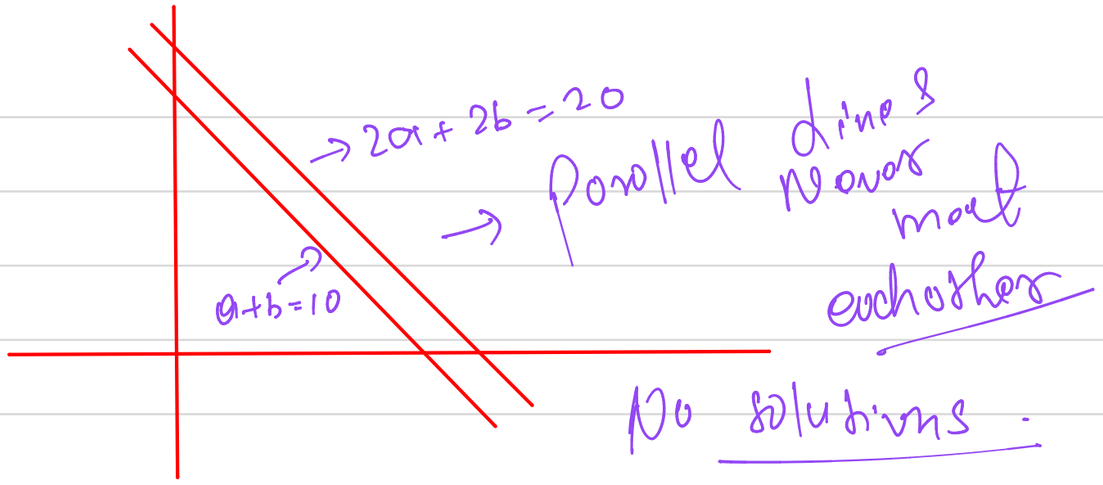
Linear equation → line

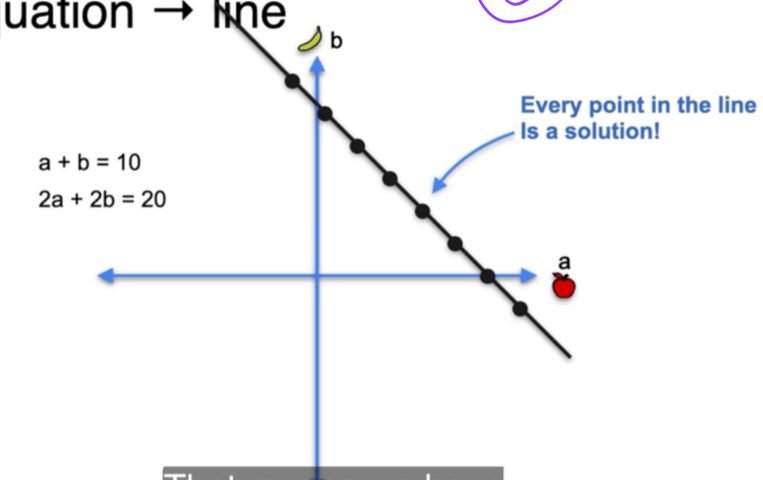


The point is precisely the unique solution to that system of equations.

1



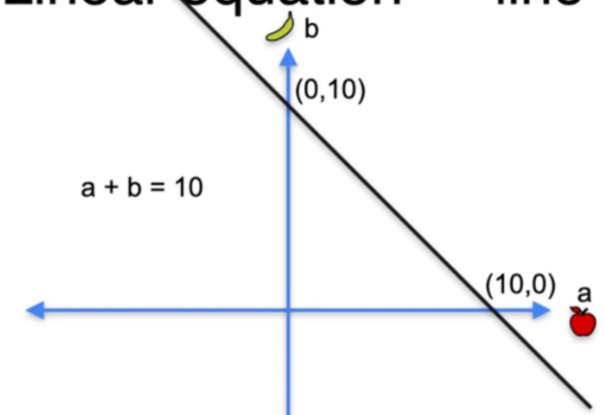
Linear equation → line

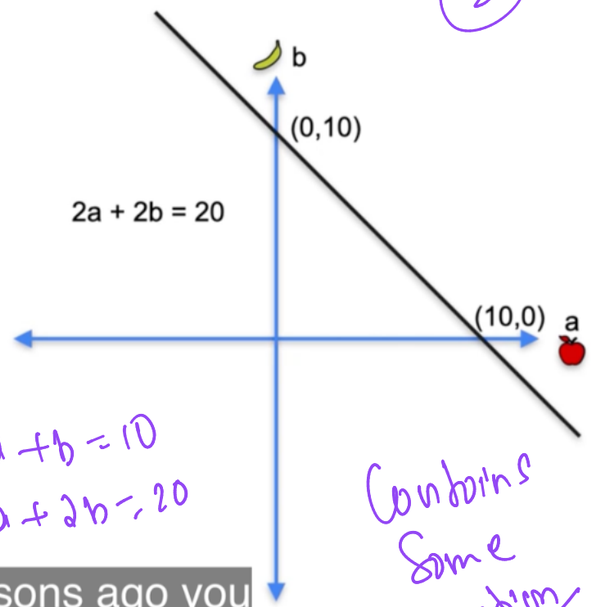


That means we have infinitely many solutions,

3

Linear equation → line



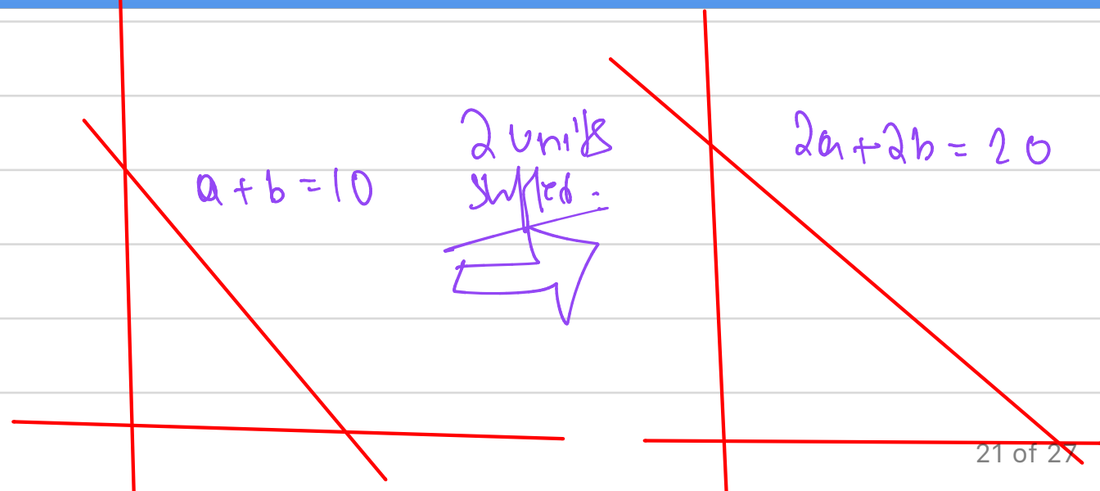


a+b=10
2a+2b=20

Contains Same Information

Recall that a few lessons ago you learned that equations a +b =10 and

2

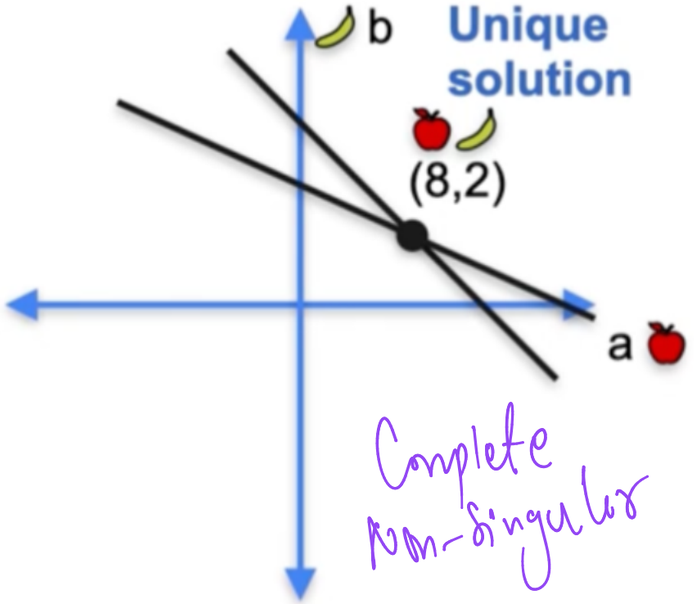


Systems of equations as lines

System 1

a + b = 10

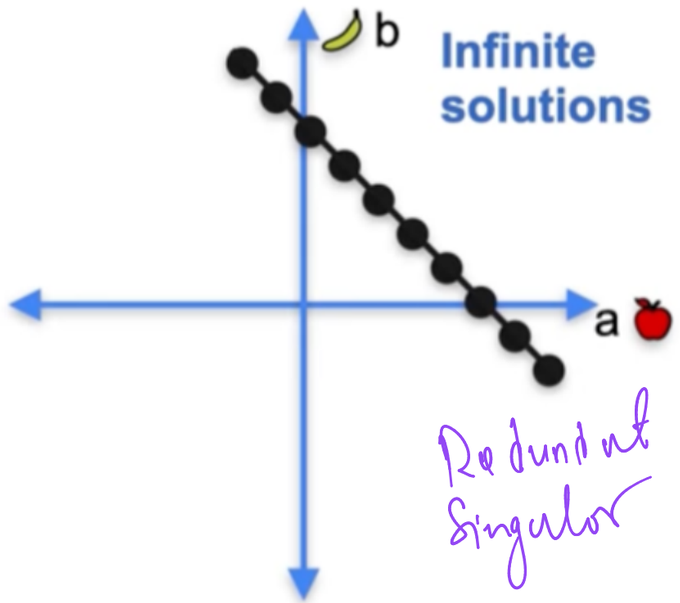
a + 2b = 12



System 2

a + b = 10

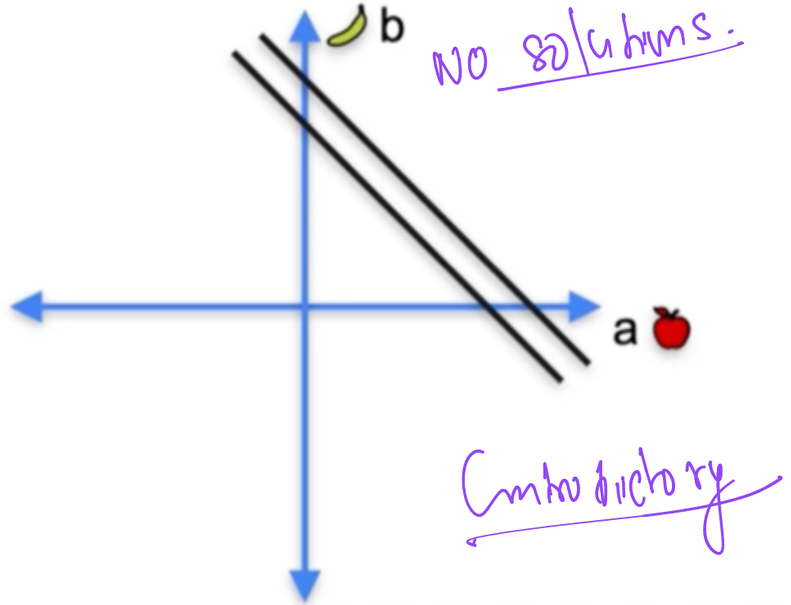
2a + 2b = 20



System 3

a + b = 10

2a + 2b = 24



which means the system has no solutions.

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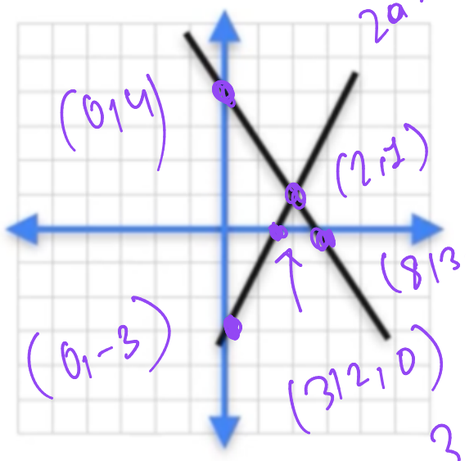
Quiz

Problem 1

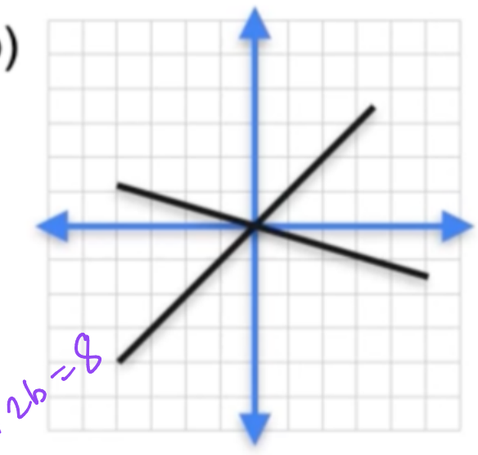
Which of the following plots corresponds to the system of equations:

- $3a + 2b = 8$
- $2a - b = 3$

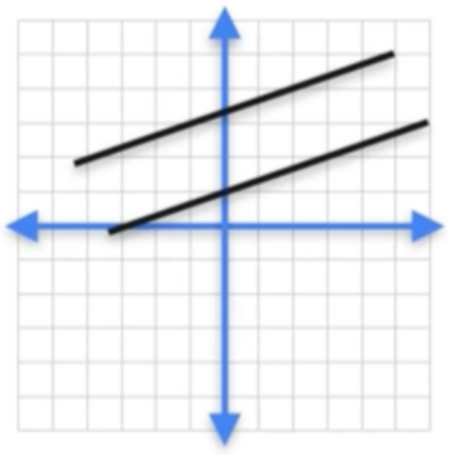
a)



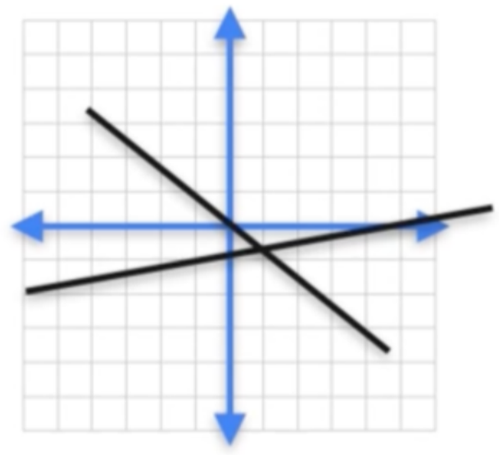
b)



c)



d)



Problem 2

Is this system singular or non-singular?

→ Non-Singular because

at (2,1) it is a intersection point.

do you conclude that the system is singular or non singular?

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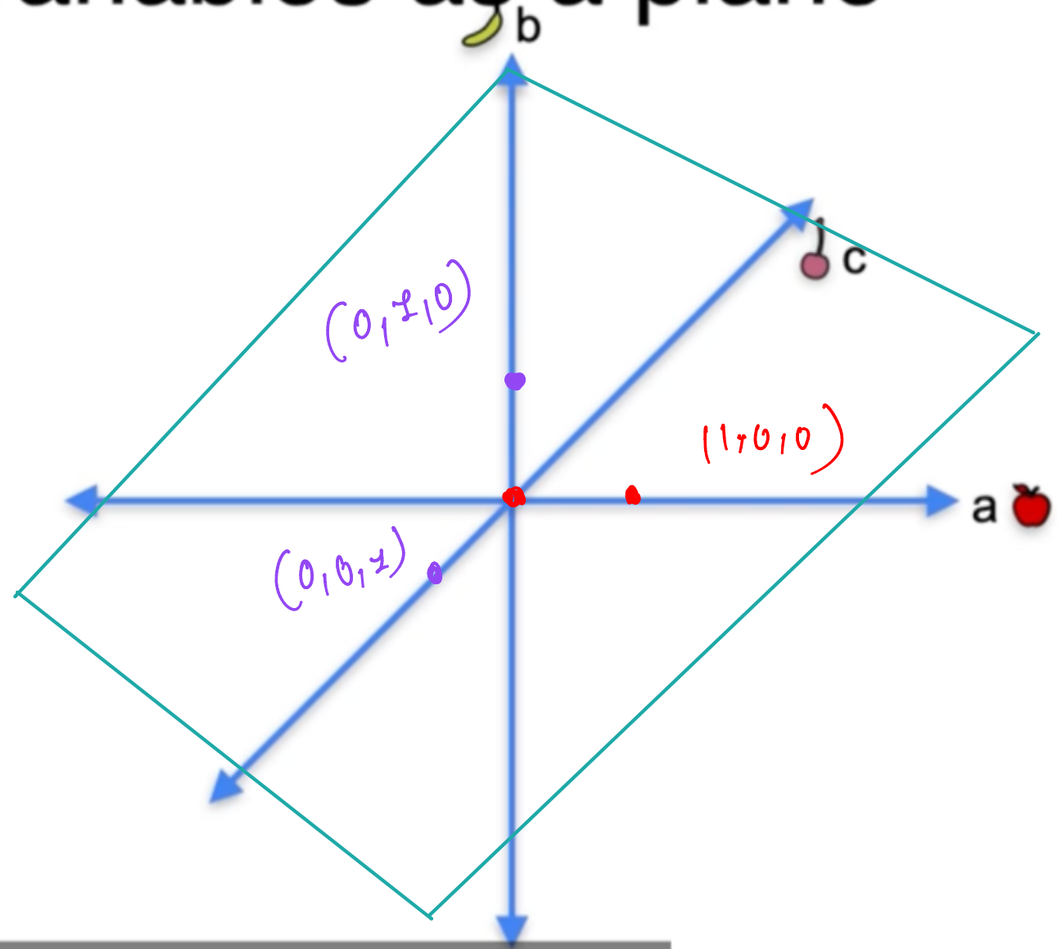
Linear equation in 3 variables as a plane

$a + b + c = 1$

$1+0+0=1$

$0+1+0=1$

$0+0+1=1$



$3a-5b+2c=0$?

$3(0)-5(0)+2(0)=0$

Passes through origin –

Well, let's look at some points that would belong to this plot.

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System 1

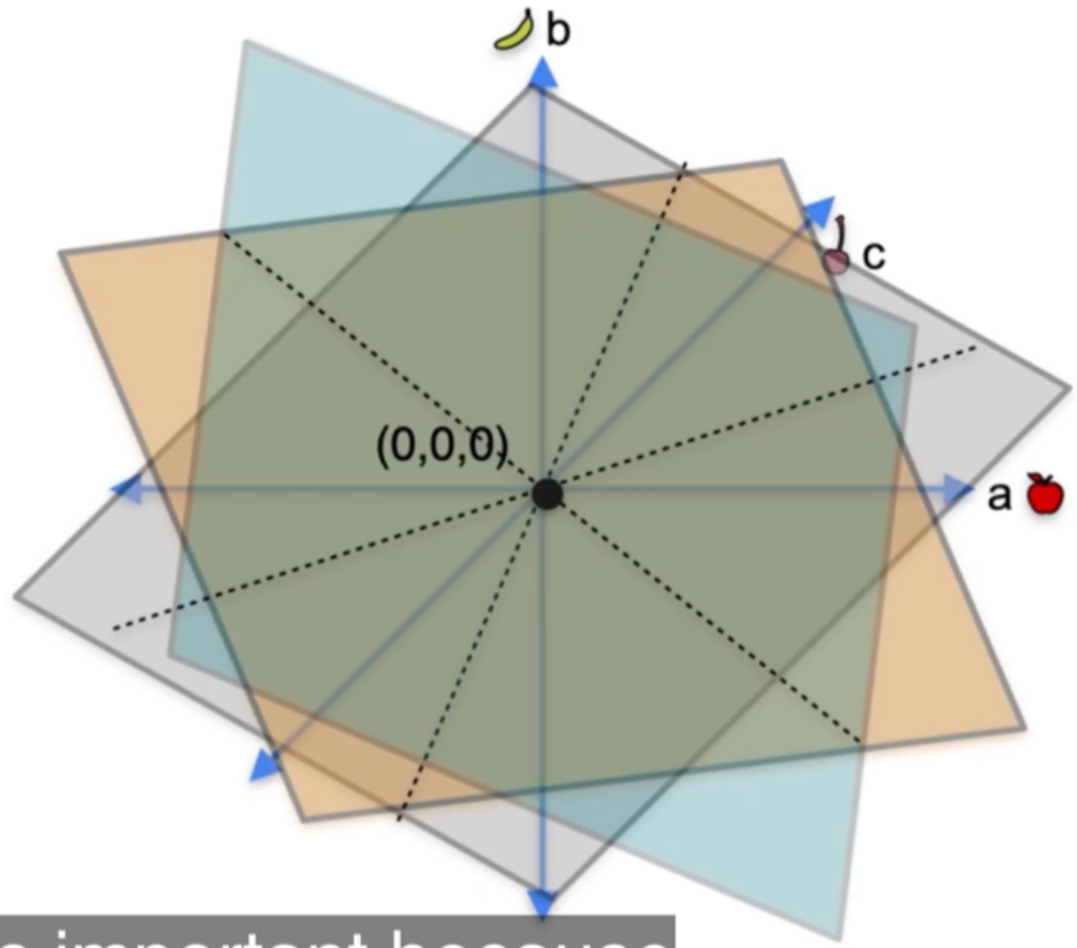
System 1

$a + b + c = 0$

$a + 2b + c = 0$

$a + b + 2c = 0$ ⇐

Unique solution (0,0,0).



So this is important because this is a non-singular system,

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System 2

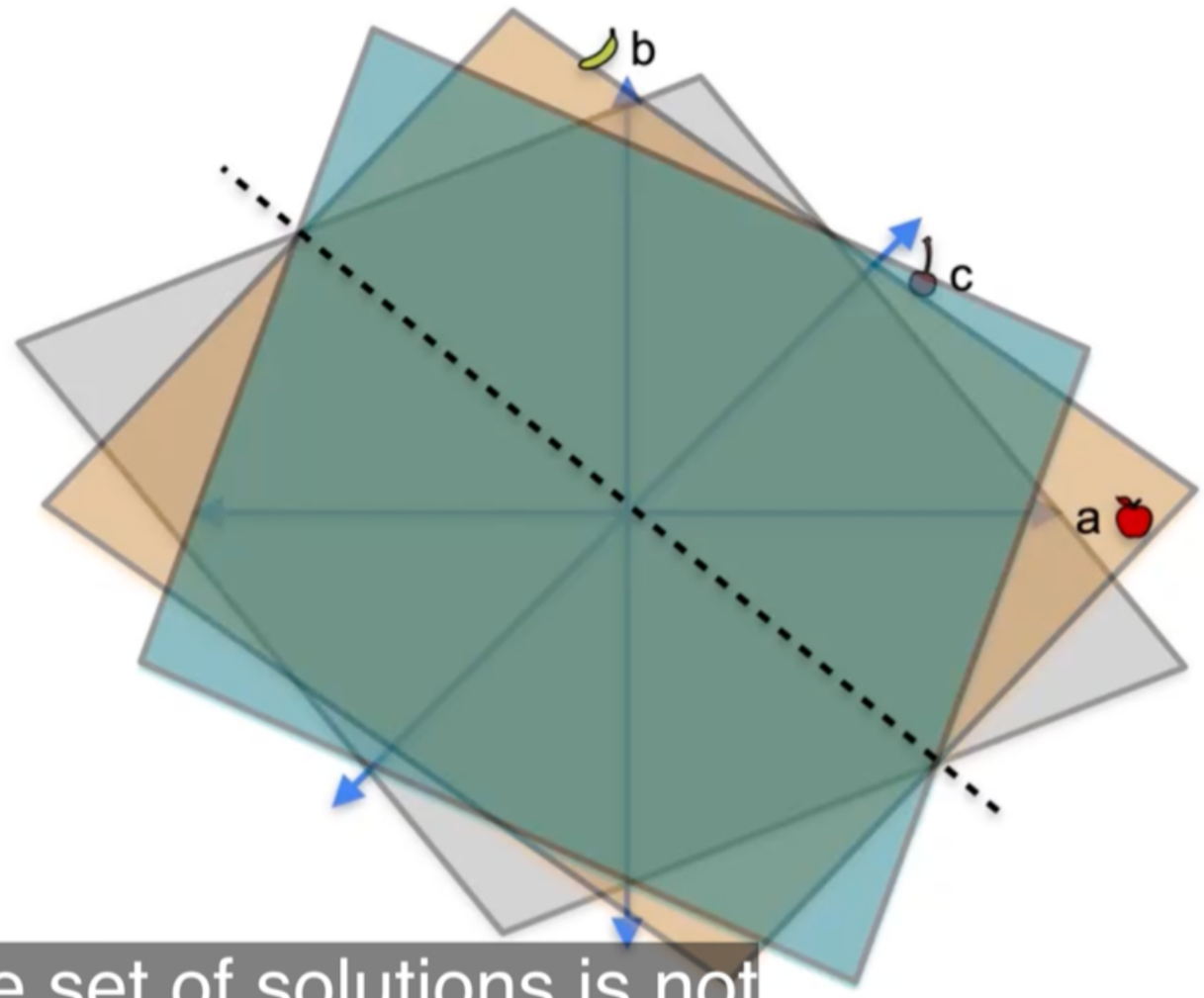
System 2

$a + b + c = 0$

$a + b + 2c = 0$

$a + b + 3c = 0$

Multiple Solutions Exist:



So the set of solutions is not just a point, it's an entire line.

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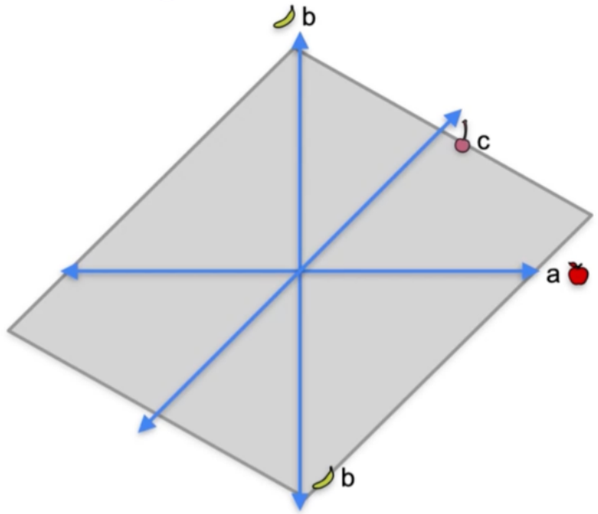
System 3

System 3

$a + b + c = 0$

$2a + 2b + 2c = 0$

$3a + 3b + 3c = 0$



So it's actually the same equation.

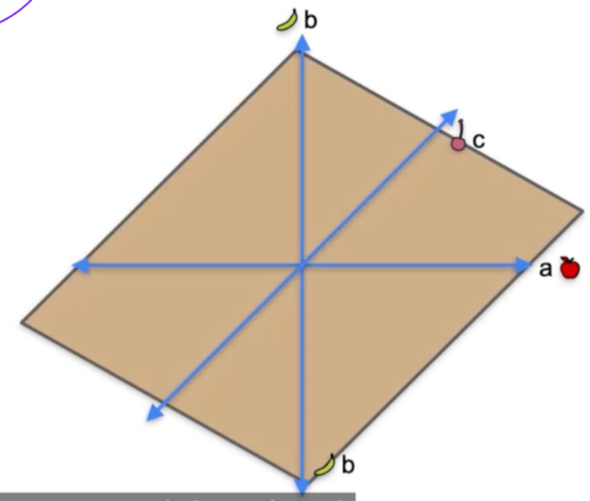
System 3

System 3

$a + b + c = 0$

$2a + 2b + 2c = 0$

$3a + 3b + 3c = 0$



So it's not surprising that it corresponds to the exact same plane.

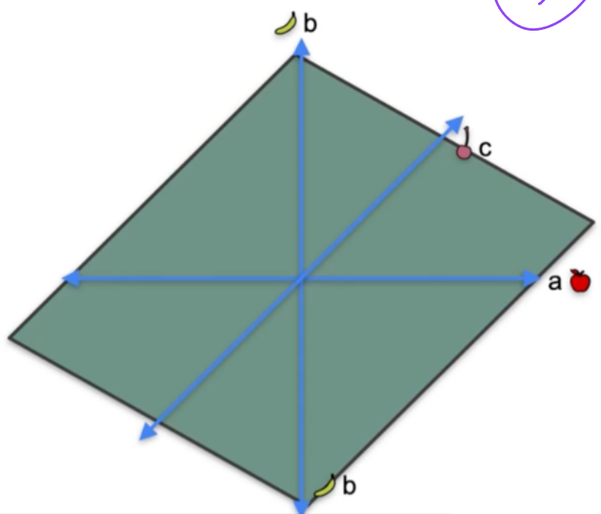
System 3

System 3

$a + b + c = 0$

$2a + 2b + 2c = 0$

$3a + 3b + 3c = 0$



Therefore, the set of solutions to this system is every single point in the plane.

Source: Linear Algebra for Machine learning and Data Science offered by Coursera Course.