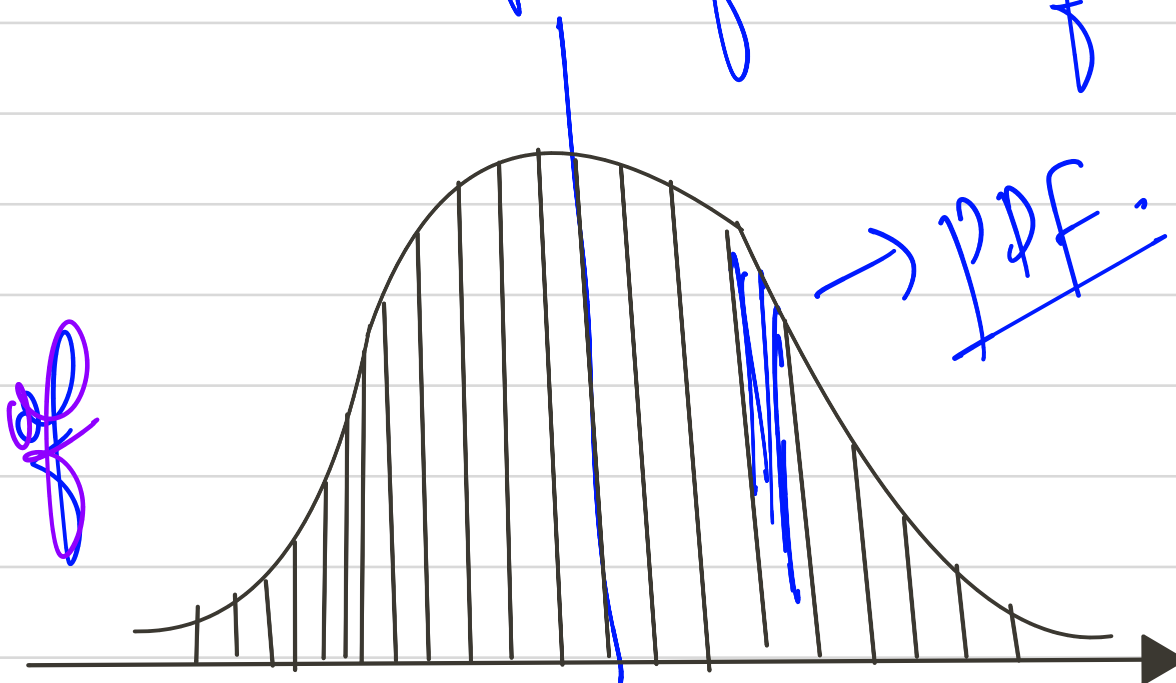
Day-75, Feb-13, 2025 (Folgun 1, 2081 B.s)

Continuous Random Variable

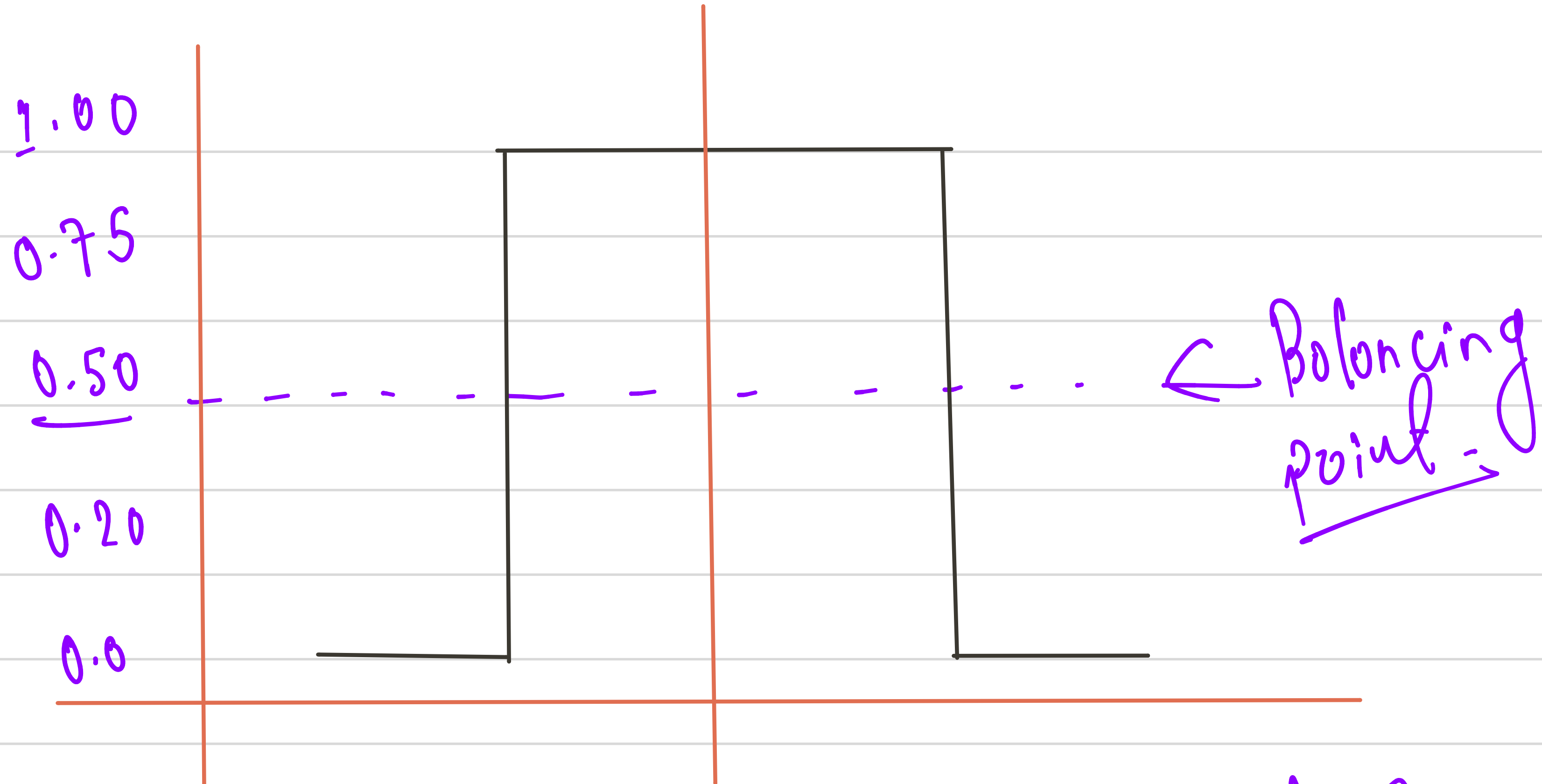
For a continuous random variable X with density f, the expected value is again exactly the center of mass of the diversity.



What is the expected value of uniform density? Do we need to balance yes we need!

Facts about expected values:

(1) Expected values are the properties of distributions.



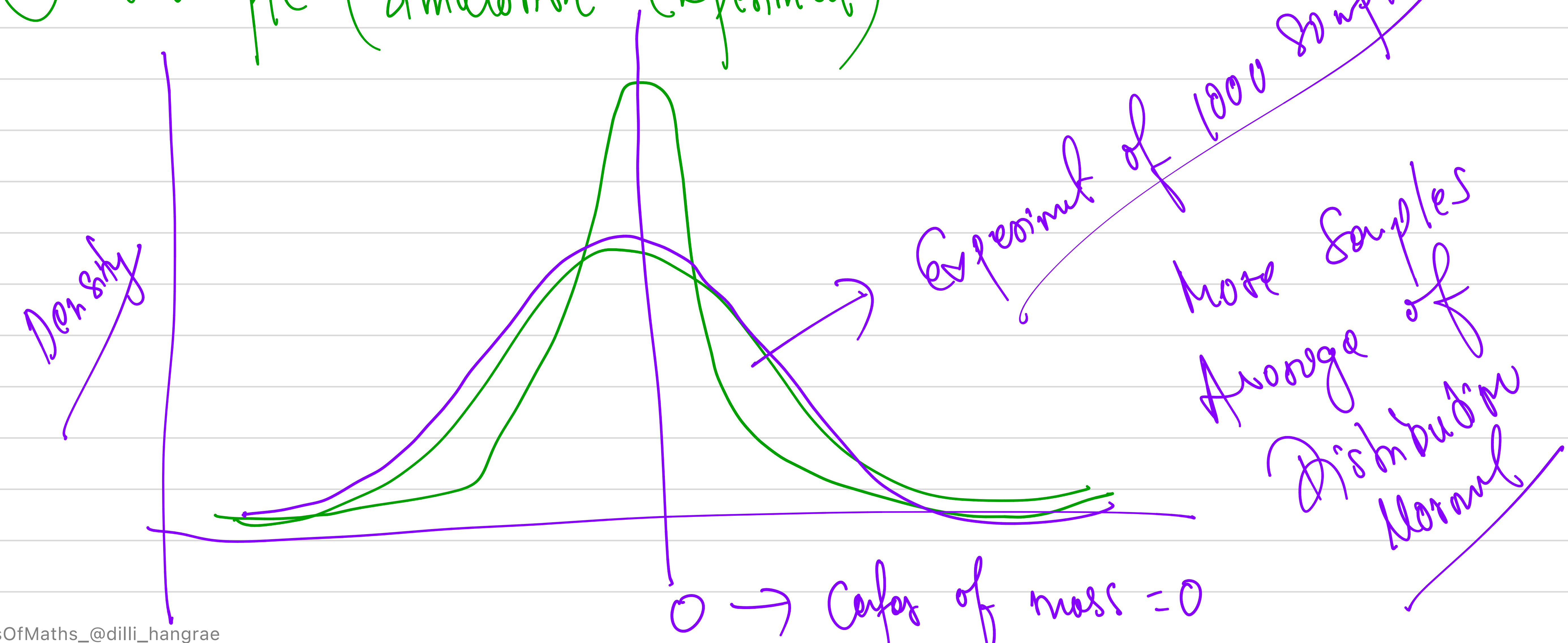
(Uniform Distribution Density)

(2) Note: the average of random variables is itself is random variable and its associated distribution

(3) The center of this distribution is the same as that of the original distribution.

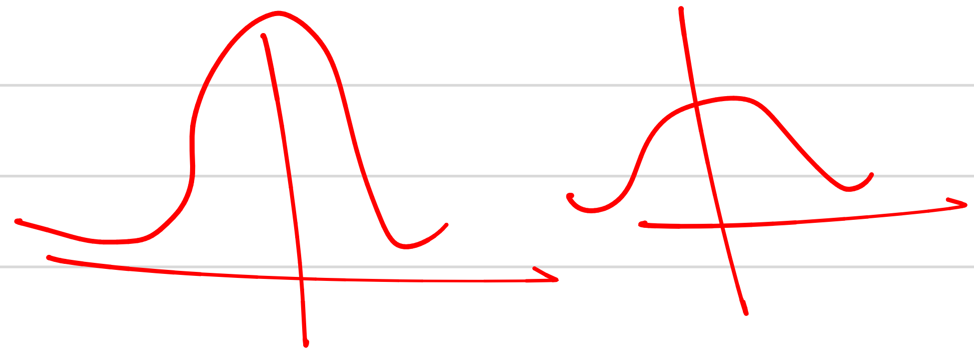
4. The sample mean is unbiased because its distribution is centered at what it's trying to estimate.

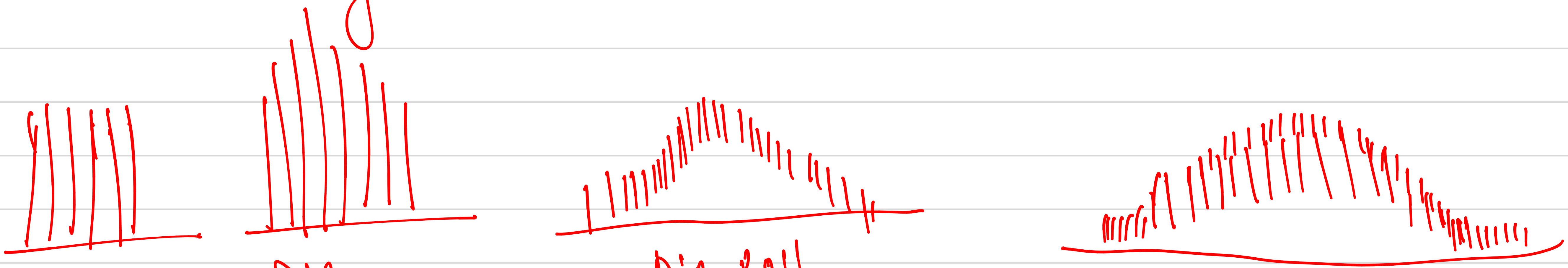
5. Example (Simulation experiment)



Average of X die rolls

→ Rolling more X balance out as Normal Distribution





Die Rolls 5 times

Die Rolls 10 times

Die Roll 25 times

50 times.

As the Average of X die Rolls it'll take the Normal distribution with the increase in Averages -

Summary:

1. Expected values are properties of distributions
2. The population mean is the center of mass of population
3. The sample mean is the center of mass of the observed data.
4. The sample mean is an estimate of the population mean
5. The sample mean is unbiased.
   - The population mean of its distribution is the mean that it's trying to estimate

The more data that goes into the sample mean, the more concentrated its density of mass function is around the population mean.

Variance:

The variance of a random variable is a measure of spread.

$$Var(X) = E\left[(X-\mu)^2\right]$$

$$\boxed{Var(X) = E\left[X^2\right] - E\left[X\right]^2}$$

The square root of the variance is called the Standard Deviation.

Example:

$$E[X] = 3.5$$

$$E[X^2] = 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + 3^2 \times \frac{1}{6} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6}$$

$$\Rightarrow 15.17$$

$$Var(X) = E[X^2] - E[X]^2$$

$$\Rightarrow 15.17 - (3.5)^2$$

$$\Rightarrow 2.92$$

What is the variance from the result of the toss of

a Coin with probability of heads (1) of p ?

Sol^n: $E[X] = 0 \times (1-p) + 1 \times p$

$= p$

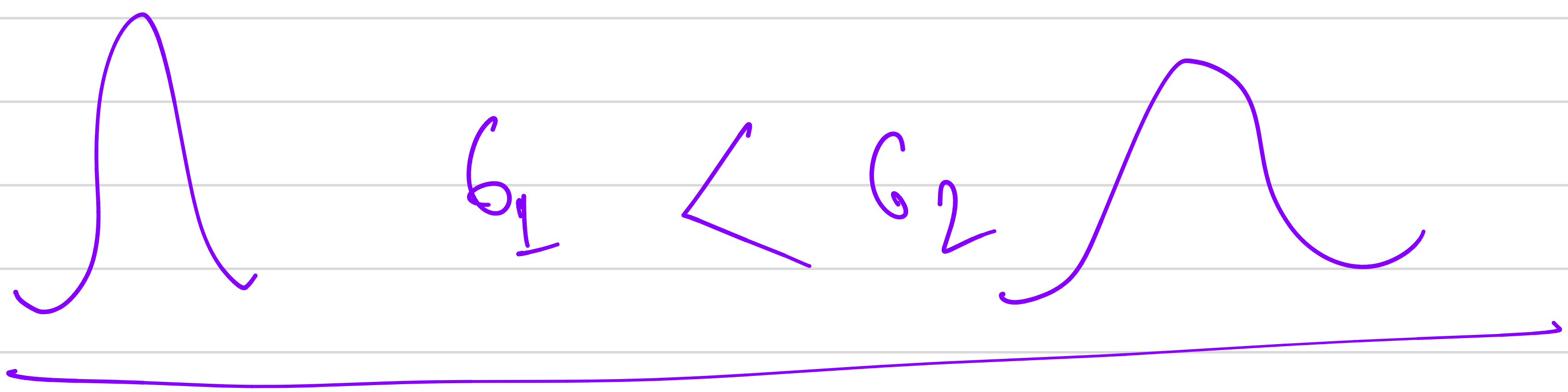
$E[X^2] = E[X] = p$

$Var(X) = E[X^2] - E[X]^2$

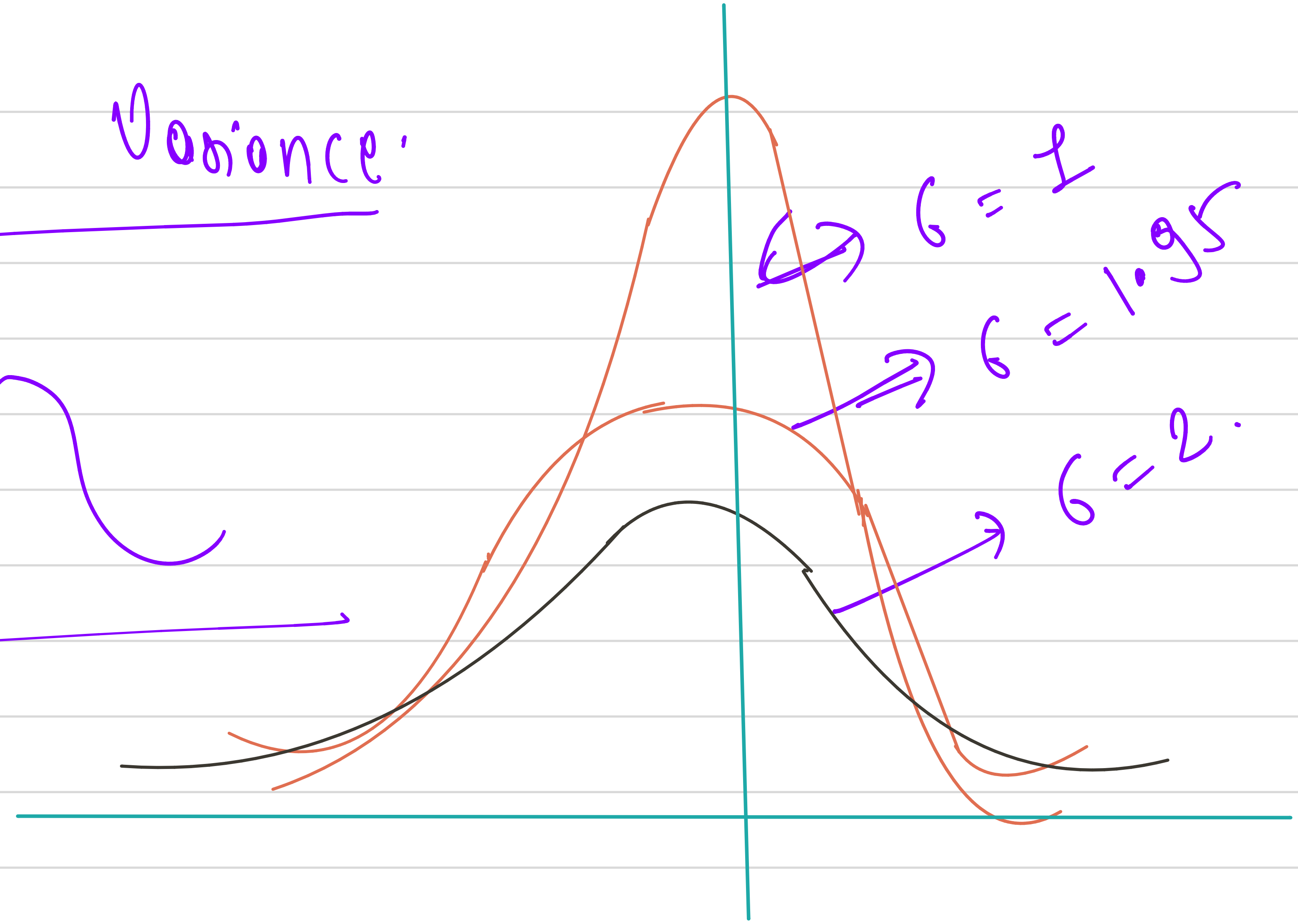
$= p - p^2$

$= p(1-p)$

Distributions with Increasing Variance:



$\sigma_1 < \sigma_2$



The Sample Variance:

$$\therefore \mathrm{Var}(X) = E[(X-\mu)^2]$$

$$\Rightarrow E[x^2] - E[x]^2$$

$$s^2 = \frac{\sum_{i=1} (x_i - \bar{x})^2}{n-1}$$ → Sample variance

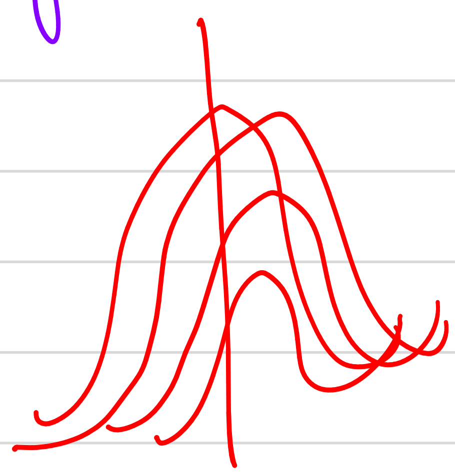
Variance of the Sample Variance:

→ Random Variable has a Population Distribution.

→ It has an associate population Distribution.

→ It's expected value is the population variance.

Simulation Experiment Examples:



Recalling the Mean:

Distribution of Sample Mean, the average of random sample from the population is itself a random variable.

$$E[\bar{X}] = \mu$$

$$Var(\bar{X}) = \frac{\sigma^2}{n}$$

Summarizing Points:

① So Imagine a population that has mean μ and variance

σ^2, so measure of spread is the sigma squared.

(2) If we take random variable take the variance, i.e. estimate of σ^2,

(3) The distribution of the sample variance is centered around σ^2/n.

→ It's logical estimate is s^2/n.

→ The logical estimate of the standard error is $s/\sqrt{n}$.

S, the standard deviation talks about how variable the population is.

$S/\sqrt{n}$ the standard error, talks about how variable averages of random samples of size n from the population are

Standard normals have variance 1, means of n standard normals have $\sigma = 1/\sqrt{n}$,

↳ where n = 1000

↳ n = 10

↳ pattern present in data distribution, different distributions are applied to different models.

Poisson Distribution:

↳ Models the probability that a certain number of events will occur during a specific time period.

↳ It is also used to represent the number of events that occur in a specific space such as distance, volume, area, time.

Data professionals use the Poisson distribution to model data such as

- Calls per hour for a customer service call center
- Visitors per hour for a website
- Customers per day of a restaurant

- Severe storms per month in a city.

Attributes of Poisson Experiment:

→ n = no. of events in the experiment can be counted.

→ $\bar{X}_n$ ⇒ mean number of events that occur during a specific time period is known.

→ Each Event is Independent.

↳ We can count the number of orders.

↳ Average of 2 orders per minute

↳ The probability of one person placing an order does not affect the probability of another person placing an order.

Poisson Distribution Formula :

$$P(X=k) = \frac{\lambda^{k} e^{-\lambda}}{k!}$$

where → λ refers to the mean number of events that occur during a specific time period

→ k refers to the number of events

→ e is a constant equal to 2.71828

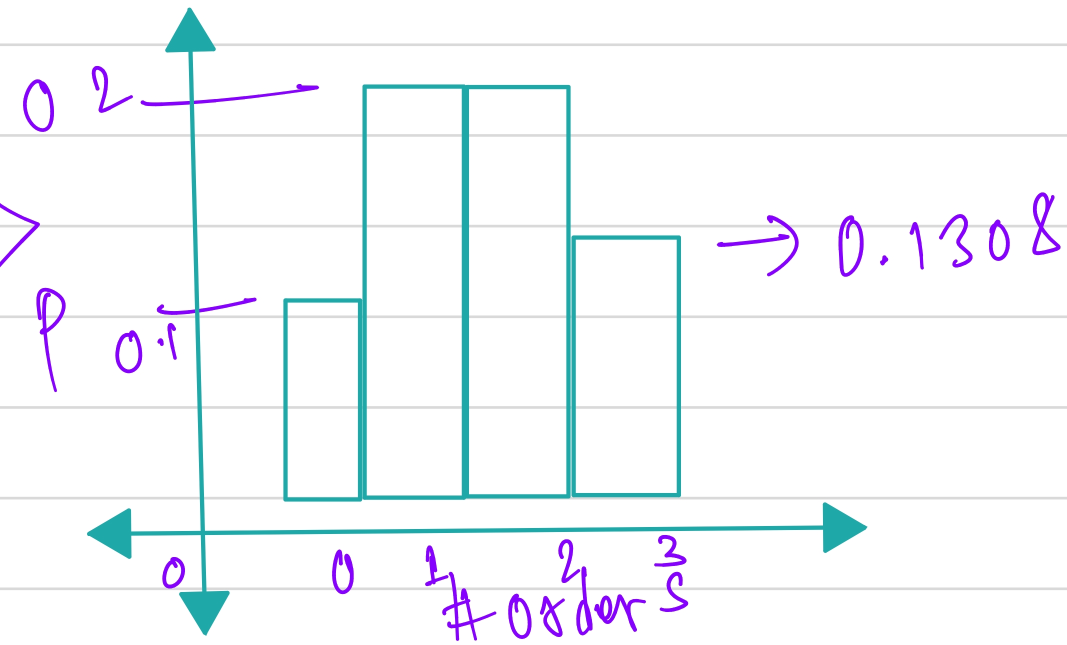
→ the exclamation point $k!$ stands for factorial.

Q. What is the probability that orders will be 0, 1, 2 for the specific point of time?

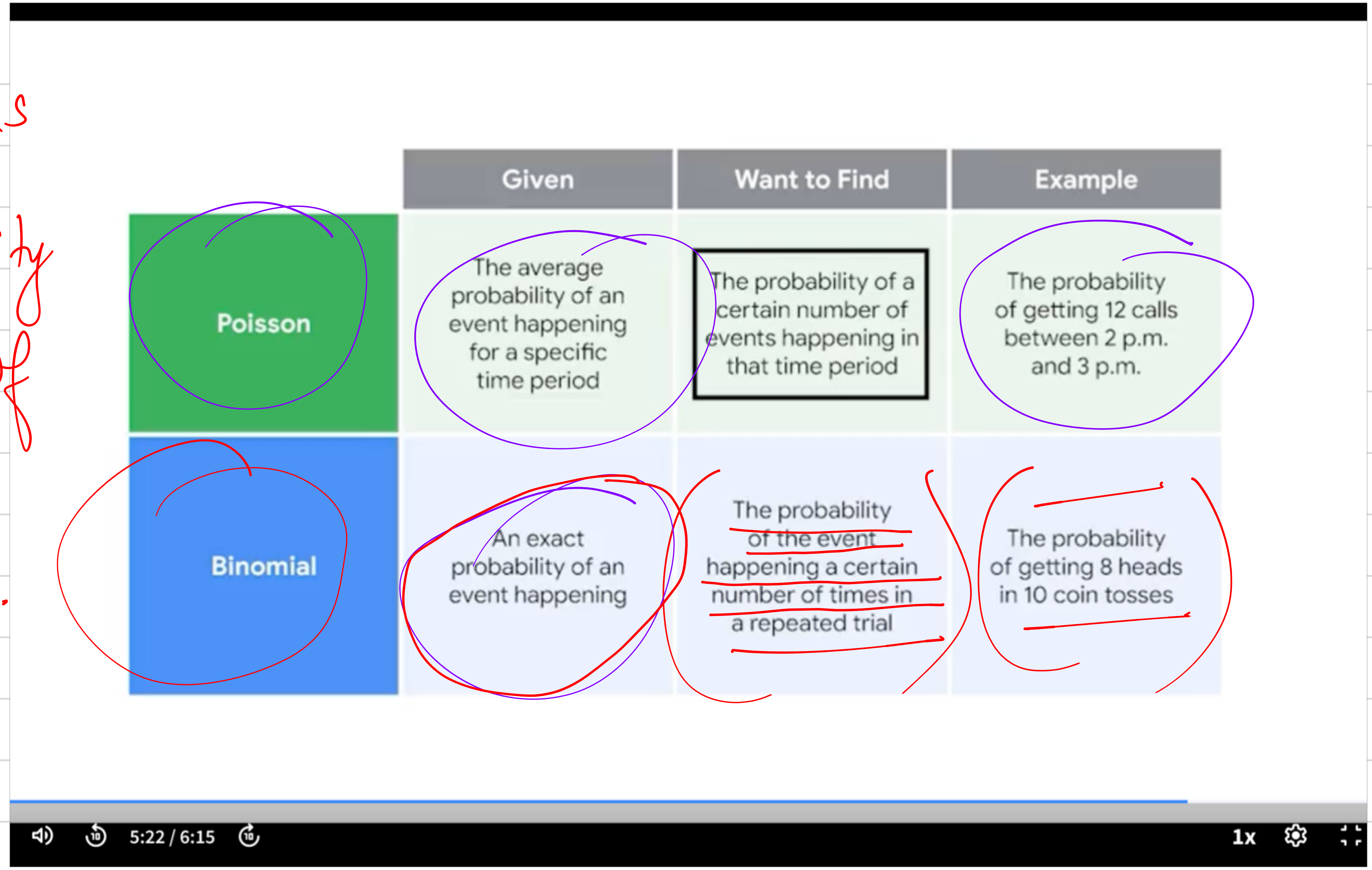
$$P(X=K) = \frac{\lambda^{K} e^{-\lambda}}{K!}$$

- $P(X=0) \Rightarrow 0.1353$ = Probability that the order 0 will be 0.1353.
- $P(X=1) \Rightarrow 0.2707$
- $P(X=2) \Rightarrow 0.2707$
- $P(X=3) = 0.1308$

$\Rightarrow$



The Poisson distributions can model the probability that certain number of events during a specific time period.



Binomial Distribution →

A discrete distribution that models the probability of events with only two possible outcomes, success or failure.

Source: Coursera, the Power of Statistics.