Day-14, Nov-29, 2024 (Mangshir-14, 2081 B.S.)

Limit at Infinity: Horizontal Asymptotes

What happens to the function as 'x' gets really big (+ve or -ve)

Sometimes the function will appoarch a specific number as x gets big

If a function f appoarches a specific number L as 'x' gets larger and larger (positive), as we say that the limit f(x) as 'x' appoarches infinity is L and write,

$$\lim_{x \to \infty} f(x) = L$$

Similarly, if f appoarches L as x becomes larger and larger negative,

$$\lim_{x \to -\infty} f(x) = L$$

In general, we use the notation $\lim\limits_{x \to \infty} f(x) = L$ to indicate that the values of $f(x)$ approaches L as x becomes larger and larger.

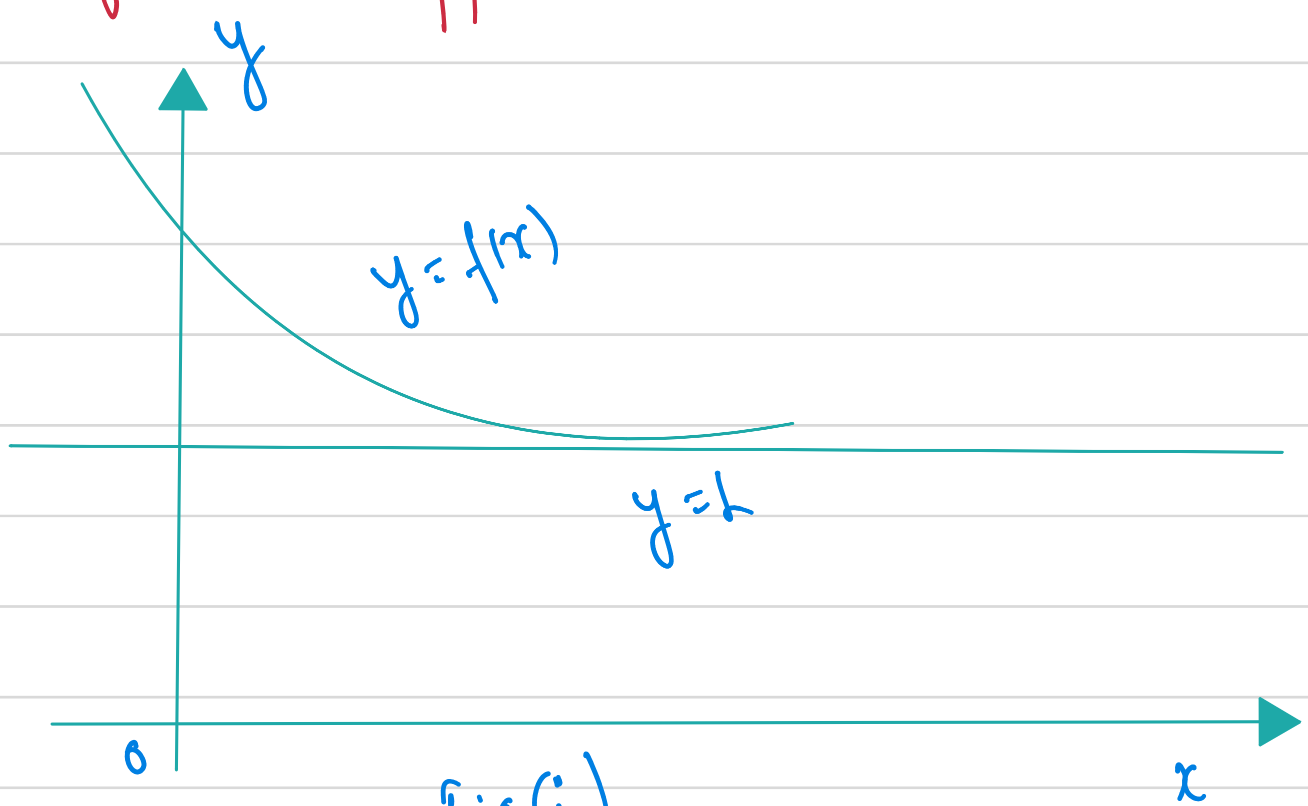


Fig (i)

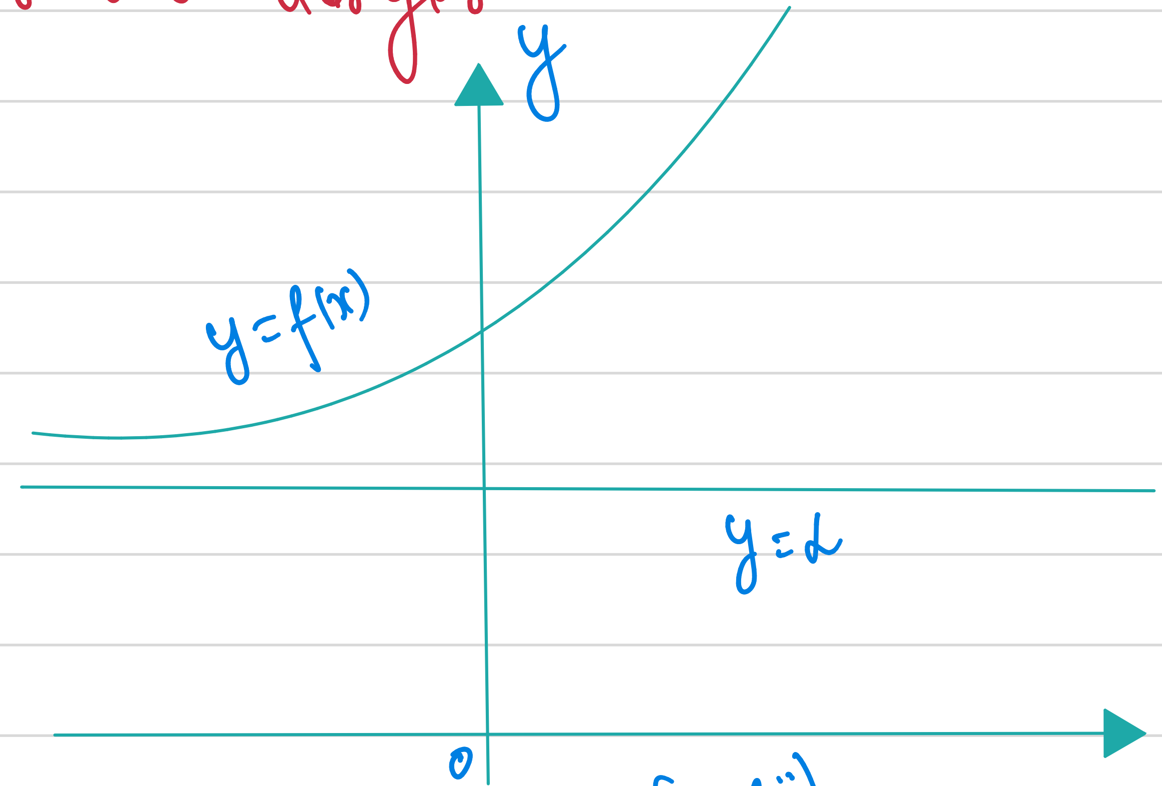


fig (ii)

$$\lim_{x \to \infty} f(x) = L \quad \text{or} \quad f(x) = L \to L \text{ as } x \to \infty$$

Horizontal Asymptotes: The line $y = L$ is called a horizontal asymptote of the curve $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = L \quad \text{or} \quad \lim_{x \to -\infty} f(x) = L$$

Both fig (i) and fig (ii) $f(x)$ flattens and approaches the line $y = L$.
So, we can call both a horizontal asymptote $f(x)$.

$$\lim_{x \to \infty} f(x) = L \quad \text{or} \quad \lim_{x \to -\infty} f(x) = L.$$

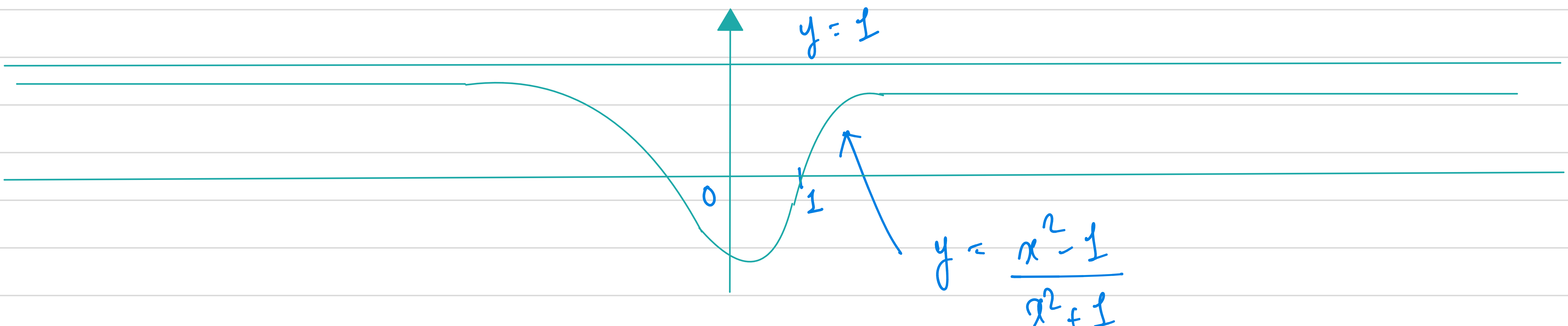
Example that illustrates the definition of limit at Infinity and horizontal Asymptote of the curve $y = f(x)$.

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

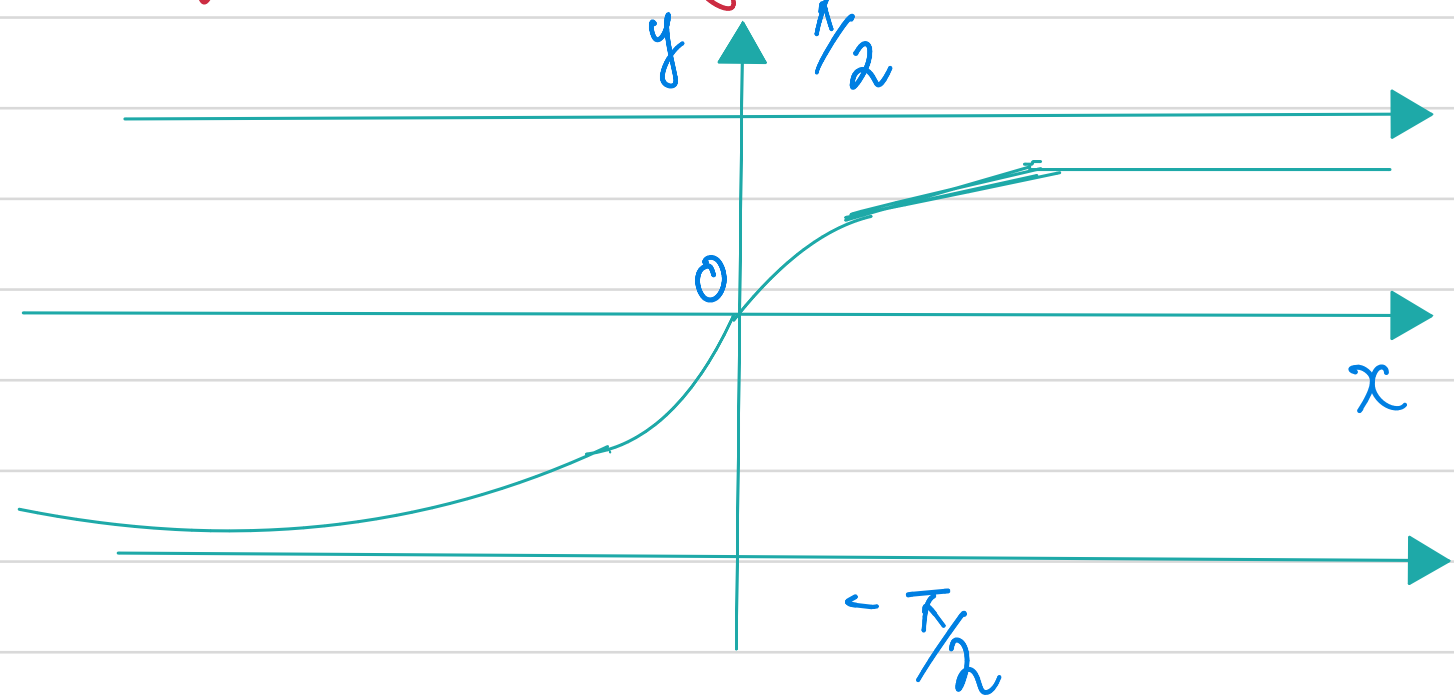
As x grows larger and larger we can see that the values of $f(x)$ gets closer and closer to 1.

In fact, it seems that we can make the values of $f(x)$ as close as we like to '1' by taking x sufficiency (may be negative or positive). This situation expressed symbolically by writing

$$\lim_{x \to \infty} \frac{x^2-1}{x^2+1} = 1$$



Example: Find the horizontal asymptotes of the curve $y = \tan^{-1} x$.



Sol^n;

$$\lim_{x \to \infty} \tan^{-1} x$$

$$\Rightarrow \tan^{-1}(\infty)$$

$$\Rightarrow \pi/2$$

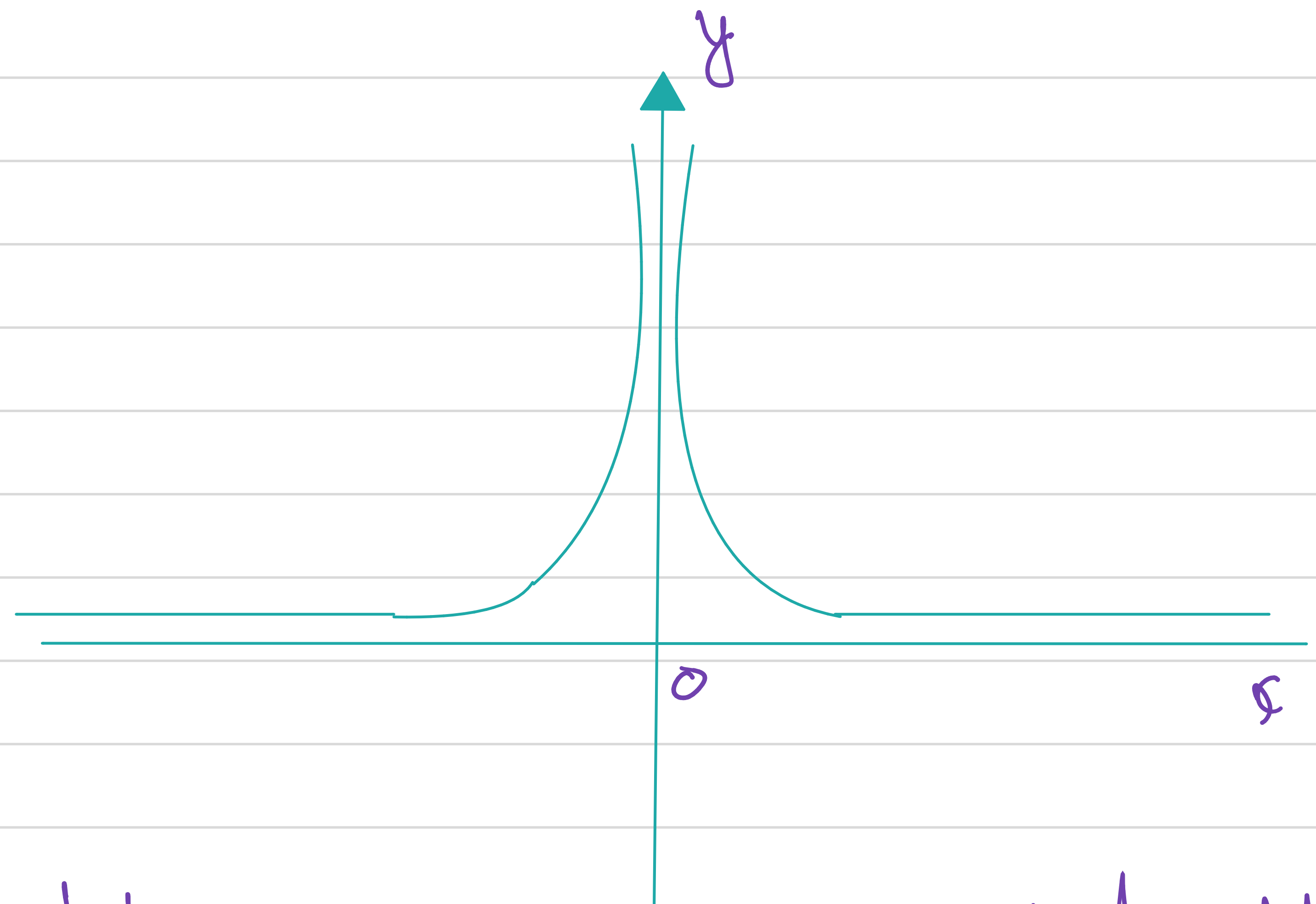
$\therefore y = \pi/2$ is a horizontal asymptote.

Again, $\lim_{x \to -\infty} \tan^{-1} x = \tan^{-1}(-\infty)$

$\Rightarrow -\pi/2$ which shows that $y = -\pi/2$ is also another horizontal asymptotes of the same function $y = \tan^{-1} x$.

Definition of Vertical Asymptote:

Let $f(x)$ be a given function. If there exists a number a such that any one of the following is true;



a. $\lim_{x \to a} f(x) = \infty$ or $-\infty$

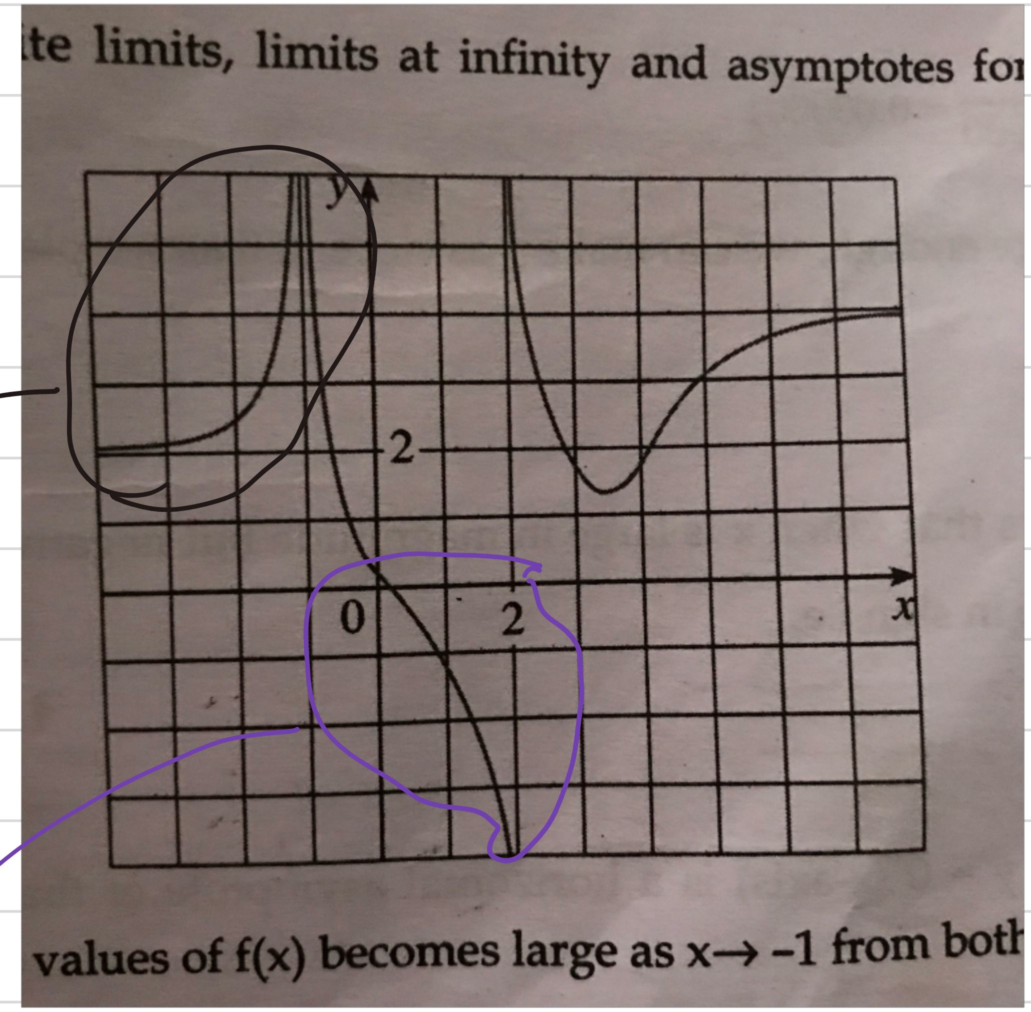
b. $\lim_{x \to a^-} f(x) = \infty$ or $-\infty$

c. $\lim_{x \to a^+} f(x) = \infty$ or $-\infty$

Then, $x = 0$ is called the vertical asymptote. and $y = 0$ is horizontal Asymptote.

Since, $\left[\lim_{x \to \infty} f(x) = h \text{ or } \lim_{x \to -\infty} f(x) = d.\right]$

Example: Find the infinite limits, limits at Infinity and asymptotes for the function f whose graph in figure.



We see that the values of $f(x)$ becomes large as $x \to -1$ from both sides, so

$$\lim_{x \to -1} f(x) = \infty .$$

Notice that $f(x)$ becomes large negative as x approaches 2 from the left, but large positive as x approaches 2 from the right. So,

$$\lim_{x \to 2^-} f(x) = -\infty \quad \text{and} \quad \lim_{x \to 2^+} f(x) = \infty$$

Both the lines $x = -1$ and $x = 2$ has infinite limits so are the vertical asymptote.