Day-80, Feb 18, 2025 (Falgun 07, 2081)

Motivation to Derivatives -

$$\text{Velocity} = \frac{\text{distance traveled}}{\text{time}}$$

[velocity between ⇒ 10 - 15 seconds] -

$$\text{velocity} = \frac{y(15) - y(10)}{5} \quad \Big] \rightarrow \text{These are given datas.}$$

$$\Rightarrow \frac{202 - 122}{5}$$

$$\Rightarrow 16\,m/s$$

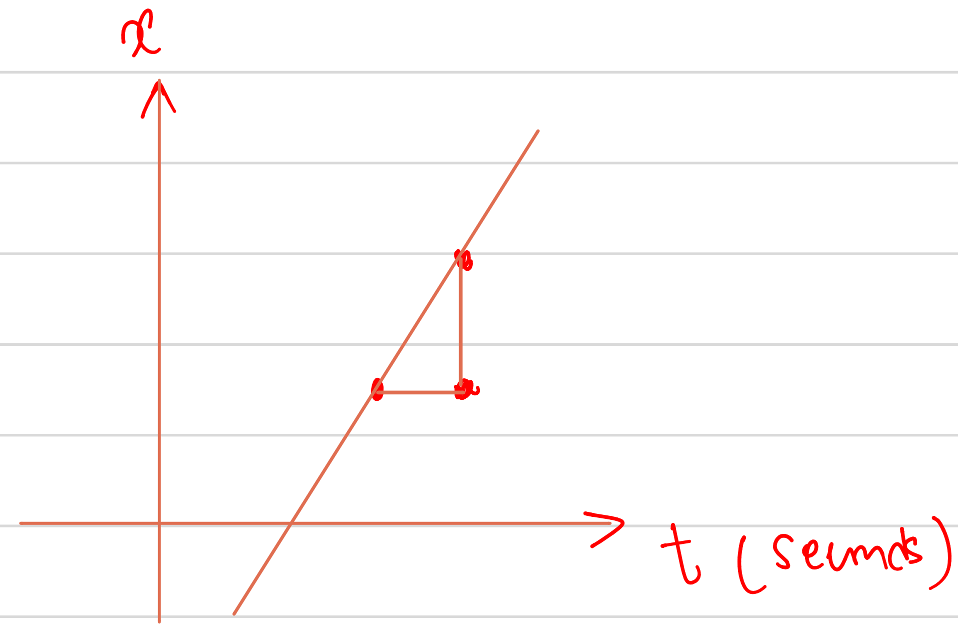
$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\Rightarrow \frac{\text{Change in distance } (\Delta x)}{\text{Change in time } (\Delta t)}$$

$$\Rightarrow \frac{x(15) - x(10)}{t(15) - t(10)}$$

$$\Rightarrow \frac{202\text{m} - 122\text{m}}{15\text{s} - 10\text{s}}$$

$$\Rightarrow \frac{80\text{m}}{5\text{s}} \quad \boxed{\text{Slope} = 16 \text{ m/s}}$$



or [The velocity is 16 m/s.]

t (s)	x (m)
10	122
11	138
12	155
13	170
14	186
15	202

t = 12.5 seconds

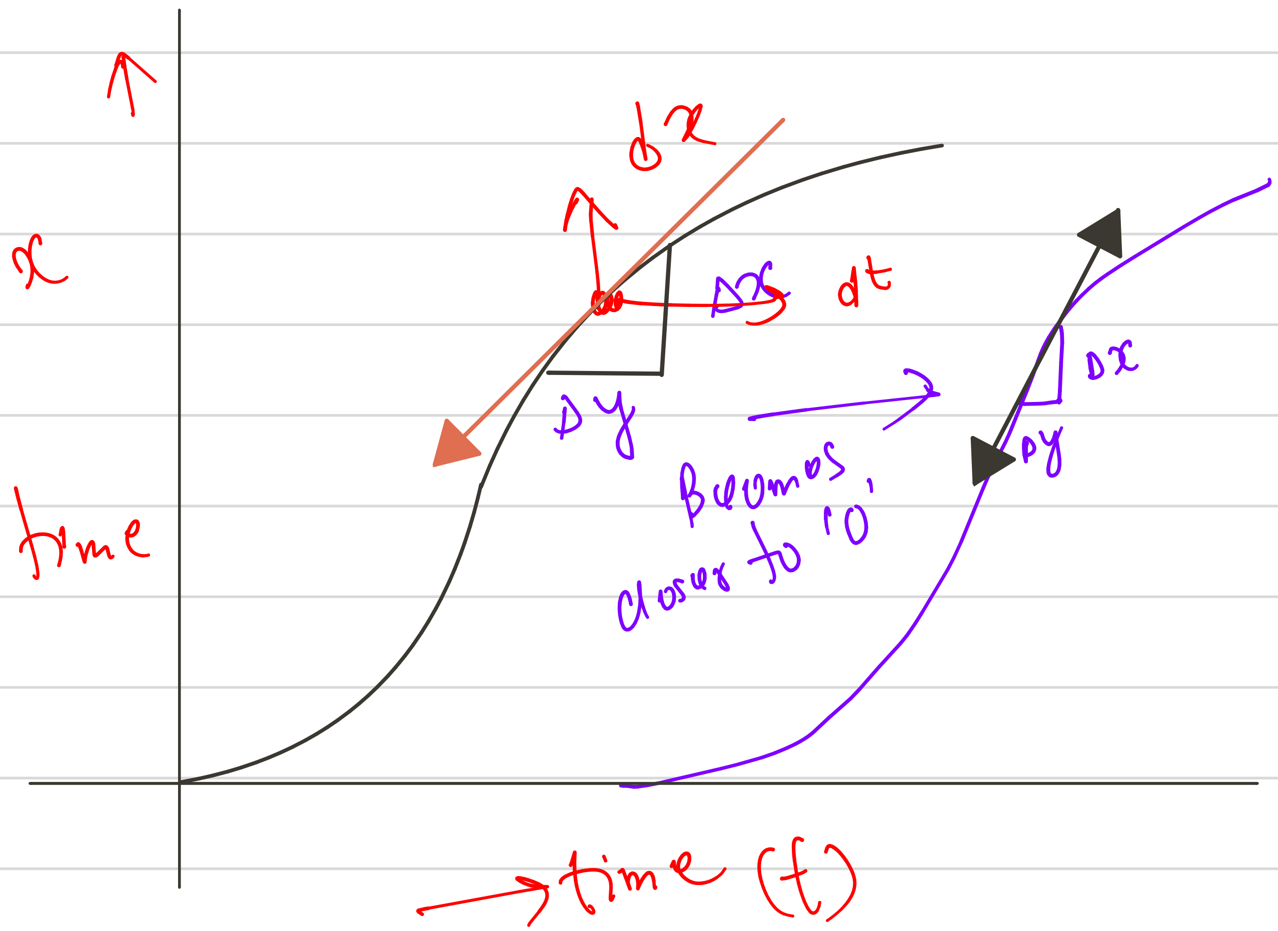
t = 12 to t = 13

$$y(13) - y(12) \,/\, 1$$

$$\Rightarrow 0.5 \text{ m/s.}$$

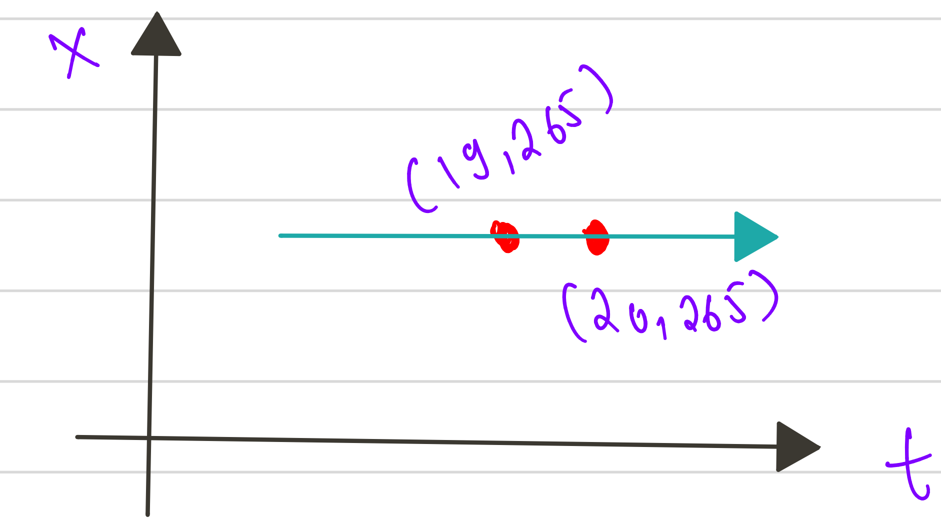
$$\text{Slope} = \frac{\Delta x}{\Delta y} \quad \text{so,} \quad \frac{dx}{dt}$$

We call instantaneous Rate of change → moving tiny tiny dx in tiny tiny time



$\text{Derivative} = \frac{dx}{dt}$ tiny tiny changes at that point.

Zero Slope

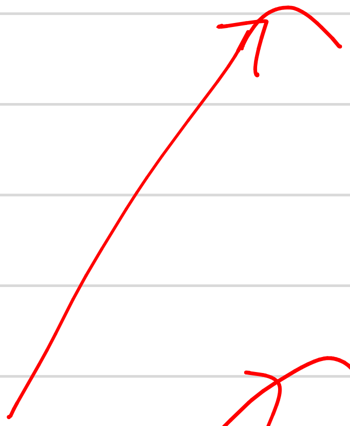


$$\text{Slope} = \frac{\Delta x}{\Delta t}$$

$$\Rightarrow \frac{x(20) - x(19)}{20 - 19}$$

$$\Rightarrow \frac{265 - 265}{1}$$

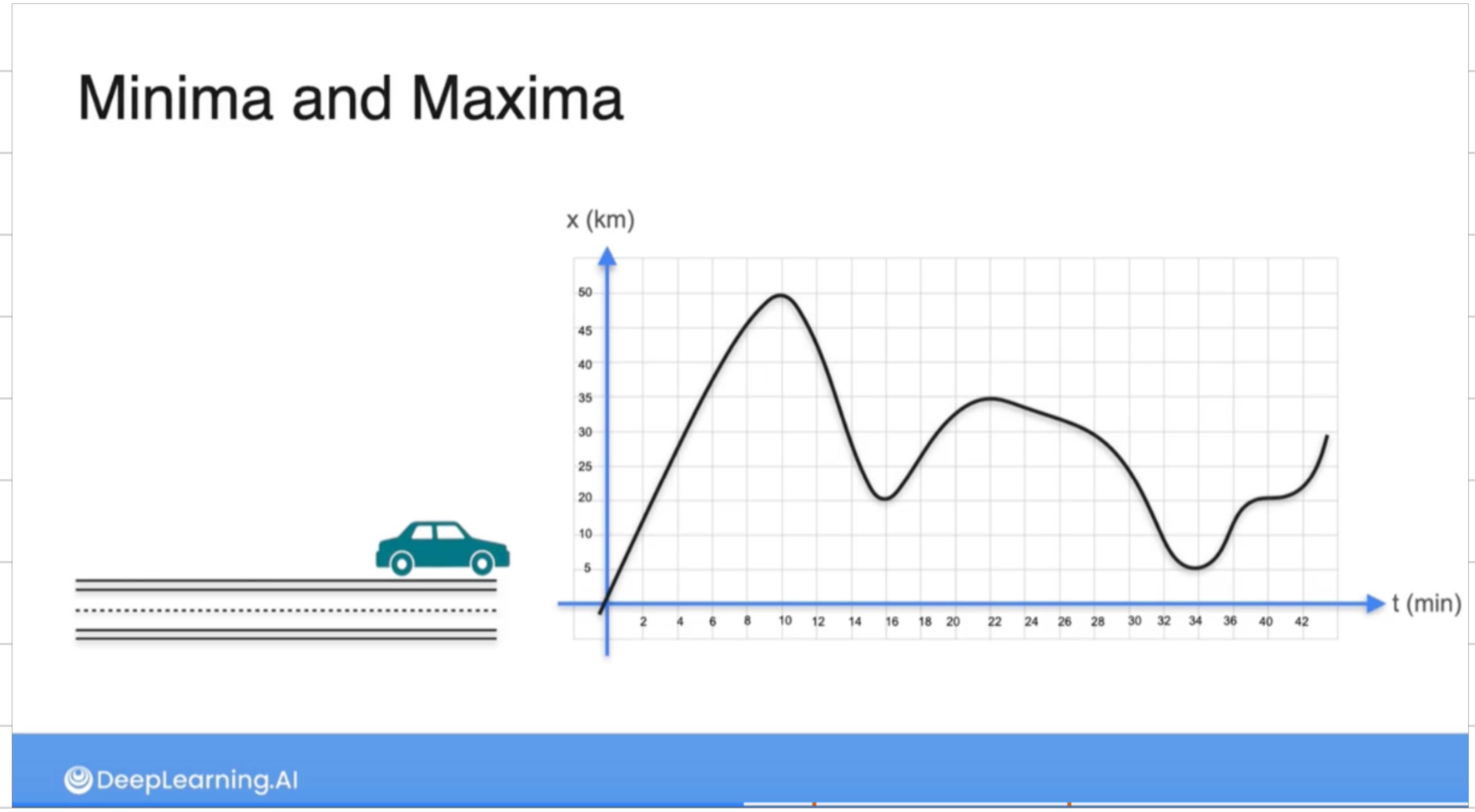
$\left[\text{Slope} = 0/1\right]$ → No Rise in Δx so Zero Slope.

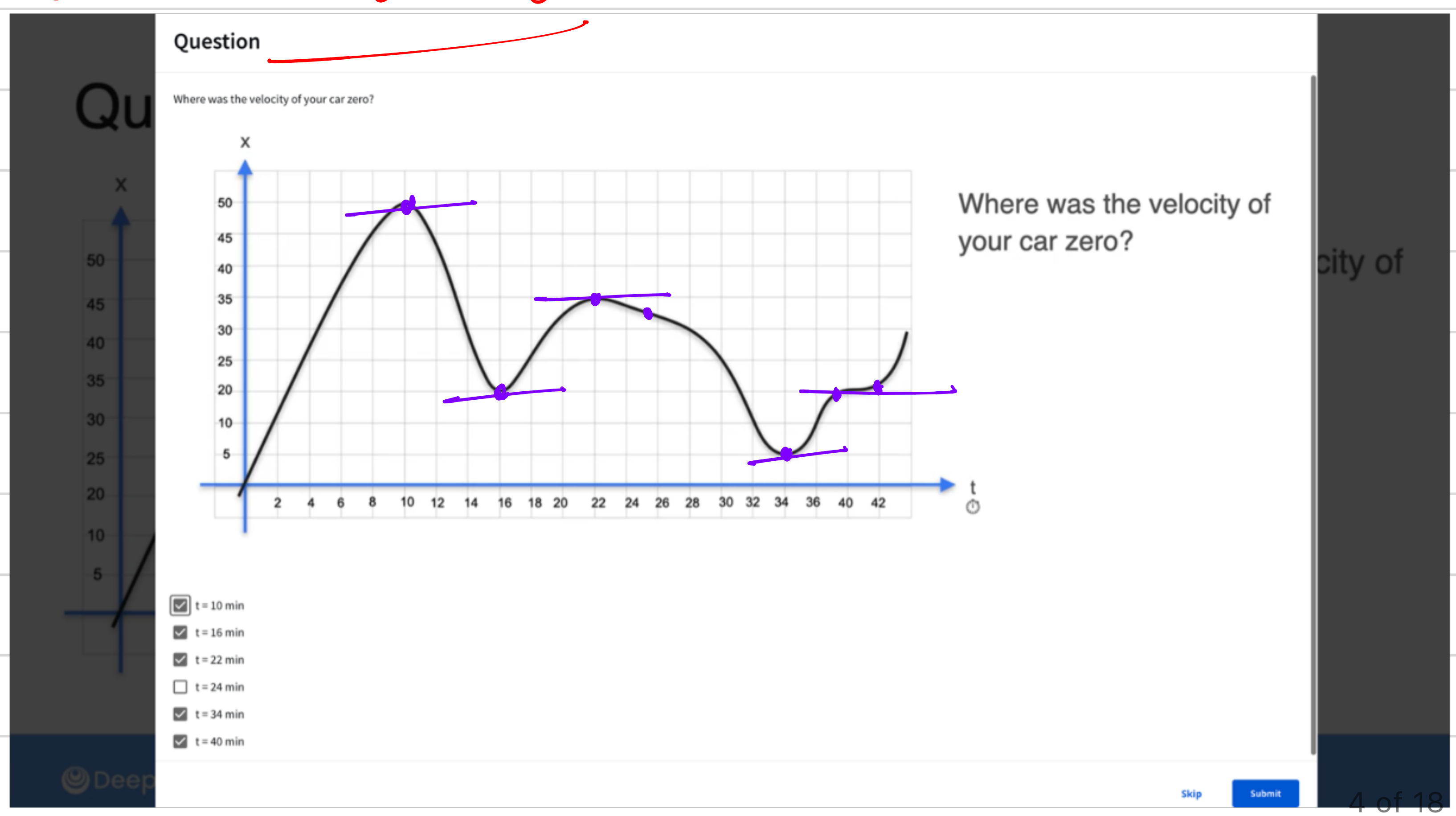


→ Car moves forward

→ Car moves backward

→ Car moves forward and so on.





→ Find the horizontal slope where the velocity is = 0 ?

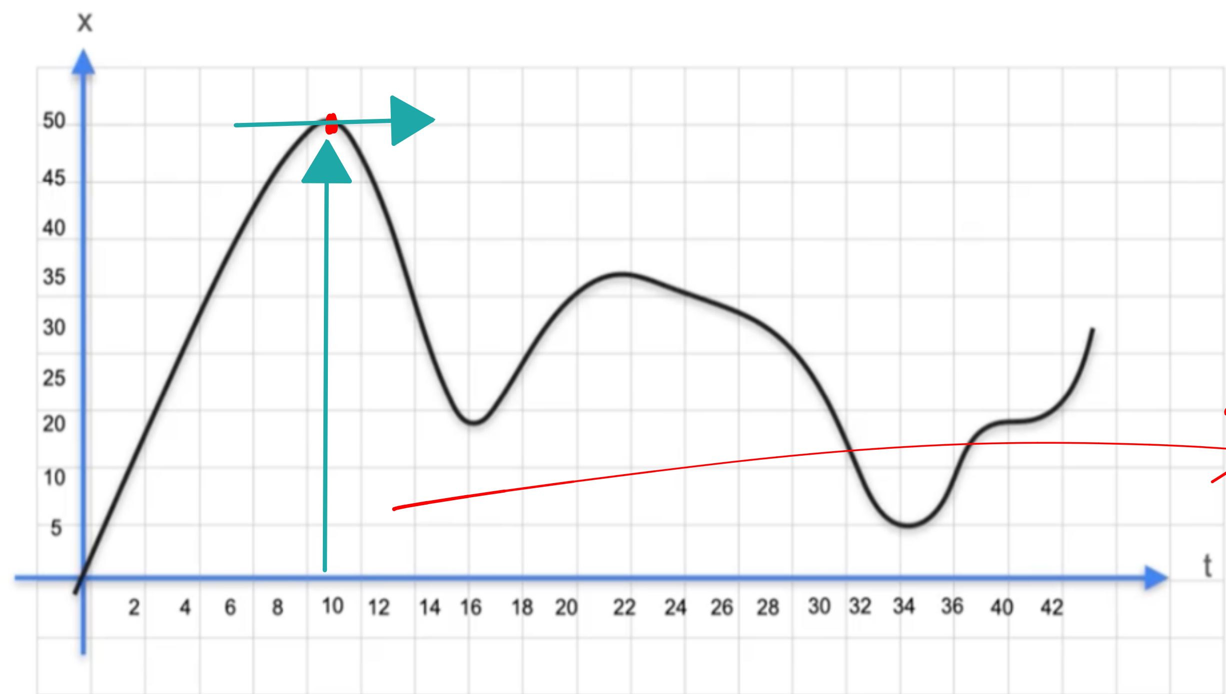
Where tangent line is Horizontal

it can be max or min ?

Local Maxima

Local Minima.

Quiz : Concept of Derivatives 3



At what time was the car farthest from its starting point?

→ Farthest

→ Highest Distance.

→ point where Car Stopped.

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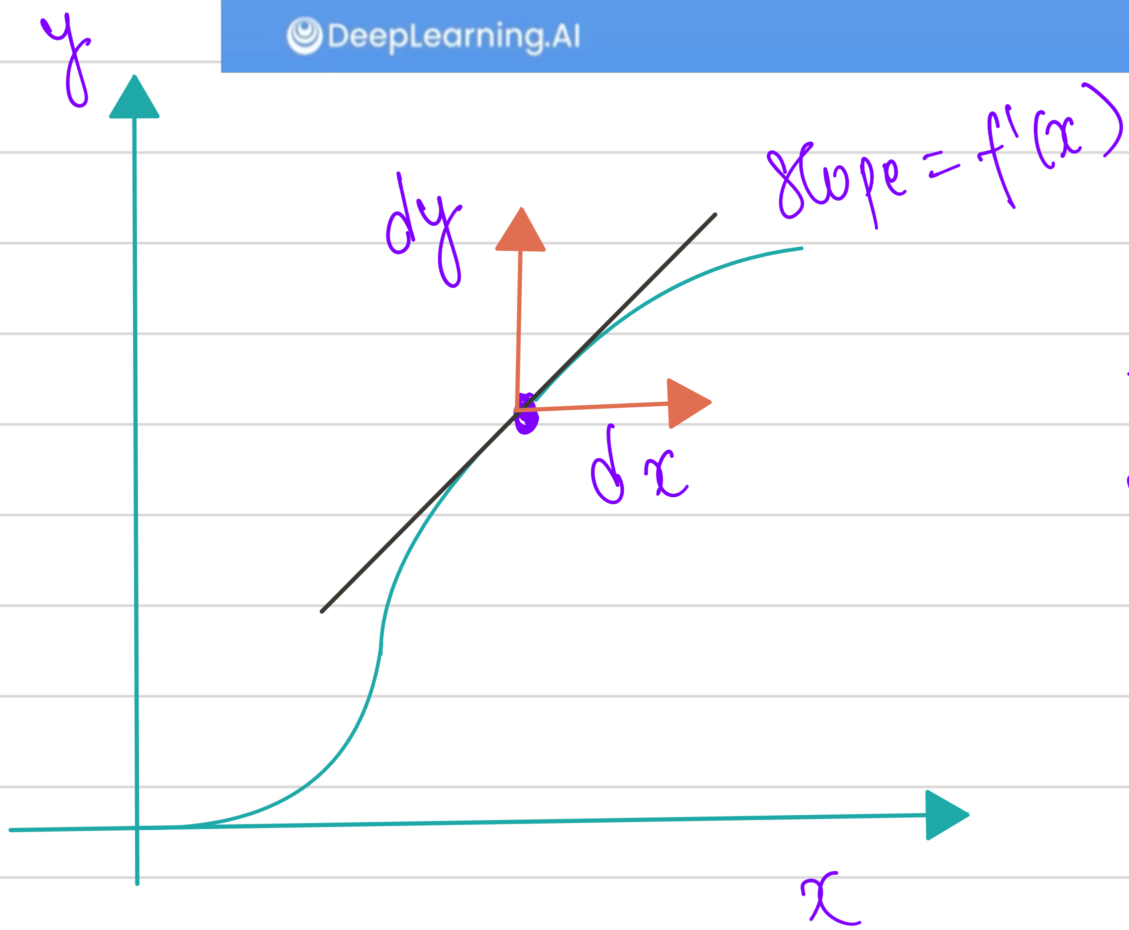
Derivatives:

Slope = $\frac{\Delta x}{\Delta t}$

Slope at a point $\Rightarrow \frac{dx}{dt}$

$y = f(x)$



→ Lagrange's And Leibniz's Notation.

$f'(x)$ → Lagrange's

$\frac{dy}{dx} = \frac{d}{dx} f(x)$ → Leibniz's Notation

Derivatives - Some Common Derivatives - Lines

$y = f(x) = C$

Slope?

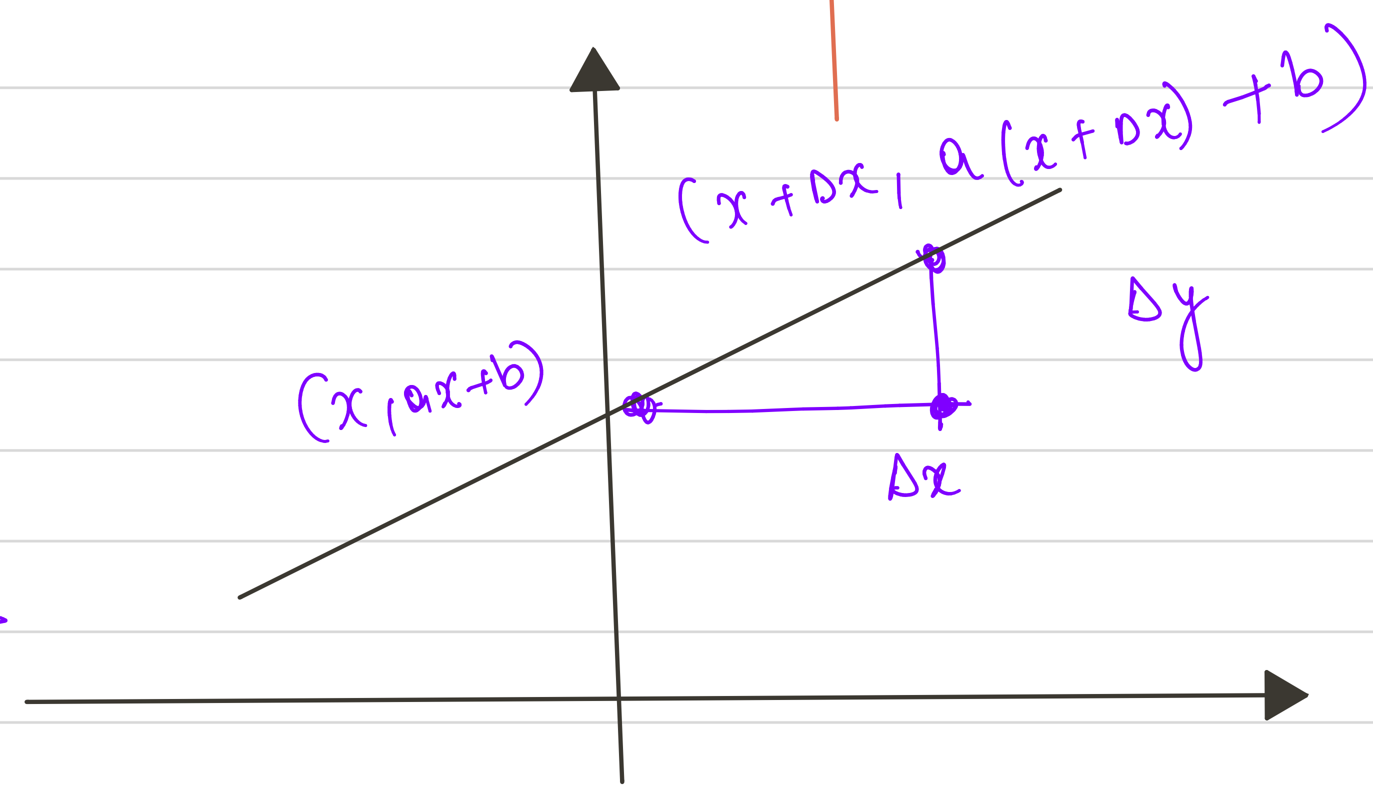
$$\frac{\Delta y}{\Delta x} \Rightarrow \frac{C - C}{x_1 - x_0} \Rightarrow 0 \rightarrow \boxed{f'(x) = 0}$$

$f(x) = ax + b$

$f'(x) \Rightarrow a$

$$\frac{\Delta y}{\Delta x} \Rightarrow \frac{\text{rise}}{\text{run}}$$

$$\frac{\Delta y}{\Delta x} = \frac{a(x + \Delta x) + b - (ax + b)}{\Delta x}$$



$$\frac{df}{dx} \cdot \frac{\Delta y}{\Delta x} \Rightarrow \frac{a(\cancel{x}+\Delta x)+\cancel{b}-(\cancel{ax}+\cancel{b})}{\Delta x}$$

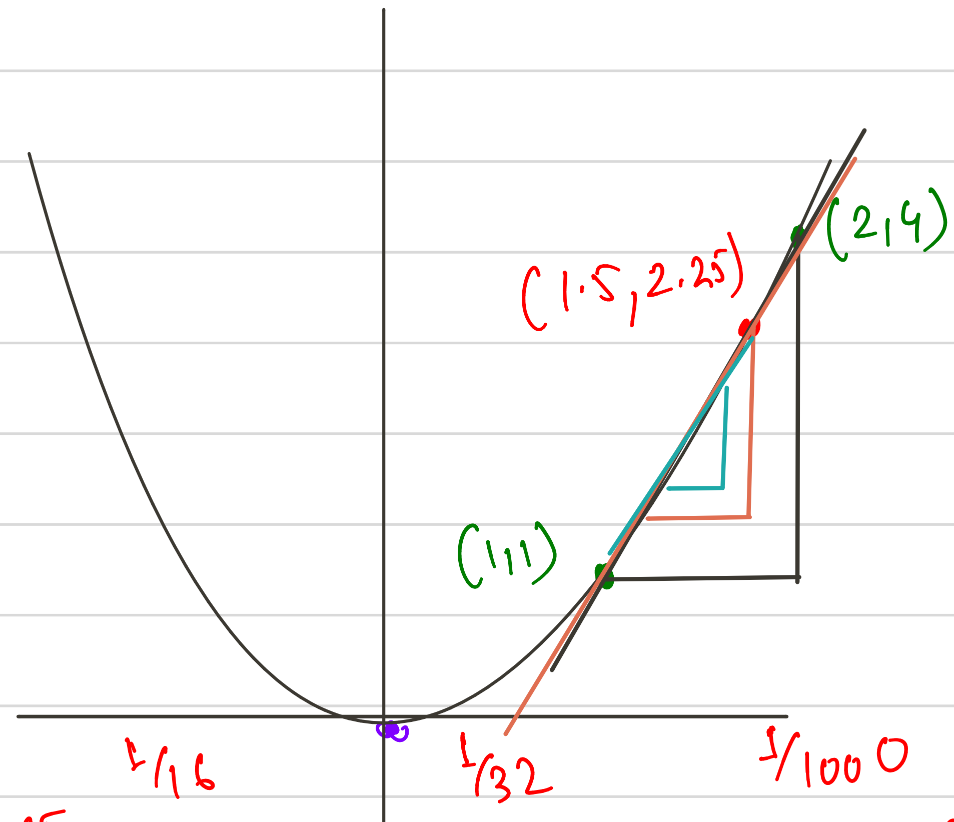
$$\Rightarrow a \cdot \frac{\Delta x}{\Delta x}$$

$$\boxed{\frac{df}{dx} \Rightarrow a}$$

Derivative of Quadratic functions

Quadratics : $y = f(x) = x^2$

Slope : $\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^2 - (x)^2}{\Delta x}$



$x = 1 \Rightarrow$

Δx	1.0	1/2	1/4	1/8	1/16	1/32	1/1000
Δf	3	1.25	0.562	0.265	0.128	—	0.002
Slope	3	2.5	2.25	2.125	2.065	- - - .	2.001

Moving towards to the 2

$f'(1) \Rightarrow \frac{d}{dx} f(1)$

$\Rightarrow 2$

$\Rightarrow 2 \times 1.$

$y = f(x) = x^2$

$\frac{\Delta f}{\Delta x} \Rightarrow \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$\Rightarrow \frac{(x+\Delta x)^2 - x^2}{\Delta x}$

$\Rightarrow \frac{\cancel{x^2} + 2\Delta x \cdot x + (\Delta x)^2 - \cancel{x^2}}{\cancel{\Delta x}}$

$\Rightarrow 2x + \Delta x$ as $\Delta x \to 0$

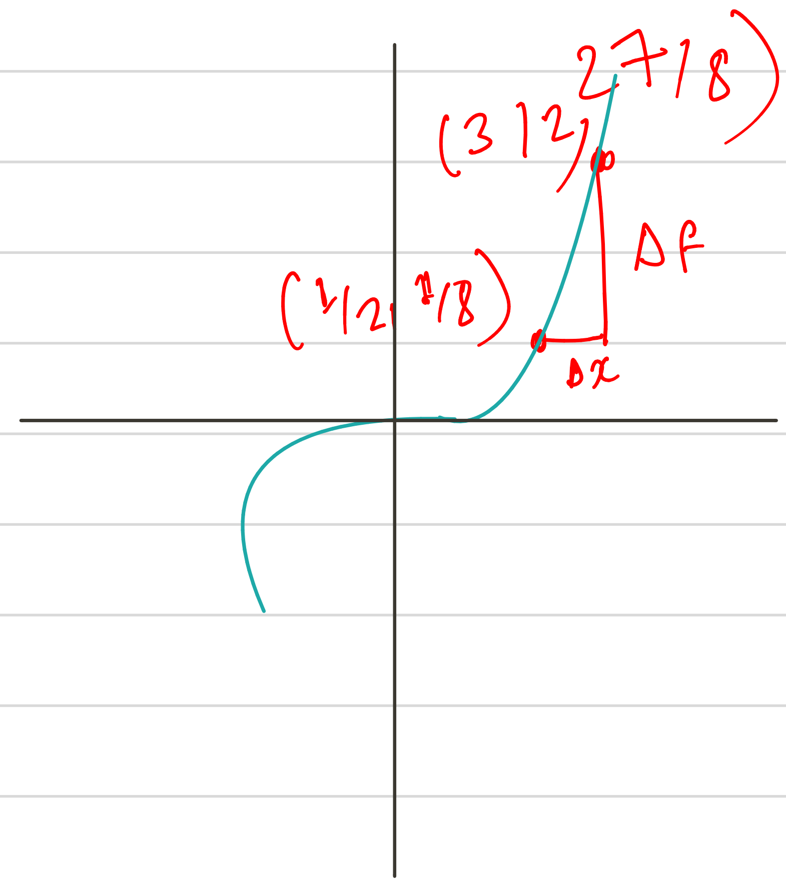
$f(x) \Rightarrow x^2 \Rightarrow f'(x) \Rightarrow 2x$

dx of Cubic functions:

Cubic: $y = x^3$

Slope $\Rightarrow \dfrac{\Delta f}{\Delta x} = \dfrac{(x+\Delta x)^3 - (x)^3}{\Delta x}$

If $x = 0.5$



$\Delta x \quad 1.0$

$\Delta f \quad 3.25$

$\left(\frac{1}{2}+1\right)^3 - \left(1/2\right)^3 = 27/8 - 1/8$

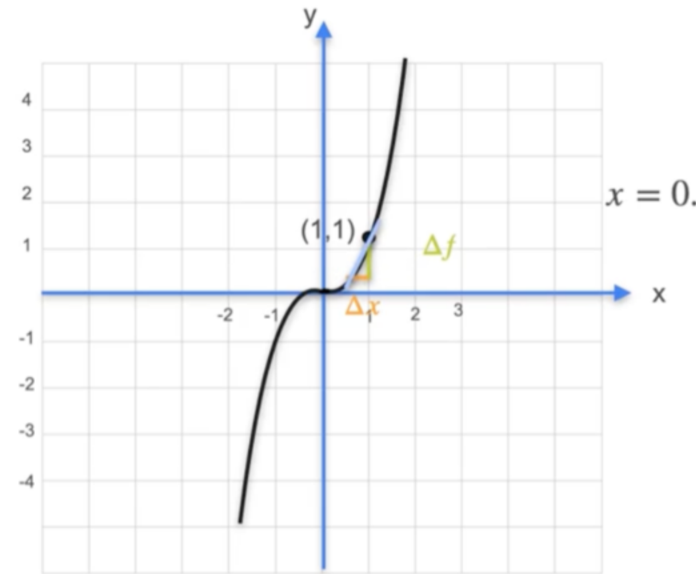
$\Rightarrow \dfrac{3.25}{1}$

$f'(0.5) = \dfrac{d}{dx} f(0.5) \Rightarrow (0.75)$

$\Rightarrow 3 \times (0.5)^2$

Derivative of Cubic Functions

1



Cubic: $y = f(x) = x^3$

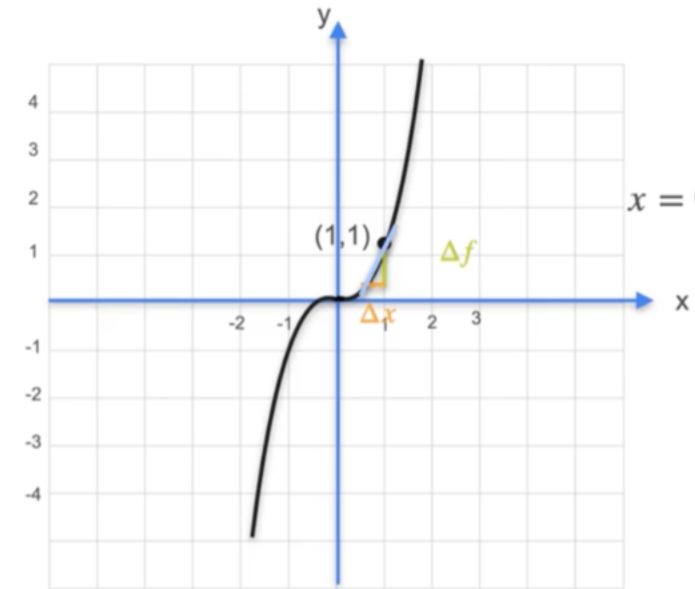
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^3 - (x)^3}{\Delta x}$

$x = 0.5$

Δx	1.0	1/2
Δf	3.25	0.86
Slope	3.25	1.75

Derivative of Cubic Functions

2



Cubic: $y = f(x) = x^3$

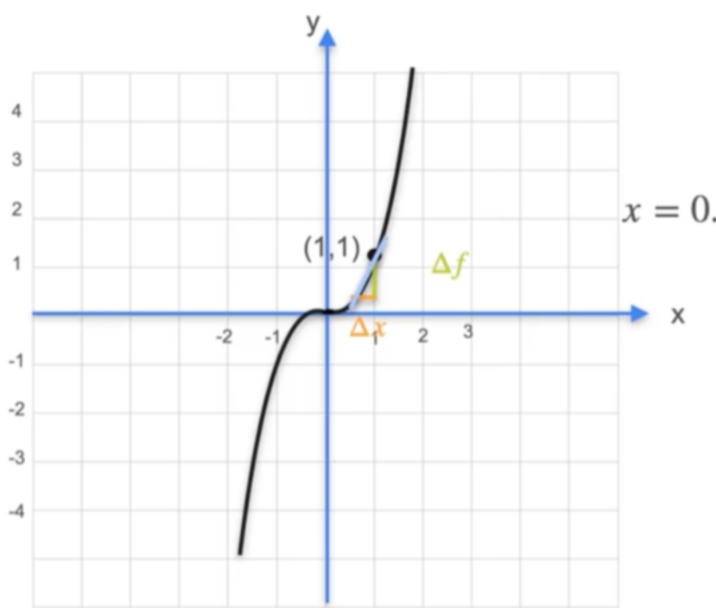
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^3 - (x)^3}{\Delta x}$

$x = 0.5$

Δx	1.0	1/2	1/4
Δf	3.25	0.86	0.30
Slope	3.25	1.75	1.188

Derivative of Cubic Functions

3



Cubic: $y = f(x) = x^3$

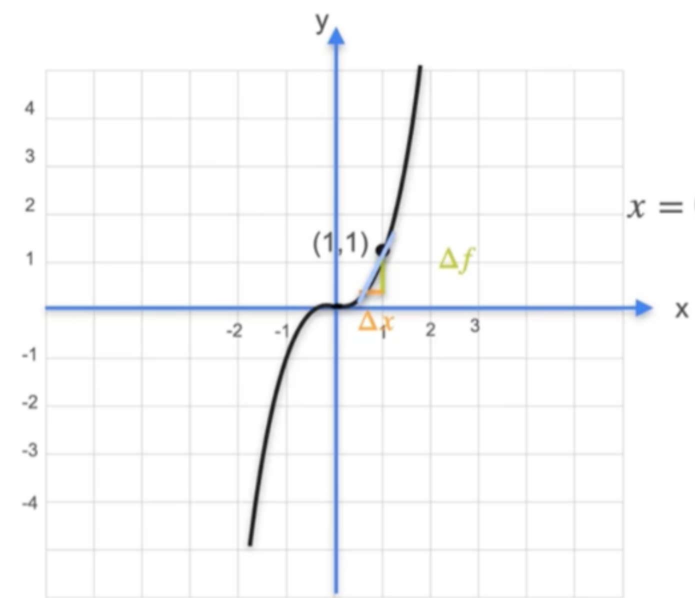
Slope: $\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^3 - (x)^3}{\Delta x}$

$x = 0.5$

Δx	1.0	1/2	1/4	1/8	1/16
Δf	3.25	0.86	0.30	0.12	
Slope	3.25	1.75	1.188	0.95	

Derivative of Cubic Functions

4



Cubic: $y = f(x) = x^3$

Slope: $\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^3 - (x)^3}{\Delta x}$

$x = 0.5$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	3.25	0.86	0.30	0.12	0.05	0.0008
Slope	3.25	1.75	1.188	0.95	0.85	0.752

Cubic: $y = f(x) \Rightarrow x^3$

Slope: $\dfrac{\Delta f}{\Delta x} = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta f}{\Delta x} \Rightarrow \frac{(x+\Delta x)^3 - x^3}{\Delta x}$$

$$\Rightarrow \frac{\cancel{x^3} + 3x(\Delta x)^{\cancel{2}} + 3x^2 \cancel{\Delta x} + (\Delta x)^{\cancel{3}} - \cancel{x^3}}{\cancel{\Delta x}}$$

$$\Rightarrow 3x\Delta x + 3x^2 + \Delta x^2$$

As $\Delta x \rightarrow 0$

$$\Rightarrow 3x^2$$

$f(x) \Rightarrow x^3$

$f'(x) \Rightarrow 3x^2$

Some Common Derivatives - Other Power Functions:

Inverse: $y = f(x)$

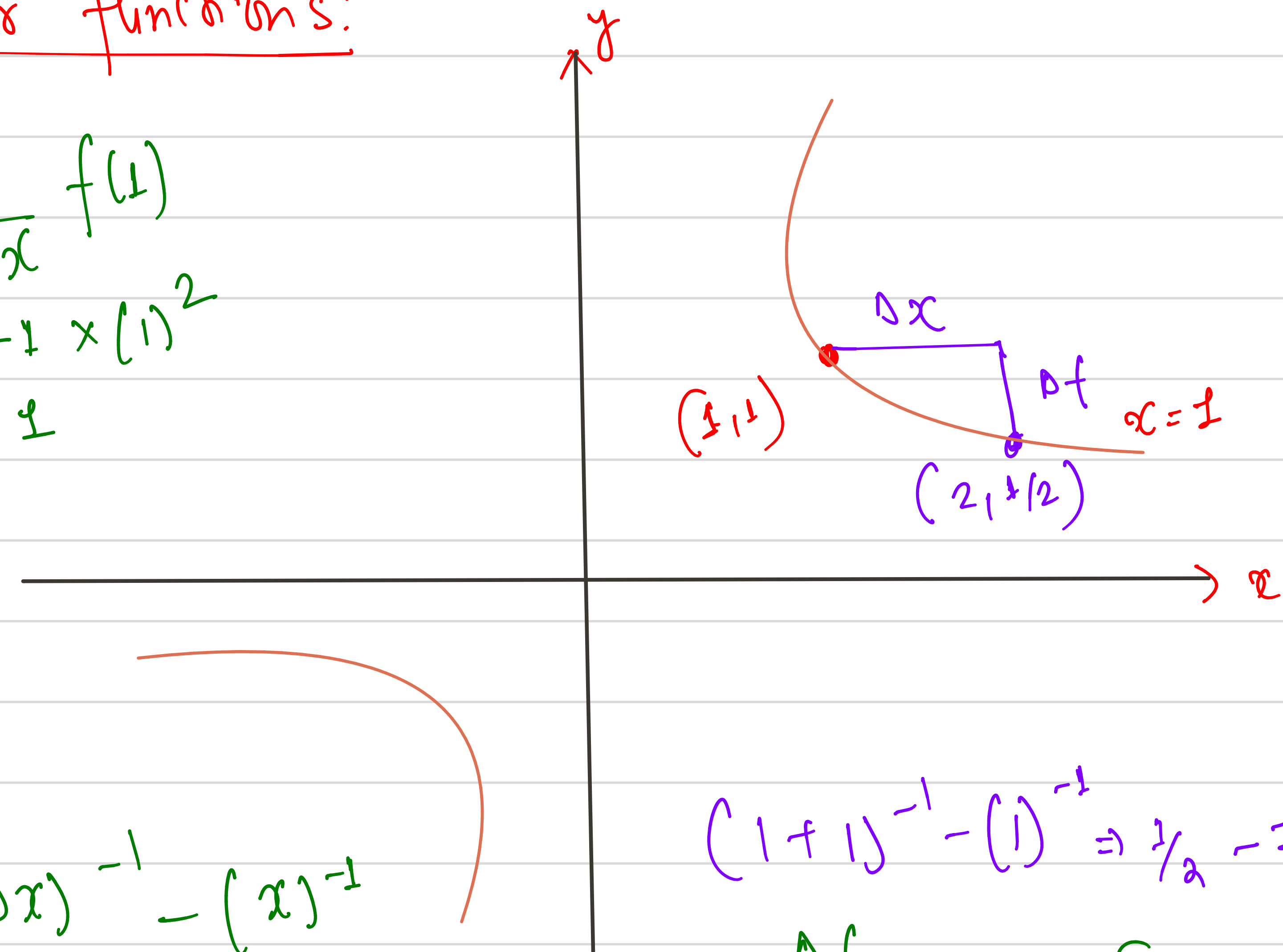
$\Rightarrow \frac{1}{x}$

$\Rightarrow x^{-1}$

$f'(1) \Rightarrow \frac{d}{\delta x} f(1)$

$\Rightarrow -1 \times (1)^{2}$

$f'(1) \Rightarrow -1$



Slope: $\frac{\Delta f}{\Delta x} \Rightarrow \frac{(x + \Delta x)^{-1} - (x)^{-1}}{\Delta x}$

When $\Delta x = 1/2$ then $\Delta f \Rightarrow (x + \Delta x)^{-1} - (x)^{-1}$

$(1+1)^{-1} - (1)^{-1} \Rightarrow 1/2 - 1$

$\frac{\Delta f}{\Delta x} \Rightarrow \frac{-0.5}{1}$

$\Delta f \Rightarrow (1 + 0.5)^{-1} - (1)^{-1}$

$\Rightarrow 2/3 - 1$

$\Delta f \Rightarrow \underline{-0.33}$

then $\frac{-0.33}{0.5} \Rightarrow -0.67$

Δx	1.0	1/2	1/4	...	
Δf	-0.5	-0.33	-0.2	--	
Slope	-0.5	-0.67	-0.89	...	-0.999

$\rightarrow$ Converges to -0.999

Slope $\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$$\Rightarrow \frac{(x+\Delta x)^{-1} - x^{-1}}{\Delta x}$$

$$\Rightarrow \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$$

$$\Rightarrow \frac{\frac{\cancel{x} - \cancel{x} - \Delta x}{x(x+\Delta x)}}{\Delta \cancel{x}}$$

$$\Rightarrow -\frac{1}{x^2 + x \cdot \Delta x}$$

$$\Rightarrow -\frac{1}{x^2} \Rightarrow -x^{-2}$$

$\frac{d}{df}(x^n) = n x^{n-1}$

$\therefore f(x) = x^{-1}$

$f'(x) \Rightarrow -x^{-2}$

Derivative of Power functions

$f(x) = x^2$
$f'(x) = 2x^1$

$f(x) = x^3$
$f'(x) = 3x^2$

$f(x) = x^{-1}$
$f'(x) = 1(-1)x^{-2}$

True for
$\frac{d}{df}(x^n) = nx^{n-1}$

Inverse Function And Its derivative

$f(x)$ $3 \xrightarrow{\text{sent to}} 4$ $f^{-1}(x)$

$3 \xleftarrow{\text{to send}} 9$

$x \xrightarrow{f} x^2 \xrightarrow{g} x$

x^2 $g(x)$ $\sqrt{x}$

$g(x)$ and $f(x)$ are inverse s

$g(x) = f^{-1}(x)$

$g(f(x)) = x$

$\sqrt{x^2} = x$ for $x > 0$

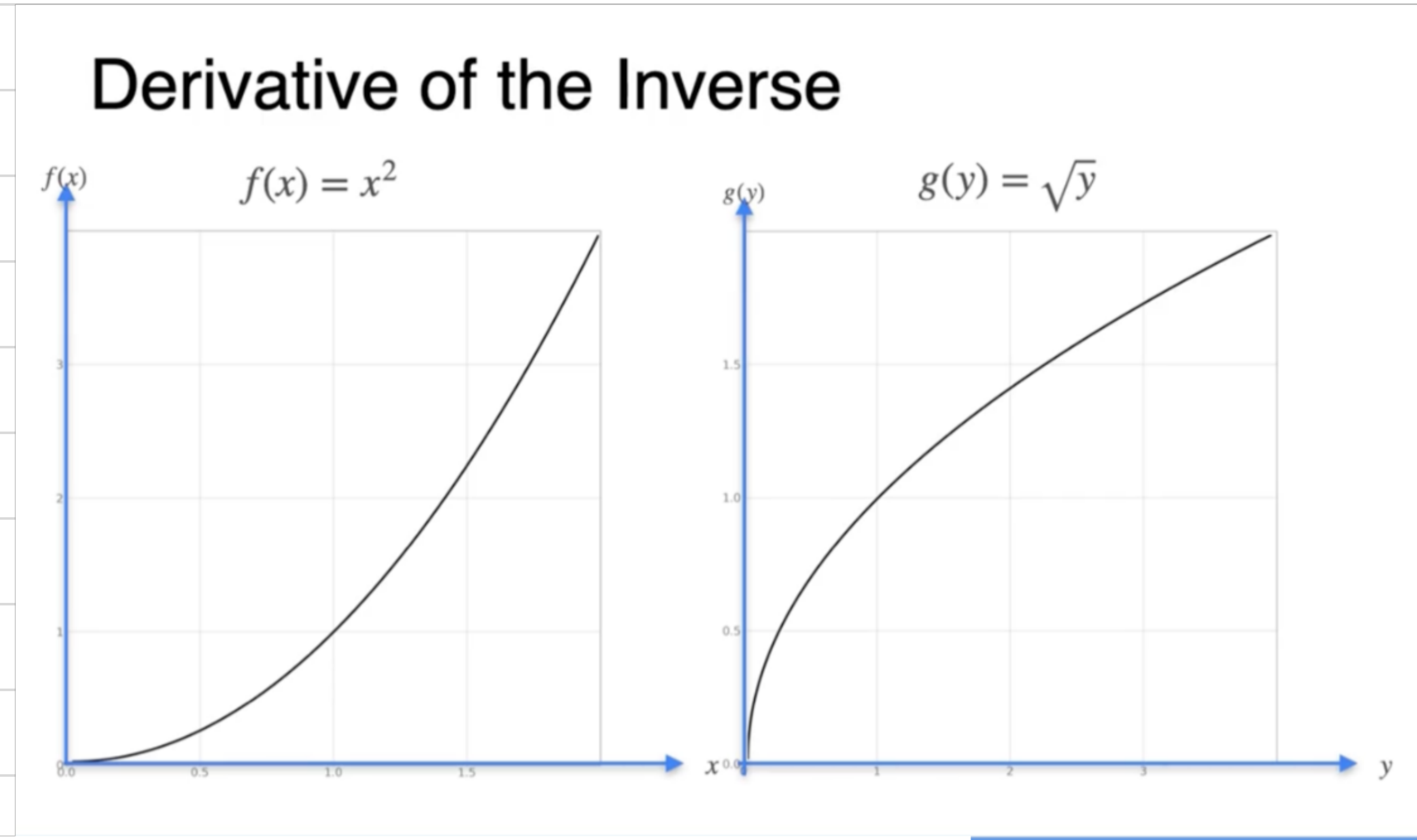
Derivative of Inverse:

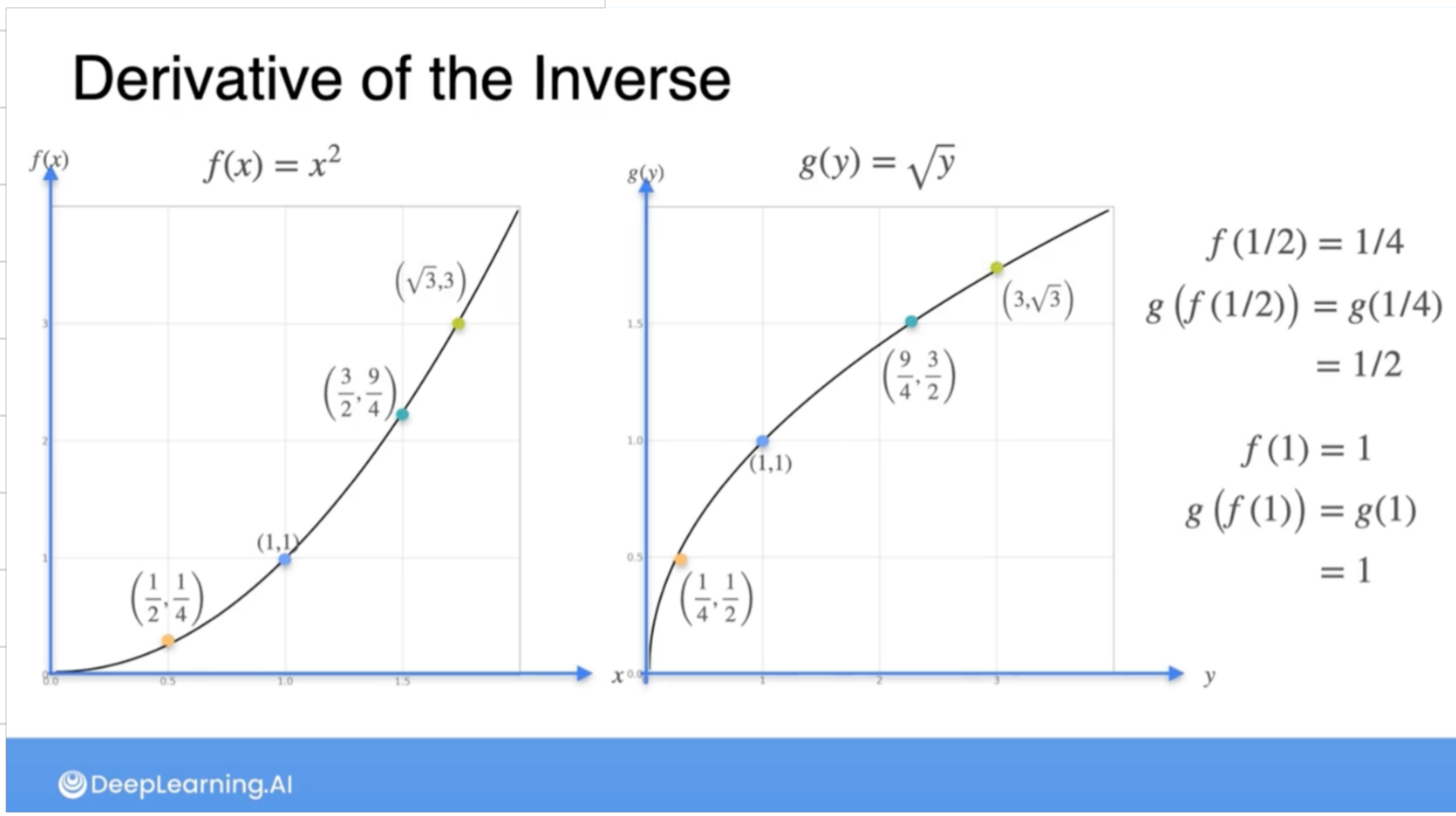
$$f(1/2) \Rightarrow 1/4$$

$$g(f(1/2)) = g(1/4) \Rightarrow 1/2$$

$$f(1) \Rightarrow 1$$

$$g(f(1)) \Rightarrow g(1) \Rightarrow 1$$



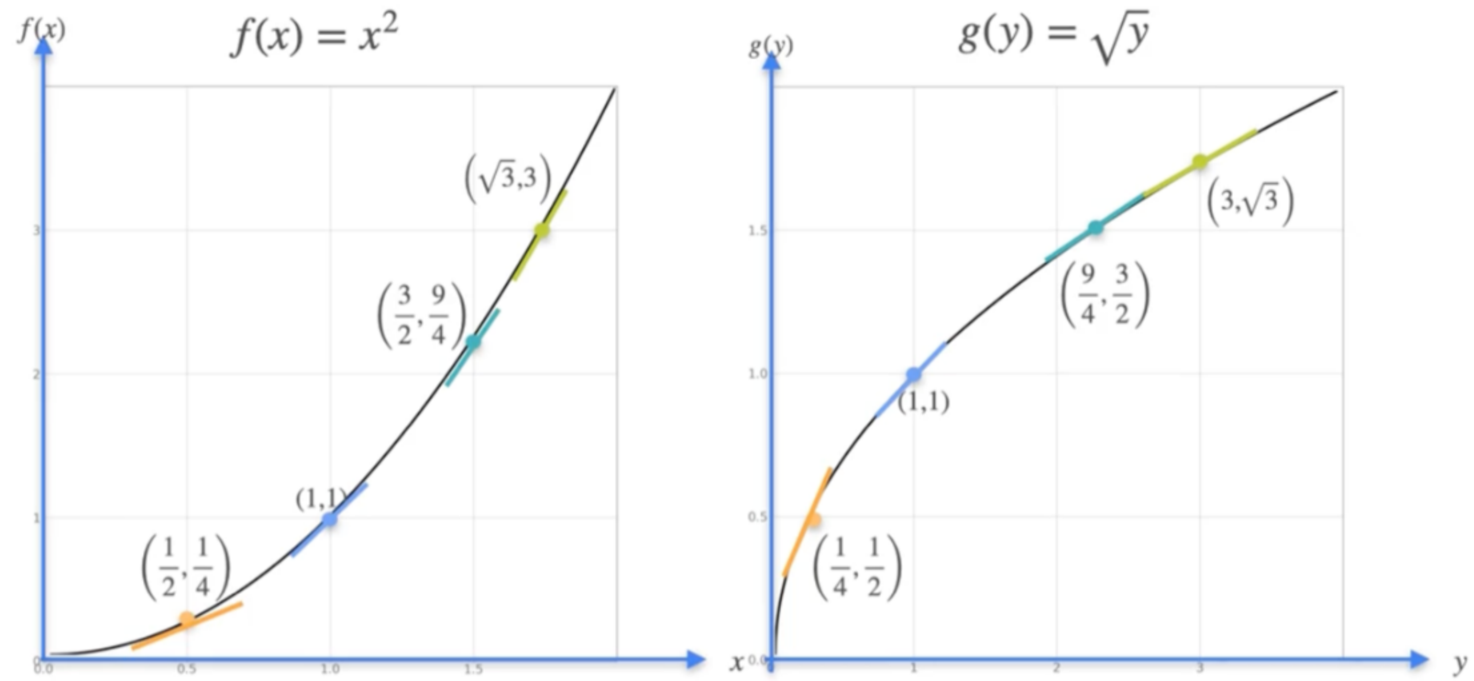


$f(x) \Rightarrow x^2$

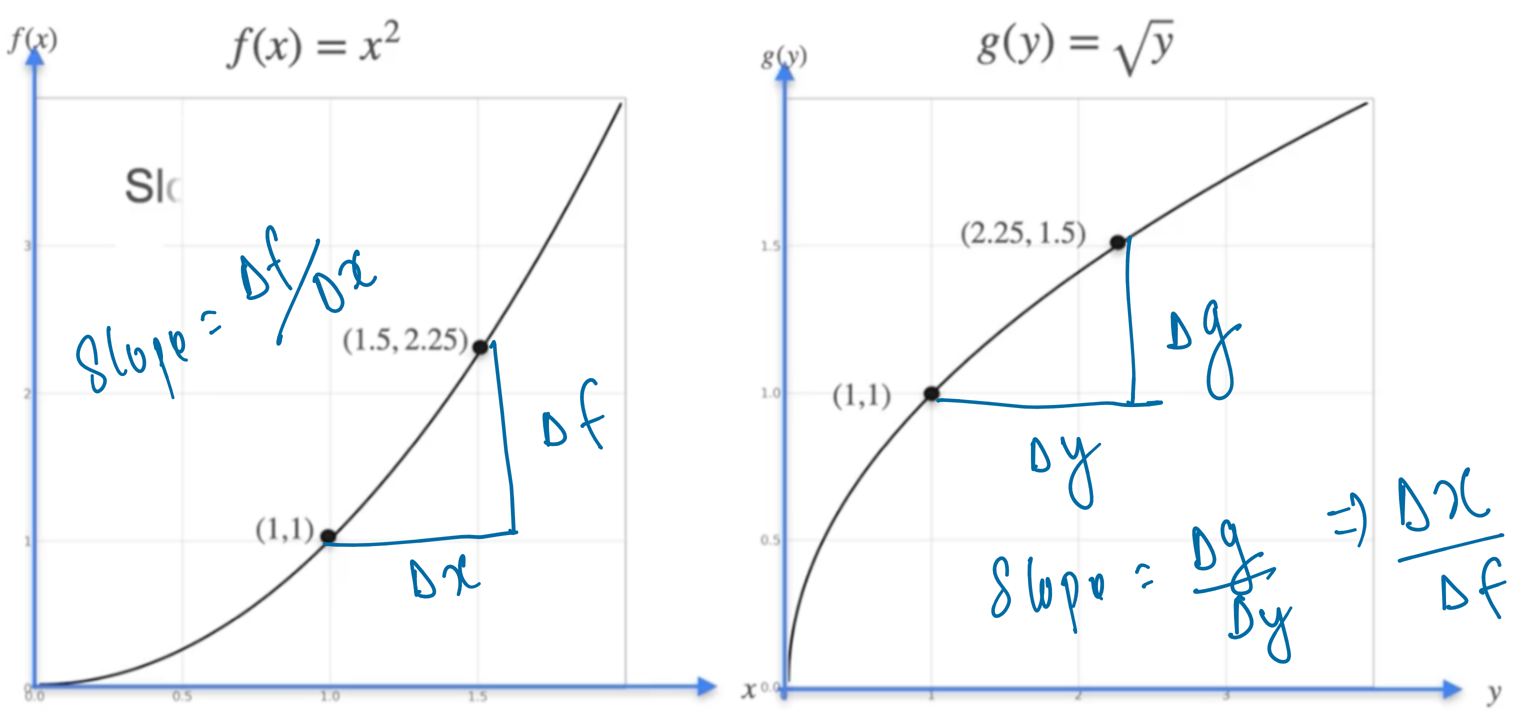
$g(y) \Rightarrow \sqrt{y}$

Both eqns slopes have interesting patterns if we observe closer.

Derivative of the Inverse



Derivative of the Inverse



Since both are Reflections of each other

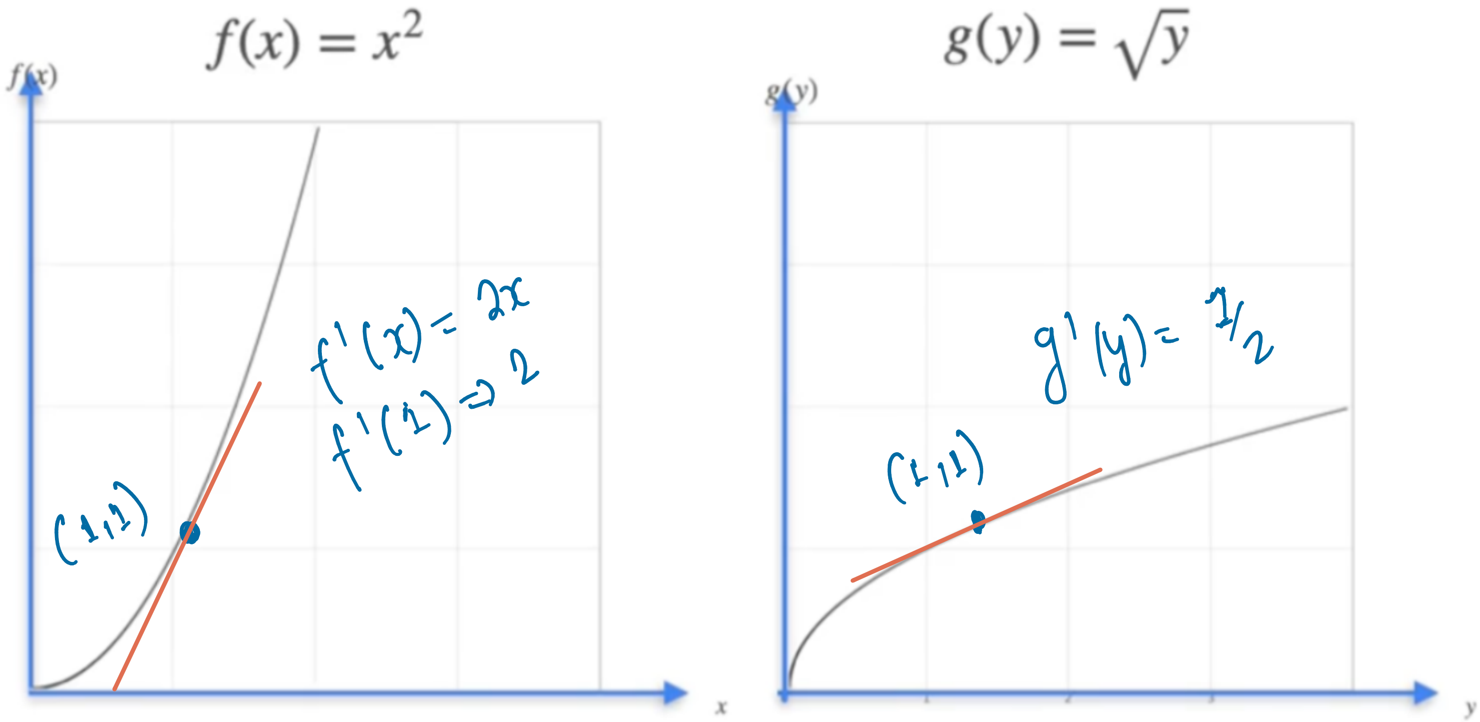
$$\frac{\Delta f}{\Delta x} = f'(x)$$

$$\frac{\Delta g}{\Delta y} = g'(y)$$

$$g'(y) \Rightarrow \frac{1}{f'(x)}$$

If f and g are inverse functions, then the derivative of g is 1 over the derivative of x.

Derivative of the Inverse



$$g'(y) = \frac{1}{f'(x)}$$

at the point (1,1)

$f(1) = 1$ $g(1) = 1$

$$g'(1) \Rightarrow \frac{1}{f'(1)}$$

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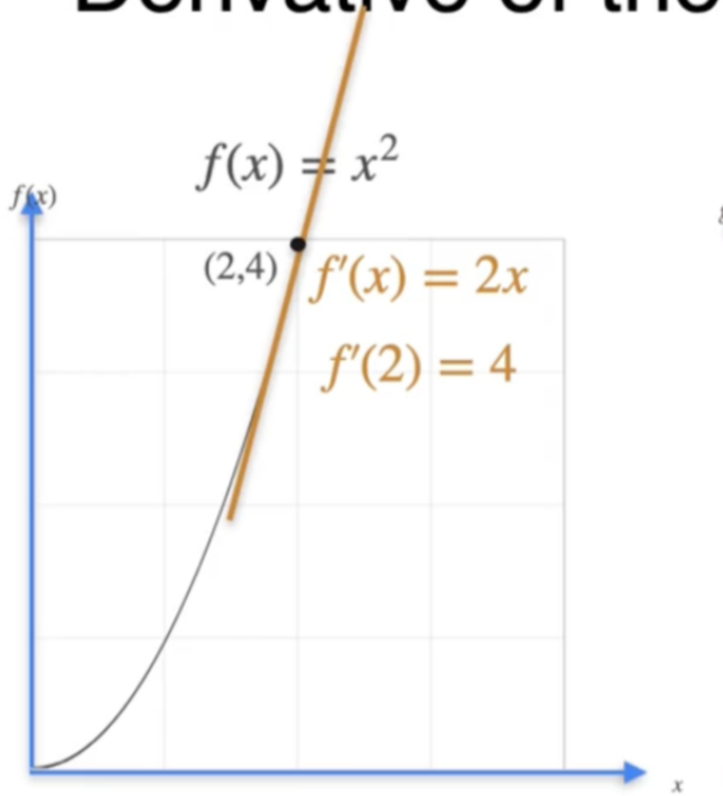
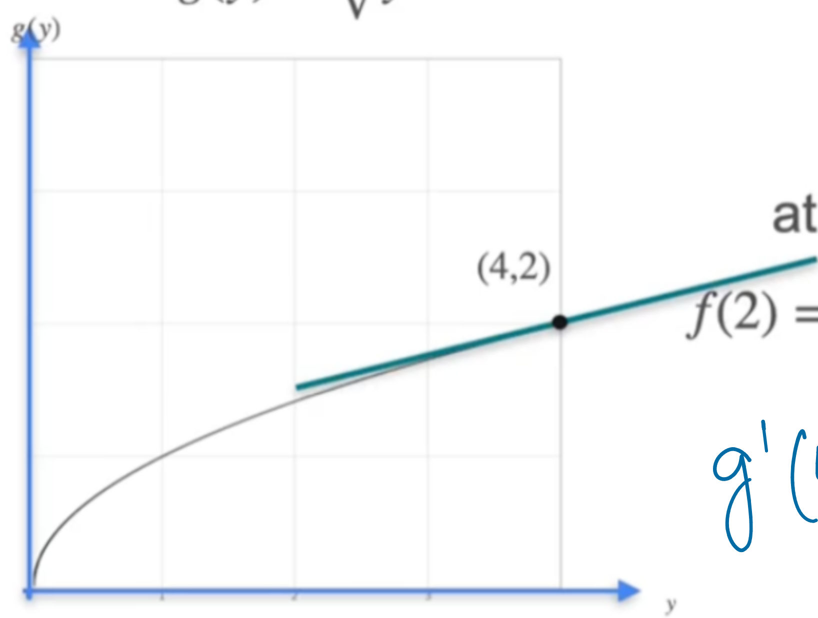
Source: Calculus for Machine Learning and Data Science Coursera Course offered by DeepLearning.

Derivative of the Inverse





$$g'(y) = \frac{1}{f'(x)}$$

at the point (2,4)

$f(2) = 4$ $\quad g(4) = 2$

$$g'(4) = \frac{1}{f'(2)}$$

$g'(y) = 1/4$

If a function does one then the inverse function is the one that undo the another.

Δx, Δy, $f'(x)$ or $f(x)$, $\frac{d}{dx} f$, $\frac{dy}{dx}$, $\frac{d^2 f(x)}{dx}$