# Day-70, Feb 8, 2025 (Magh 26, 2081)

Probability:

→ It is used to understand what patterns in nature are "real" and which are due to chance.

→ Tool that let us make an inference from a sample to a population.

→ It represents the relative frequency of outcomes after infinitely many repetitions.

Probability is a tool that quantify outcomes of an experiment whose exact values cannot be predicted with certainty.

# Example

from a 52 cards, draw one card. What is the probability without putting in back, draw a second card

1. What is the p of getting a Queen in the first draw?
2. What is the 'p' of getting a Diamond in the second draw?

# Example Prevalence of any incident outcomes.

probabilistic Model, Law of Conditional Probability,

# Probabilistic Model:

→ The Sample Space $\Omega$: the set of all outcomes from an experiment.

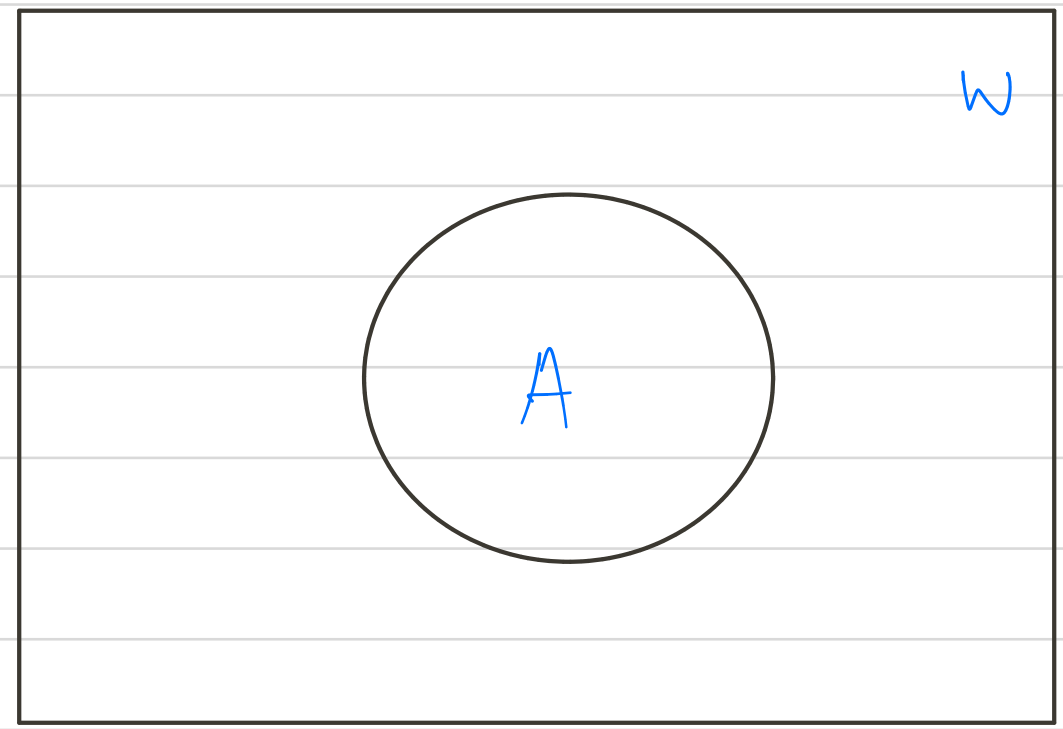
→ The probability law: which assigns a number $P(A)$ to a set $A$ of possible outcome.

Where $A$ is called an Event. An Event is a subset of the Sample Space $\Omega$ or a collection of possible outcome.

- $P(A)$ is called the Probability of $A$

  ⇒ $P(A)$ encodes our knowledge or belief about the "likelihood" of $A$.

▷ Interpretation in terms of frequency.




$$P(A) = \frac{|A|}{|W|}$$

assuming the events in W are finite, discrete and equally likely.

<u>Examples of Events:</u>

- Draw an ace from a pack of cards
    - ▷ Sample Space: All possible draws
    - ▷ Event: $E = \{ 1\ \text{Heart}, 1\ \text{Spade}, 1\ \text{Club}, 1\ \text{Diamond} \}$
    - ▷ $P(E) = ??$

• Draw two spades from a pack of cards.

$E = \{ (1S, 2S), (1S, 3S) \; --- \; (2S, 1S) \; -- \}$

$\Rightarrow \{ (xS, yS) \mid x \in 1 \,..\, 13, \; y \in 1, 2, -- 13$
$x \neq y \}$

Example: Probability of getting an even number when rolling a dice

• The Sample Space : $\Omega$ : $\{1, 2, 3, 4, 5, 6\}$

• $A = \{2, 4, 6\}$ represents an event of getting an even number

$$P(A) = \frac{\text{Number of elements in } A}{\text{Total number of elements in } \Omega}$$

$$\Rightarrow 3/6 \quad \Rightarrow 0.5$$

Note: that in this experiment each outcome is equally likely

Hence, we call the dice 'fair'.

# Probability Axioms: Things to keep in Mind

① (Non-Negativity): For every event A

$$P(A) \geq 0$$

② (Additivity): If A and B are two disjoint events then the probability of their union satisfies.

$$P(A \cup B) = P(A) + P(B)$$

More generally, if $A_1, A_2 \dots$ is a sequence of disjoint events, then

$$P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$$

(8) Normalization: The probability of the entire sample space $\Omega$ is 1.

$$P(\Omega) = 1.$$

**Definition:**

For two events A and B where $P(B) > 0$, the conditional probability of A given B is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Remark:**

For discrete events conditional probability can be estimated as-

$$P(A|B) = \frac{C(A, B)}{C(B)}$$

$$\Rightarrow \frac{\#\text{ of both } A \,\&\, B \text{ in the sample}}{\#\text{ of event } B}$$

## Independence:

When A is independent of B.

$$P(A|B) = P(A) \quad \text{—— eqn (i)}$$

By the definition of conditional probability.

$$P(A \cap B) = P(B) \cdot P(A/B) \quad \text{—— eqn (ii)}$$

Combining eqn (i) and eqn (ii) we get. A is independent of B if and only if

$$P(A \cap B) = P(A) \cdot P(B)$$

- A is independent of B if and only if B is independent of A.
- We usually simply say A and B are independent.

# Draw one card without putting it back,

① first draw $= \frac{4}{52}$
$P(Q)$

② Second draw $= \frac{13}{51}$
only ~~Diamond.~~ $P(D)$

③ Compute $P(Q \cap D) \Rightarrow P(Q) \cdot P(D)$

$\Rightarrow \frac{4}{52} \times \frac{13}{51}$

$$\boxed{P(Q \cap D) \Rightarrow \frac{1}{51}}$$

Q. Compute $P(D|Q) = \frac{P(Q \cap D)}{P(Q)} \Rightarrow \frac{1}{51} \times \frac{51}{13} \rightarrow \frac{1}{13}.$

# Total Probability Theorem

Let $A_1, A_2 \ldots A_n$ be disjoint events that form a partition of the sample and assume that $P(A_i) > 0$ for all $i$. Then for any event $B$.

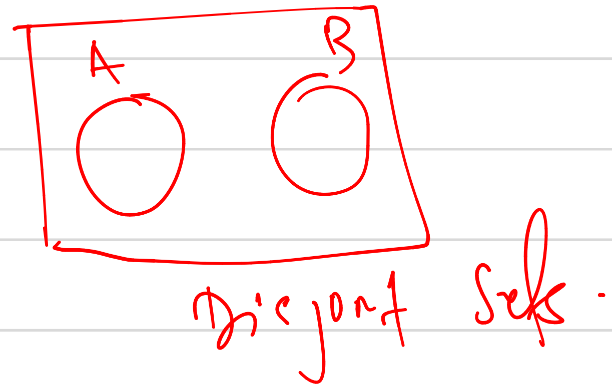
$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \cdots + P(A_n \cap B)$$

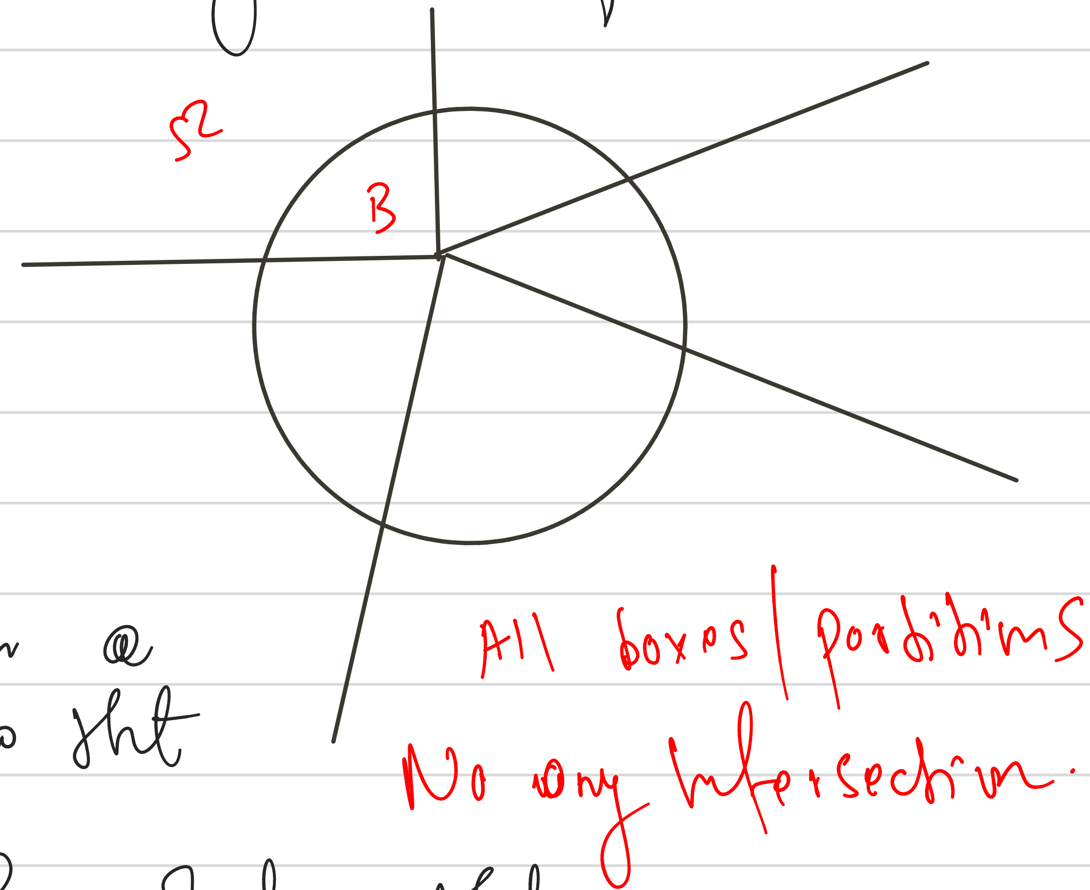
$$\Rightarrow P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P(B|A_2) + \cdots + P(A_n) \cdot P(B|A_n)$$

$$\Rightarrow \sum_{i=1}^{n} P(A_i) \cdot P(B|A_i)$$

Recall that $A_1, A_2, \ldots A_n$ is a partition of $B$ if $A_i$ are disjoint and $B = A_1 \cup A_2 \ldots \cup A_n$.

So, the visualization of total probability or we say law of Total Probability –




# Bayes Rule:

Let $A_1, A_2 \ldots A_n$ be disjoint events that form a partition of the sample space, and assume that $P(A_i) > 0$ for all $i$. Then for any event B such that $P(B) > 0$.

$$P(A_i \mid B) = \frac{P(A_i)\, P(B \mid A)}{P(B)}$$

$$\Rightarrow \frac{P(A_i)\, P(B \mid A)}{\sum_{i=1}^{n} P(A_i) \cdot P(B \mid A_i)} \qquad \alpha \quad P(A_i) \cdot P(B \mid A_i).$$

Where $P(B)$ is the total Probability Theorem.

## Implication of Bayes Rules:

- Prevalence of Diabetes in Nepal is 25%. Let D be an event that a person has diabetes

$$P(D) = 0.25$$

- Number of men with diabetes is 100,000 in Banepa. Total # of Men in the area is 250,000. While the total population is 500,000. Let M be the event that a random person in the city is a man, i.e.

$$P(M) = \frac{250{,}000}{500{,}000} \Rightarrow 0.5$$

and $P(D|M) \Rightarrow \frac{75000}{250000}$

$\Rightarrow 0.4$

$P(D) \cdot P(M) \Rightarrow 0.25 \times 500,000$

$= 75000$

$\Rightarrow \frac{P(D \cap M)}{P(M)}$

$\Rightarrow \frac{P(D) \cdot P(M/D)}{P(M)}$

$\Rightarrow 0.4$.

$P(M/D) = P(M)$

For Disjoint sets.

Q. What is the probability that a random person with diabetes in Banepa is a man? $P(M/D) = ?$

Using Bayes Rule,

$$P(M/D) = \frac{P(M \cap D)}{P(D)}$$

$$\Rightarrow \frac{P(D/M) \cdot P(M)}{P(D)}$$

$P(D/M) = P(D)$

$$\Rightarrow \frac{(0.4 \times 0.5)}{0.25}$$

$P(D/M) \Rightarrow$ Calculated $\underline{0.4}$.

$P(M) \Rightarrow 0.5$

$\Rightarrow 0.8$

$P(D) \Rightarrow 0.25$ (given)

Source:

NAAMII : Introduction to Probability - Part I YouTube.

# Statistical Inference:

The process of generating conclusions about a population from a noisy Sample.

Example: Predicting and analyzing the weatherforecast, MRI of Medical Report Analysis and Prediction.

There are more than one way to think about the Inference and Probability.