Day-18, Dec-3, 2024 (Mangshir-18, 2081 BS.)

Even and Odd function. Symmetry:

A function $y=f(x)$ is an even function if $f(-x)=f(x)$, and odd function if $f(-x)=-f(x)$ for every value of x.

Note that the graph of even function is symmetrical about y-axis and the graph of odd function is symmetrical about origin.

Examples on Even and odd function: Symmetry:

i) $f(x)=x^{-4}$

Since $f(x)=x^{-4} \Rightarrow 1/x^4$

So, $f(-x) = \frac{1}{(-x)^4} \Rightarrow \frac{1}{x^4} = f(x)$

So the given $f(x) = x^{-4}$ is even function.

ii) $f(x) = x^3$

$f(-x) = (-x)^3 \Rightarrow -x^3$

$\Rightarrow -f(x)$

$\therefore$ The given function $f(x) = x^3$ is an odd function.

iii) Since,

$f(x) = |x| + 5$

so, $f(-x) = |-x| + 5 \Rightarrow |1||x| + 5$

$\Rightarrow |x| + 5$

$f(-x) \Rightarrow f(x).$

$\therefore$ the given function $f(x) = x^3$ is odd function.

iii) $f(x) = x + 1$

Since, $f(-x) = -x + 1$

$\Rightarrow -(x - 1) \neq f(x)$

and $f(-x) = -x + 1$

$\Rightarrow -(x - 1) \neq -f(x)$

$\therefore$ the given $f(x) = x + 1$ is neither even nor odd.

Example: If f and g are even functions, prove that $f+g$ also even function.

Let f and g are even so $f(-x) = f(x)$ and $g(-x) = g(x)$.

Suppose $f+g = h$;

So, $h(-x) = f+h(-x)$

$\Rightarrow f(-x) + g(-x)$

$\Rightarrow f(x) + g(x)$

$\Rightarrow (f+g)x$

$\Rightarrow h(x)$ So h is even function.

$\therefore f+g$ is a even function.

$h(-x) = h(x)$ $\qquad f + g = h.$

$\Rightarrow f(x) + g(x)$

$\Rightarrow f(-x) + g(-x)$

$\Rightarrow (f+g)x.$

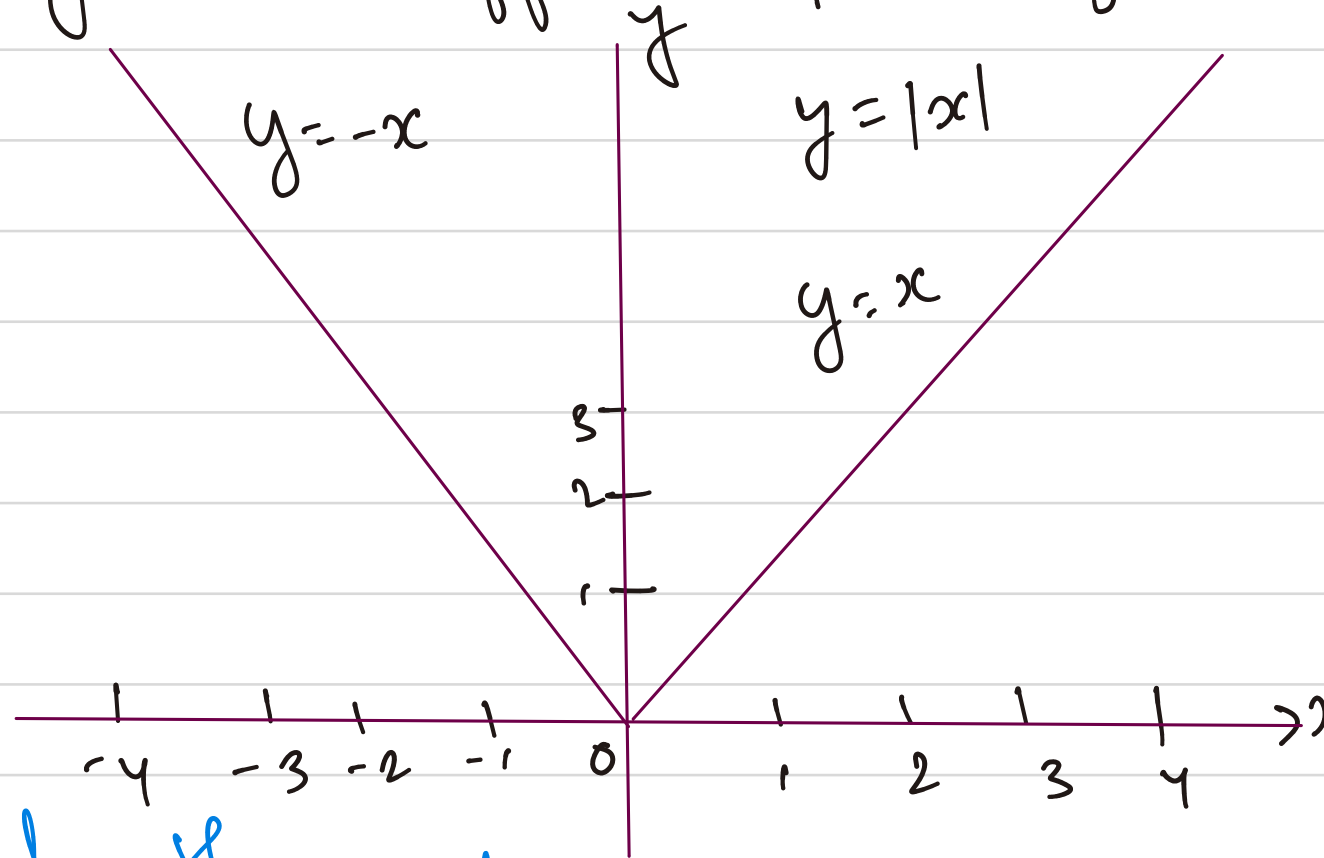
$\Rightarrow h(x).$

Piecewise Defined function:

A piecewise function is a function whose formula changes depending on different part of its domain. A simple example is absolute value function, which is

$$f(x) = |x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

A function whose formula changes depending on different parts of its domain is a piecewise function.



Example: A function defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

Evaluate $f(-3)$, $f(-1)$ and $f(1)$ and sketch the graph.

Here given,

$$f(-3) = 1 - (-3) \quad (\text{taking } f(x) = 1 - x)$$

$$\Rightarrow 1 + 3 \quad \Rightarrow 4$$

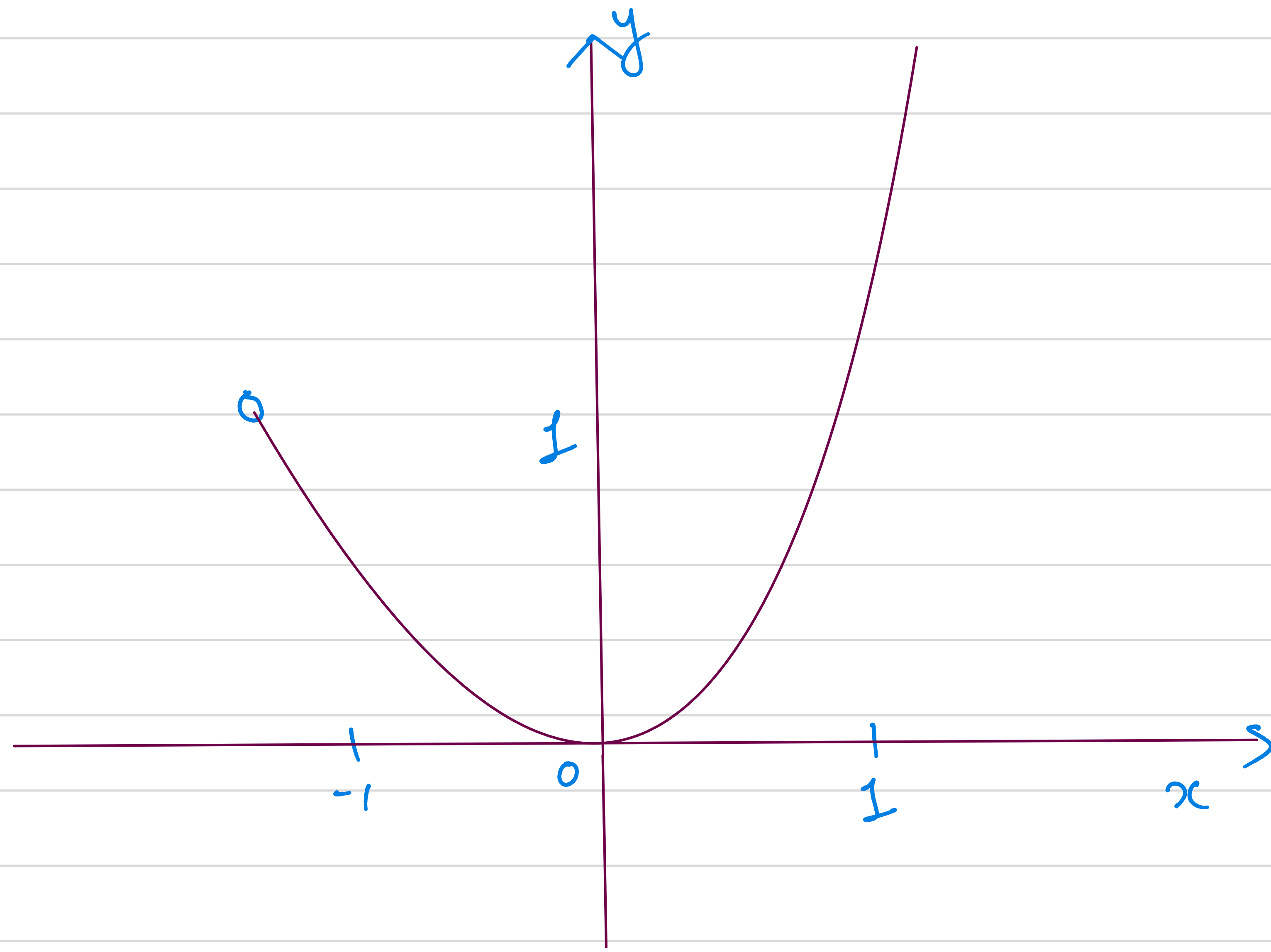
$f(-1) = 1-(-1)$

$\Rightarrow 1+1$

$\Rightarrow 2$

$f(1) \Rightarrow 1^2$ (we take $f(x) = x^2$)

$\Rightarrow 1$



for the graph of $f(x) = 1-x$

ie. $y = 1-x$, is straight line

put $x = -1$, $y = 2$ and $x = -2$, $y = 3$

Hence $(-1, 2)$ and $(-2, 3)$ lies on the line $y = 1-x$,

for the graph of $f(x) = x^2$ is Parabola.

Linear Mathematical Model:

A mathematical model is a method of simulating real life situation with mathematical equation to forecast their future behaviour. The mathematical model described in term of four stages.

Stage 1 (Formulation):

On given real problem by using the dependent (x, y) and independent variable establish an equation called mathematical model.

Stage 2 (Solving the Model):

We use mathematical model to solve the mathematical model. Calculus is the primary tool of analysis in this text (100 days).

Stage 3 (Interpretation):

After the mathematical model has been solved any conclusions that may be obtained from solving are applied to the original real world problem.

Stage 4 (Testing):

In this stage, the model is tested by gathering new data to check the accuracy of any predictions inferred from the analysis. If the predictions are not confirmed then the assumptions of model are adjusted and modeling process is repeated.

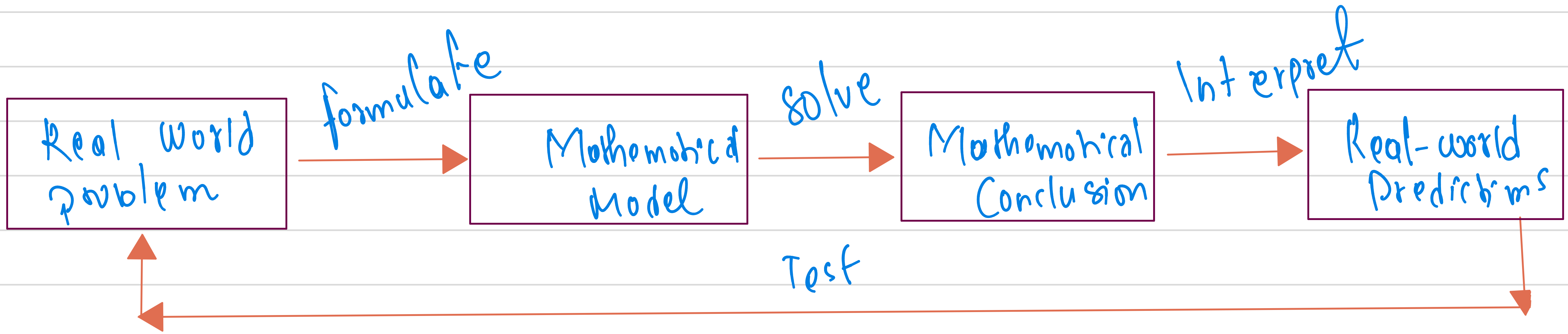


Fig. Block Diagram of Linear Mathematical Model.

Q. A dry air moves upward, it expand and cools. If the ground temperature is 20°C and the temperature at a height of 1 Km is 10°C.

a) Express the Temperature T (in °C) as a function of the height h (in km) assuming that a linear model is appropriate.

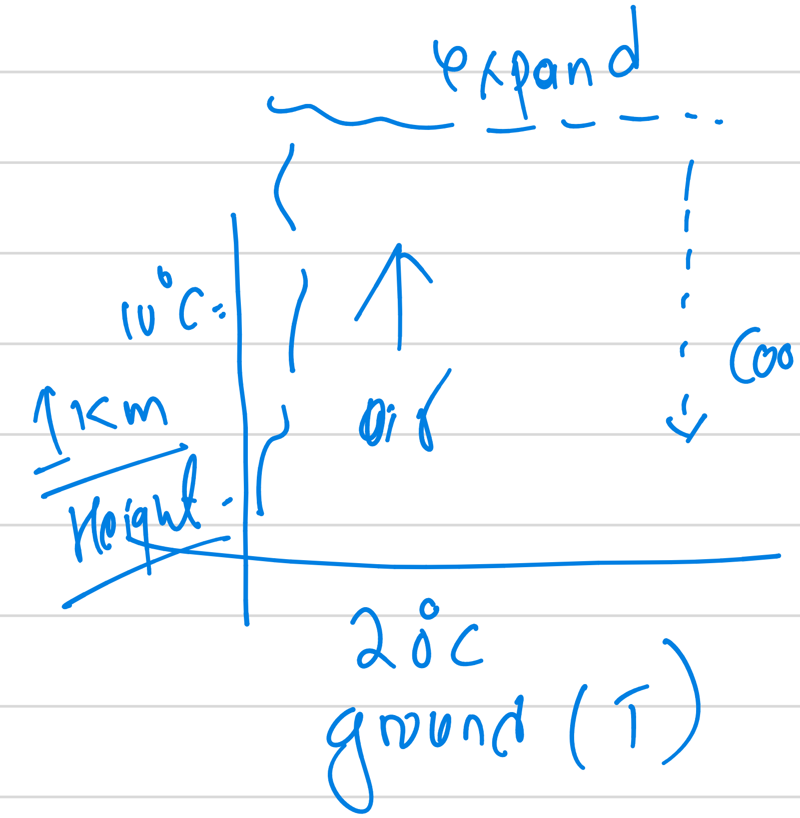
b) What slope represent?

c) What is the temperature at a height of 25 km?

Sol^n:

(a) Since Linear Model so,

$$T = mh + b \quad \text{—— eqn (i)}$$



where T is the temperature at height h,

m is the slope of model

b is T intercept (i.e. temperature at ground level)

Given that $T = 20$ when $h = 0$

(ii) $b = 20$

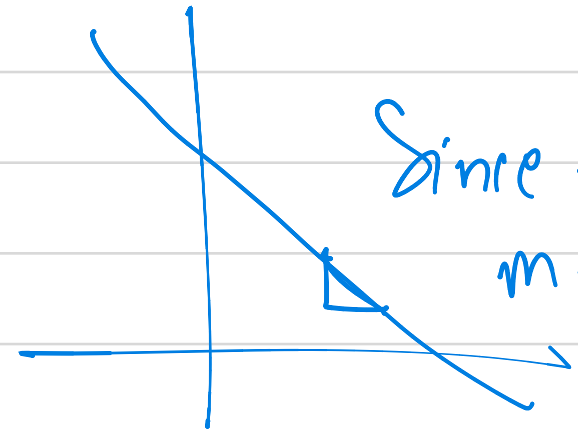
Again, given $T = 10$ when $h = 1$ So eqn (i) -

$$10 = m1 + 20$$

$$\boxed{m = -10}$$

So (i) becomes (eqn (i) becomes) -

$$\boxed{T = -10h + 20 \text{ is a required model.}}$$



Since $m = -ve$

It is $-ve$ slope.

(b) When slope $m = -10$

Rate of change of temperature with respect to height $= \dfrac{dT}{dh}$

$\Rightarrow -10$ i.e. when 1 Km height is increased then temperature is decrease by $10^\circ C$.

(c) eqn (i) becomes $T = -10h + 20$

$= -10 \times 2.5 + 20$

$(T = -5^\circ C)$

at 2.5 Km from ground the temperature is $-5^\circ C$.