

Day - 58, Jan - 2nd, 2025 (Magh 14, 2085 B.S.)

A Communication Problem



Communication channel

Noise

Interference from other devices

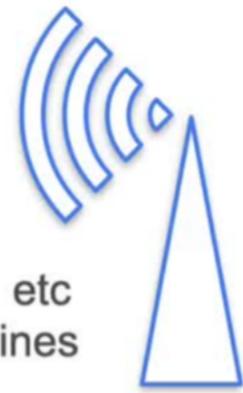
Obstructions like walls, trees, etc.

Atmospheric conditions: rain, humidity, etc

Electrical interference, i.e. from power lines

Others

Message sent: 10010



Message Received:
10010 + \mathcal{Z}

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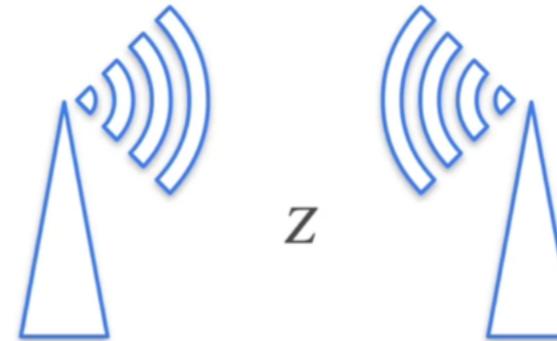
⇒ We assume the noise \mathcal{Z} follows

the normalization $[0, 1]$ and

Dispersion of Noise Signal

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Chi-Square Distribution



What is the power of the noise in the channel

$$W = Z^2$$

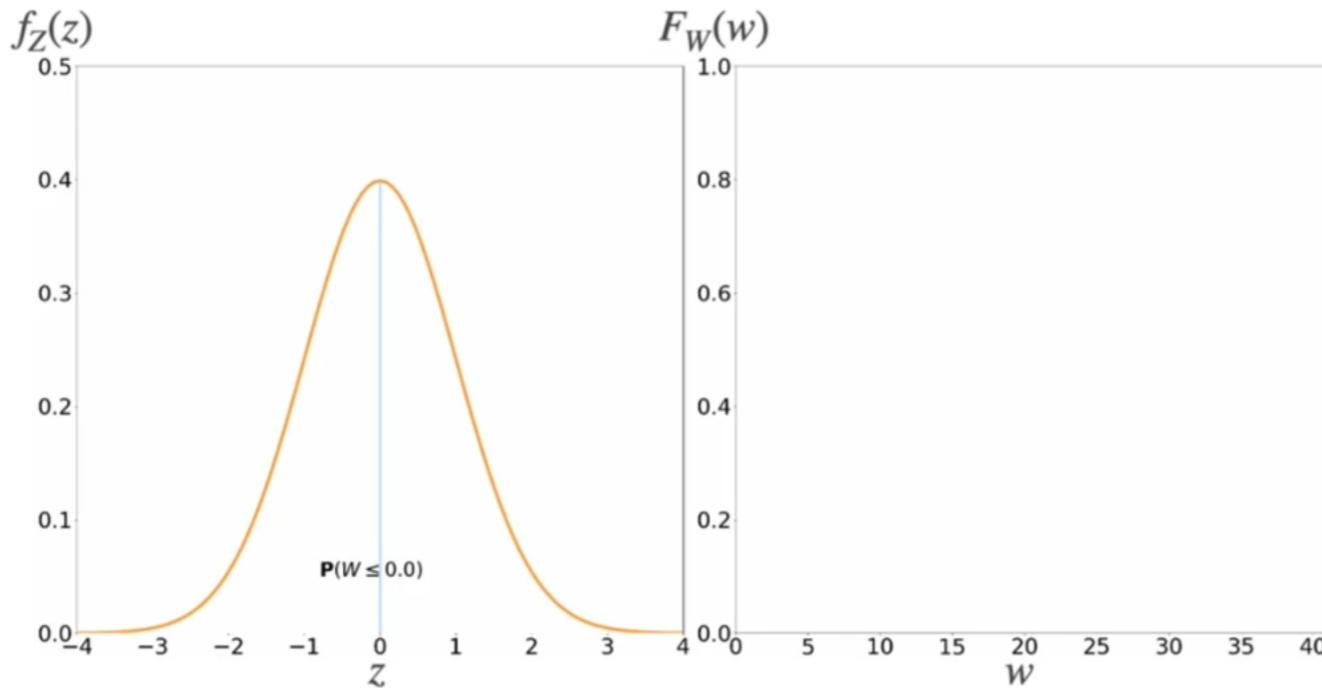
What is the distribution of W ?

The communication channel has noise with a standard normal distribution

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Viewing the graph we get,

Chi Square Distribution



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$$F_W(\omega) = P(W \leq \omega)$$

$$\Rightarrow P(Z^2 \leq \omega)$$

$$\Rightarrow P(|Z| \leq \sqrt{\omega})$$

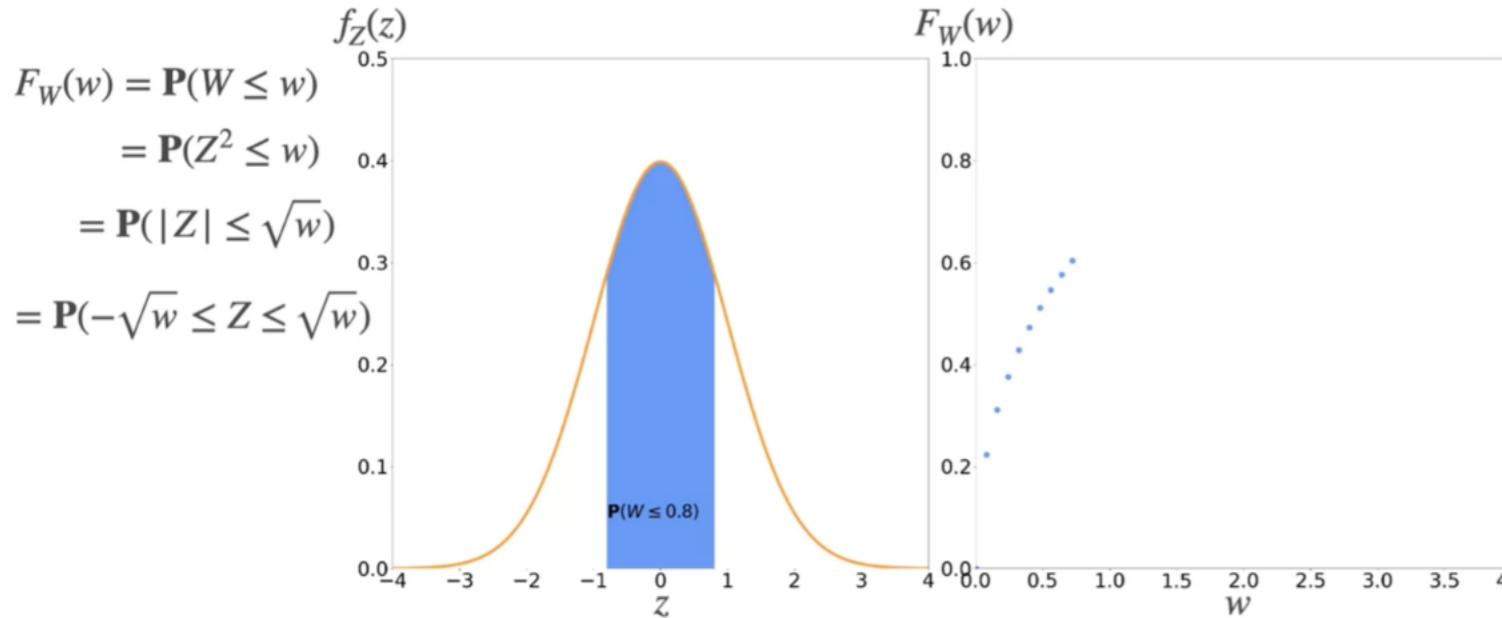
$$\Rightarrow P(-\sqrt{\omega} \leq Z \leq \sqrt{\omega})$$

Let's try to see it graphically. Each value of W can be achieved with two different values of Z , which are negative square root of W , and square root of W , even more, the probability that capital W 's and W is the area under the PDF curve on the Gaussian between these two numbers. You can get the CDF for W by finding these areas for each possible value W . Notice that for small values of W , you gain area at a much quicker rate. This is because the Gaussian distribution concentrates probability around 0. This is known as the Chi-squared distribution with one degree of freedom.

Sum of Squared of Independent Standard Normal Random Variables
distribution is Chi-Squared Distribution.

Chi Square Distribution

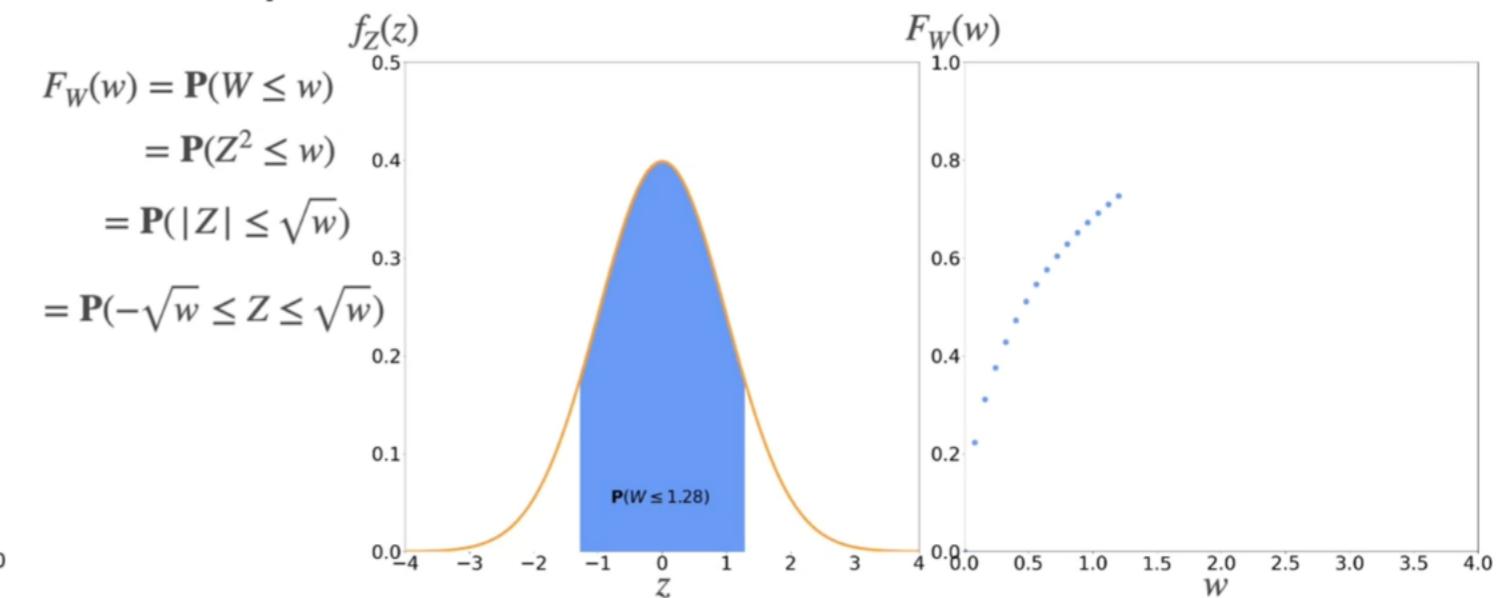
1



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Chi Square Distribution

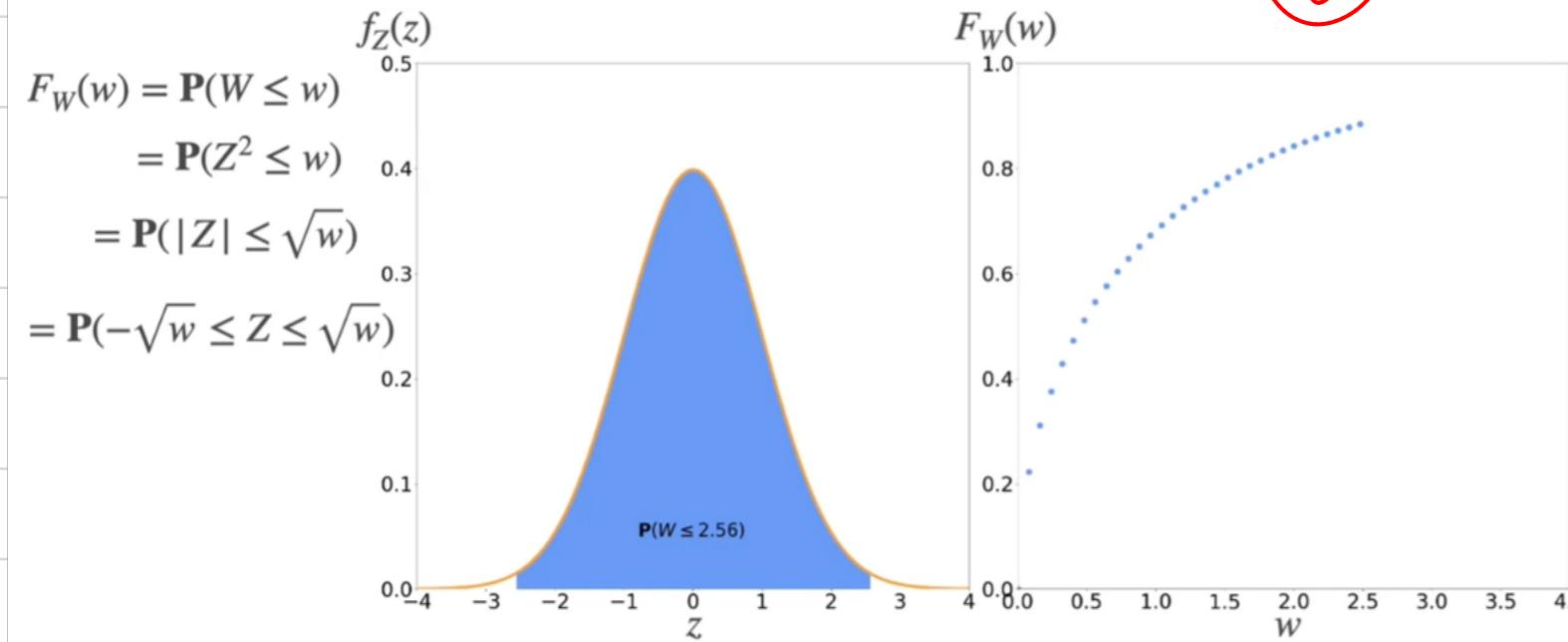
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Chi Square Distribution

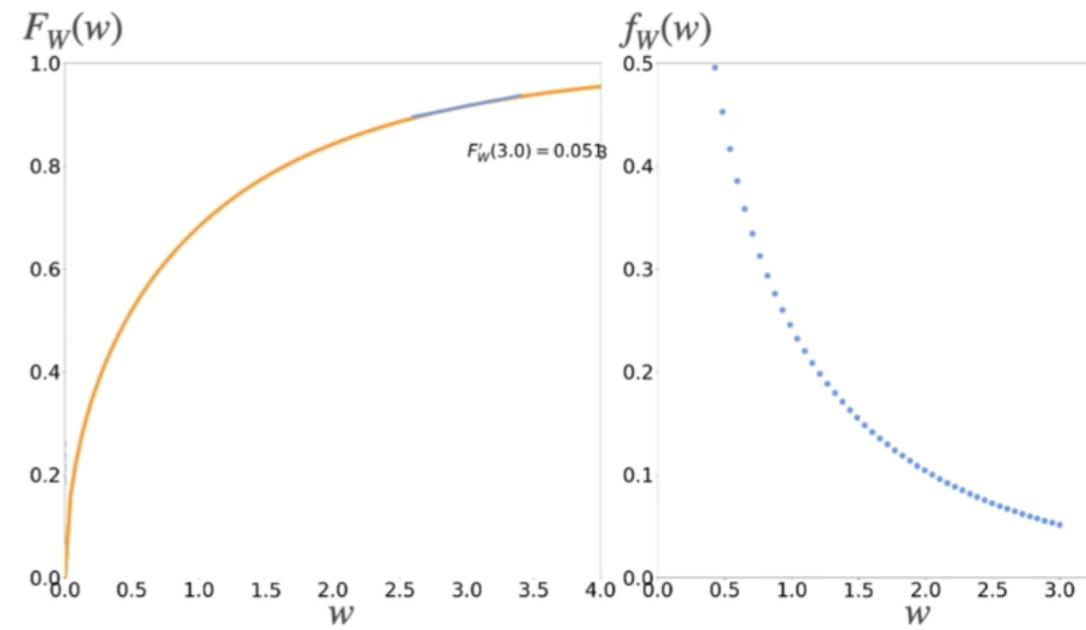
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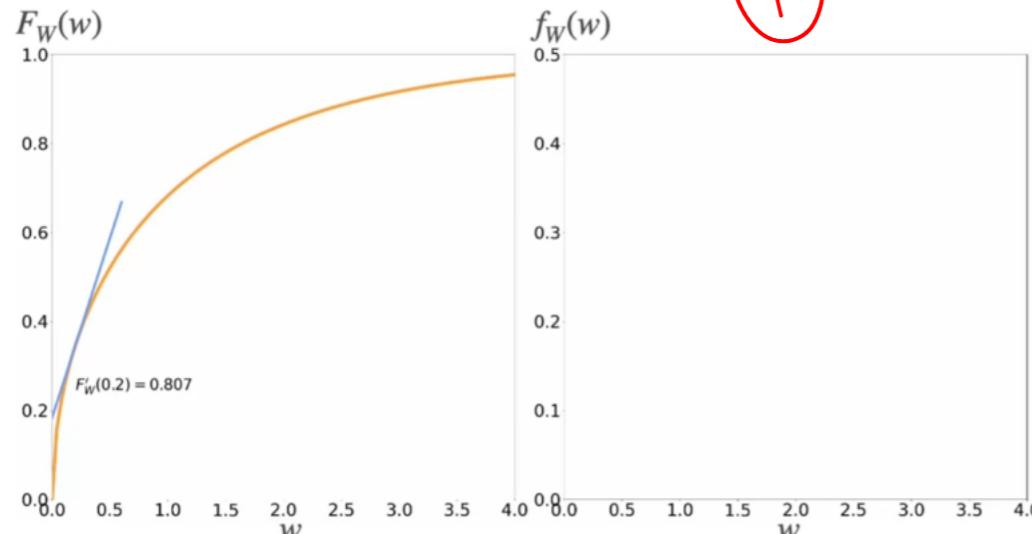
Chi Square Distribution

$$f_W(w) = F'_W(w)$$



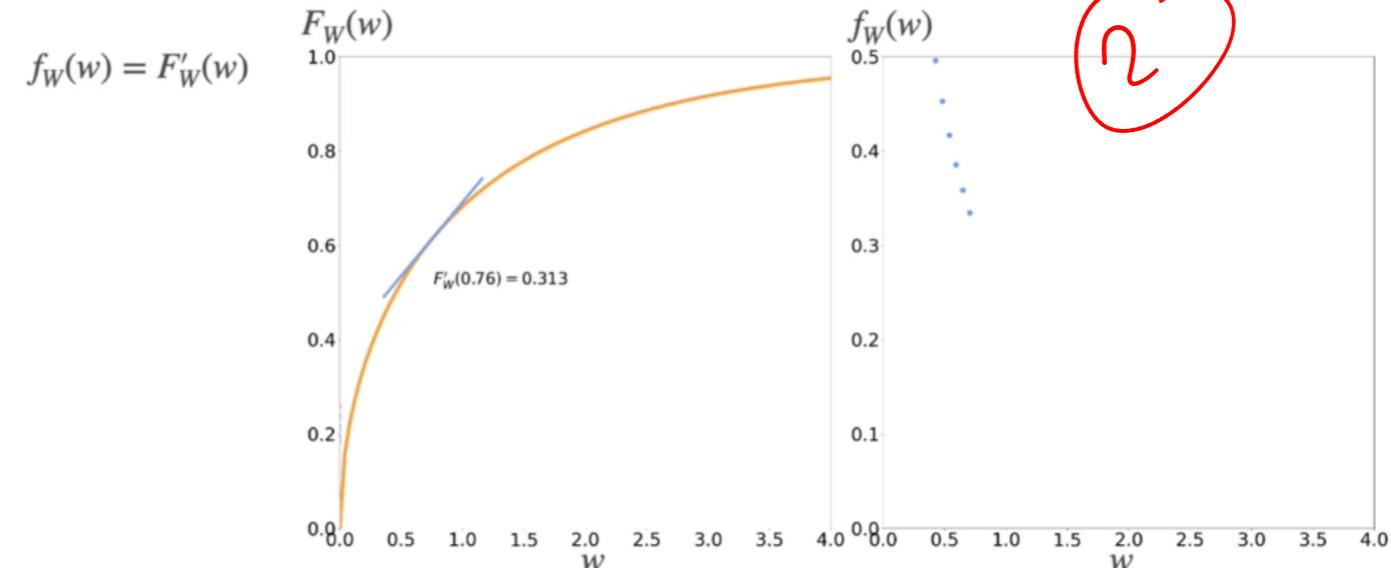
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Chi Square Distribution



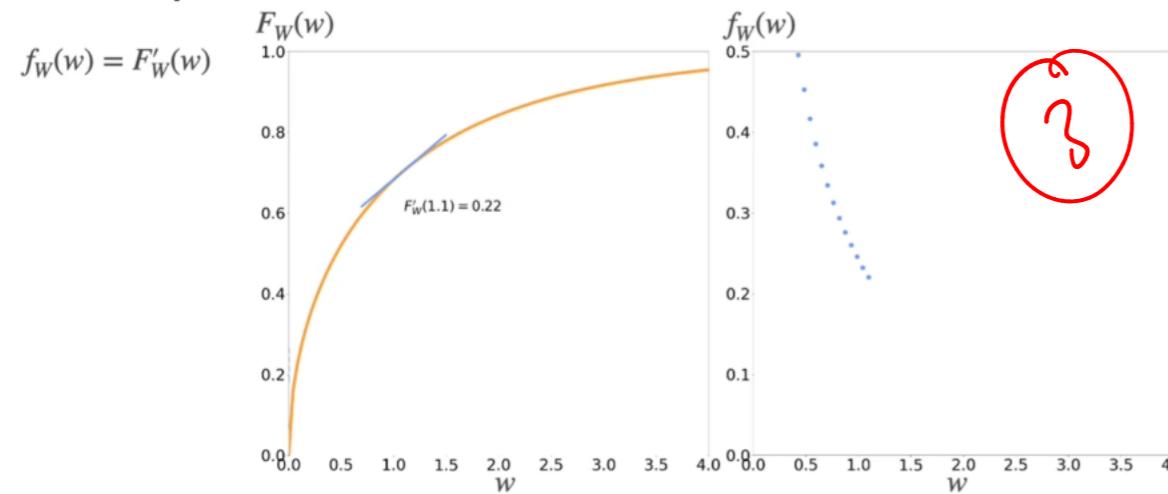
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Chi Square Distribution

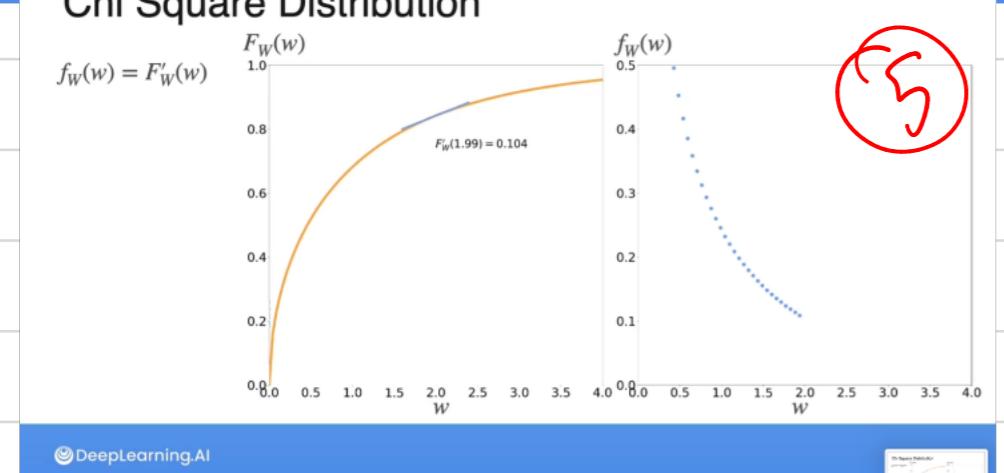


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Chi Square Distribution

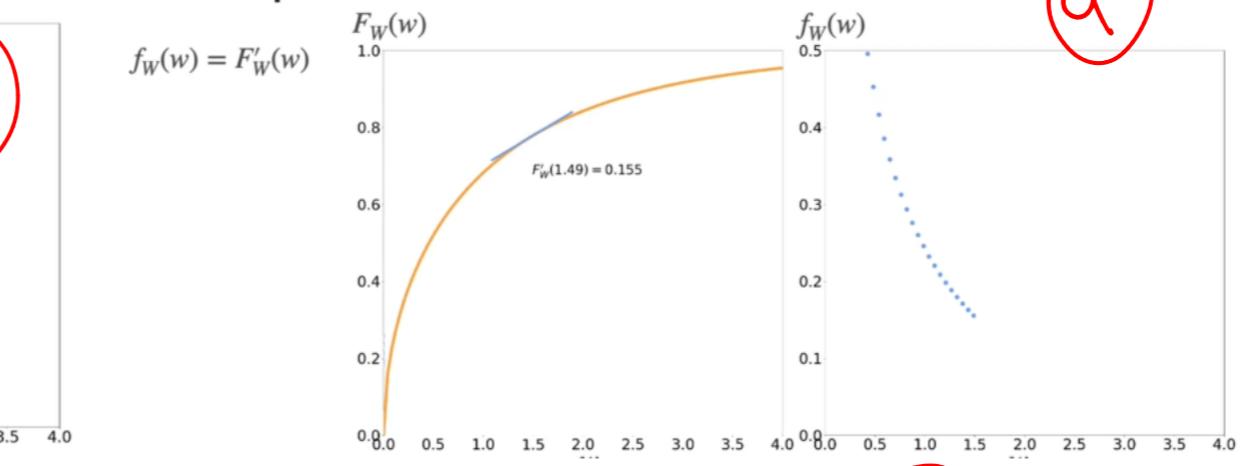


Chi Square Distribution

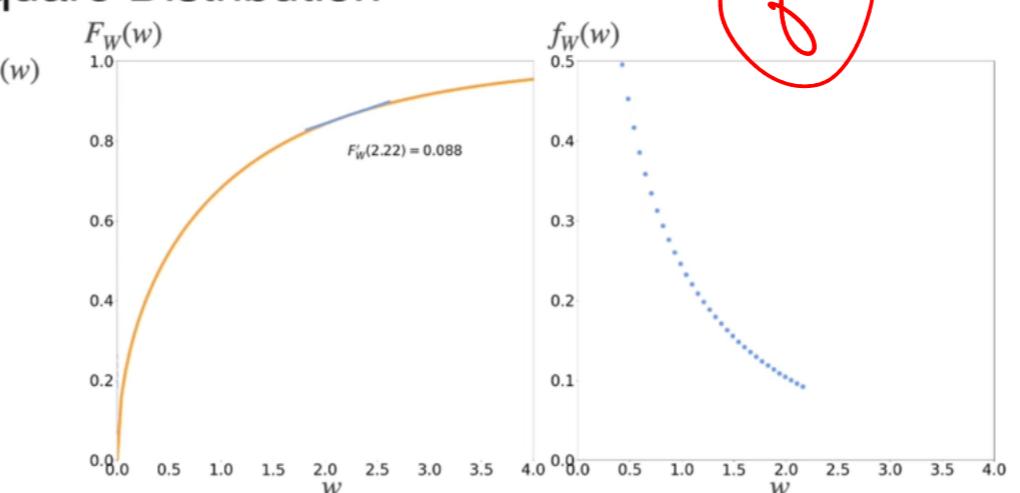


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Chi Square Distribution



Chi Square Distribution



derivative of
CDF?
Slope-

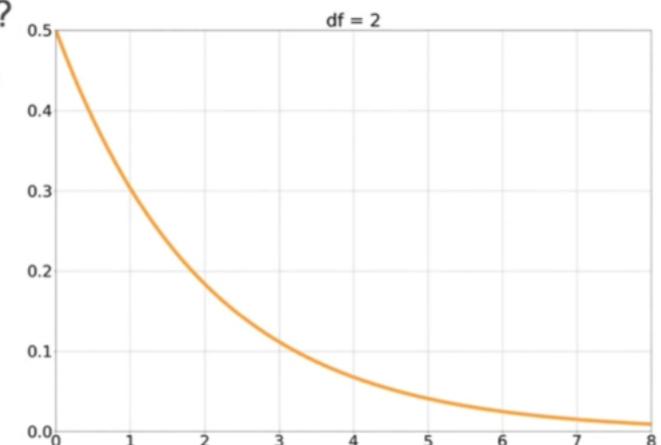
Well, notice that again, this is the sum of many independent standard normal variables squared and is set to follow a Chi-square distribution with k degrees of freedom and notice that as k increases, the PDF is more spread and becomes more and more symmetrical.

Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df



Chi-Square Distribution

Accumulated power over 2 transmissions?

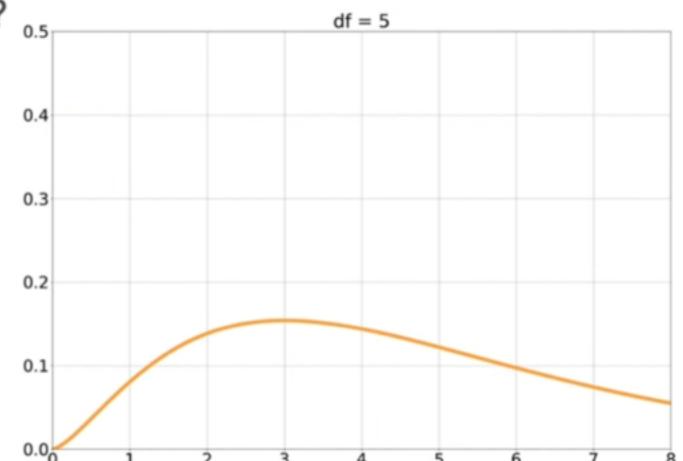
$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square with 5 df



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Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

Accumulated power over 5 transmissions?

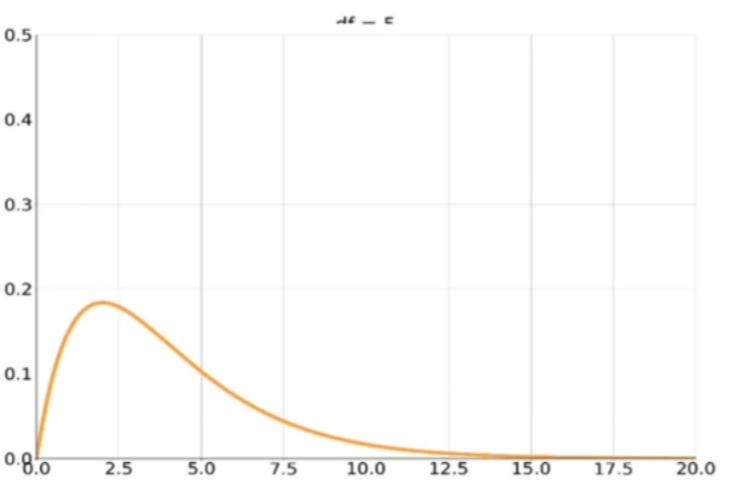
$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square with 5 df

Accumulated power over k transmissions?

$$W_k = \sum_{i=1}^k Z_i^2$$

Chi-Square with k df



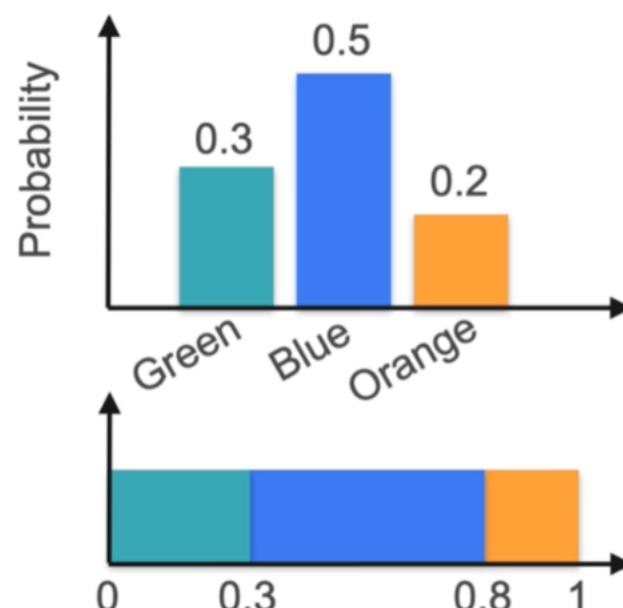
K degrees of freedom increases the PDF becomes more spread and more symmetrical.



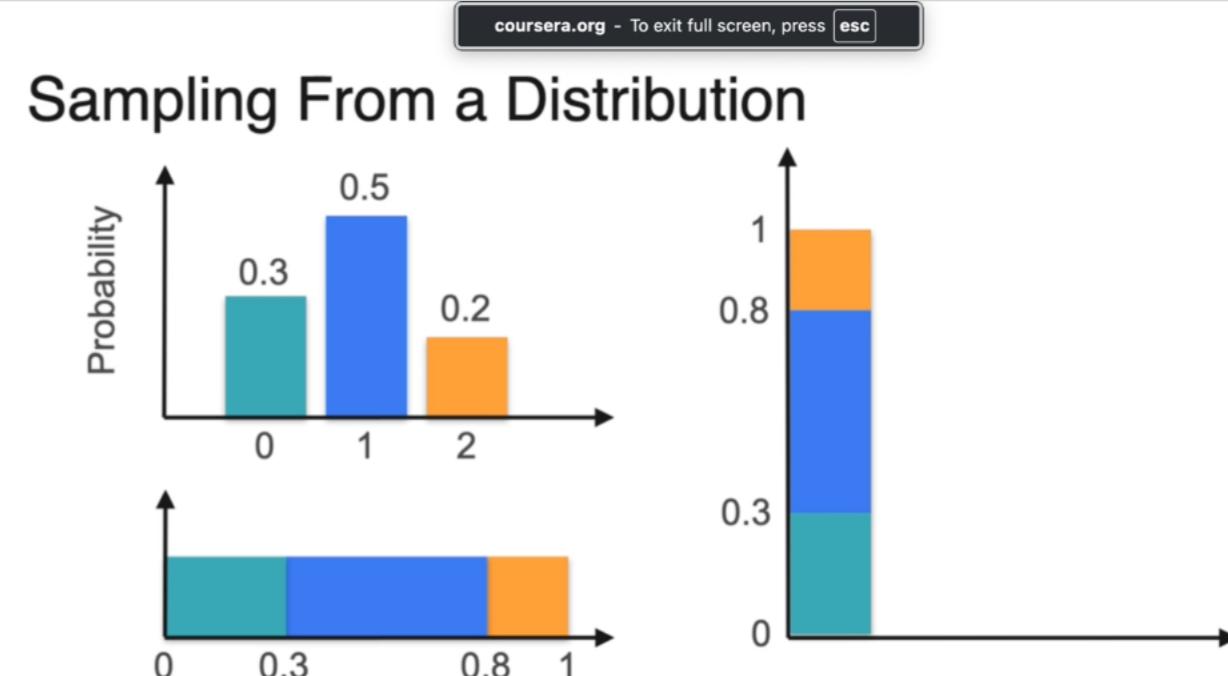
Sampling from Distributions:

- By sampling, I mean picking points that have the probabilities given by the original distribution. So this is a very important concept in probability and in machine learning. Sampling probability distribution.

Sampling From a Distribution

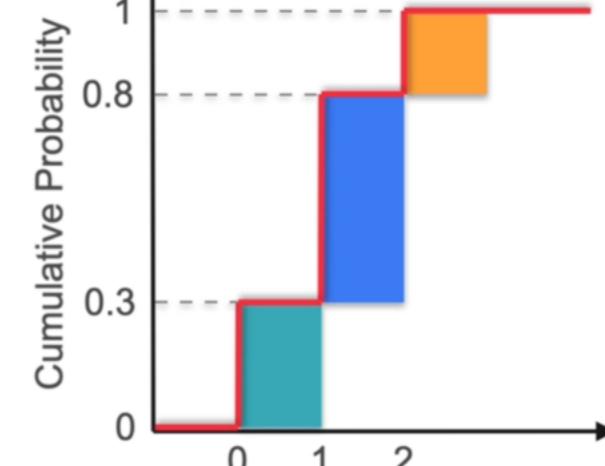
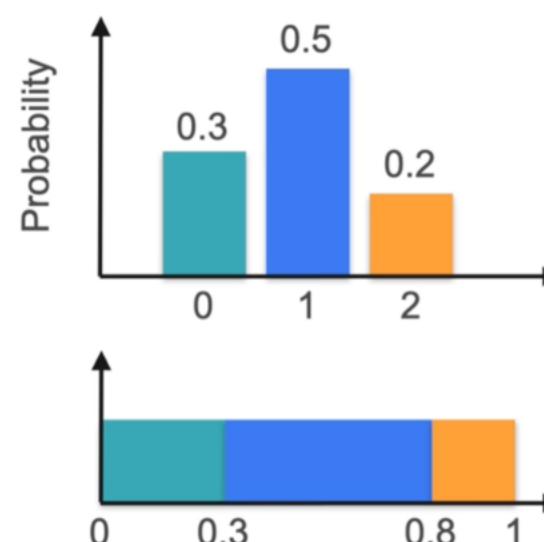


- Step 1: generate a random number between 0 and 1
- Step 2: find out which interval the number belongs to
 - [0, 0.3)
 - [0.3, 0.8)
 - [0.8, 1]
- Step 3: Assign an outcome based on the interval



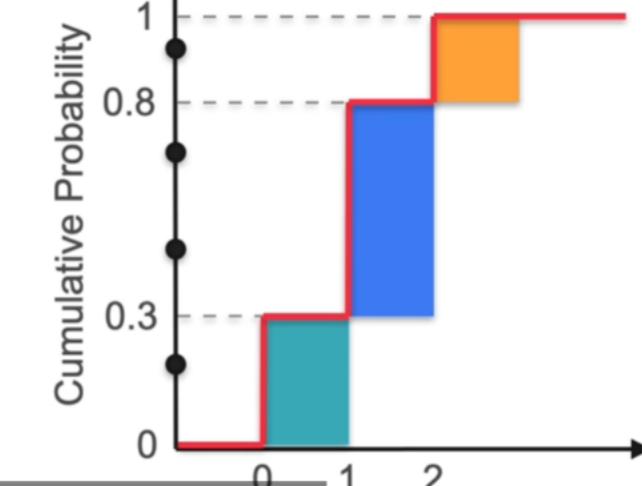
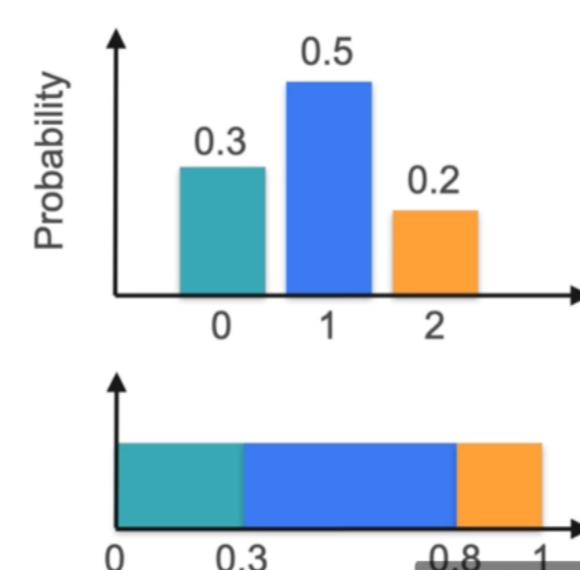
In the fig XX if it is difficult to compute CDF of Normal distribution, so the Group Level represents Interval and we take uniform points and find the Probability distribution.

Sampling From a Distribution



1

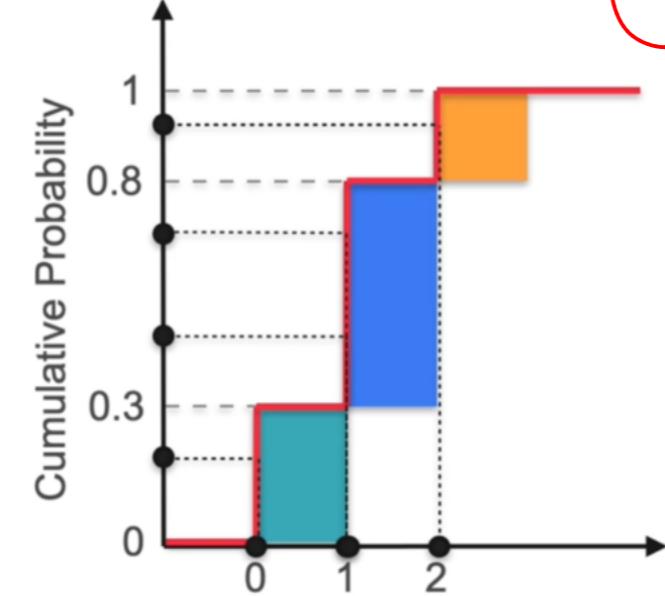
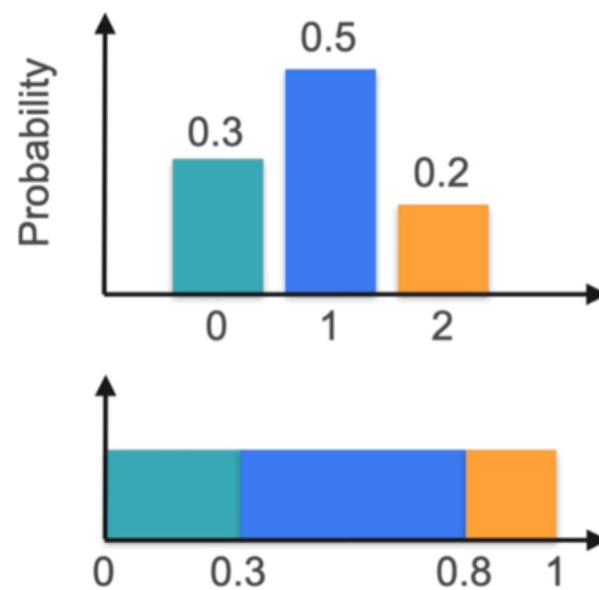
Sampling From a Distribution



2

So these four points are sampled uniformly and then you simply

Sampling From a Distribution



3

Sampling From a Normal Distribution

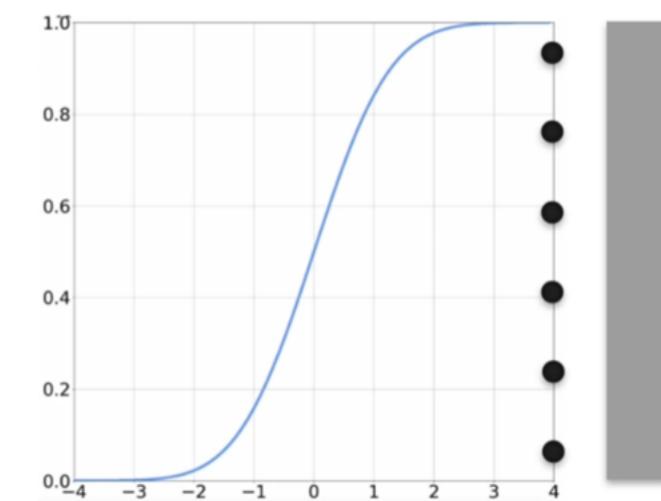
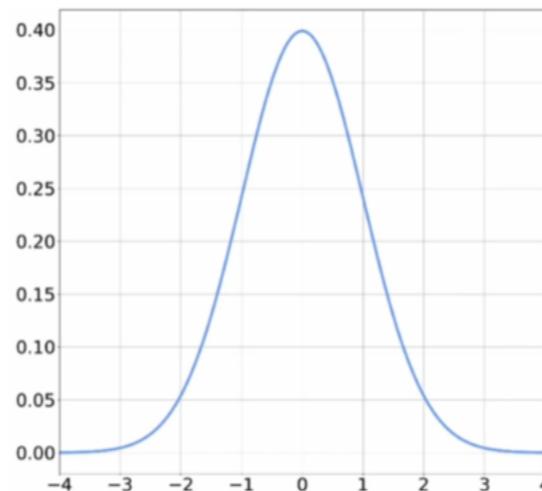
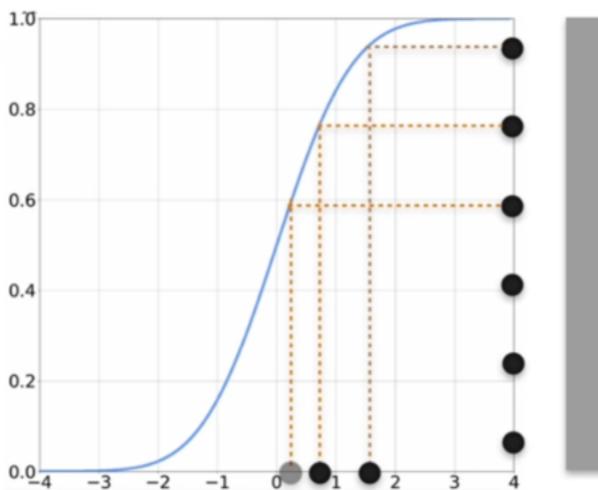
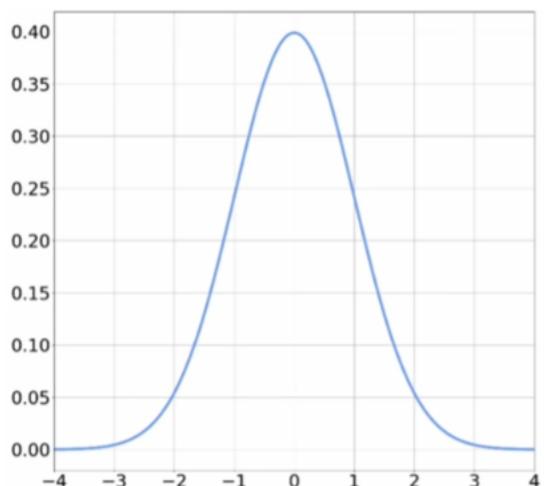


fig. XY

And now all we do is we take a look at where this hits

Sampling From a Normal Distribution

①



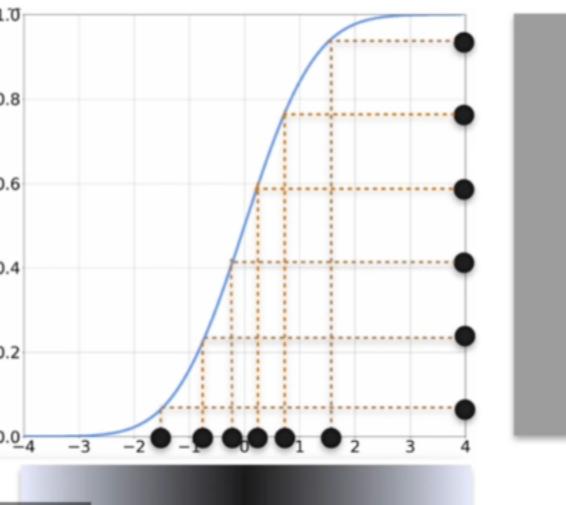
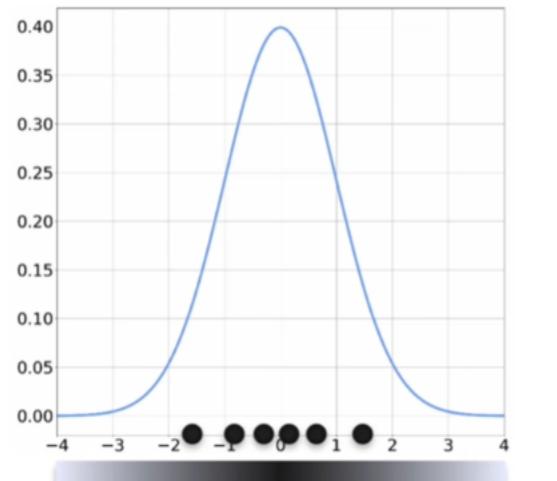
So these points over here
are actually distributed

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Sampling From a Normal Distribution

②

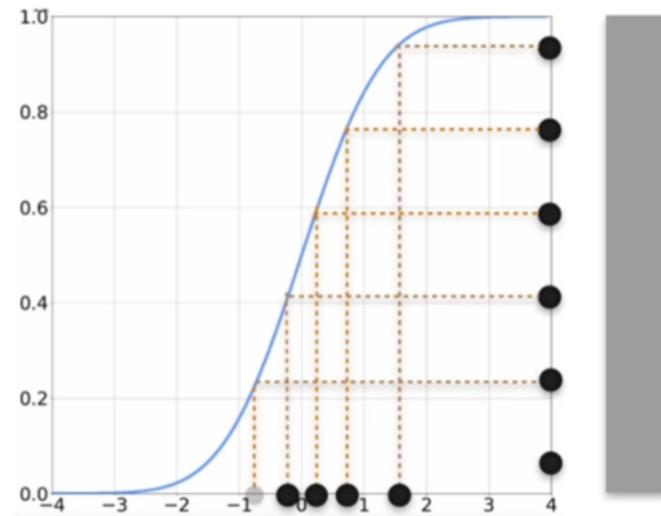
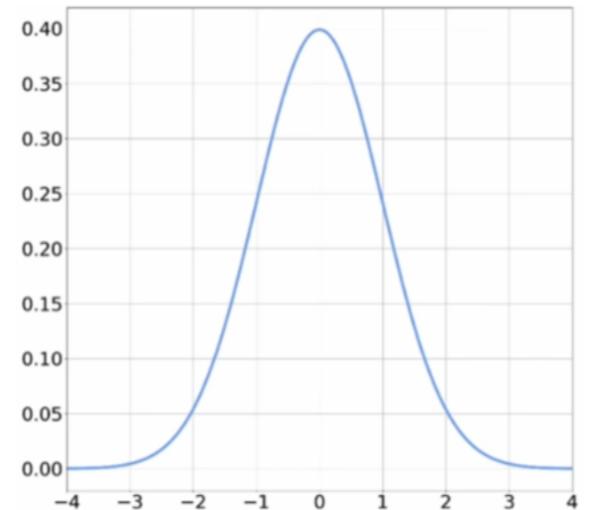


that's the exact
distribution that they're taken from.

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Sampling From a Normal Distribution

③

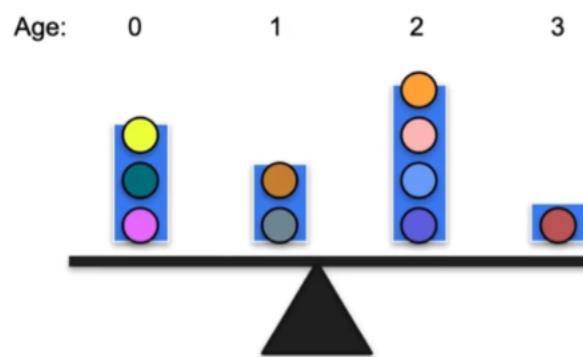


based on the normal
distribution on the left.

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Expected Value ($E(x)$)
Mean, Median and Mode
describe the central distribution
of data.

Mean: Example



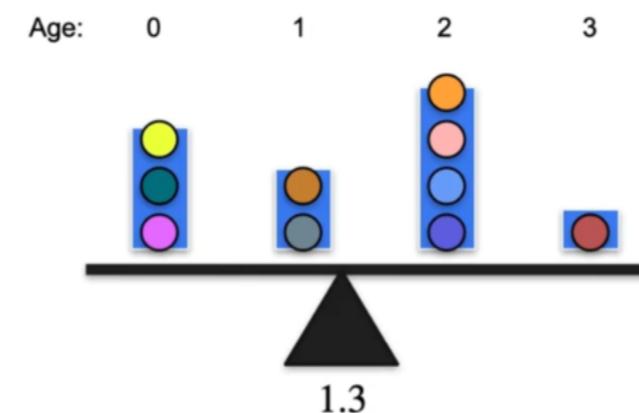
$$\frac{0 + 0 + 0 + 1 + 1 + 2 + 2 + 2 + 3}{10}$$

$$= \frac{13}{10} \\ = 1.3$$

This sums up to 13 over 10 which equals 1.3.

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Children in a Room



$$\frac{0 + 0 + 0 + 1 + 1 + 2 + 2 + 2 + 3}{10}$$

$$= \frac{13}{10} = 1.3$$

$$= \frac{3 \cdot 0 + 2 \cdot 1 + 4 \cdot 2 + 1 \cdot 3}{10}$$

Weighted average

$$= \frac{3}{10} \cdot 0 + \frac{2}{10} \cdot 1 + \frac{4}{10} \cdot 2 + \frac{1}{10} \cdot 3 = 1.3$$

is the weighted average of the values of the variable where the

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Expected Value: Motivation Example 1

You play a game with a friend



You win 10 dollars

You win nothing

Long term: $0.5 \cdot \$10 + 0.5 \cdot \$0 = \$5 \rightarrow$ You expect to win \$5 on average
 $\mathbb{E}[X] = 5$

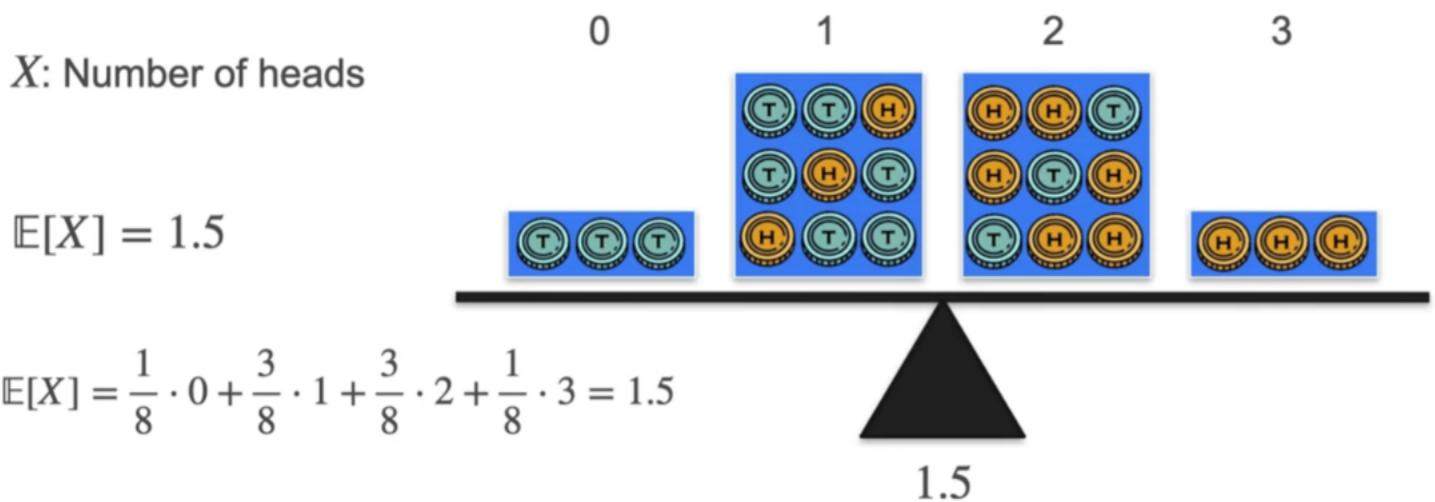
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Expected Value: Motivation Example 1



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Expected Value: Motivation Example 2



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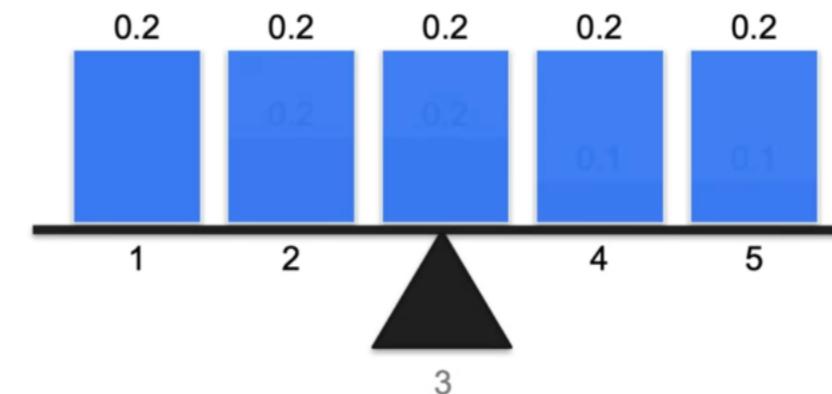
In the case of PMF

$$P_X(x) = P(X=x)$$

$$E[X] = \sum_x x P_X(x)$$

Expected Value

Ideal Case:



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Expected Value: Discrete Case

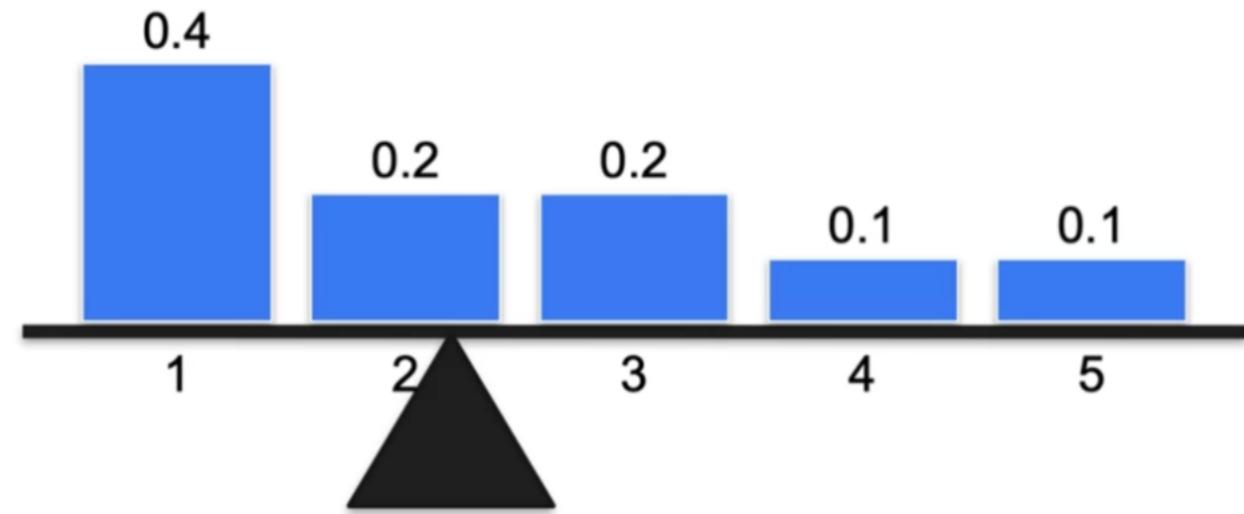
X a discrete random variable

PMF of X
 $p_X(x) = P(X=x)$

$$E[X] = \sum_x x p_X(x)$$

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Expected Value



equilibrium point towards it.

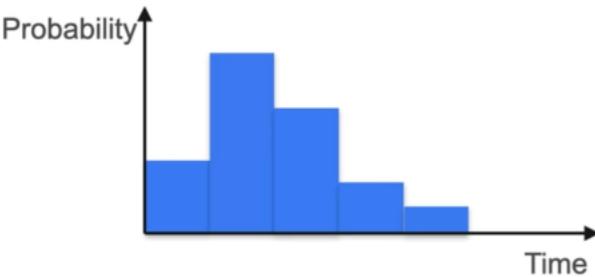
In this case,

-We can't just increase one probability without adjusting (increase or decrease) others

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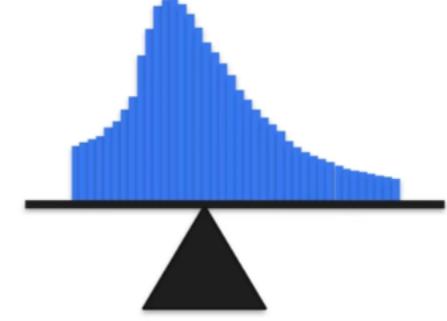
Expected Value - Continuous



For this case,
it balances somewhere around

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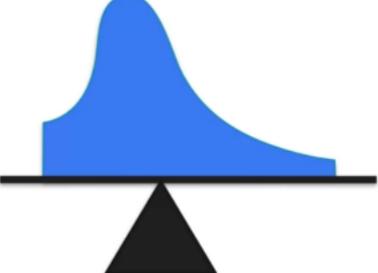
Expected Value - Continuous



talk about the distribution in your
sample more precisely and you can

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Expected Value - Continuous



you get a continuous distribution.

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Expected Value - Continuous

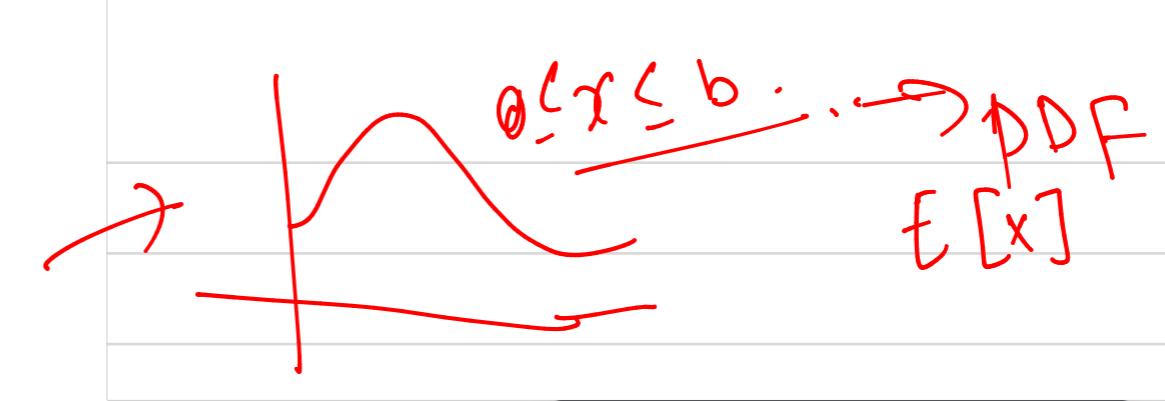
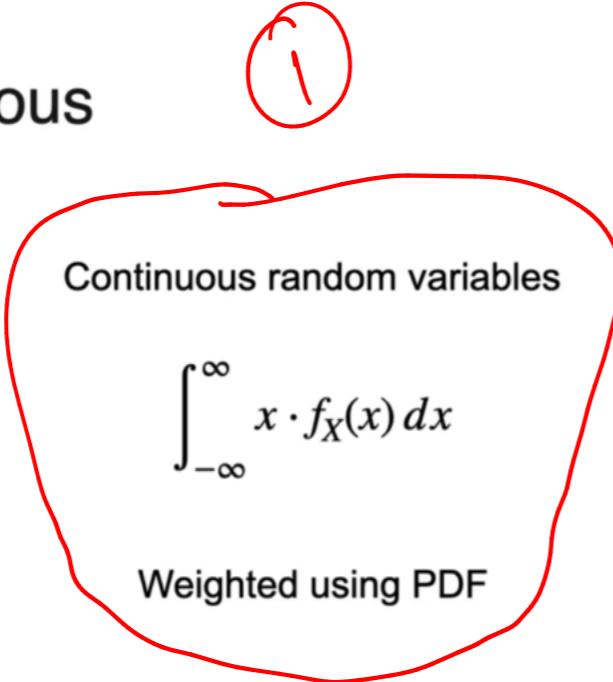
Discrete random variables

$$\mathbb{E}[X] = \sum_x x p_X(x)$$

Weighted using PMF

the possible values of x , but in the discrete case you weight the sum

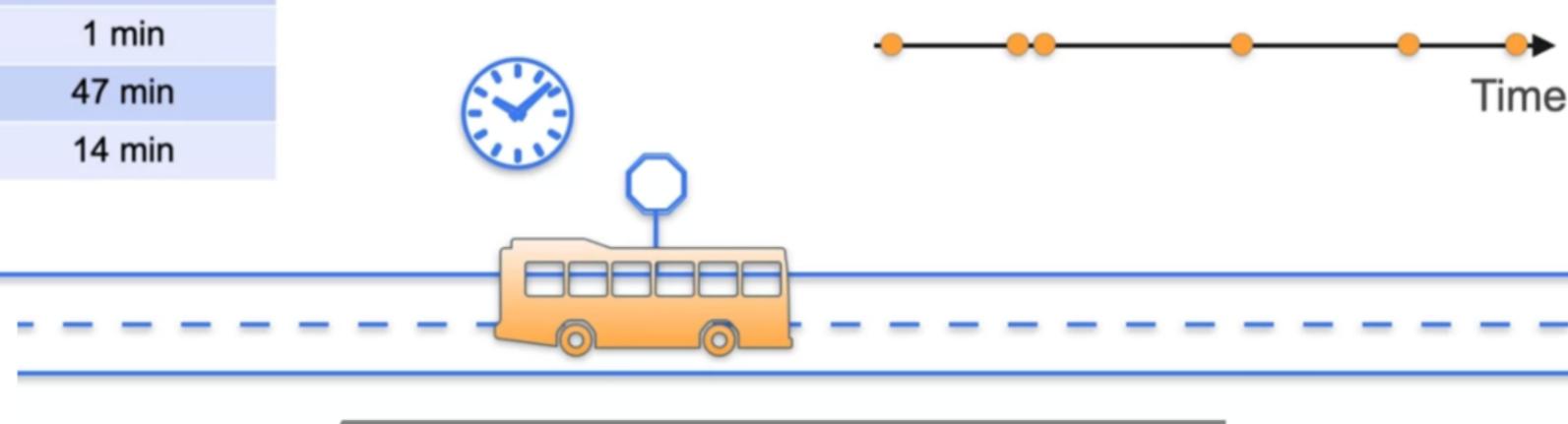
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Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min

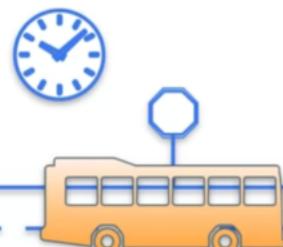
② Rocking| Knocking records



Expected Value

Waiting Time
15 min
32 min
58 min
1 min
47 min
14 min
37 min
8 min
29 min
55 min
...

Average = 30

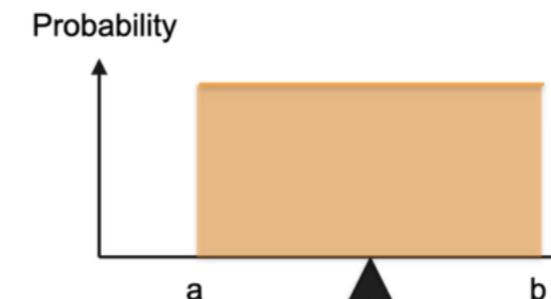


point can land pretty much anywhere in that interval equally.

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Expected Value: Uniform Distribution

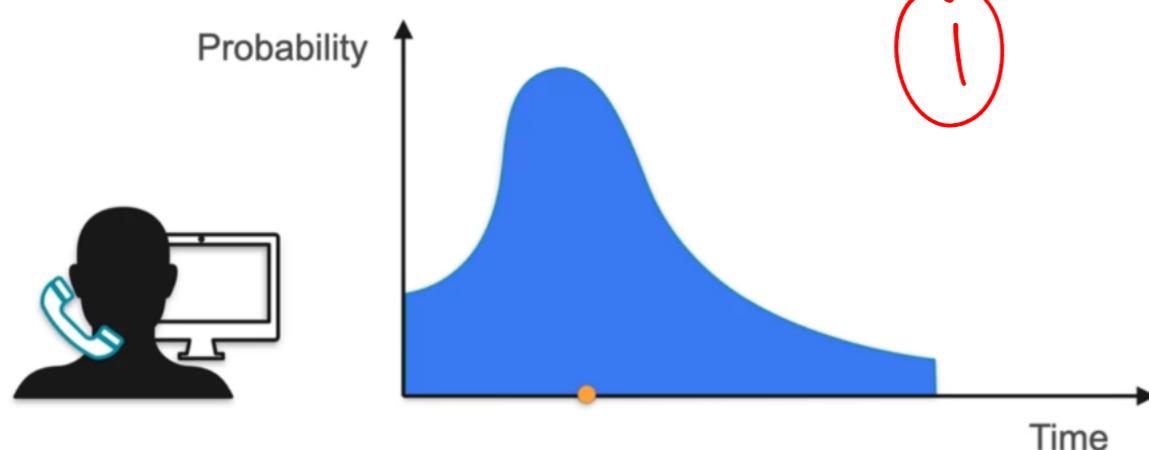
$$\text{Average} = \frac{a + b}{2}$$



balancing it in the middle which is the point a

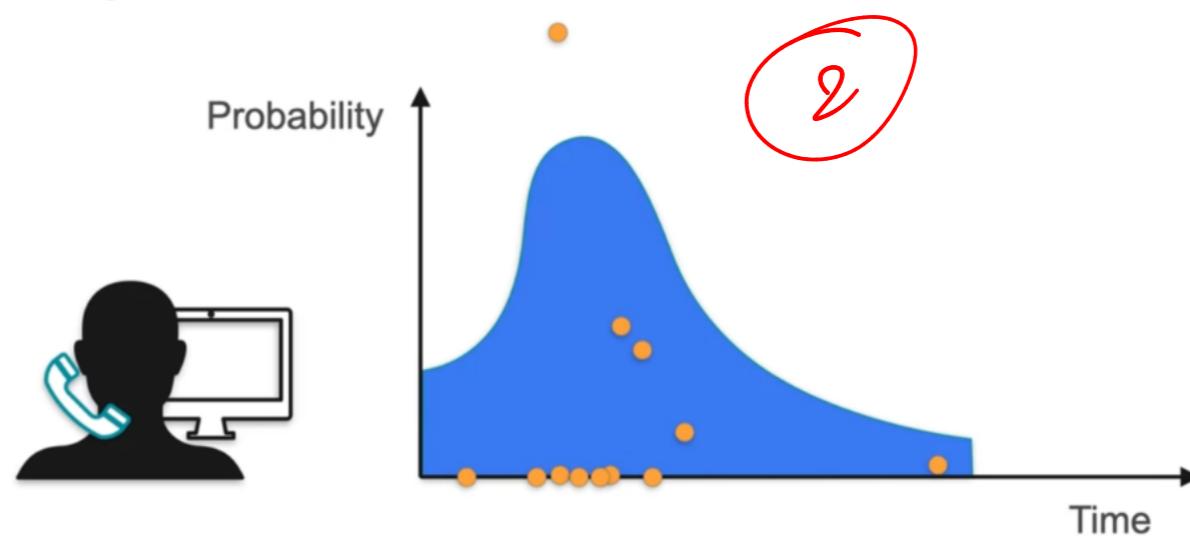
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Expected Value



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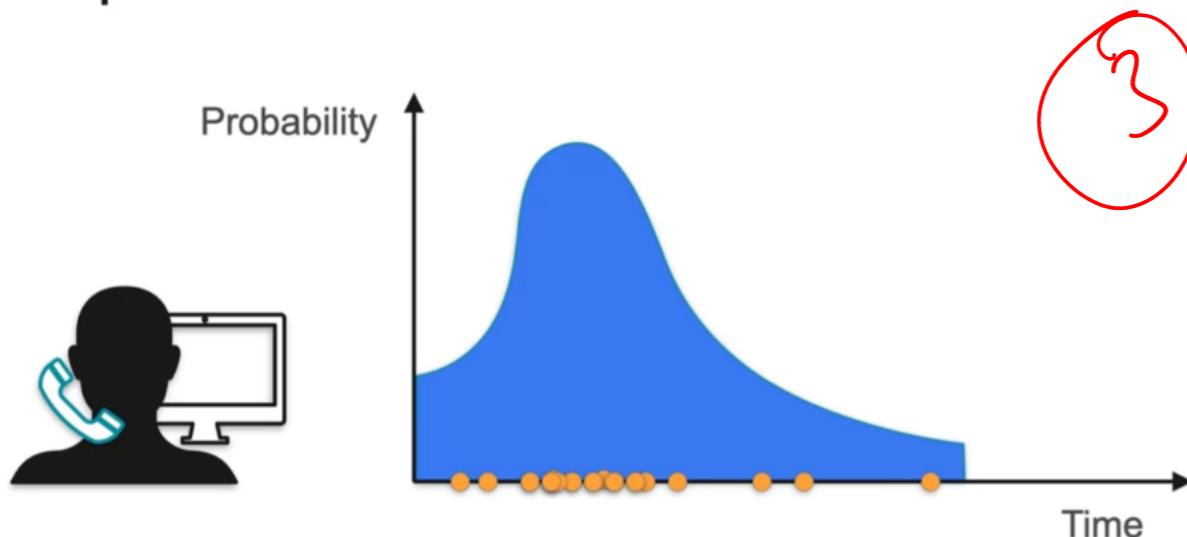
Expected Value



If you make more calls
and record how long you wait,

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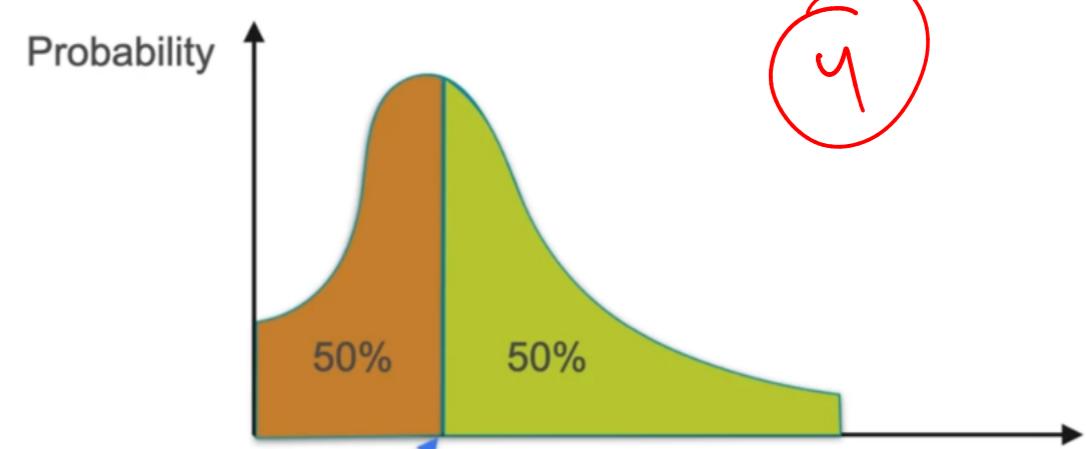
Expected Value



with relatively more dots where the
probability density function

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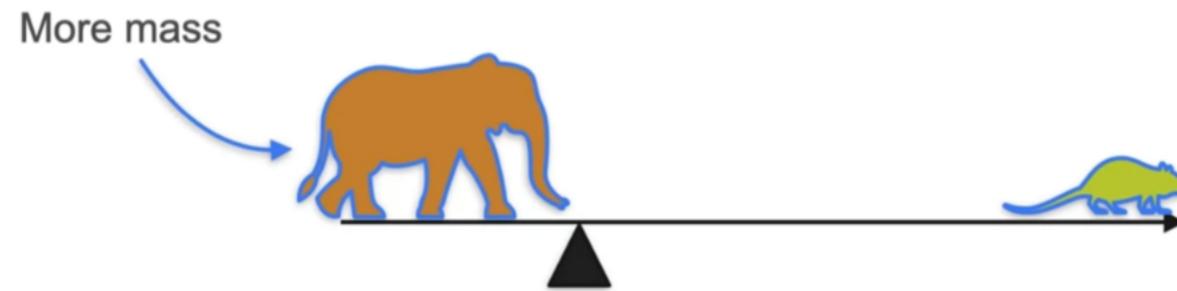
Expected Value: Common Misconception



Not the mean

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Expected Value: Common Misconception



Even though the elephant has more mass,
the mouse is much,

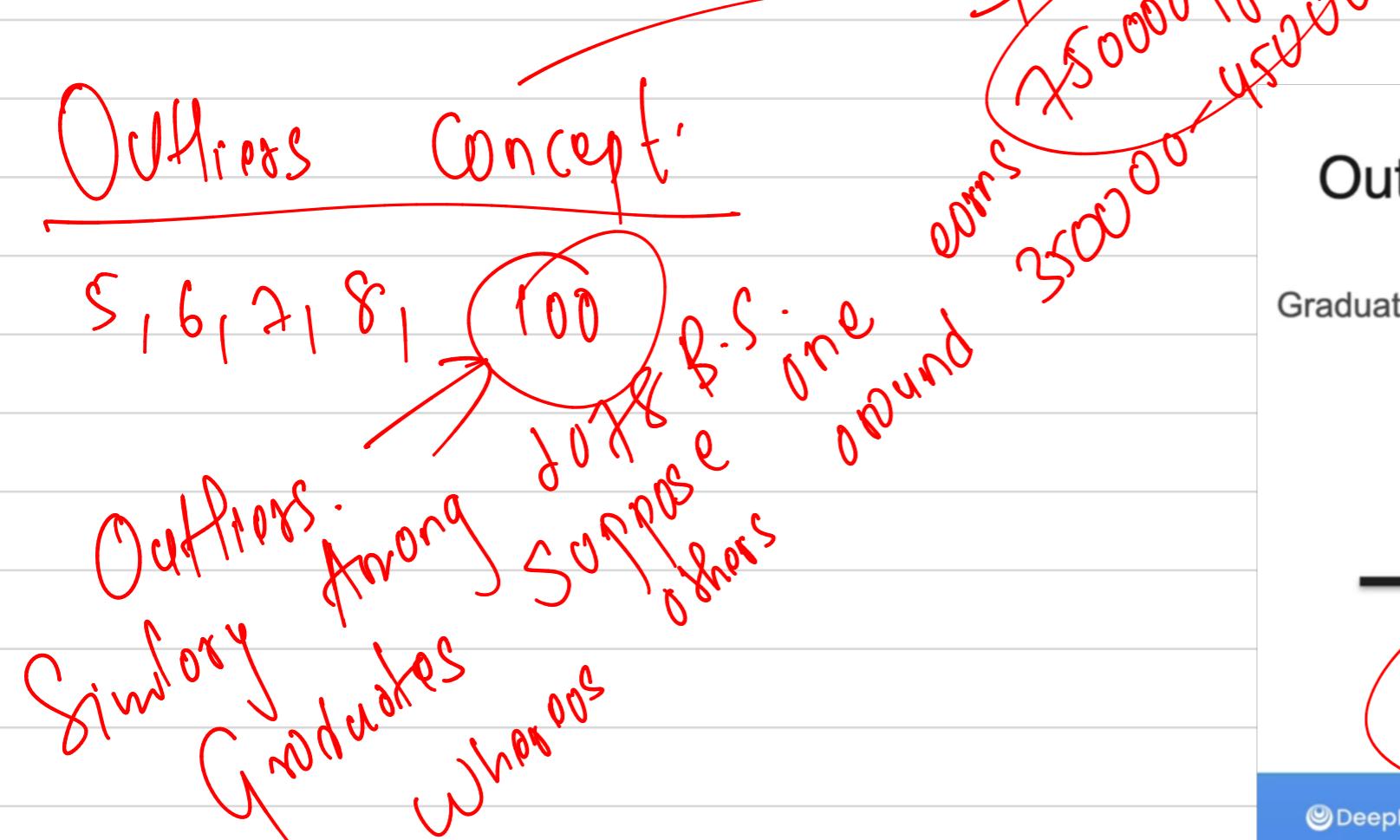
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Expected Value

- $E[X]$
- Mean / Balancing point
- Defined for discrete and continuous random variables
- Weighted average of the PMF / PDF

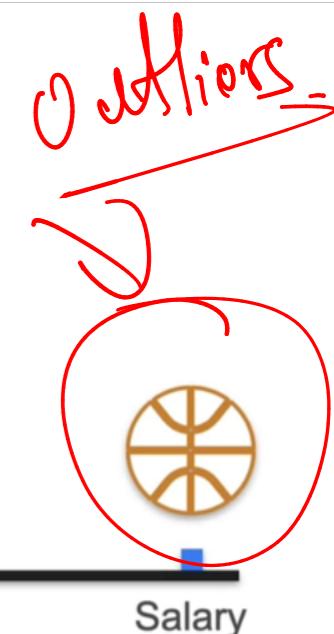
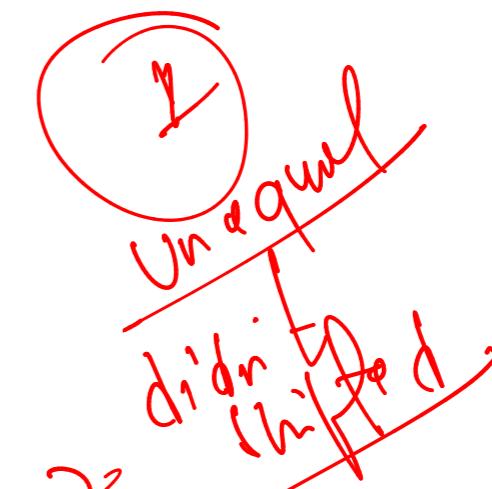
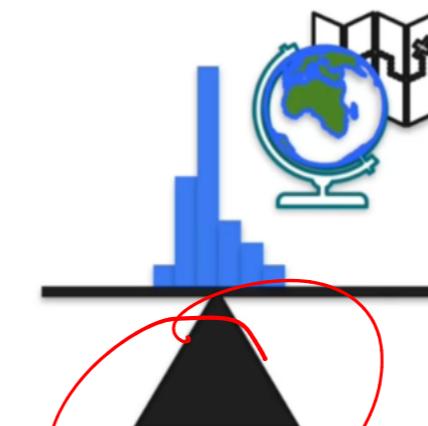
random variables, and you can think
of it as the

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Outliers

Graduates

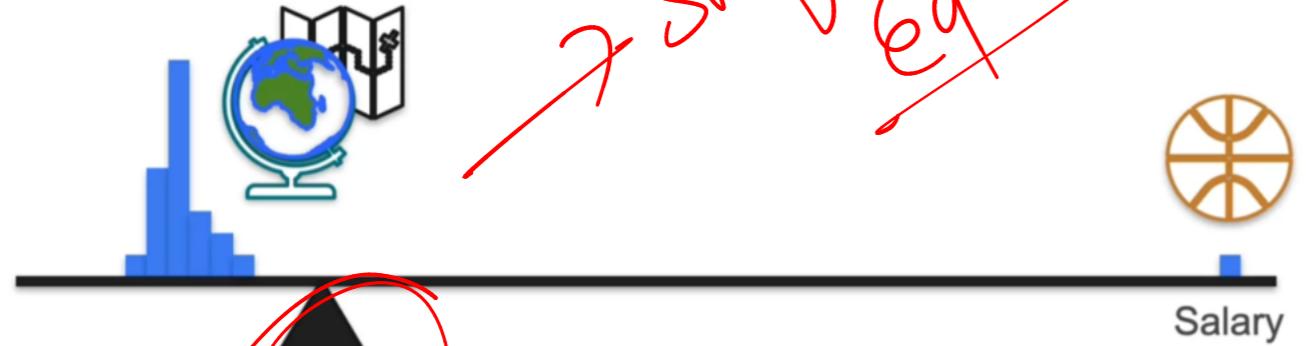


However Michael Jordan is
all the way here

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Outliers

Graduates

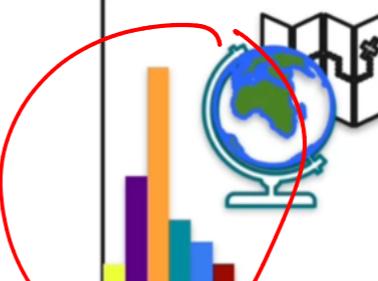


It got skewed by that one point on the right.

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Outliers

Graduates



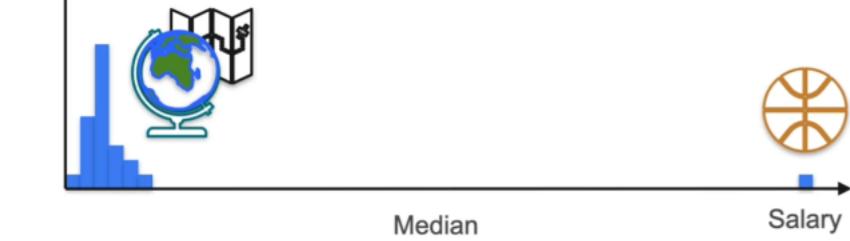
Salary

(3)



Salary

Graduates

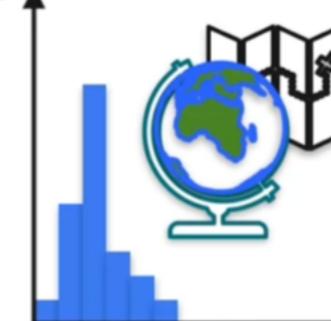


one person to this list that is the median now if the

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Median

Graduates

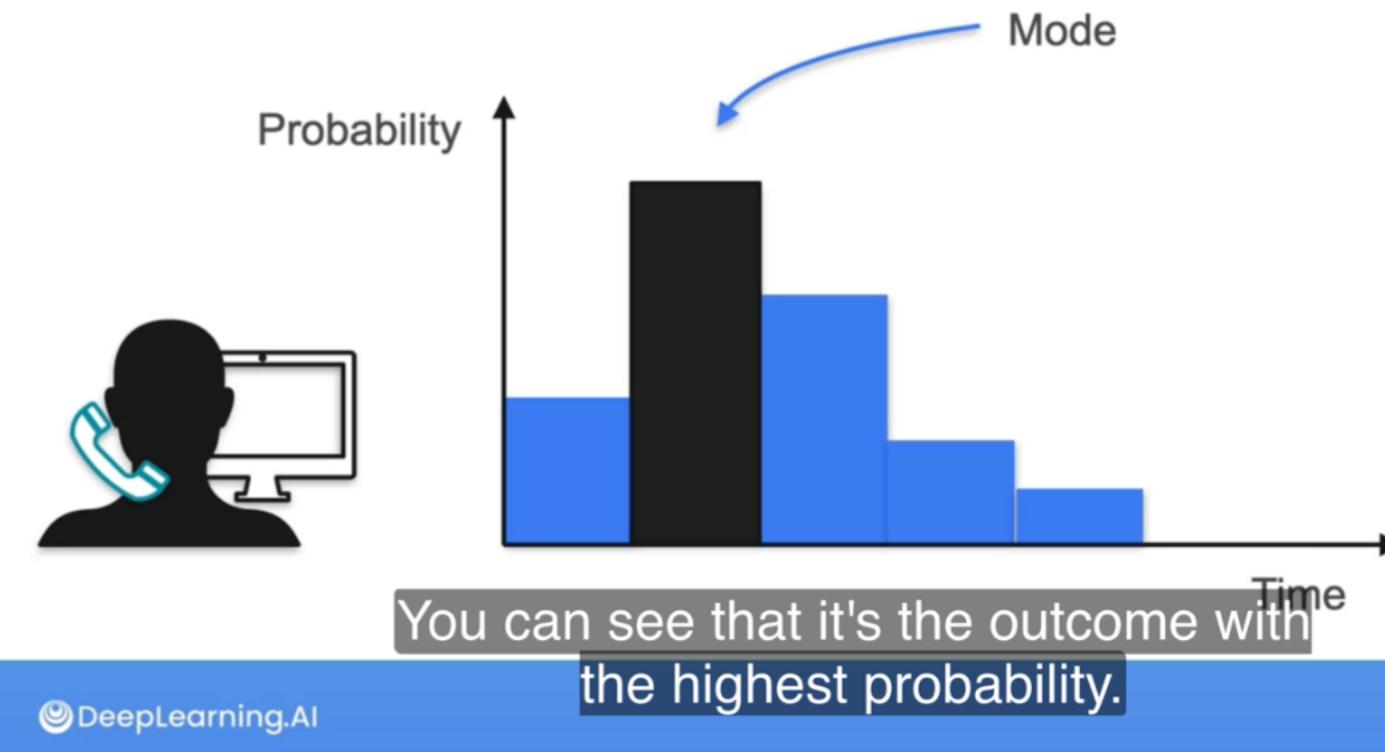
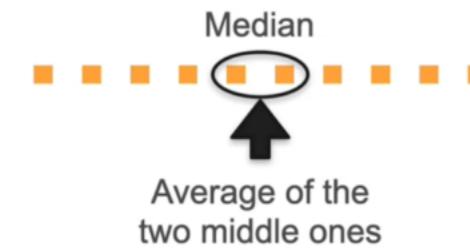


Salary

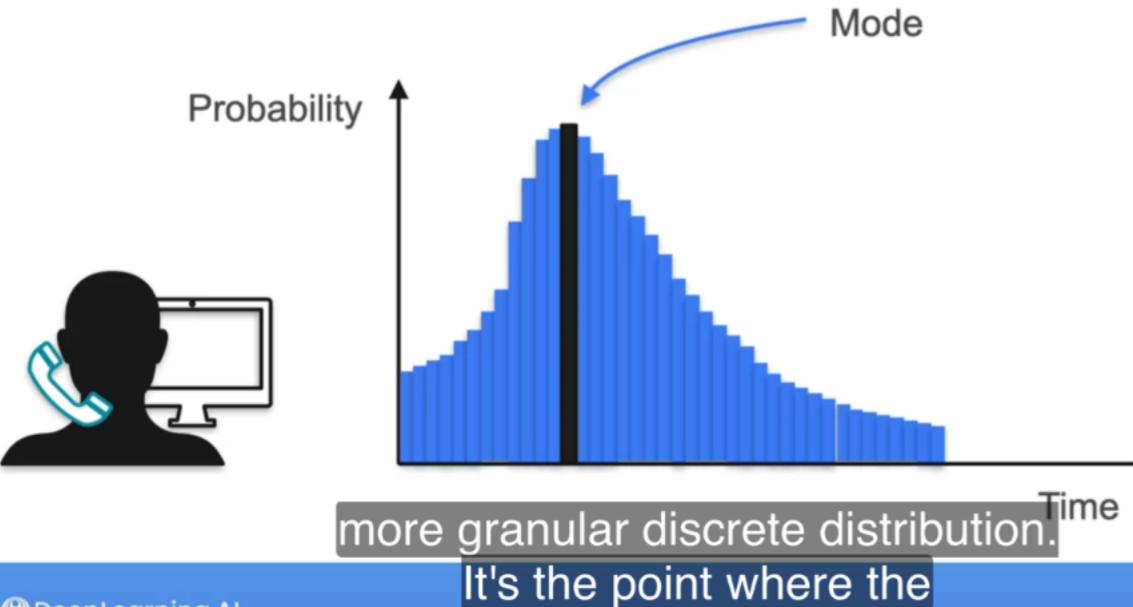
So now it's just the position that matters and once we do that

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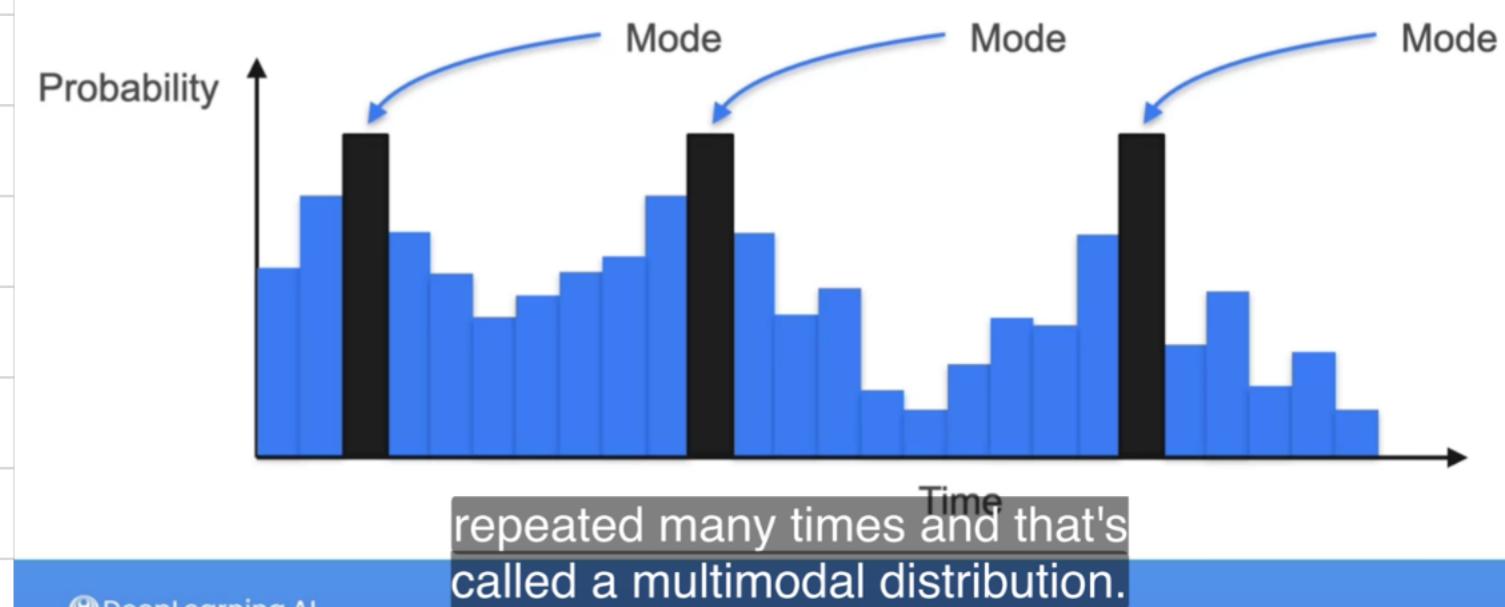
Median



Mode

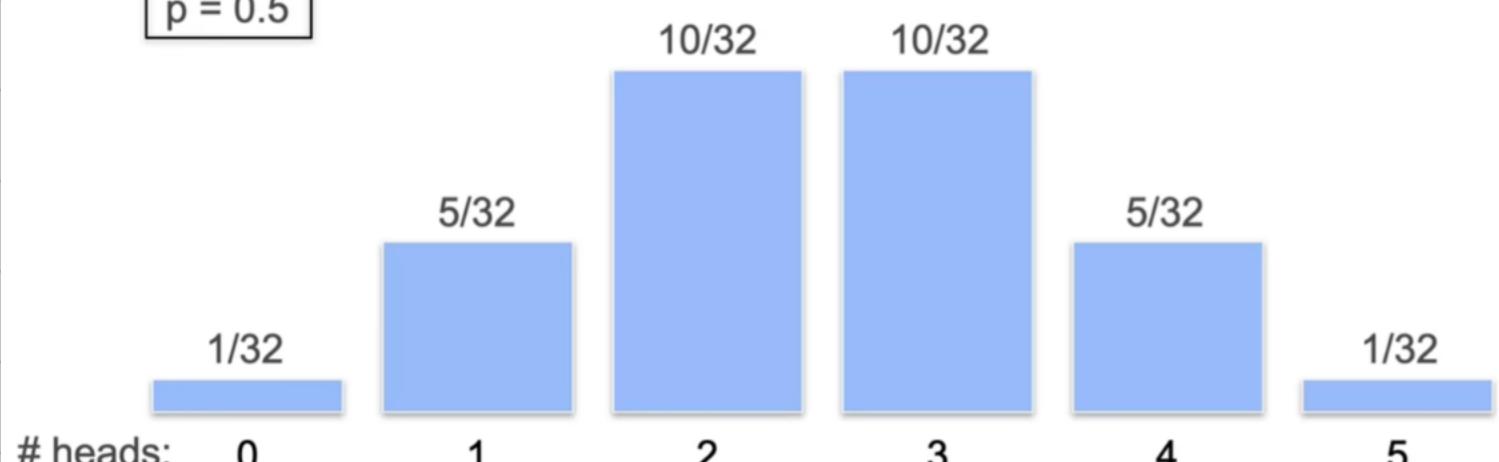


Mode: Multimodal Distribution



Mean, Median and Mode in Binomial Distribution

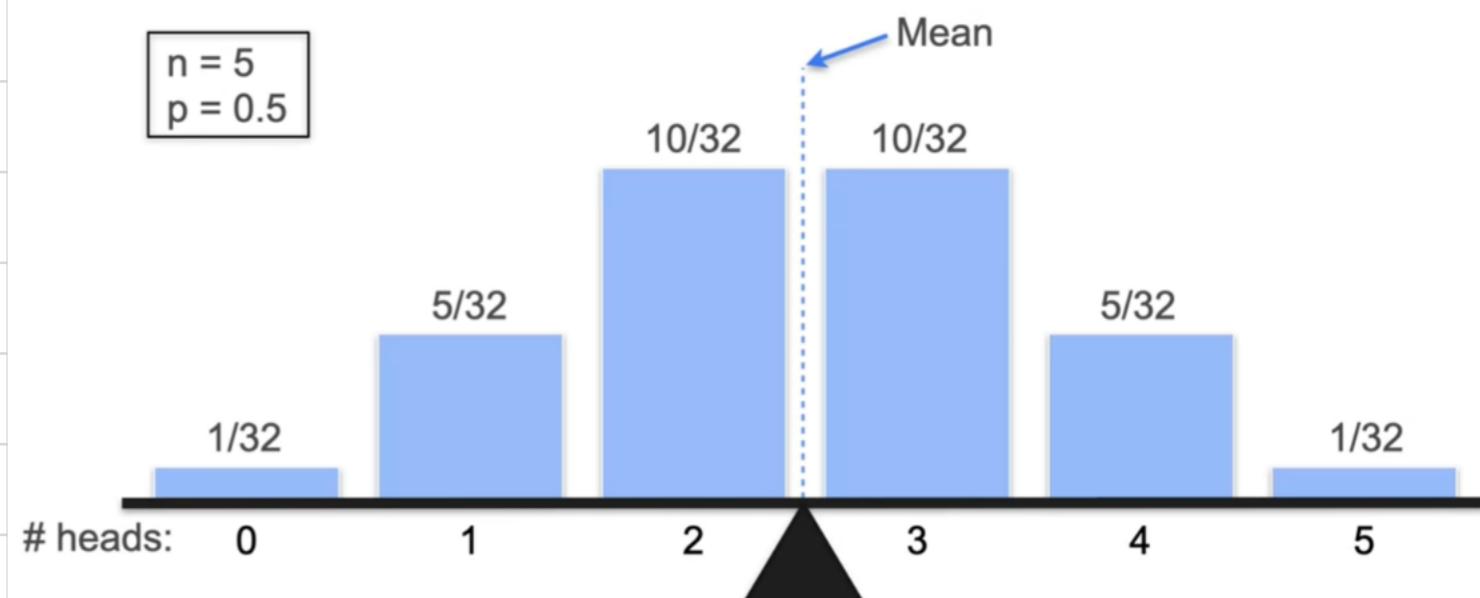
$n = 5$
 $p = 0.5$



Now, let's look at a real example, the binomial distribution.

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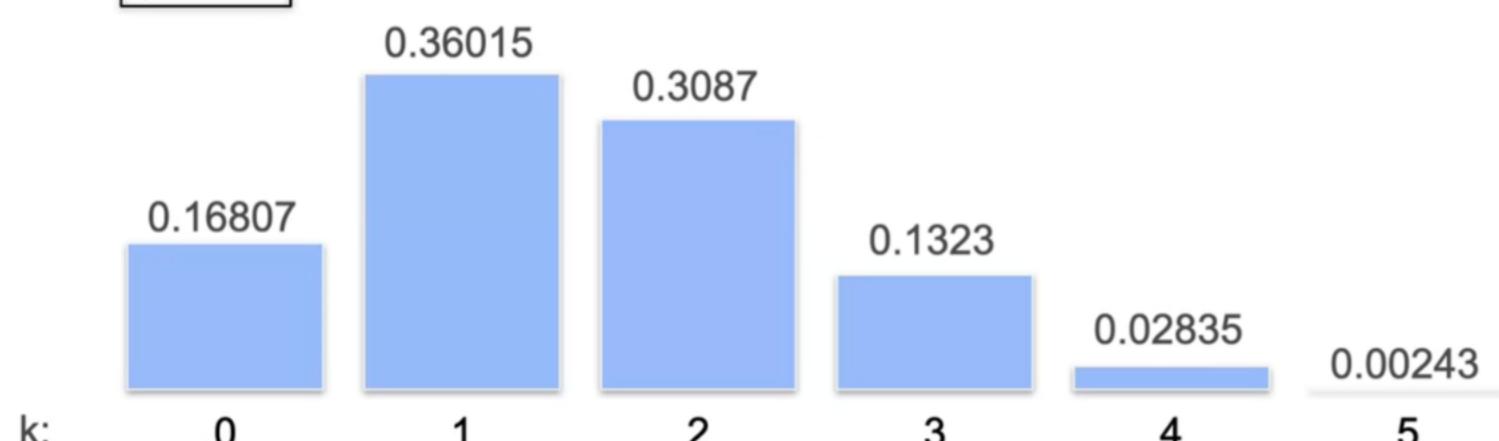
$n = 5$
 $p = 0.5$



The mean is at a value of 2.5.

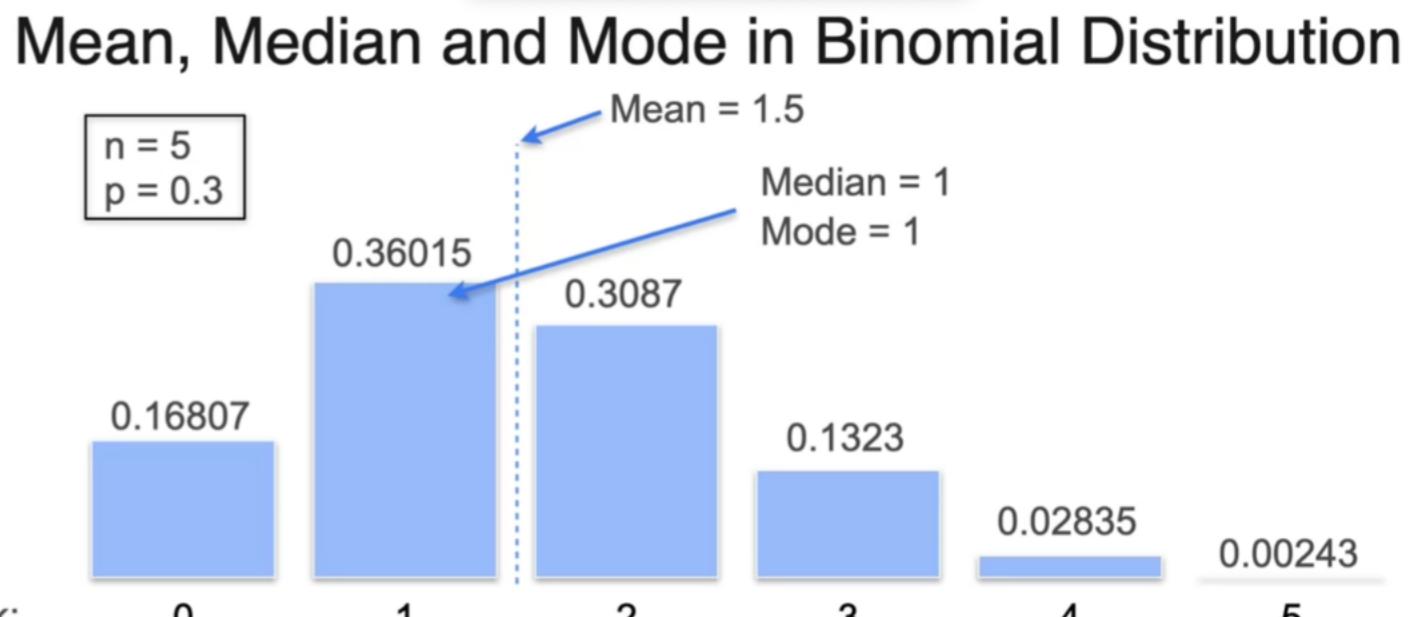
Mean, Median and Mode in Binomial Distribution

$n = 5$
 $p = 0.3$



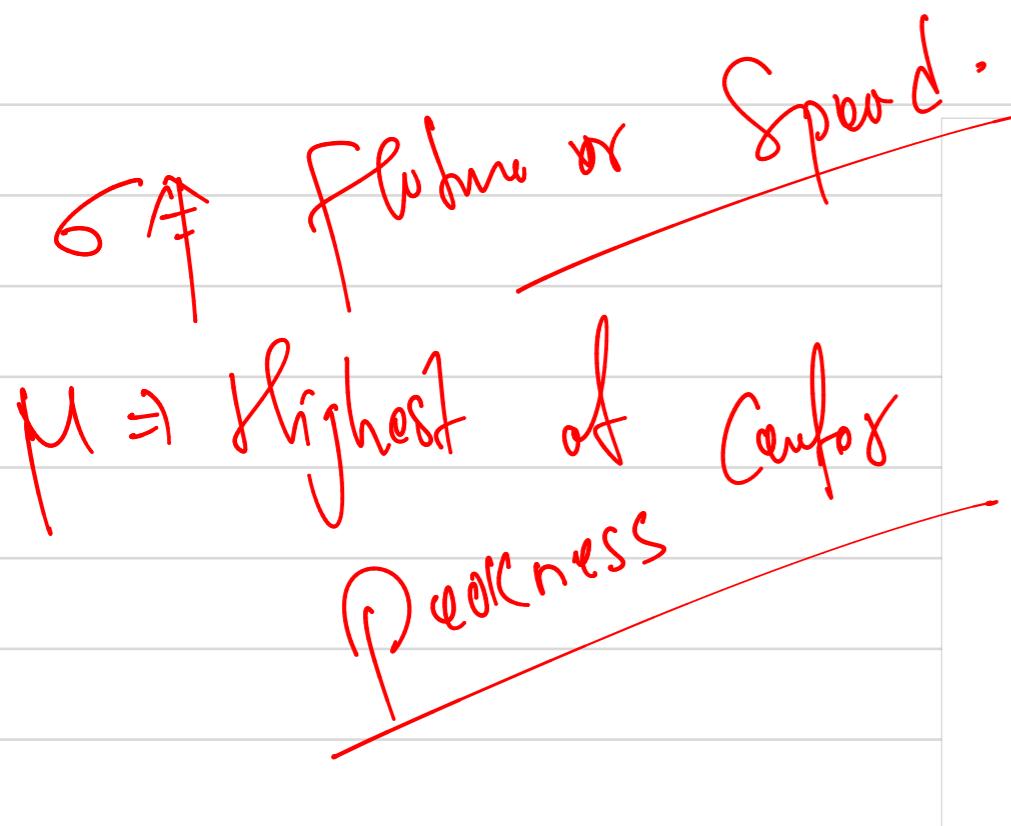
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$n = 5$
 $p = 0.3$

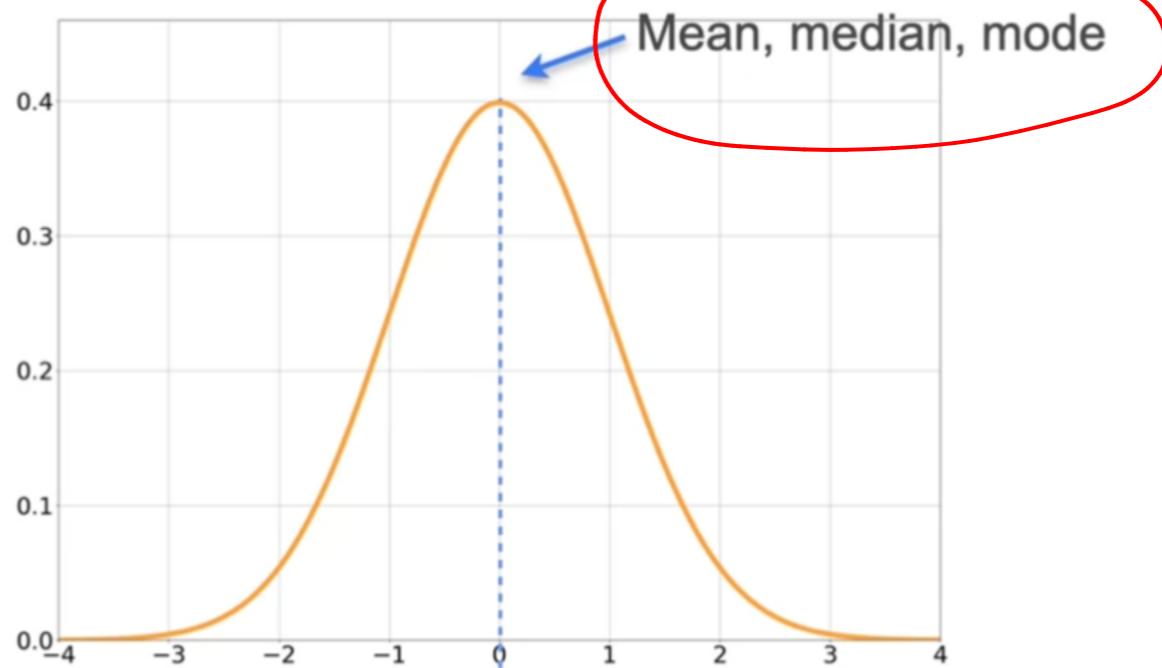


And mode is again the outcome with the highest probability,

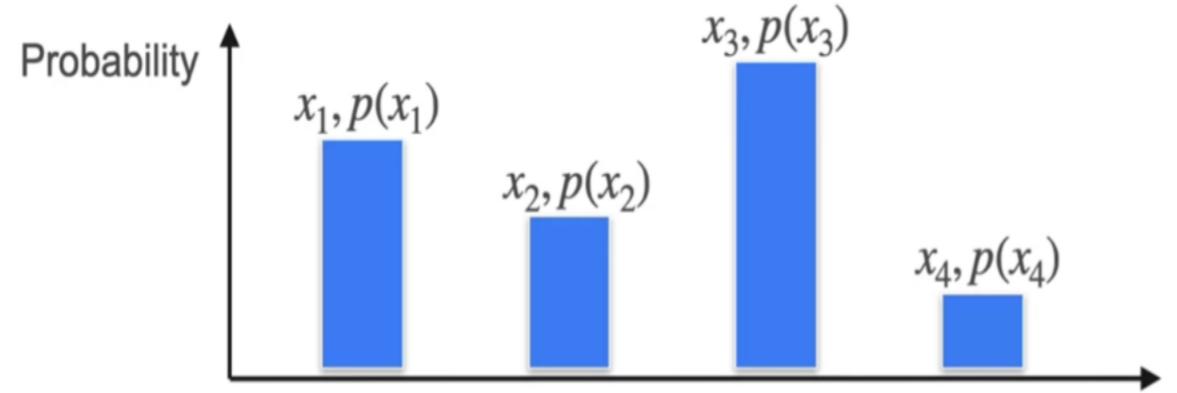
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Mean, Median and Mode in Normal Distribution



Expected Value of a Function



$$\mathbb{E}[X] = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$

$$E[g(X)] = g(x_1)p(x_1) + g(x_2)p(x_2) + g(x_3)p(x_3) + g(x_4)p(x_4)$$

You just replace the x

which is the highest one.
So for a normal distribution,

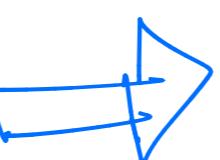
$$\begin{aligned} E[X] &= x_1 p(x_1) + x_2 p(x_2) + \\ &\quad x_3 p(x_3) + x_4 p(x_4) \\ &\text{replace } x_i \text{ with } g(x_i) \end{aligned}$$

Expected Value of a Function

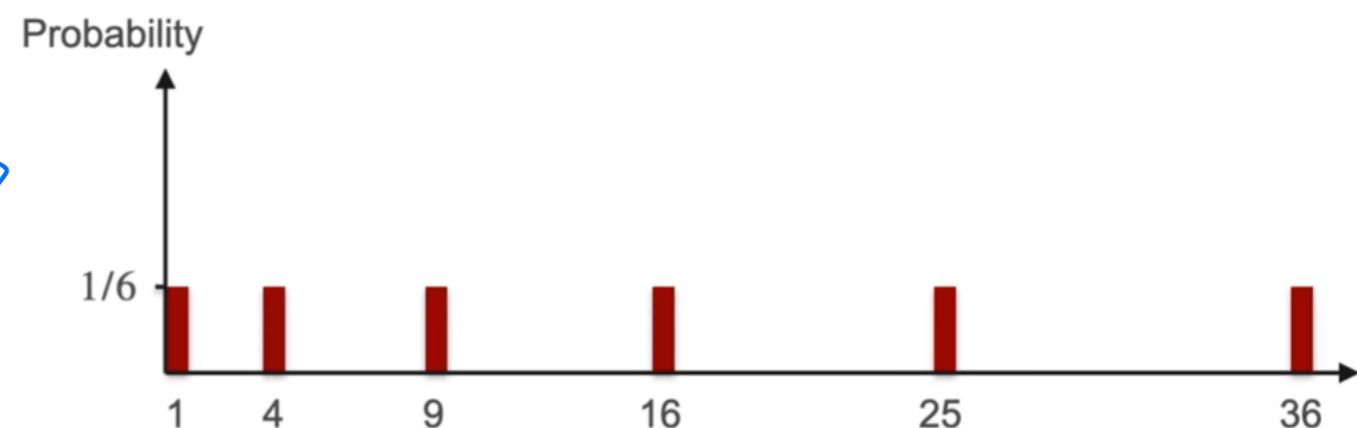
Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6
Square:	1	4	9	16	25	36

What is the fair price
to pay for this game?

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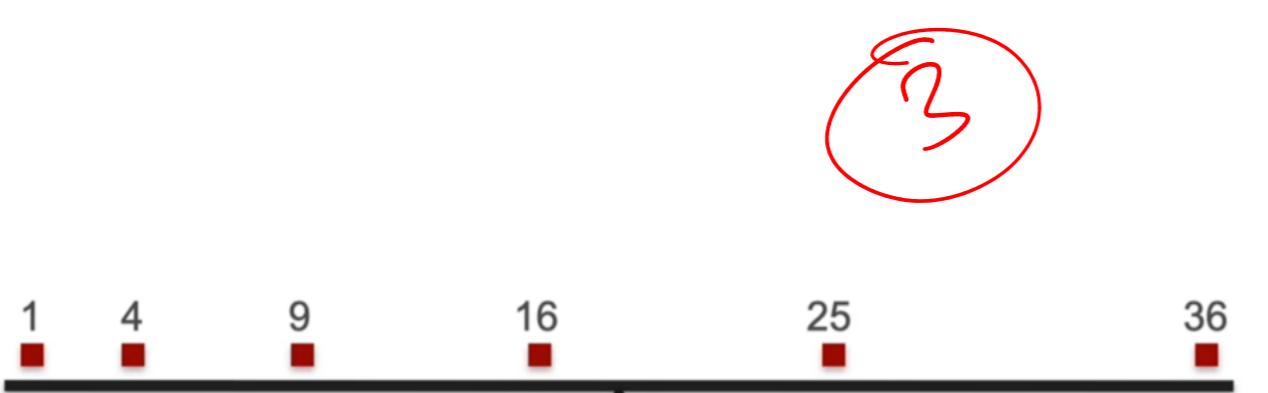
Expected Value of a Function



1
2

You can plot the distribution of

Expected Value of a Function



you can simply take the
average of these numbers,

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Expected Value of a Function

$$\frac{1 + 4 + 9 + 16 + 25 + 36}{6} = \frac{91}{6} = \frac{1^2}{6} + \frac{2^2}{6} + \frac{3^2}{6} + \frac{4^2}{6} + \frac{5^2}{6} + \frac{6^2}{6}$$

$$= \mathbb{E}[X^2]$$



You have actually calculated
the expected value of x^2 .

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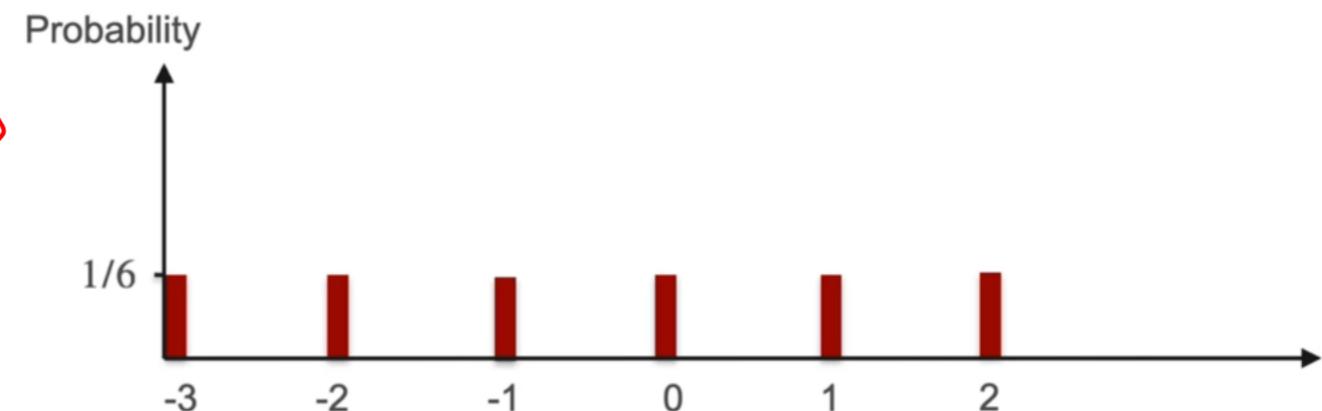
Expectation of Linear Function

Probability:	1/6	1/6	1/6	1/6	1/6	1/6
Roll:	1	2	3	4	5	6
Double:	2	4	6	8	10	12
Wins	-3	-2	-1	0	1	2

Which means your wins are -3,

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Expected Value of a Function



your payoffs and try to balance it on a scale,

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Expected Value of a Function

$$\frac{(2 \cdot 1 - 5) + (2 \cdot 2 - 5) + (2 \cdot 3 - 5) + (2 \cdot 4 - 5) + (2 \cdot 5 - 5) + (2 \cdot 6 - 5)}{6} = \frac{-3}{6} = -0.5$$



the value rolled minus the five dollars you started with.

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Expected Value of a Function

$$\frac{2 \cdot (1+2+3+4+5+6) + (-5)}{6} = \frac{-3}{6} = -0.5$$



Now, $1+2+3+4+5+6/6$ is simply

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We can use only:

numbers in a and b

Therefore, we can use any values for 'a' and 'b' to create different linear transformations of the expected value, each with its own scaling and shifting effects.

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Sum of Expectations



Win \$1

Win \$1 \$2 \$3 \$4 \$5 \$6



Win nothing

$$\mathbb{E}[X_{coin}] = \$0.5$$

$$\mathbb{E}[X_{dice}] = \$3.5$$

$$\mathbb{E}[X] = \mathbb{E}[X_{coin}] + \mathbb{E}[X_{dice}] = \$0.5 + \$3.5 = \$4$$

In general: $\mathbb{E}[X_1 + X_2] = \mathbb{E}[X_1] + \mathbb{E}[X_2]$
the variables is equal to
the sum of the expectations.

Expected Value of a Function

$$\mathbb{E}[2 \cdot X + (-5)] = \frac{2 \cdot (1 + 2 + 3 + 4 + 5 + 6)}{6} + (-5) = \frac{-3}{6} = -0.5$$
$$= 2 \cdot \mathbb{E}[X] + (-5)$$



In general:

$$\mathbb{E}[aX + b] = a\mathbb{E}[X] + b$$

But also the expected value of a

$$\mathbb{E}[x_1 + x_2] = \mathbb{E}[x_1] + \mathbb{E}[x_2]$$

$\left[\text{Coin} \right] \cdot \text{Win } -\frac{1}{2}$

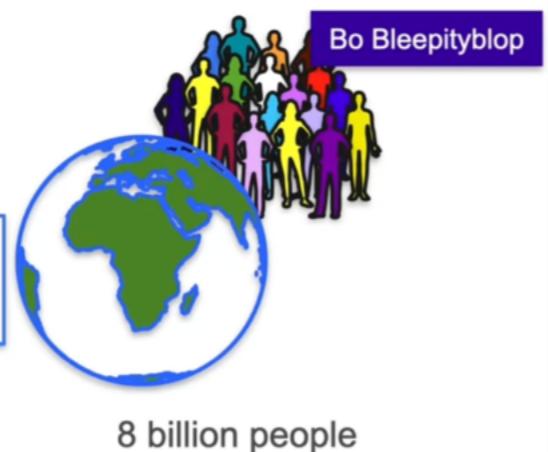
$\left[\text{Dice} \right] \cdot \text{lose } \Rightarrow \frac{21}{6}$

Sum of Expectations



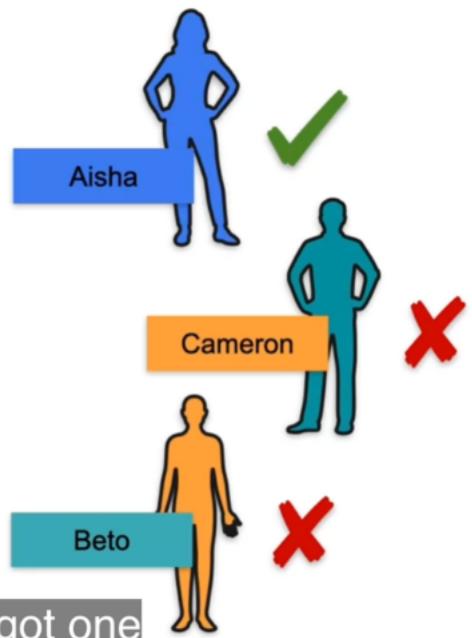
1

Expected number of correct assignments?



Bo Bleepityblob who's going to be given their own name.

Sum of Expectations



This time, we got one right and two wrongs.

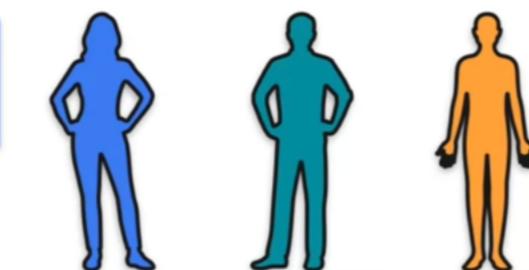
Sum of Expectations



Average
1

Correct

3
1
1
0
0



Aisha	Beto	Cameron
Aisha	Cameron	Beto
Beto	Aisha	Cameron
Beto	Cameron	Aisha
Cameron	Aisha	Beto
	Beto	Aisha

calculate because there's going to be

Sum of Expectations



$$\begin{aligned} \mathbb{E}[\text{Matches}] &= \mathbb{E}[A] + \mathbb{E}[B] + \mathbb{E}[C] \\ &= 1/3 + 1/3 + 1/3 \\ &= 1 \end{aligned}$$

Average
1

the expected number of matches is going to be one.

Sum of Expectations



n people ($n = 8$ billion)

$$\mathbb{E} [\text{Matches}] = \mathbb{E} [X_{\text{person}_1}] + \mathbb{E} [X_{\text{person}_2}] + \dots + \mathbb{E} [X_{\text{person}_n}] = \frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n} = n \cdot \frac{1}{n} = 1$$

In general:

$$\mathbb{E} [X_1 + X_2 + \dots + X_n] = \mathbb{E} [X_1] + \mathbb{E} [X_2] + \dots + \mathbb{E} [X_n]$$

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So, the Sum of Expectations is
among n people $8 = \text{billion}$

Variance:

So it turns out that expected value, although it tells us a lot about the distribution, it doesn't tell us the whole story. For example, two distributions may have the same expected value, but one of them can be very narrow and the other one can be very wide. This is captured by something called the variance

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Variance Motivation: Fair Price To Play the Game

You play a game with a friend

Game cost:

\$0

You win 100 dollars

You lose 100 dollars

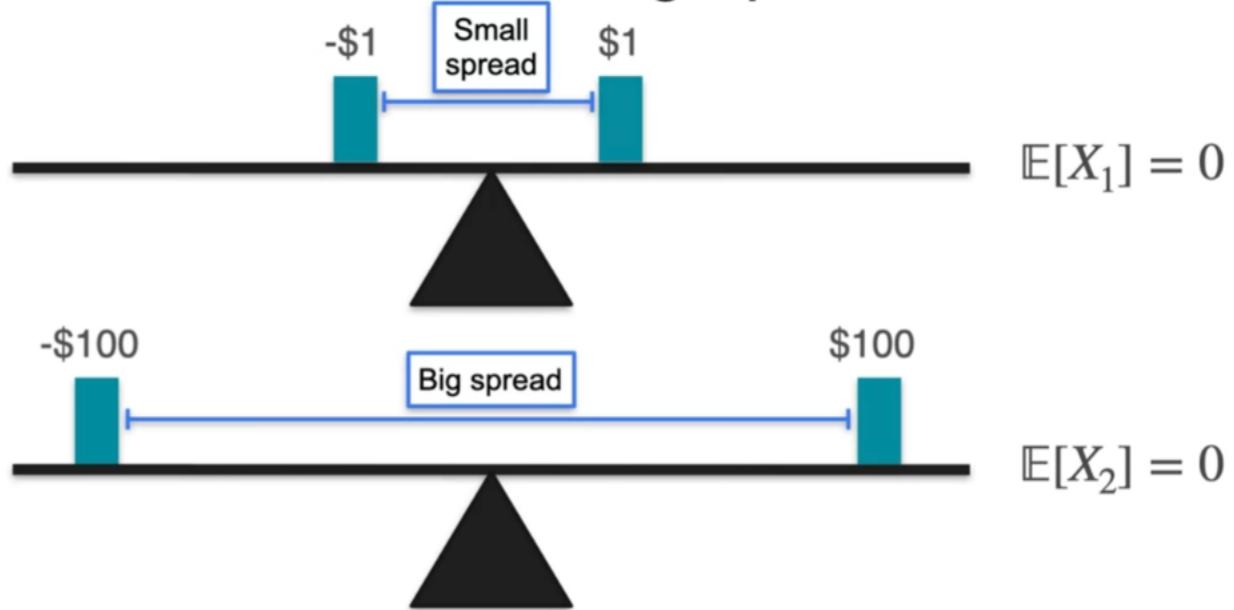
What is the fair amount of money to pay to play this game?

Variance Motivation: Measuring Spread



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Variance Motivation: Measuring Spread

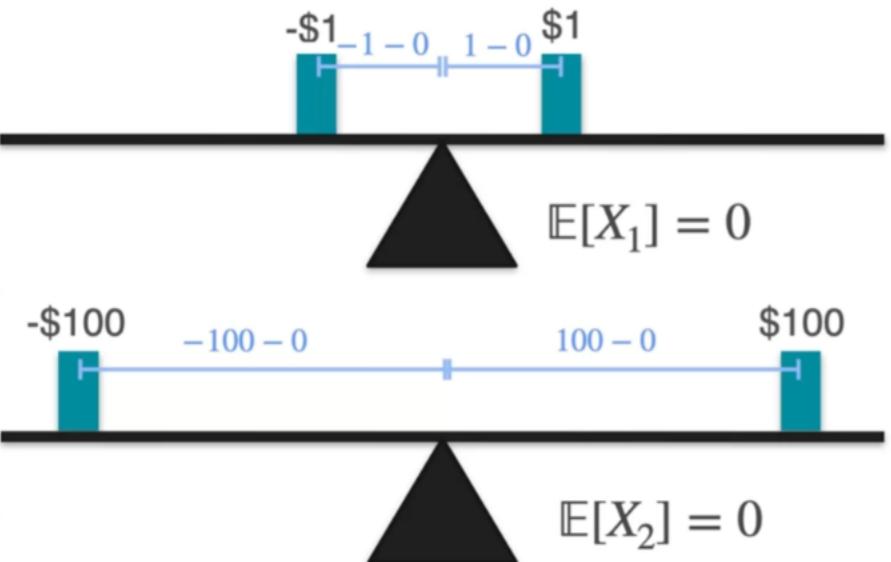


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Variance Motivation: Measuring Spread

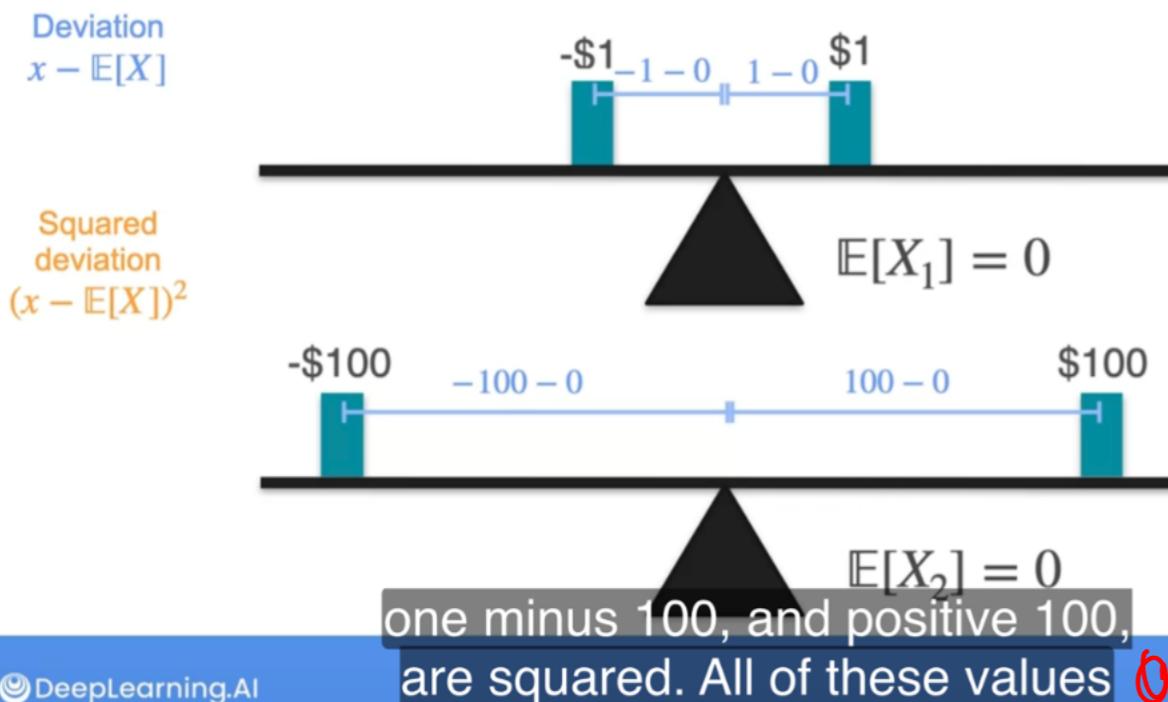
Deviation
 $x - E[X]$ Expected Deviation
 $E[x - E[X]]$

$$\frac{-1 + 1}{2} = 0$$

Absolute deviation
 $|x - E[X]|$ 

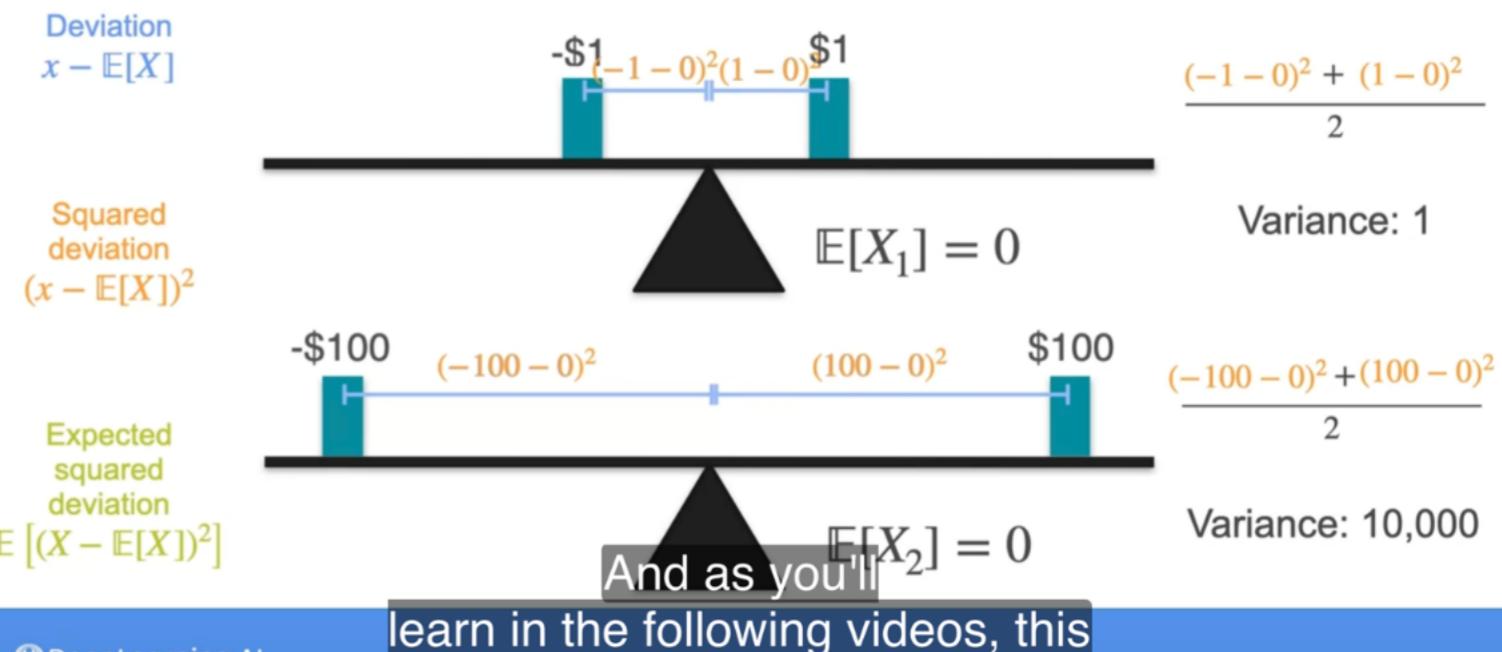
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Variance Motivation: Measuring Spread



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Variance Motivation: Measuring Spread



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Variance Formula

$$\text{Var}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2]$$

1. Find X's mean
2. Find the deviation from that mean for every value of X
3. Square those deviations
4. Average those squared deviations

"Average squared deviation"

Hence, we always use squared deviations instead of absolute deviation only.

Variance Motivation: Centering With Mean

Game 1



You win 2 dollars



You lose 2 dollars

They have the same variance

Game 2



You win 3 dollars



You lose 1 dollar

(1)

Variance Motivation: Centering With Mean

Game 1



You win 2 dollars



You lose 2 dollars

$$\mathbb{E}[X_1] = \frac{1}{2} \cdot (-2) + \frac{1}{2} \cdot 2 = 0$$

(2)

Game 2



You win 3 dollars



You lose 1 dollar

$$\mathbb{E}[X_2] = \frac{1}{2} \cdot (-1) + \frac{1}{2} \cdot 3 = 1$$

Variance Motivation: Centering With Mean

Game 1



You win 2 dollars



You lose 2 dollars

$$\mathbb{E}[X_1] = 0$$

$$\frac{1}{2}(-2-0)^2 + \frac{1}{2}(2-0)^2 = 4$$

Different price, but same spread

Game 2



You win 3 dollars



You lose 1 dollar

$$\mathbb{E}[X_2] = 1$$

$$\frac{1}{2}(-1-1)^2 + \frac{1}{2}(3-1)^2 = 4$$

(3)

Both have same

Spread or

Variance →

Variance Formula

$$Var(X) = \mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2 - 2\mathbb{E}[X]X + \mathbb{E}[X]^2]$$

$$\begin{aligned}\mathbb{E}[X+Y] &= \mathbb{E}[X] + \mathbb{E}[Y] \\ &= \mathbb{E}[X^2] - \mathbb{E}[2\mathbb{E}[X]X] + \mathbb{E}[\mathbb{E}[X]^2] \\ &= \mathbb{E}[X^2] - 2\mathbb{E}[X]\mathbb{E}[X] + \mathbb{E}[X]^2 \\ &= \mathbb{E}[X^2] - 2\mathbb{E}[X]^2 + \mathbb{E}[X]^2 \\ &= \mathbb{E}[X^2] - \mathbb{E}[X]^2\end{aligned}$$

$\mathbb{E}[\text{constant} \cdot X] = \text{constant} \cdot \mathbb{E}[X]$
 $\mathbb{E}[X] \text{ is a constant}$
 $\mathbb{E}[\text{constant}] = \text{constant}$

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Properties of the Variance

Probability: $1/6$ $1/6$ $1/6$ $1/6$ $1/6$ $1/6$

Roll: 1 2 3 4 5 6



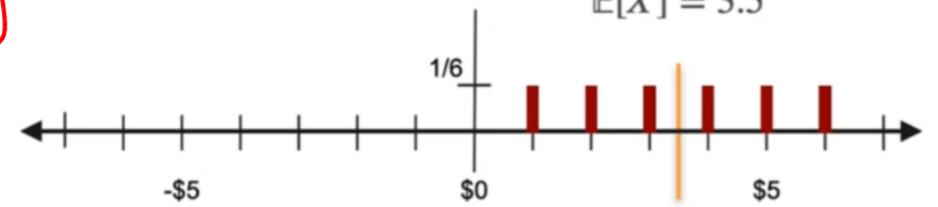
Win Double: \$2 \$4 \$6 \$8 \$10 \$12

Net Amount: -\$3 -\$1 \$1 \$3 \$5 \$7

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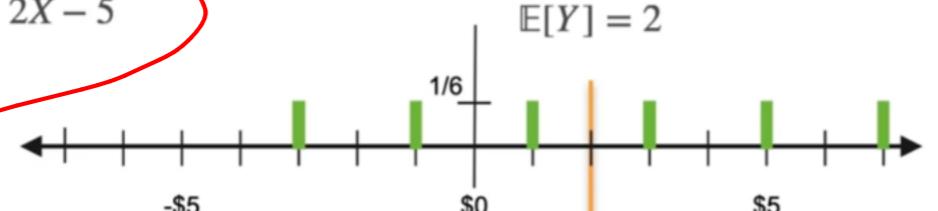
Properties of the Variance

Dice roll is random variable: X



Net winnings is random variable: $Y = 2X - 5$

$$Var(Y) = Var(2X - 5) = 4Var(X)$$

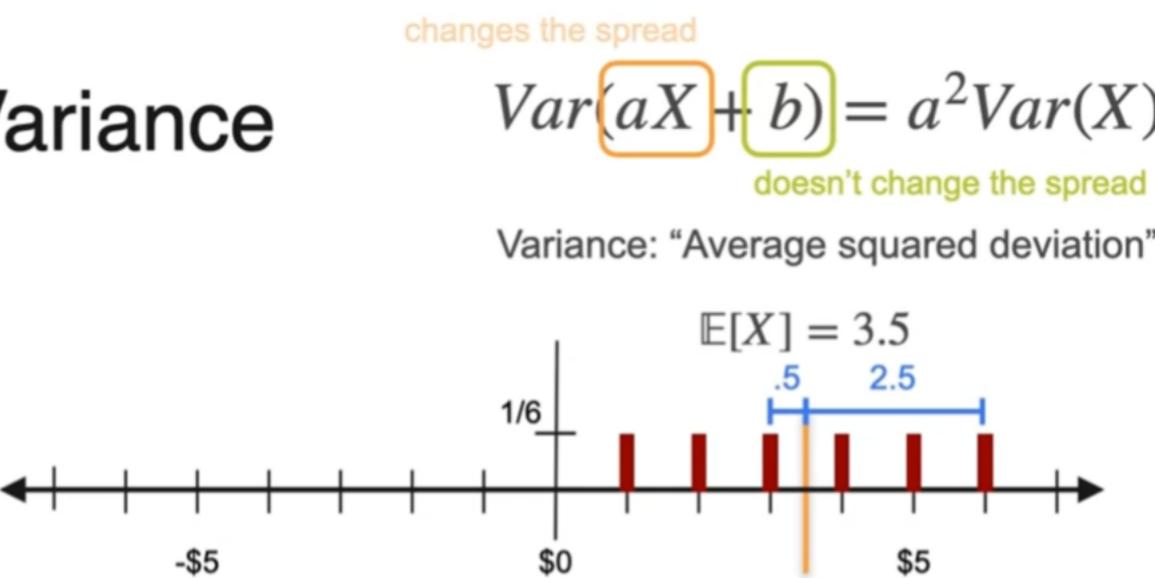


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So, here $a=2$ and
 $b = -5$ for this
close and the
formula is
 $\text{Var}(aX + b) = a^2 \text{Var}(X)$

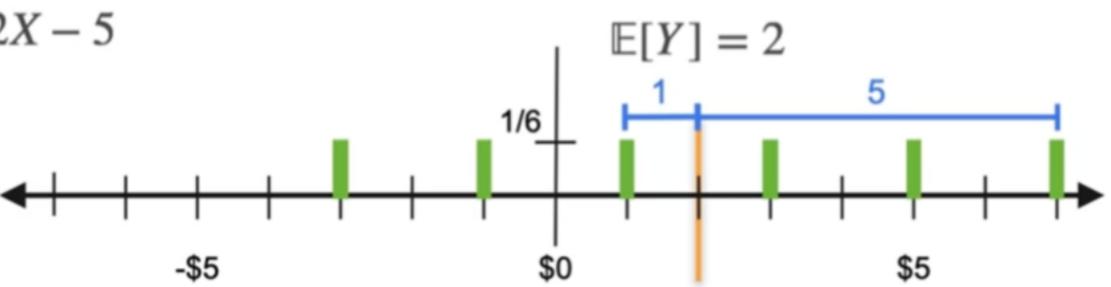
Properties of the Variance

Dice roll is random variable: X



Net winnings is random variable: $Y = 2X - 5$

$$\text{Var}(Y) = \text{Var}(2X - 5) = 4\text{Var}(X)$$



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So, next is Standard Deviation?

So the variance is a pretty useful way to measure the spread of a distribution. However, it has one small drawback, the units. Imagine if your distribution has, for example, the height of people. So, these heights are measured in meters or feet, and the expected value is also measured in meters or feet. However, the variance is measured in meters squared or feet squared. And this is not useful. So what do we do? We take the square root, we call that the standard deviation. So the standard deviation is a pretty useful way to measure the spread of a distribution using the same units of the distribution

1

Standard Deviation

$$\text{Var}(X) = \mathbb{E}[(X - \mu)^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

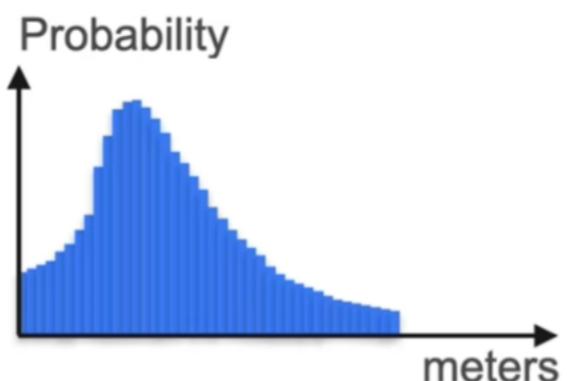
Say X is measured in meters.

Then $\mathbb{E}[X]$ is measured in meters.

Then $\text{Var}(X)$ is measured in meters².

Then $\sqrt{\text{Var}(X)}$ is measured in meters.

Let's call $\text{std}(X) = \sqrt{\text{Var}(X)}$, the *standard deviation* of X



2

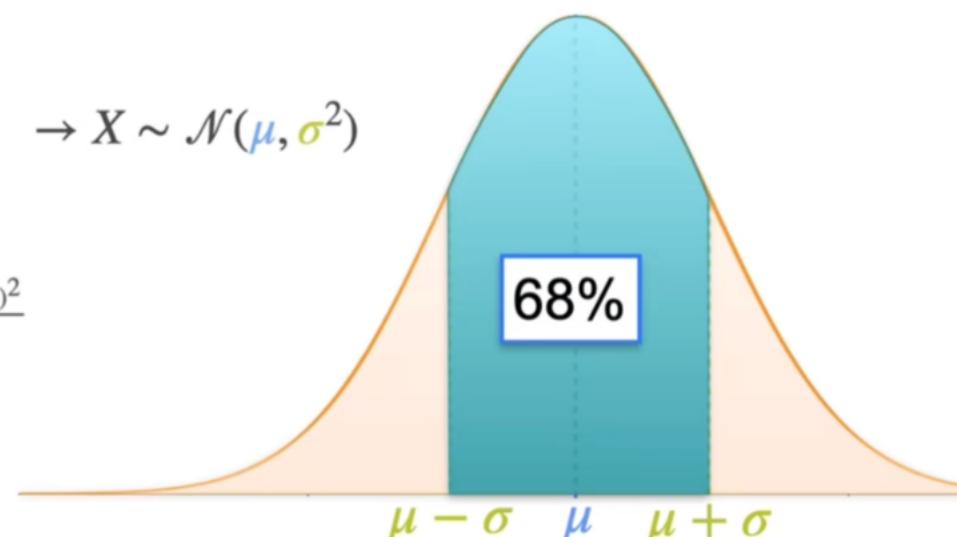
Normal Distribution: 68-95-99.7 Rule

Parameters:

- μ : center of the bell
- σ : spread of the bell

$$\rightarrow X \sim \mathcal{N}(\mu, \sigma^2)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$



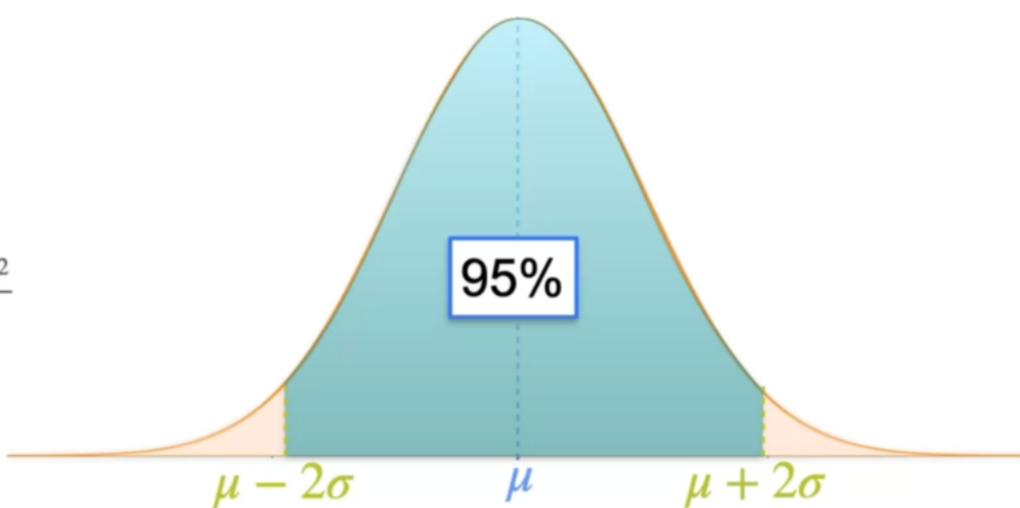
Left's evolve the
Gaussian Distribution
Probability distribution

Normal Distribution: 68-95-99.7 Rule

Parameters:

- μ : center of the bell
- σ : spread of the bell

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

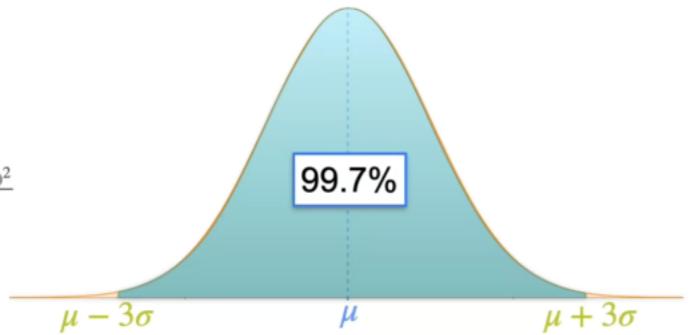


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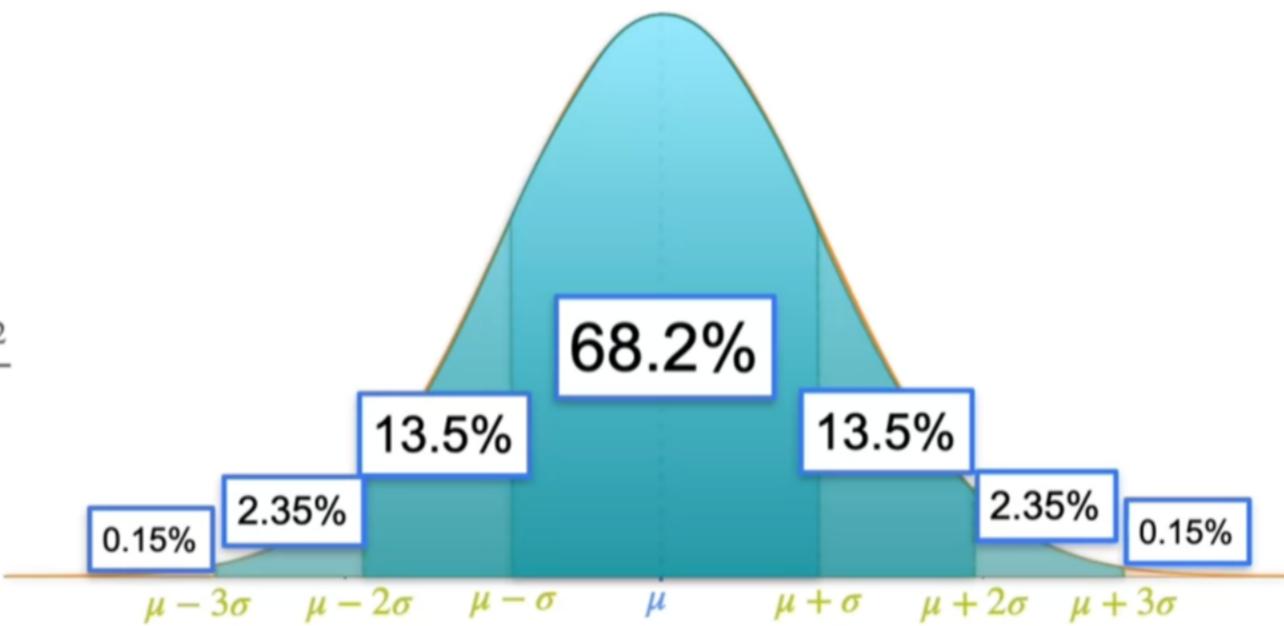
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