Day-60, Jan-29, 2025 (Magh 16, 2081 B.S.)

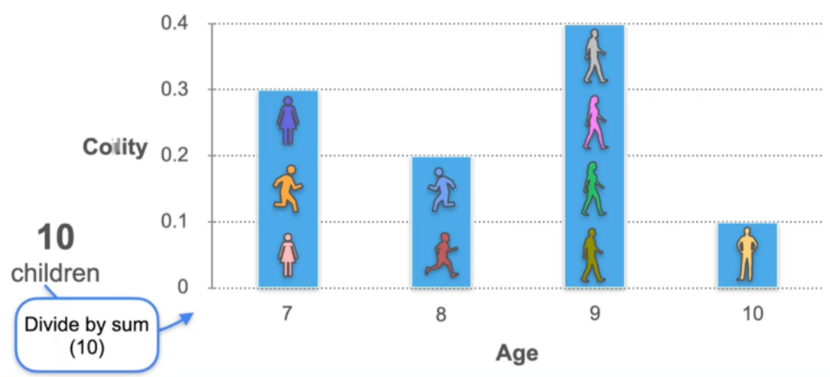
Joint Distributions (Discrete) → Distribution of Height & Age Grp.

Joint Distributions (Discrete): Example 1

Age (Year)	Count
7	3
8	2
9	4
10	1

10 children

Joint Distributions (Discrete): Example 1



Joint Distributions (Discrete): Example 1

Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Height (in)	Count
45	1
46	2
47	2
48	0
49	3
50	2

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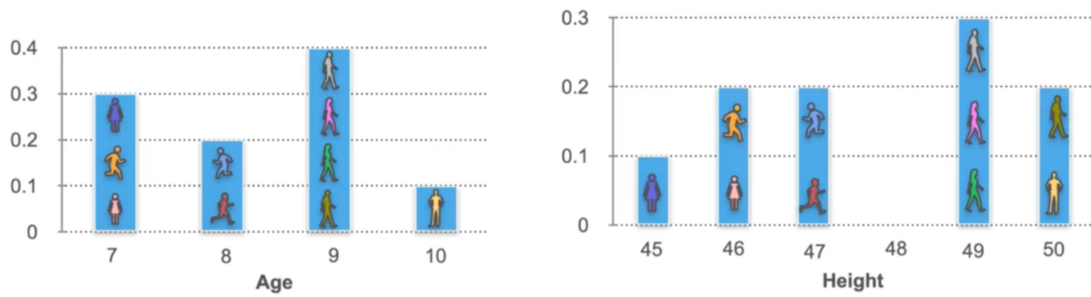
Age and Height

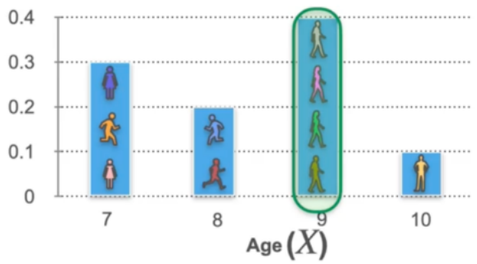
Q. What is the probability that a child is 9 years old and 49 inches tall?

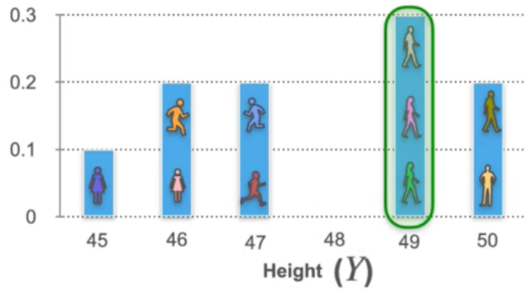
⇒ Use PMF $\frac{3}{10}$ ⇒ 0.3

Joint Distributions (Discrete): Example 1

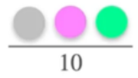


Joint Distributions (Discrete): Example 1





What is the probability that a child is 9 years old and 49 inches tall?



$$= \frac{3}{10}$$

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Joint Distributions (Discrete): Example 1

What is the probability that a child is 9 years old and 49 inches tall?

$$p_{XY}(9, 49) = \mathbf{P}(X = 9, Y = 49) = \frac{\text{●●●}}{10} = \frac{3}{10}$$

$$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$$

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$P_{XY}(x,y) = P(X=x, Y=y)$

Arranging in the table property.

Joint Distributions: Example 1

All probabilities for all possible combinations of X and Y

Age (X) \ Height (Y)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

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Divide by 10 we get 1st M.F.

Joint Distributions: Example 1

Age (years):	7	7	7	8	8	9	9	9	9	10
Height (in):	45	46	46	47	47	49	49	49	50	50

Age \ Height	45	46	47	48	49	50
7	1	2	0	0	0	0
8	0	0	2	0	0	0
9	0	0	0	0	3	1
10	0	0	0	0	0	1

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Numerical Example:

See the probability of a child is 7 and 46 inches tall?

$P_{XY}(7, 46) = 2/10 \Rightarrow 0.2$

Joint Distributions: Example 1

What is the probability that a child is 8 and 48 inches tall?

$p_{XY}(x, y) = \mathbf{P}(X = x, Y = y)$

$p_{XY}(8, 48) = \mathbf{P}(X = 8, Y = 48)$

$p_{XY}(8, 48) = 0$

$p_{XY}(7, 46) = \frac{2}{10}$

Height (Y)

Age (X)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

Probabilities

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Joint Distributions (Discrete): Example 2

X

the number rolled on the 1st dice

$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

Y

the number rolled on the 2nd dice

$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

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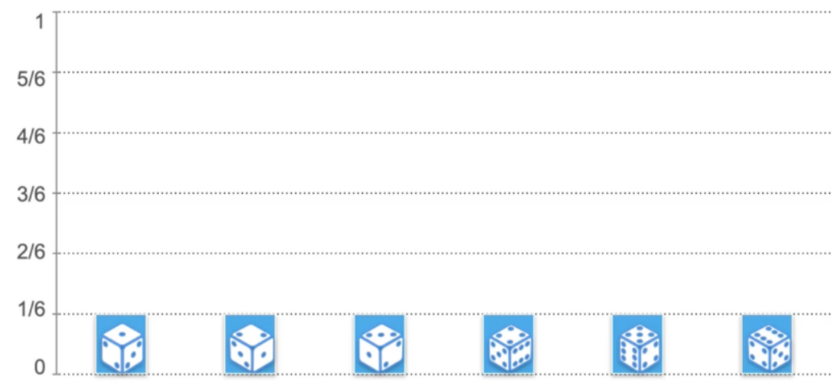
Transformed into plot

Joint Distributions: Example 2



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Joint Distributions: Example 2

X: the number rolled on the 1st dice

Y: the number rolled on the 2nd dice

X and Y are independent

$\mathbf{P}(x)$	1/6	1/6	1/6	1/6	1/6	1/6
$\mathbf{P}(y)$	1/6	1/6	1/6	1/6	1/6	1/6

$$p_{XY}(\square, \square) = \mathbf{P}(\square) \cdot \mathbf{P}(\square) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

X \ Y	1	2	3	4	5	6
1					↓	
2	→	→	→	→	1/36	
3						
4						
5						
6						

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Thus for independent discrete random variables:

$$P_{XY}(x, y) = P(X = x, Y = y)$$
$$= P(x) \cdot P(y)$$

Joint Distributions (Discrete): Example 2

X □

the number rolled on the 1st dice

$X = 4$

Y □ + □

sum of the two dice

$Y = 4 + 5$

→ PMF = ?

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Joint Distributions - Example 3

Y: Sum of both dice

□	2	3	4	5	6	7
□	3	4	5	6	7	8
□	4	5	6	7	8	9
□	5	6	7	8	9	10
□	6	7	8	9	10	11
□	7	8	9	10	11	12

PMF for Dice Rolls.

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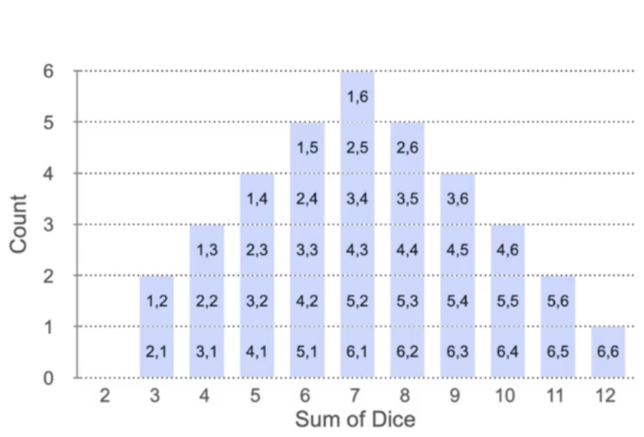
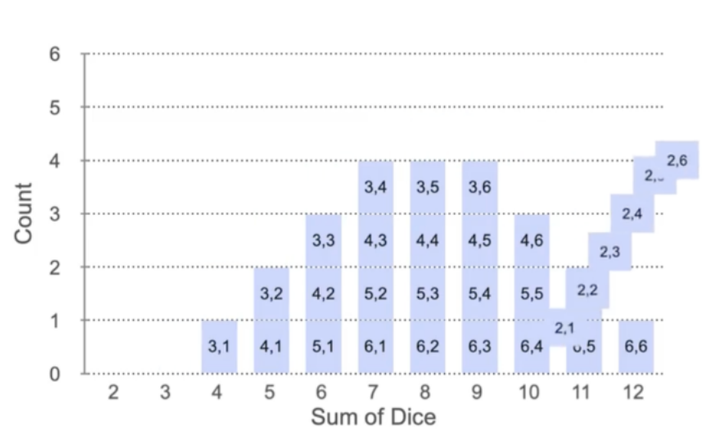
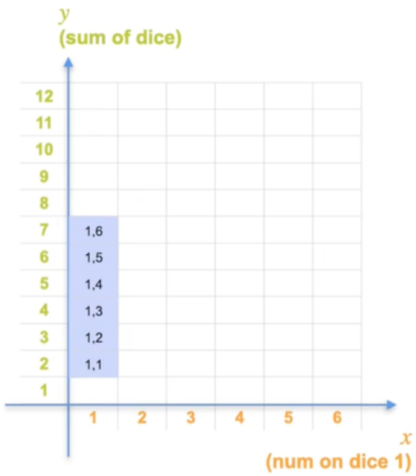
What About Histogram?

Joint Distributions: Example 3

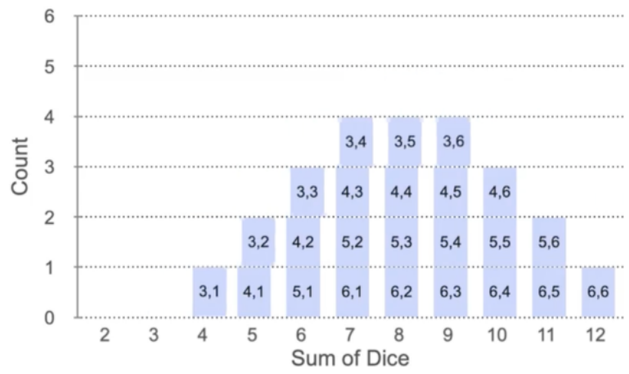


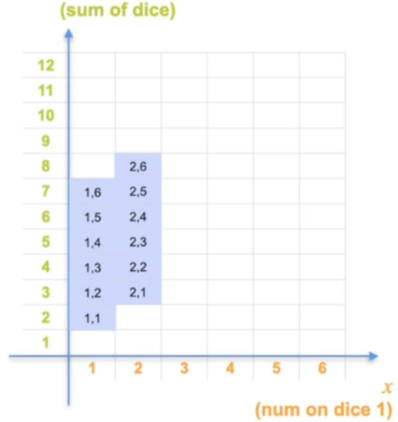
Joint Distributions: Example 3



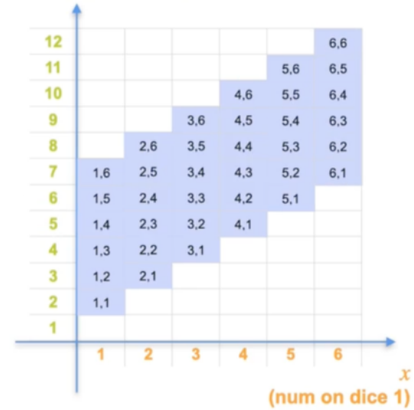


Joint Distributions: Example 3

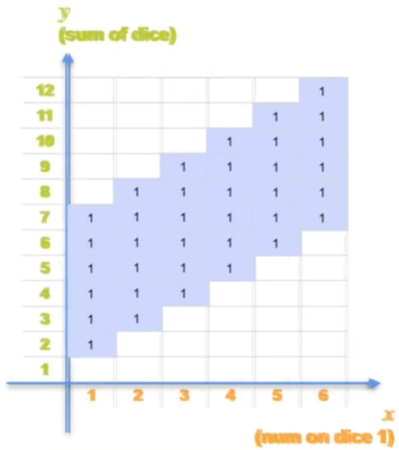




Joint Distributions: Example 3



Joint Distributions: Example 3



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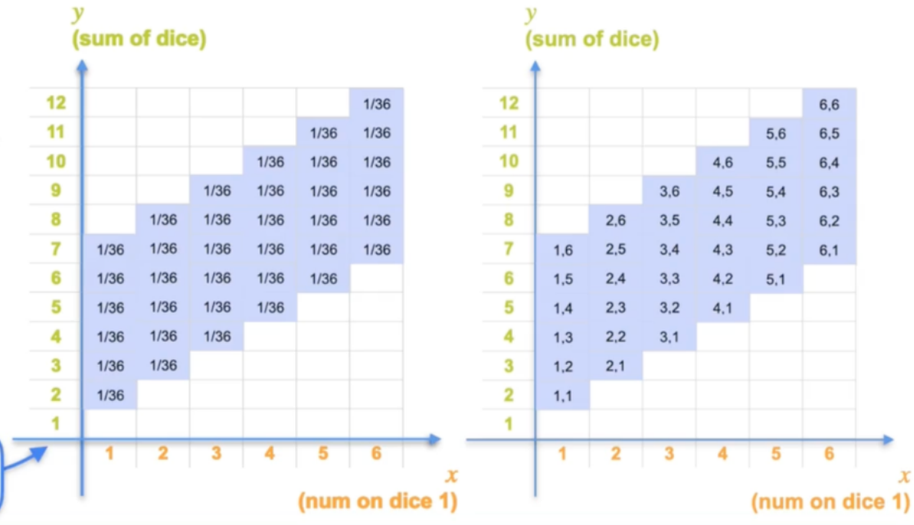
Joint Distributions: Example 3

Joint Distribution for X and Y

36 possible outcomes

Divide by sum (36)



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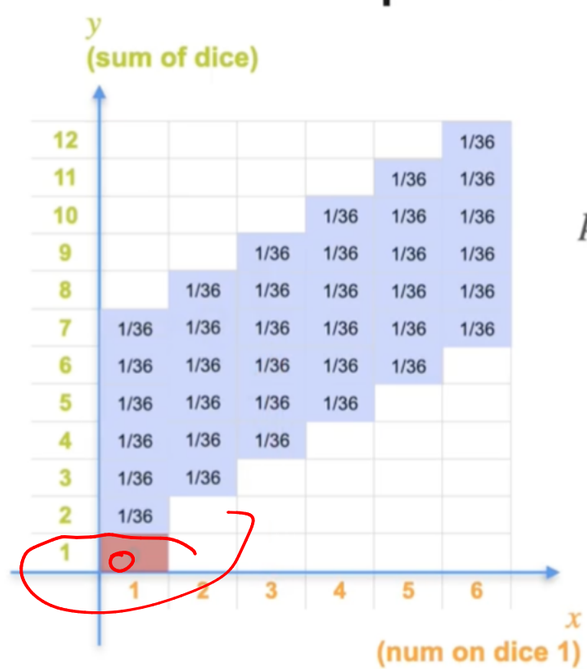
O Probability of sum of dice and num on dice is 1

O Probability

Joint Distributions: Example 3

Joint Distribution for X and Y



$$p_{XY}(3,7) = \mathbf{P}(X=3, Y=7) = \frac{1}{36}$$

$$p_{XY}(1,1) = \mathbf{P}(X=1, Y=1)$$

Strictly Continuous Distributions:

Joint Continuous Distributions

1

X : age of a child in year

Y : discrete values of height of child in inches

X : the number rolled on the 1st dice

Y : the number rolled on the 2nd dice

X and Y are Discrete Random Variables

X : the number rolled on the 1st dice

Y : sum of both dice

What about when X and Y are Continuous Random Variables?

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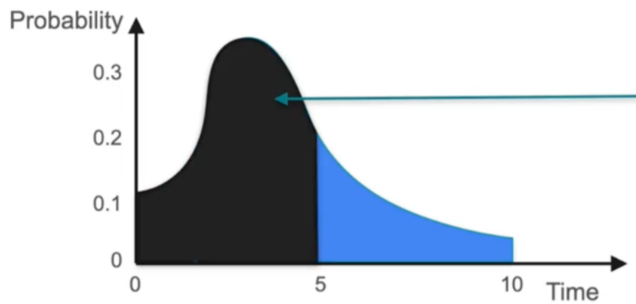
Joint Continuous Distributions

2

X variable: Waiting time



$\mathbf{P}(X$ between 0 and 5 mins)
Area occupied by this region

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Joint Continuous Distributions

X

Waiting time
before a call is picked up
[0 - 10 minutes]

2.4 minutes

1.5 minutes

Both variables are continuous

Y

Customer
satisfaction rating
[0 - 10]

0.0

5.7

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Joint Continuous Distributions

3

X

Waiting time
before a call is picked up
[0 - 10 minutes]

Y

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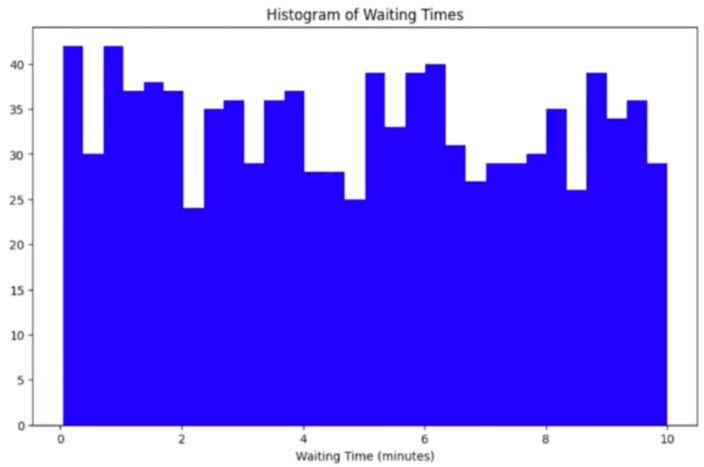
Joint Continuous Distributions: Dataset

X variable: Waiting time (mins)
0 - 10 mins

1000 customers

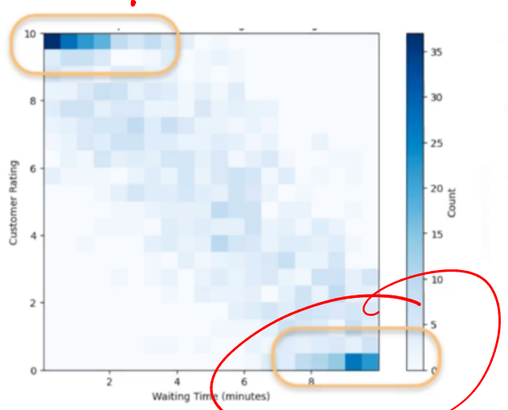


Joint Continuous Distributions: Dataset

Y variable: Satisfaction rating
0 - 10

1000 customers

Joint Continuous Distributions: Dataset

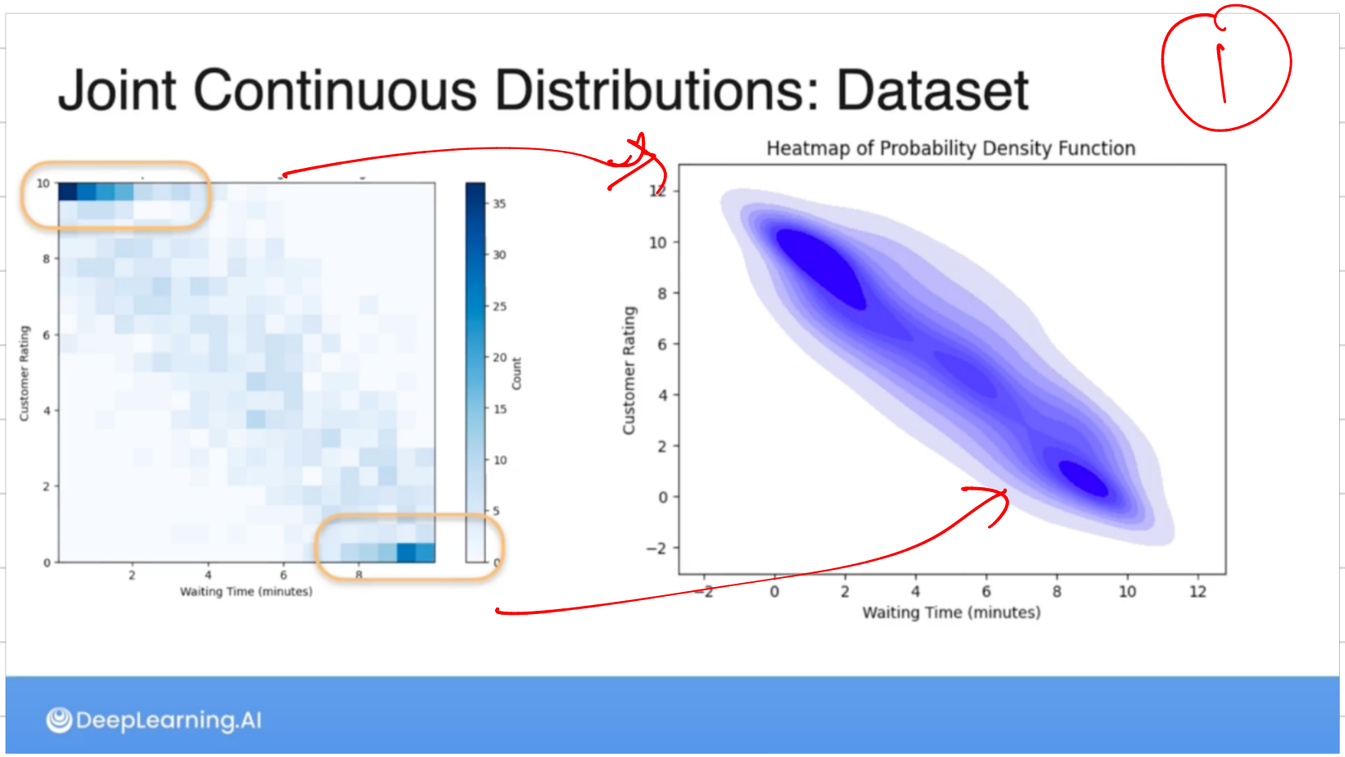


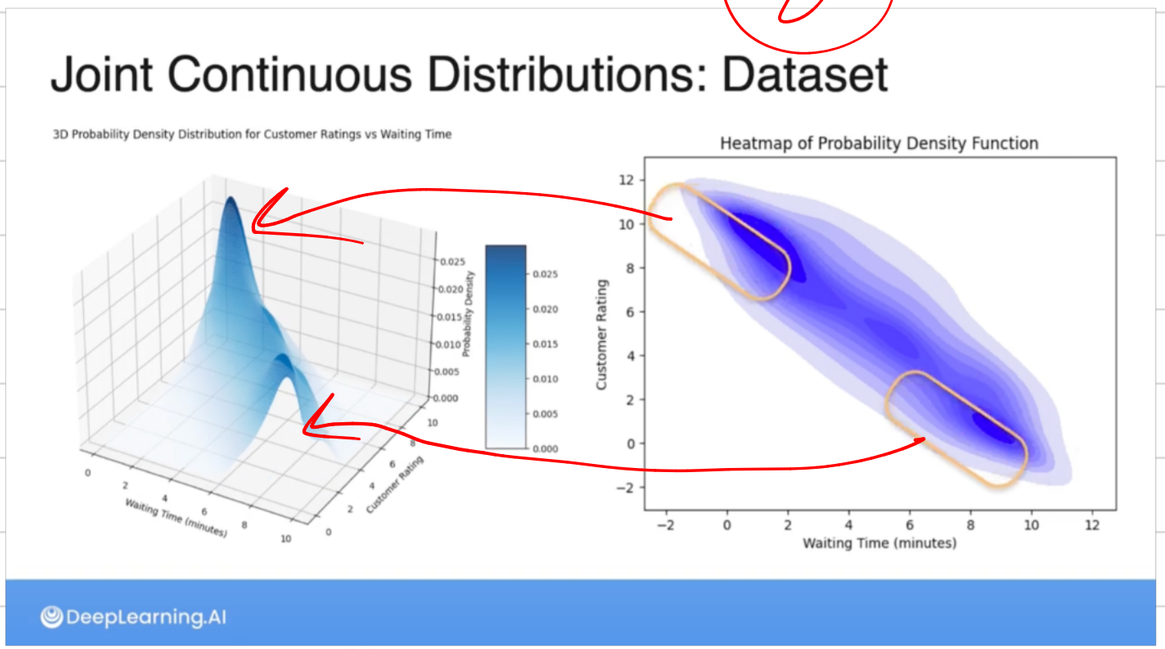
Joint Continuous Distributions: Dataset

X variable: Waiting time (mins)
0 - 10 mins

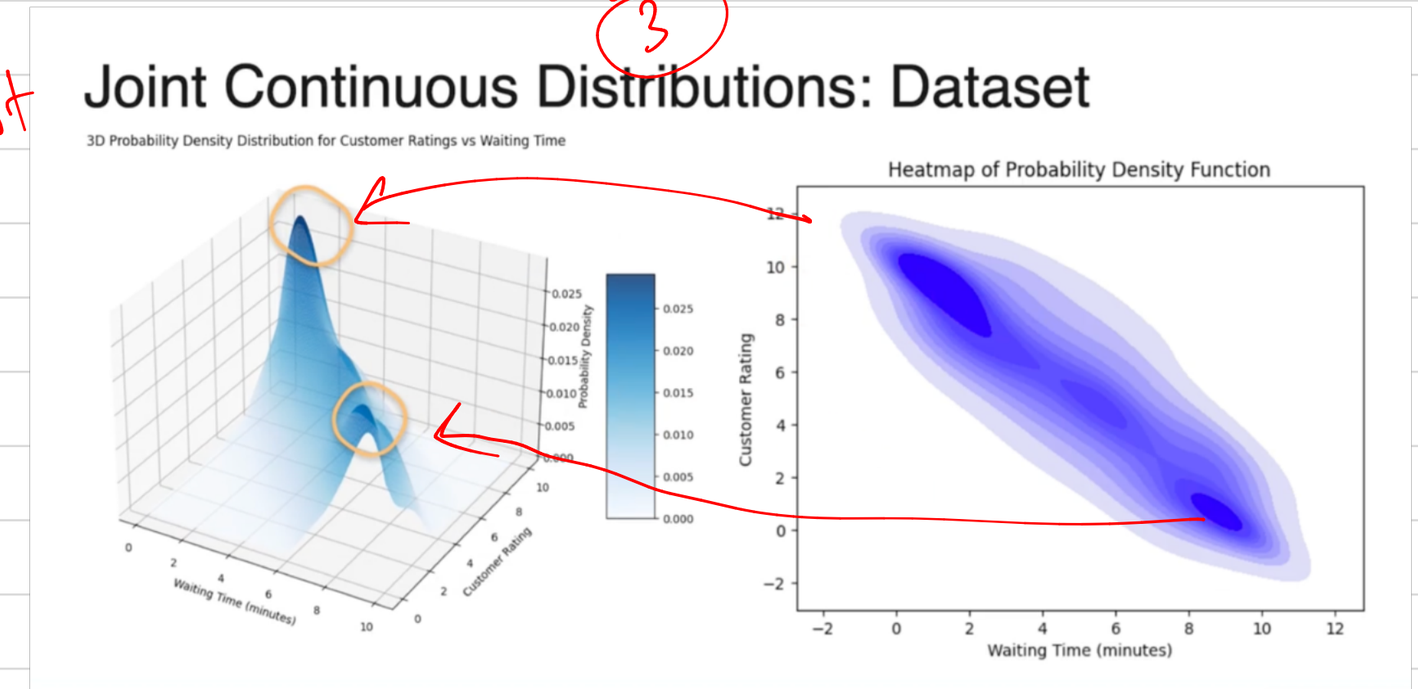
Y variable: Satisfaction rating
0 - 10

1000 customers





So the 2D and 3D plot shows the high Satisfaction and Low Satisfaction when combined.



Joint Continuous Distributions: Dataset

X variable: Waiting time (mins)
0 - 10 mins

Y variable: Satisfaction rating
0 - 10

1000 customers

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Expected Value

X variable: Waiting time (mins)
0 - 10 mins

Y variable: Satisfaction rating
0 - 10

1000 customers

$\mathbb{E}[X] =$ 4.903 minutes

Before the call is picked up 4.903 minutes.

X = waiting times.

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Expected Mean or balancing the dataset

$E[Y] = 5.280$.

Expected Value

X variable: Waiting time (mins)
0 - 10 mins

Y variable: Satisfaction rating
0 - 10

1000 customers

$\mathbb{E}[X] =$ 4.903 minutes

$\mathbb{E}[Y] =$ 5.280

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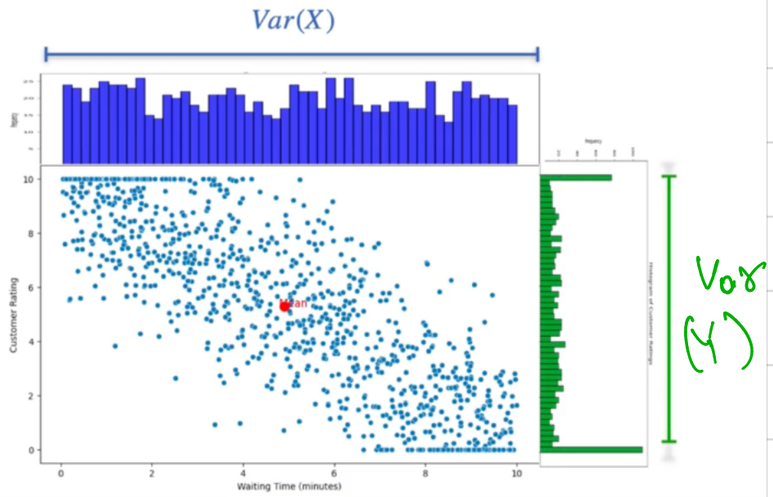
Variances



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Variances

$\mathbb{E}[X] = 4.903$ minutes

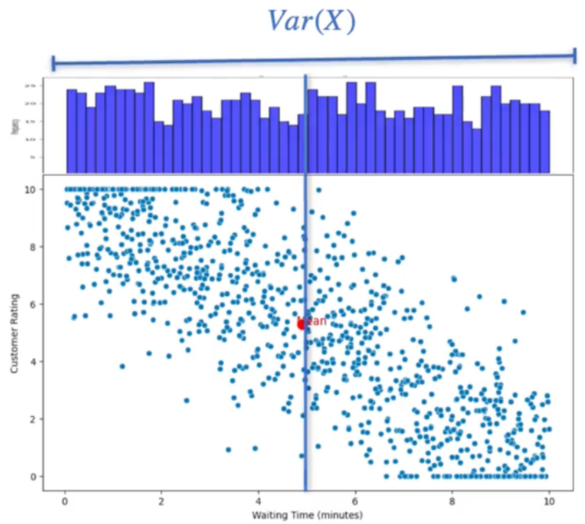
$\mathbb{E}[X^2] = 32.561$

$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

$= 32.561 - 4.903^2$

$= 8.526$

Variance of X



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Variances

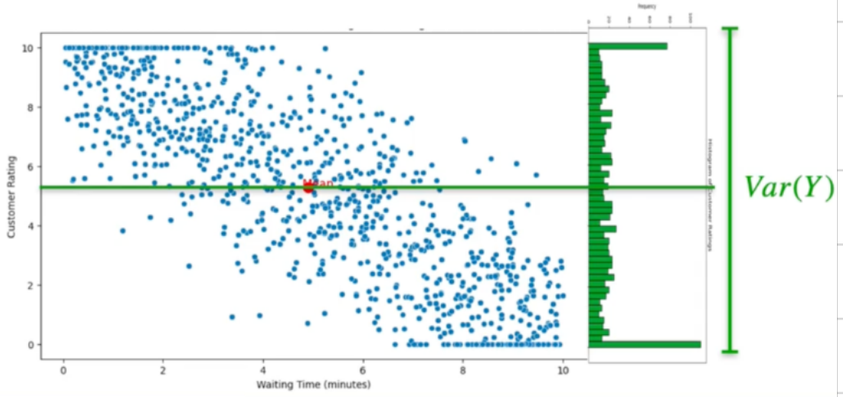
$\mathbb{E}[Y] = 5.280$

$\mathbb{E}[Y^2] = 38.037$

$Var(Y) = \mathbb{E}[Y^2] - \mathbb{E}[Y]^2$

$= 38.037 - 5.280^2$

$= 10.163$



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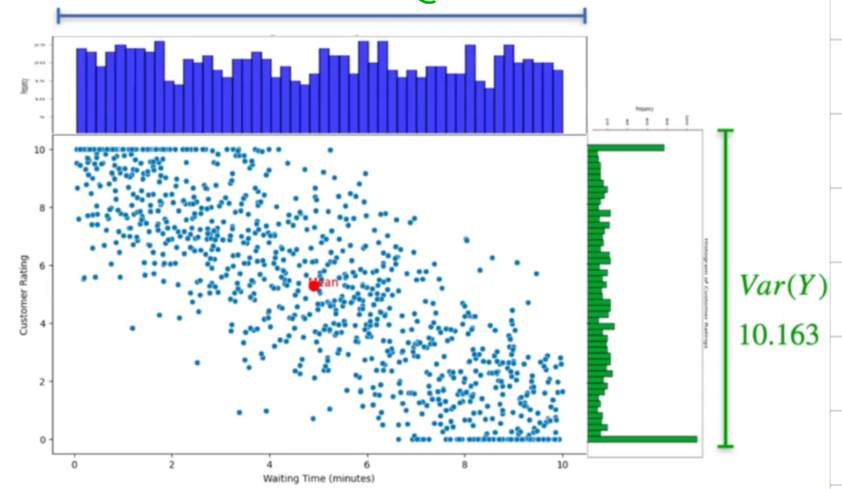
Variances



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→ Discrete JD Assign P to individual outcomes, while Continuous JD Assign P to ranges of values

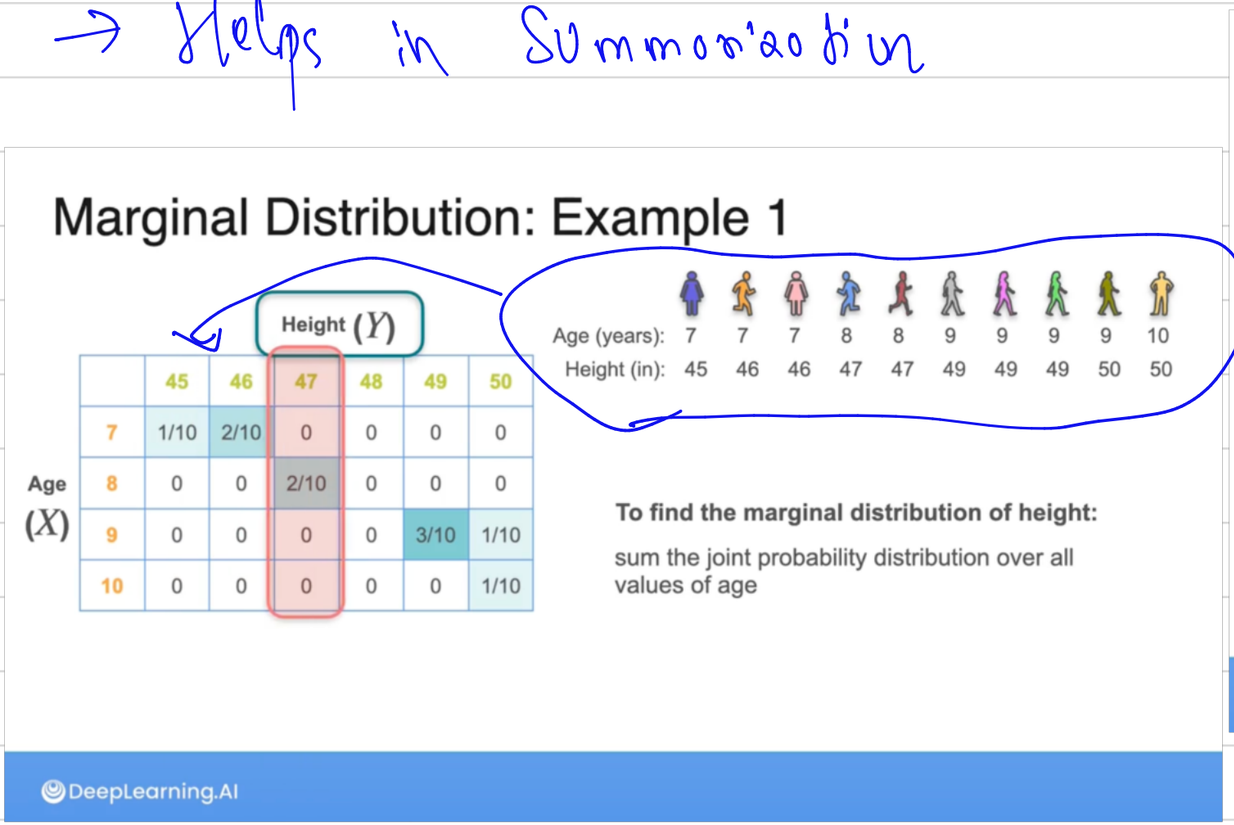
→ Continuous JD have an Uncountably infinite number of possible outcomes

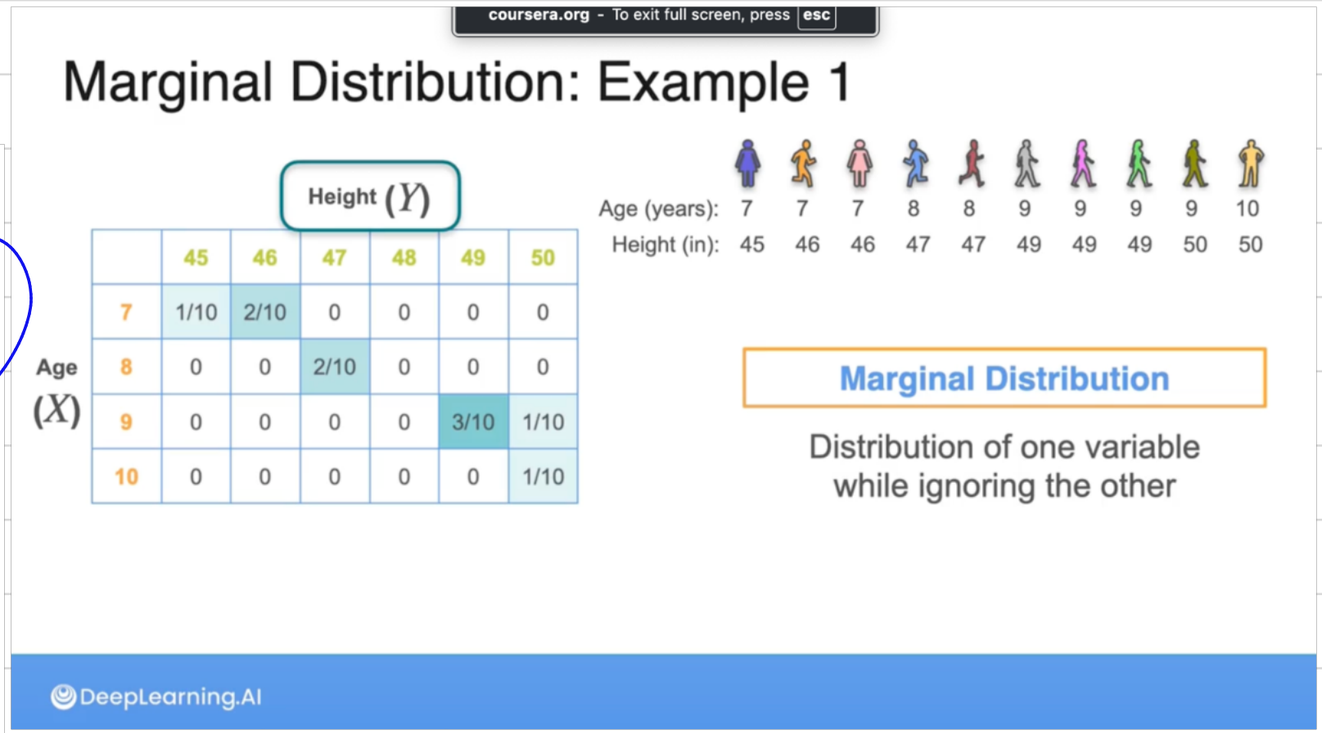
Marginal and Conditional Distribution:

→ Slicing the distribution from the full set distribution under the specific requirements is known as Conditional Distribution.

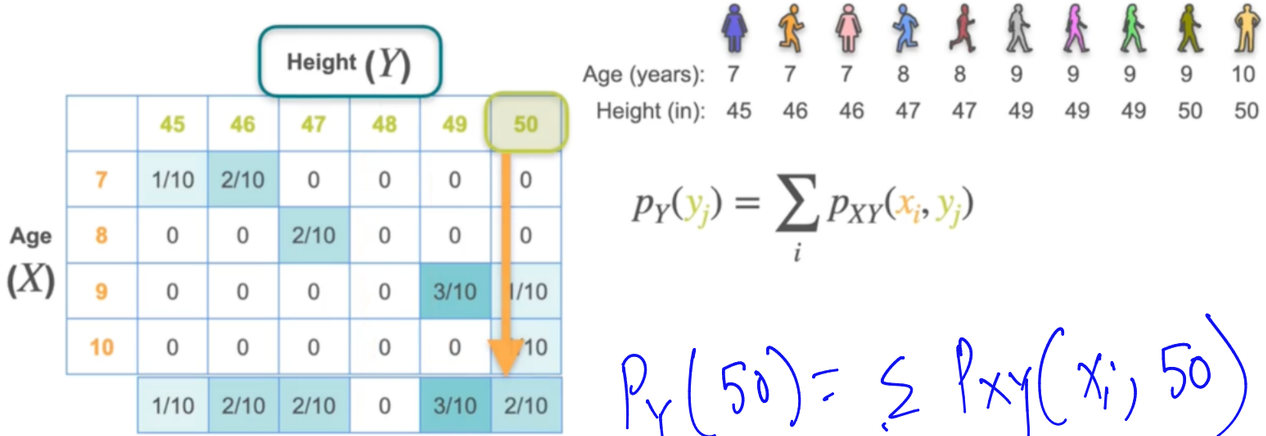
→ Marginal Distribution: Distribution of one variable while ignoring the other.

→ Helps in Summarization





Marginal Distribution: Example 1



Age (years): 7 7 7 8 8 9 9 9 9 10
Height (in): 45 46 46 47 47 49 49 49 50 50

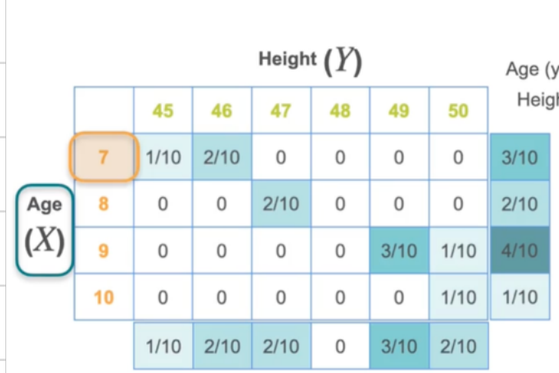
$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j)$$

$P_Y(50) = \sum P_{XY}(x_i, 50)$

$\Rightarrow 2/10$

Let's talk about age (X)

Marginal Distribution: Example 1

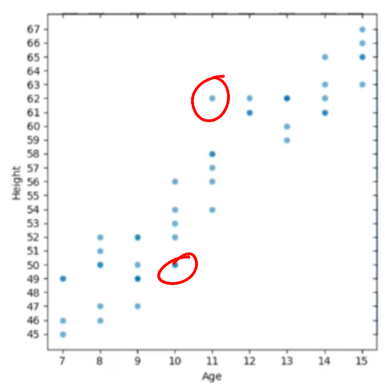


Age (years): 7 7 7 8 8 9 9 9 9 10
Height (in): 45 46 46 47 47 49 49 49 50 50

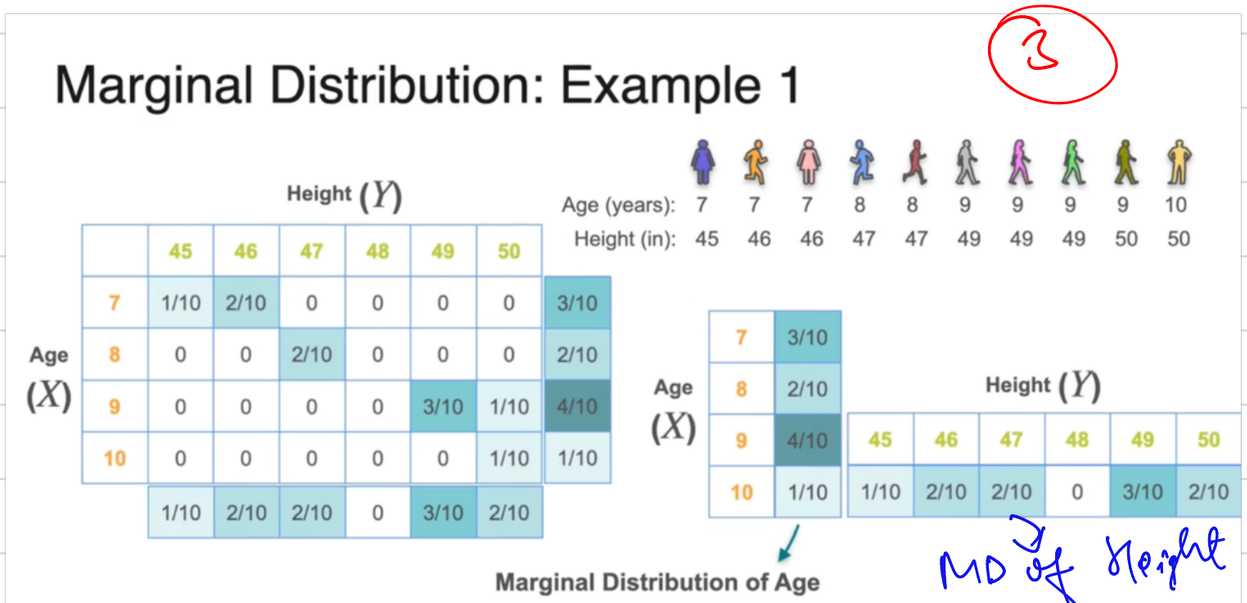
$$p_X(x_i) = \sum_j p_{XY}(x_i, y_j)$$

Marginal Distribution: Example 1

Age and Height Dataset for 50 children



Marginal Distribution: Example 1



MD of Age : $\sum_i P_{XY}(x_i, x_j)$

Same for Height -

So Now what about

X: the number rolled on the 1st dice

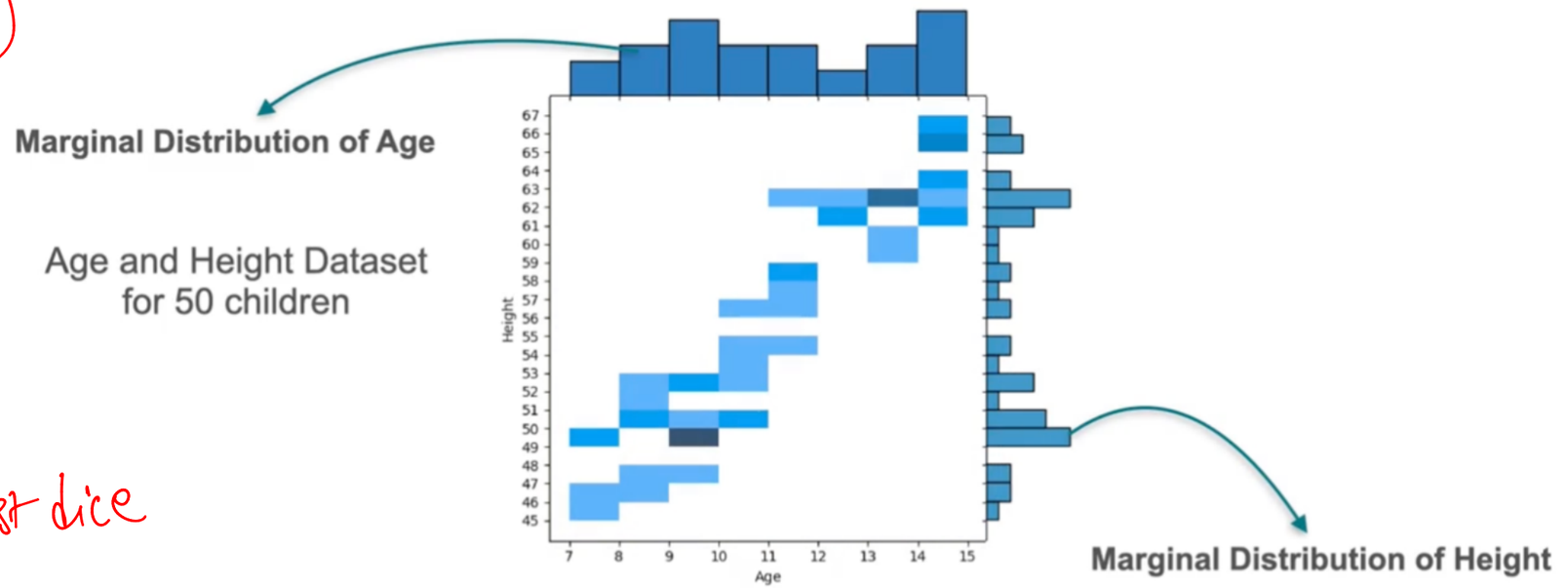
Y: the number rolled on the 2nd dice.

X	Y
1/6	1/6
1/6	1/6
1/6	1/6
1/6	1/6
1/6	1/6
1/6	1/6

X: the number rolled on the 1st dice

Y: Sum of the two dice

Marginal Distribution: Example 1



Marginal Distributions: Example 2

X \ Y	1	2	3	4	5	6	
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6
	1/6	1/6	1/6	1/6	1/6	1/6	

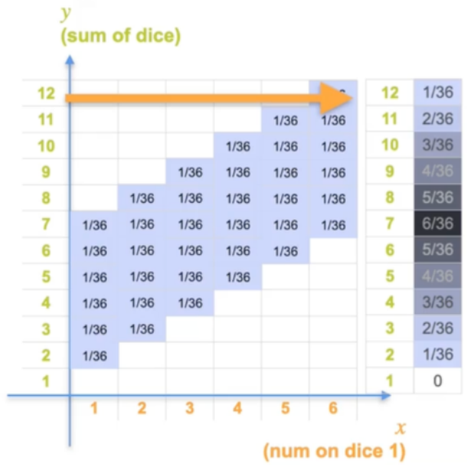
X: the number rolled on the 1st dice

Y: the number rolled on the 2nd dice

$P_X(x_i) = 1/6$

$P_Y(y_j) = 1/6$

Marginal Distributions: Example 3



X : the number rolled on the 1st dice

Y : sum of the two dice

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Marginal Distributions: Example 3

X : the number rolled on the 1st dice

Y : sum of the two dice

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Marginal Distributions: Example 3

X : the number rolled on the 1st dice

Y : sum of the two dice

$$p_Y(y_j) = \sum_i p_{XY}(x_i, y_j) = ?$$

$$p_Y(10) = ? \quad = \frac{3}{36}$$

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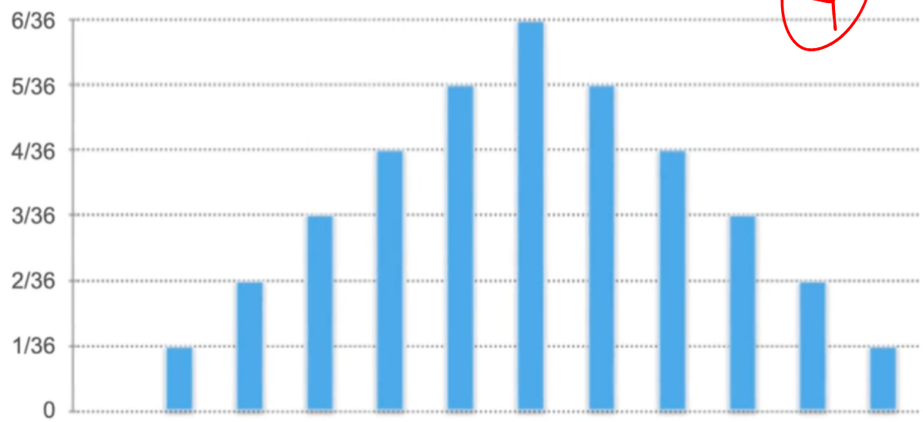
Marginal Distributions: Example 2



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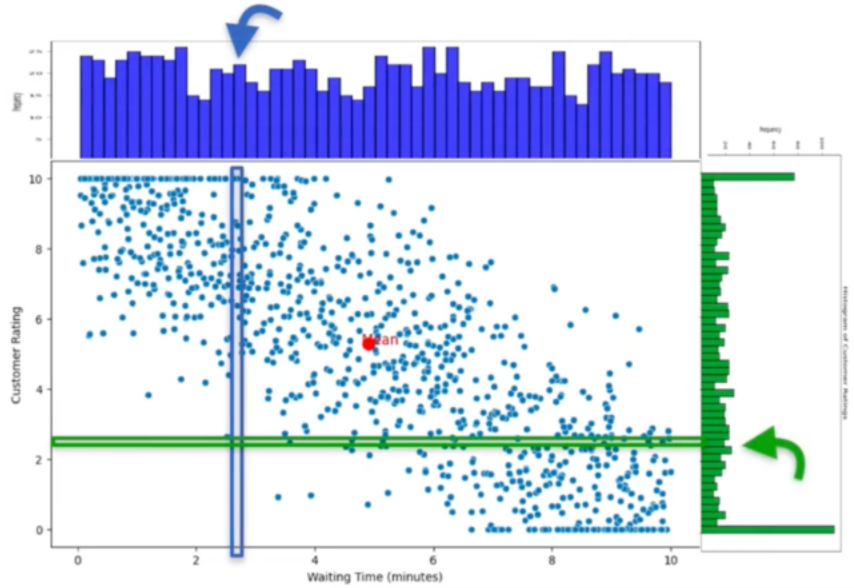
Marginal Distributions



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Continuous Marginal Distribution

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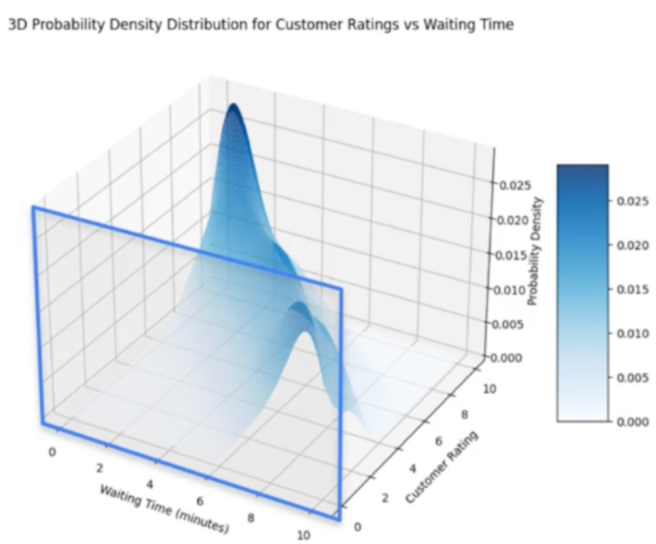
Continuous Marginal Distribution



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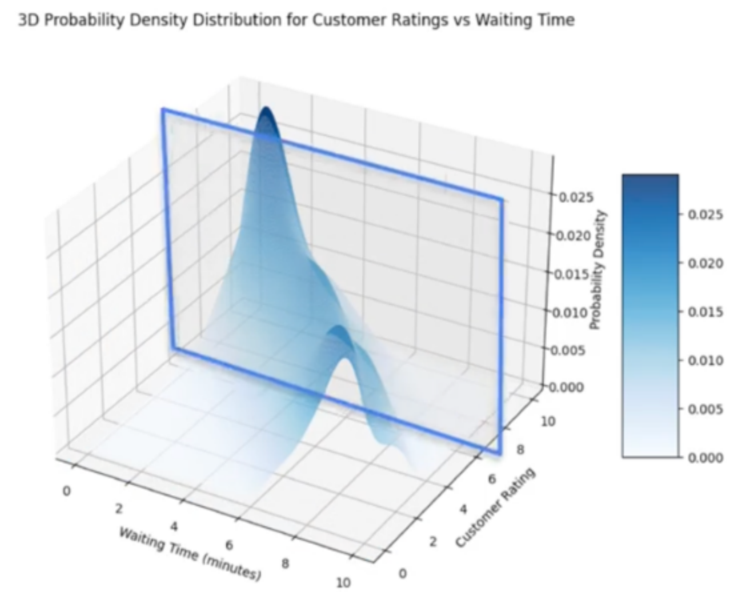
Continuous Marginal Distribution



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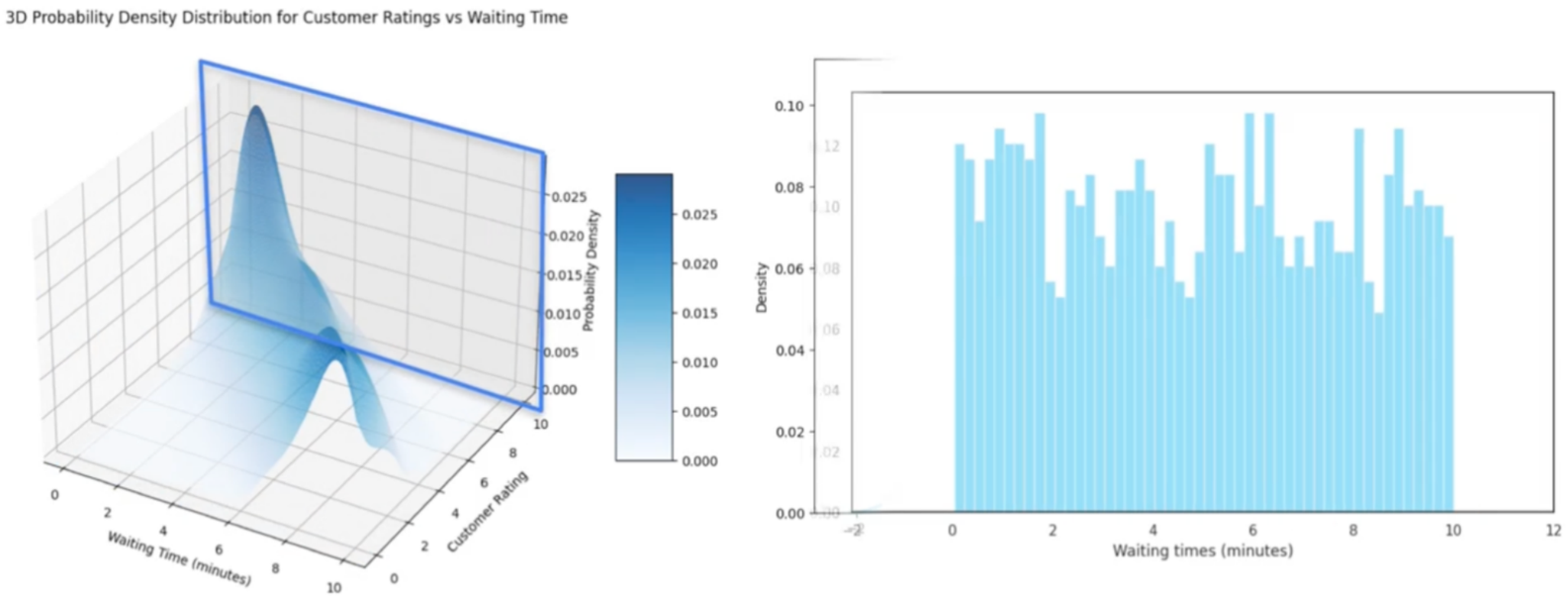
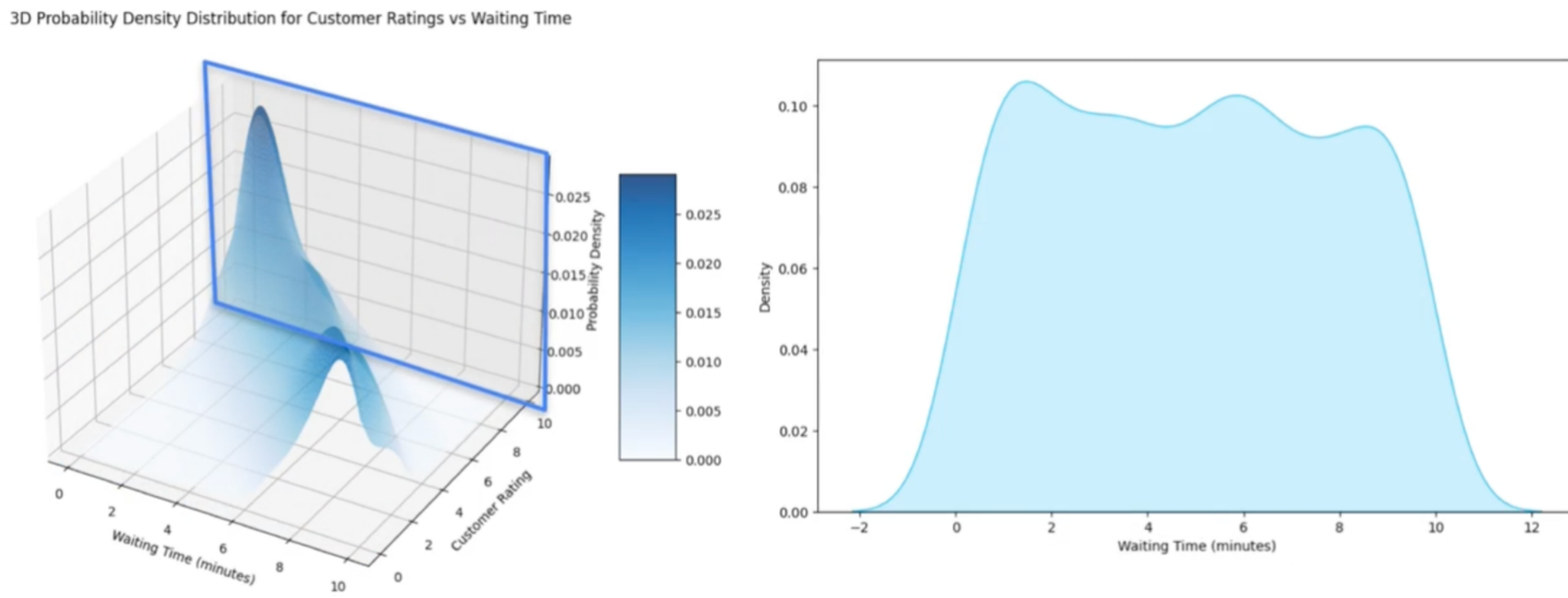
Continuous Marginal Distribution



So we Sum Σ the probability of one variable ignoring others in 1-D - in a plot

So the final Distribution looks like this - in 3D plot -

Continuous Marginal Distribution

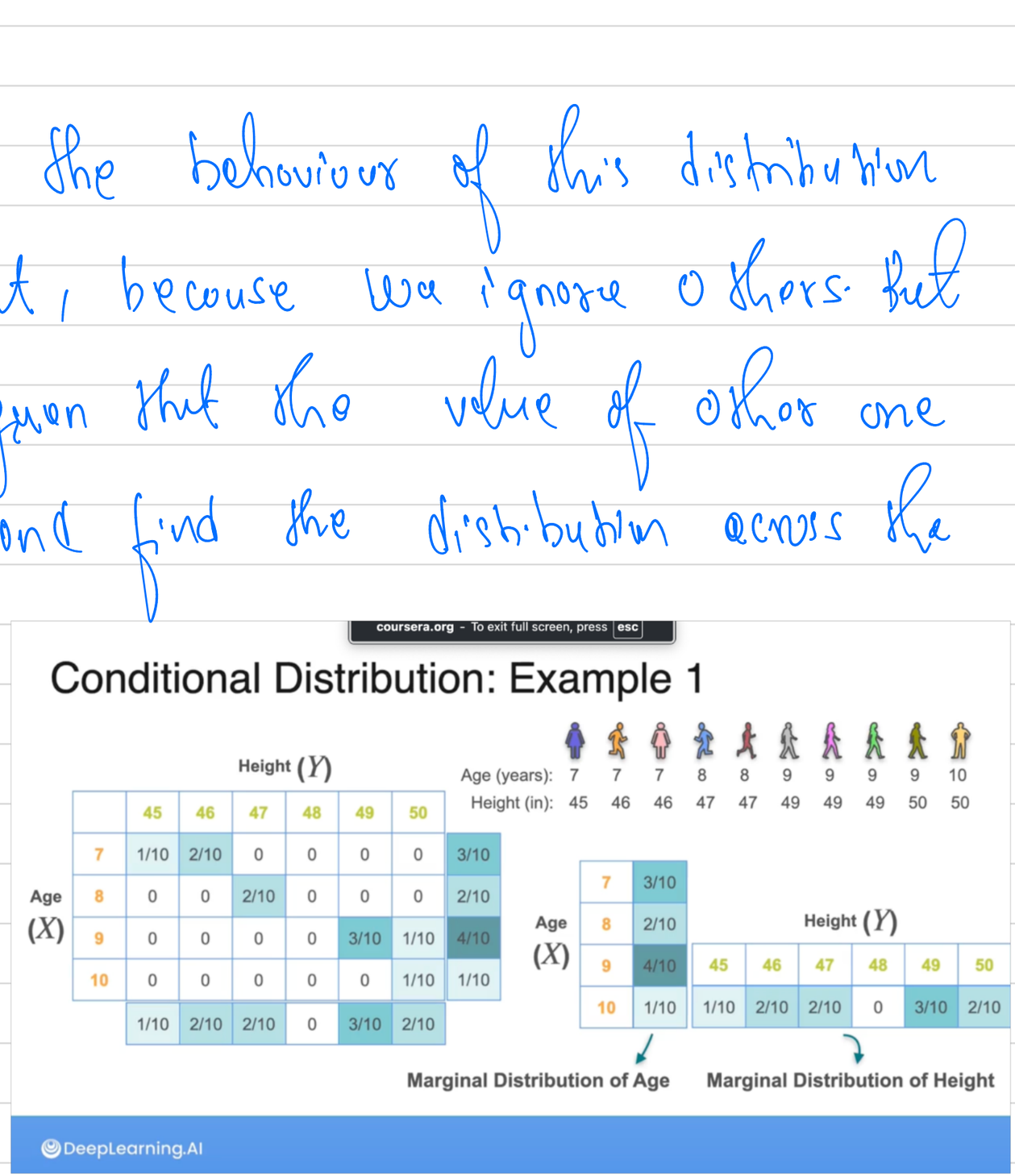


Conditional Distribution ?

With a MD, we're summarizing the behaviour of this distribution across only one variable, say height, because we ignore others. But in CD one must be observed given that the value of other one is known. Using only age 9 and find the distribution across the height variable.

If age is 9, what is the distribution across the height variable?

$$P_{Y|X=9}(y) = P(Y=y \mid X=9)$$



Conditional Distribution: Example 1

Height (Y)

Age (X)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

If age = 9, what is the distribution across the height variable?

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

So, $P(Y=49 \mid X=9) = 3/4$, how?

$P(A \mid B) = P(A) \cdot P(B/A)$

$P(Y=49 \mid X=9) = \dfrac{P(X=9, Y=49)}{P(X=9)}$

$\Rightarrow \dfrac{3/10}{4/10} = 3/4$

$P(Y=49 \mid X=9) = 3/4$

Conditional Distribution: Example 1

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

Height (Y)

Age (X)	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

$P_{Y|X=x}(y) = \dfrac{P_{XY}(x,y)}{P_X(x)}$.

$3/10 + 1/10$ gives $4/10$ and is not 1 so we must normalize

everything has to add to one.

Conditional Distribution: Example 1

$$p_{Y|X=9}(y) = \mathbf{P}(Y = y \,|\, X = 9)$$

Height (Y)

Age (X)	45	46	47	48	49	50
9	0	0	0	0	3/10	1/10

Normalize

Divide by row sum

9	0	0	0	0	3/4	1/4

$$\mathbf{P}(A, B) = \mathbf{P}(A) \cdot \mathbf{P}(B \,|\, A)$$

$$\mathbf{P}(X = 9, Y = 49) = P(X = 9) \cdot P(Y = 49 \,|\, X = 9)$$

$$P(Y = 49 \,|\, X = 9) = \frac{\mathbf{P}(X = 9, Y = 49)}{P(X = 9)}$$

Row sum

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3}{4}$$

$$\mathbf{P}(Y = 49 \,|\, X = 9) = \frac{3/10}{4/10} = \frac{3}{4}$$

That's this 3/4 we obtained over here.

Question

Using the table below, find the value of $P(Y = 46|X = 7)$

Height (Y)

Age (X)	45	46	47	48	49	50
7	1/10	2/10	0	0	0	0
8	0	0	2/10	0	0	0
9	0	0	0	0	3/10	1/10
10	0	0	0	0	0	1/10

① $P(Y = 46|X = 7) \Rightarrow 2/3$

- $P(Y = 46|X = 7) = \frac{1}{3}$
- $P(Y = 46|X = 7) = \frac{1}{10}$
- ⑤ $P(Y = 46|X = 7) = \frac{2}{3}$

Skip Continue

Discrete Conditional Distribution: Formula

②

Joint PDF of X and Y

Conditional PDF of Y $\quad p_{Y|X=x}(y) = \frac{p_{XY}(x, y)}{p_X(x)}$

Marginal distribution of X

and this is the marginal distribution of X.

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Conditional Distributions: Example 2

③

Dice 1: 1/6 1/6 1/6 1/6 1/6 1/6

Dice 2: 1/6 1/6 1/6 1/6 1/6 1/6

$$p_{Y|X=4}(y = 1) = \frac{p_{XY}(x = 4, y = 1)}{p_X(x = 4)} = \frac{1/36}{1/6} = \frac{1}{6}$$

X \ Y	1	2	3	4	5	6	Sum
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6

and you get 1/6, which is precisely what

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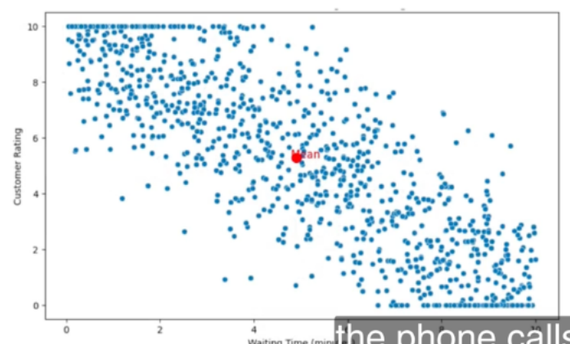
④

Conditional Distributions: Example 4



the phone calls with waiting time and customer rating.

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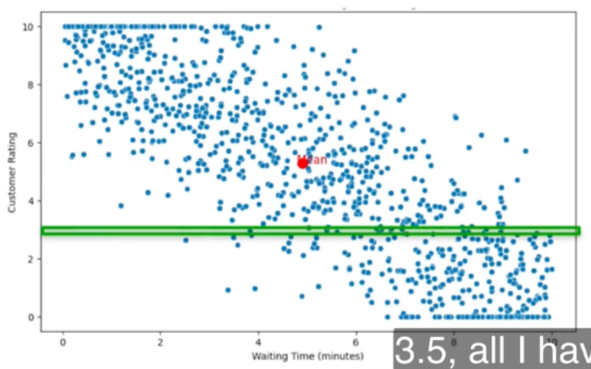
Conditional Distributions: Example 4



1

3.5, all I have to do is take this slice over here.

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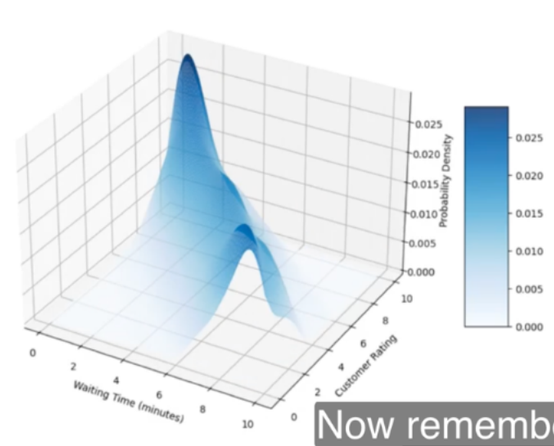
Continuous Conditional Distribution

3D Probability Density Distribution for Customer Ratings vs Waiting Time



2

Now remember that the density function was like this.

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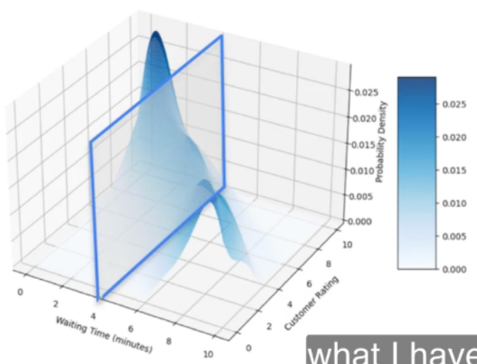
Continuous Conditional Distribution

3D Probability Density Distribution for Customer Ratings vs Waiting Time

Probability distribution for rating given that waiting time was 4 minutes



3

what I have to do is take a slice at a waiting time of

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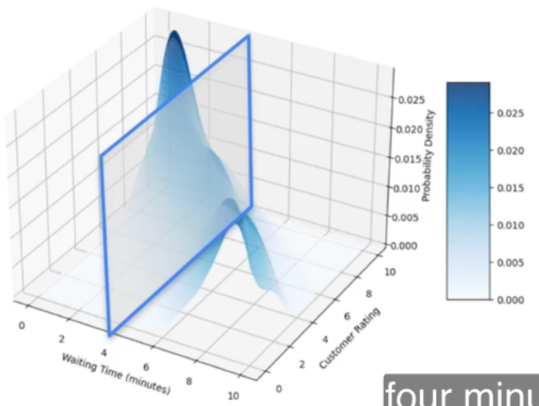
Conditional PDF of Y $f_{Y|X=x}(y) =$

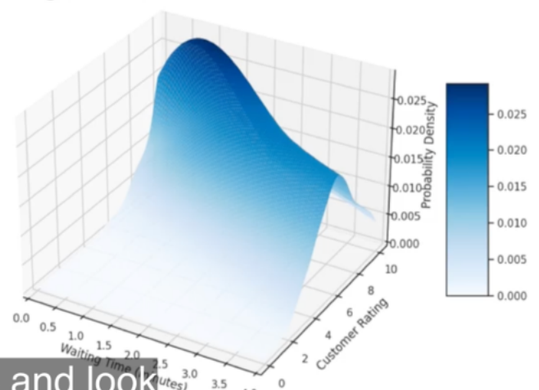
Continuous Conditional Distribution

4

3D Probability Density Distribution for Customer Ratings vs Waiting Time



Probability distribution for rating given that waiting time was 4 minutes



four minutes and look at the plot in there.

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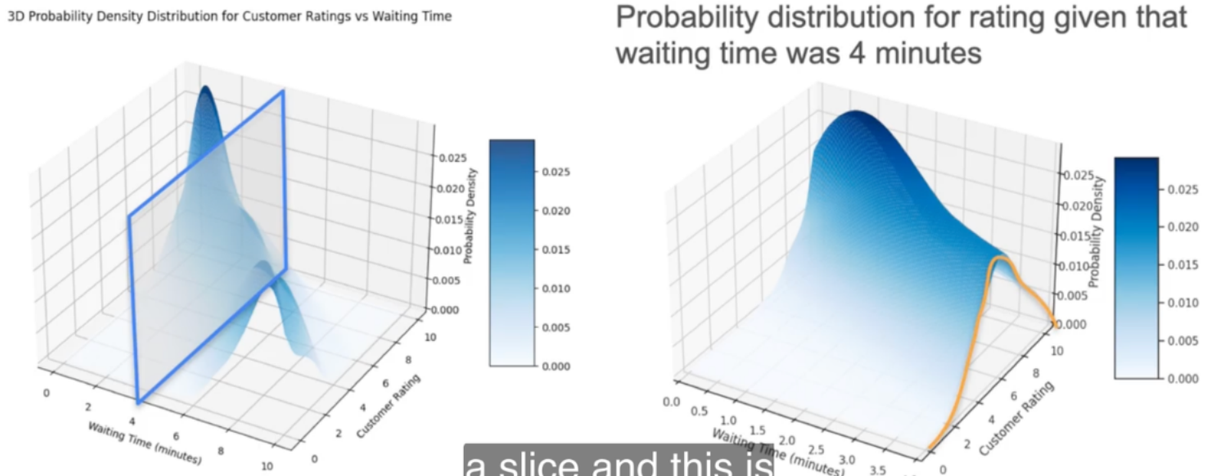
$\frac{f_{XY}(x,y)}{f_X(x)}$ → joint pdf of X and Y

→ pdf of X

Continuous Conditional Distribution



a slice and this is
the slice over here,

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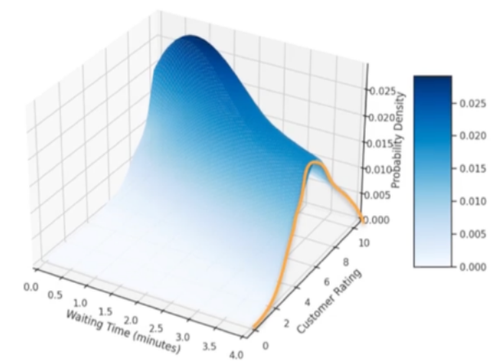
Continuous Conditional Distribution



and the curve that you obtain on

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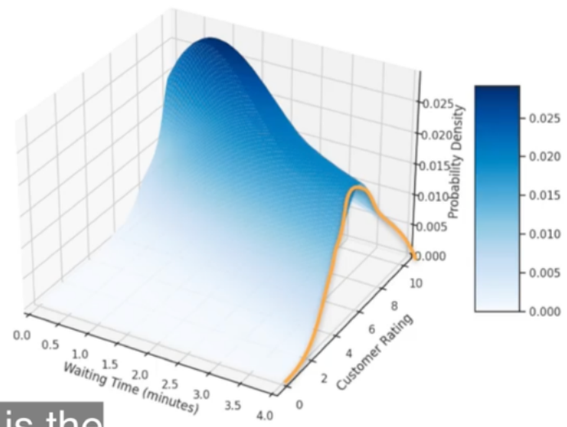
But we have to normalize it so,

Continuous Conditional Distribution



this slice is the
conditional distribution.

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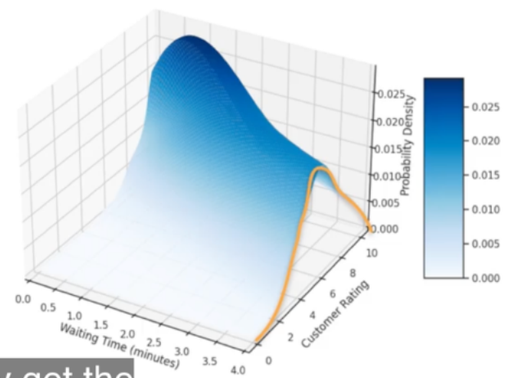
Continuous Conditional Distribution



Conditional PDF of y given x = 4

you actually get the
conditional PDF of y given x=4.

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