Day-31, Dec-19, 2024 (Poush 4, 2081 B.S.)

Linear Equations in Linear Algebra

Definition: A linear equation in the variables $x_1, x_2, x_3 \cdots x_n$ is on equation in the form

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots\cdots + a_nx_n = b$$

Where the coefficients $a_1, a_2, a_3 \cdots a_n$ and the value of b are real or complex numbers.

$y = mx + C$ or $y = b_0 + b_1x$ are examples of linear equation.

Real numbers ⇒ rational $(\frac{1}{2}, -\frac{3}{4}, \frac{7}{2})$, irrational $(\pi, \sqrt{2}, e)$,
integers $(-3, -2, \dots 1, 2, \dots)$, whole numbers $(0, 1, 2, 3 \dots)$
natural numbers $(1, 2, 3, \dots \infty)$

Complex Numbers ⇒ An extension of real numbers, incorporating the imaginary unit 'i' so $i = \sqrt{-1}$. Imaginary unit i is defined as the Square root of -1.

System of linear Equation: A collection of one or more linear equations, is a system of linear equations or a linear system

$$x_1 + 5x_2 = 7$$
$$2x_1 + 7x_2 = 5$$

are the examples of linear System.

Solution of the System of linear Equations

A solution of the system is a list of values $(x_1, x_2 \cdots x_n)$ of numbers that satisfies the given input.

Example: $x_1 + 5x_2 = 7$ ——— eqn (i)

$2x_1 + 7x_2 = 5$ ——— eqn (ii)

$\Rightarrow x_1 = 7 - 5x_2$

$\Rightarrow 2(7 - 5x_2) + 7x_2 = 5$

$\Rightarrow 14 - 10x_2 + 7x_2 = 5$

$\Rightarrow +3x_2 = +9$ $\quad (x_2 = 3)$

$\Rightarrow 7 - 5 \times 3$

$\Rightarrow -8$

$x_1 = (-8)$

$x_2 = (3)$

So,

$(x_1, x_2) = (-8, 3)$ has solution.

Solutions of linear System

→ No solution (Inconsistent System)

→ Contain exactly one solution

→ Contains more than one solution

→ Infinite number of solution

[Consistent System]

Geometrical / Graphical Representation of Consistency of LE

→ linear Equation ≃ Straight line.

→ No. of Straight line ≃ linear System

→ System of linear Equations is consistent if the equations have a common intersection point.

⇒ If they donot intersected at a common point then the system is inconsistent.

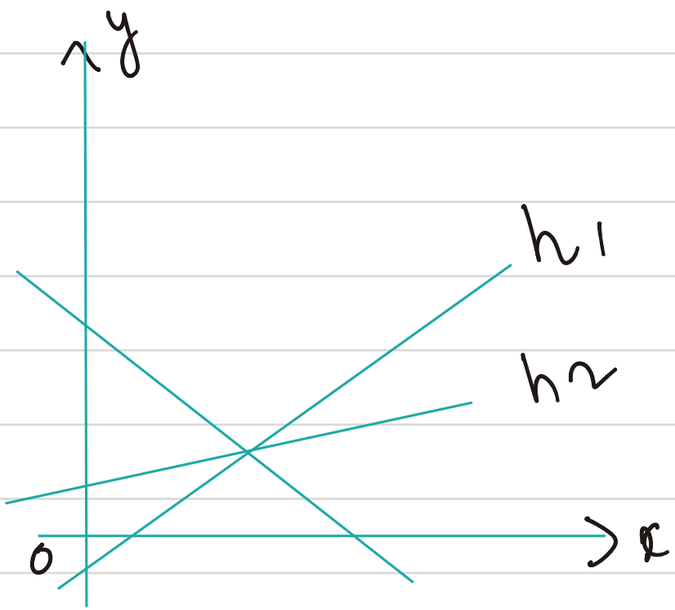


fig (a)

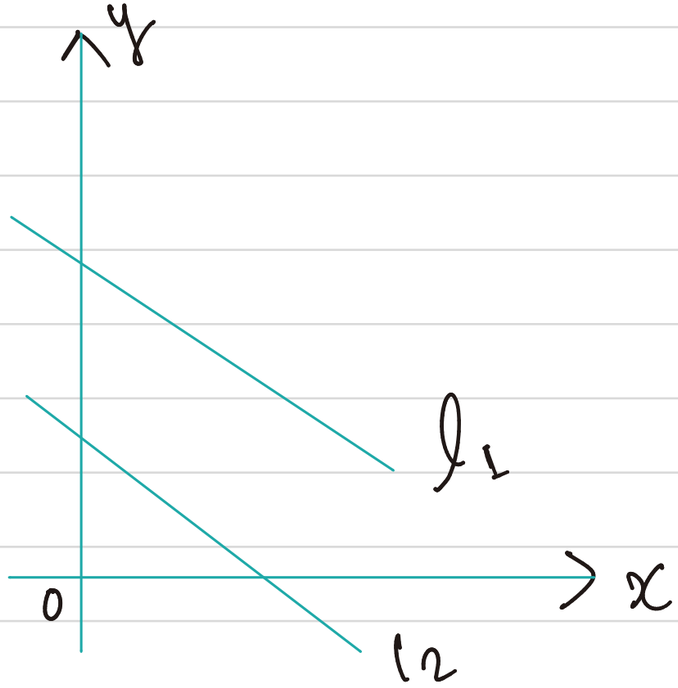


fig (b)

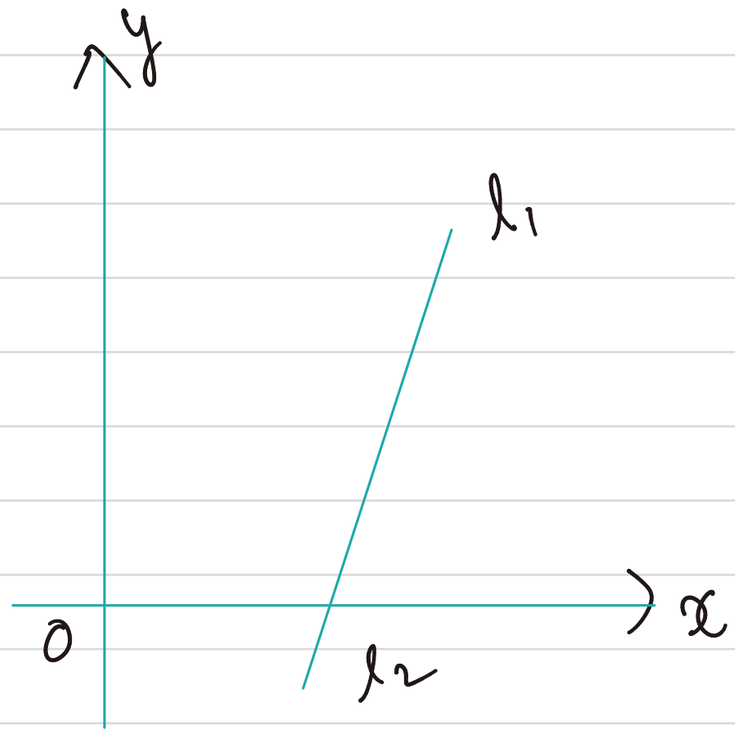


fig (c)

fig (a) → Consistent and has exactly one solution because the lines are intersect at a point.

Fig (b) → parallel, so the system of equations l_1 and l_2 is inconsistent in figure (b).

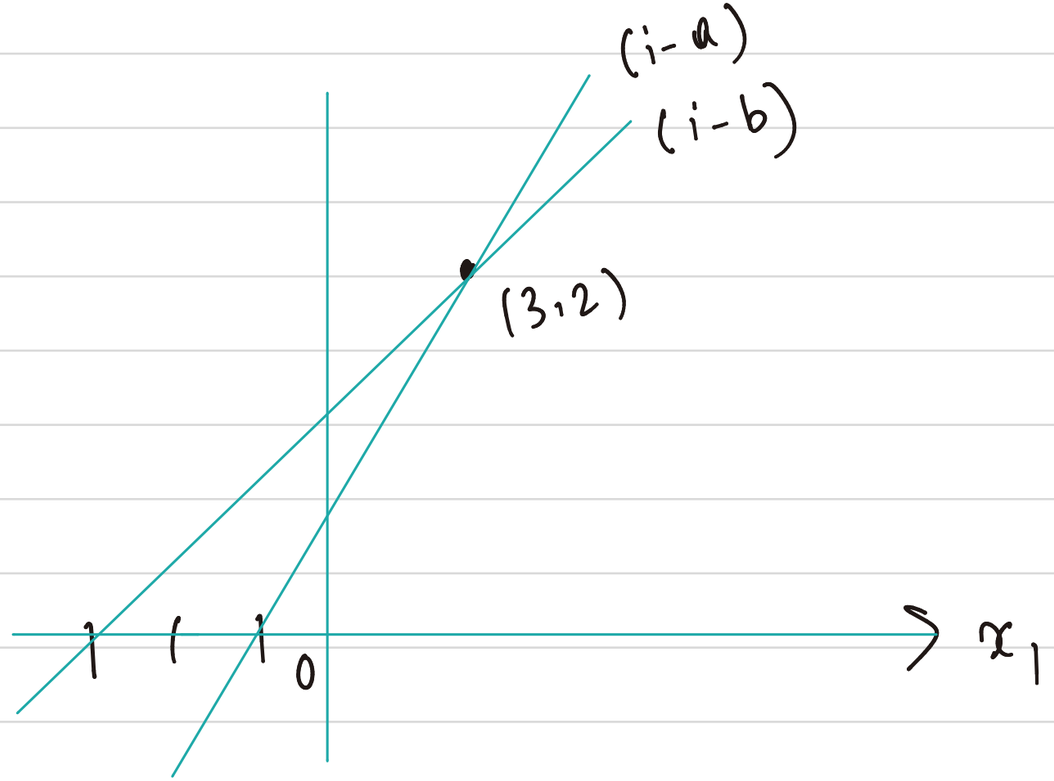
fig (c) → the line l_1 and l_2 overlap which means the lines have infinitely many points of intersect. So, it has infinitely many solutions and is consistence.

Examples: Consider a system

$$x_1 - 2x_2 = -1$$
$$x_1 - 3x_2 = -3$$

line satisfy by the point (3, 2). So, the system is consistent

and has exactly one solution because the lines intersect each other only at a point.



Matrix Notation of the System

A matrix form of Coefficients and the Constant values of a linear system is Known as matrix notation of the System

Augmented Matrix → Matrix Notation involves the Coefficients of linear System as well as Constant value

Coefficient of Matrix → Matrix Notation involves the Coefficient of variables then the matrix is called Coefficient matrix.

Examples:

$$x_2 + 4x_3 = -5$$

$$x_1 + 3x_2 + 5x_3 = -2$$

$$3x_1 + 7x_2 + 7x_3 = 6$$

In the form of Matrix Notation with coefficient each variable-

$$\begin{bmatrix} 0 & 1 & 4 \\ 1 & 3 & 5 \\ 3 & 7 & 7 \end{bmatrix}$$ is the coefficient of matrix

And, the matrix notation of the system —

$$\left[\begin{array}{ccc:c} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{array}\right]$$ is the Augmented Matrix.

Solve the system of linear equation by elementary Row operation.

$$x_1 - 3x_3 = 8$$
$$2x_1 + 2x_2 + 9x_3 = 7$$
$$x_2 + 5x_3 = -2$$

The matrix notation of the system is -

$$\left[\begin{array}{ccc:c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\left[\begin{array}{ccc:c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array}\right]$$

$R_3 \to 2R_3 - R_2$

$$\left[\begin{array}{ccc:c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 0 & -5 & 5 \end{array}\right]$$

Taking the equation form of the matrix notation is -

$$x_1 - 3x_3 = 8$$

$$2x_2 + 15x_3 = -9$$

$$-5x_3 = 5.$$

we get $\boxed{x_3 = -1}$

$$x_1 + 3 = 8$$

$$\boxed{x_1 = 5}$$

$$\therefore (x_1, x_2, x_3) = (5, 3, -1)$$

REFERENCE:

Binod Prasad Dhakal, Ph.D, Ramesh Gautam et al 2075, Mathematics - II, KEC Publication and Distribution (P.) Ltd