Day-34, Dec-22, 2024 (Poush 7, 2081 B.S).

Matrix Algebra

If A is $m \times n$ matrix in which A has m rows and n-columns. For instance, the 2×3 matrix can be written as

$$A = \begin{bmatrix} 5 & 6 & 7 \\ 4 & 3 & 2 \end{bmatrix}$$

m = row
n = column
2×2.

The entry a_{ij} be the $(i,j)^{th}$ entry of matrix. If A is $m \times n$ matrix then we write it in matrix form as,

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & & \vdots & & \vdots \\ a_{m1} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix} \rightarrow \text{Row}$$

$\downarrow$
Column

Also matrix can be written as –

$$A = [a_1 \ a_2 \ \cdots\cdots \ a_n]$$

Since a_{ij} be an entry of A so the $i=j$ is called diagonal entries of A

Matrix Operations:

① Sums and Scalar Multiples:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & & & \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & & \vdots & & \\ a_{m1} & & a_{mj} & & a_{mn} \end{bmatrix}$$

Sum of matrices A and B of same size (means same row and column) be the sum of corresponding entries of the matrices.

If A be a matrix and r be any scalar then the multiple of rA is the multiple of each entries of A.

Let's take an example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 5 & 7 \\ 9 & 11 & 13 \end{bmatrix}$$

Let $\gamma = 2$ then

$$\gamma A = 2 \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$$

Subtraction:

$$A - B = A + (-1)B$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + (-1)\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} -2 & -3 & -4 \\ -5 & -6 & -7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Theorem: Let A, B and C be the matrices of the same size with r and s be scalars.

1. $A + B = B + A$
2. $A + 0 = A$
3. $(A + B) + C = A + (B + C)$
4. $r(A + B) = rA + rB$
5. $r(sA) = (rs)A$
6. $(r + s)A = rA + sA$

Matrix Multiplication

If A is $m \times n$ and B is $n \times p$ matrix. Let the columns of B are $b_1, b_2, \dots b_p$.

$(\underline{m \times n = n \times p}) \to$ only multiplication possible.

then the product AB is matrix of order $m \times p$ where columns are $Ab_1, Ab_2, Ab_3 \cdots Ab_p$ i.e.

$$AB = A\begin{bmatrix} b_1 & b_2 & \cdots & b_p \end{bmatrix}$$

$$= \begin{bmatrix} Ab_1 & Ab_2 & \cdots & Ab_p \end{bmatrix}$$

$$[m \times n = n \times m] \qquad [3 \times 2 = 2 \times 1]$$

Possible and we use method of AB

Compare AB where $A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}$

Here, the columns of B are,

$$b_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

Then,

$$Ab_1 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 8+3 \\ 4-5 \end{bmatrix} \Rightarrow \begin{bmatrix} 11 \\ -1 \end{bmatrix}$$

2×2 $\quad$ 2×1

$$Ab_2 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 13 \end{bmatrix}$$

$$Ab_3 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 21 \\ -9 \end{bmatrix}$$

Now,

$$AB = A \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} Ab_1 & Ab_2 & Ab_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & 21 \\ -1 & 13 & -9 \end{bmatrix}$$

Transpose of Matrix: If A is $m \times n$ then the transpose of the given matrix A is denoted by A^T and that is $n \times m$ matrices

Example:

$$A = \begin{bmatrix} 2 & 5 & 6 \\ 3 & 4 & 1 \end{bmatrix}. \text{ Compute } A^T.$$

$$A^T = \begin{bmatrix} 2 & 3 \\ 5 & 4 \\ 6 & 1 \end{bmatrix}$$

Theorem: Let A be an $m \times n$ matrix, and let B and C have sizes for which the indicated sums and products are defined

① $A(BC) = (AB)C$ Associative law of multiplication

② $A(B+C) = AB + AC$ (left Distributive)

③ $(B+C)A = BA + CA$ Right Distributive

④ $r(AB) = (rA)B$ $\Rightarrow A(rB)$ for any Scalar

(e) $I_m A = A I_n$ (identity for matrix multiplication).

Note: It is not always $AB = BA$ sometimes $AB \neq BA$

Let's verify.

$$A = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 10+4 & 0+3 \\ 6-8 & 0-6 \end{bmatrix} = \begin{bmatrix} 14 & 3 \\ -2 & -6 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 & +2 \\ 29 & -2 \end{bmatrix}$$

Since $AB \neq BA$.

Theorem: Let A and B denote matrices whose sizes are appropriate for the following sums and products:

a) $(A^T)^T = A$

b) $(A+B)^T = A^T + B^T$

c) For any scalar r, $(rA)^T = rA^T$

d) $(AB)^T = B^T A^T$

REFERENCE: Binod Prasad Dhakal et al, 2075 Mathematics II B.Sc. CSIT Second Semester, KEC Publication and Distribution, Kathmandu Nepal.