Day-21, Dec-6, 2024 (Mangshir - 21, 2081)

Instantaneous Velocity

Suppose a particle is moving in a straight line AB. Then the distance described increases with time. So, the distance 's' can be considered to be a function 'f' of the time 't', and $s = f(t)$.

At times 't' and '$t+\Delta t$', suppose the particle is at the points P and Q respectively such that $AP = s$ and $AQ = s + \Delta s$.

Then,

$$PQ = AQ - AP$$
$$= S + \Delta S - S$$
$$\Rightarrow \Delta S$$

Also, $\Delta s = s + \Delta s - s$
$$= f(t+\Delta t) - f(t)$$

Thus, the average velocity V_{av}, during the time interval $(t, t+\Delta t)$ is

$$V_{av} = \frac{\Delta S}{\Delta t}$$

$$\Rightarrow \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

Now as $\Delta t \to 0$, Q tends to P. So the instantaneous velocity 'v' of the particle at 'P' or in time 't' is the limit to which V_{av} tends as $\Delta t \to 0$, and

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$\left[v = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t} \right]$$

Quick Intuitiative Idea on Derivative:

A function $y = f(x)$ is said to be differentiable with respect to x, if the limit,

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ exists

This limit, if it exists is called the derivative or the differential coefficient of $f(x)$ with respect to 'x'.

The derivative or the differential coefficient of $f(x)$ is denoted by $f'(x)$, $\frac{dy}{dx}$ or $\frac{df}{dx}$.

The differential coefficient or the derivative of $f(x)$ for the value $x = a$ is denoted by $f'(a)$. So

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Note: The process of finding the derivative of a function which is defined by an equation is called differentiation.

Derivative:

Let $y = f(x)$ be a continuous function defined on some interval I. Let Δx be the small increment in x and Δy be the corresponding increment in y. Then the derivative of 'y' w.r.t. x is denoted by $\frac{dy}{dx}$ and is defined by

$$\boxed{\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}}$$

OR

$$\boxed{\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

provided the limit on R.H.S. exists.

The derivative of $y = f(x)$ w.r.t. x.

$$\boxed{\frac{dy}{dx} \text{ or } f'(x) \quad \frac{d}{dx}[f(x)]}$$

Applications of Derivative:

The derivative is used to calculate the slope of the tangent and velocity of a moving particle at some specified point. Hence, we have

1. The slope of the tangent on the curve $y = f(x)$; at the point (x, y) is given by the differential coefficient $\frac{dy}{dx}$ at that point.

2) The velocity of a particle describing the path given by $s = f(t)$ at a time 't' is the differential coefficient of the function w.r.t. ie. $\frac{ds}{dt}$ always represents the velocity of the particle at particular time 't'.

The differentiation of the function is denoted by $\frac{d}{dx}$.

The result of operation is called the derivative or the differential Coefficient of the function.

Definition: Let the function f be defined in the interval (a, b). Then the derivative or the differential coefficient of the function f at a point x, of the interval is defined to be the limiting value of

$$\boxed{\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

Again if 'a' is fixed point then, the derivative of $f(x)$ at $x = a$ denoted by $f(a)$ is defined by -

$$\boxed{\lim_{x \to a} \frac{f(x) - f(a)}{\Delta x}}$$ Provided that the limit exists.

With the symbols used to denote the derivative of f with respect to 'x' are,

$$\boxed{f'(x), \quad \frac{d\,f(x)}{dx}, \quad y', \quad \frac{dy}{dx}}$$