Day-6, Nov-21, 2024 (Mongshir-7, 2081 B.S.)

Continuity in Interval

A function $f(x)$ is said to be continuous in an open interval (a,b), if it is continuous at every point in (a,b).

A function $f(x)$ is said to be continuous in the closed interval $[a,b]$, if it is continous at every point of the open interval (a,b) and if it is continuous at the point 'a' from the right and continuous at the point 'b' from the left.

i.e. $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$

Definition: A function $f(x)$ is said to be continuous at point $x = a$. if $\lim\limits_{x \to a} f(x) = f(a)$

i.e. limiting value = functional value of the function $f(x)$ at $x = a$.

Existence of limit:

Left Hand limit $\lim\limits_{x \to a-0} f(x)$ = Right Hand limit of the function $f(x)$ at $x = a$. $\lim\limits_{x \to a+0} f(x)$

Alternatively.

A function $f(x)$ is said to be continuous at $x = a$, if any $\varepsilon > 0$, however small, there exists a $\delta > 0$ such that

$$|f(x) - f(a)| < \varepsilon \quad \text{wherever} \quad |x - a| < \delta.$$

Example: $\lim\limits_{x \to 2} (2x^2 + 3x - 14)$

$\Rightarrow 0$

Example: $f(x) = \begin{cases} 2x+3 & \text{for } x < 1 \\ 4 & \text{for } x = 1 \\ 6x-1 & \text{for } x > 1 \end{cases}$

Left Hand Limit at $x = 1$.

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} (2x+3) \Rightarrow 5$

$x < 1$

Right Hand Limit at $x = 1$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} 6x - 1 \Rightarrow 5$

$x > 1$

$\boxed{\therefore \text{Left Hand} = \text{Right Hand}}$

$\therefore$ So, $\lim\limits_{x \to 1} f(x)$ exists and $\lim\limits_{x \to 1} f(x) = 5$

But at $x = 1$ for $f(1) = 4$ So, $f(x)$ is not continuous at $x = 1$.

Example of Continuous function:

A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x < 3 \\ 2kx & \text{for } x \geq 3 \end{cases}$$

find the value of K so that $f(x)$ is continuous at $x=3$.

Soln: $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (x^2 - 1)$

$\Rightarrow 8$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2kx = 6k \quad \Rightarrow 6k = 6k$

$\therefore f(3) = 6k$

So, $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$

$$2Kx = 8$$

$$6K = 8$$

$$\boxed{K = 4/3}$$

$$6K = 6K$$

$$6 \times 4/3 = 6 \times 4/3$$

So, the $f(x)$ is continuous at $x = 3$.

Continuity Theorem:

Let $f(x)$ and $g(x)$ be two continuous function at $x = a$, then we have:

i) $f(x) \pm g(x)$ is continuous at $x = a$

ii) $f(x) \cdot g(x)$ is continuous at $x = a$

iii) $\dfrac{f(x)}{g(x)}$ is continuous at $x = a$, Provided $g(a) \neq 0$.

iv) $\sqrt[n]{f(x)}$ is continuous at $x = a$, Provided $f(a) \geq 0$ and 'n' is even.

Properties of Continuous functions:

i) Let $f(x)$ be continuous on a closed interval $[a, b]$. Suppose $f(a)$ and $f(b)$ have opposite signs. Then there exists a points $c \in (a, b)$ and such that $f(c) = 0$.

ii) Let $f(x)$ be continuous function on a closed interval $[a, b]$. Suppose K is any number between $f(a)$ and $f(b)$. Then there exists a point $c \in (a, b)$ such that $f(c) = K$.

iii) A continuous function on a closed interval is always bounded.

iv) A continuous function on a closed interval has a maximum and minimum value on the interval.

REFERENCES:

D.R. Bajracharya et. al, 2014, Basic Mathematics Grade XI, Sukunda Pustak Bhawan, Kathmandu

Multiple features in Linear Regression

x_1	x_2	x_3	x_4	x_5
1	3	5	7	9
2	4	6	8	10

So, if $x_1^2 \Rightarrow 2$

$x_2^1 \Rightarrow 3$

x_i are the features

w is the weight

b is the bias or the cost of House price (base price)

So, $y = x_i w_i + b$

Vectorization:

$$f_{(\vec{w}, b)}(\vec{x}) = \vec{w} \cdot \vec{x} + b$$

$\vec{w}$ and $\vec{x}$ are the vector (list of numbers we say).