Day-9, Nov 24, 2024, (Mangshir 09, 2081 BS)

Definition:

Let f be a function defined on some open interval that contains the number a except possibly at a itself. Then we say that the limit of $f(x)$ as x approaches 'a' is L, and we write,

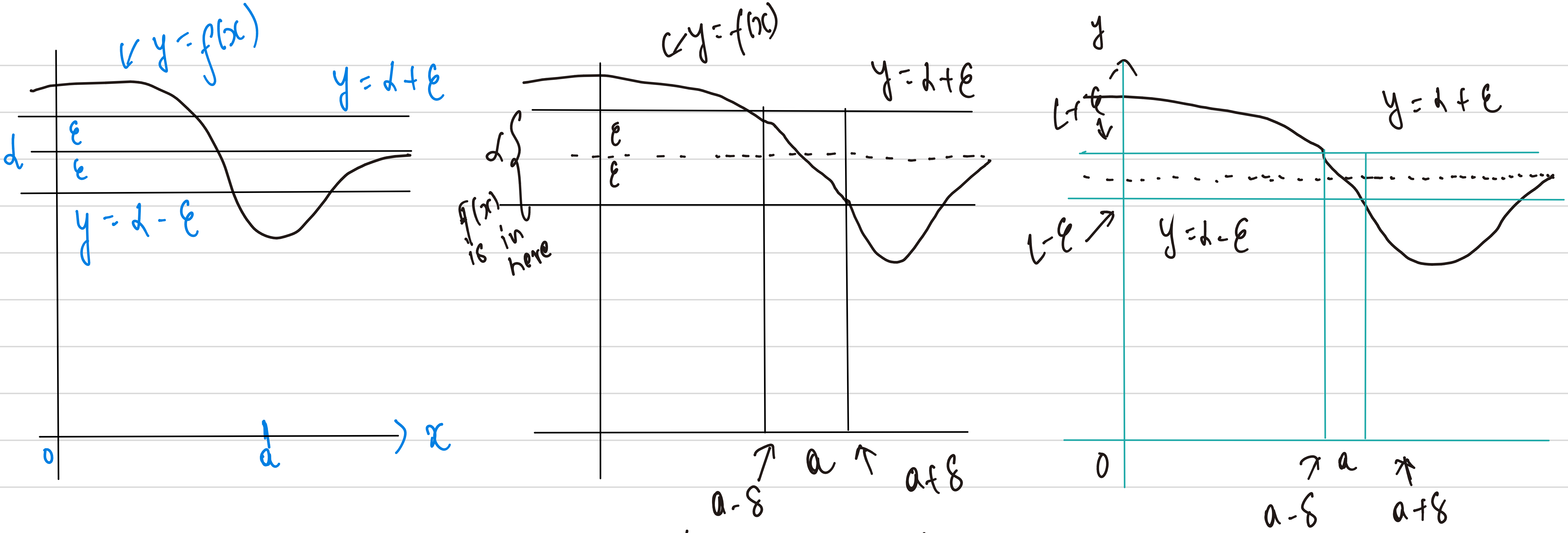
$$\lim_{x \to a} f(x) = L$$ if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that if $\underline{0 < |x - a| < \delta}$ then $\underline{|f(x) - L| < \epsilon}$

Since $|x-a|$ is the distance from 'x' to 'a' and $|f(x)-L|$ is the distance from $f(x)$ to L, and Since 'ε' can be arbitrarily small, the definition of a limit can be expressed in words as follows:

$\lim_{x \to a} f(x) = L$ means that the distance between $f(x)$ and L can be made arbitrarily small by taking the distance from x to a is sufficiently small (but not zero).

Alternatively, $\lim_{x \to a} f(x) = l$ means that the values of $f(x)$ can be made as close as we please to l by taking 'x' close enough to a (but not equal to a).

Alternatively, $\lim_{x \to a} f(x) = l$ means for every $\epsilon > 0$ (however it is small enough) we can find $\delta > 0$ such that x lies in the open interval $(a-\delta, a+\delta)$ then $f(x)$ lies in the open interval $(l-\epsilon, l+\epsilon)$ which is illustrated in the following figures.



Where x is in here

$(x \neq a)$

Example 1: Use a graph to find a number δ such that -

if $|x - 1| < \delta$ then $|(x^3 - 5x + 6) - 2| < 0.2$

if $|x-1| < \delta$ then -

$$|(x^3-5x+6)-2| < 0.2$$

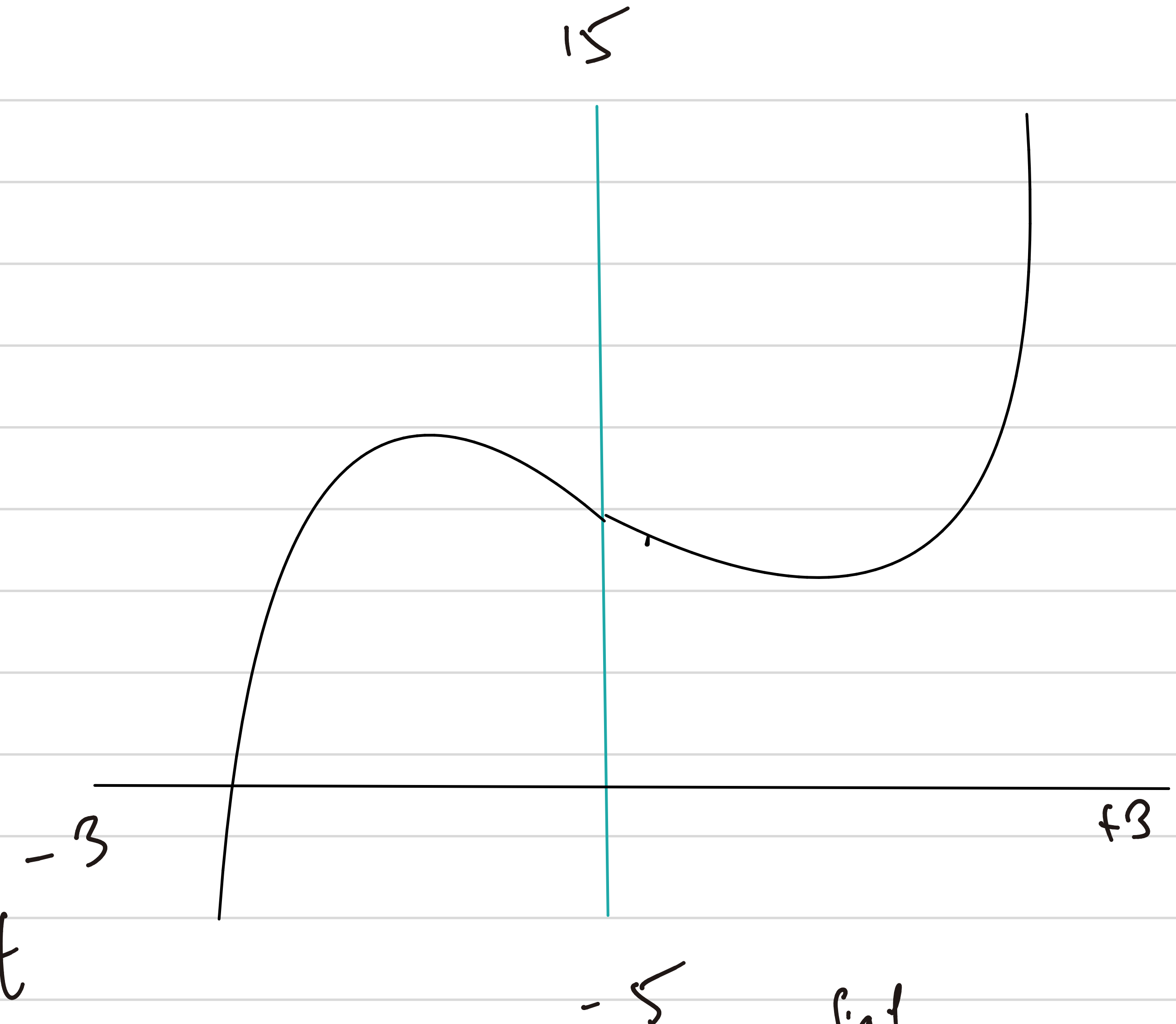


fig 1

Alternatively,

find a number δ that corresponds to $\epsilon = 0.2$, in the definition of a limit for the function $f(x) = x^3 - 5x + 6$ with $a = 1$ and $d = 2$.

Sol^n

A graph of f is shown in fig 1 our concentration is near the point (1.2)

$$|(x^3-5x+6)-2| < 0.2$$

or, $$1.8 < x^3-5x+6 < 2.2$$

$[y = x^3-5x+6]$ find the values of x for which the curve $y=x^3-5x+6$ lies between the horizontal lines $y=1.8$ and $y=2.2$.

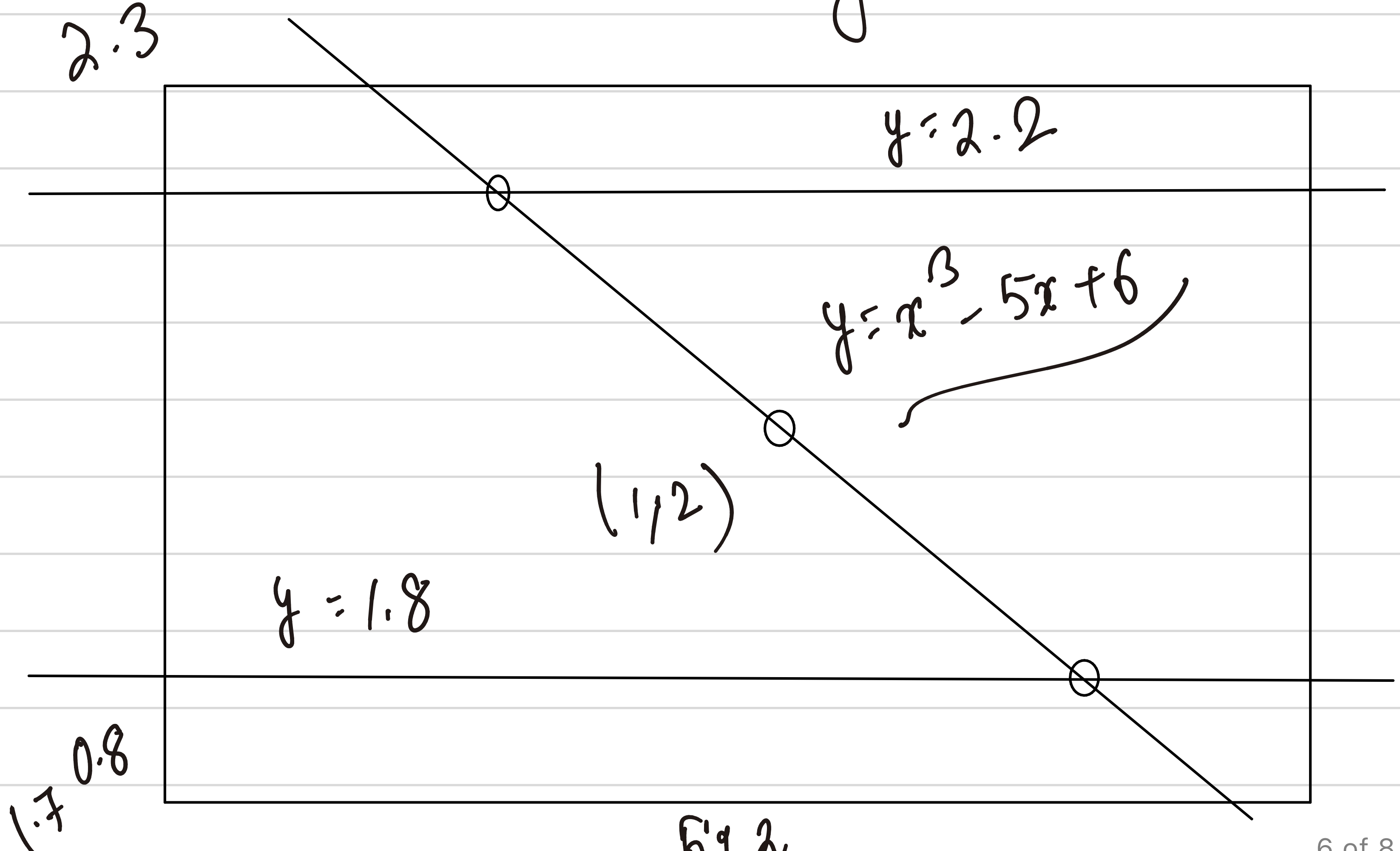


Fig 2

So, the conclusion is by keeping x within 0.08 of 1, we are able to keep $f(x)$ within 0.2 of 2!

<u>Numerical Examples:</u> prove that $\lim_{x \to 3} 4x - 5 = 7$

if $0 < |x-3| < \delta$, then $|4x - 5 - 7| < \varepsilon$

$|4x - 12| < \varepsilon$

$|x - 3| < \varepsilon/4$

$\therefore \delta = \varepsilon/4$ (or any smaller positive number).

For any +ve number ε, we choose $\delta = \varepsilon/4$ such that $0 < |x-3| < \delta$

$|4x - 5 - 7| = |4x - 12|$

$= 4|x-3| < 4$

$\delta \Rightarrow 4 \times \varepsilon/4$

$\boxed{\delta \Rightarrow \varepsilon}$

$\therefore \lim_{x \to 3} 4x - 5 \Rightarrow 7$

Example: Prove that $\lim_{x \to 3} x^2 = 9$

we have to show $\forall\, \varepsilon > 0\ \exists\ \delta > c$ such that $0 < |x-3| < \delta$

$\Rightarrow |x^2 - 9| < \varepsilon$

Now $|x^2 - 9| \Rightarrow |x-3|\,|x+3| < \delta |x+3| < \delta c = \varepsilon$

$\therefore \delta = \varepsilon/c$