Day - 57, Jan - 26, 2026 (Magh 13, 2082 B.S.).

An Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \quad = \frac{1}{32}$$

$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \quad = \frac{1}{32}$$

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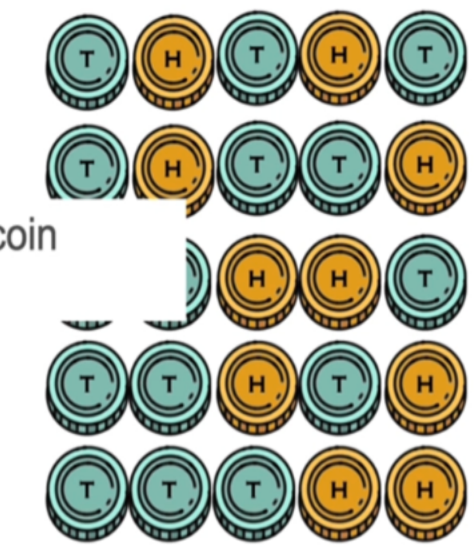
An Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

10 ways to have 2 heads in 5 coin tosses



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An Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

$$10 = \frac{5!}{2!(5-2)!} = \binom{5}{2}$$

Binomial coefficient

Number of ways you can get 2 heads in 5 coin tosses

Binomial Coefficient

In general:

$\binom{n}{k}$ counts all the combinations for landing k heads in n coin tosses

Property:

$$\binom{n}{k} = \binom{n}{n-k}$$

k heads is the same thing as obtaining n-k tails.

Binomial Coefficient

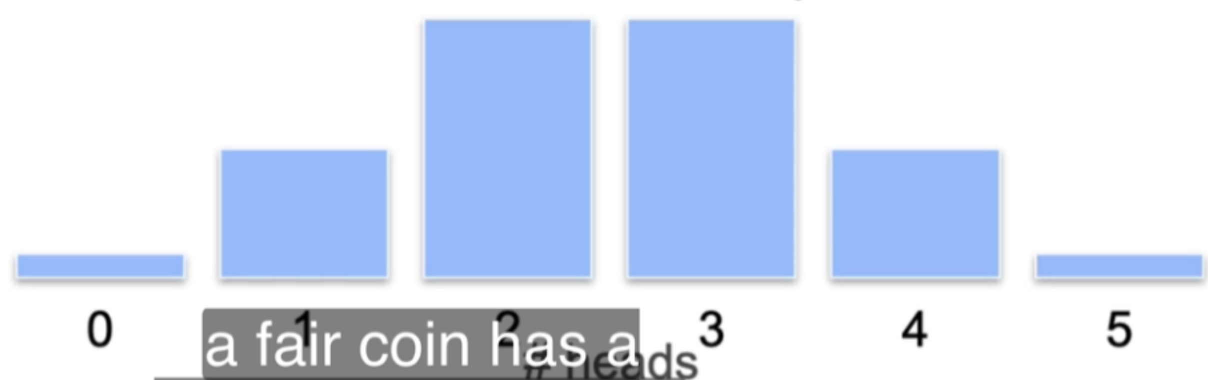
In general:

$\binom{n}{k}$ counts all the combinations for landing k heads in n coin tosses

Property:

$$\binom{n}{k} = \binom{n}{n-k}$$

The PMF with a fair coin is symmetrical



a fair coin has a symmetrical shape.

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

Probability of seeing $5 - x$ tails

All the possible orders

$$\boxed{\binom{5}{x}}\boxed{p^x}\boxed{(1-p)^{5-x}}$$

Probability of seeing x heads

so you need to take into account
all the possible orders,

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

X follows a binomial distribution

$$X \sim Binomial(\boxed{5},\boxed{p})$$

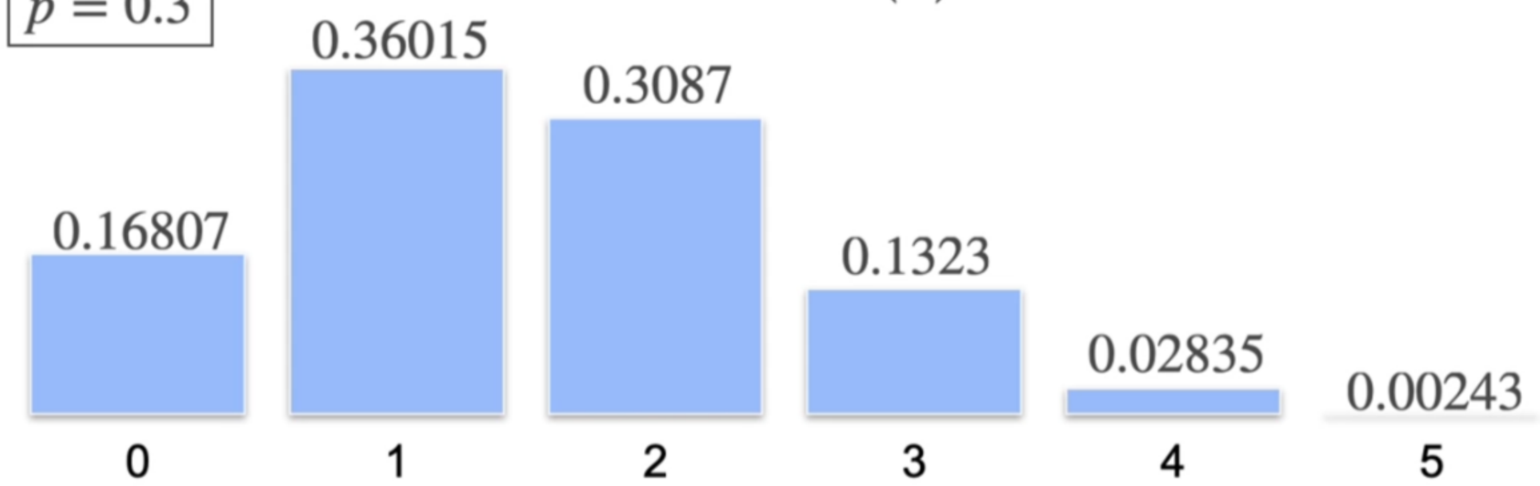
Number of flips $\qquad \mathbf{P}(H)$

We denote it as
binomial of 5 and p,

Binomial Distribution

$$\boxed{\begin{array}{c} n = 5 \\ p = 0.3 \end{array}}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.3^k 0.7^{5-k}$$



greater chances of seeing
a small number of heads,

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Binomial Distribution

General PMF for X : number of heads in n coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in n tosses

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0,1,2,\ldots,n$$

$X \sim Binomial(n, p)$

n and p are called the **parameters** of the binomial distribution

n is the number of tosses,

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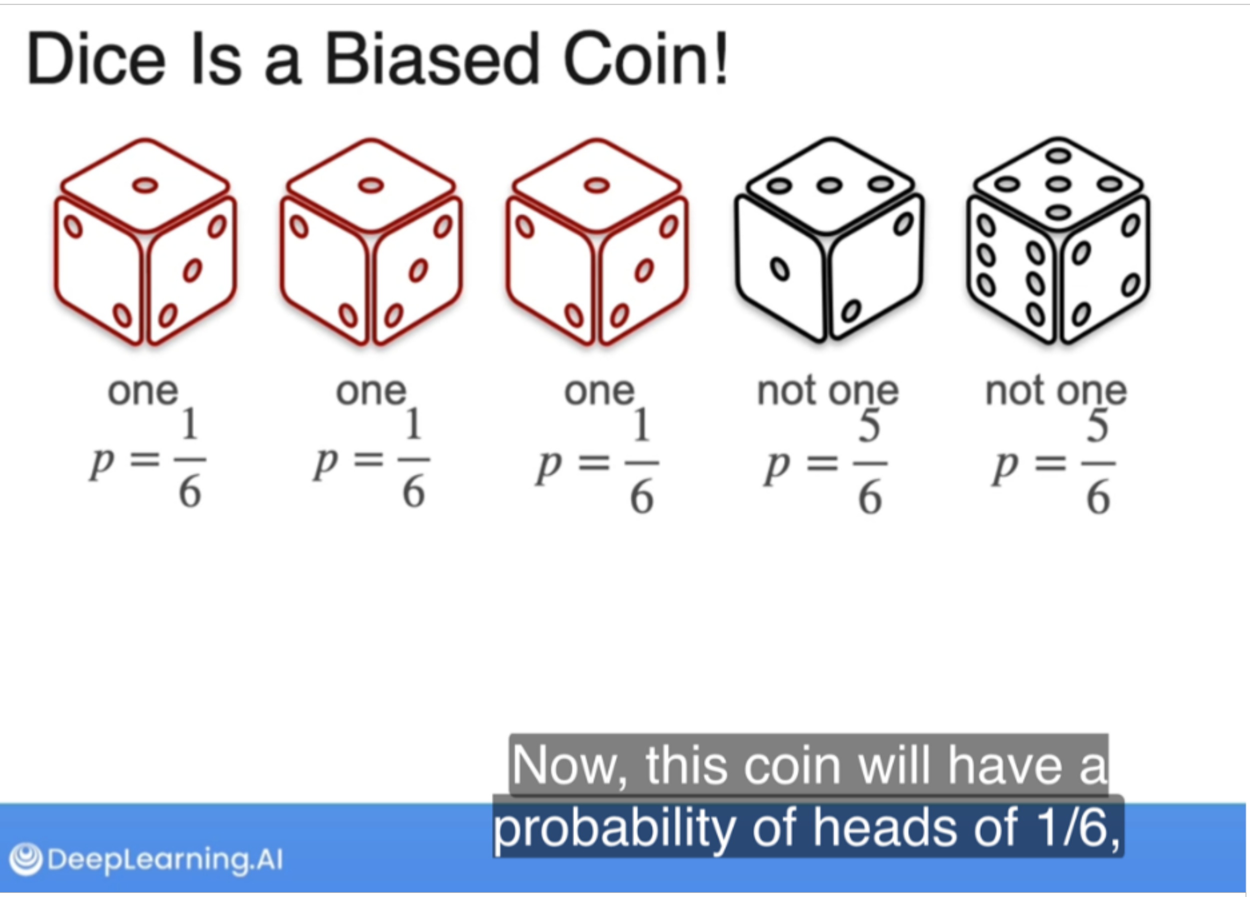
Dice Is a Biased Coin!
one
$p = \frac{1}{6}$
one
$p = \frac{1}{6}$
one
$p = \frac{1}{6}$
not one
$p = \frac{5}{6}$
not one
$p = \frac{5}{6}$
Now, this coin will have a
probability of heads of 1/6,
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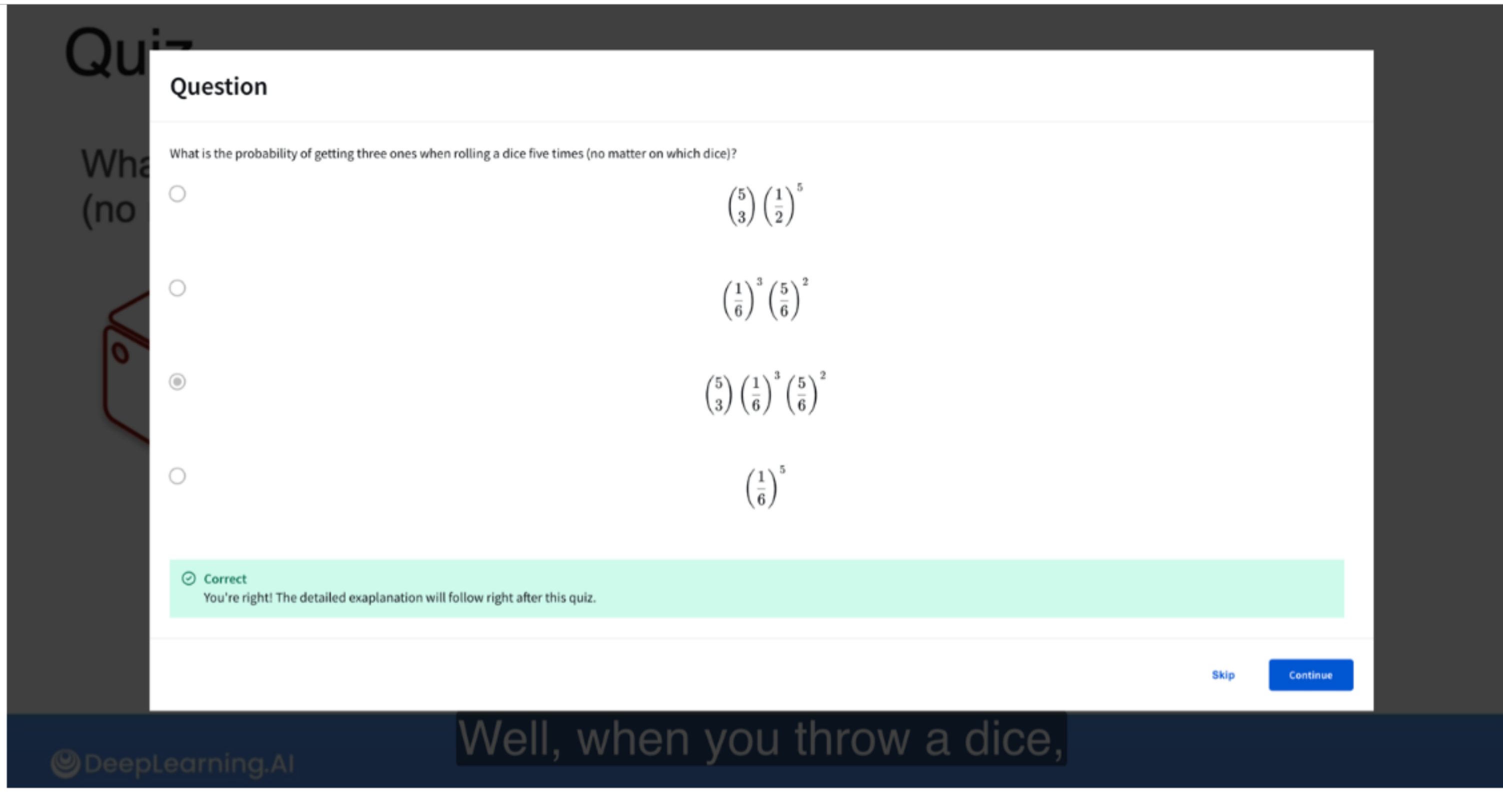
Question
What is the probability of getting three ones when rolling a dice five times (no matter on which dice)?
$\binom{5}{3}\left(\frac{1}{2}\right)^5$
$\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2$
$\binom{5}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2$
$\left(\frac{1}{6}\right)^5$
Correct
You're right! The detailed exaplanation will follow right after this quiz.
Skip
Continue
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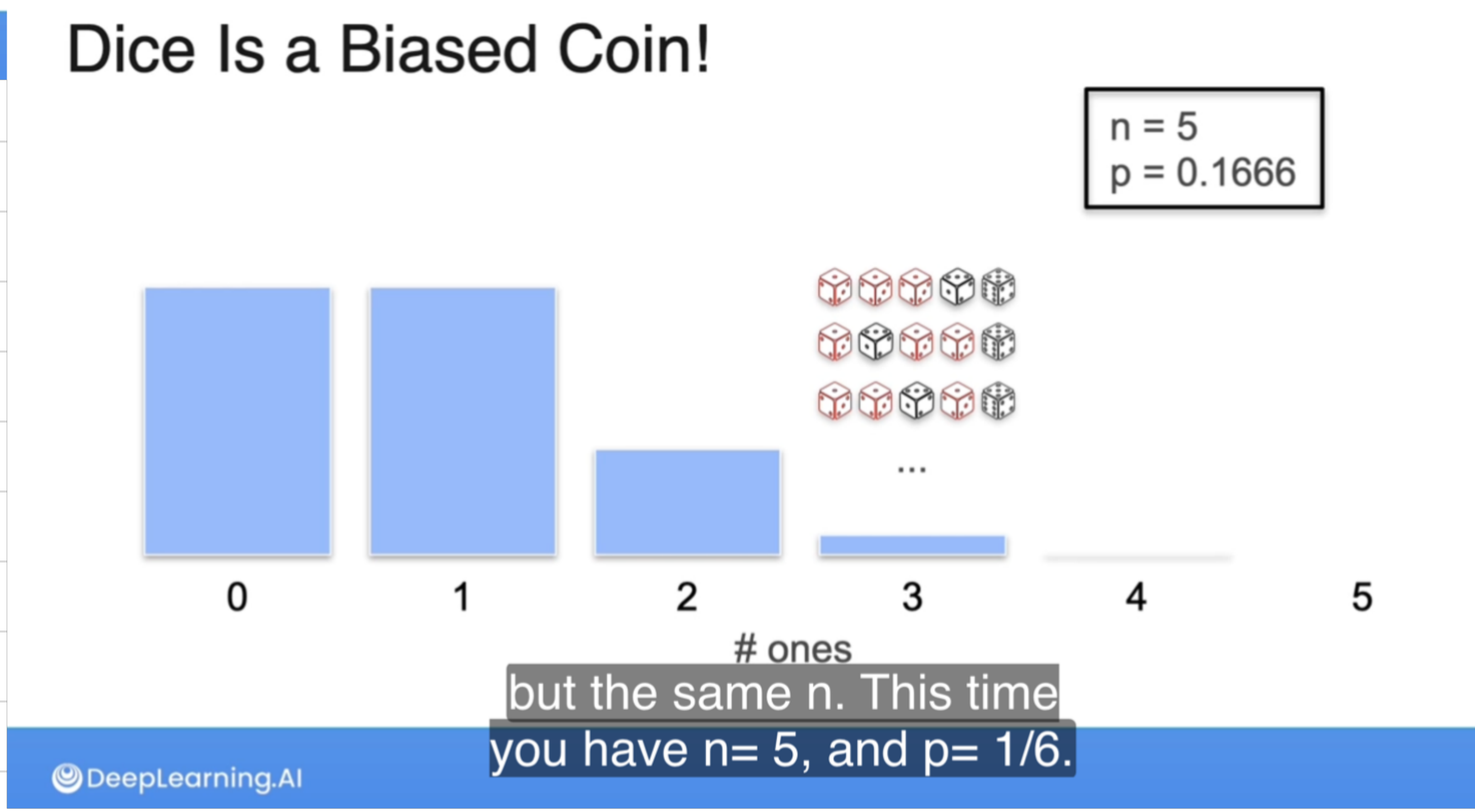
Well, when you throw a dice,


Dice Is a Biased Coin!
n = 5
p = 0.1666
...
0
1
2
3
4
5
ones
but the same n. This time
you have n= 5, and p= 1/6.
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Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options → $n-1$ options → $n-2$ options → … → $n-(k-1)$options

Ordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

…

Some of them repeat

$$n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))$$

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Question

If X is the number of times we get a 1 when rolling a dice ten times, then

$$X \sim \text{Binomial}(n,p)$$

Where n, p is equal to:

- (selected) n = 10, p = 1/6
- n = 6, p = 1/6
- n = 10, p = 1/2
- n = 10, p = 6

Correct
Correct. The probability of getting a 1 when throwing one dice is 1/6. And n is the total of experiments (10 times).

Skip | Continue

$$p(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Where:
n: number of trials
k: number of successes
p: probability of success
$(1-p)$ Probability of failure

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options	3 options	2 options	1 option

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

…

4,3,2,1

$$4\cdot3\cdot2\cdot1 = 4!$$

For five numbers:

$$5\cdot4\cdot3\cdot2\cdot1 = 5!$$

General solution: $k!$

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k different objects.

Binomial Coefficient

Pick 1st number — Pick 2nd number — Pick 3rd number — Pick k-th number

n options $\to$ $n-1$ options $\to$ $n-2$ options $\to \dots \to$ $n-(k-1)$options

Ordered sets of length k

$1,2,3,4,\dots,k$
$2,1,3,4,\dots,k$
$5,1,3,k,\dots,2$ $\quad n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))$
$n,1,3,4,\dots,2$
$\dots$

was n times n-1 all
the way to times n-k-1.

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Binomial Coefficient

Pick 1st number — Pick 2nd number — Pick 3rd number — Pick k-th number

n options $\to$ $n-1$ options $\to$ $n-2$ options $\to \dots \to$ $n-(k-1)$options

Ordered sets of length k $\longrightarrow$ Unordered sets of length k

$1,2,3,4,\dots,k$
$2,1,3,4,\dots,k$
$5,1,3,k,\dots,2$
$n,1,3,4,\dots,2$
$\dots$

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))}{k!} = \frac{n!}{(n-k)!\,k!} = \binom{n}{k}$$

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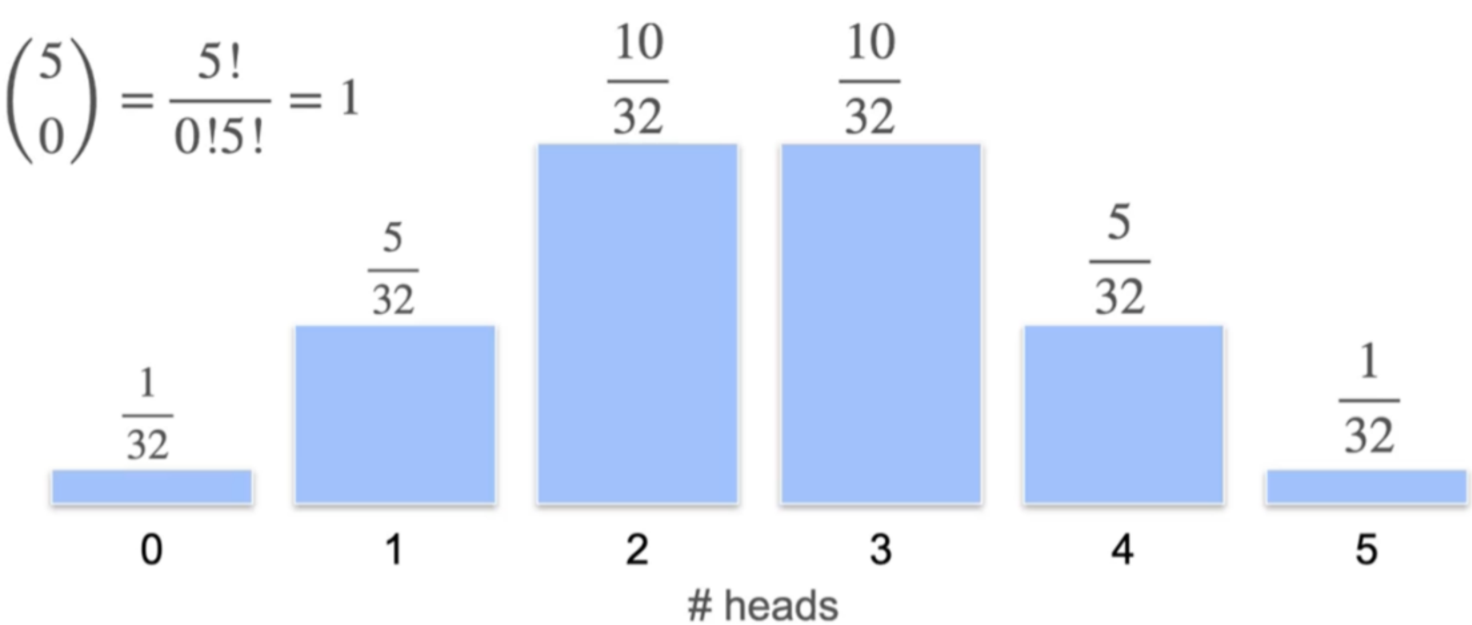
Binomial Distribution models the number of successes in 'n' independent Bernoulli trials, where each trial has two outcomes: (with probability p) and without 1-p.

Binomial Distribution: Fair Coins

$$\binom{5}{0} = \frac{5!}{0!5!} = 1$$



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Biased Coins

30 %

70 %

$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.00243$

$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.01323$

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→ A biased coin is a coin where the probabilities of landing on heads or tails are not equal, unlike a fair coin. p of head cannot be 0.5.

Binomial Coefficient often denoted as $\binom{n}{k}$ represents the number of ways to choose k items from n items without considering the order.

Biased Coins

$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5 \cdot 0.7^0$

$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 = 0.3^4 \cdot 0.7^1$

$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 = 0.3^3 \cdot 0.7^2$

$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^2 \cdot 0.7^3$

$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^1 \cdot 0.7^4$

$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^0 \cdot 0.7^5$

$= 0.3^k \cdot 0.7^{n-k}$

And here we have all the other ones

0.3 to the 4 times 0.7 to the 1,

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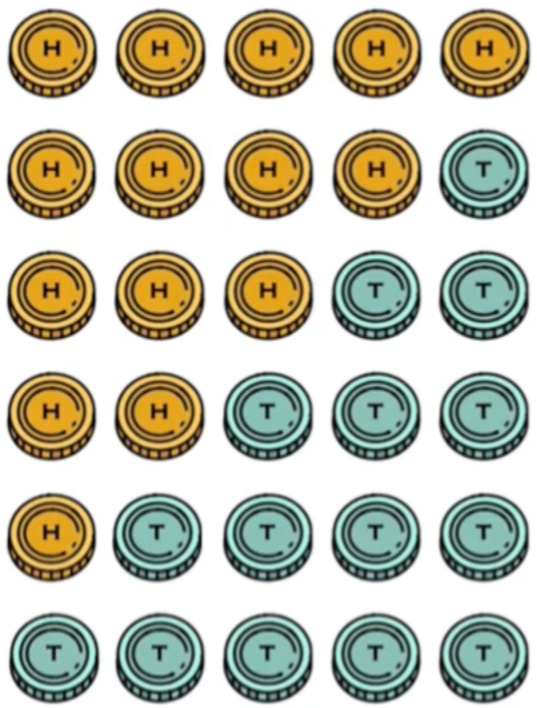
Biased Coins



$0.3^5 \cdot 0.7^0$

$0.3^4 \cdot 0.7^1$

$0.3^3 \cdot 0.7^2$

$0.3^2 \cdot 0.7^3$

$0.3^1 \cdot 0.7^4$

$0.3^0 \cdot 0.7^5$

$$= 0.3^k \cdot 0.7^{n-k} \Rightarrow \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

Account for all possible orders of heads and tails

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And that's where n choose k comes

$n \Rightarrow$ no. of trials

$k \Rightarrow$ no. of favourable outcomes.

Question

Throwing a 4-sided fair dice and observing if it lands in 2 or not might be modeled as a Bernoulli distribution with p equals to:

- 1/4
- 1/6
- 1/2
- 3/4

Correct

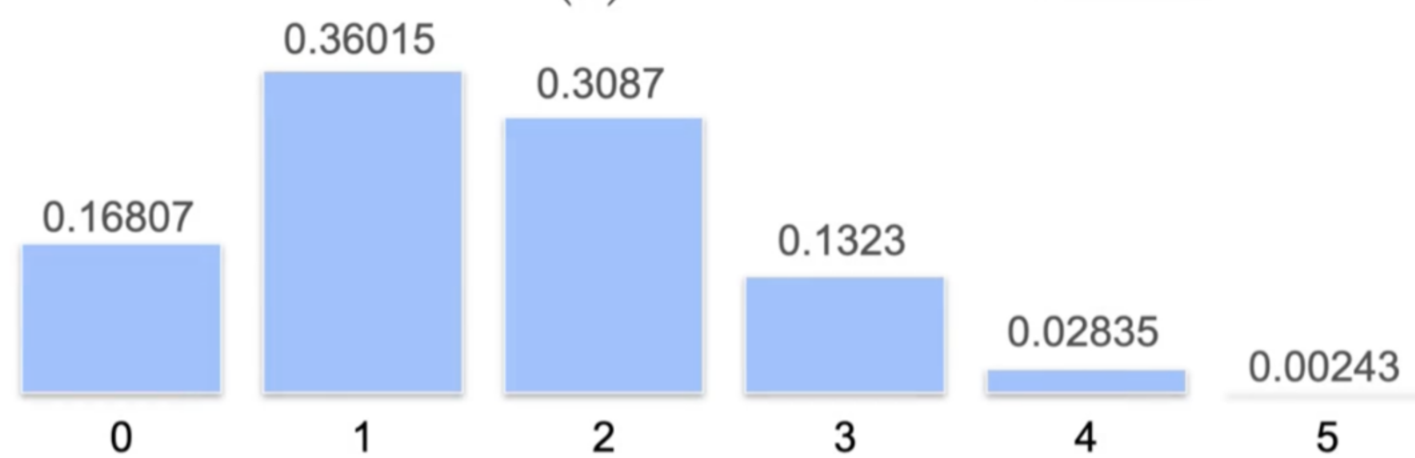
Correct! In a 4-sided fair dice, there is a 1/4 probability of landing in each face.

Skip Continue

Binomial Distribution

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

n = 5
p = 0.3



Except that now you can calculate these probabilities using the formula that you

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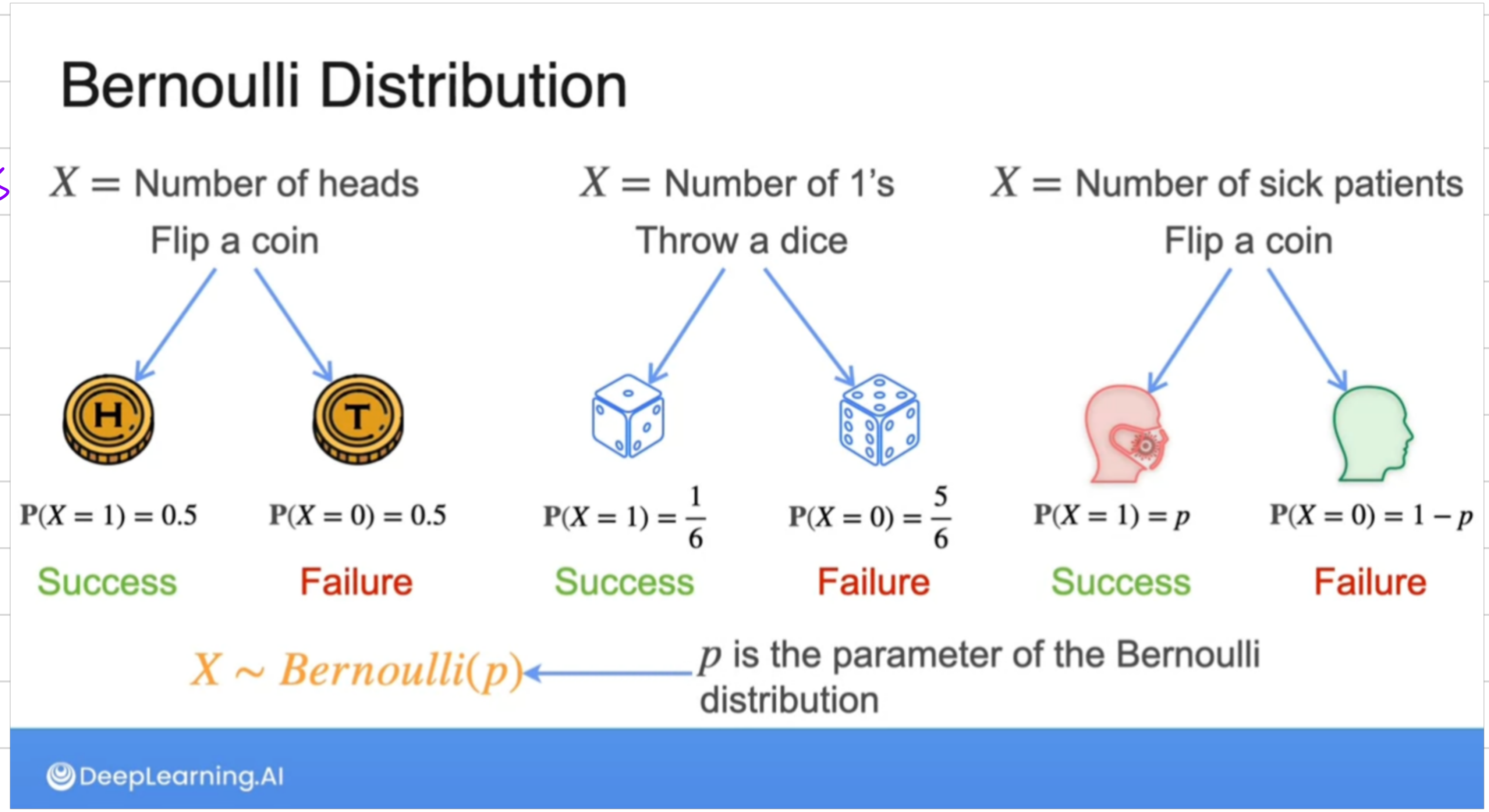
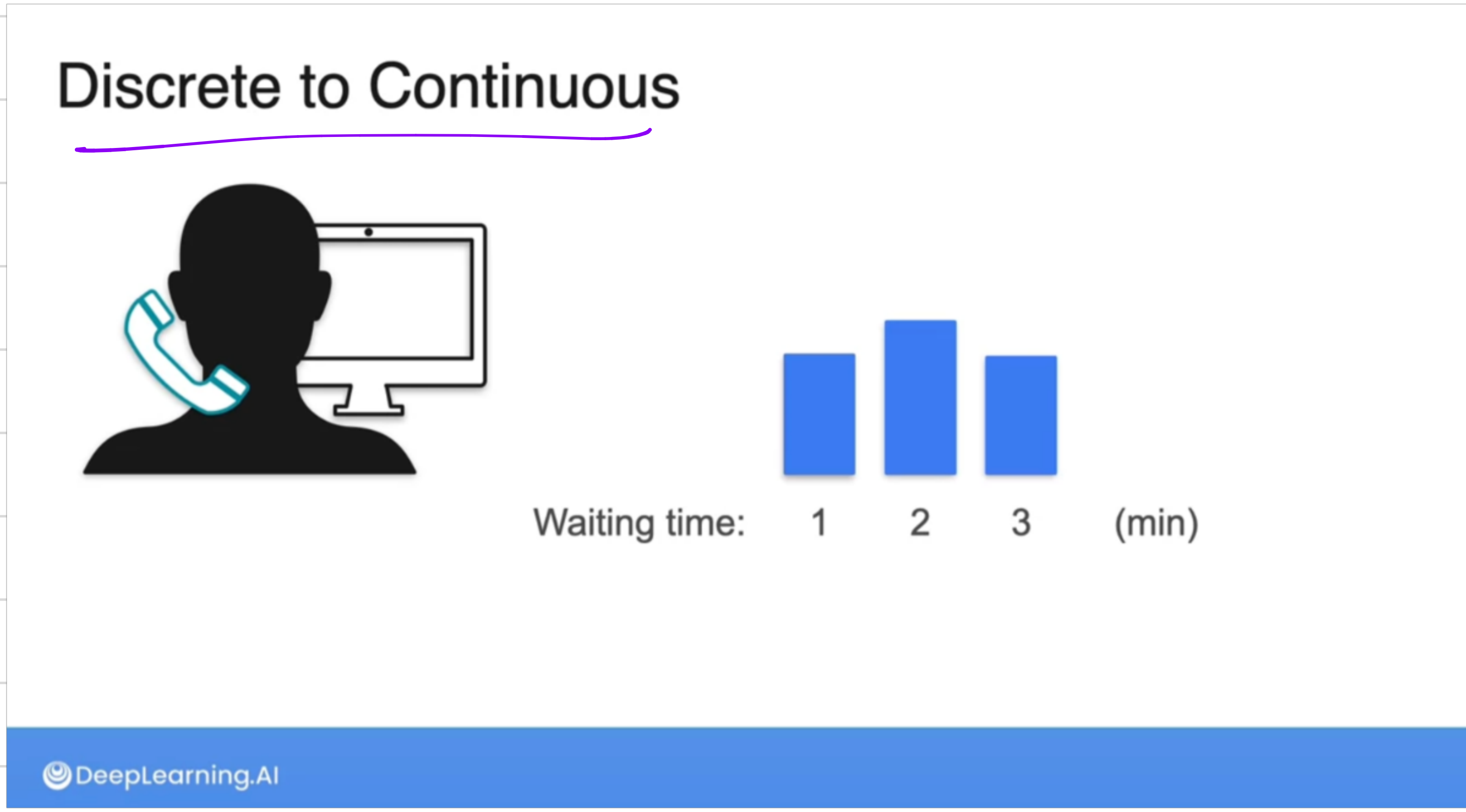
Bernoulli Distribution represents the outcome of a single trial of an experiment where there are only two possible outcomes:

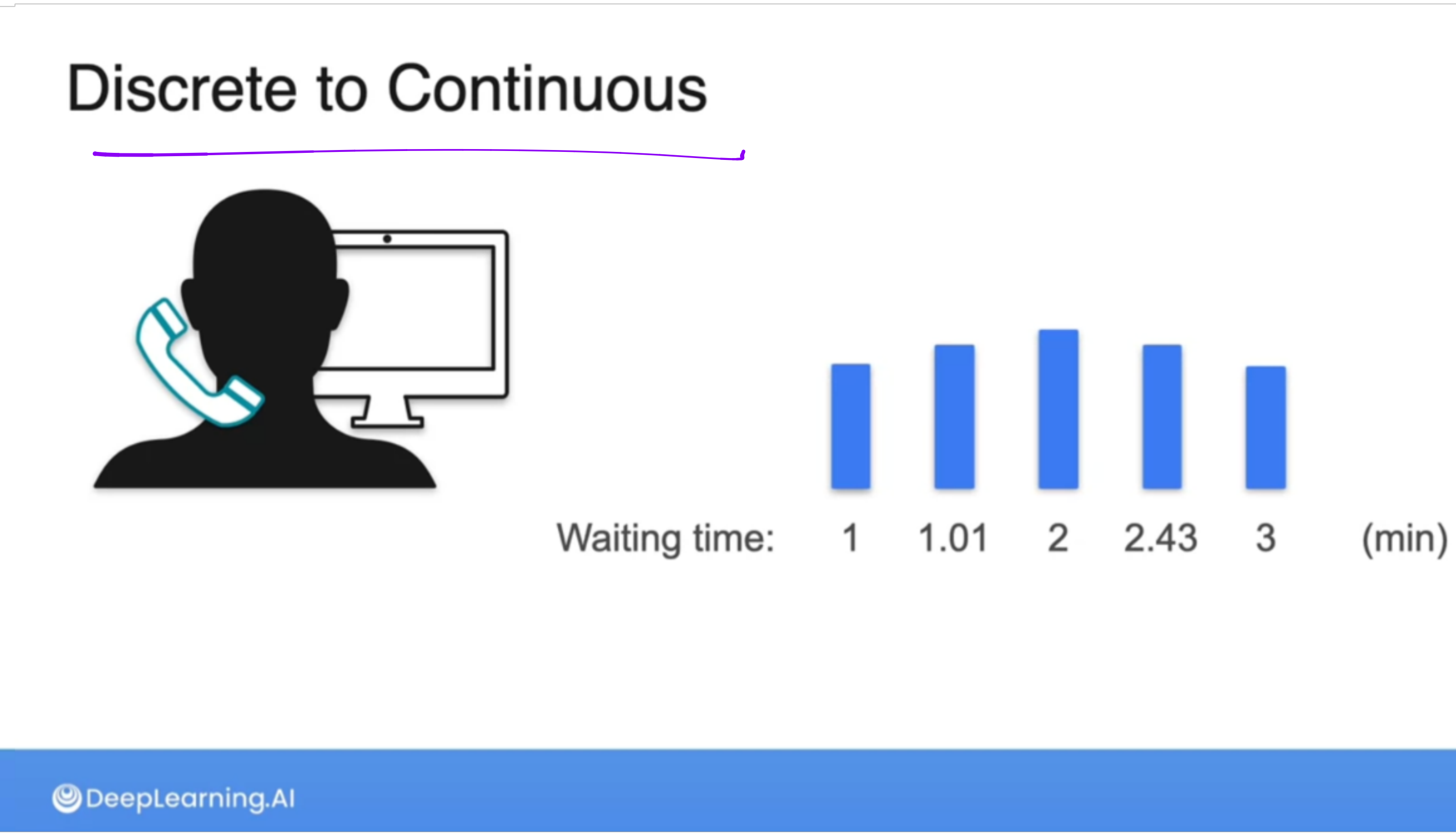
Success (often denoted as 1) and failure (often denoted as 0):

→ Success (1) with p

→ failure (0) with $1-p$.



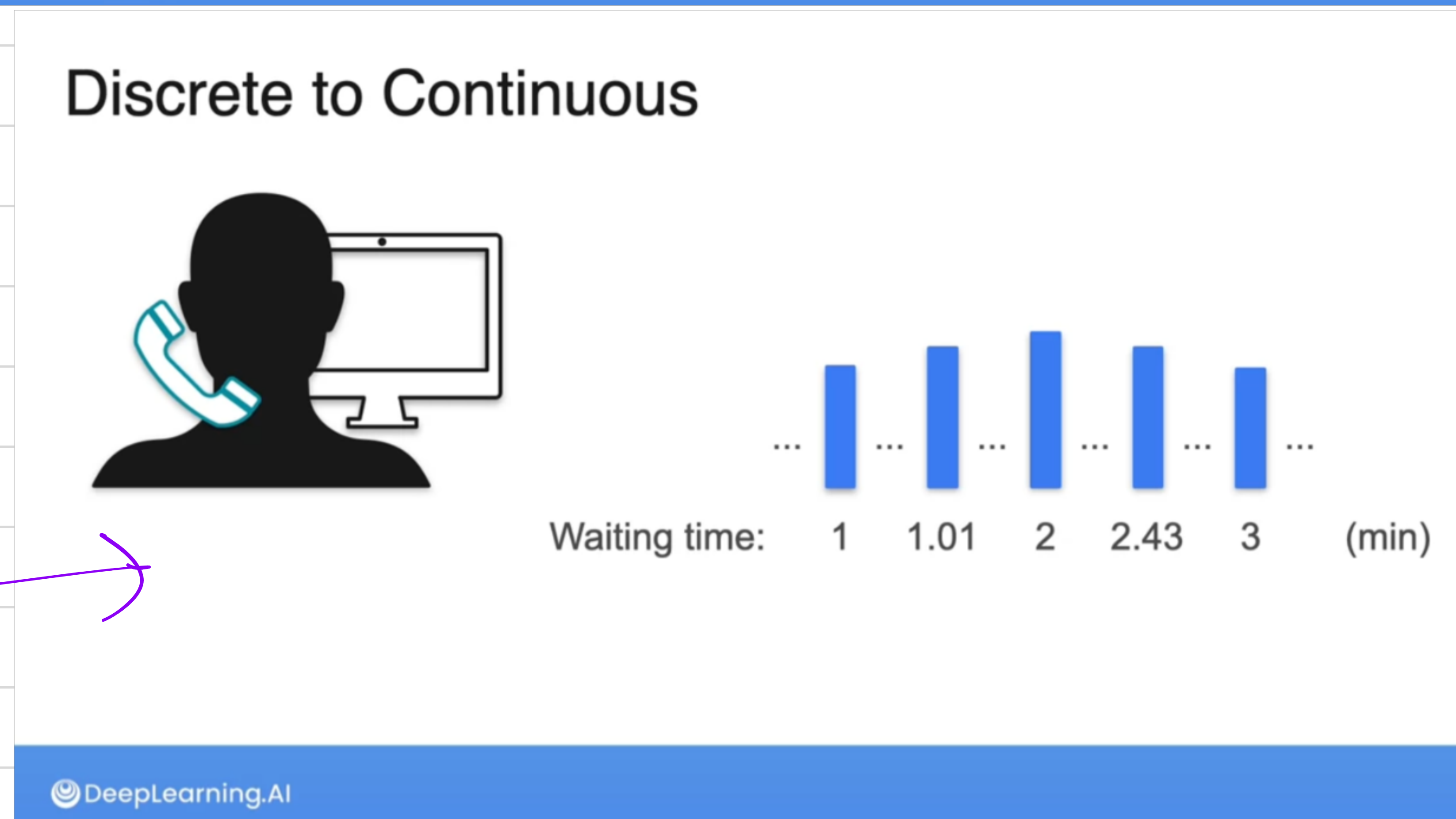




[1 – 1.01]

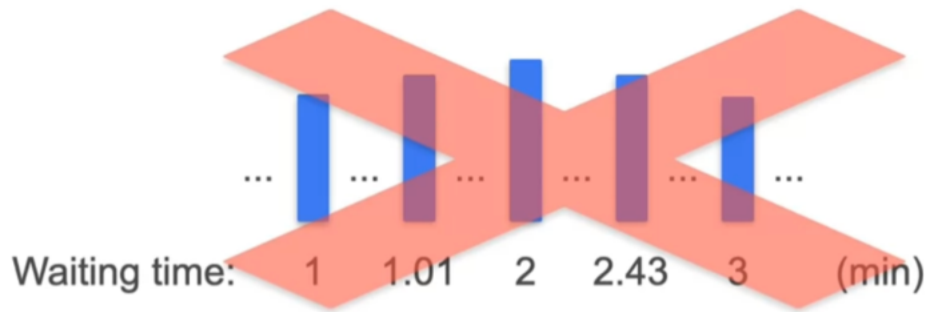
[2 – 2.43]

there are infinitely large numbers between 1 and 2.



Discrete to Continuous

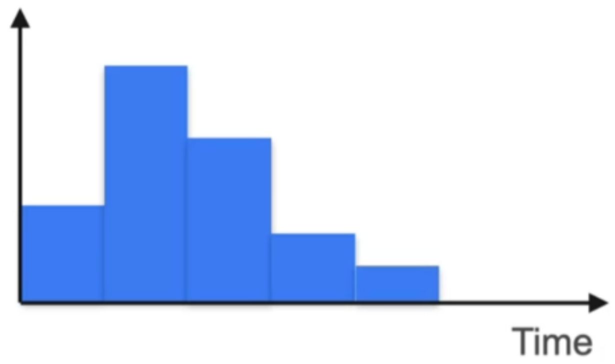
This is a continuous distribution!



What is the probability that you will wait EXACTLY one minute for the call?

Mixed both Discrete & Continuous But here ∞.

Discrete to Continuous



P(between 0 and 1 mins)
P(between 1 and 2 mins)
P(between 2 and 3 mins)
P(between 3 and 4 mins)
P(between 4 and 5 mins)

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Transformation flow
Discrete Distributions
becomes
Continuous Distributions

Discrete to Continuous

P(between 0 and 0.5 mins)
P(between 0.5 and 1 mins)
P(between 1 and 1.5 mins)
⋮
P(between 3.5 and 4 mins)
P(between 4 and 4.5 mins)
P(between 4.5 and 5 mins)

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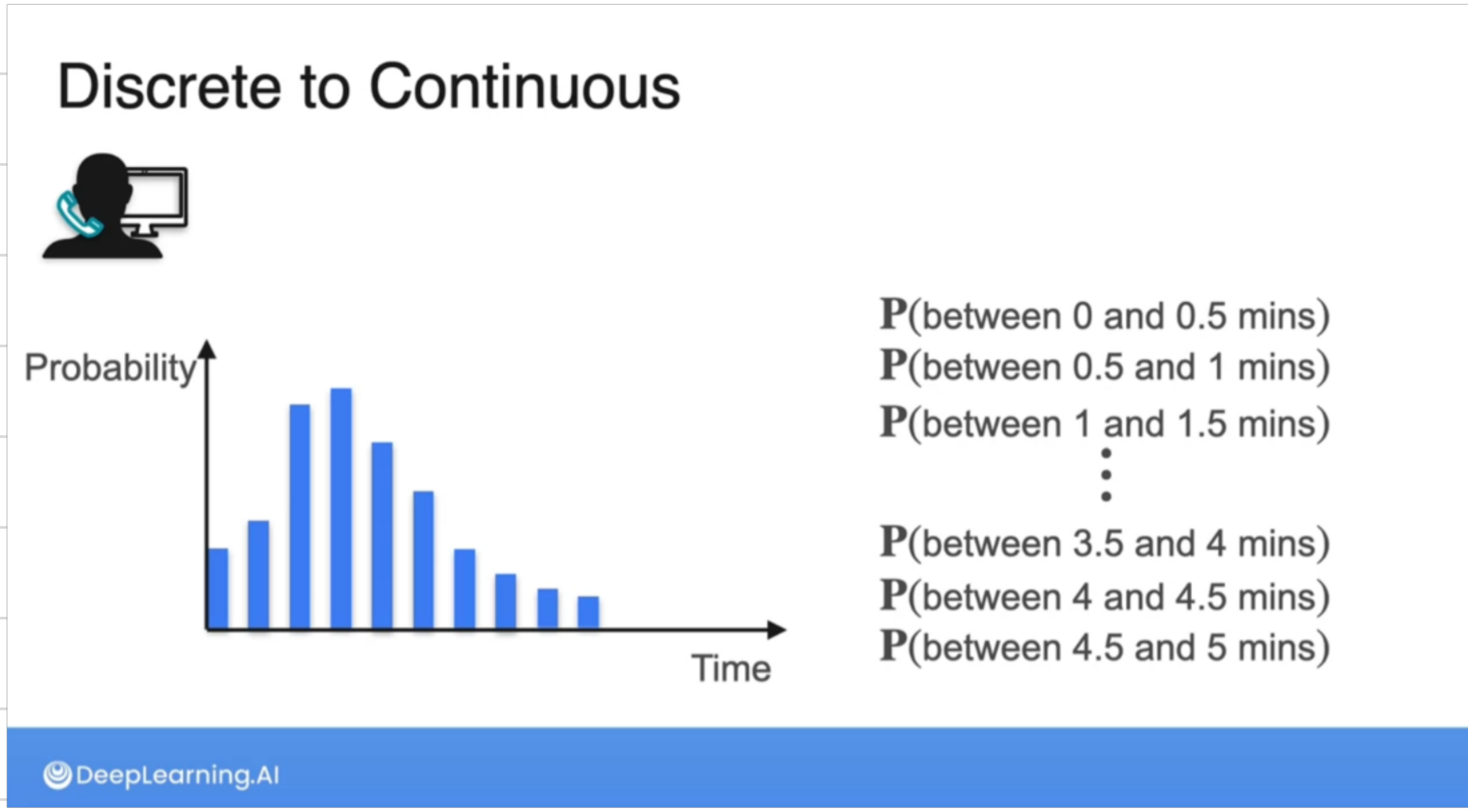
Discrete to Continuous
Probability
Time
P(between 0 and 0.5 mins)
P(between 0.5 and 1 mins)
P(between 1 and 1.5 mins)
⋮
P(between 3.5 and 4 mins)
P(between 4 and 4.5 mins)
P(between 4.5 and 5 mins)
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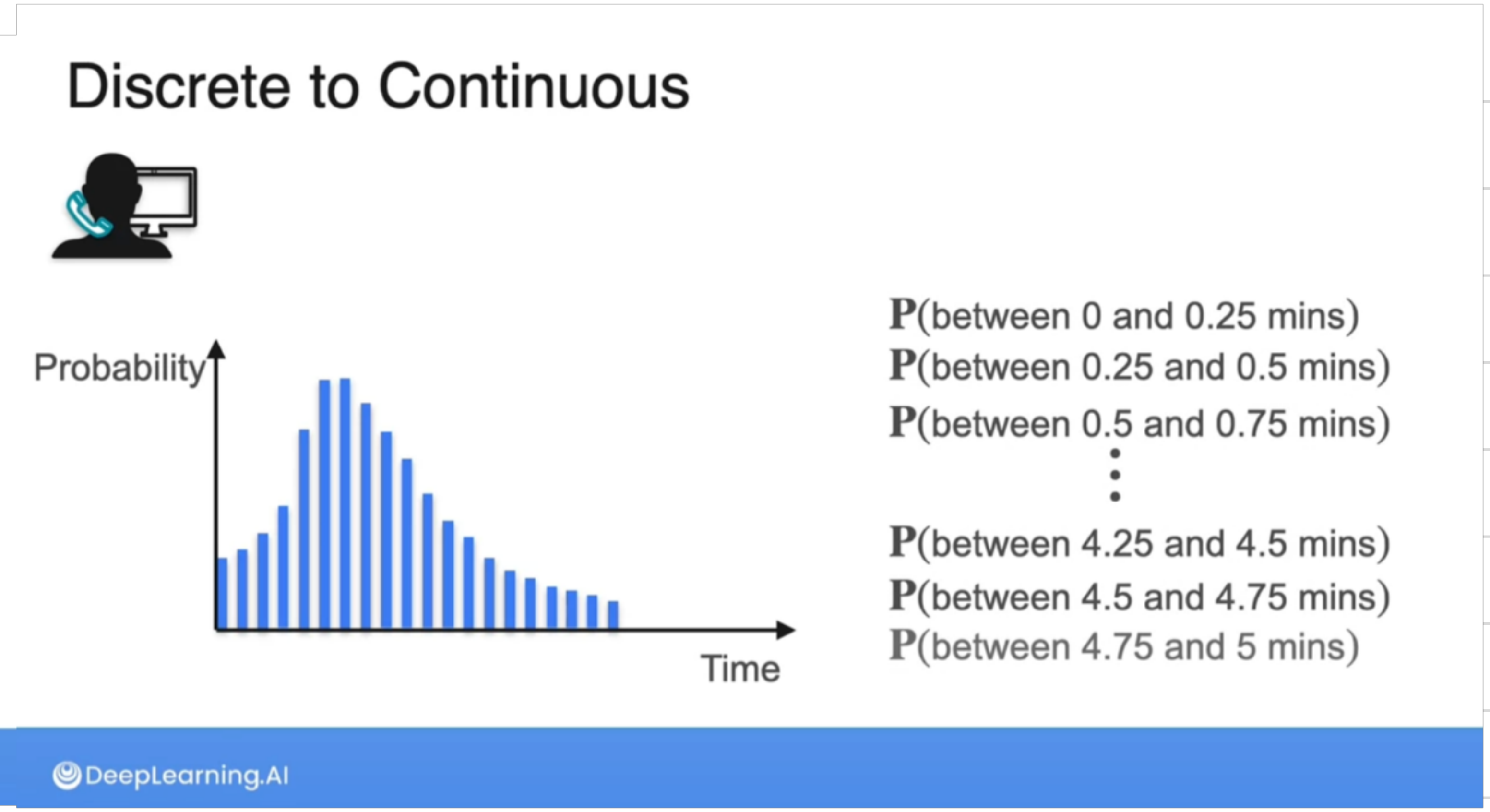
Discrete to Continuous
Probability
Time
P(between 0 and 0.25 mins)
P(between 0.25 and 0.5 mins)
P(between 0.5 and 0.75 mins)
⋮
P(between 4.25 and 4.5 mins)
P(between 4.5 and 4.75 mins)
P(between 4.75 and 5 mins)
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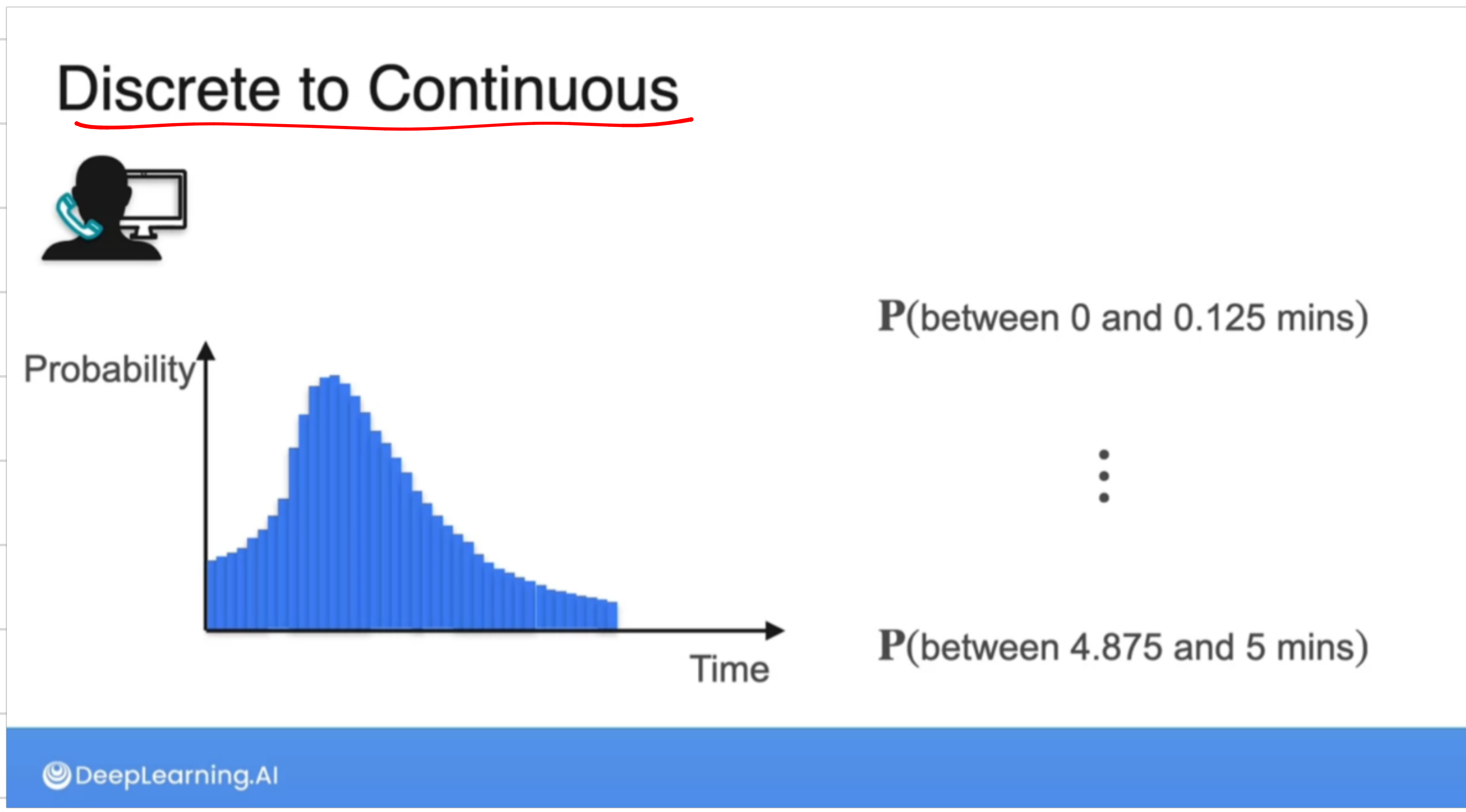
Discrete to Continuous
Probability
Time
P(between 0 and 0.125 mins)
⋮
P(between 4.875 and 5 mins)
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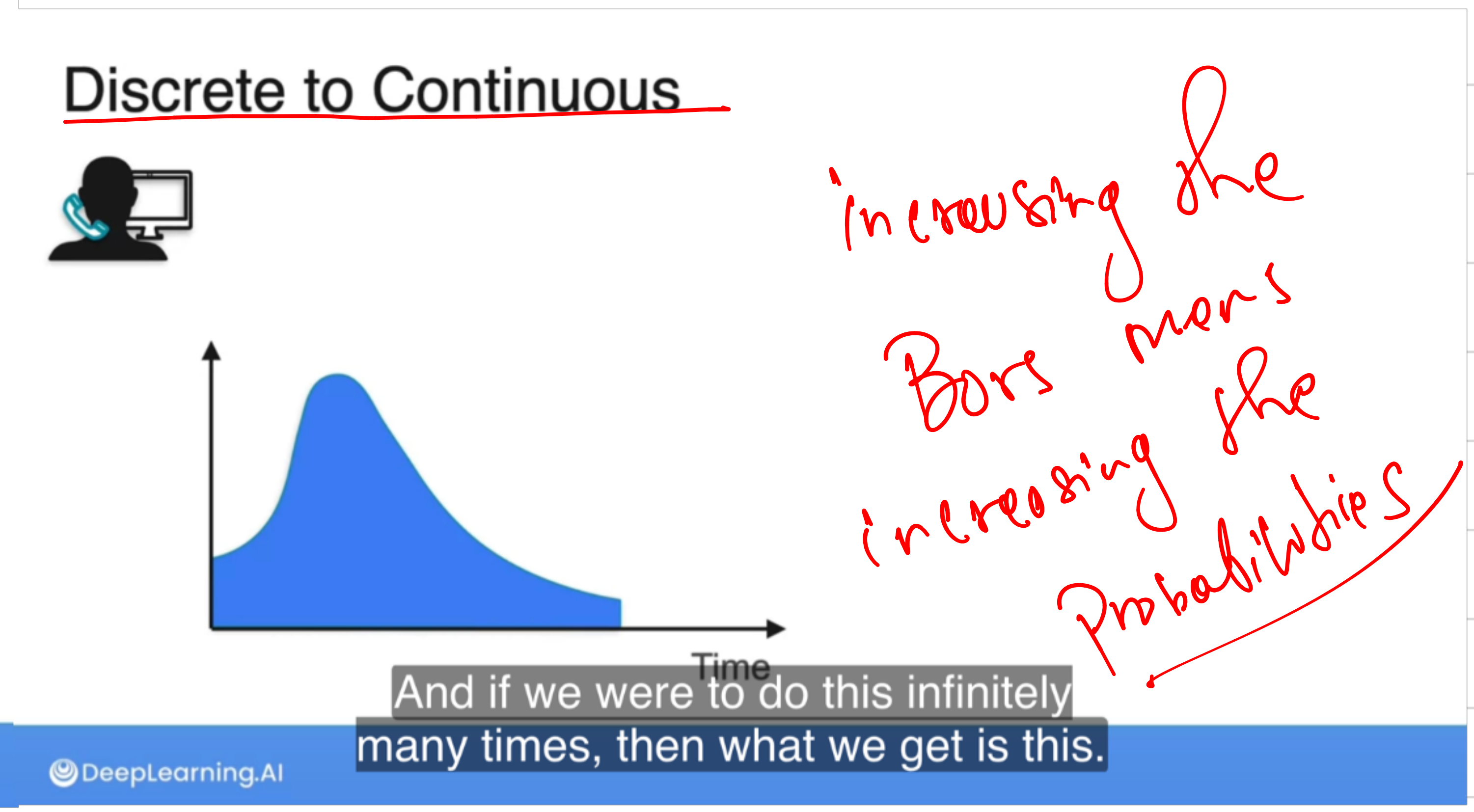
Discrete to Continuous
Time
increasing the Bars means increasing the Probabilities
And if we were to do this infinitely
many times, then what we get is this.
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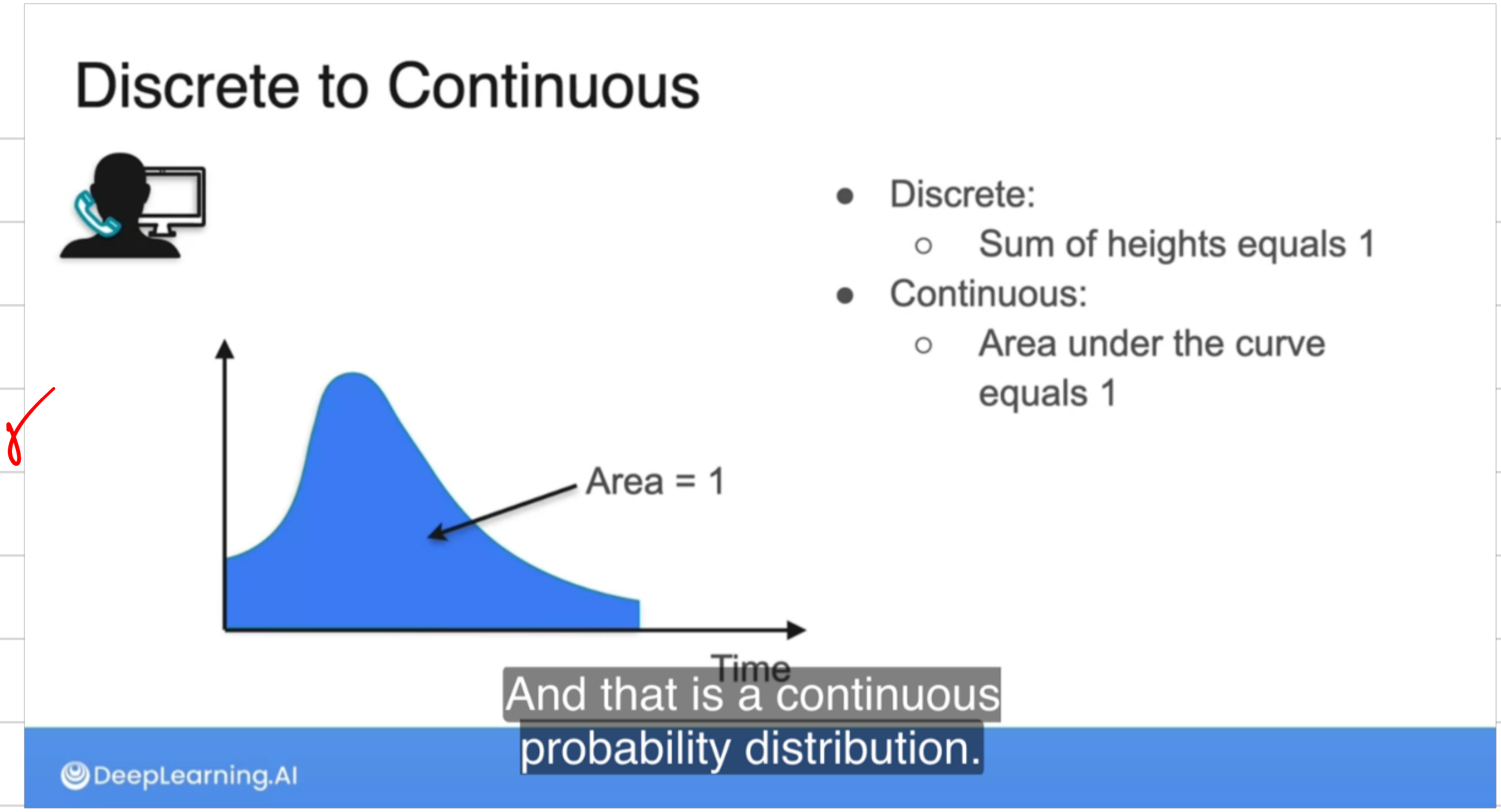
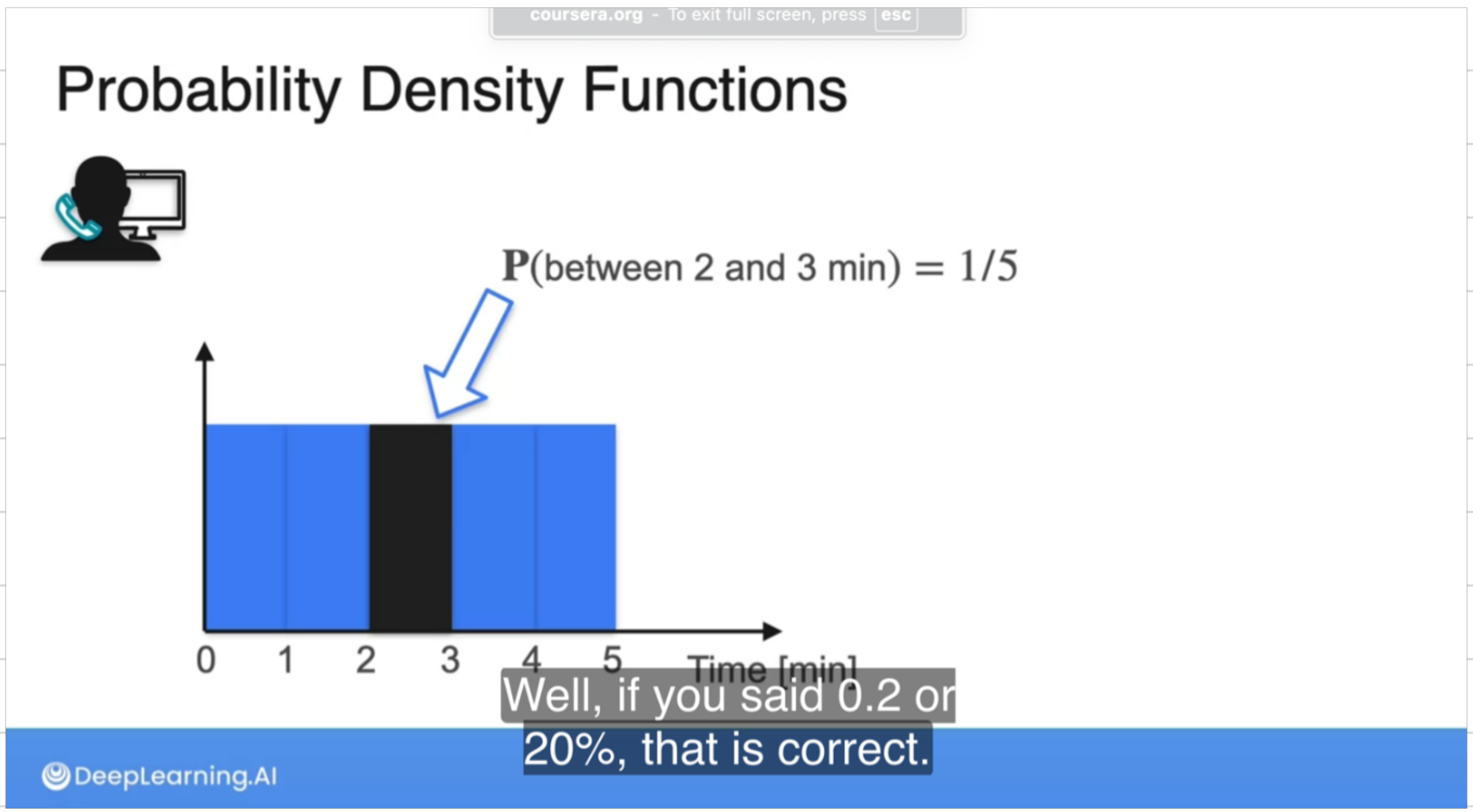
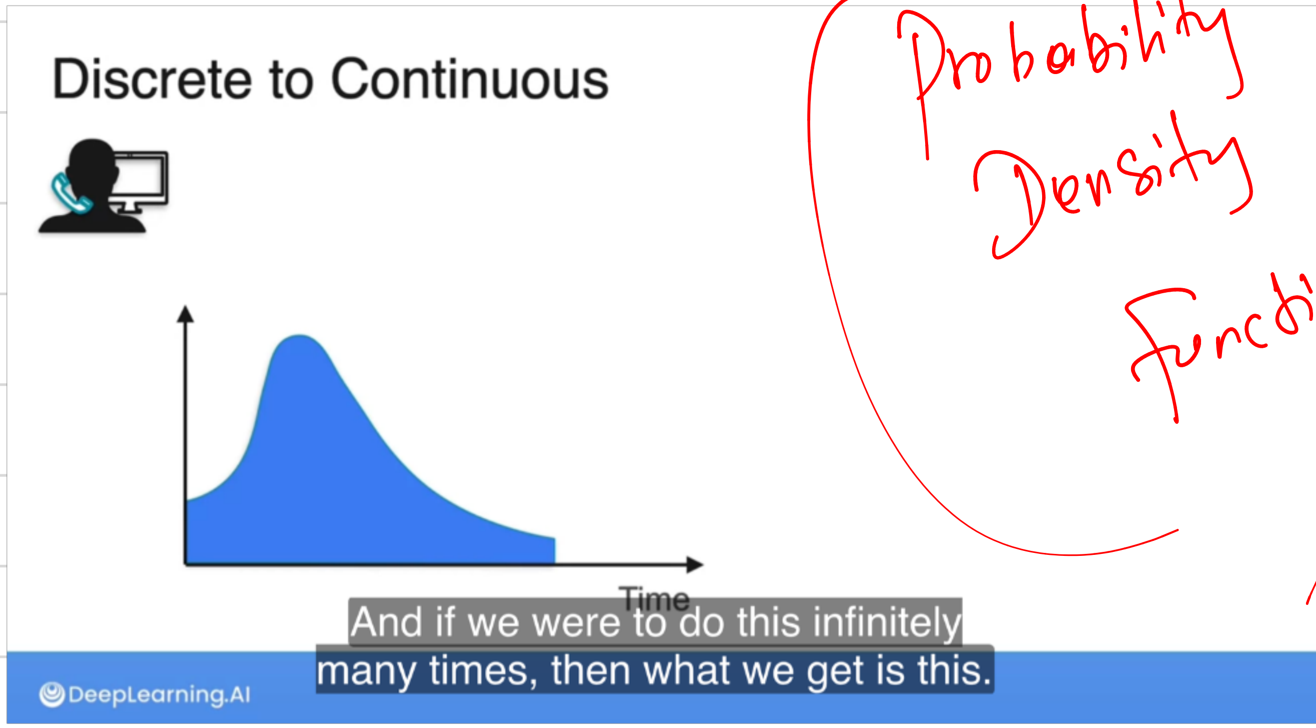
Both Discrete and Continuous have value under f. (Area = 1)



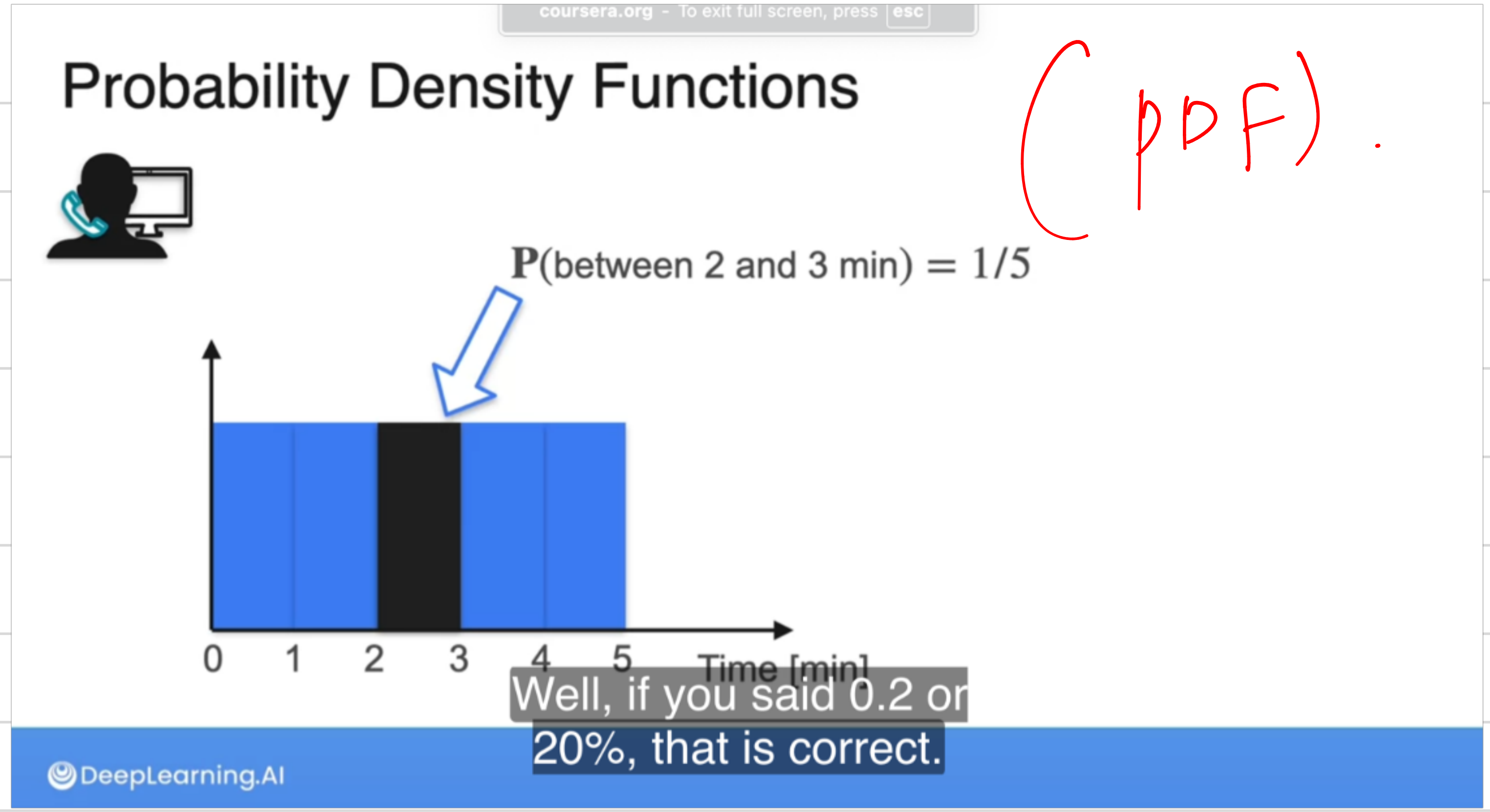


Probability (2 and 3 min) = 1/5

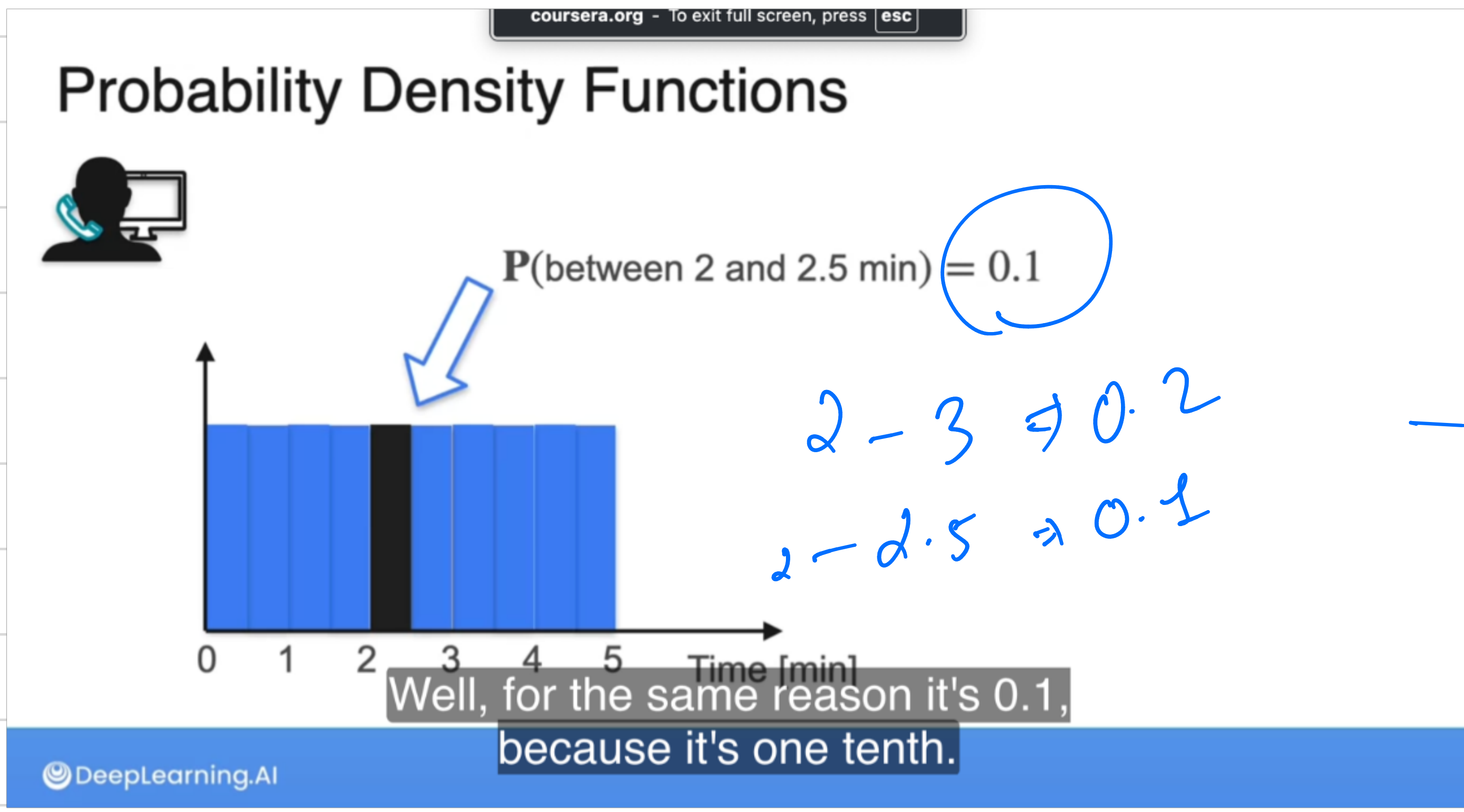
Since there are 5 events.



Probability Density Function



(PDF).

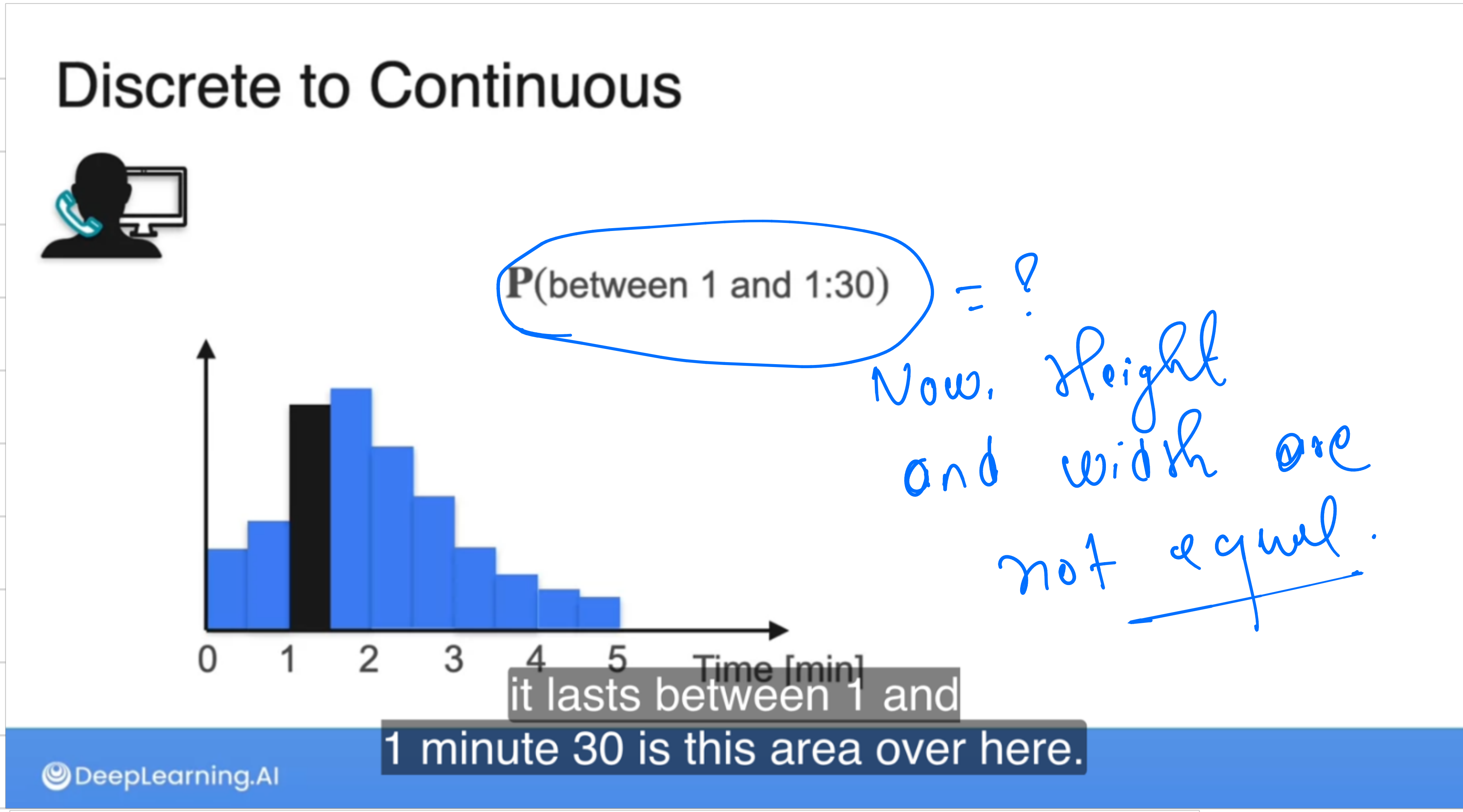


2 - 3 ⇒ 0.2

2 - 2.5 ⇒ 0.1

→ PDF ⇒ Since there are 5 and it is discrete event.

→, We're looking height, width we can say area [all equal].


Discrete to Continuous
P(between 1 and 1:30)
= ?
Now, Height and width are not equal.
0 1 2 3 4 5 Time [min]
it lasts between 1 and
1 minute 30 is this area over here.
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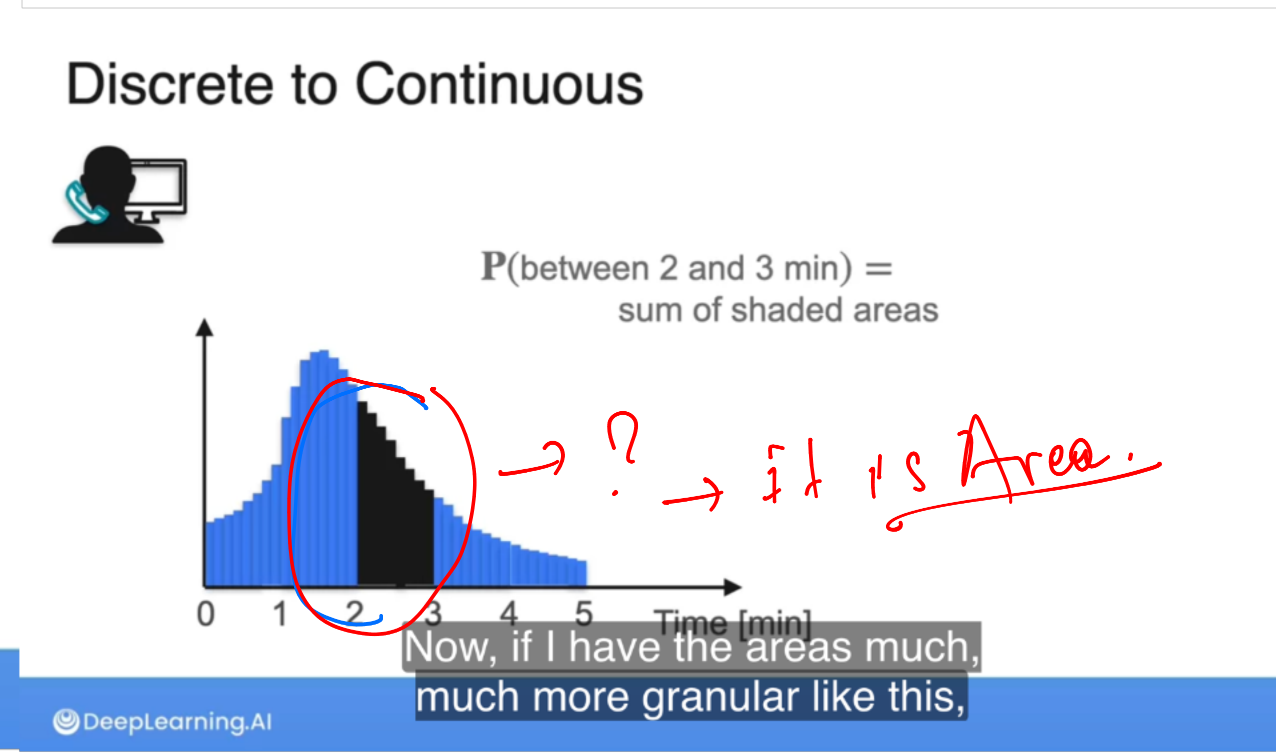
Discrete to Continuous
P(between 2 and 3 min) =
sum of shaded areas
→ ? → it's Area.
0 1 2 3 4 5 Time [min]
Now, if I have the areas much,
much more granular like this,
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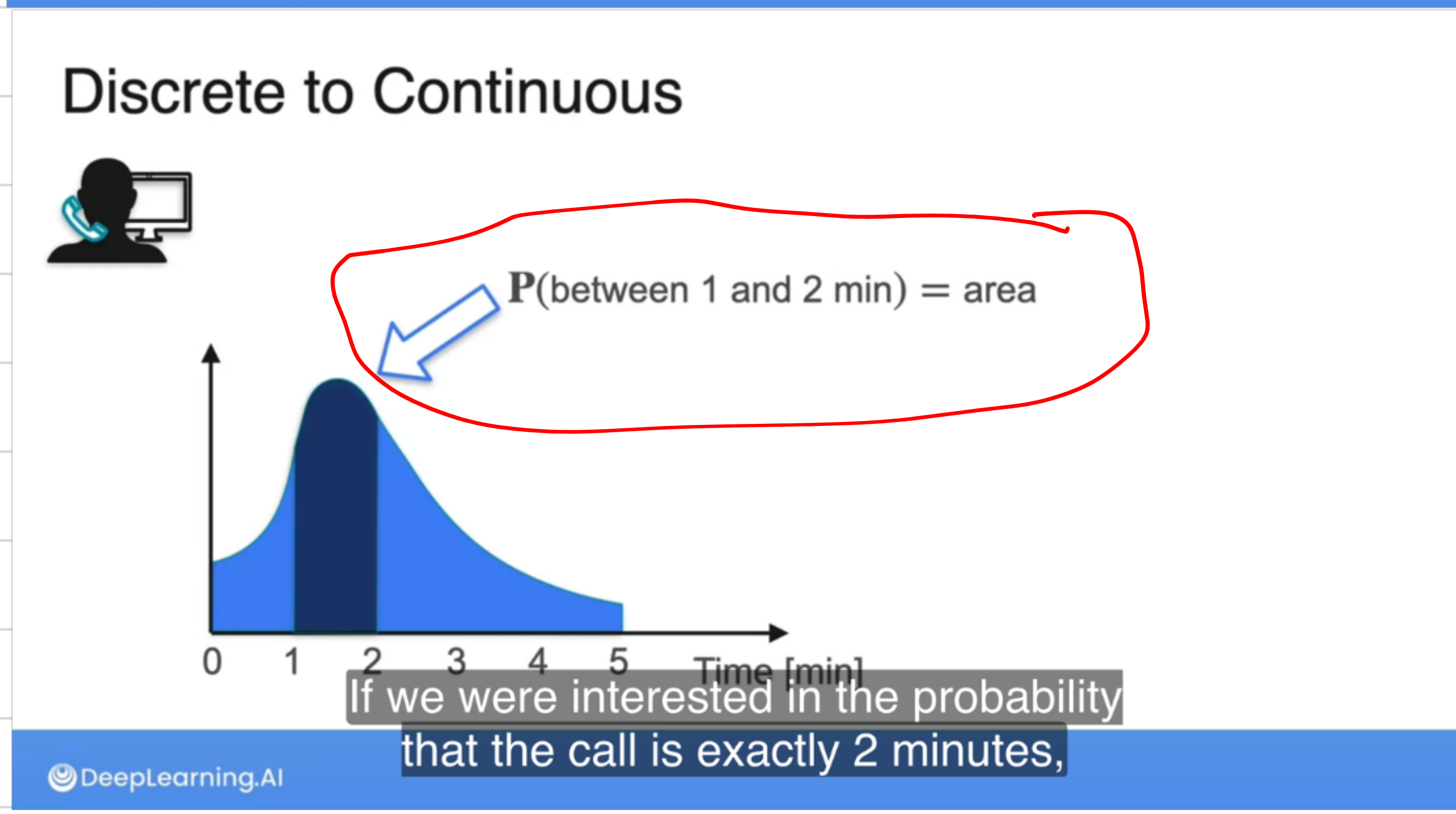
Discrete to Continuous
P(between 1 and 2 min) = area
0 1 2 3 4 5 Time [min]
If we were interested in the probability
that the call is exactly 2 minutes,
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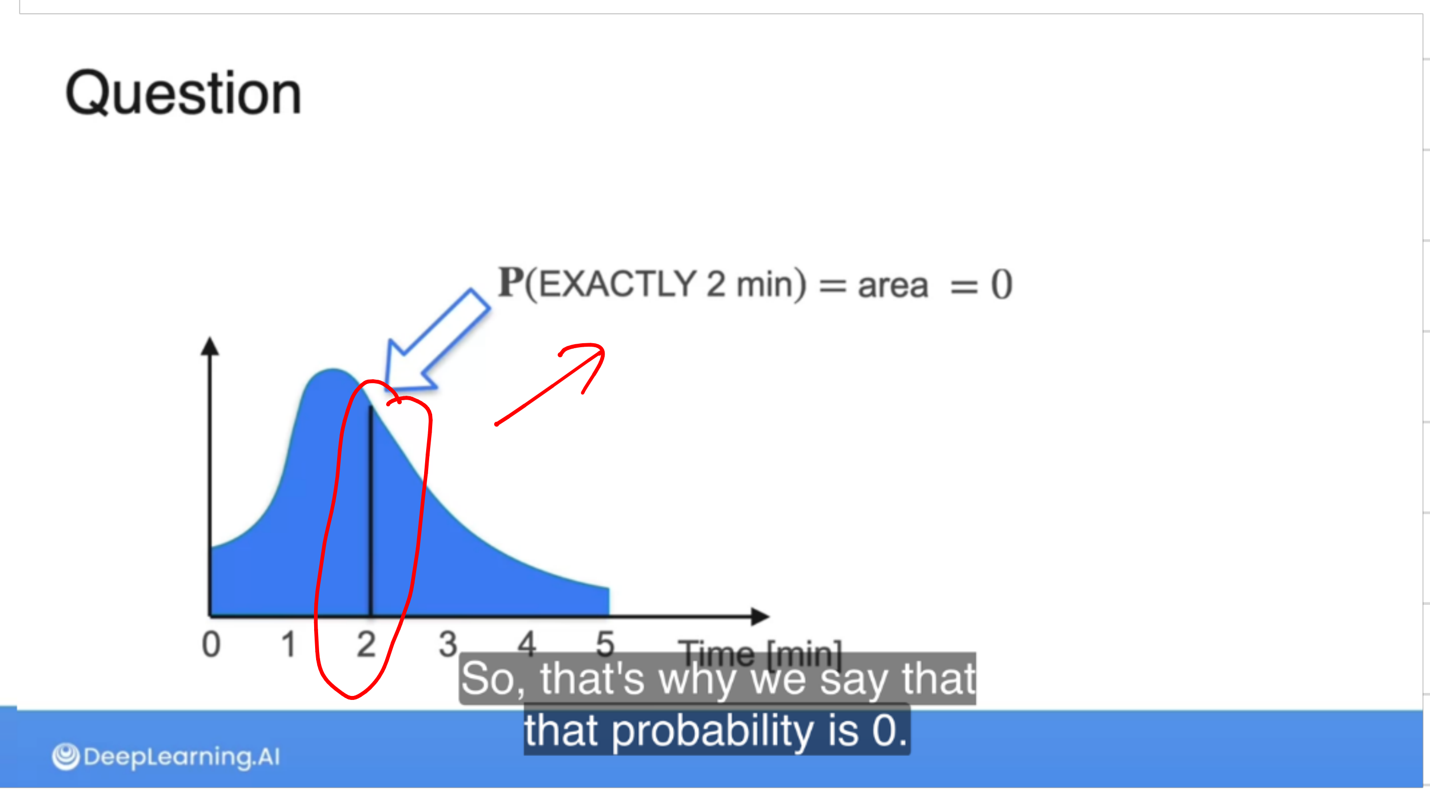
Question
P(EXACTLY 2 min) = area = 0
0 1 2 3 4 5 Time [min]
So, that's why we say that
that probability is 0.
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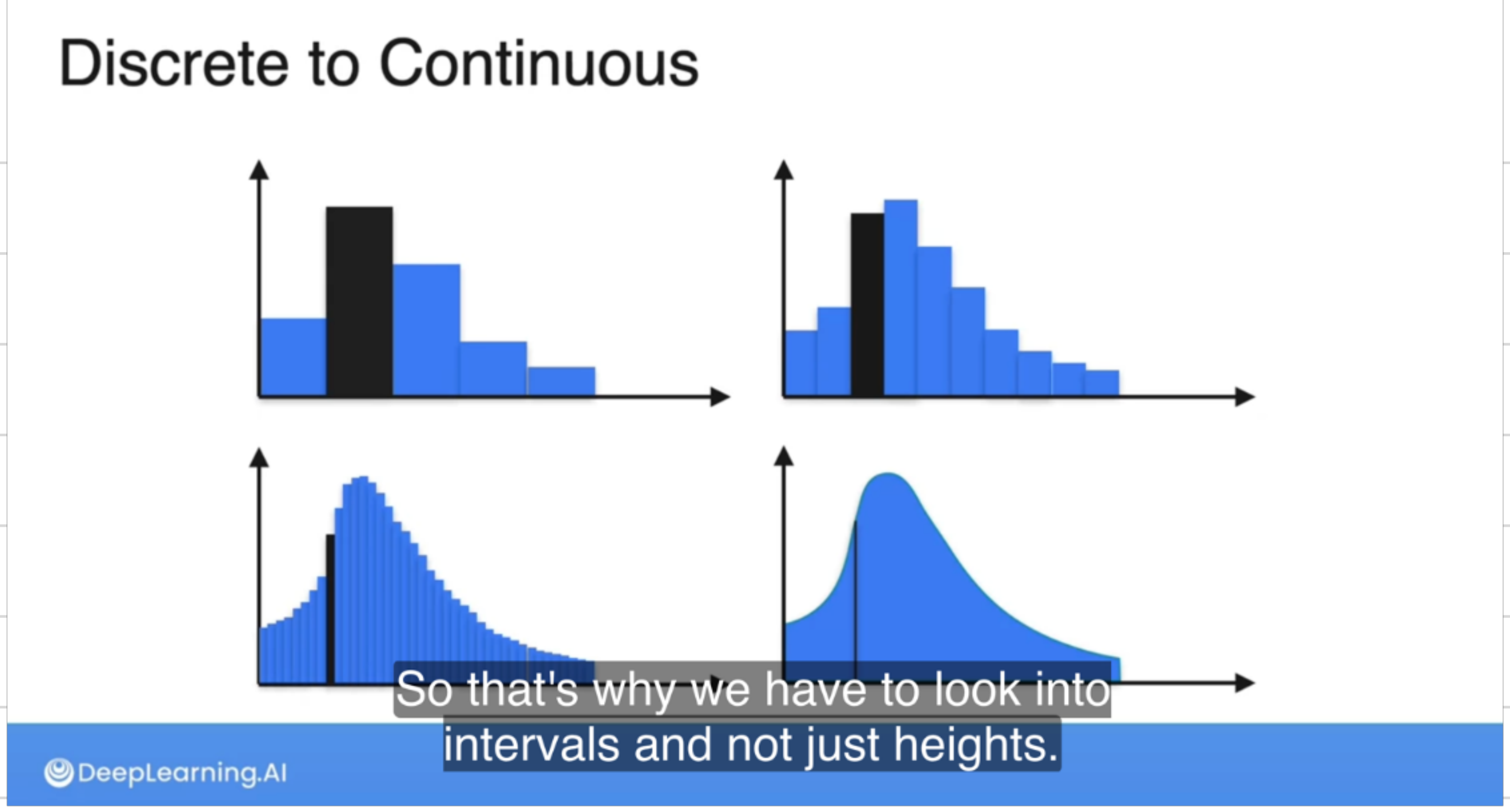
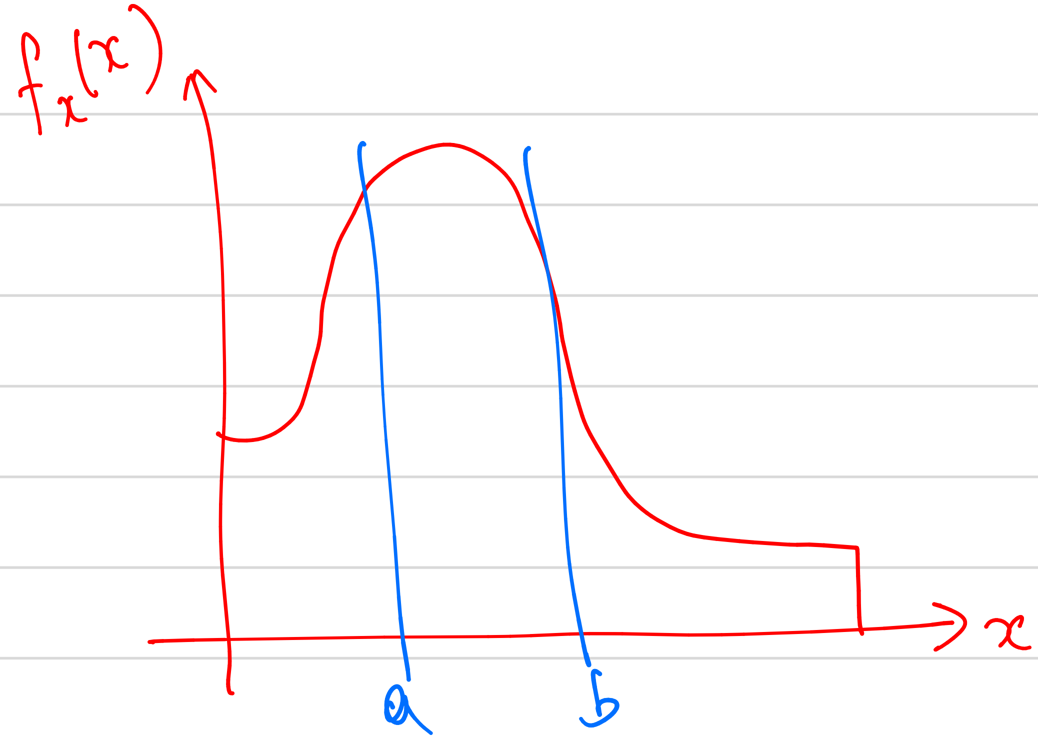




Probability Density Function (PDF)

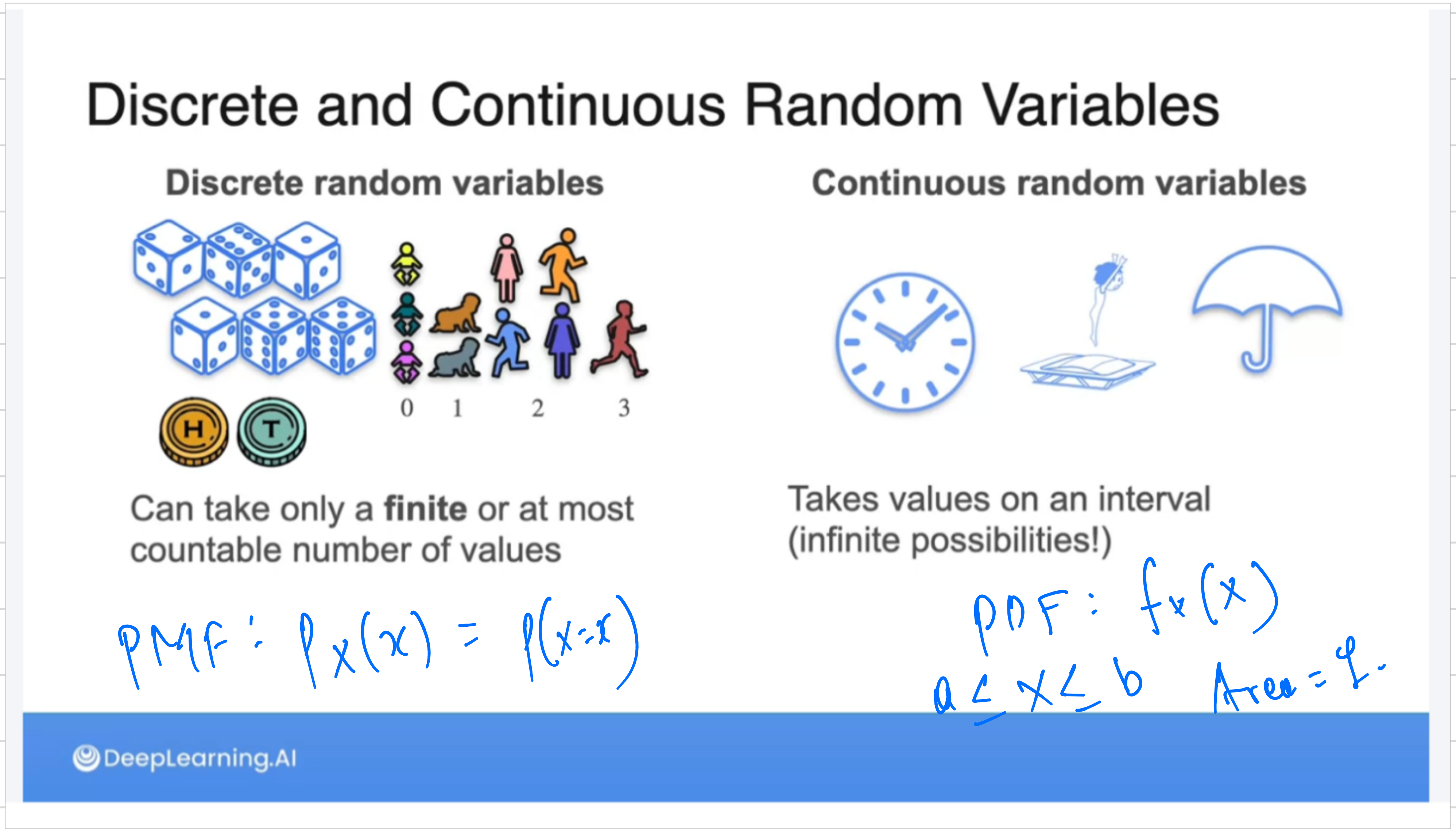
$f_X(x)$

Tells you the rate you accumulate probability around each point. Only defined for continuous variables!.

$P(a < X < b)$ = area under $f_X(x)$

$f_X(x)$ needs to satisfy:

- It is defined for all numbers
- $f_X(x) \geq 0$
- $f_X(x) = 1$



PMF: $p_X(x) = P(X=x)$

PDF: $f_X(x)$

$a \le X \le b$ Area = 1.

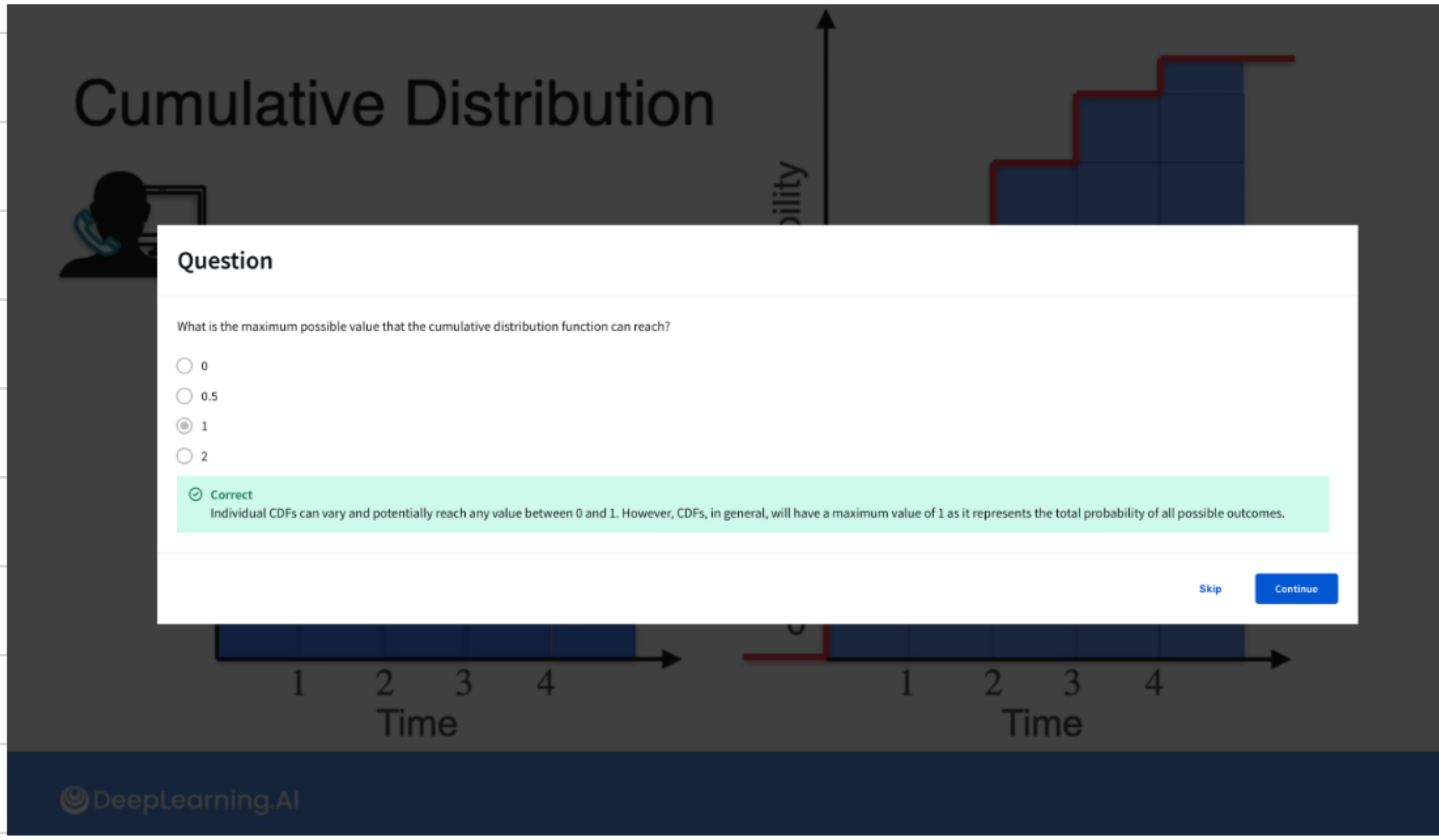
RV ⇒ Random Variables.

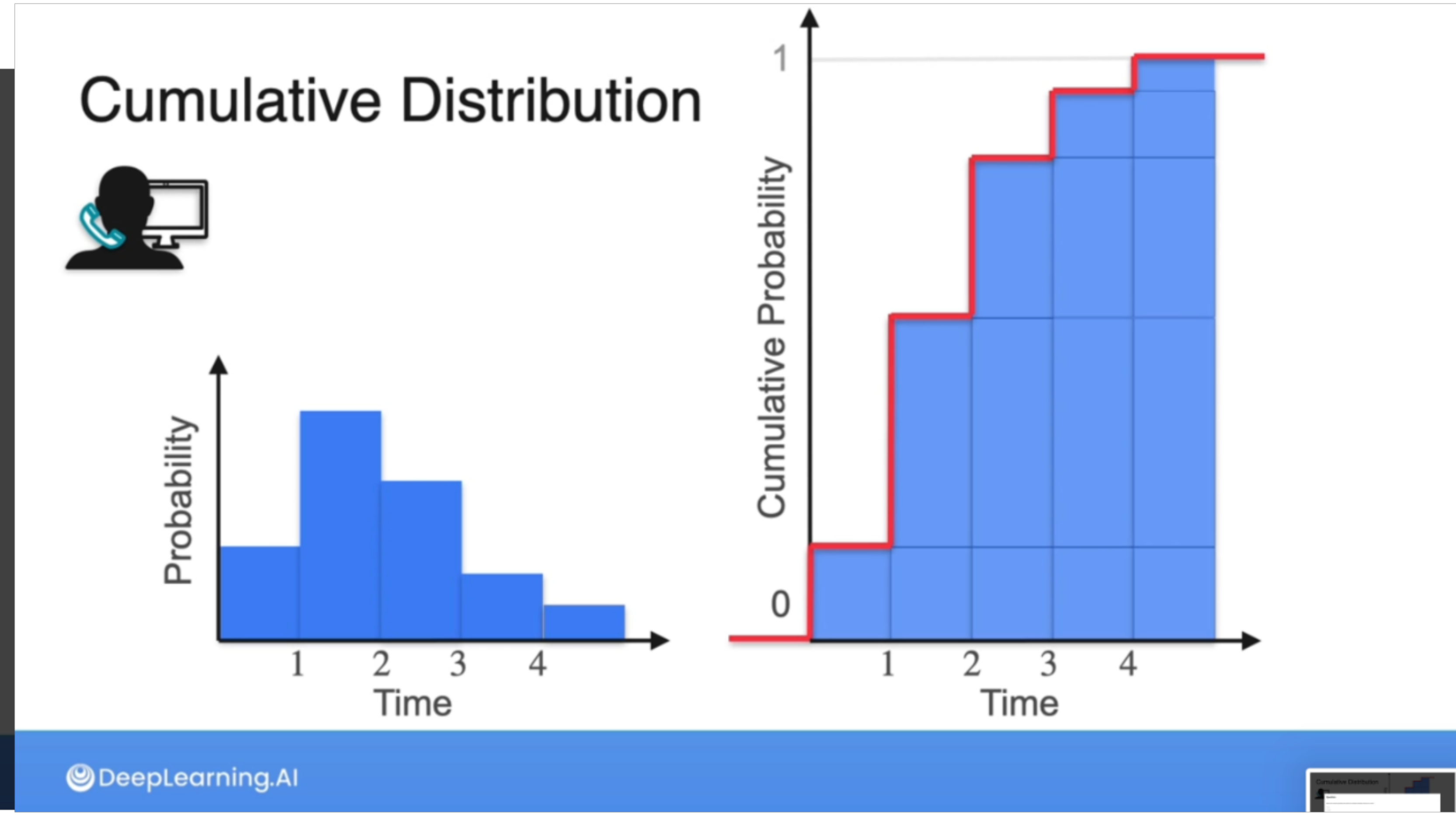
PMF ⇒ Discrete RV.

PDF ⇒ Continuous Random Variables.

Cummulative Distribution function:

CDF is the one that shows the actual probabilities that the event is between 0 and certain number of event.





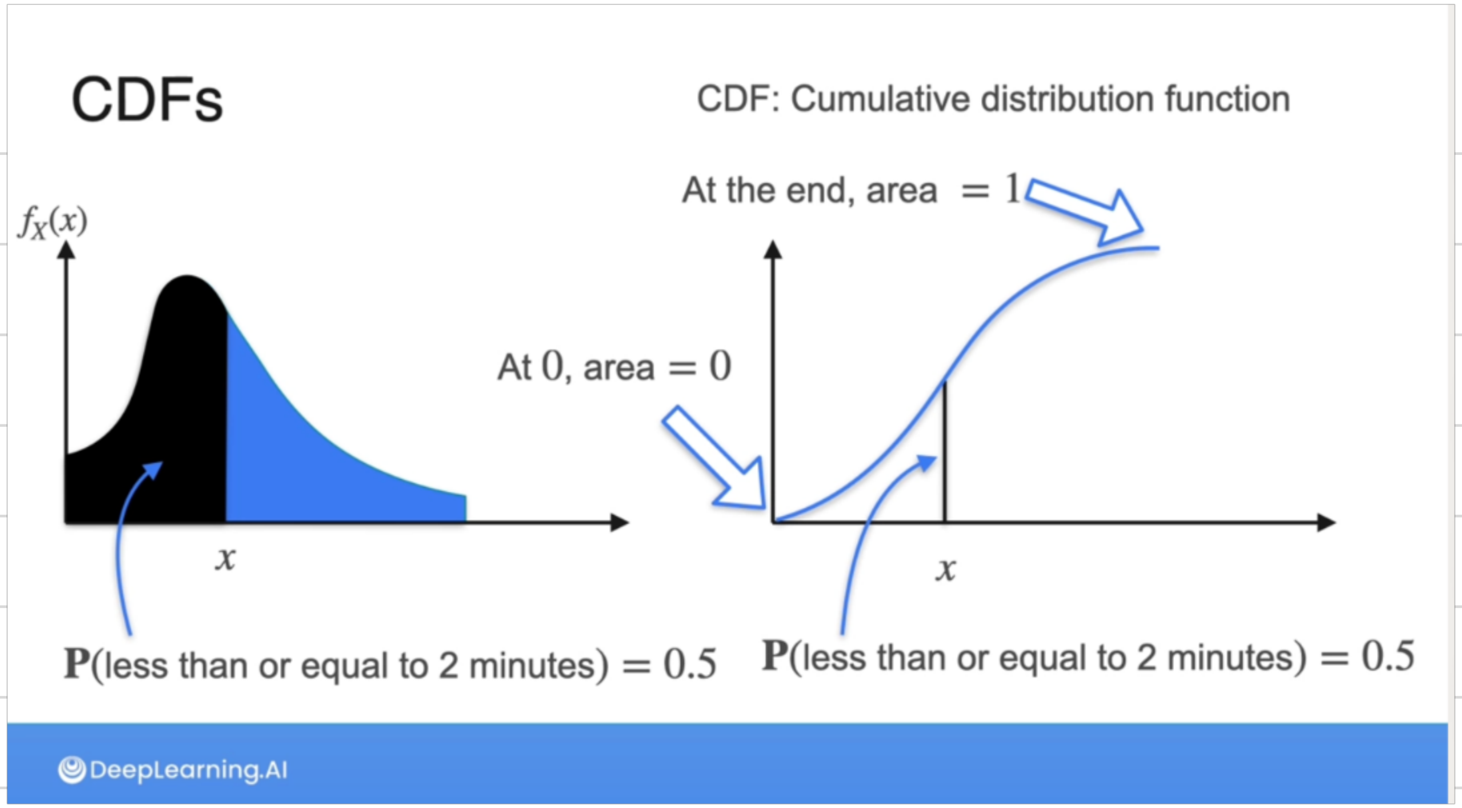
So, the p between 0 - 1 or the 1-2, and 2-3 and so on untill the Probability is 1. which means we start from 0 to 1.

← At the end, area = 1

P(less than or equal to 2 min) = 0.5

At 0, area = 0

P(less than or equal to 2 min) = 0.5



The CDF shows how much probability the variable has accumulated until a certain value

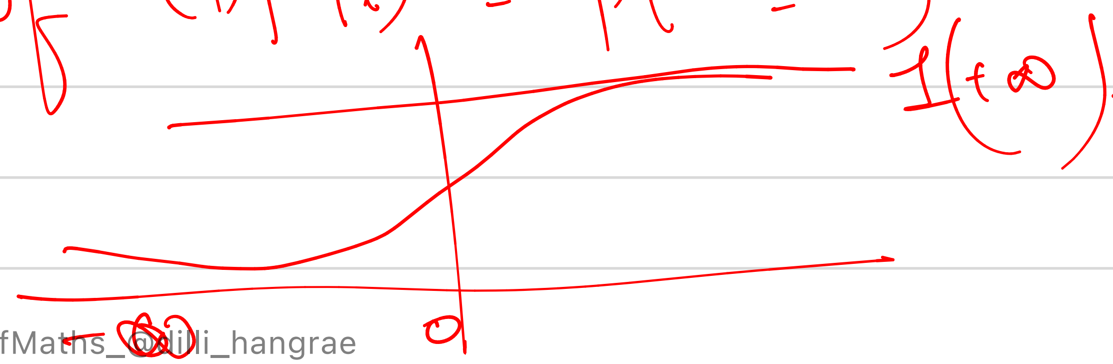
$CDF(x) = P(X \le x) \leftarrow$ It is defined for every real number

$[-\infty \text{ to } +\infty]$

$F_X(x) = P(X \le x)$

F sub (capital Y). $F_Y(x)$

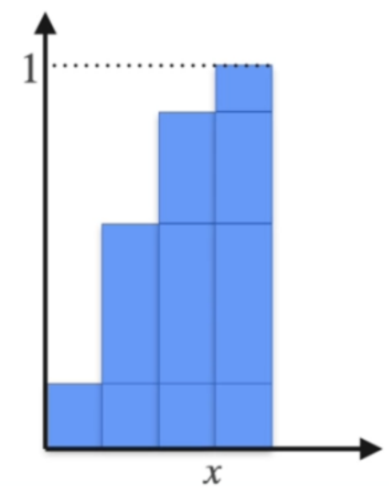
We get continuous function on curve of $CDF(x) = P(X \le x)$

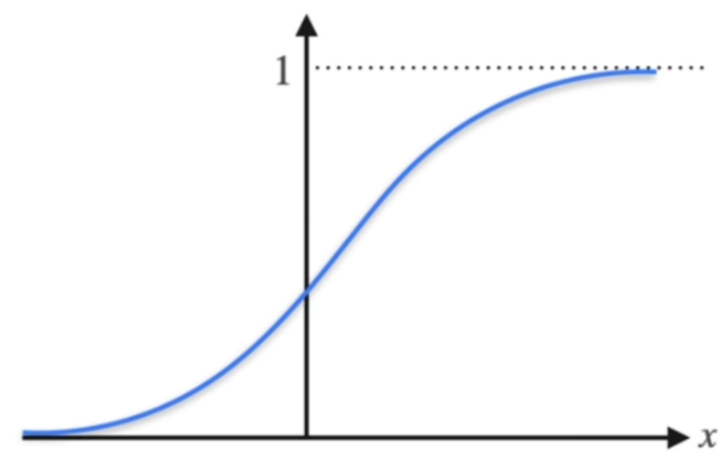


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that $F_X(x)$ $\text{CDF}(x) = \mathbf{P}(X \le x) \longleftarrow$ It is defined for every real number





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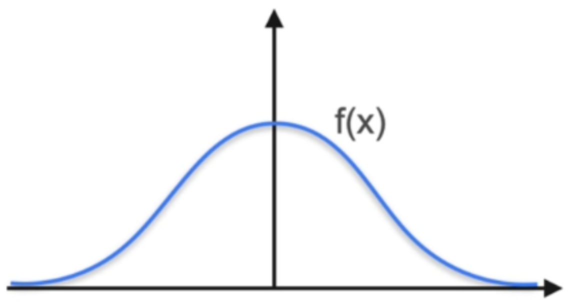
$0 \leq F_X(x) \leq 1$ —

[Left endpoint is 0
Right Endpoint is 1]

Never Decreases

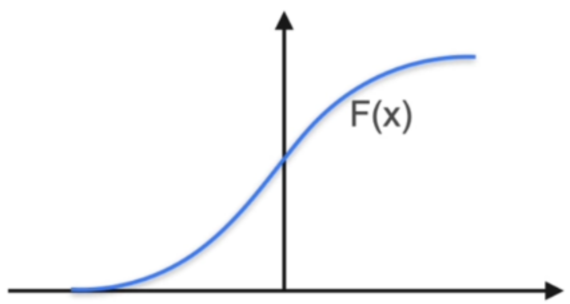
PDF and CDF Summary

PDF



- area = 1
- Always positive

CDF



- left "endpoint" is 0
- right "endpoint" is 1
- (endpoints can be at infinity)
- Always positive and increasing

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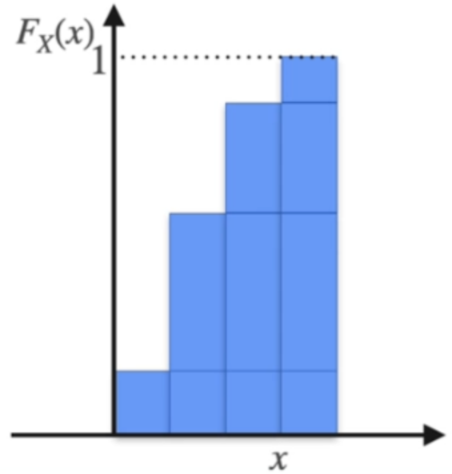
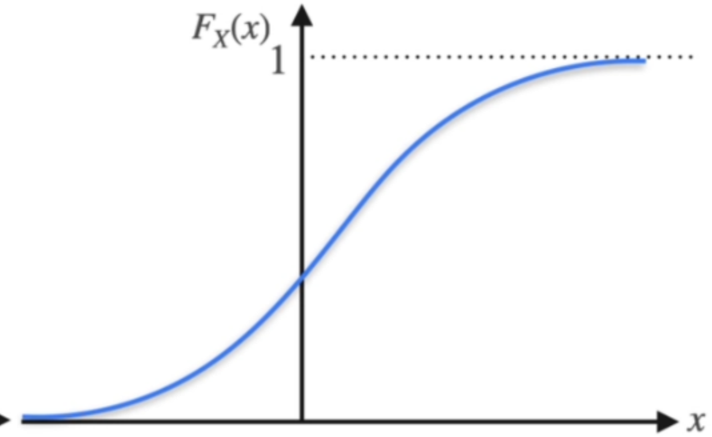
Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that $F_X(x) = \mathbf{P}(X \leq x)$ ← It is defined for every real number





Properties

- $0 \leq F_X(x) \leq 1$
- Left "endpoint" is 0
- Right "endpoint" is 1
- **Never decreases**

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Seems PDF follows the gausian Distribution Whereas CDF is like Sigmoid function S Curve.

PDF => Area under the Curve or the Density for Continuous Random Variable Probabilities.

Uniform Distribution

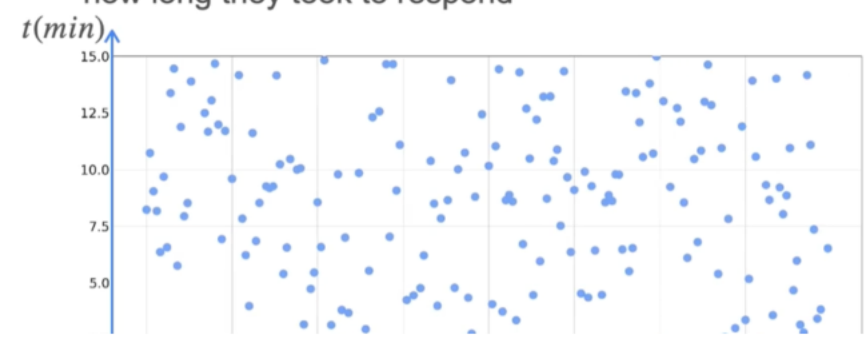
Graph looks like Homogenous in the Interval from 0-10 minutes. Sometimes I take 1 min, 5, 3, 2, 7, 9, 10 min as waiting time for bus arrival.

Uniform Distribution: Motivation

You're calling a tech support line.
They can answer any time between zero and 15 minutes and if they don't answer in this time, the line is disconnected.

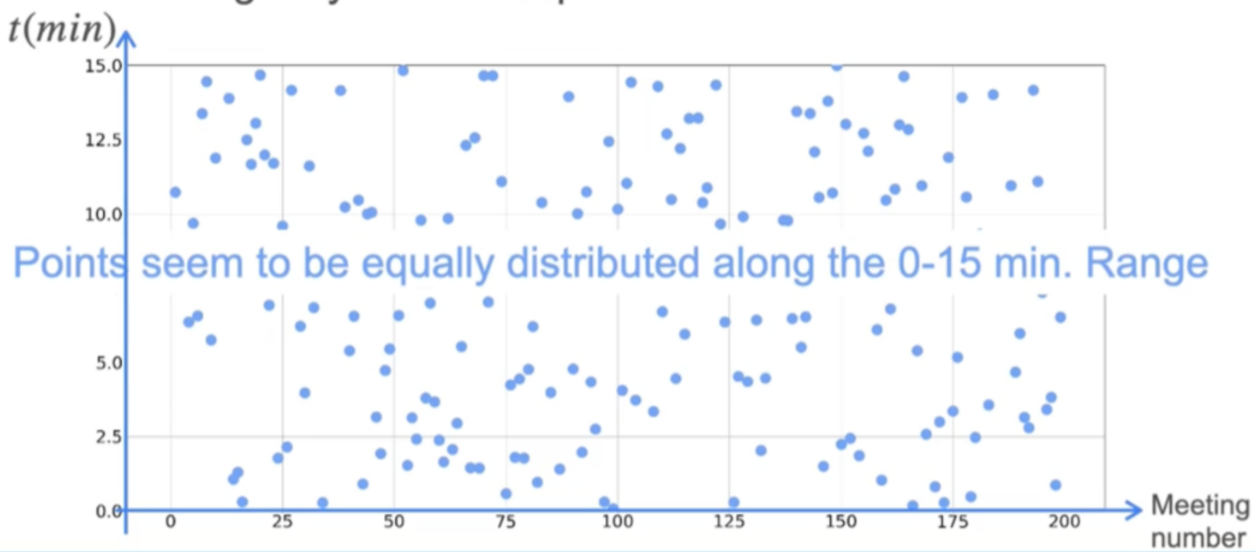
Uniform Distribution: Motivation

Last 200 times you called them, you took down notes of how long they took to respond

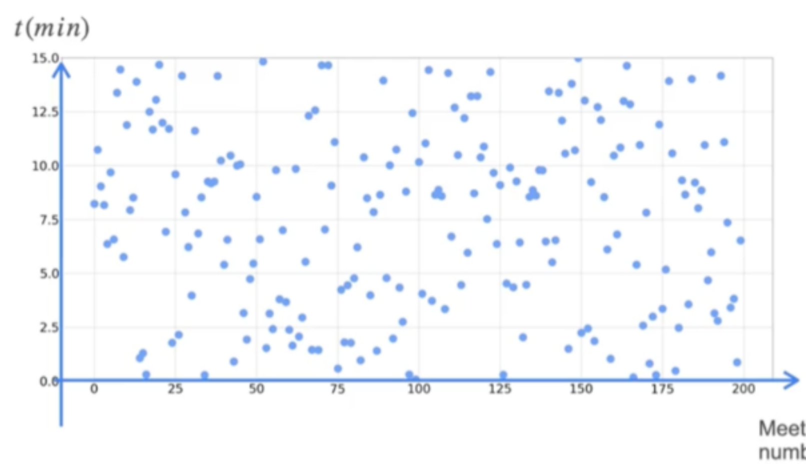


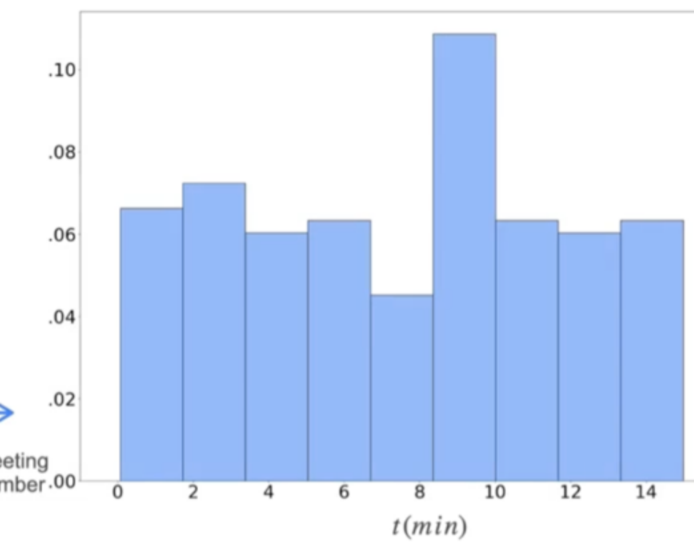
Uniform Distribution: Motivation

Last 200 times you called them, you took down notes of how long they took to respond



Uniform Distribution: Motivation





Uniform Distribution: Motivation

T: time (in minutes) you have to wait

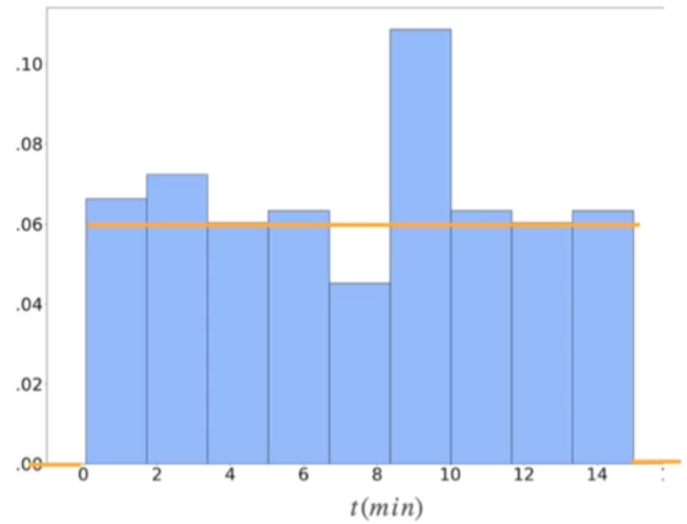
Any value between 0 and 15 minutes must have the same frequency of occurrence.

↓

The pdf must be constant for all values in the interval (0,15)

↓

Which constant? $\longrightarrow 15 \times h = 1 \longrightarrow h = \frac{1}{15} = 0.06$



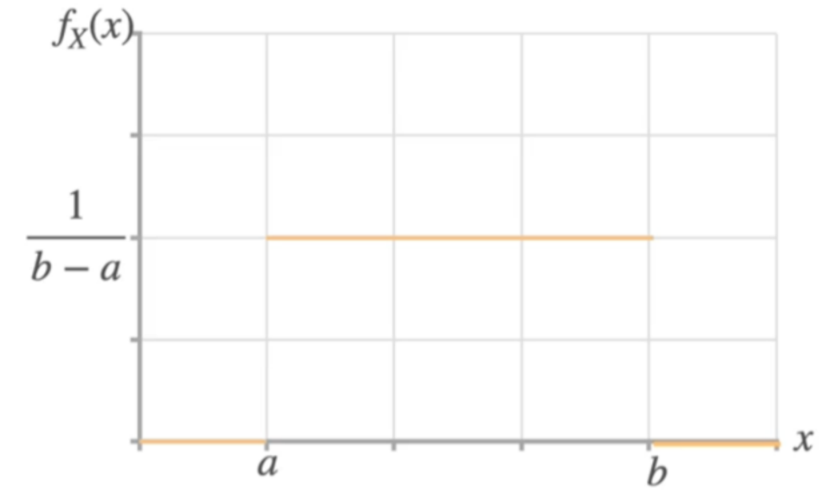
Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

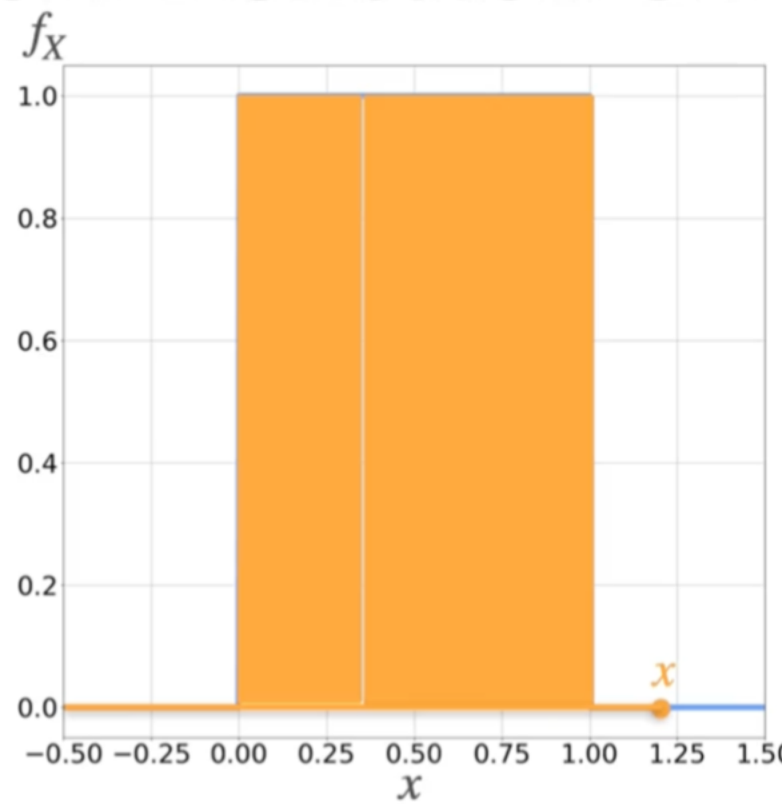
- a: beginning of the interval
- b: end of the interval

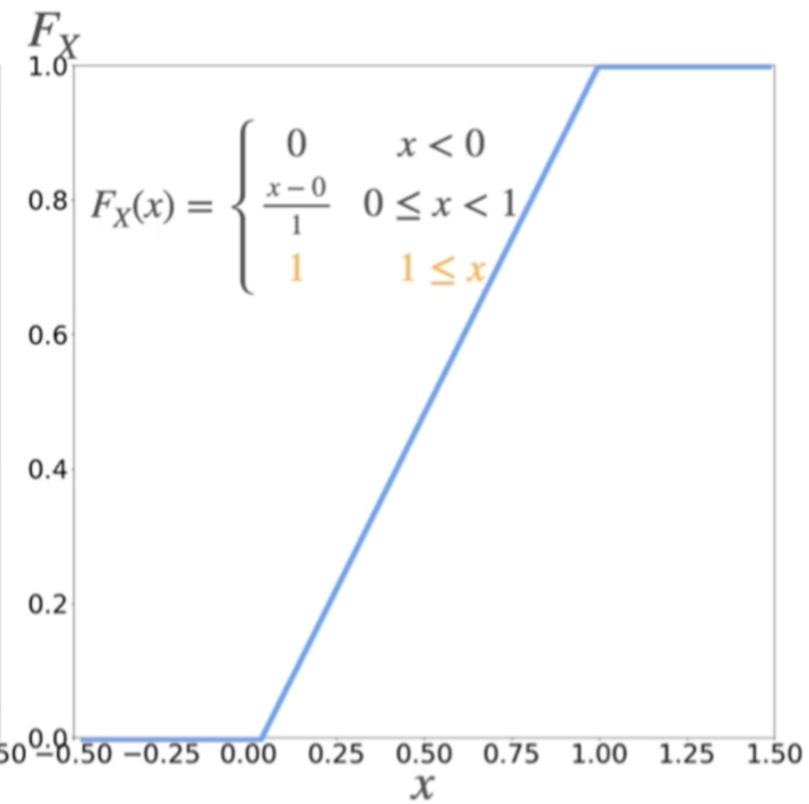
$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & x \notin (a,b) \end{cases}$$



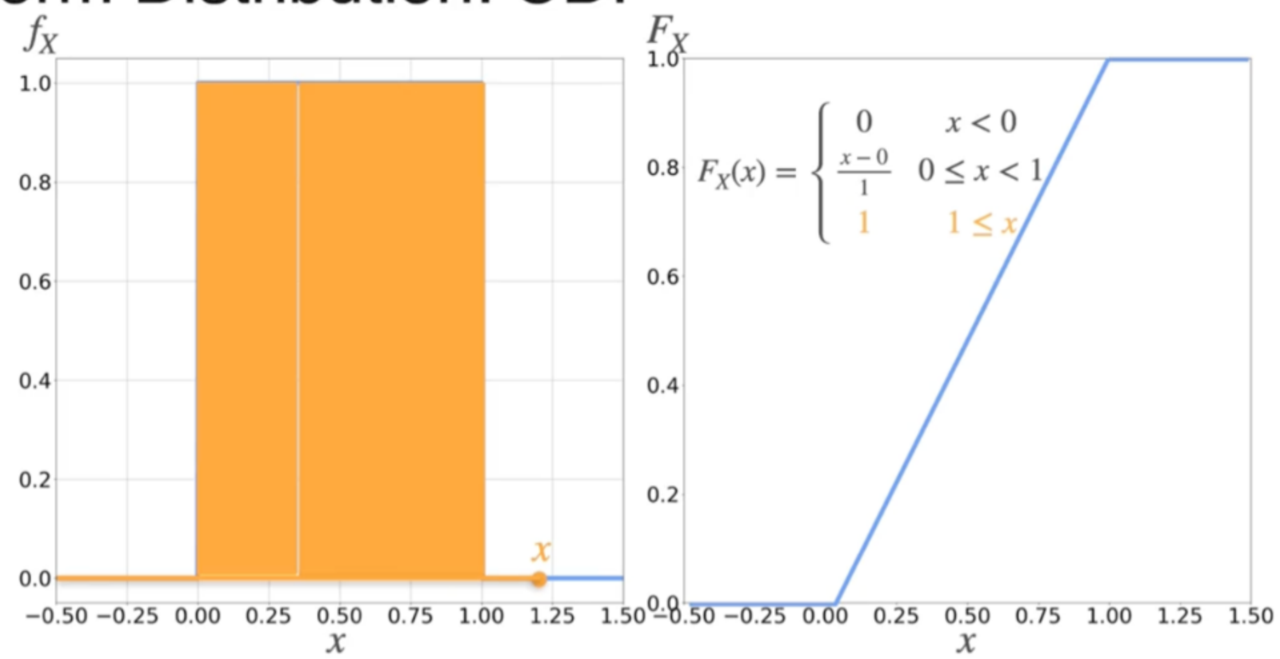
$$F_X(x) = \begin{cases} 0 \; ; & x < 0 \\ \frac{x-0}{1} & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

Uniform Distribution: CDF





Uniform Distribution: CDF



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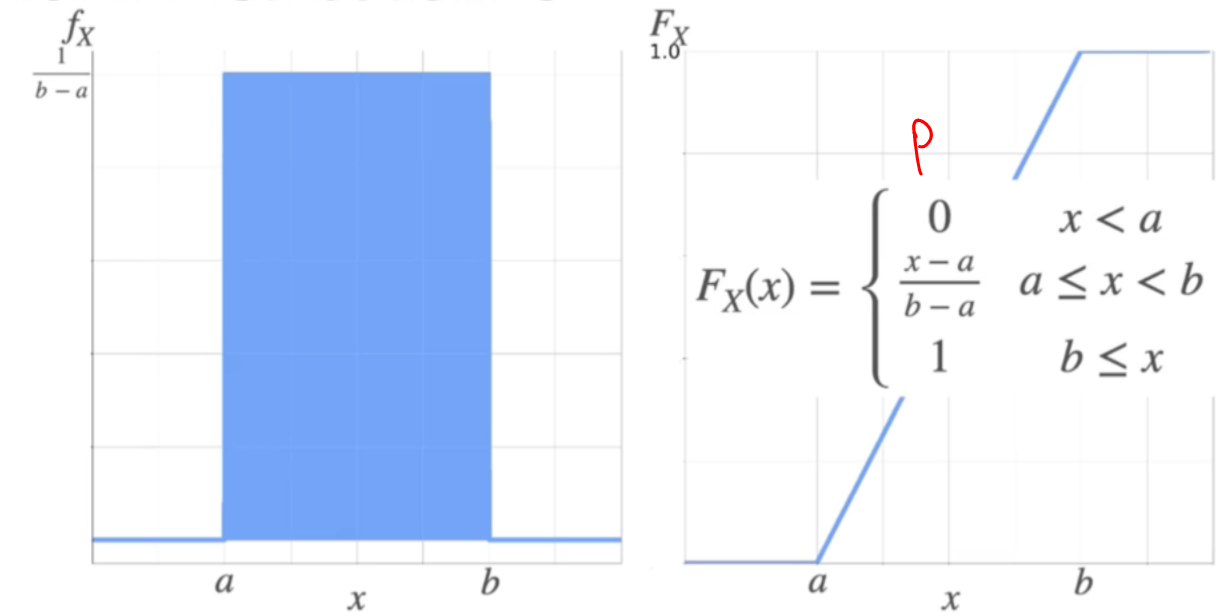
Uniform Distribution: CDF



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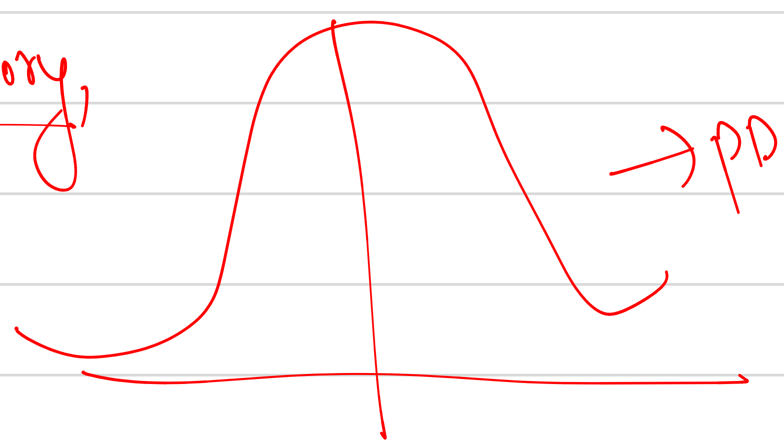
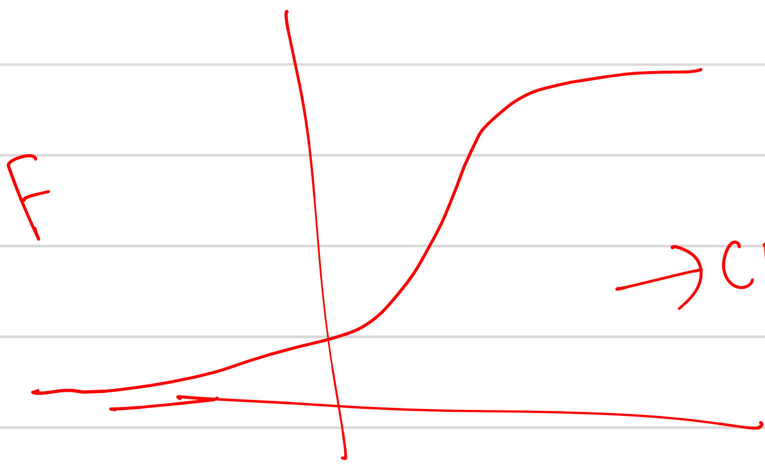
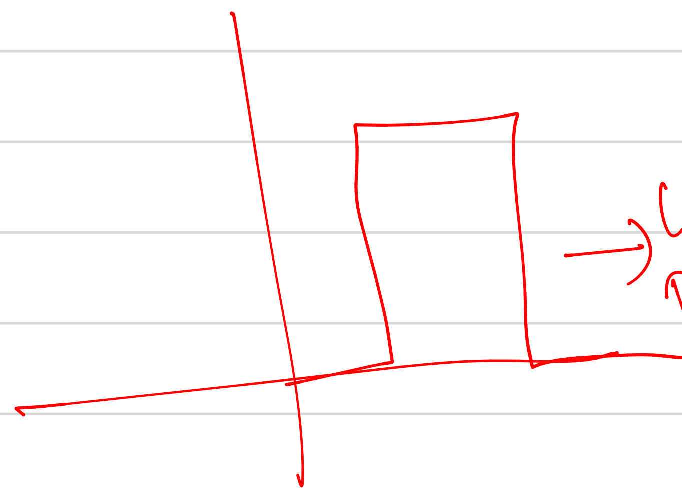
So In Summary,



→ PDF



→ CDF



→ Uniform Distribution

→ Area represents P Under the Continuous.

P starts from 0 to 1.

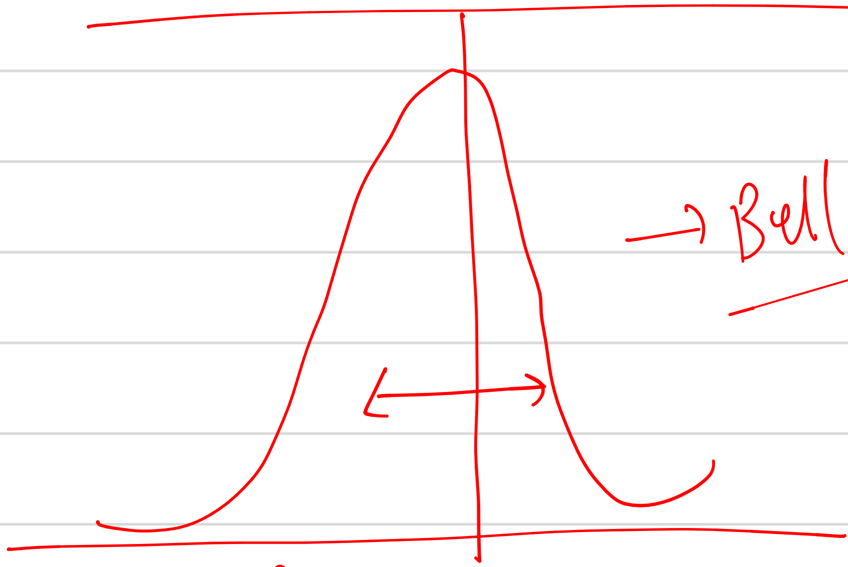
Eg: bus waiting time

→ PMF for Discrete.

Normal or Gaussian Distribution:

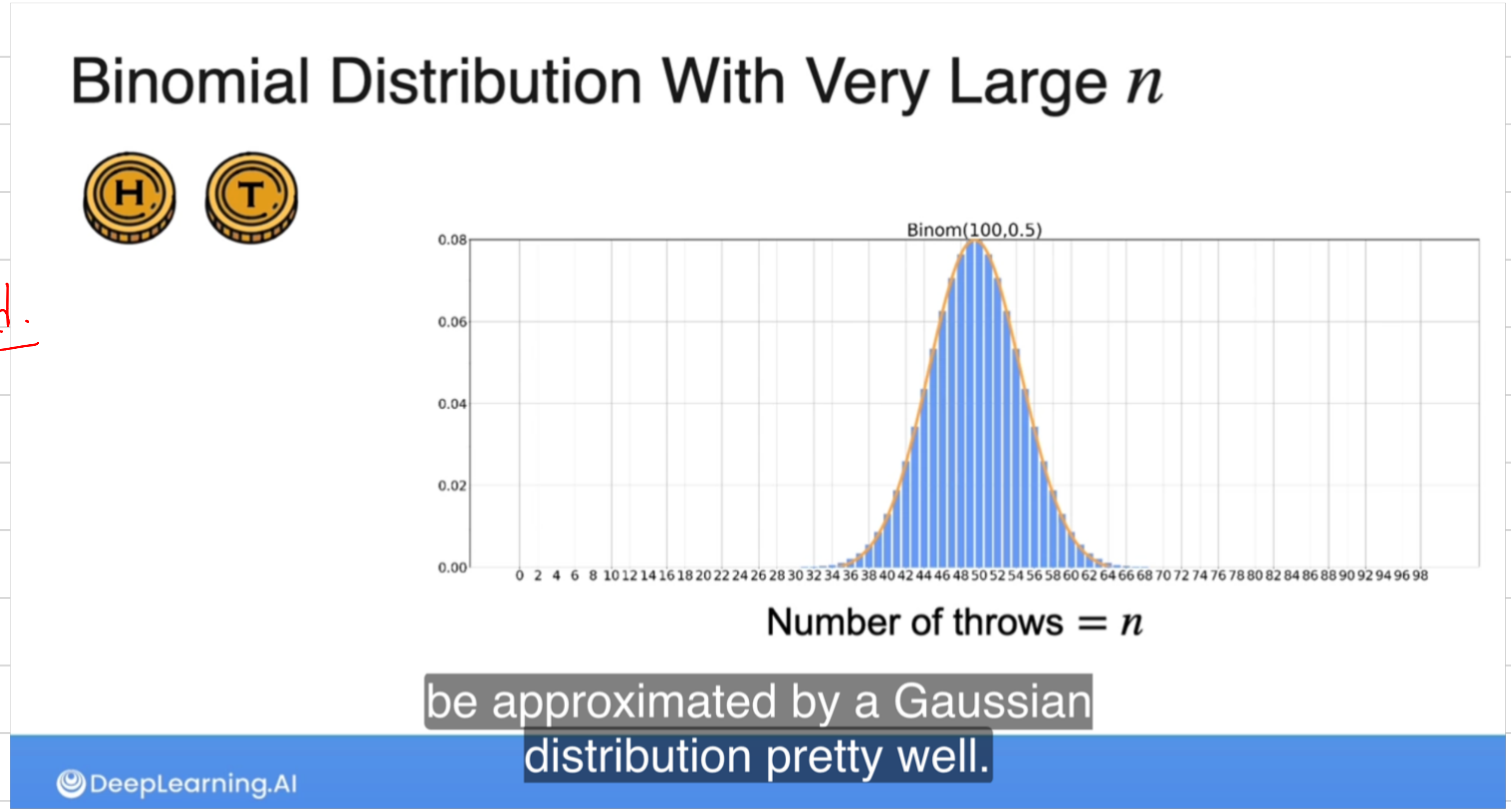
Let's take, $e^{-\frac{x^2}{2}}$

which can be bell-shaped.



→ Bell Curve.

$\sigma = 1$



$e^{-\frac{1}{2}\left(\frac{x-2}{3}\right)^2}$

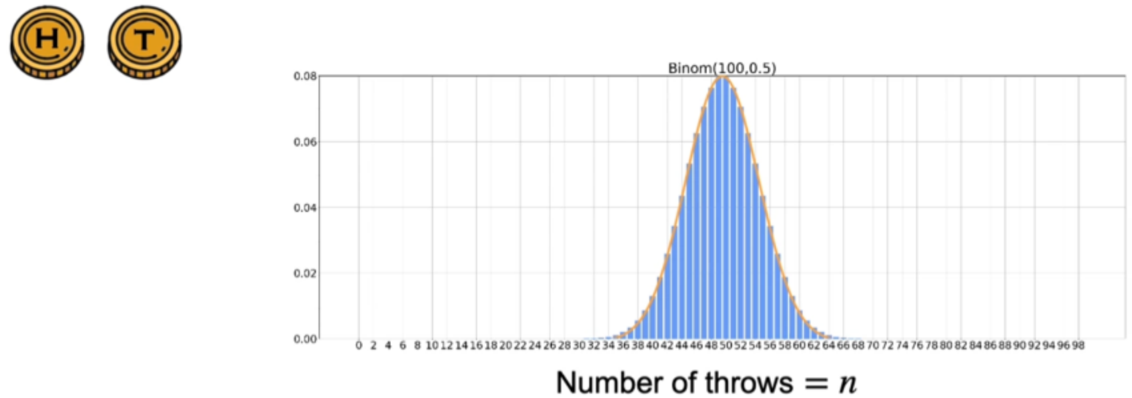
(σ tell how curve is thick.)

We have two parameters Mean = μ and Std. deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$\sigma \Rightarrow$ Flatness or wideness
whereas μ is the place where dist is centered.

Binomial Distribution With Very Large n



be approximated by a Gaussian distribution pretty well.

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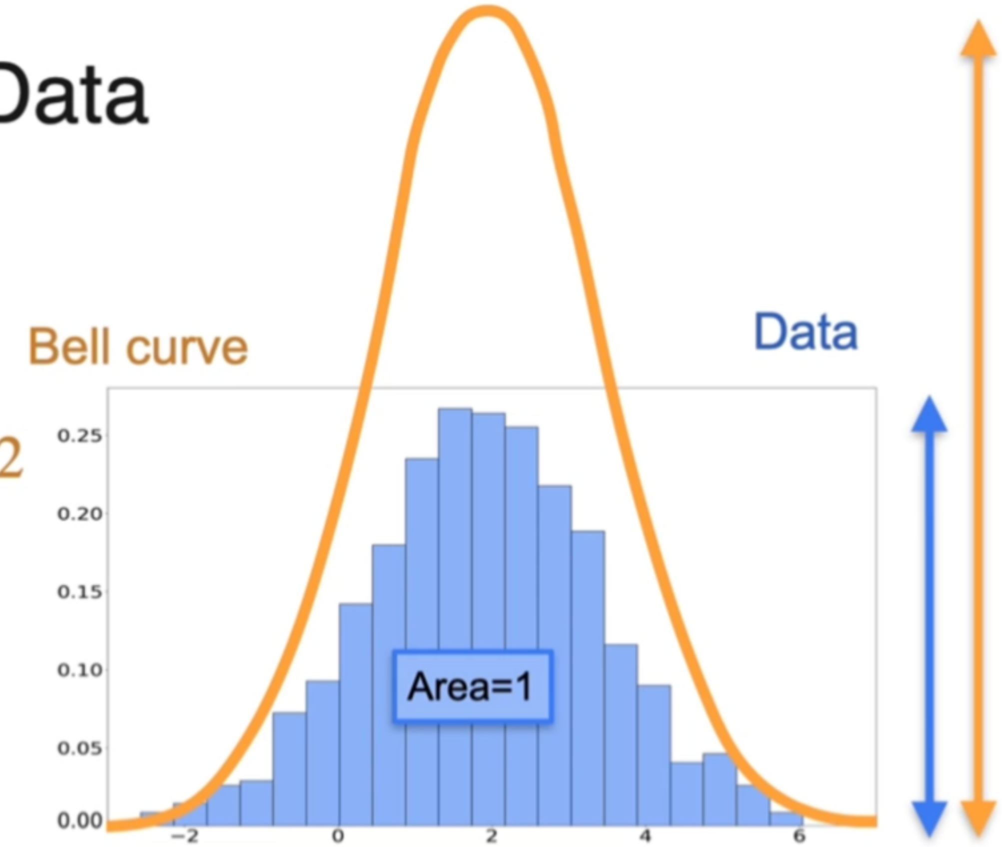
Bell Shaped Data

$$e^{-\frac{1}{2}\left(\frac{x-2}{3}\right)^2}$$

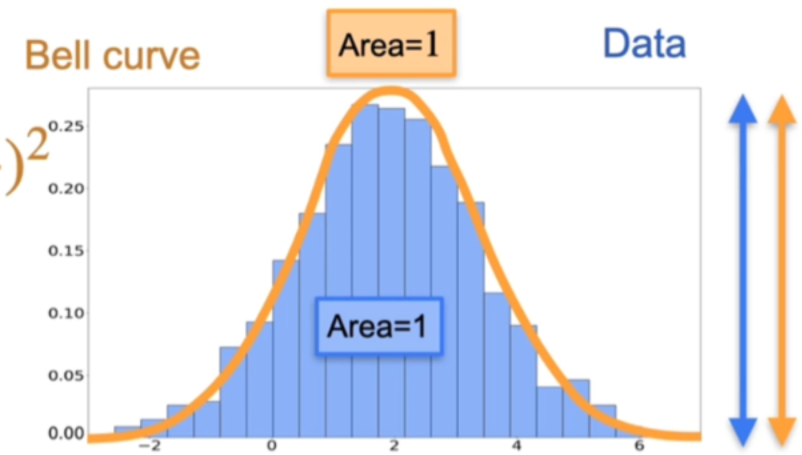


The area under the blue curve is 1 because it's a probability distribution, and

Making Adjustments we get,

Bell Shaped Data

$$\frac{1}{3\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-2}{3}\right)^2}$$



So that's the formula for the Gaussian or normal distribution.

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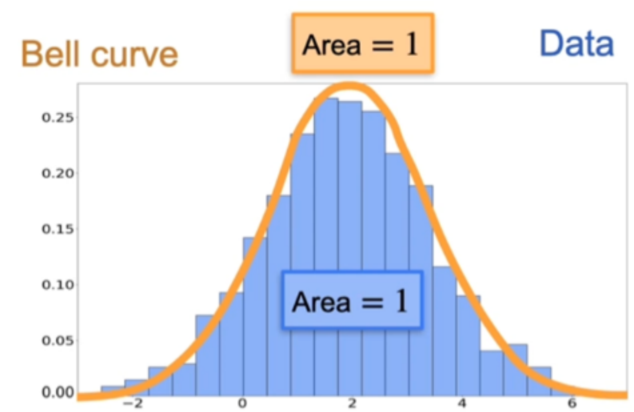
Bell Shaped Data

Mean = μ

Standard deviation = σ



And if you want the mean to be mu and the standard deviation to be sigma,

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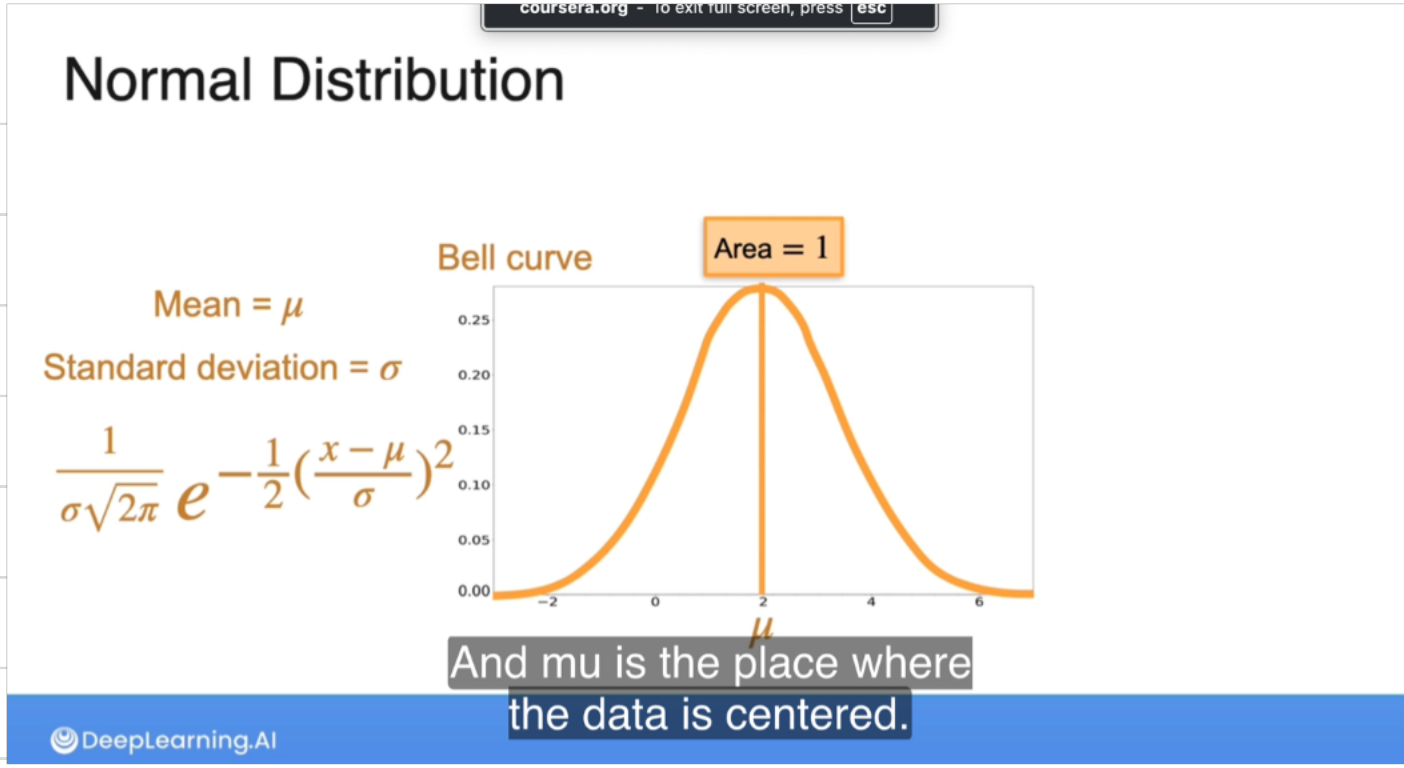
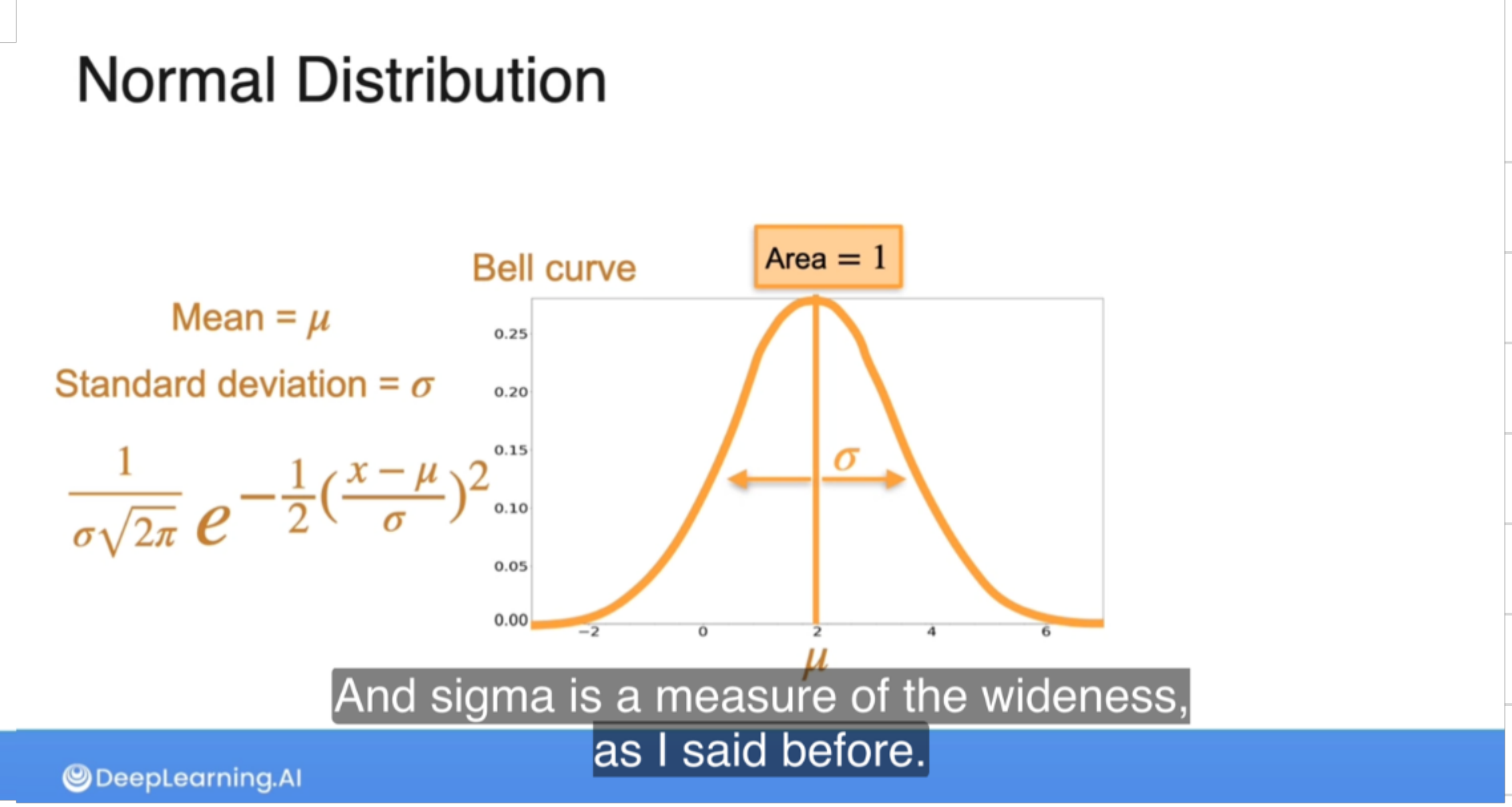




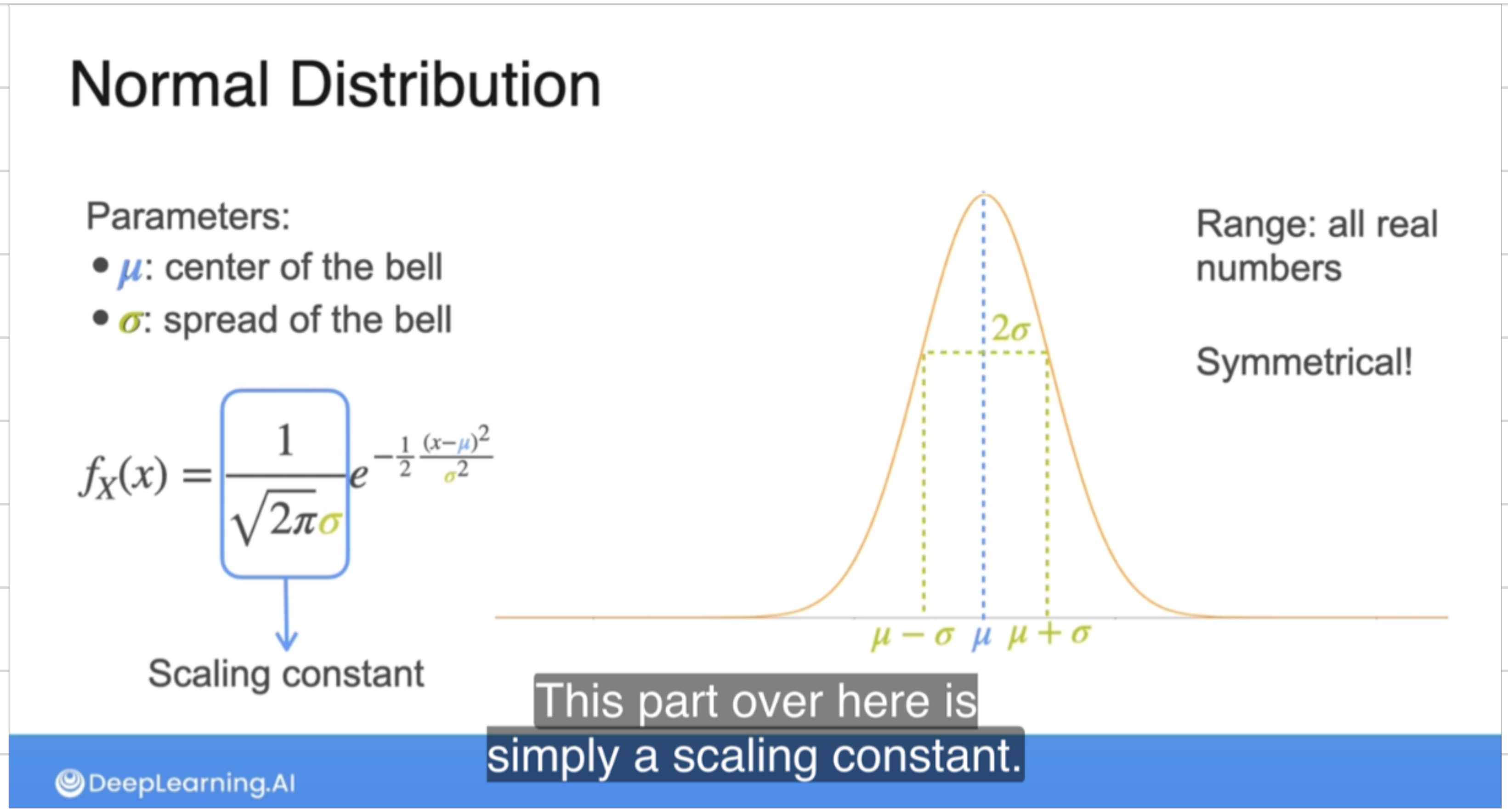
Parameters

μ : center of the bell

σ : spread of the bell

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma}} \cdot e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Scaling Constant



So, depending upon μ and σ we have,

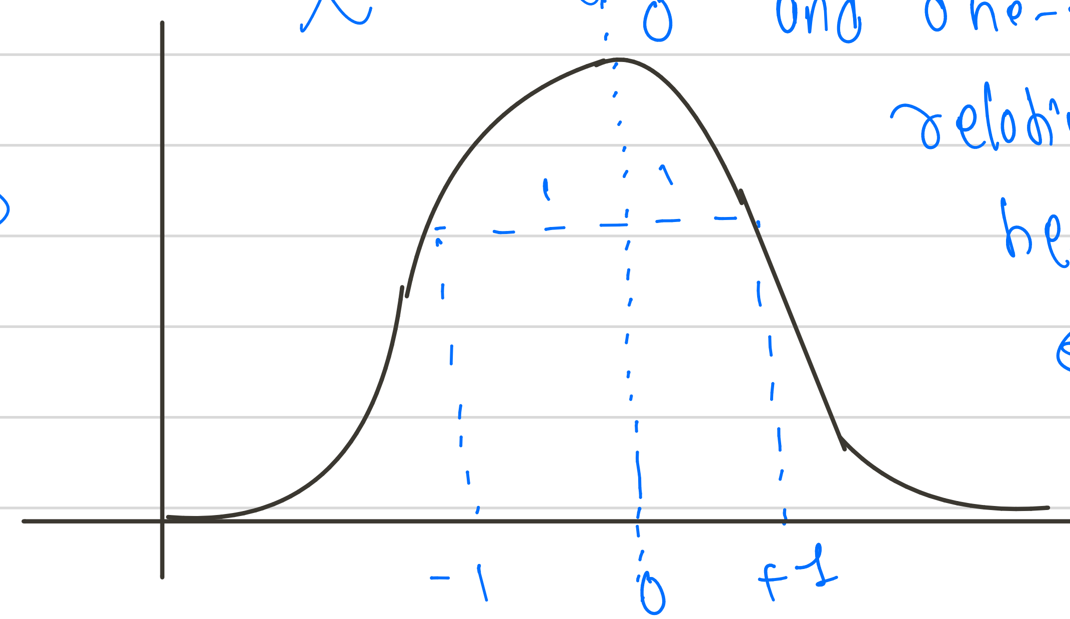
So if we have Random Variable X with this PDF we can write

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{2\sigma^2}} \Rightarrow X \sim N(\mu, \sigma^2)$$

— σ have to be always +ve.

$X \sim N(0,1)^2$ and one-to-one relationship between σ and μ.

Here, $\mu = 0$ and $\sigma = 2$



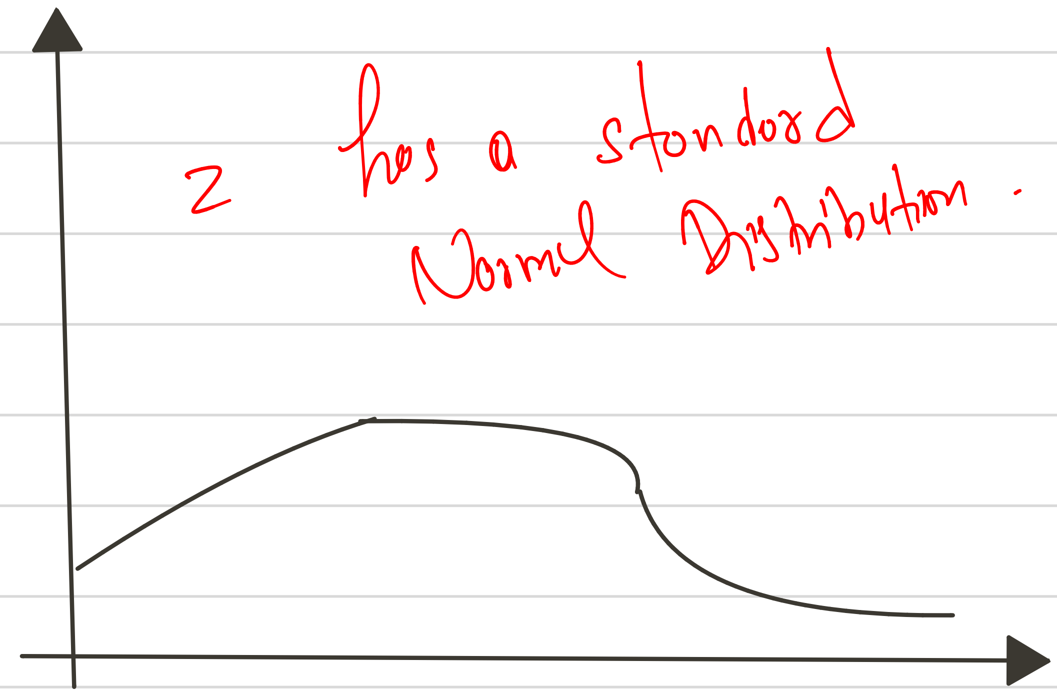
Standardization.

Convert any Normal Distribution to the standard one.

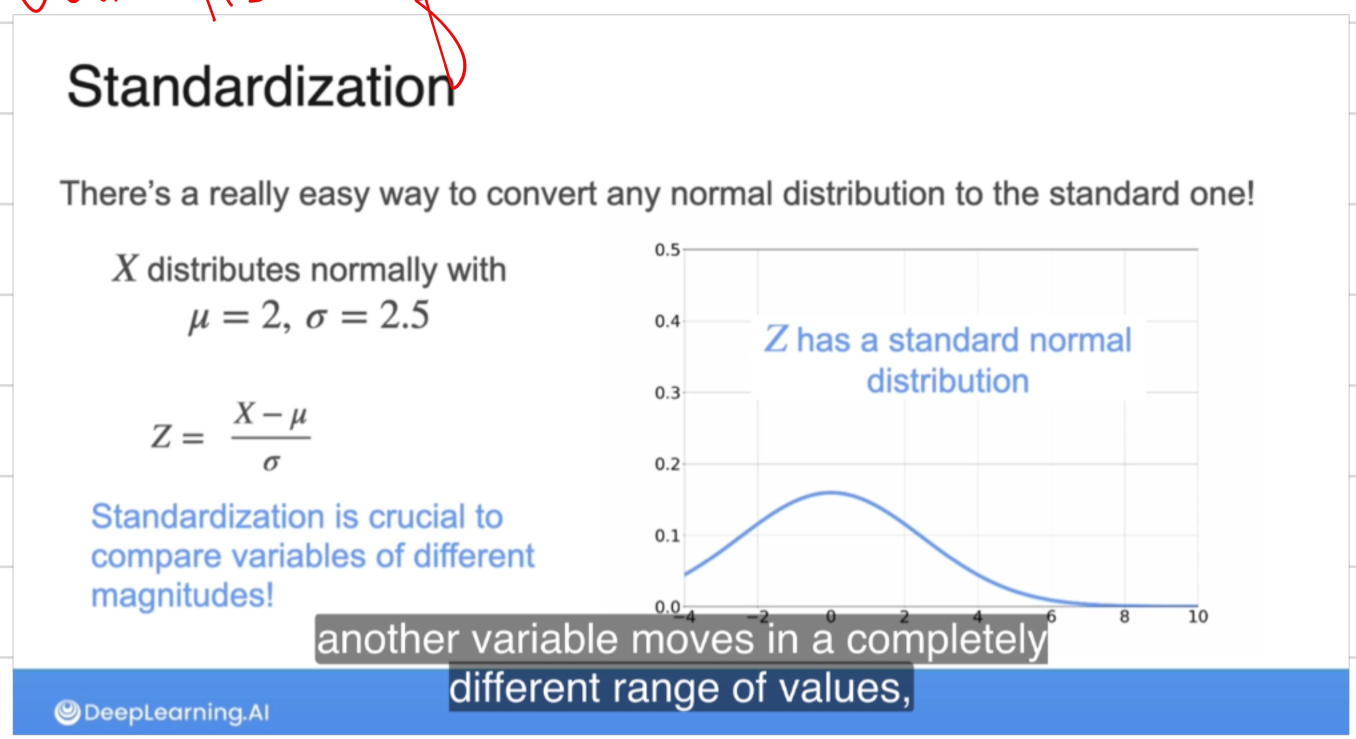
X distributes normally with

$\mu = 2, \ \sigma = 2.5$

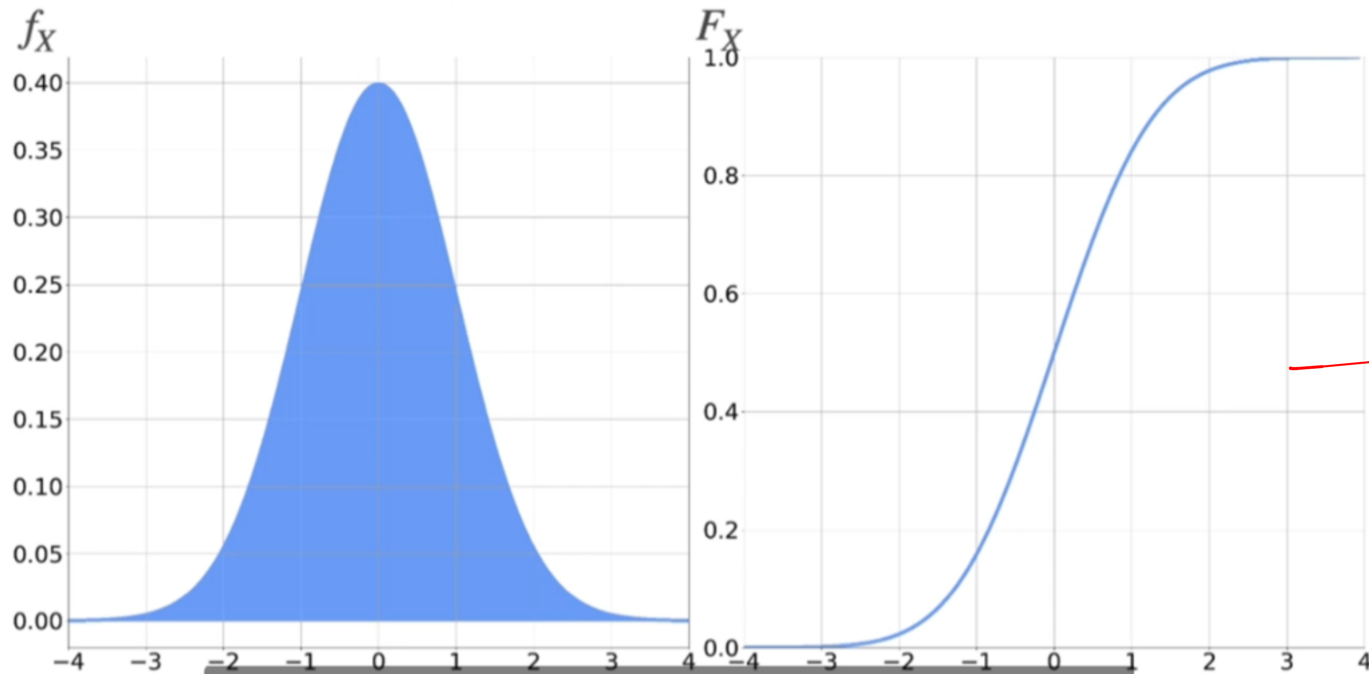
$$Z = \frac{X - \mu}{\sigma}$$



Standardization is crucial to compare variables of different magnitudes!



What Does the CDF Look Like?



As usual, it starts at 0 on the very left,
which is now the point at infinity,

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Math can't be done by hand we use computer

→ Area computing is hard.

Source:

Coursera Probability & Statistics for Machine Learning.

Normal Distribution: Applications

Height

Weight

IQ

Noise in a communication channel

In general, characteristics that are the sum of many independent processes

Many models in ML are designed under the assumption that the variables follow a normal distribution

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