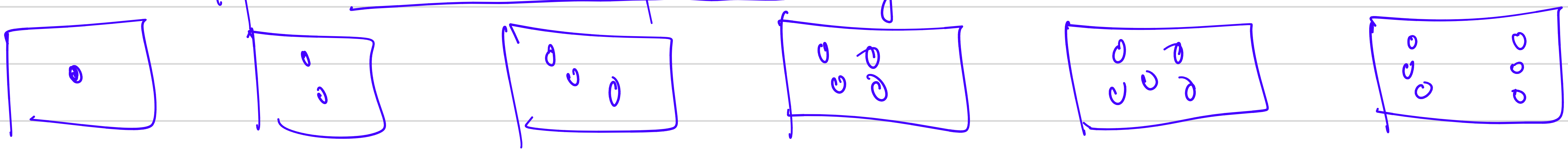
Day-73, Feb-12, 2025 (Magh 29, 2081)

Conditional Probability.

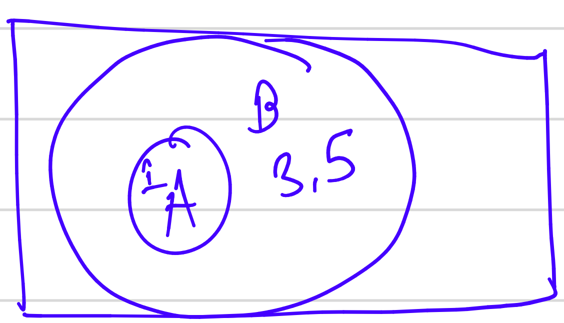


$A = \{1\}$ $\qquad$ $B = \{1, 3, 5\}$

$P(\text{one given that roll is odd}) = P(A|B)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B) = P(A)$ because



$$\Rightarrow \frac{P(A)}{P(B)} \Rightarrow \frac{1/6}{3/6} \Rightarrow 1/3$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{P(A) \cdot P(B)}{P(B)}$$

→ Where let B be an event so that $P(B) > 0$.

$$= P(A)$$

Bayes' Rule (Bayesian Theorem:

Compute $P(B|A)$ when we have $P(A|B)$ so,

$$P(B|A) = \frac{P(A|B)\cdot P(B)}{P(A|B)\,P(B) + P(A|B^c)\,P(B^c)}$$

mostly used in diagnostic test.

Diagnostic tests:

Let + and − be the events that the result of a diagnostic test is positive or negative respectively.

Let D and D^c be the event that the subject of the test has or doesn't have the dataset respectively.

* Sensitivity $= P(+ \mid D)$
* Specificity $= P(- \mid D^c)$

For an example, in an HIV blood test, we could take people who we know to have disease and apply the diagnostic test to that blood. Similarly the people not having diseases.

More Definitions

(1) positive predictive value $= P(D \mid +)$

probability of having Disease given that the disease is positive

(2) Negative Predictive value $= P(D^c \mid -)$

(3) Prevalence of disease $= P(D)$

Example:

Sensitivity of 99.7 %.

Specificity of 98.5 %.

Population with a 1 % Prevalence of HIV

$P(D \mid +)$

#Source: John Hopkins University Inferential Statistics Coursera Course.

Using Bayes' Formula:

$$P(D|+) = \frac{P(+|D) \cdot P(D)}{P(+|D) \cdot P(D) + P(+|D^c) \cdot P(D^c)}$$

$$\Rightarrow \frac{P(+|D) \cdot P(D)}{P(+|D) \cdot P(D) + \{1 - P(-|D^c)\}\{1 - P(D)\}}$$

$$\Rightarrow \frac{.997 \times .001}{.997 \times 0.001 + .015 \times .999} \Rightarrow 0.062$$

Likelihood Ratios:

$$P(D|+) = \frac{P(+|D)\cdot P(D)}{P(+|D)\cdot P(D) + P(+|D^c)\cdot P(D^c)}$$

population with a 1% prevalence of HIV,

$$P(D^c|+) = \frac{P(+|D^c)\cdot P(D^c)}{P(+|D)P(D) + P(+|D^c)P(D^c)}$$

$$P(D^c|+) \Rightarrow \frac{P(+|D^c)\cdot P(D^c)}{P(+|D)\,P(D) + P(+|D^c)\cdot P(D^c)}$$

Now dividing the eqns

$$P(D|+) = \frac{P(+|D) \cdot P(D)}{P(+|D) \cdot P(D) + P(+|D^c)\,P(D^c)} \quad \text{— eqn (i).}$$

$$P(D^c|+) = \frac{P(+|D^c) \cdot P(D^c)}{P(+|D)\,P(D) + P(+|D^c) \cdot P(D^c)} \quad \text{— eqn (ii)}$$

Now eqn (i) and eqn (ii) dividing we get —

Likelihood Ratios:

$$\frac{P(D|+)}{P(D^c|+)} \Rightarrow \frac{P(+|D)\cdot P(D)}{P(+|D^c)\cdot P(D^c)}$$

When we take a probability and divide it by $1-p$ = odds

$$\frac{P(D|+)}{P(D^c|+)} = \frac{P(+|D)}{P(+|D^c)} \cdot \frac{P(D)}{P(D^c)}$$

→ Diagnostic test Result Likelihood Ratio.

→ odds of disease in the absence of the test result.

Odds of disease given a positive test Result.

$$\frac{P(D \mid +)}{P(D^c \mid +)} = \frac{P(+ \mid D)}{P(+ \mid D^c)} \times \frac{P(D)}{P(D^c)}$$

post-test odds of D = DLR$_+$ × pre-test odds of D

HIV example Revisited:

[Specificity of 98.5%] [Sensitivity of 99.7%]

$$DLR_+ = .997 / (1 - .985)$$

$\Rightarrow$ 66 times or pre-test odds are

66 times more supported by the data than the hypothesis of no disease.

$$DLR_{-} = (1 - .997) / .985 \approx 0.03$$

Post-test odds of disease in the light of negative test result is now 0.3% that of the pre-test odds of disease. In other words, the hypothesis of disease is supported 0.003 times that of the hypothesis of the absence of disease given the negative test result.

Brief Idea of Independence in Probability:

→ Event A is Independent of event B if

$$P(A \cap B) = P(A) \cdot P(B)$$

Example:

$A = \{\text{Head in flip } 1\} \sim P(A) = 0.5$

$B = \{\text{Head in flip } 2\} \sim P(B) \Rightarrow 0.5$

$A \cap B = \{\text{Head in flips } 1 \text{ and } 2\}$

$P(A \cap B) = P(A) \cdot P(B) \Rightarrow 0.25$

Example: Prevalence of Sudden Infant death Syndrome of 1 out of 8,543,

$$\left(\frac{1}{8543}\right)^2$$

Multiplying Probabilities without Independence

i.e. $P(A_1 \cap A_2)$ is not necessarily equal to $P(A_1) \cdot P(A_2)$

Random Variables are Independent (Default Model for Random Samples)

Source: The power of Statistics, Coursera Course.

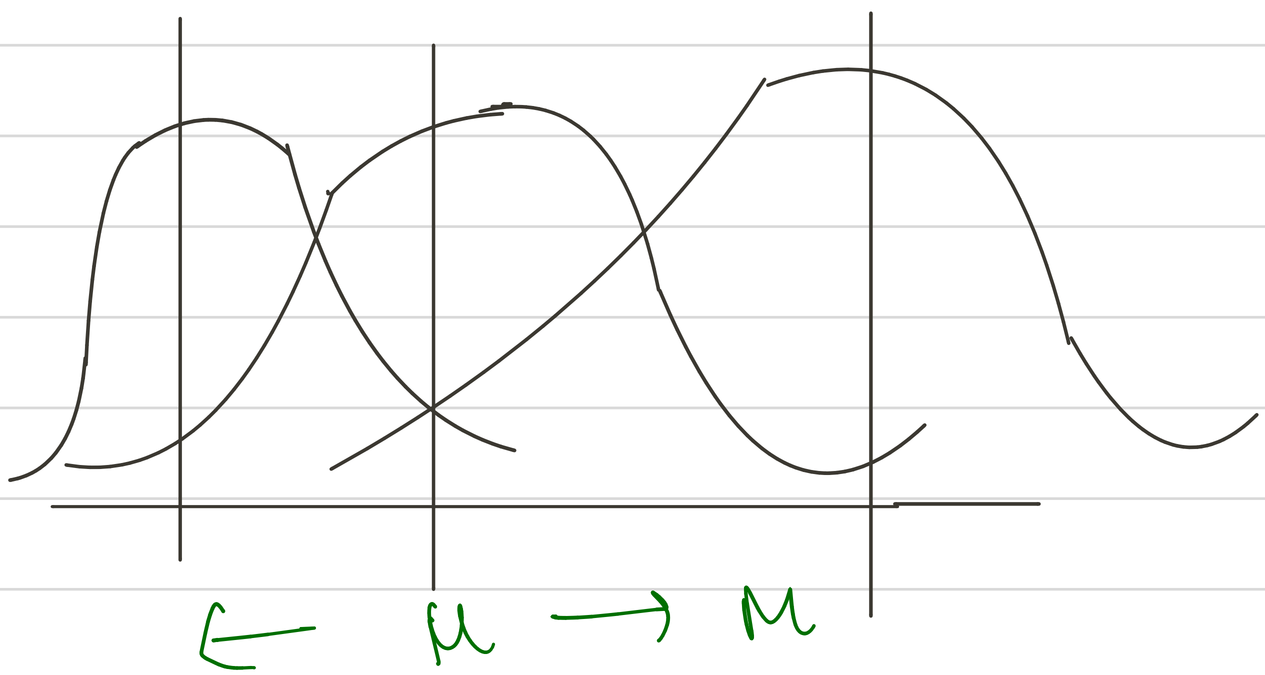
Expected Values:

$$E[X] = \sum_x x\, p(x).$$

→ The process of making conclusions about populations from noisy data that was drawn from it.

→ The most useful characterization are so called expected value.

→ Mean is a characterization of its center.



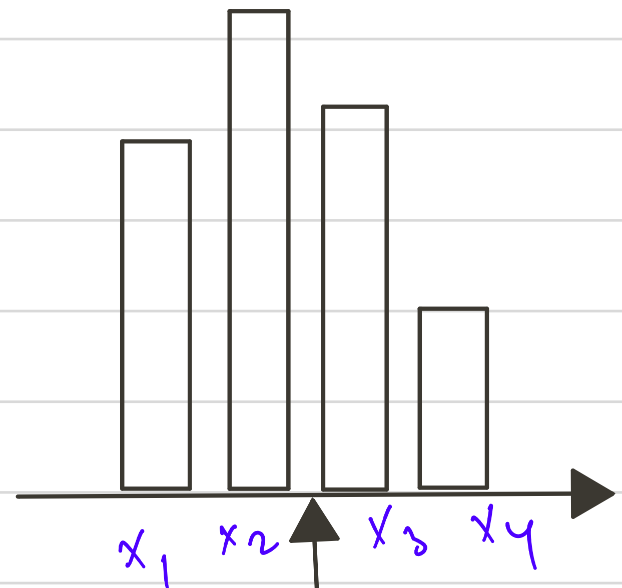
→ The variance and standard deviation are characterizations of how spread out it is

→ Our sample expected values (the sample mean and variance) will estimate the population versions

→ Expected value or mean of a random variable is the center of its distribution.

$E[X]$ represents the center of mass of a collection of locations and weights, $\{x, p(x)\}$.

$$E[X] = \sum_x x\, p(x).$$



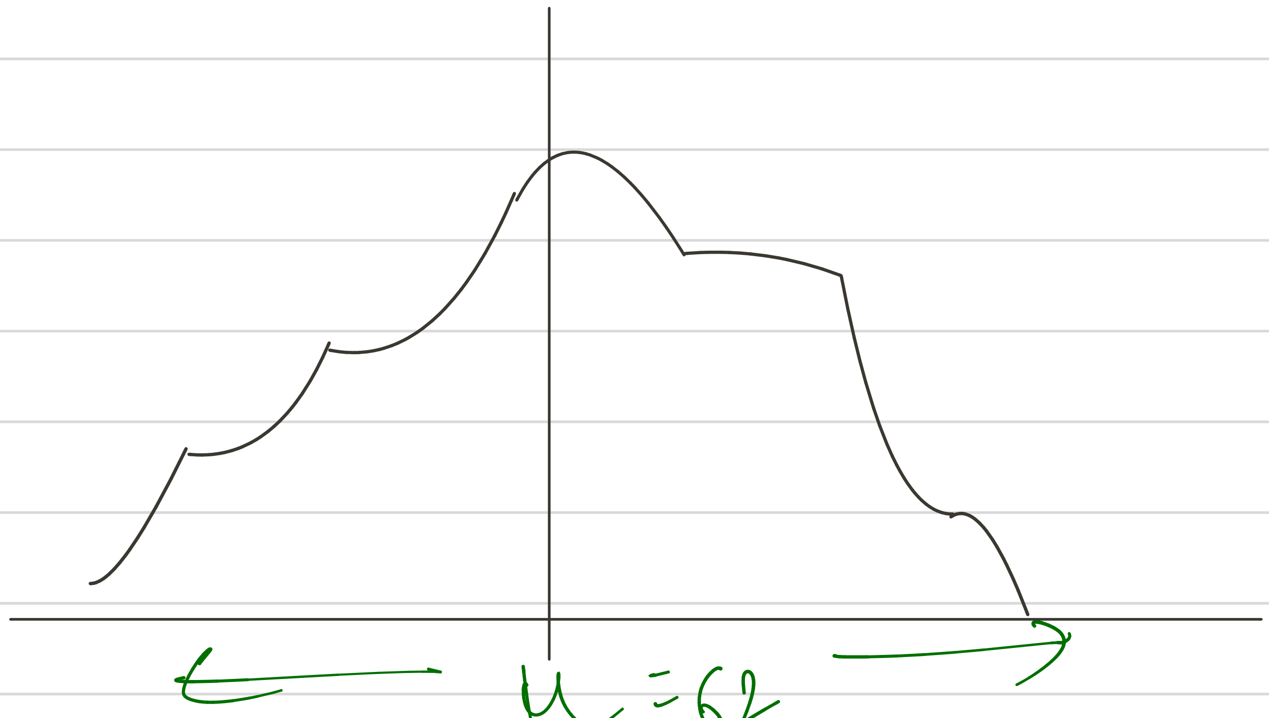
Height is $p(x)$.

Mean is the center of mass.

Sample Mean estimates the population mean.

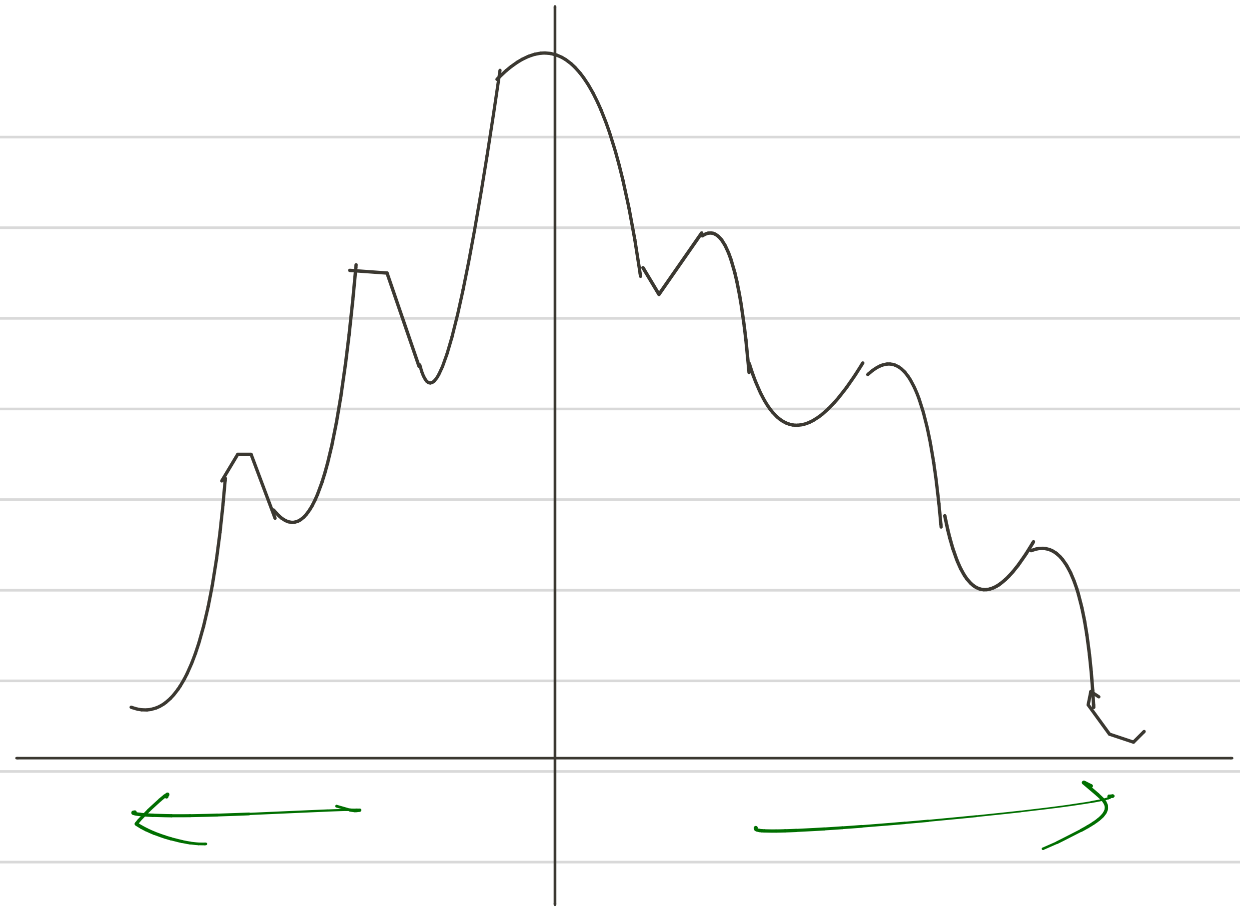
$$\bar{X} = \sum_{i=1}^{n} x_i\, p(x_i)$$

Which is the center of mass of the data is the empirical mean.



$\mu = 62$

Suppose. MSE = 43.403



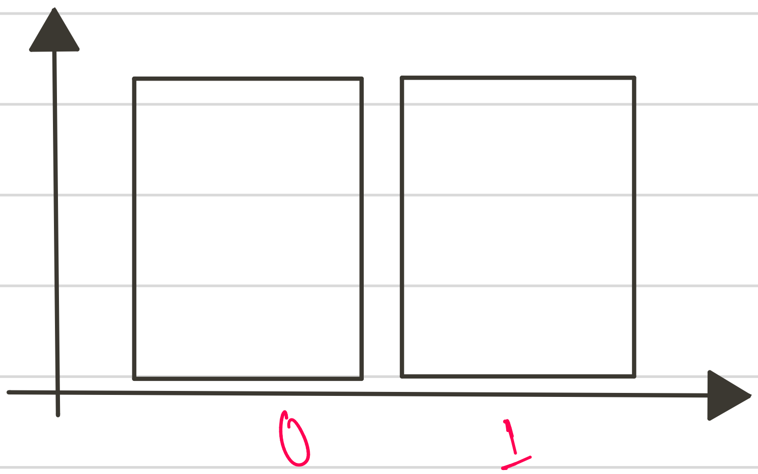
μ

→ MSE helps to balance the distribution as ↑ μ and ↓ MSE.

Example of a Population Mean.

Suppose a coin is flipped and X declared 0 or 1 corresponding to a head or a tail. What is the expected value of x? So, again the expected value is the property of the population.

$\mathbb{E}[X] = .5 \times 0 + .5 \times 1 \Rightarrow .5$



Q. What about a biased coin?

$\Rightarrow$ Suppose that a random variable X, is so that $p(X=1) = p$ and $p(X=0) = (1-p)$

$\mathbb{E}[X] = 0 * (1-p) + 1 * p$

$\Rightarrow p$

Q. What about a dice? Suppose that a dice is rolled, and X is the number that is face up, what is the expected value of X?

$\mathbb{E}[X] \Rightarrow 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$

$\Rightarrow 3.5$

Expanded Version of Bayes Theorem

$$\therefore P(A|B) = \frac{P(B|A) * P(A)}{P(B|A) * P(A) + P(B|A^c) * P(A^c)}$$

→ This version of Bayes Theorem are used for quality Control and testing diagnostic medical tests.

→ False positives and False Negatives are the Errors Account for the probability test.

False Positive: Test Result that indicates Something is present when it really is not. Pam email as email, Software testing & medical testing.

False Negative: Test result that indicates something is not present when it really is. Spam email as pam email, applied to quality control tests and other kinds of tests.

Evaluate the Accuracy of a Diagnostic test:

- 1% of the population has the medical condition
- If a person has the condition, there's a 90% chance that the test is positive.
- If a person does NOT have the condition, there is still a 2% chance that the test is positive.

Some have allergy after the test means updating the posterior probability.

- Prior probability = the probability that a person has the medical condition.

- posterior probability = the probability that the condition is present GIVEN that the test is positive.

- Event A = actually having the medical condition
- Event B = testing positive

  - $P(A) = 1\%$ (allegry)
  - $P(B|A) = 95\%$ (person having allegry tested positive)
  - $P(B|\text{not } A) = 2\%$ (person not having allegry tested negative)

Having not Allegry

$$P(A') = 1 - P(A)$$

$$\Rightarrow 1 - 0.01$$

$$\Rightarrow 0.99$$

$$\Rightarrow 99\%$$

Bayes' theorem (Basic Version)

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

Alternatively we can use expanded version if not calculating $P(B)$.

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B|A) * P(A) + P(B|A^c) * P(A^c)}$$

$$\Rightarrow \frac{0.95 * 0.01}{0.95 * 0.01 + 0.02 * 0.99} \Rightarrow 0.324.$$

$P(A|B) \Rightarrow 32.4\%$. means P of having Allergy given that the test is positive is 32.4%.