Day-2, Nov-17, 2024. (Mangshir 2, 2081 B.S.)

Limit of a function

using the concept of the limit of sequence to understand the meaning of the limit of a function.

$$y = f(x) = 2x + 3 \longrightarrow \text{eqn (i)}$$

Suppose x can be sequence of values [0.5, 0.75, 0.9, 0.99, 0.99 ......
then the $y = f(x)$ can be 4, 4.5, 4.8, 4.9 .... go nearer to 5 when 'x' is very near to 1. So, when 'x' is sufficiently close to 1. $f(x)$ is very close to 1.

Again if the sequence of x are $[2, 1.5, 1.25, 1.1, 1.01, 1.001, 1.0001 \ldots$ whose limit is 1.

Then $y = f(x)$ be $7, 6, 5.5, 5.2, 5.02 \ldots\ldots$ go nearer to 5 when x is very close to 1.

<u>Definition:</u> A function $f(x)$ is said to tend to a limit 'l' when $x \to a$ if the numerical difference between $f(x)$ and l can be made as small as we please by making sufficiently close to 'a' and we write,

$$\lim_{x \to a} f(x) = l$$

Meaning of Infinity (∞)

Let us consider the function $y = f(x) \Rightarrow \frac{1}{x}$

Now, taking sequence of values of x to be [1, 0.5, 0.1, 0.01, 0.001, 0.0001, 0.00001 ....
whose limit is 0 then $f(x)$ are

$y = f(x)$ will be large enough as it becomes

$y = f(x)$ are [1, 2, 10, 100, 1000, 10000, ...
which go on increasing]

So, $\left[\lim_{x \to 0} \frac{1}{x} = \infty \right]$ $f(x)$ tends to infinity $f(x) \to \infty$ as $x \to 0$

Infinity as a limit of function

Let $f(x)$ be a function of 'x'. Making 'x' is sufficiently close to 'a' if the value of $f(x)$ obtained is greater than any per-assigned number however, large, we say that the limit of $f(x)$ is infinity as 'x' tends to 'a'. Symbolically we write,

$$\left[\lim_{x \to a} f(x) = \infty\right] \text{ if } f(x) > a$$

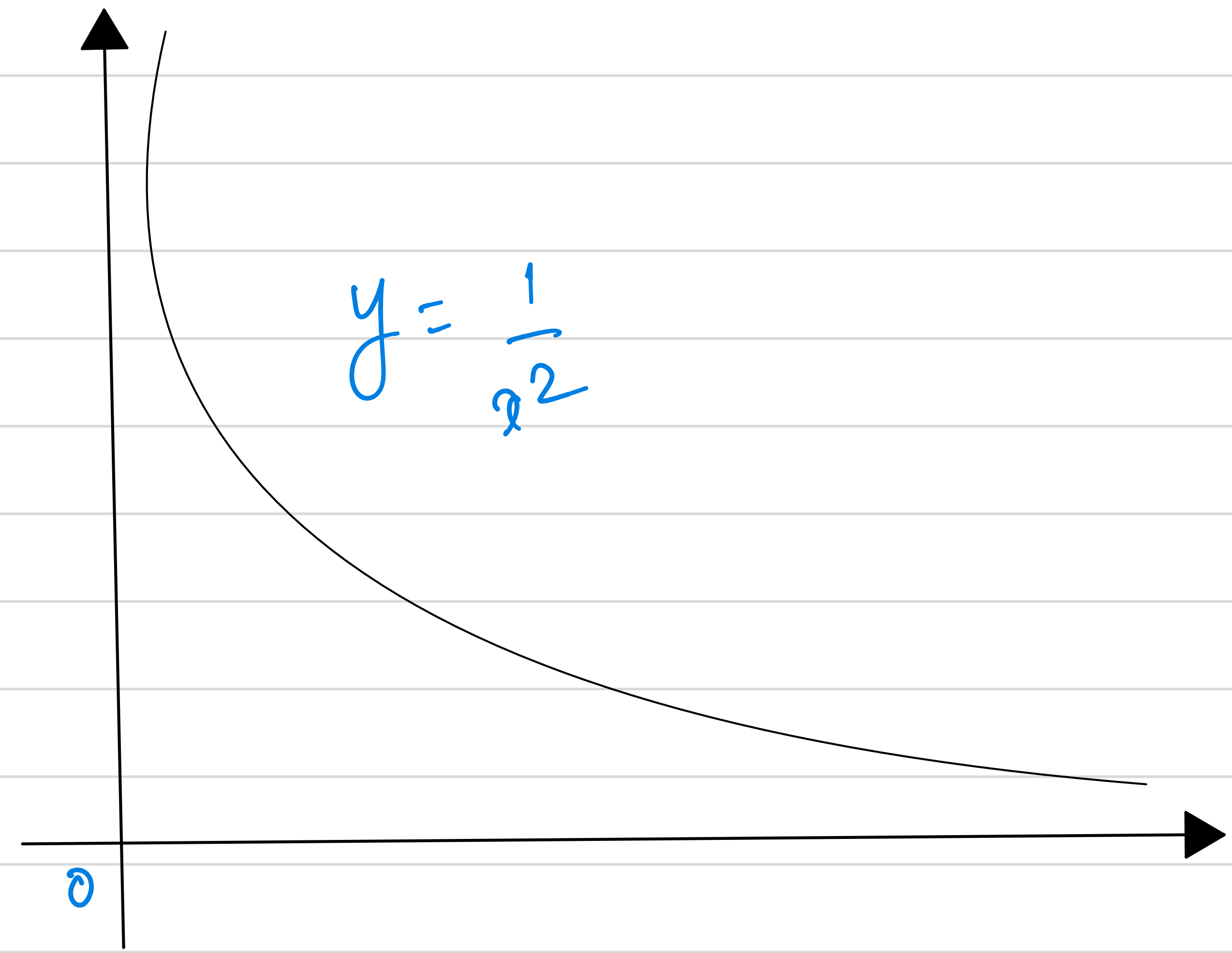
What if we take larger 'x' then the $f(x)$ will be ?

→ $f(x)$ will be smaller

→ So there is inverse relationship between 'x' and $f(x)$.

Example taking equation $y = \frac{1}{x^2}$

x	1	10	100	1000	...	..	..
$f(x)$	1	0.01	0.001	0.0001	..	..	...



So, mathematically

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{1}{x^2} \Rightarrow 0$$

A function $f(x)$ is said to tend to 'l' when $x \to \infty$ if $f(x)$ can be made close to 'l' when 'x' is greater than any pre-assigned number, however large. Symbollically, we write $\lim\limits_{x \to \infty} f(x) = l$

Limit Theorems

Let $f(x)$ and $g(x)$ be two functions of 'x' such that $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} g(x) = m$, then we have the following theorems on limits.

i) The limit of the sum (or difference) of the functions $f(x)$ and $g(x)$ is the sum (or difference) of the limits of the functions i.e.

$$\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

$$\lim_{x \to a} [f(x) \pm g(x)] \Rightarrow l \pm m$$

ii) The limit of the product of the functions $f(x)$ and $g(x)$ is the product of the limits of the functions i.e.

$$\lim_{x \to a} [f(x) \cdot g(x)] \Rightarrow \left(\lim_{x \to a} f(x) \right) \cdot \left(\lim_{x \to a} g(x) \right)$$

$$\Rightarrow l \cdot m$$

iii) The limit of the quotient of the function $f(x)$ and $g(x)$ is the quotient of the limits of the functions, provided that the limit of the denominator is not zero i.e.

$$\lim_{x \to a} \frac{f(x)}{g(x)} \Rightarrow \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \Rightarrow \frac{l}{m} \quad [\because m \neq 0]$$

provided $\lim_{x \to a} g(x) = m \neq 0$

iv) The limit of the 'nth' root of a function $f(x)$ is the nth root of the limit of function i.e.

$$\lim_{x \to a} \sqrt[n]{f(x)} \Rightarrow \sqrt[n]{\lim_{x \to a} f(x)}$$

$$\left[\lim_{x \to a} \sqrt[n]{f(x)} \Rightarrow \sqrt[n]{l} . \right]$$

(+, -, *, / and $\sqrt{\ }$ operations theorem)

REFERENCES -

i) D.R. Bajracharya, R.M. Shrestha, M.B. Singh, Y.B. Shhapit & B.C. Bajracharya, 2014, Basic Mathematics Grade XI (3rd Edition), Sukunda Pustak Bhawan, Kathmandu