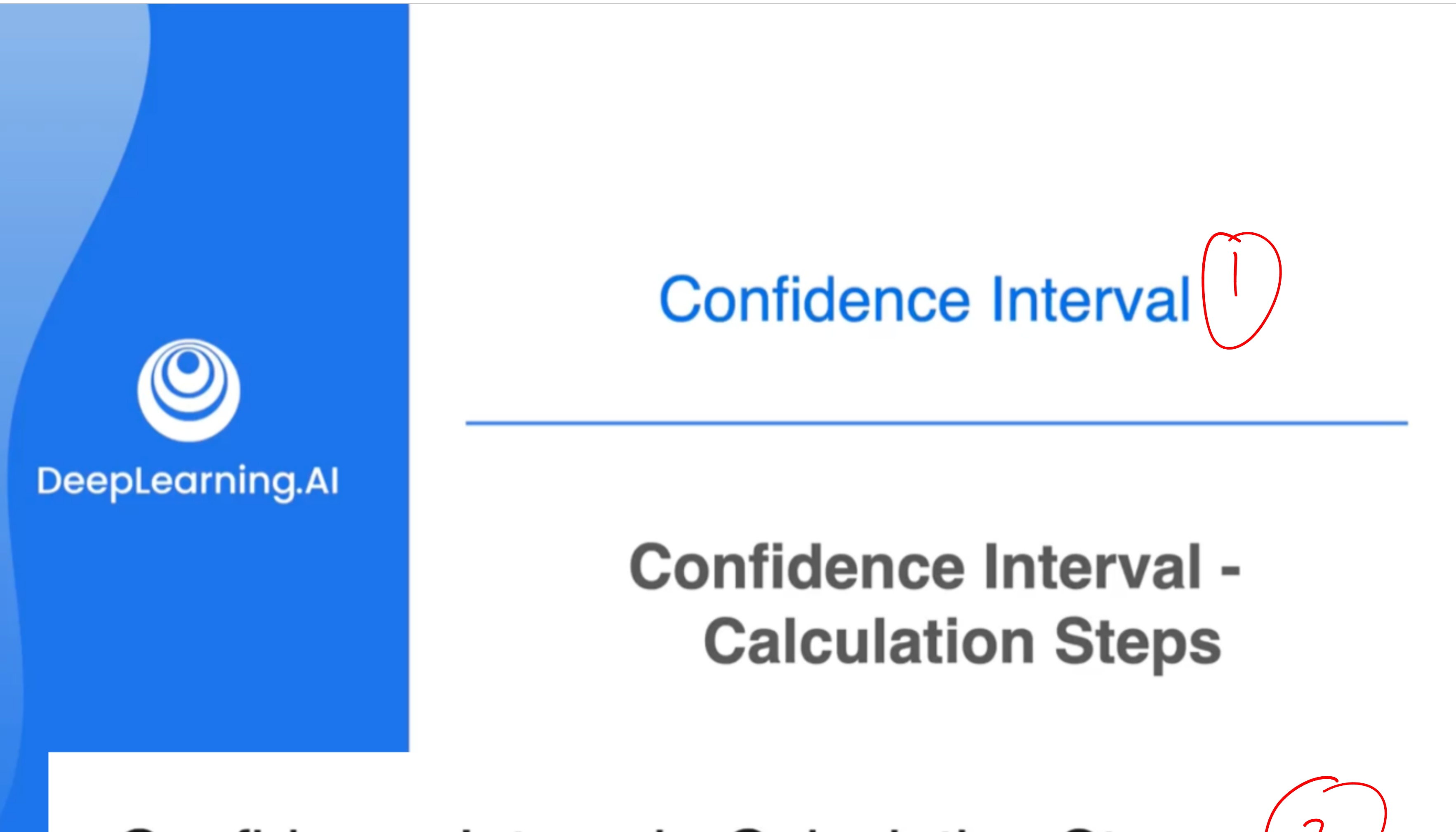
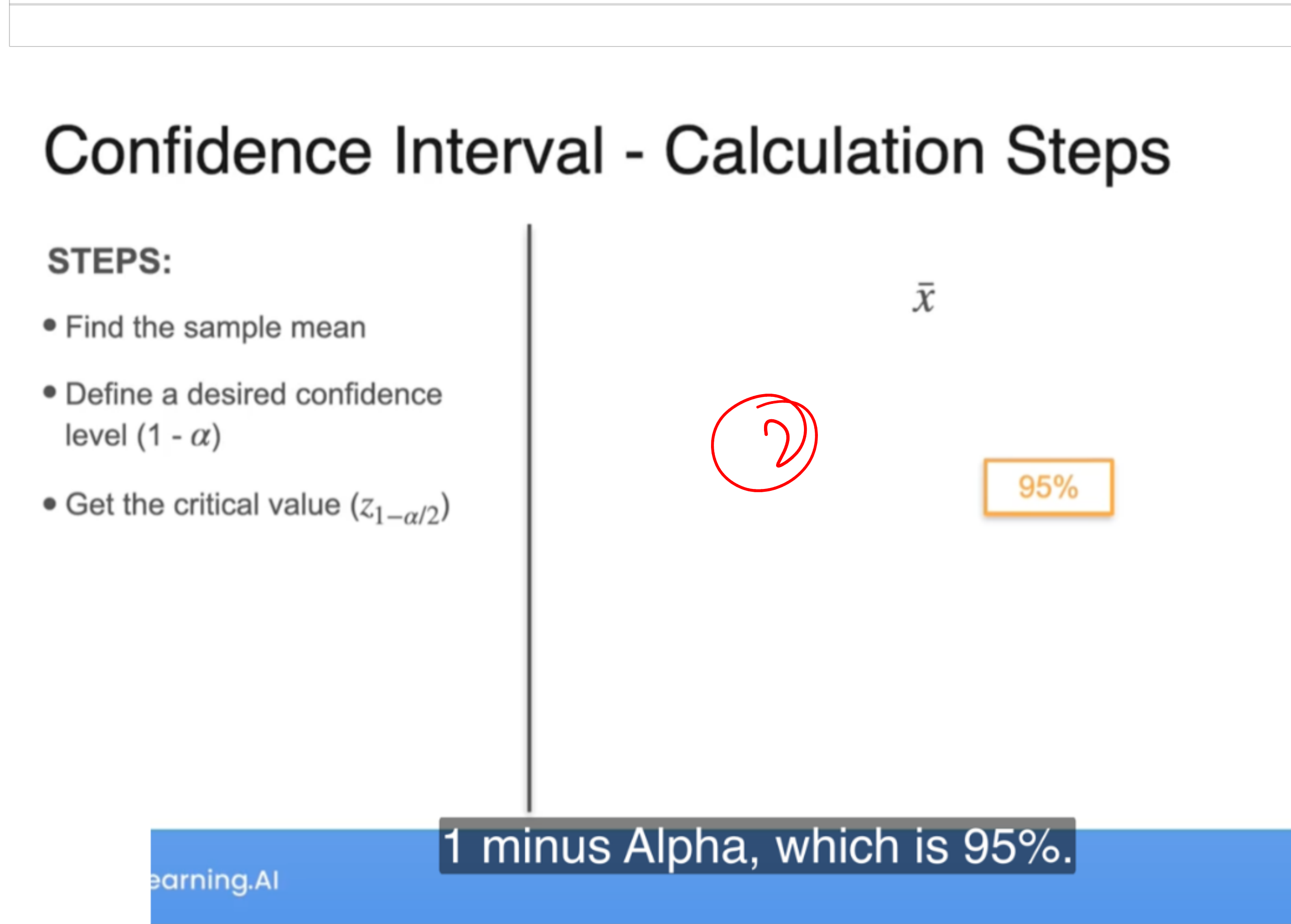
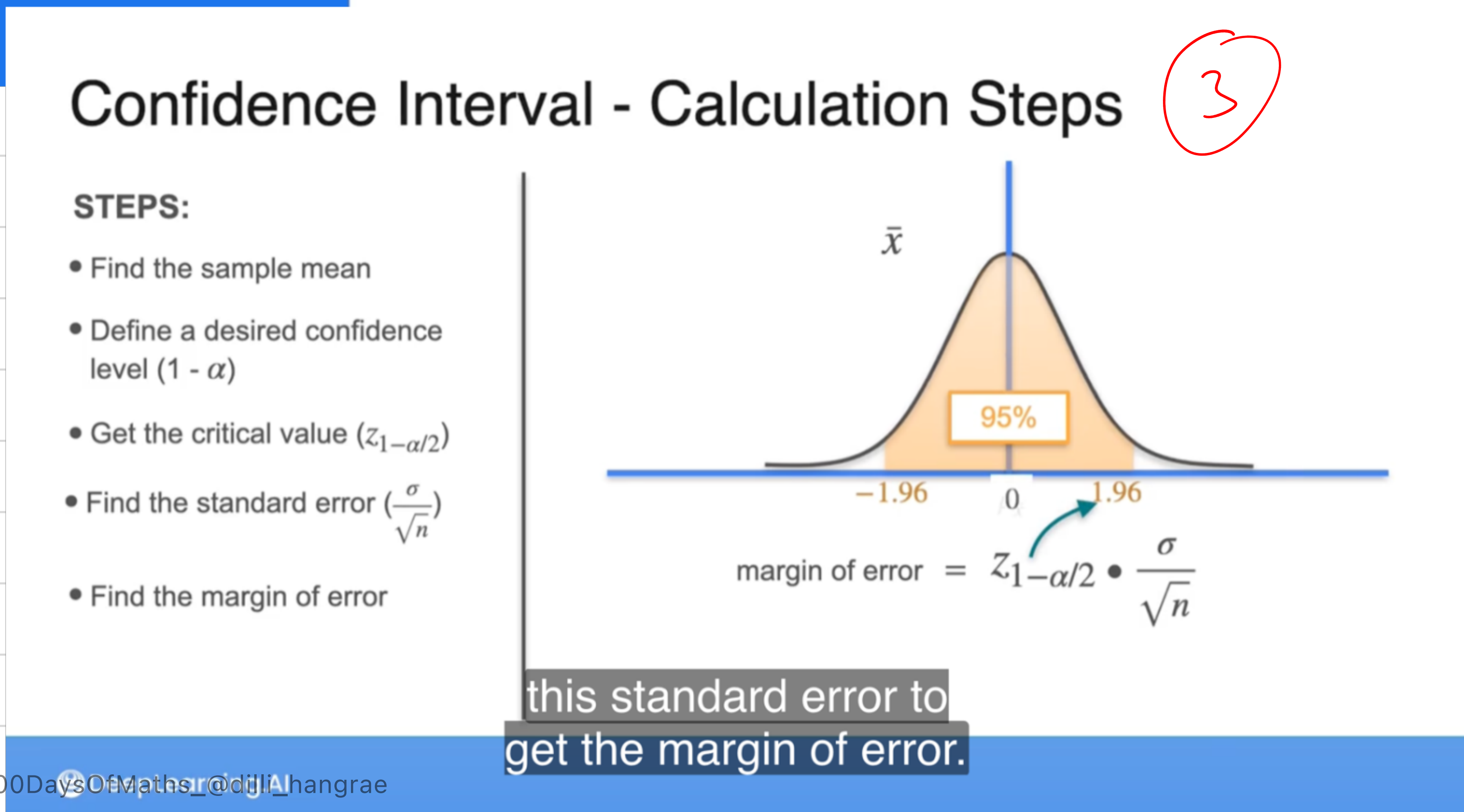
Day- 66, Feb 4, 2025 (Magh 22, 2081 B.S.)

1. Confidence Intervals- Calculation Steps
2. Confidence Intervals - Example.
3. Calculating the Sample Size
4. Difference between Probability & Confidence
5. Unknown Standard Deviation
6. Confidence Intervals for Proportion
7. Defining Hypothesis, Type I and II Errors







Assumptions for Valid Confidence Intervals:

Random Sample:
The sample must represent the population without bias.

Sample Size:
Ideally, n>30(Central Limit Theorem applies).

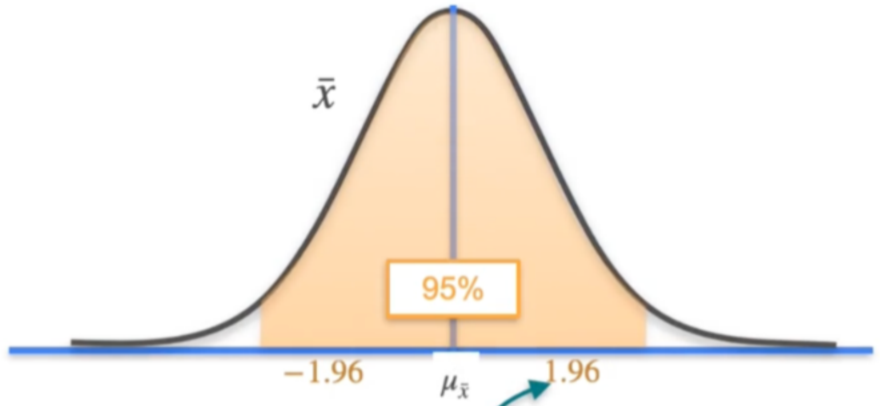
Distribution:
If $n < 30$, the population should be approximately normal.

4

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)
- Get the critical value ($z_{1-\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error



$$\text{margin of error} = z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

this standard error to
get the margin of error.

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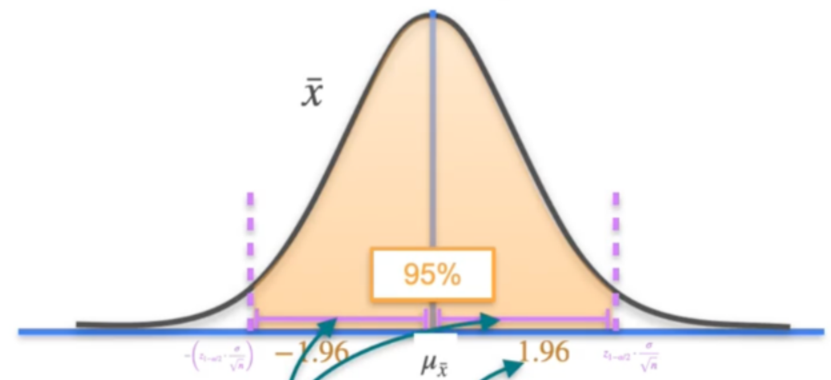
Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)
- Get the critical value ($z_{1-\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean



$$\text{margin of error} = z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

the sample mean to obtain

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Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)
- Get the critical value ($z_{1-\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
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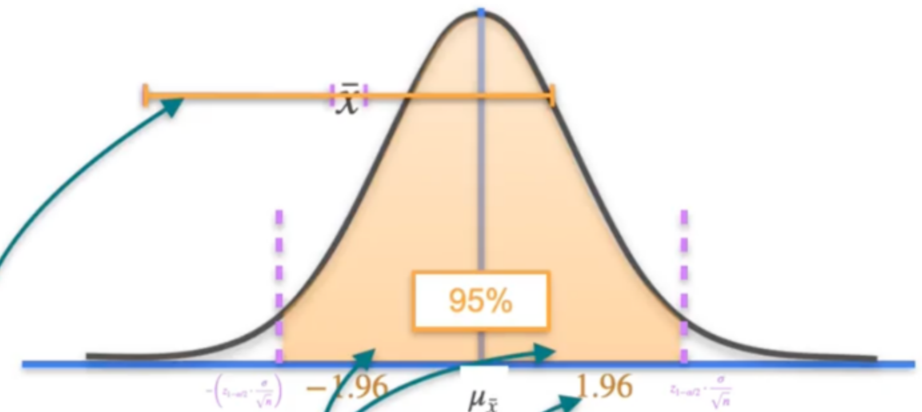

$$\text{margin of error} = z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$\text{confidence interval} = \bar{x} \pm z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

And there you have
it. That's how

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Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level (1 - α)
- Get the critical value ($z_{1-\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Assumptions

- Simple random sample
- Sample size > 30 or population is approximately normal

be sure to follow
these assumptions.

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Confidence Interval

Confidence Interval - Example

2

Confidence Interval Calculation Example:

1. **Sample Details:**
   - Population size = 6,000
   - Sample size (n) = 49
   - Sample mean ($\bar{x}$) = 170 cm
   - Population standard deviation (σ) = 25 cm
   - Confidence level = 95% (Z = 1.96)
2. **Margin of Error (ME):**

$$ME = Z \times \frac{\sigma}{\sqrt{n}} = 1.96 \times \frac{25}{\sqrt{49}} = 1.96 \times \frac{25}{7} = 1.96 \times 3.57 \approx 7 \text{ cm}$$

3. **Confidence Interval:**

$$170 \pm 7$$

Lower bound = 170 - 7 = **163 cm**

Upper bound = 170 + 7 = **177 cm**

Conclusion:

The 95% confidence interval for the average height of Statistopia's adults is **163 cm to 177 cm**. We're 95% confident the true population mean lies within this range.

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Confidence Interval - Example

Statistopia

6,000 adults

Random Selection

49

$\bar{x} = 170cm$

$\sigma = 25cm$

So let's find a 95% confidence interval

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Confidence Interval - Example

Statistopia

6,000 adults

Random Selection

49

$\bar{x} = 170cm$

$\sigma = 25cm$

Calculate a 95% confidence interval for the average height of adults on Statistopia.

for the average height of adults on Statistopia.

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Confidence Interval - Example

Random Selection

49

$\sigma = 25cm$

95% $\rightarrow z_{1-\alpha/2} = 1.96$

$$\text{margin of error} = z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}} = 1.96 \bullet \frac{25}{\sqrt{49}} = 1.96 \bullet \frac{25}{7} = 7$$

Therefore, the confidence interval

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Confidence Interval - Example

Random Selection

2500

$\sigma = 10cm$

95% $\rightarrow z_{1-\alpha/2} = 1.96$

Confidence Interval

$170cm \pm$ margin of error

margin of error $= 7$

Confidence Interval

$170cm - 7 = 163cm$

$170cm + 7 = 177cm$

163 centimeters and 177 centimeters.

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Question

Which statement is an accurate interpretation of the confidence interval $163cm < \mu < 177cm$ at a 95% confidence level when estimating the average height in Statistopic?

- ○ There is a 95% chance that the average height in Statistopic falls within the range of 163cm to 177cm.
- ◉ We are 95% confident that the true average height in Statistopic is between 163cm and 177cm.
- ○ The average height in Statistopic is definitively between 163cm and 177cm.

✓ Correct

Great job! The confidence interval provides a range of values within which we are 95% confident that the true population parameter (average height in this case) lies.

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Confidence Interval

Calculating Sample Size

2

This example highlights the relationship between **sample size** and **margin of error** in confidence interval calculations. Here's a simple, concise version:

Key Idea:

If you want a **smaller margin of error** (like 3 cm instead of 7 cm), you need to increase the sample size.

Example Breakdown:

- **Population size:** 6000 adults
- **Sample size:** Initially 49
- **Sample mean:** 170 cm
- **Standard deviation:** 25 cm
- **Critical value (z for 95% confidence):** 1.96
- **Desired margin of error:** 3 cm

Calculation:

Using the margin of error formula:

$$\text{Margin of Error} = z \times \frac{\sigma}{\sqrt{n}}$$

Substitute the values and solve for n:

3

Calculation:

Using the margin of error formula:

$$\text{Margin of Error} = z \times \frac{\sigma}{\sqrt{n}}$$

Substitute the values and solve for n:

$$3 \geq 1.96 \times \frac{25}{\sqrt{n}}$$

Rearranging and solving:

$$n \geq \left(\frac{1.96 \times 25}{3}\right)^2$$

$$n \approx 267$$

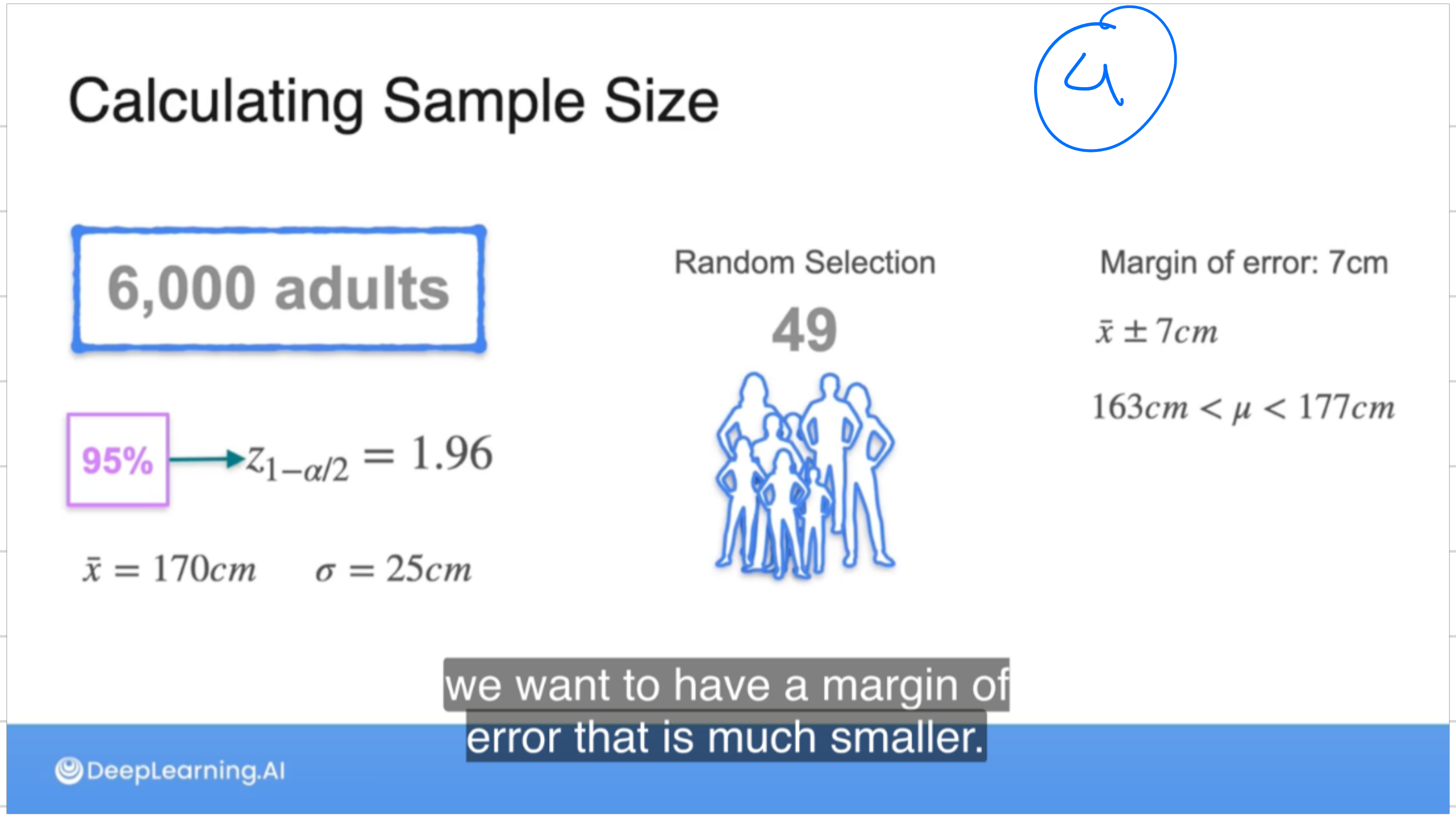
Conclusion:

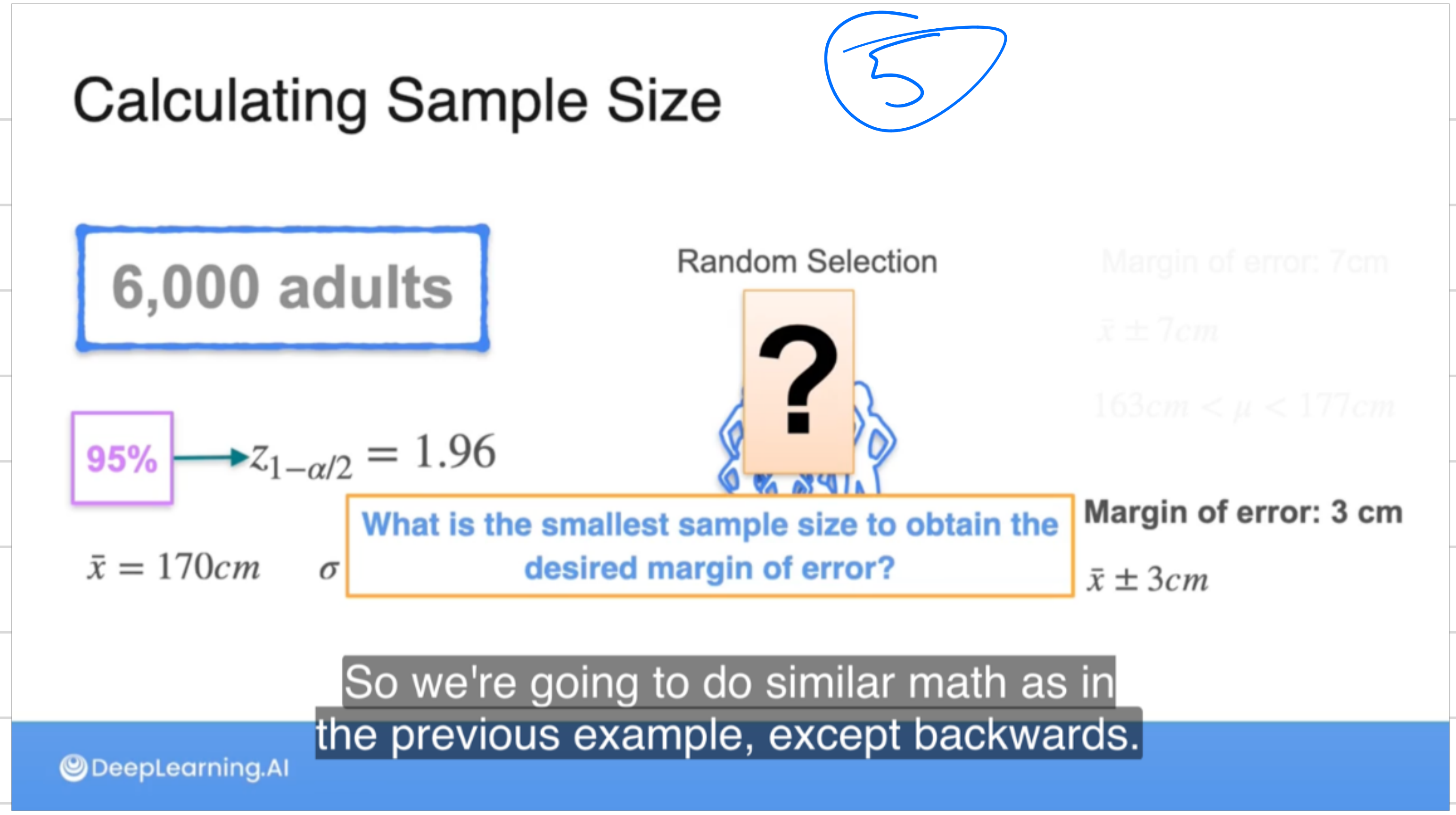
To achieve a margin of error of 3 cm, you need at least **267 adults** in the sample.

This formula can be generalized as:

$$n \geq \left(\frac{z \times \sigma}{\text{Margin of Error}}\right)^2$$

This helps determine the required sample size for any desired confidence level and margin of error.





Question

6

What is the smallest sample size to obtain the desired margin of error of $3cm$, given $\sigma = 25cm$, $\bar{x} = 170cm$, and $z_{1-\alpha/2} = 1.96$? **Round to the nearest whole number.**

Note: $\text{margin of error} = z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

- ○ 11
- ○ 16
- ○ 30
- ◉ 267

Correct

Great job! By working backward to find n, you should walk backward from the margin of error formula to get $n = \left(\frac{z_{1-\alpha/2} \cdot \sigma}{\text{margin of error}}\right)^2$. Continue the video to hear the full explanation. Continue the video to hear the full explanation.

Skip Continue

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Calculating Sample Size

6,000 adults

95% $\rightarrow z_{1-\alpha/2} = 1.96$

$\bar{x} = 170cm \qquad \sigma = 25cm$

Margin of error: 3 cm

$$\text{margin of error} = z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

Now, replacing all the values
we know into our inequality,

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Calculation of Sample Size.

$$n \geq \left(\frac{z \times \sigma}{MOE} \right)^2$$

8

Calculating Sample Size

$$\text{margin of error} = z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$$

$$n \geq \left(\frac{1.96 \times 25}{3} \right)^2$$

$$n \geq \left(\frac{z_{\alpha/2} \cdot \sigma}{MOE} \right)^2$$

We want the sample to be at least
one minus z of alpha over two,

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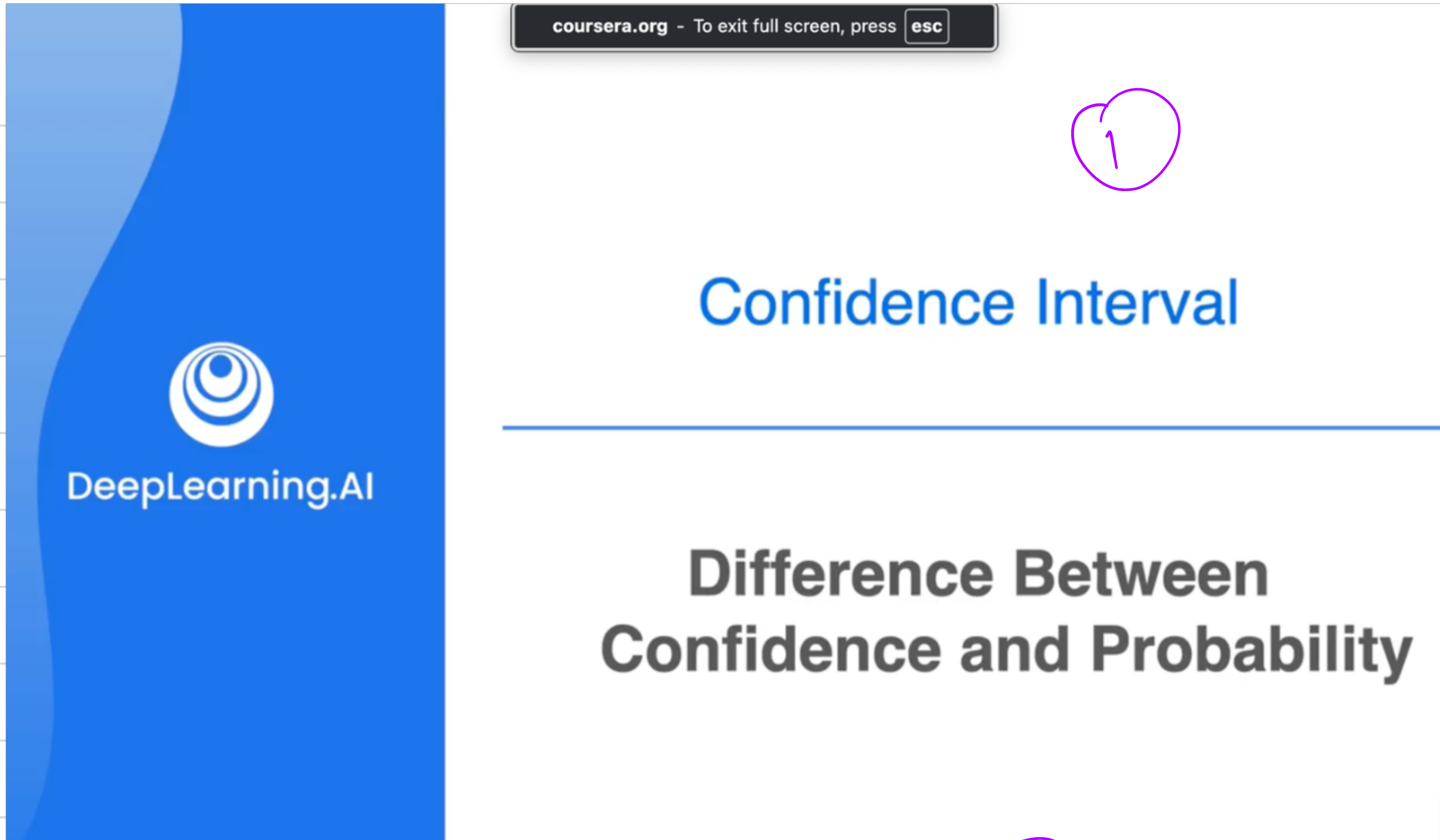

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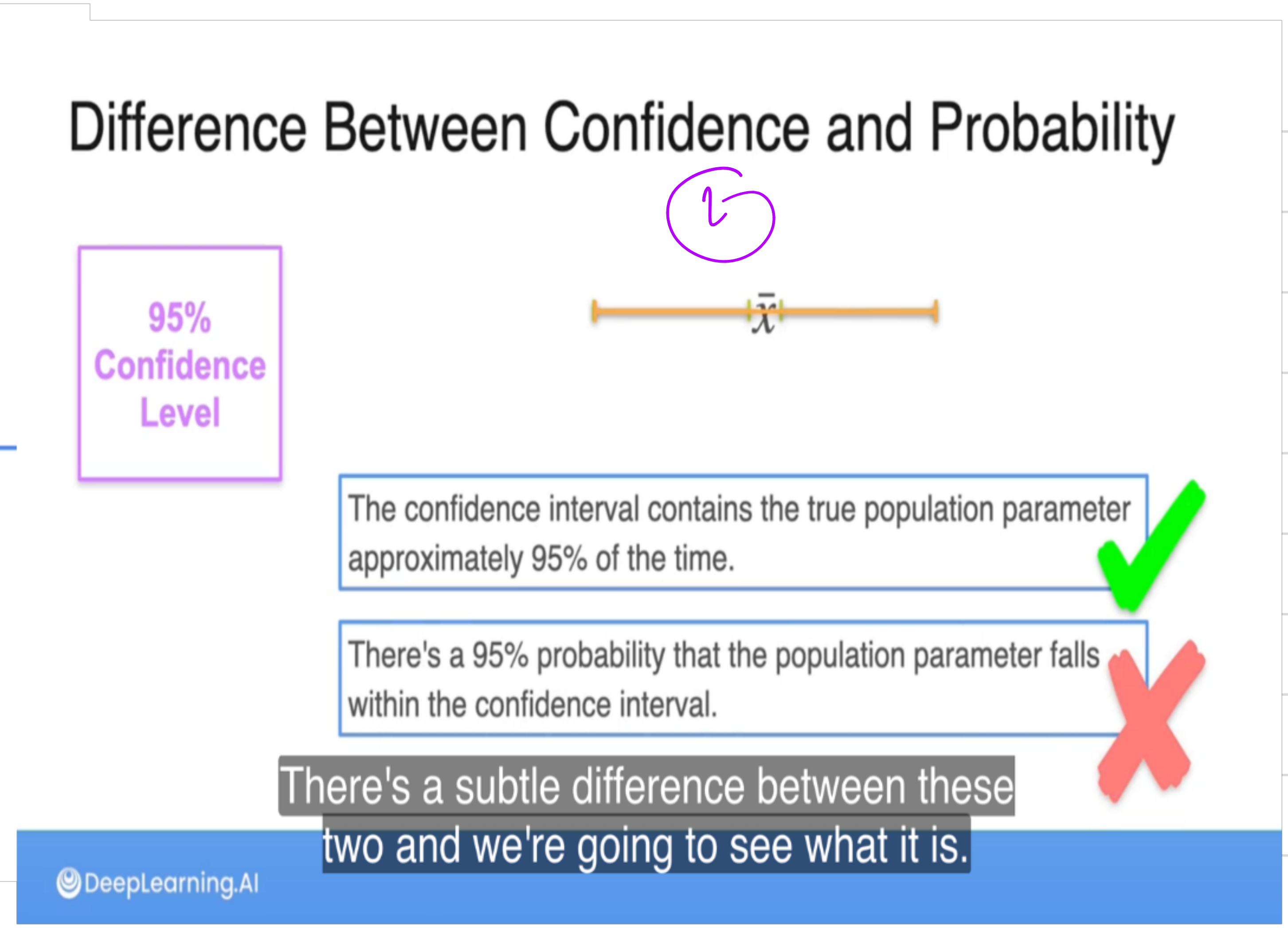

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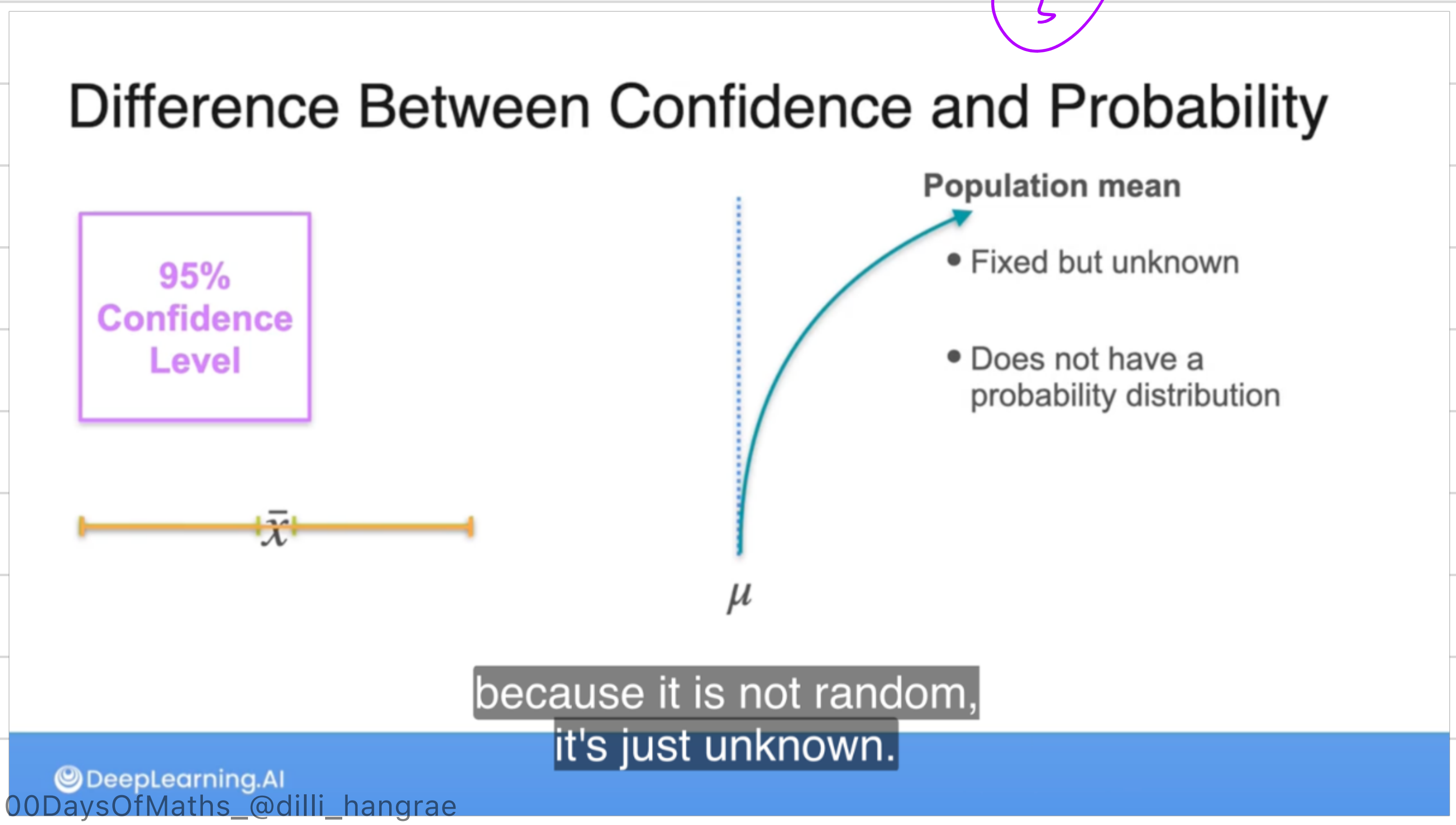


Why Degrees of Freedom Matter

Smaller degrees of freedom → fatter tails (greater variability).
Larger degrees of freedom → t-distribution approaches the normal distribution.

Practical Implication
In real-world data analysis, we often don’t know the population standard deviation.
Therefore, using the t-distribution with the appropriate degree of freedom is crucial for reliable confidence intervals, especially when sample sizes are small.

Key Points:

Population Mean (μ):
- Fixed but unknown.
- Does not have a probability distribution.
- It's either inside the confidence interval or not—there's no "probability" for this.

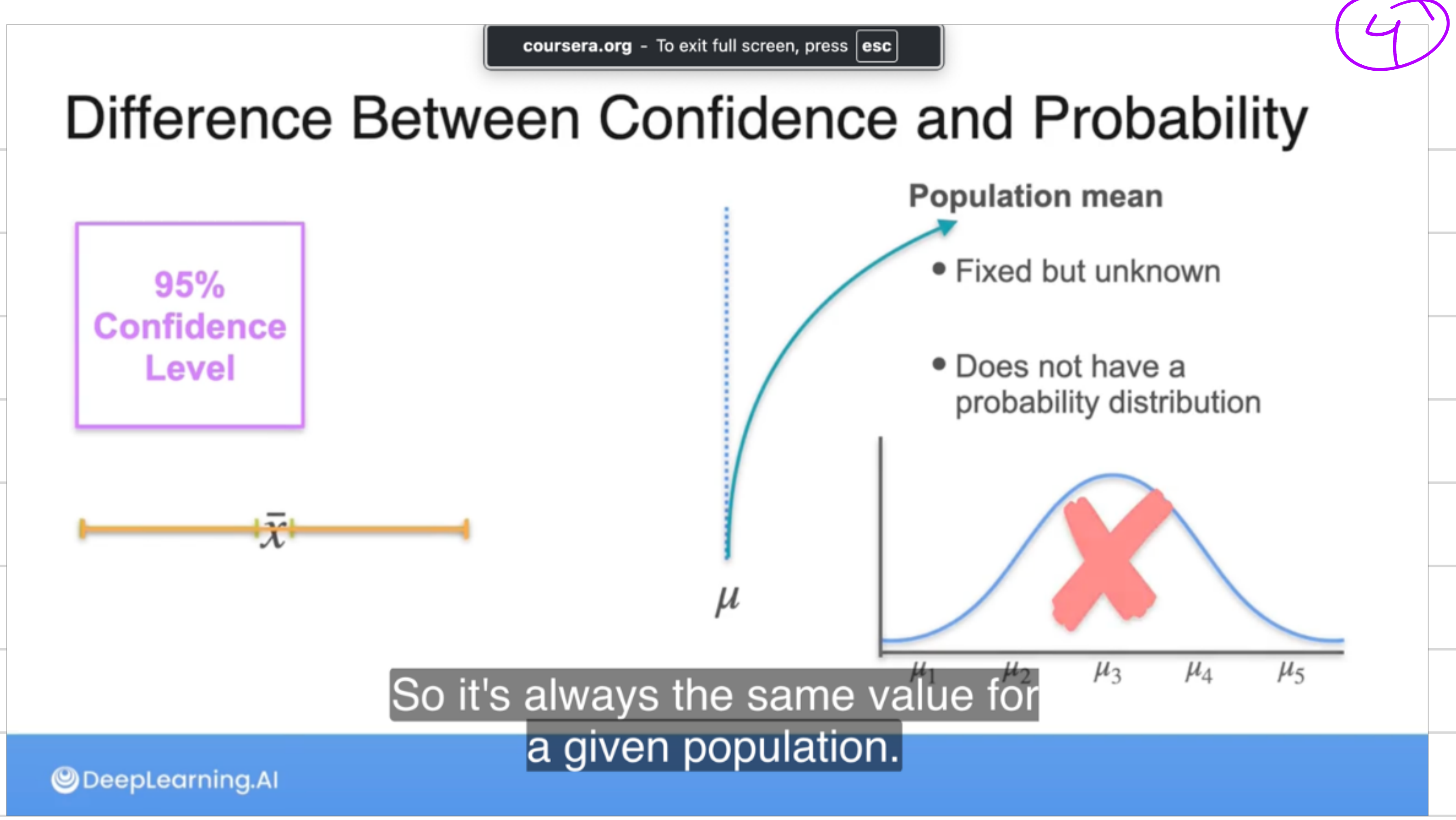
Confidence Interval:
- Based on the sampling distribution of the sample mean ($\bar{x}$).
- If you repeat the sampling process many times, 95% of the confidence intervals will contain the true population mean (μ).

Correct Interpretation:
- "The confidence interval contains the true population mean 95% of the time (over many samples)."

Incorrect Interpretation:
"There is a 95% probability that the population mean lies within this specific interval."
This is wrong because μ is fixed, not random.

Analogy:
Imagine shooting arrows at a target (the true mean). The confidence interval is like drawing a circle around where most of your arrows hit. Saying "95% of intervals contain the target" refers to how often your aim is accurate, not the probability of the target being in one circle—it's either in or out.

Takeaway:
Confidence intervals describe the reliability of the process, not the probability of μ being in a single interval.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Difference Between Confidence and Probability
95% Confidence Level
$\bar{x}$
μ
Population mean
Fixed but unknown
Does not have a probability distribution
μ_1 μ_2 μ_3 μ_4 μ_5
So it's always the same value for a given population.
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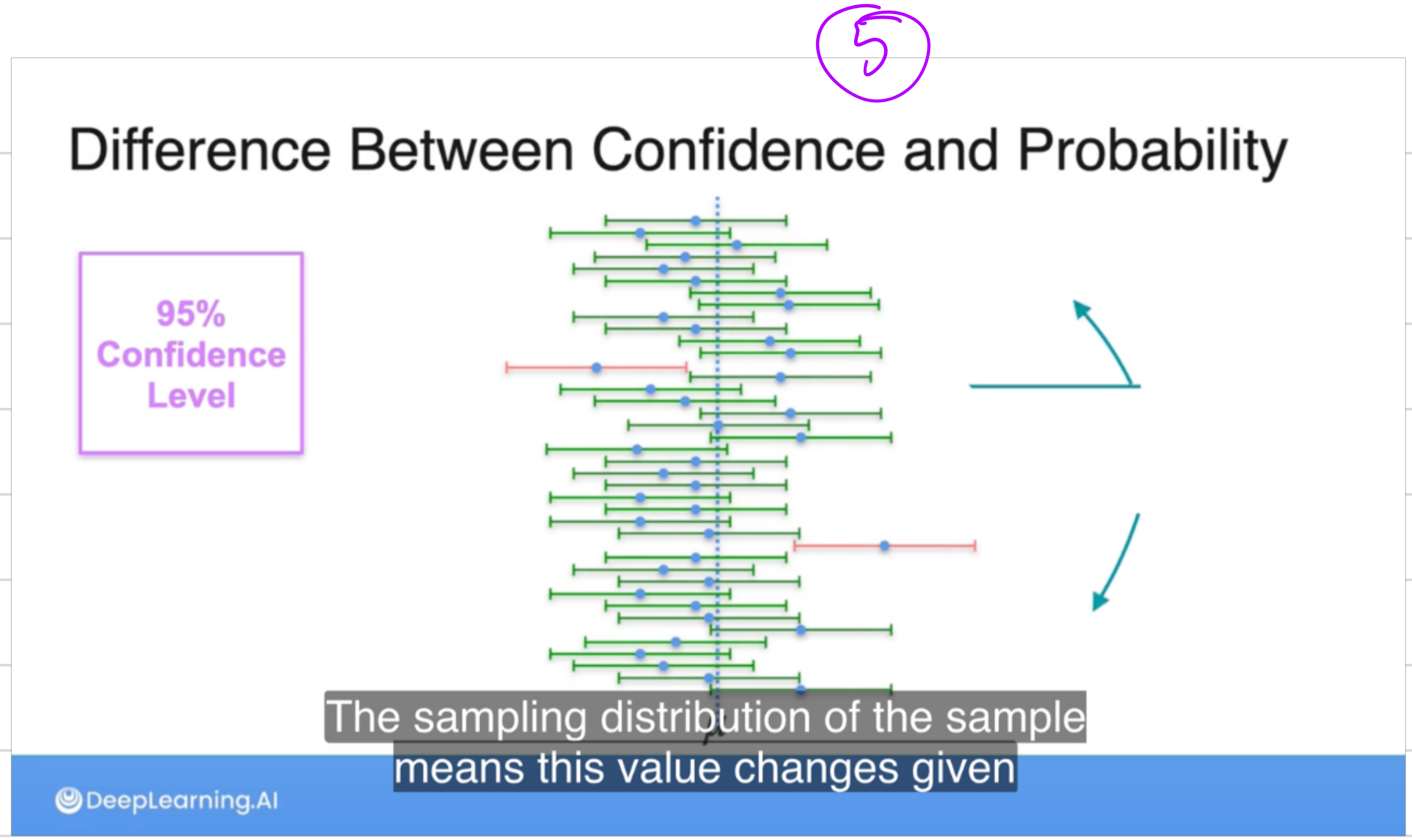
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Difference Between Confidence and Probability
95% Confidence Level
The sampling distribution of the sample means this value changes given
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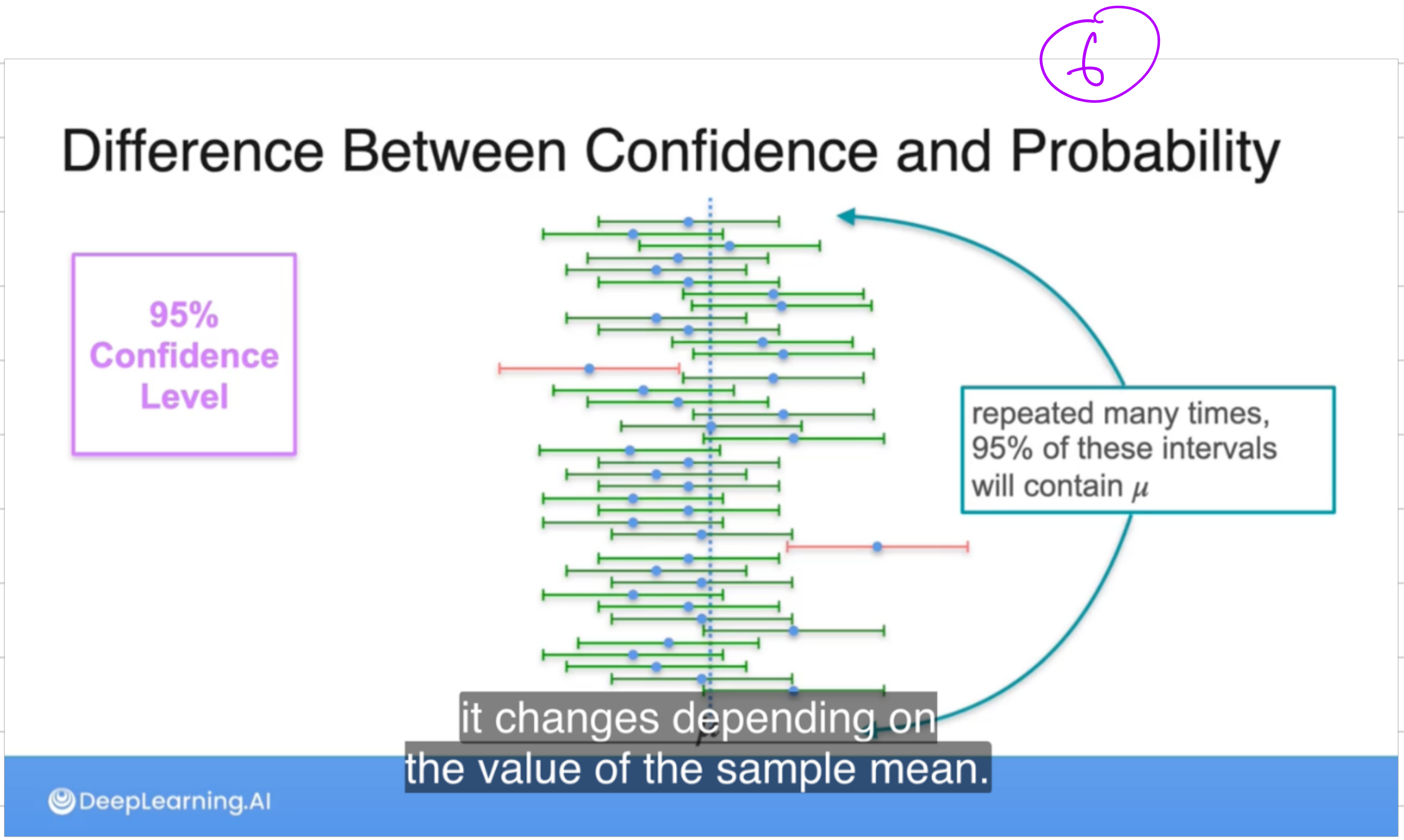
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Difference Between Confidence and Probability
95% Confidence Level
repeated many times, 95% of these intervals will contain μ
it changes depending on the value of the sample mean.
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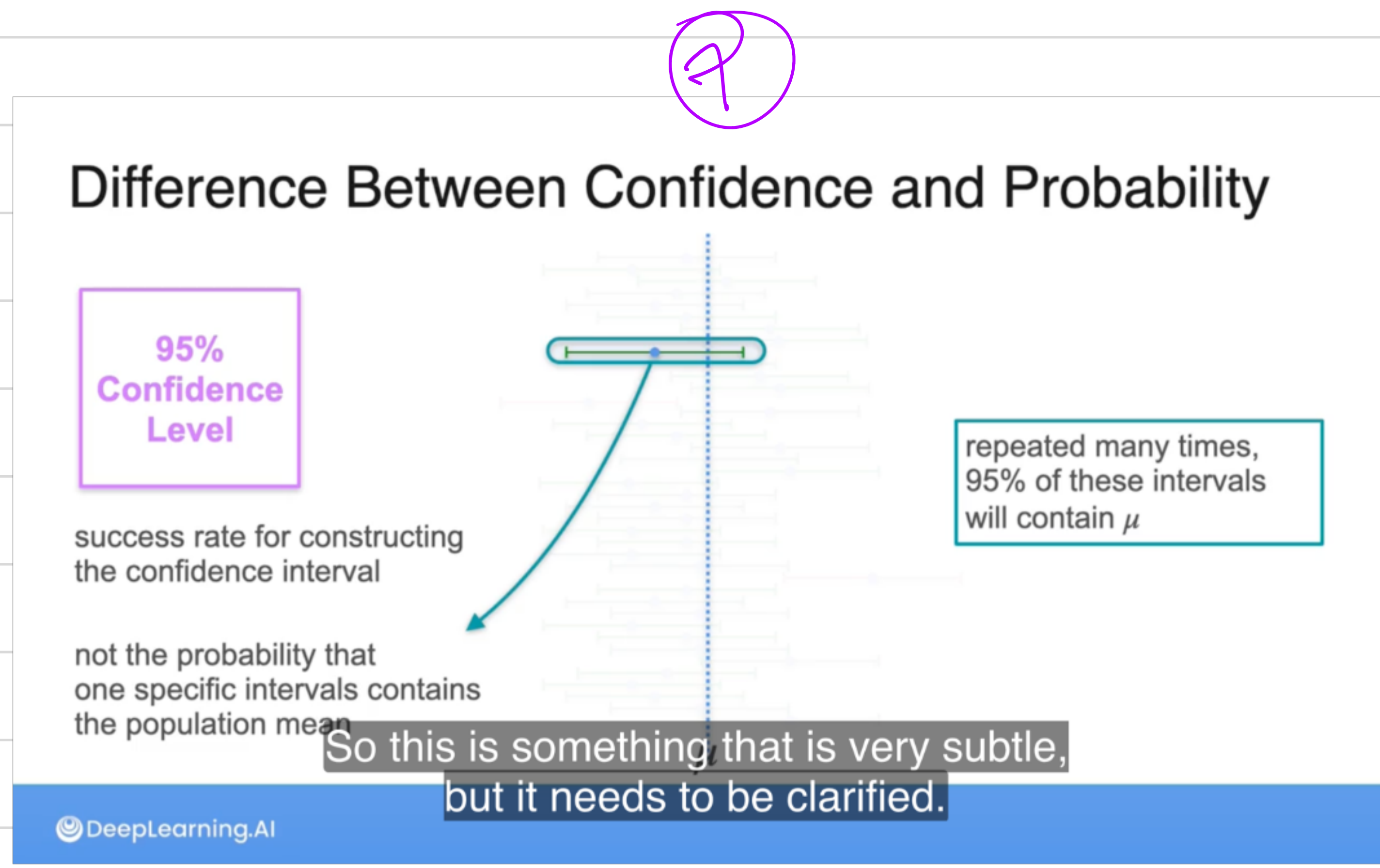
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Difference Between Confidence and Probability
95% Confidence Level
success rate for constructing the confidence interval
not the probability that one specific intervals contains the population mean
repeated many times, 95% of these intervals will contain μ
So this is something that is very subtle, but it needs to be clarified.
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Confidence Interval

Confidence Interval (Unknown Standard Deviation)

The explanation highlights a significant concept in inferential statistics: dealing with unknown population standard deviation and how the **Student's t-distribution** helps handle this scenario. Here's a simplified recap and practical understanding:

2

Key Takeaways

1. **Known vs Unknown Population Standard Deviation**
   - **Known Sigma (σ)**: Use the **Z-score** with the **Normal Distribution**.
   - **Unknown Sigma**: Use the **Sample Standard Deviation (s)** with the **Student's t-distribution**.
2. **Why the t-distribution?**
   - The t-distribution accounts for additional variability when estimating the population standard deviation using a sample.
   - It has **fatter tails** compared to the normal distribution, meaning it accounts for more variability and uncertainty.
3. **T-score vs Z-score**
   - **Z-score** assumes less uncertainty (known population standard deviation).
   - **T-score** adapts for sample variability and provides more accurate confidence intervals when σ is unknown.
4. **Degrees of Freedom (df)**
   - Degrees of freedom (df) = $n - 1$, where n is the sample size.
   - The larger the sample size, the closer the t-distribution becomes to the normal distribution.

Confidence Interval - *t* Distribution

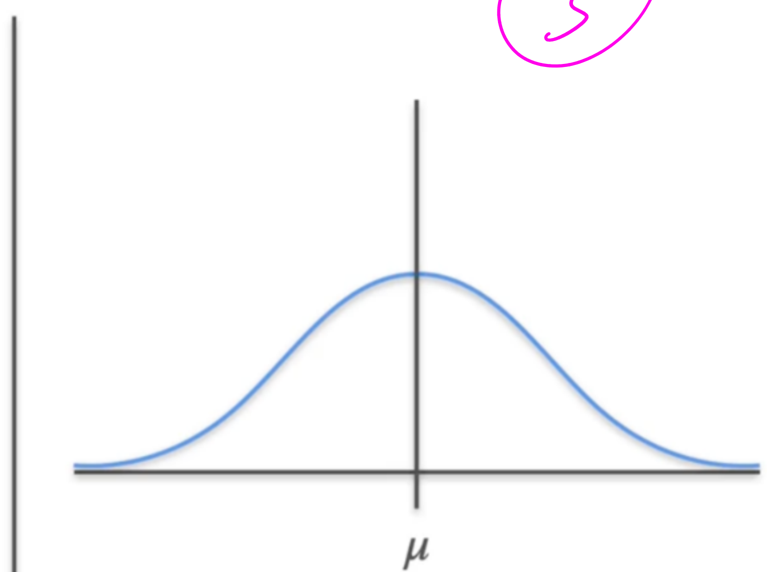
3

known σ ? → s

$$\bar{x} \pm \boxed{z_{1-\alpha/2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \dashrightarrow \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

normal distribution



However, this quantity is no longer a normal distribution.

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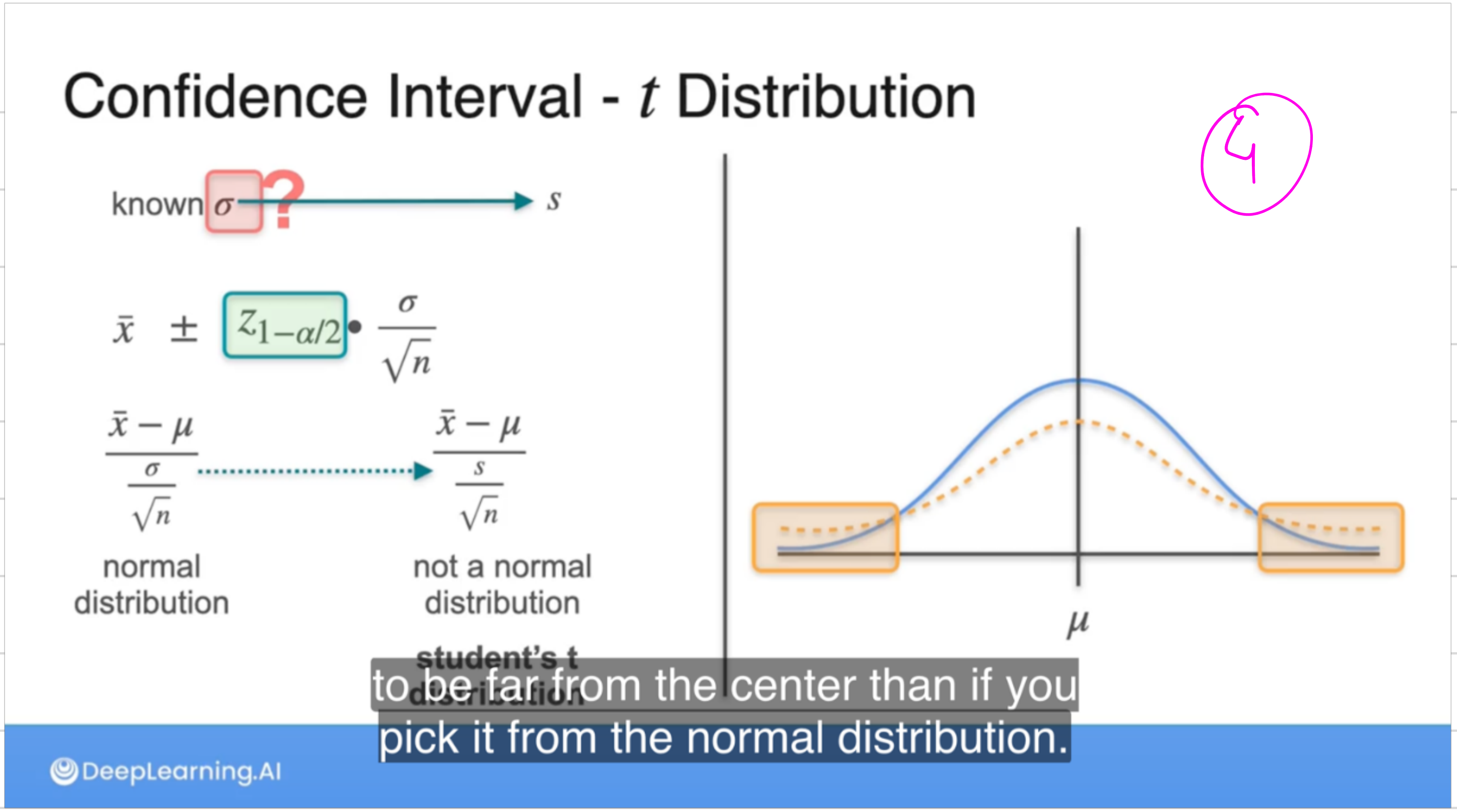
Confidence Interval - t Distribution
4
known σ ? s
$\bar{x} \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$
$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$
$\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$
normal distribution
not a normal distribution
student's t distribution
μ
to be far from the center than if you
pick it from the normal distribution.
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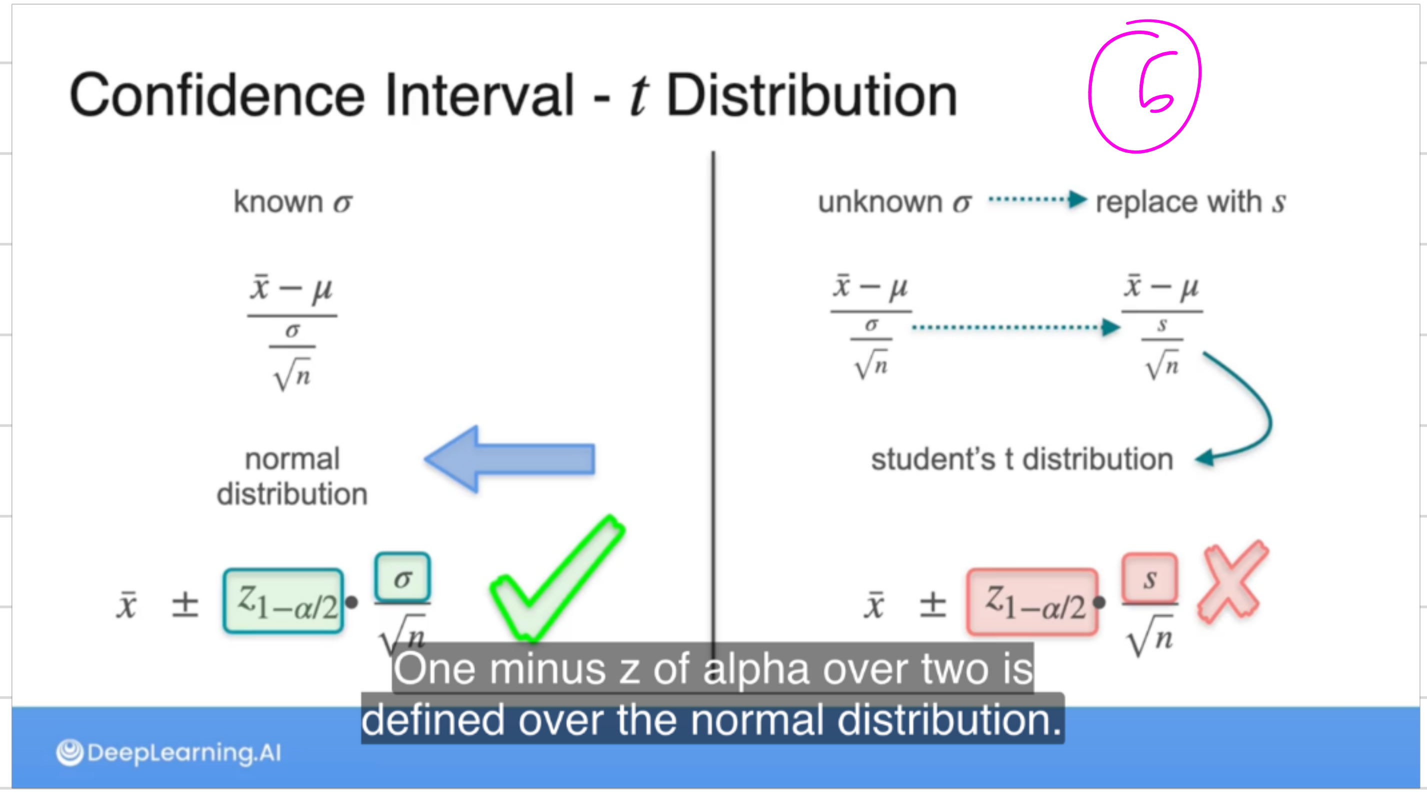
Confidence Interval - t Distribution
5
known σ
unknown σ replace with s
$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$
$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$
$\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$
normal distribution
student's t distribution
$\bar{x} \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$
$\bar{x} \pm z_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}}$
One minus z of alpha over two is
defined over the normal distribution.
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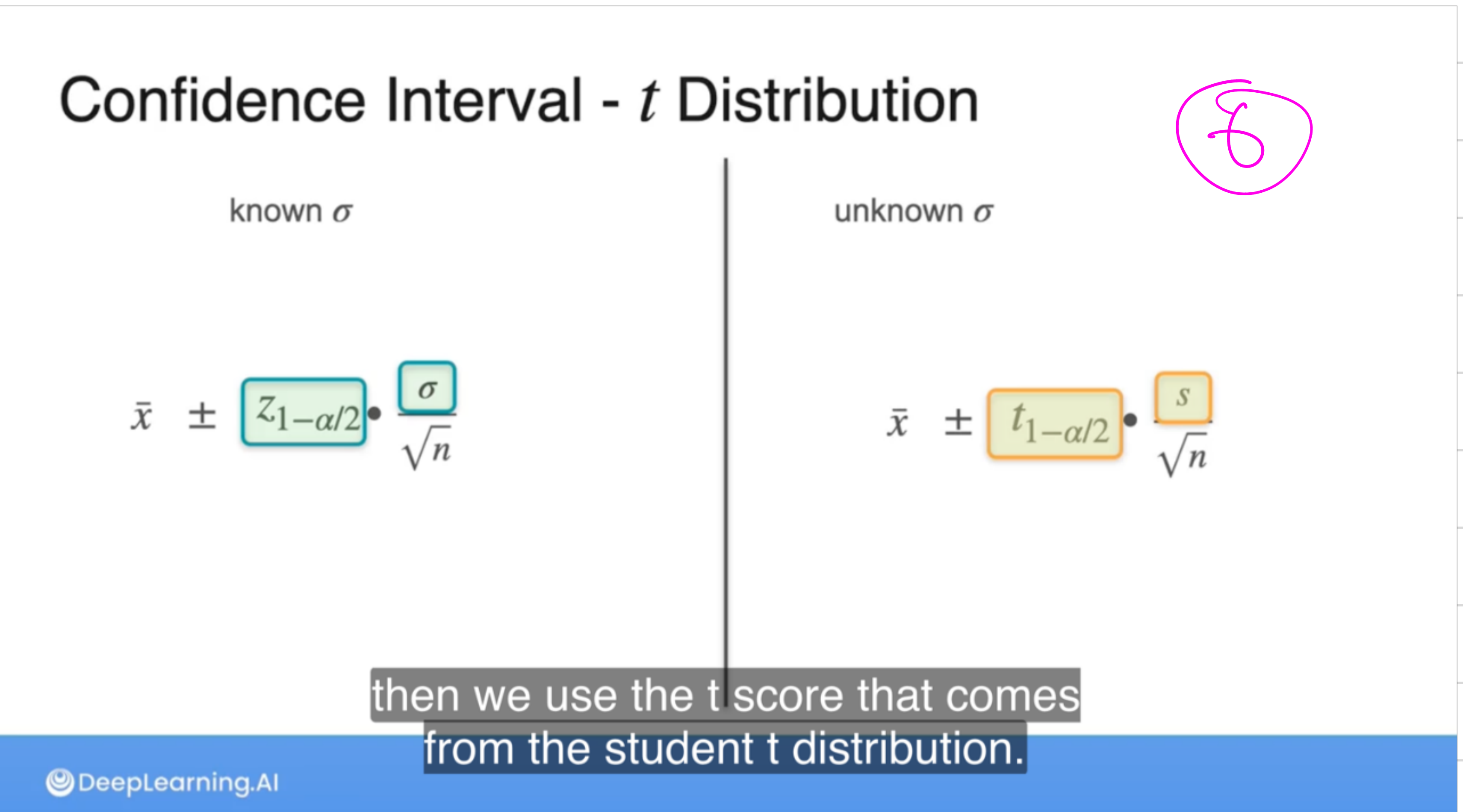
Confidence Interval - t Distribution
6
known σ
unknown σ
$\bar{x} \pm z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$
$\bar{x} \pm t_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}}$
then we use the t score that comes
from the student t distribution.
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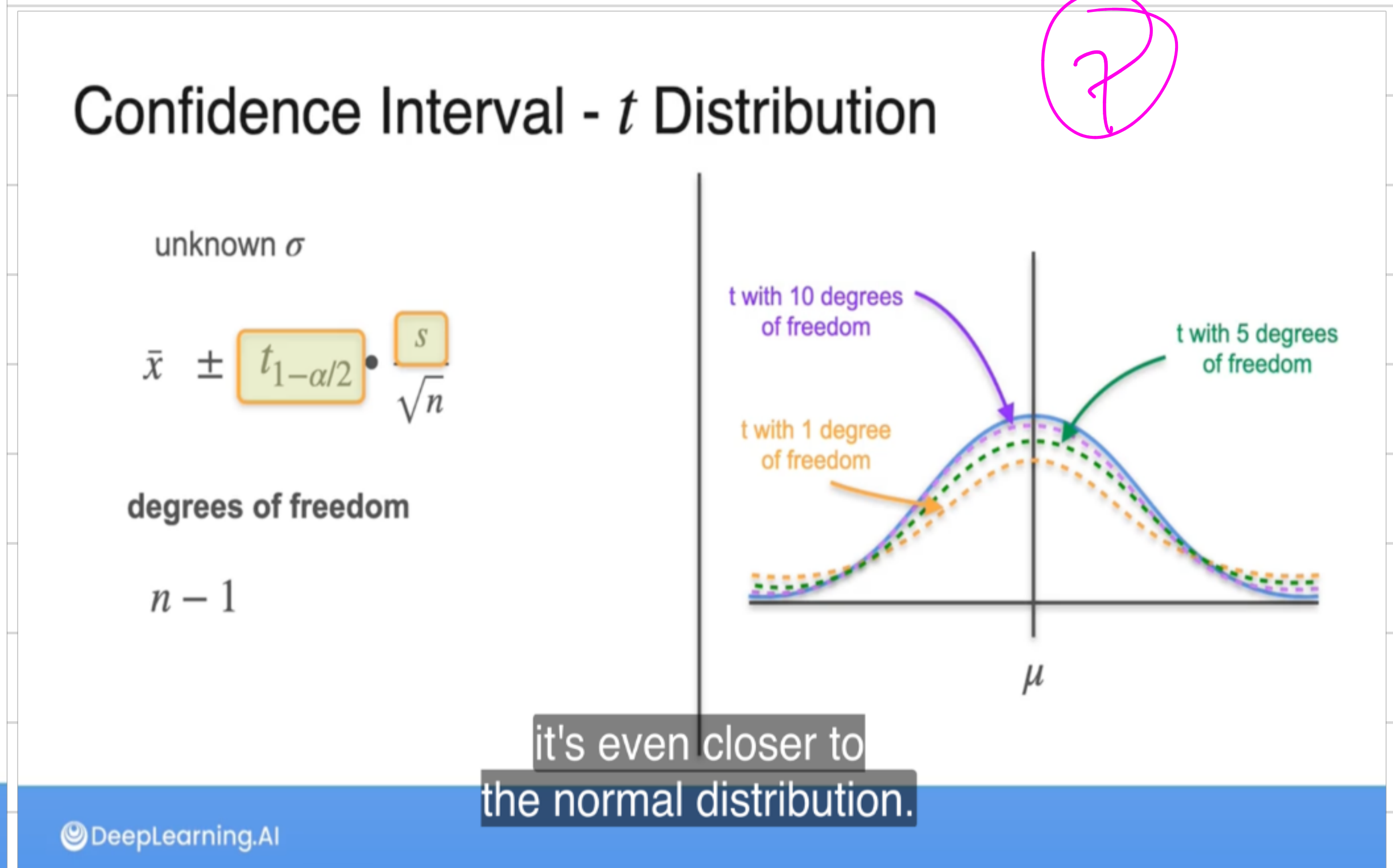
Confidence Interval - t Distribution
7
unknown σ
$\bar{x} \pm t_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}}$
degrees of freedom
$n-1$
t with 10 degrees of freedom
t with 5 degrees of freedom
t with 1 degree of freedom
μ
it's even closer to
the normal distribution.
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Confidence Interval for Proportions

Confidence Interval for Means

confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{1-\alpha/2} \bullet \frac{\sigma}{\sqrt{n}}$

Confidence Interval for Proportions

$x = 24$

$n = 30$

$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$

How do you calculate a 95% confidence interval for this sample proportion?

this sample proportion?

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Confidence Interval for Proportions

Confidence Interval for Means

confidence interval $= \bar{x} \pm$ margin of error

margin of error $= z_{1-\alpha/2} \bullet \boxed{\frac{\sigma}{\sqrt{n}}}$

Confidence Interval for Proportions

$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\,\%$

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{1-\alpha/2} \bullet \boxed{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$

standard error

So, quick recap.

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Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= z_{1-\alpha/2} \bullet \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

margin of error $= 1.96 \bullet \sqrt{\frac{0.8(1-0.8)}{30}}$

margin of error $= 0.14$

$n = 30 \qquad \hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

$\boxed{95\%} \longrightarrow z_{1-\alpha/2} = 1.96$

And that gives us a margin of error of zero point 14.

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Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval $= \hat{p} \pm$ margin of error

margin of error $= 0.14$

confidence interval $= 0.8 \pm 0.14$

$0.66 < p < 0.94$

$66\,\% < p < 94\,\%$

$n = 30 \qquad \hat{p} = 80\,\% = 0.8$

Calculate a 95% confidence interval for this sample proportion

$\boxed{95\%} \longrightarrow z_{1-\alpha/2} = 1.96$

So.

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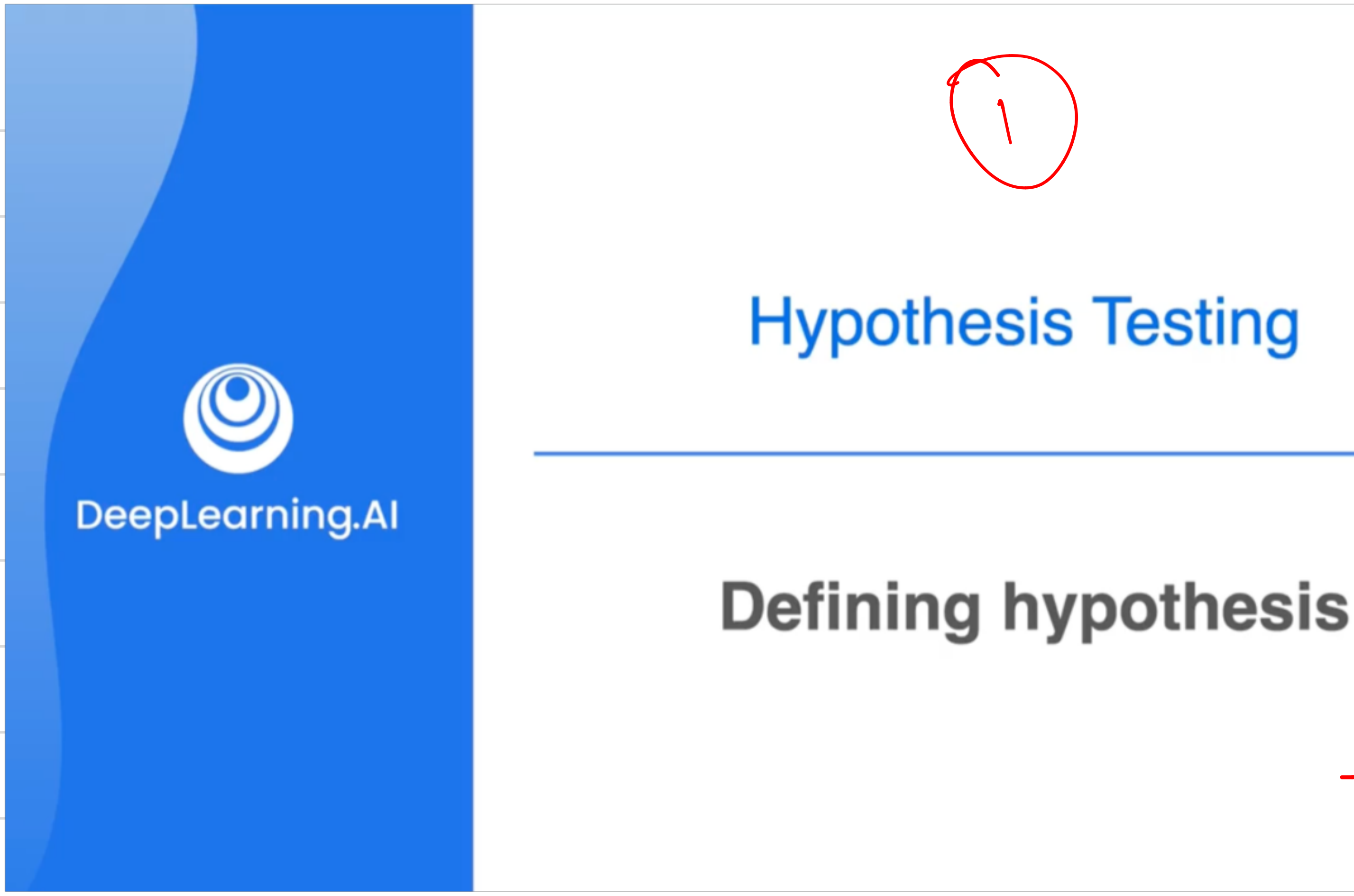

Evidence-Based Decision Making
If sufficient evidence (such as spammy phrases like "earn extra cash" or "winner") is found against the null hypothesis, you reject H_0 and accept H_1. However, failure to reject H_0 doesn't prove H_0 is true; it only means there isn't enough evidence for H_1.

Why the Asymmetry Matters
In hypothesis testing, rejecting the null hypothesis has more definitive implications than failing to reject it.
In the spam example, rejecting H_0 sends an email to spam, but not rejecting it doesn't guarantee that the email is genuinely important.

Practical Applications (A/B Testing)
One important use case of hypothesis testing is A/B Testing, where companies compare two versions of a product (like different website layouts) to see which performs better based on collected data.

Key Takeaways from Hypothesis Testing

Definition:
Hypothesis testing is a statistical method to decide whether a belief about a population is likely true based on sample data.
Important Concepts

Null Hypothesis (H_0)
The default assumption that nothing unusual is happening.
In the spam detection example: "The email is ham (not spam)."

Alternative Hypothesis (H_1)
The competing statement you're trying to prove.
In this example: "The email is spam."

Motivation

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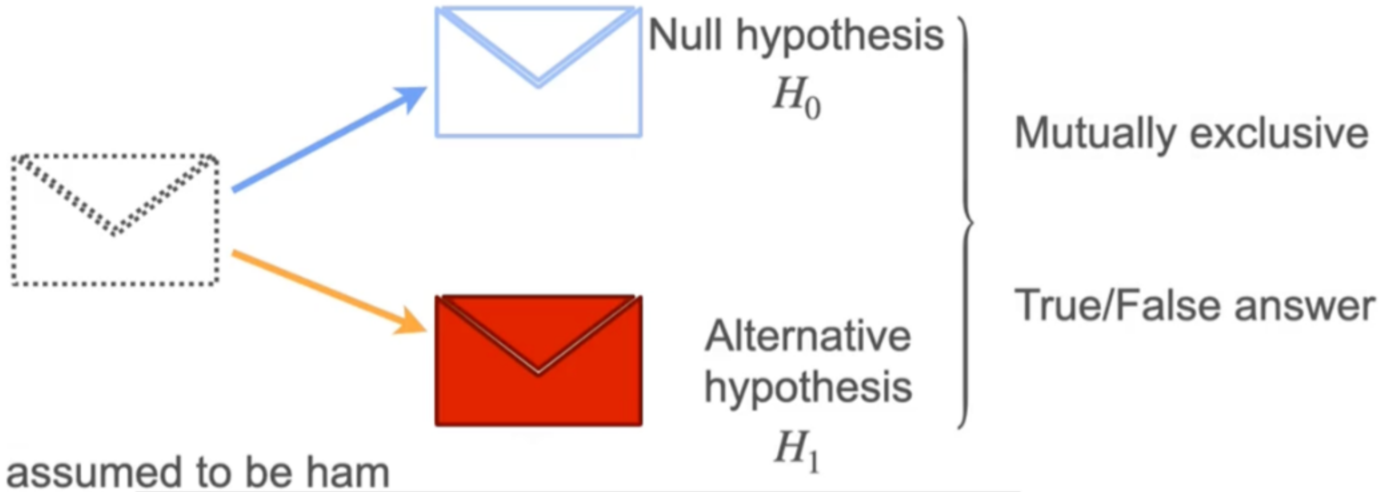

Because the email can't be ham and spam at the same time.

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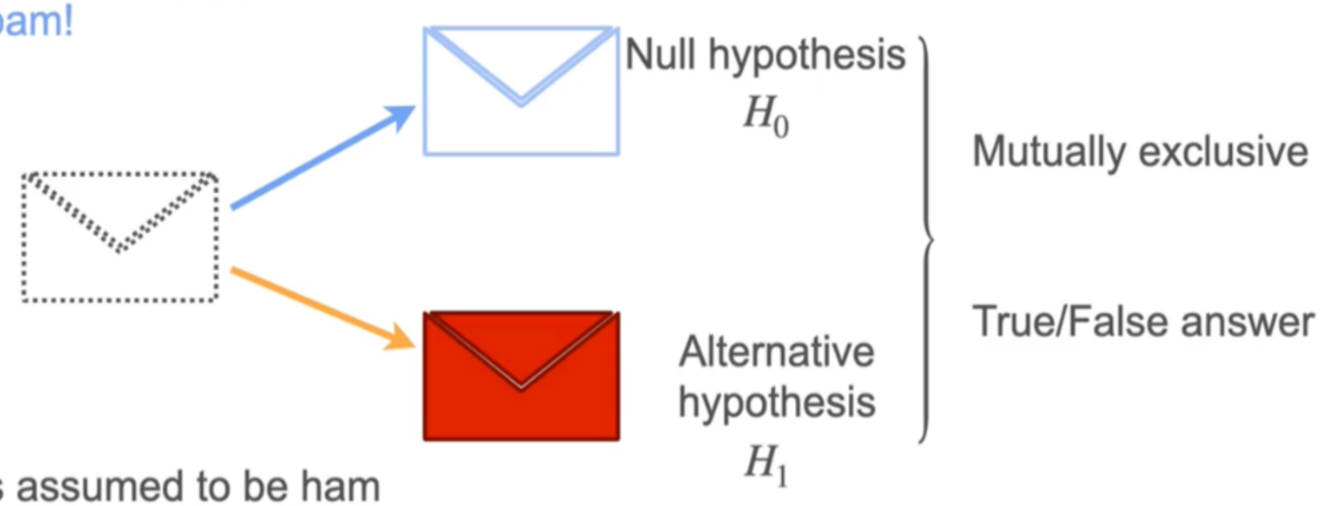
Motivation

3

When rejecting that the email is not spam, you are accepting that the email is spam!

By failing to reject that the email IS spam, you are **not** accepting that it's ham



then you can reject the null hypothesis.

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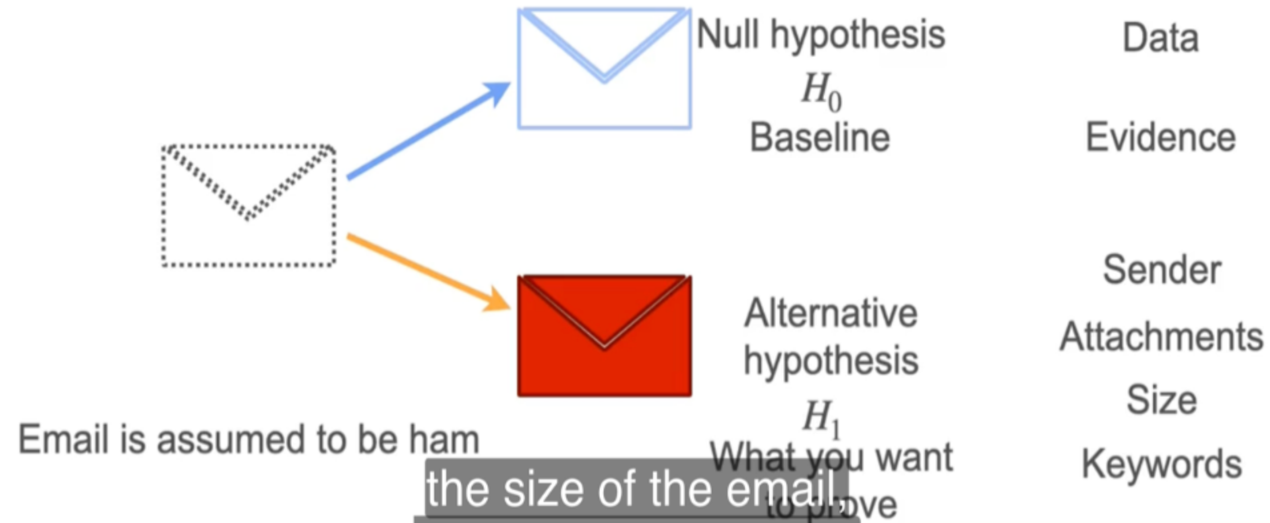
Motivation

4

Not labeling the email spam, doesn't mean the email is ham!



the size of the email, certain keywords, etc.

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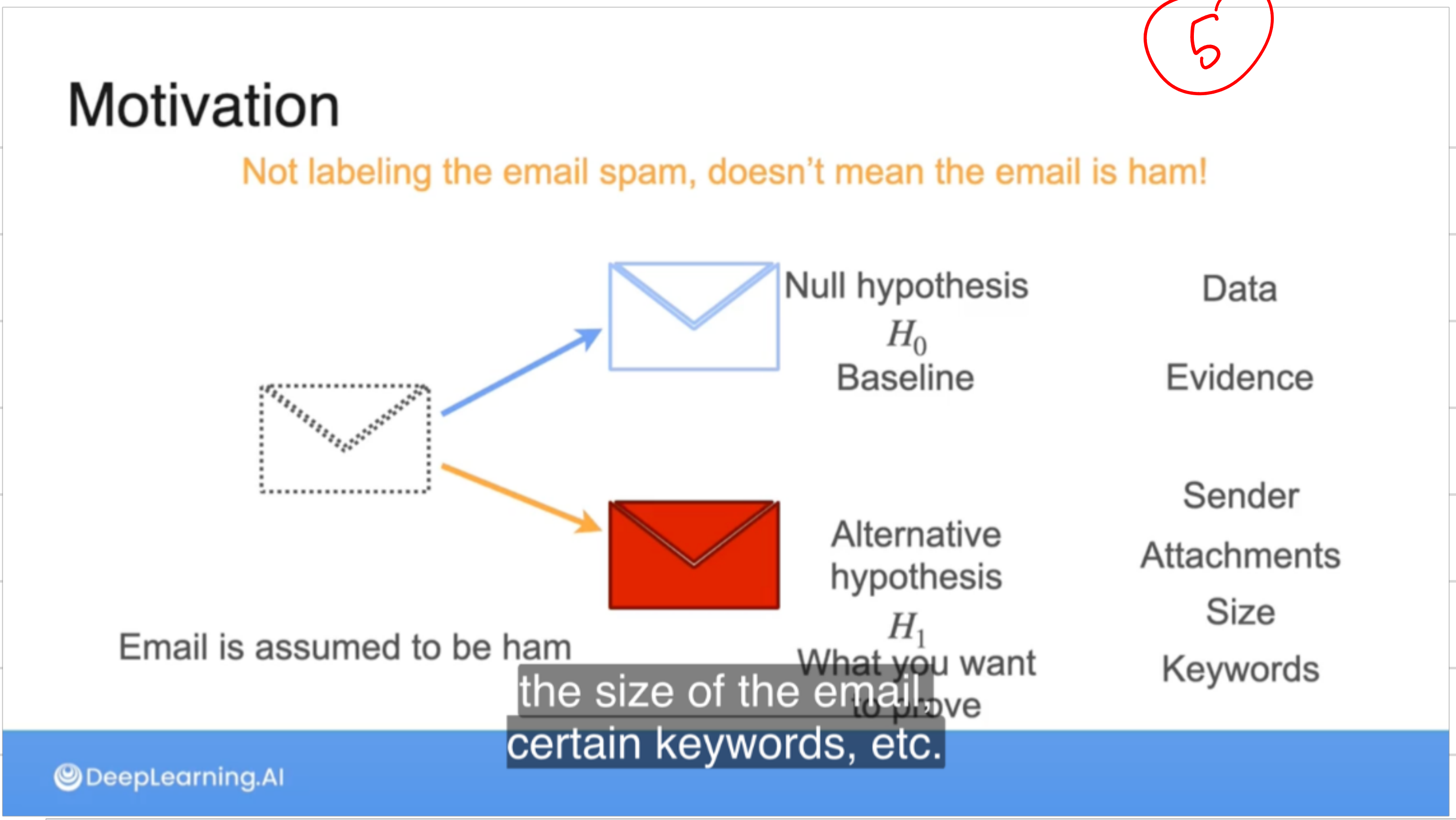

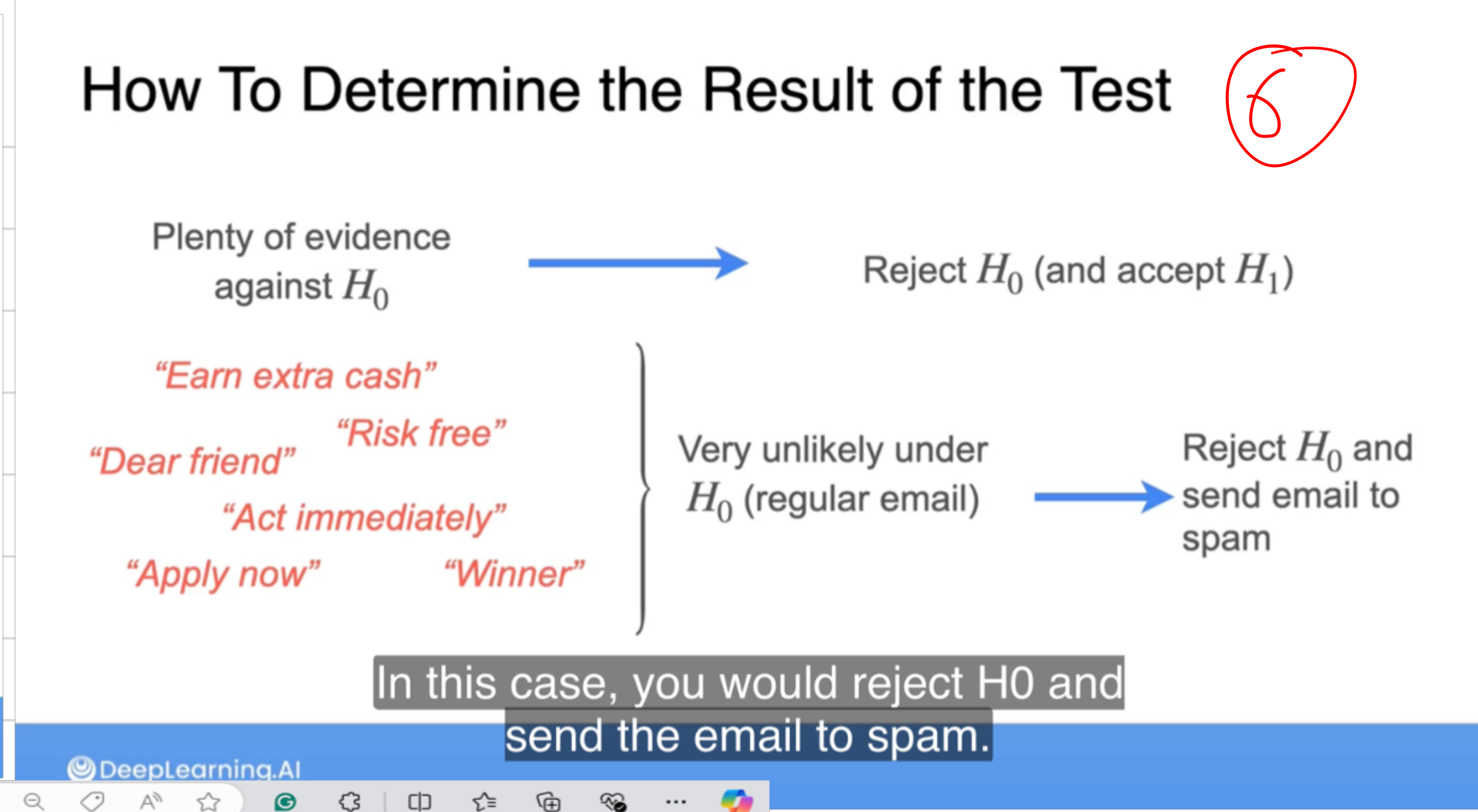

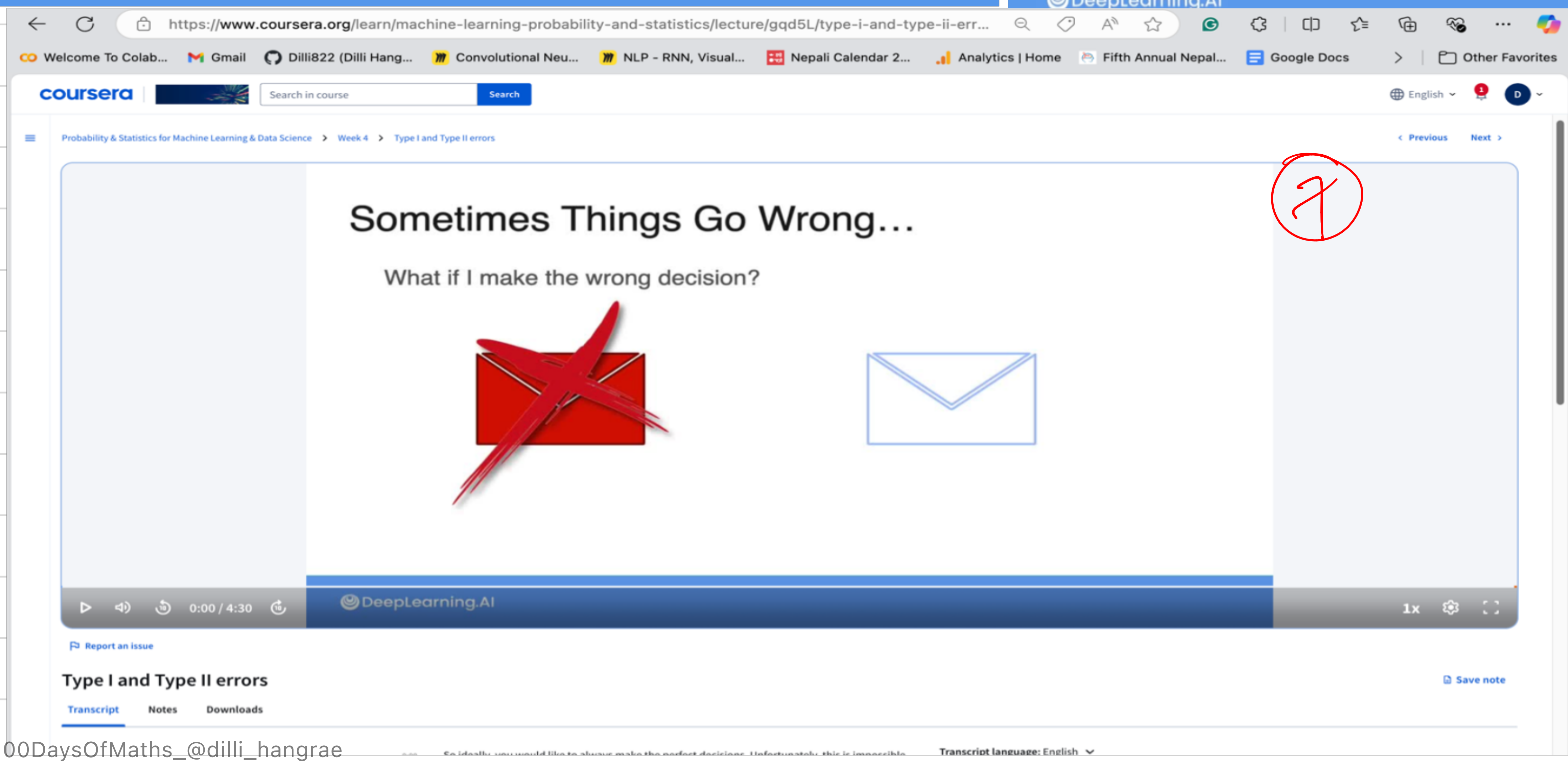

Type I and
Type II error.

Sometimes Things Go Wrong…

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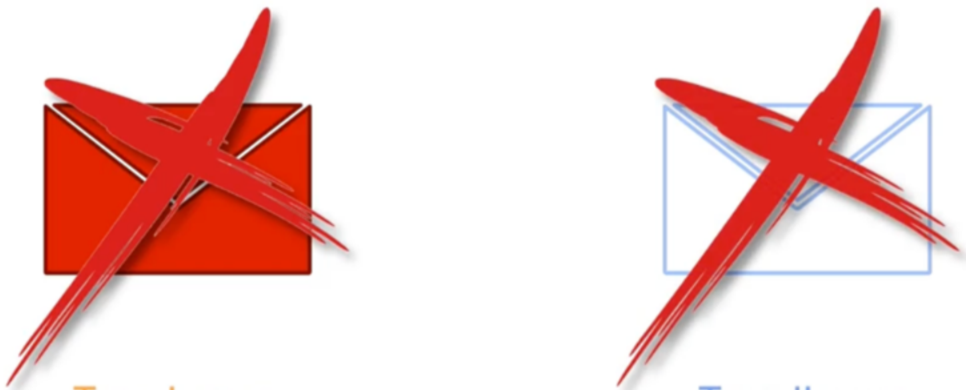
What if I make the wrong decision?



Type I error
(False positive)

Type II error
(False negative)

ham to be a regular email.

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A type I error occurs when we incorrectly reject the null hypothesis when it is actually true, while a type II error occurs when we fail to reject the null hypothesis when it is actually false.

Type I and Type II Errors

9

Decision	Reality	
	H_0 True (Not spam)	H_0 False (Spam)
Reject H_0 (Decide spam)	Type I error	Correct
Don't reject H_0 (Decide not spam)	Correct	Type II error

even though Mail is not Spam, System Decide Spam but don't send to Direct inbox

That means you took a spam email and sent it to the inbox.

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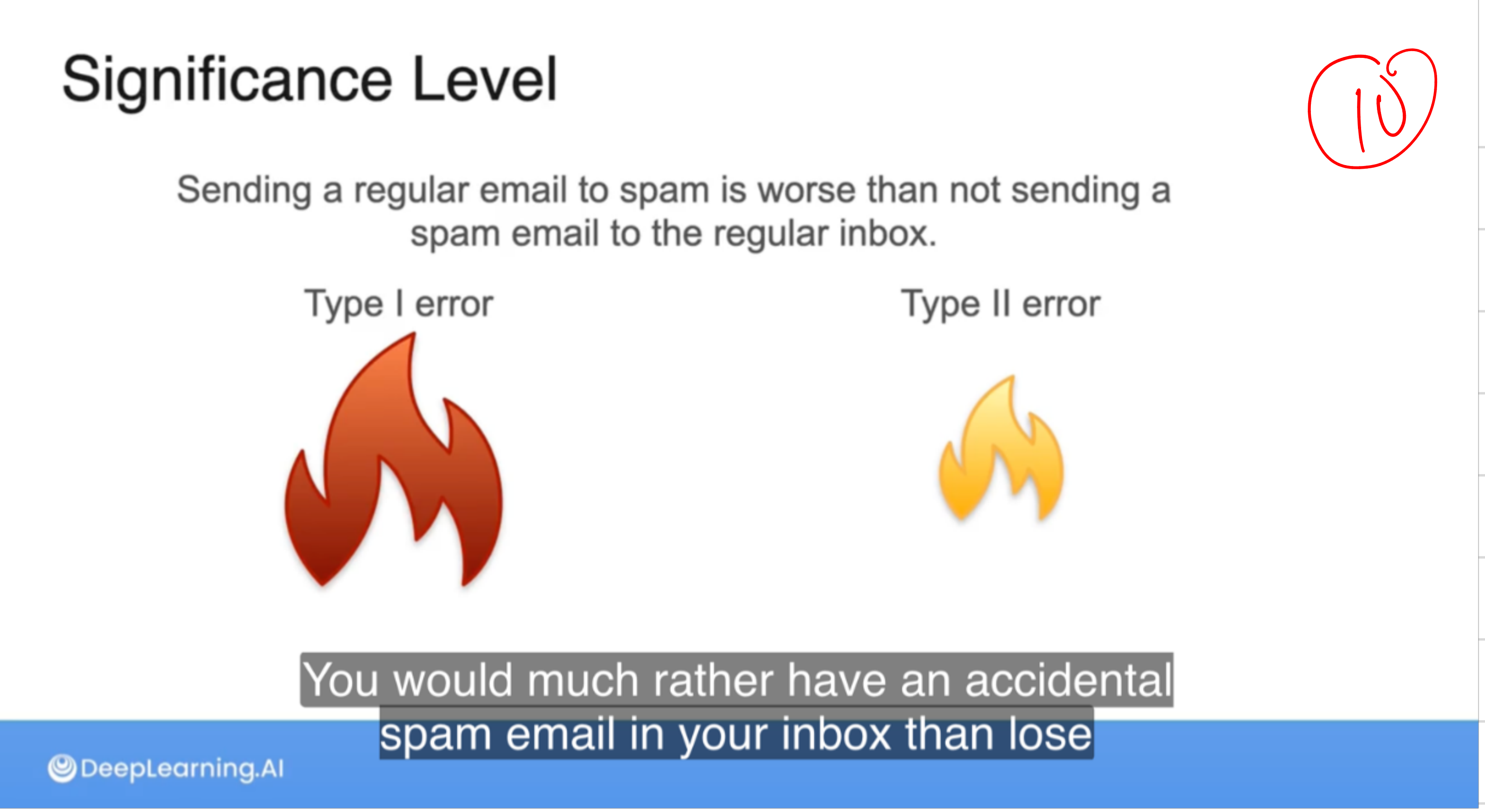
10
Significance Level
Sending a regular email to spam is worse than not sending a spam email to the regular inbox.
Type I error
Type II error
You would much rather have an accidental
spam email in your inbox than lose
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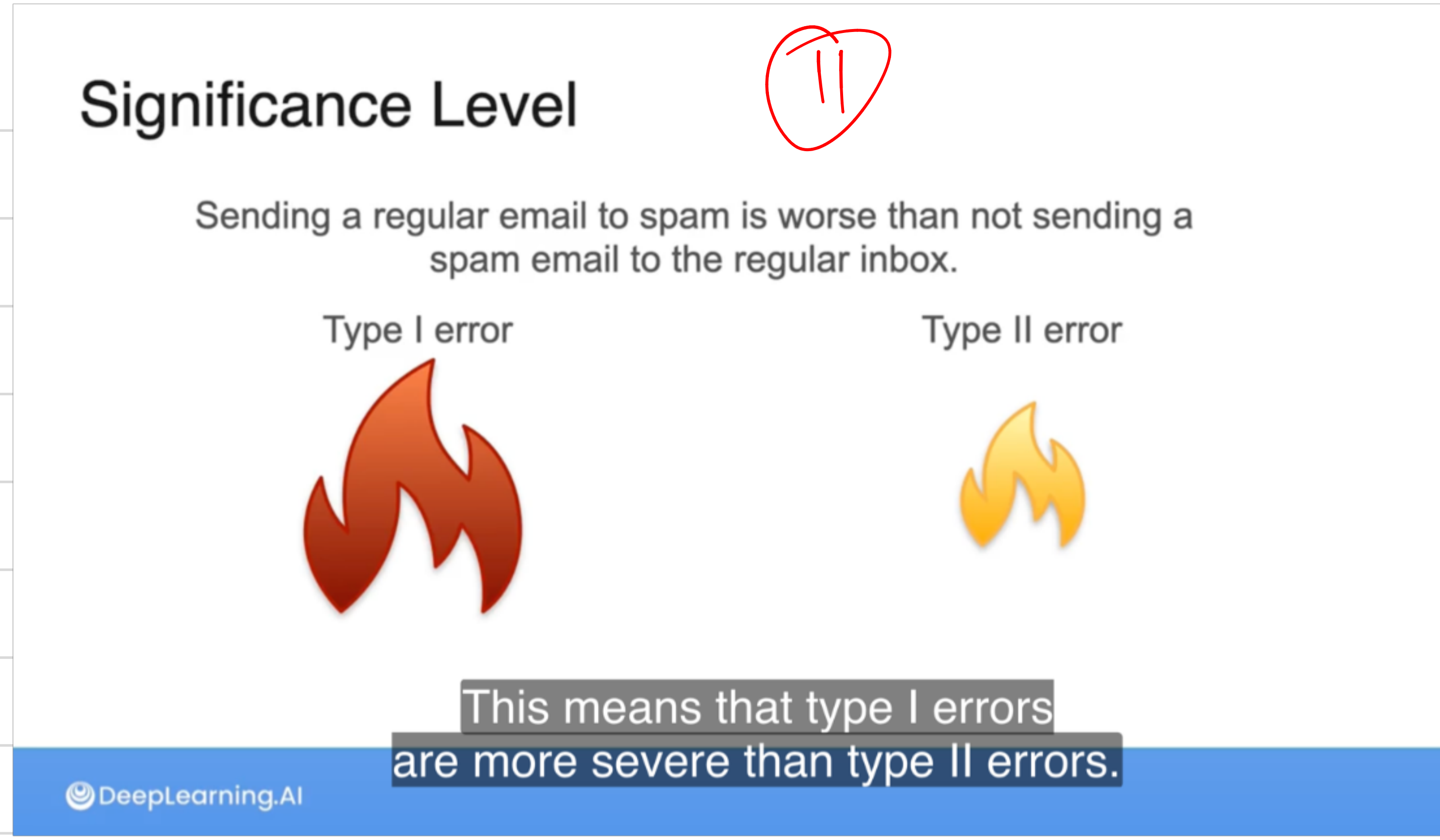
11
Significance Level
Sending a regular email to spam is worse than not sending a spam email to the regular inbox.
Type I error
Type II error
This means that type I errors
are more severe than type II errors.
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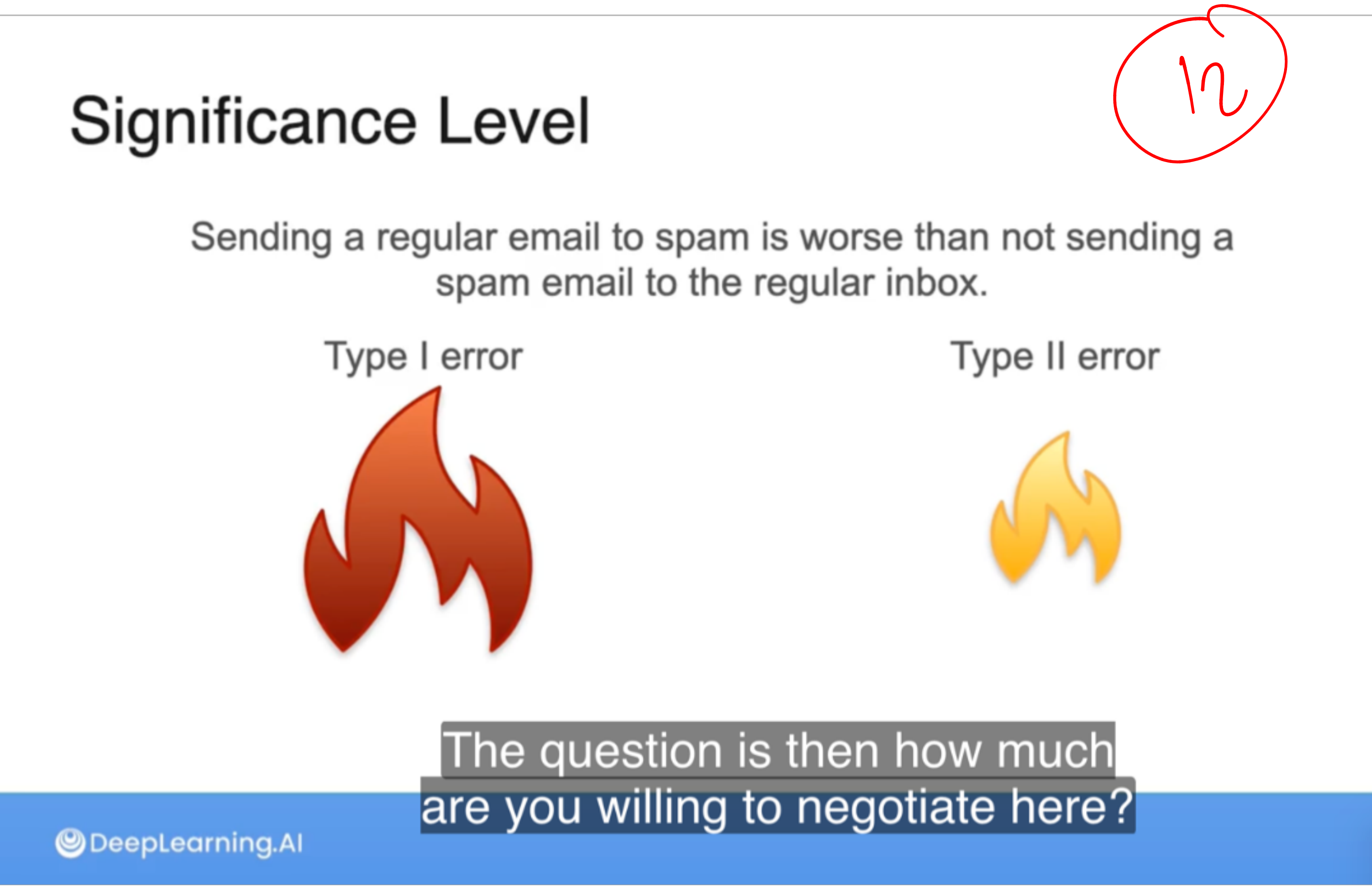
12
Significance Level
Sending a regular email to spam is worse than not sending a spam email to the regular inbox.
Type I error
Type II error
The question is then how much
are you willing to negotiate here?
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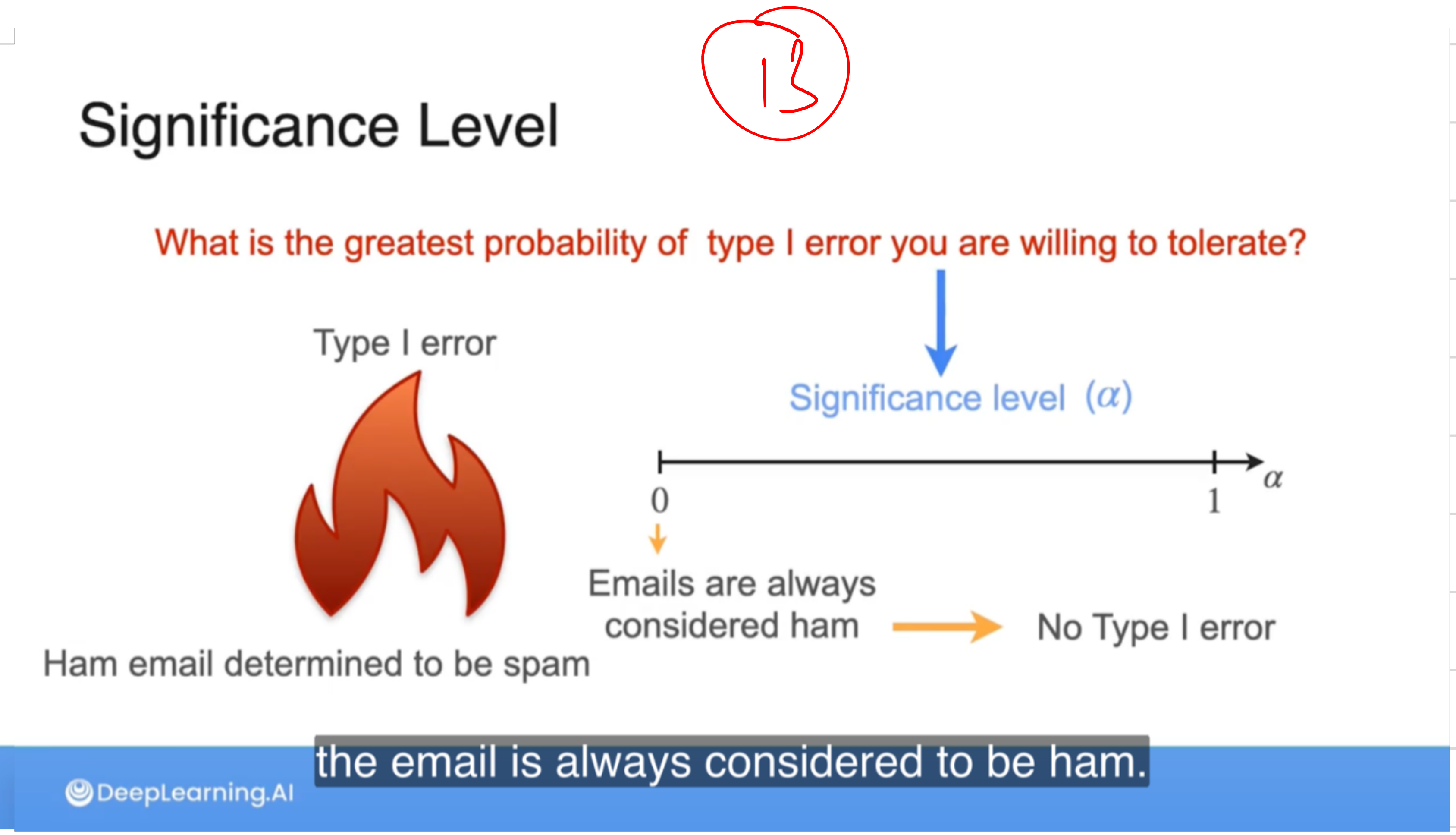
13
Significance Level
What is the greatest probability of type I error you are willing to tolerate?
Type I error
Significance level (α)
0
1
α
Emails are always considered ham
No Type I error
Ham email determined to be spam
the email is always considered to be ham.
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Significance Level

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What is the greatest probability of type I error you are willing to tolerate?

Significance level (α)

Type I error

Ham email determined to be spam

0 — 1 — α

Emails are always considered ham → No Type I error

the email is always considered to be ham.

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Significance Level

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Significance level (α)

0 — 1 — α

0: Every email is considered ham → No Type I error

1: Every email is considered spam → Every time an email is ham you make a Type I error

Of course, both extremes are terrible decision makers.

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Significance Level

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Significance level (α)

$\alpha = 0.05$

0 — 1 — α

0: Every email is considered ham → No Type I error

Both options are awful decision makers

You want a small α, but not zero

Catch:

↓Type I error ↑Type II error

1: Every email is considered spam → Every time an email is ham you make a Type I error

As we said, you want alpha to be as small as possible.

Source: Coursera Probability & Statistics for Machine Learning & Data Science.

Type I and Type II Errors in Hypothesis Testing

Type I Error (False Positive)

Occurs when you reject the null hypothesis (H_0) when it is actually true.

You assume there is an effect or difference when there isn't one.

Example: A spam filter marks a legitimate email as spam (declaring spam when it's actually not).

Probability: Denoted by α (alpha), which is the significance level (commonly set at 0.05).

Type II Error (False Negative)

Occurs when you fail to reject the null hypothesis when it is actually false.

You assume there is no effect or difference when one actually exists.

Example: A spam filter allows a spam email into the inbox (declaring it as ham when it's actually spam).

Probability: Denoted by β (beta).

Summary Table

Error Type.	Decision Made	Truth
Type I	Reject H_0 (Declare Spam)	H_0 True (Email is Ham)
Type II	Fail to Reject H_0 (Declare Ham)	H_0 False (Email is Spam)

Key Points

Minimizing Errors: Lowering α reduces Type I error but may increase Type II error, and vice versa.

Balancing Errors: Choose the significance level based on the consequences of each error (critical in medical tests, security systems, etc.).