Day - 92, Mar 2, 2025 (Falgun 18, 2081)

① Introduction: PCA

② Singularity and Rank of Linear Transformations

③ Determinant as an area

④ Determinant of a product

⑤ Determinant of Inverses

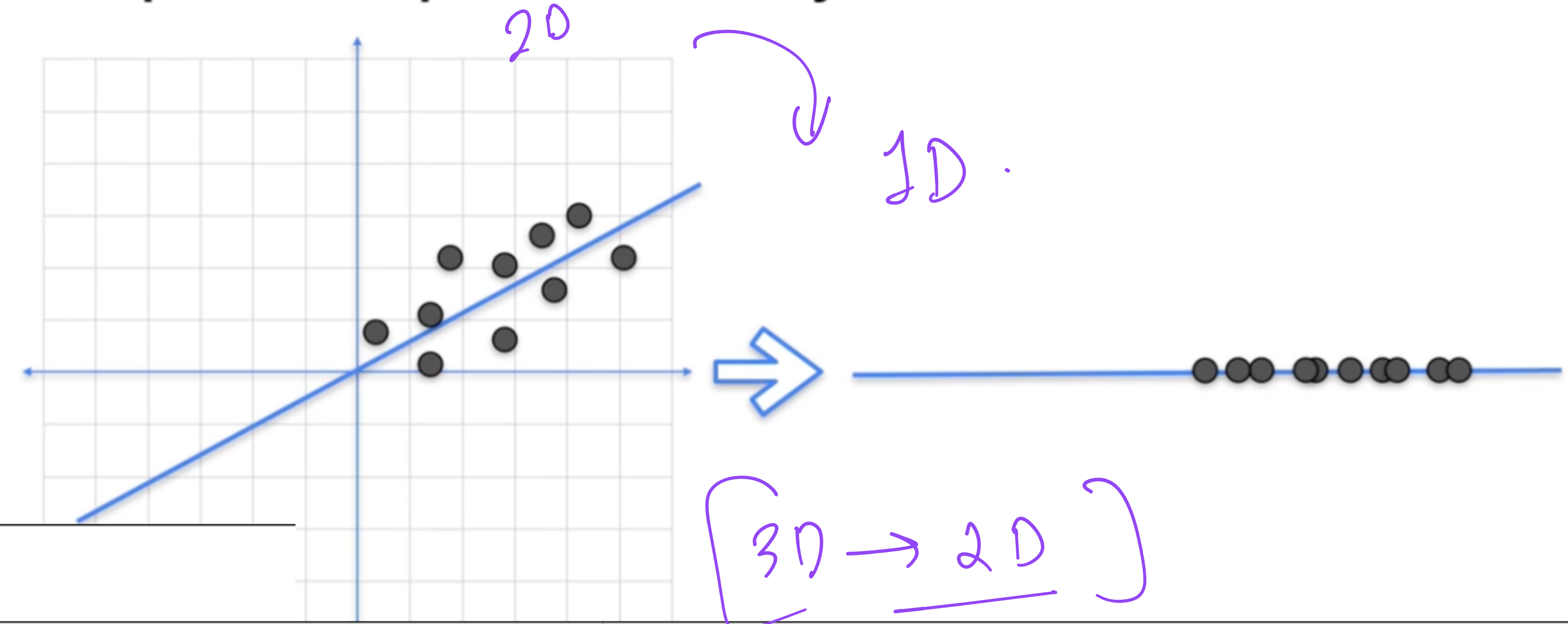
⑥ Bases in Linear Algebra, Span in Linear Algebra, Eigen bases

⑦ Eigen values, Eigen Vectors

Principal Component Analysis:

- Reduces dimensions (columns) of dataset
- Preserve as much information as possible

Principal Component Analysis



What to expect?

Linear transformation

1 → Non singular | Singular

2 → Determinant as area

3 → properties of determinant

Characterize by transformation.

Characterize your transformation

The first thing you'll learn is if the transformation is singular or not.

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What to expect?

Linear transformation

1 Singular / Non-singular

2 Determinant as area

3 Properties of determinant

Characterize your transformation

1 Basis

2 → $\boxed{\text{Span}}$

3 → Eigen Values and eigen vectors

PCA

to any point in a space.

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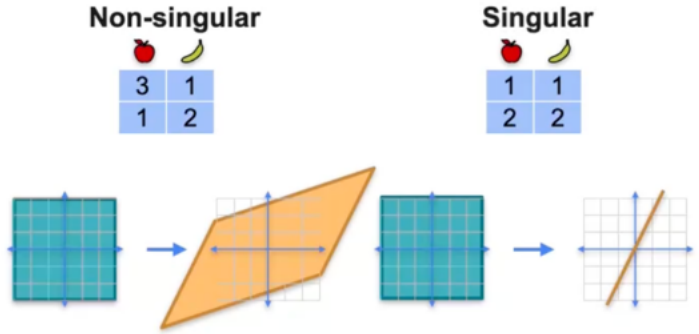
①

What to expect?

- Linear transformation
- 1 Singular / Non-singular
- 2
- 3
- Characterize your transformation



earlier in the course.

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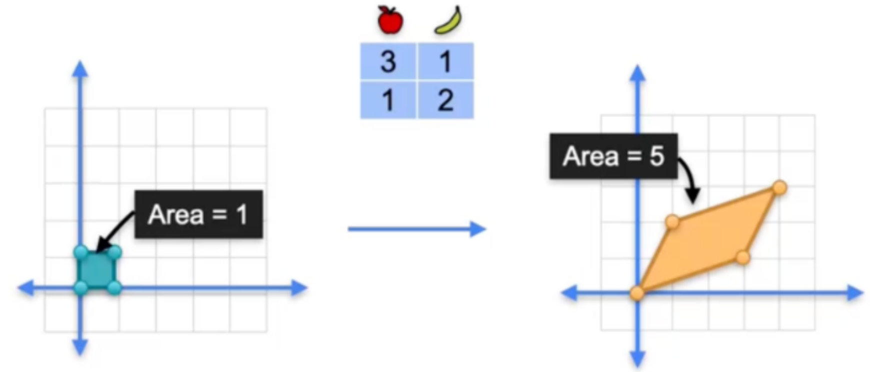
②

What to expect?

- Linear transformation
- 1
- 2 Determinant as area
- 3
- Characterize your transformation



shrinking that particular transformation produces.

③

What to expect?

- Linear transformation
- 1
- 2
- 3 Properties of determinant
- Characterize your transformation

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -3 & 3 \end{bmatrix}$$

Det = 5 ✓   Det = 3 ✓   Det = 15 $= 5 \cdot 3$ ✓

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Det = 5 ✓   Det = $0.2 = \frac{1}{5}$ ✓

if you have a cascade of different transformations one after another.

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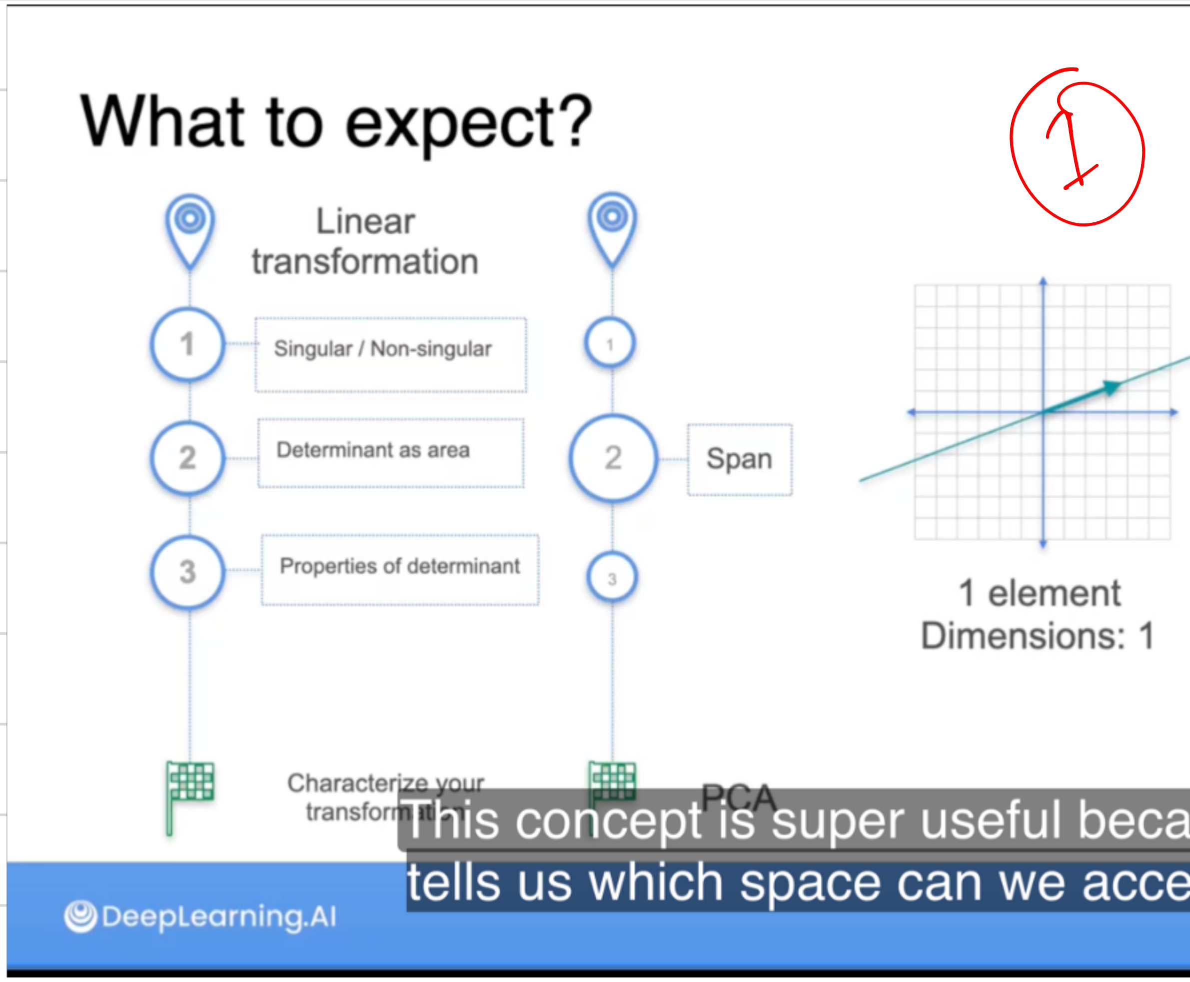
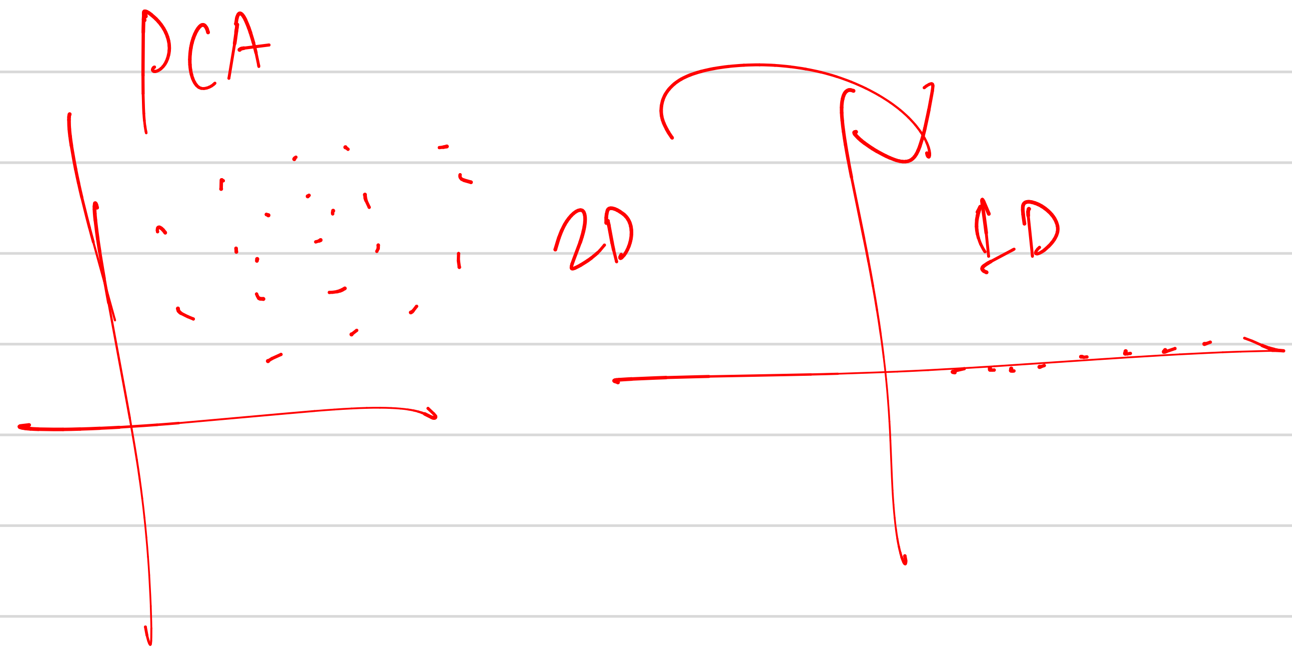
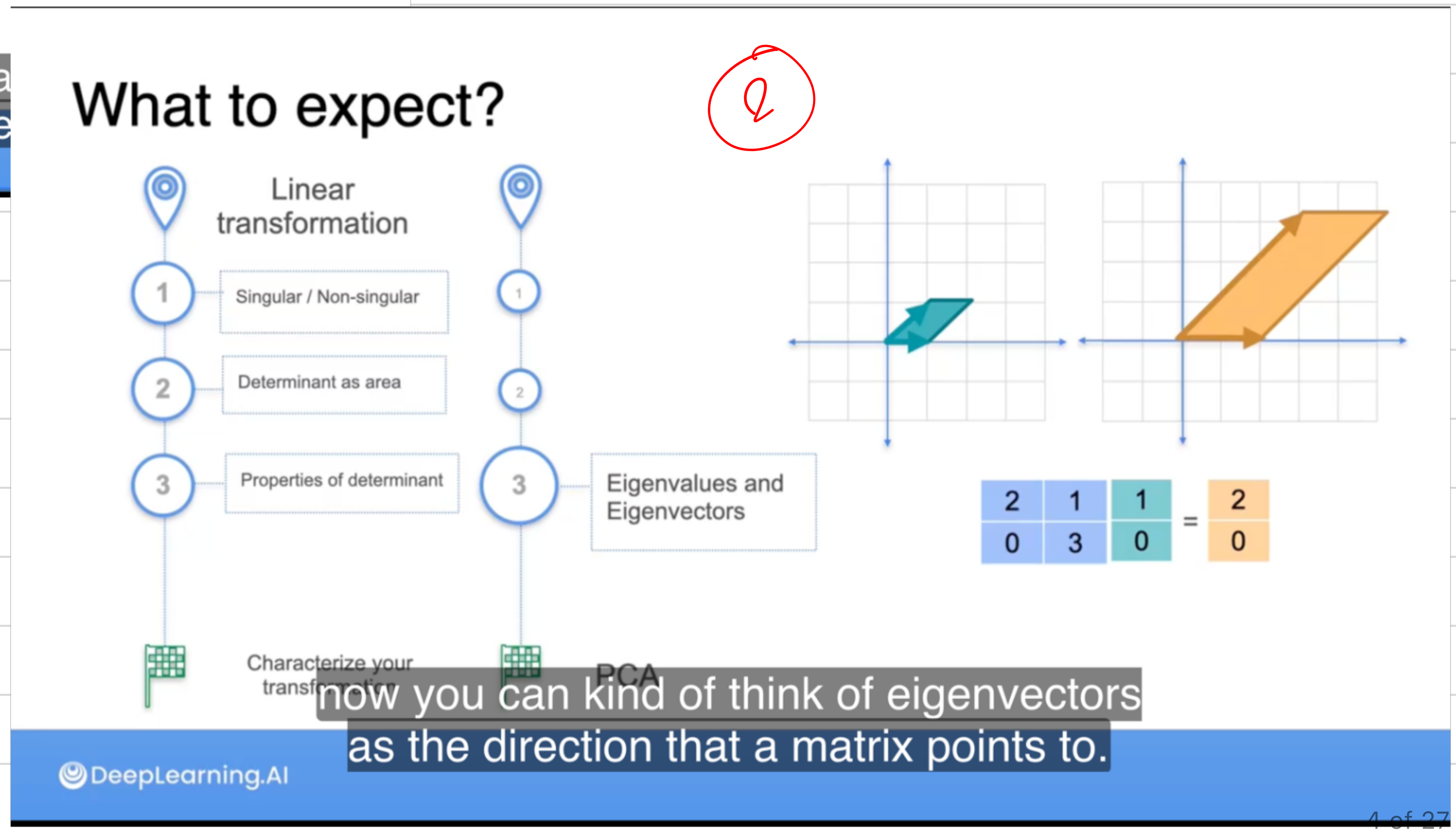


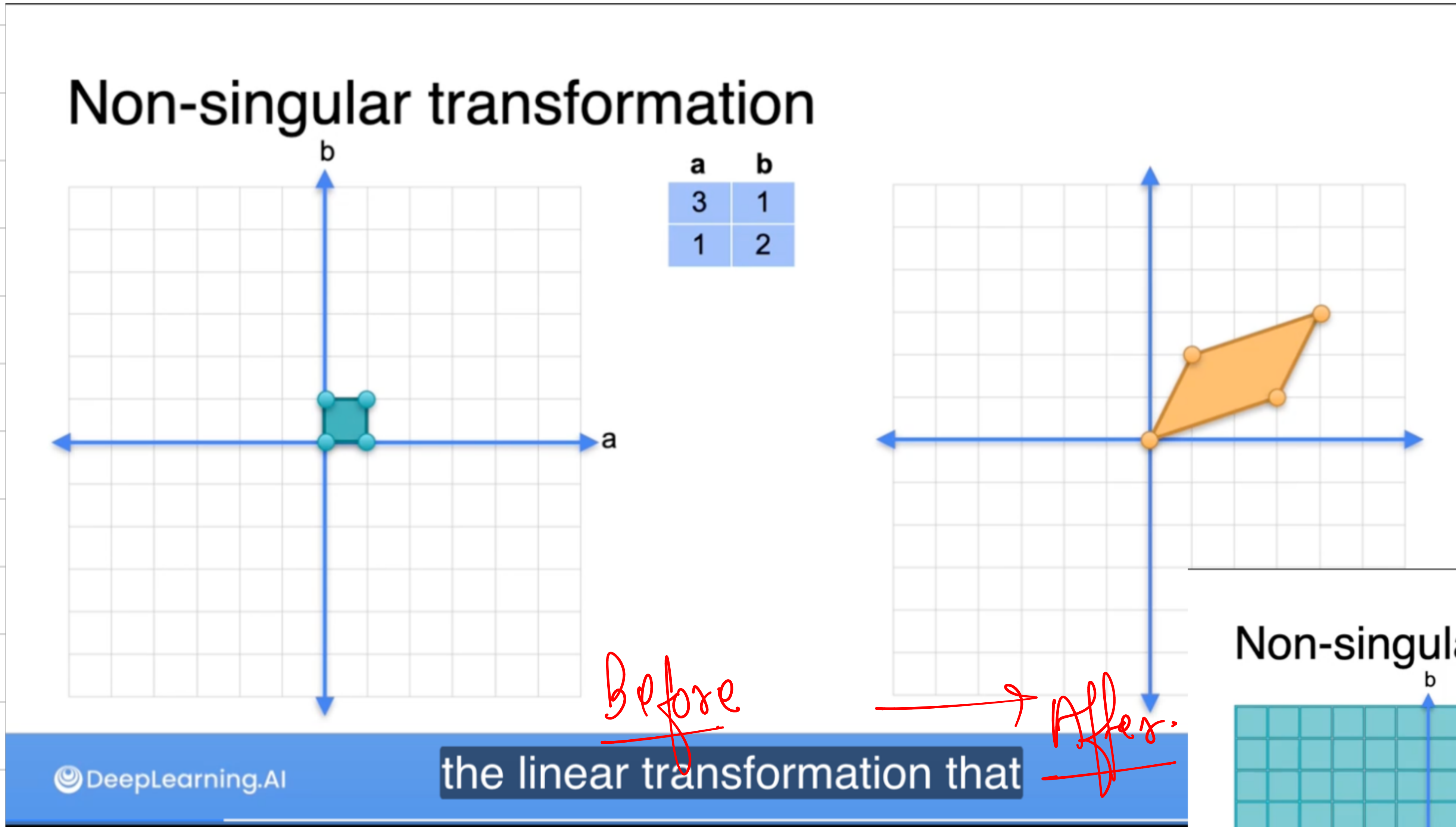


1 dimension vector Span
a straight line

Whereas 2 dimension have
different 2D and co-ordates.

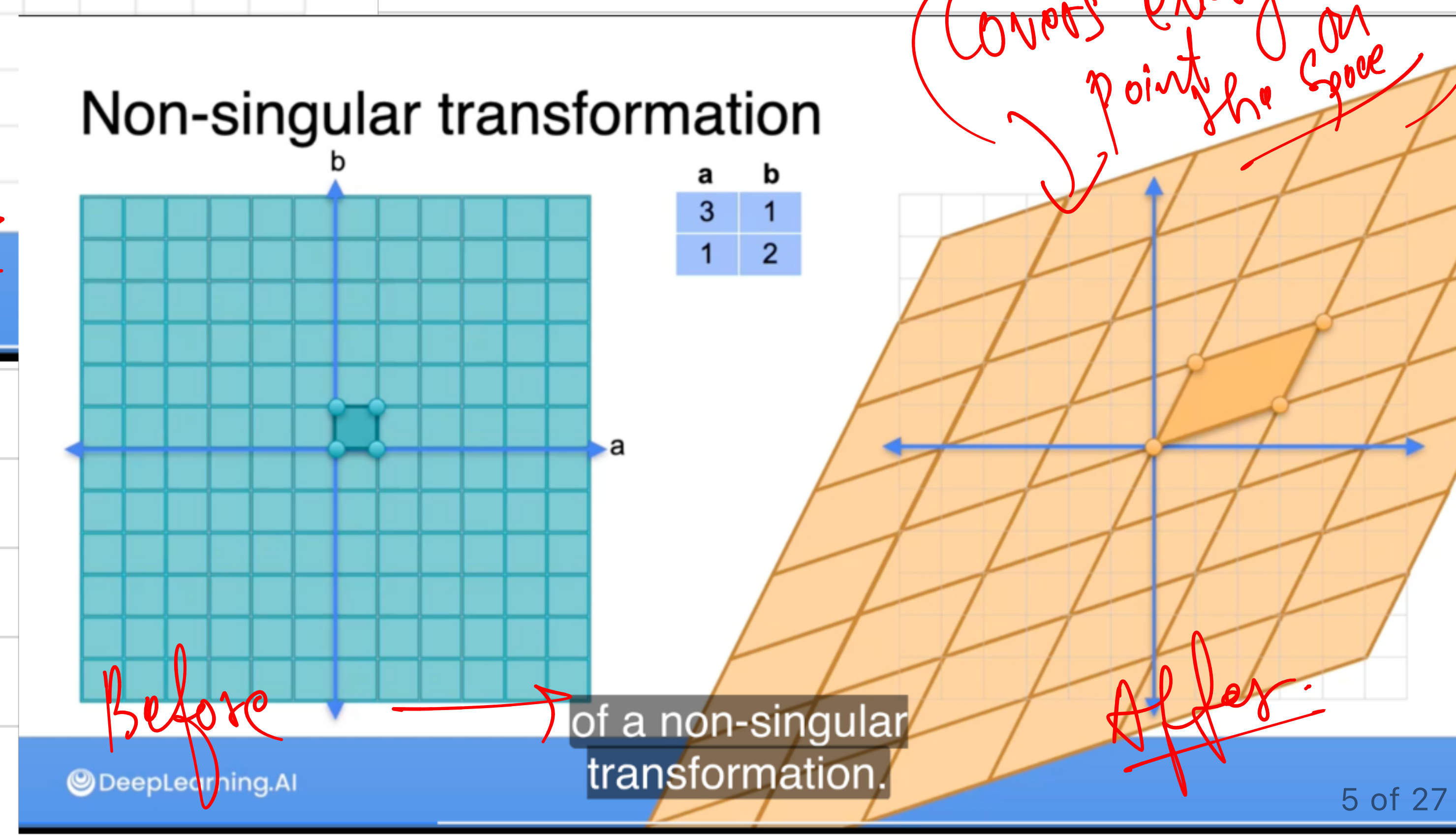


Singularity and Rank of Linear Transformations:



Before → After.

↳ Points Covered is called Image transformation.



(Covers every point on the space)

Before → After.

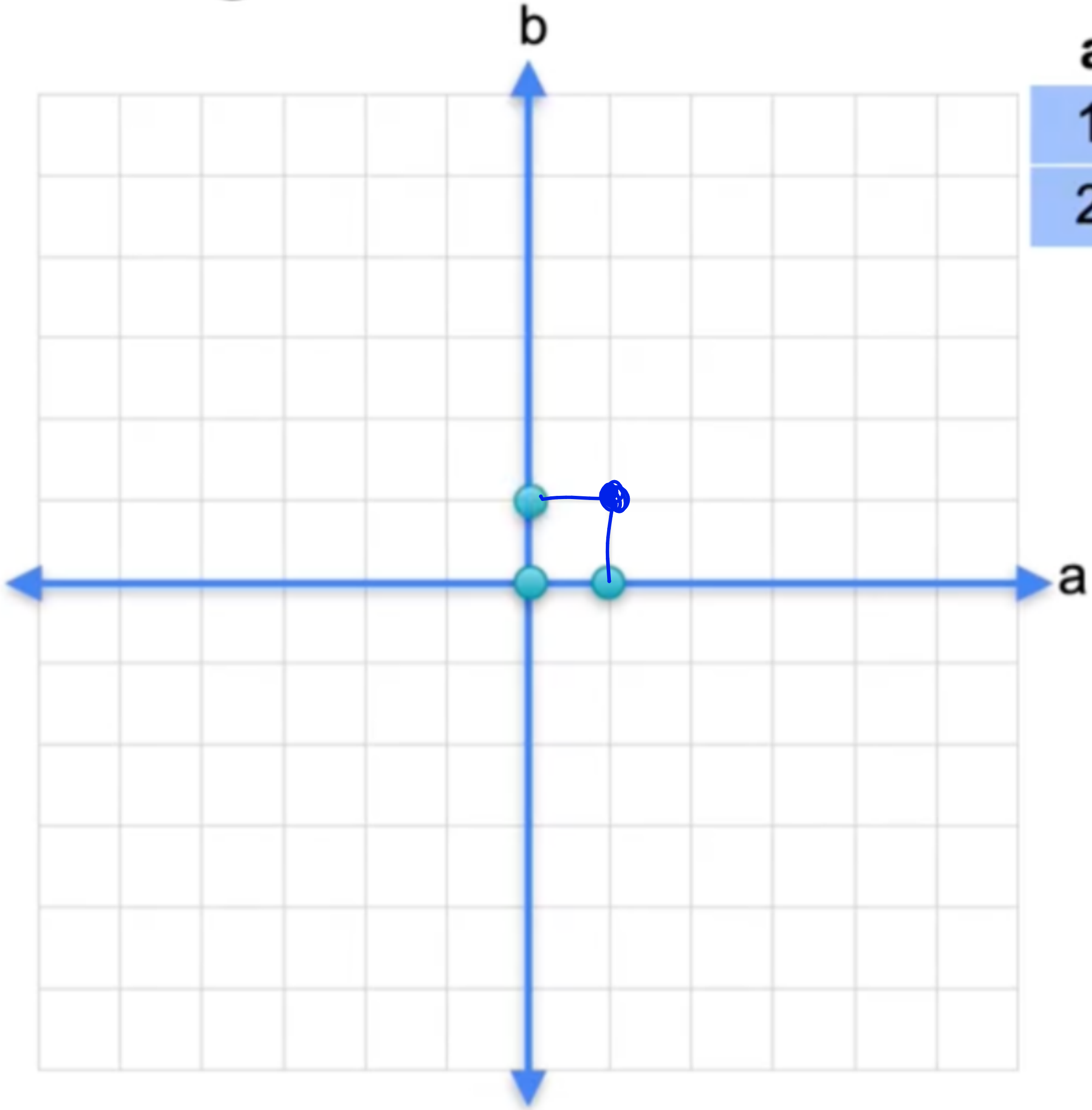
Visualizing How

↳ rectangular to Parallelogram.

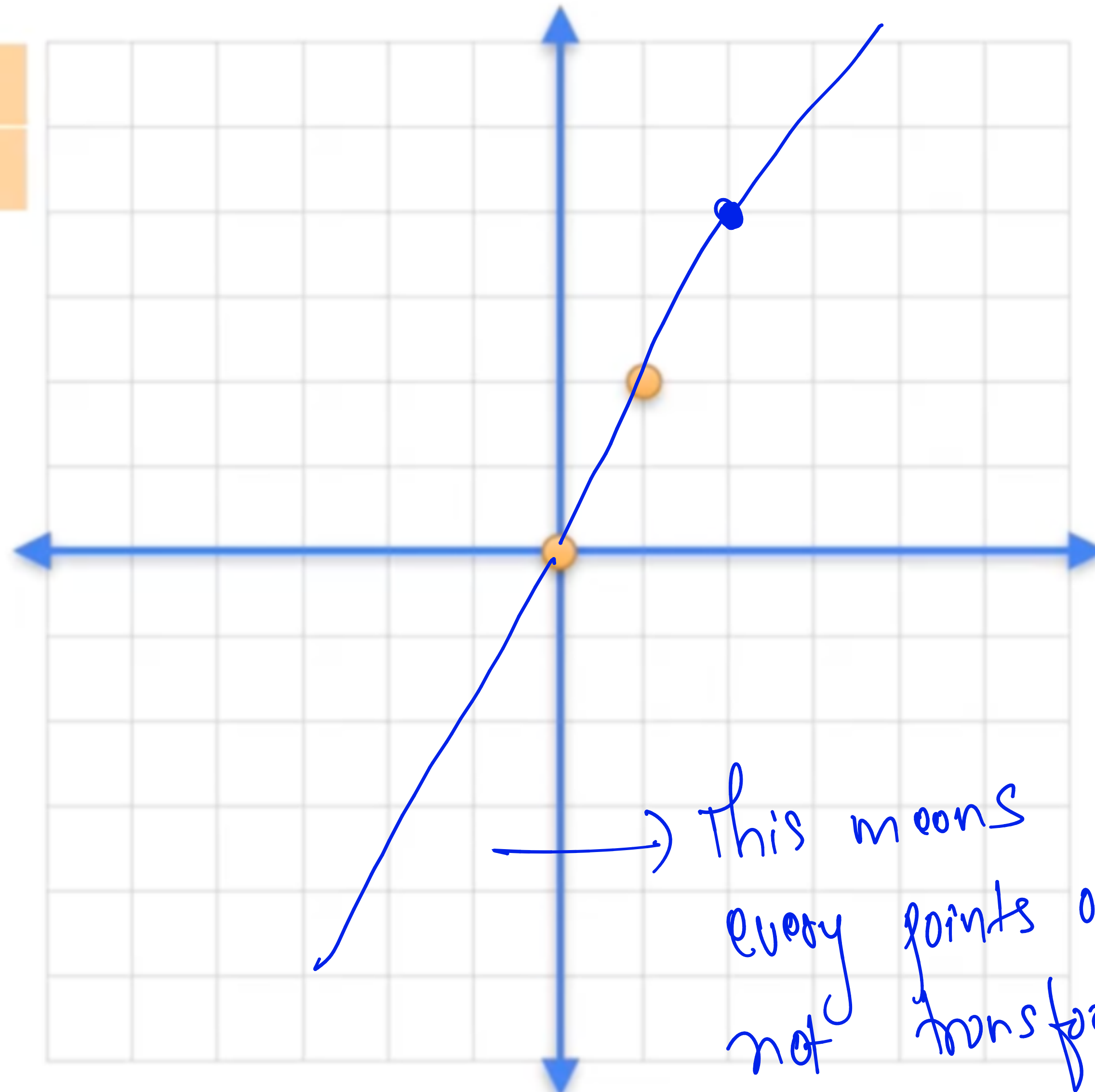
Singular transformation

(Not Able to Cover All points | positions).

$$\begin{array}{cc} a & b \\ \end{array}$$
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



(0,0) → (0,0)
(1,0) → (1,2)
(0,1) → (1,2)
(1,1) → (2,4)



→ This means every points are not transformed like Parallelogram.

multiply it by the vector 0,1.

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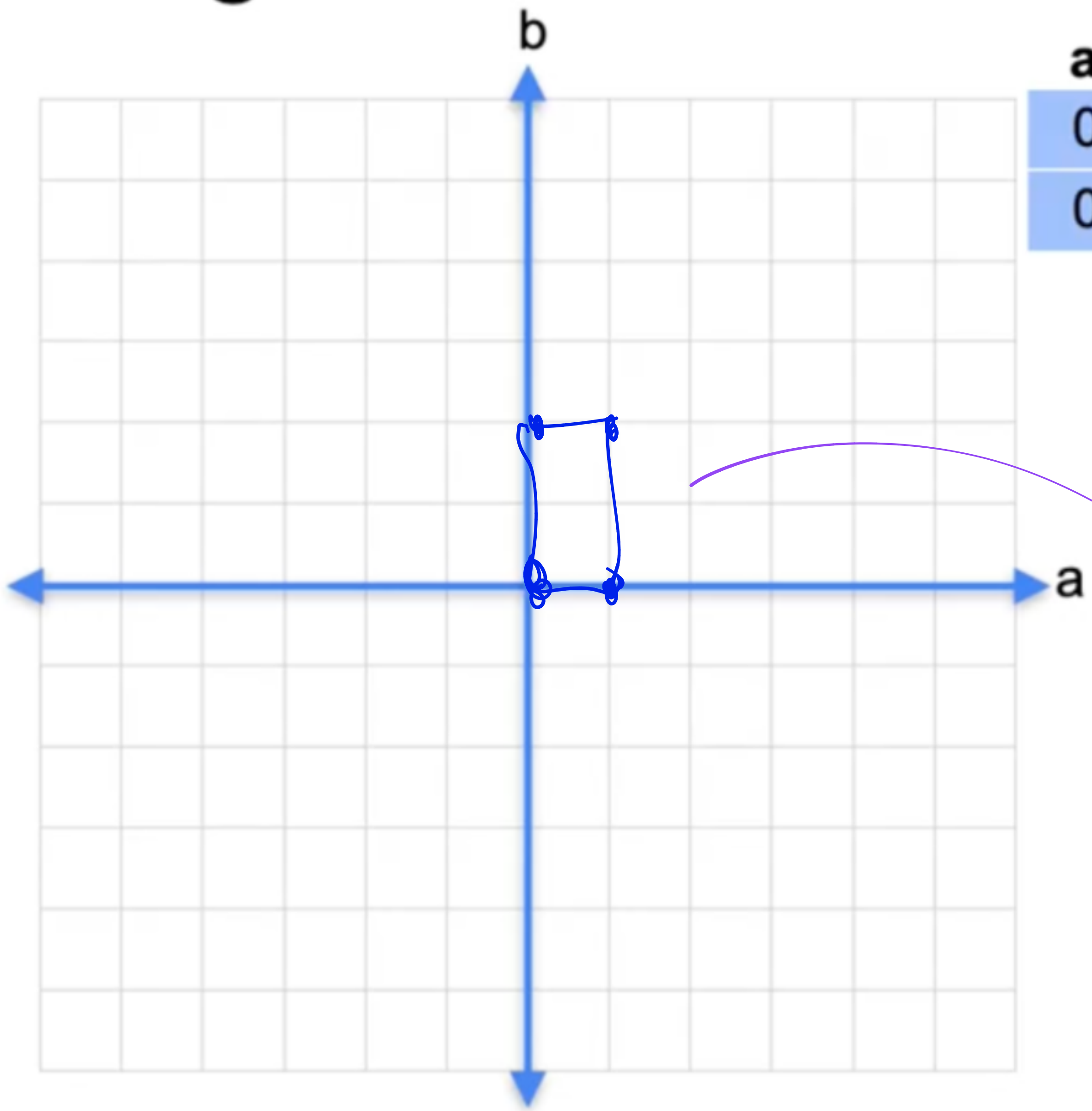
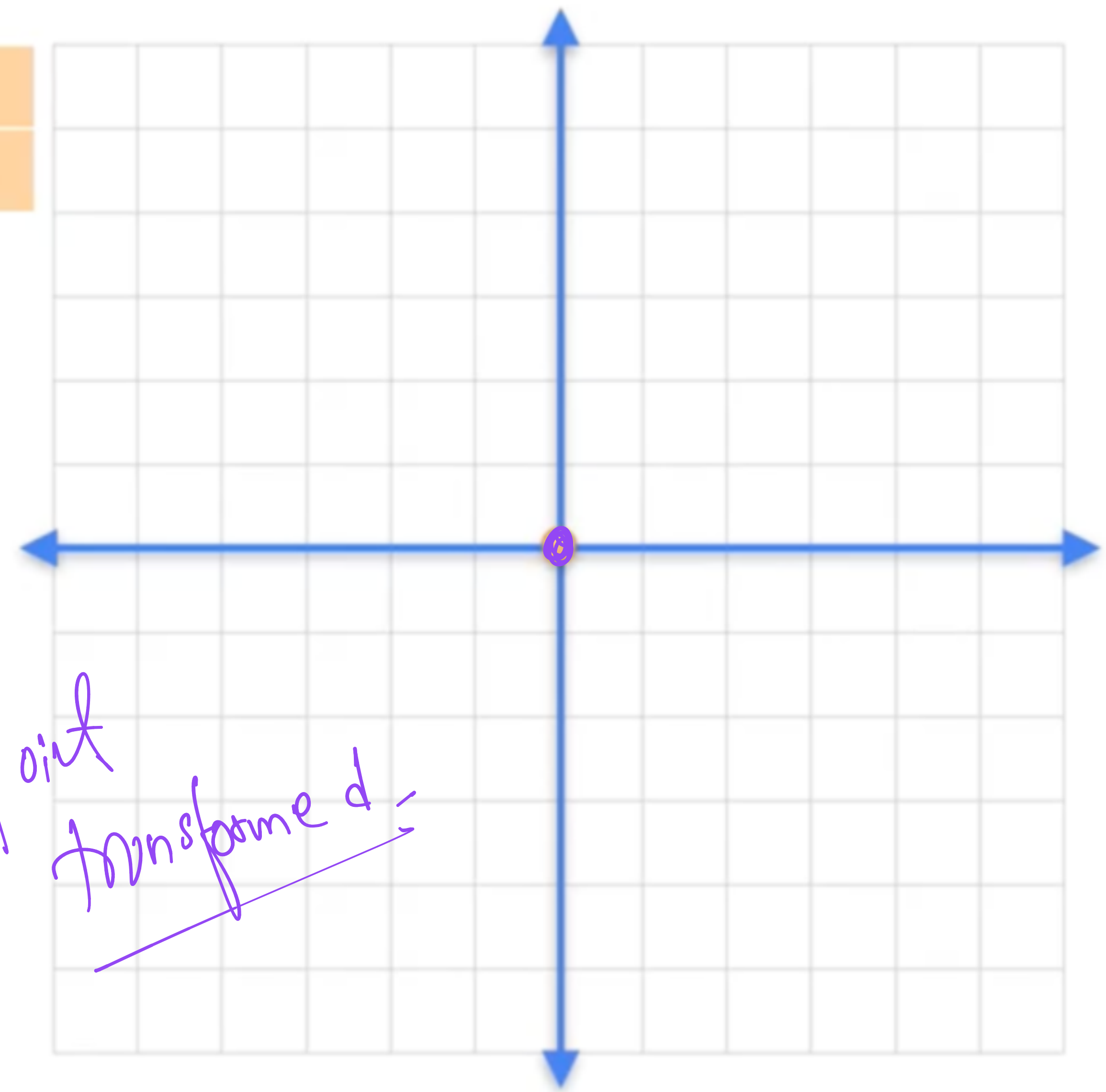
Singular transformation



$$\begin{array}{cc} a & b \end{array}$$
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Send →

Only one point transformed.



0,0, which is the origin.

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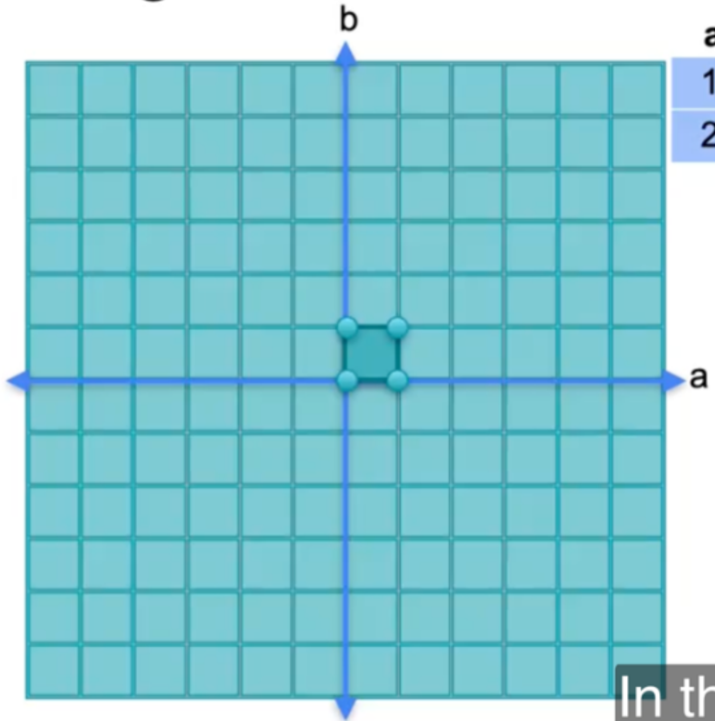
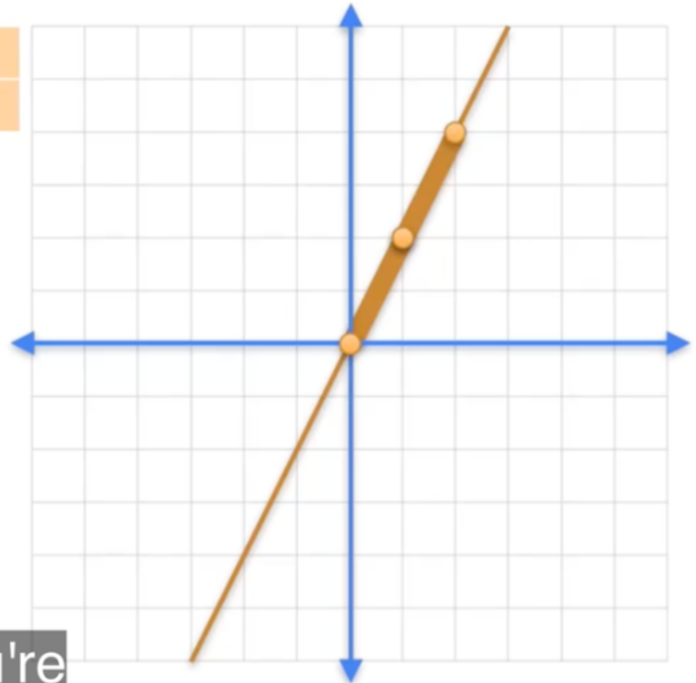
Singular transformation



$$\begin{array}{cc} a & b \end{array}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (1,2)
(0,1) → (1,2)
(1,1) → (2,4)



In this case, you're covering a line.

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→ Non-Singular transforms every point and is covering portions.

→ Whereas Singular are not.

(1) → Dimension → 2

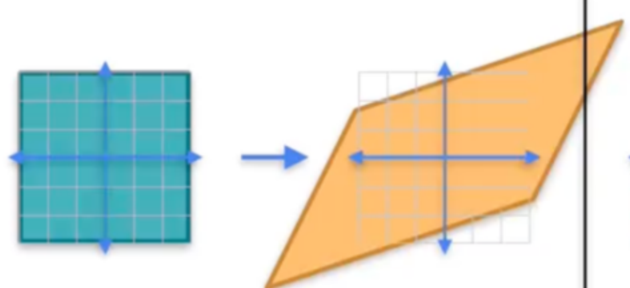
(2) Dimension → 1

(3) " → 0

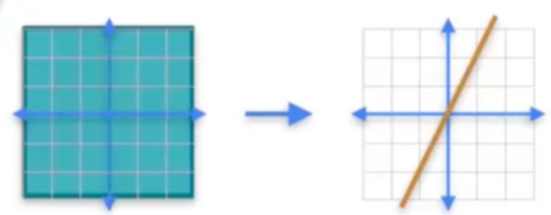
of the Image of Linear Transformation

Singular and non-singular transformations

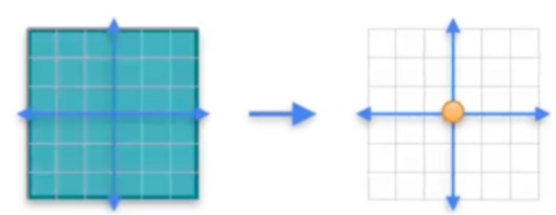
Non-singular		Singular		Singular	
a	b	a	b	a	b
3	1	1	1	0	0
1	2	2	2	0	0



(1)



(2)



(3)

so it's even more singular.

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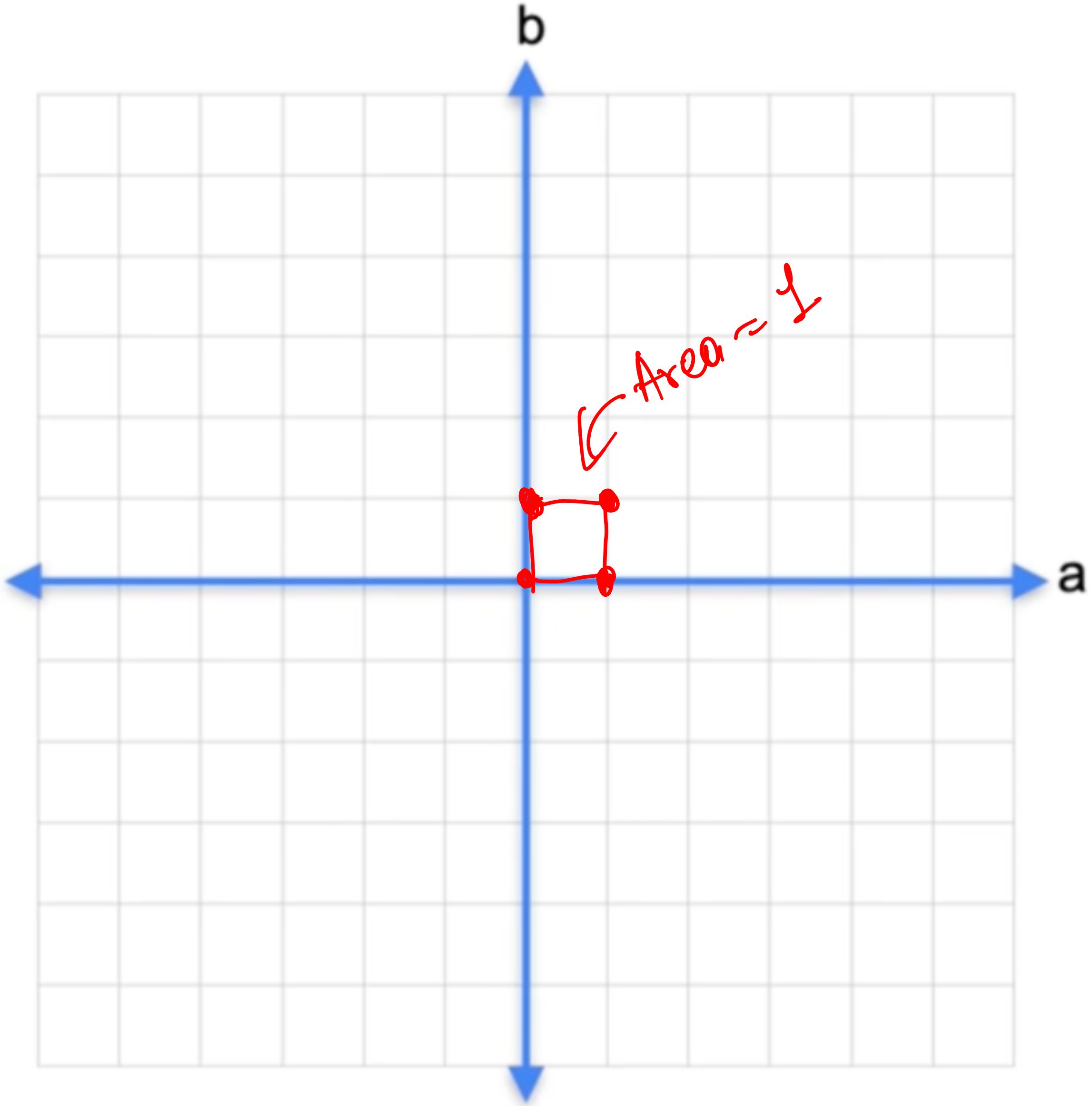
Determinant as an Area:

Determinant as an area



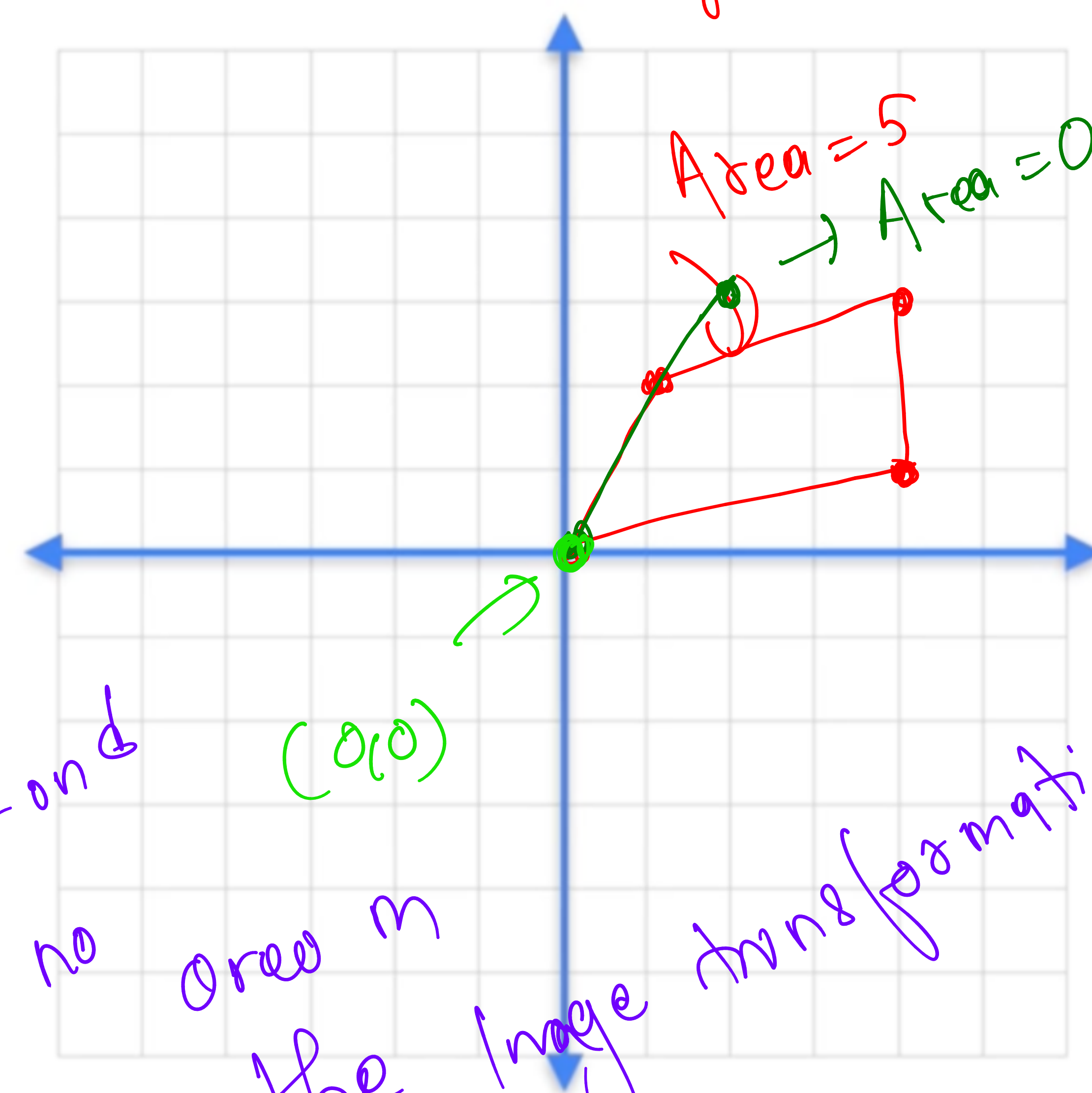
a	b
3	1
1	2

$[3 \cdot 2 - 1 \cdot 1 \Rightarrow 5]$

$|D| = 5$

● → Singular

● → Non-Singular



Determinant $|D| = 0$

For Singular and has no area in the image transformation

It's determinant is 3 times

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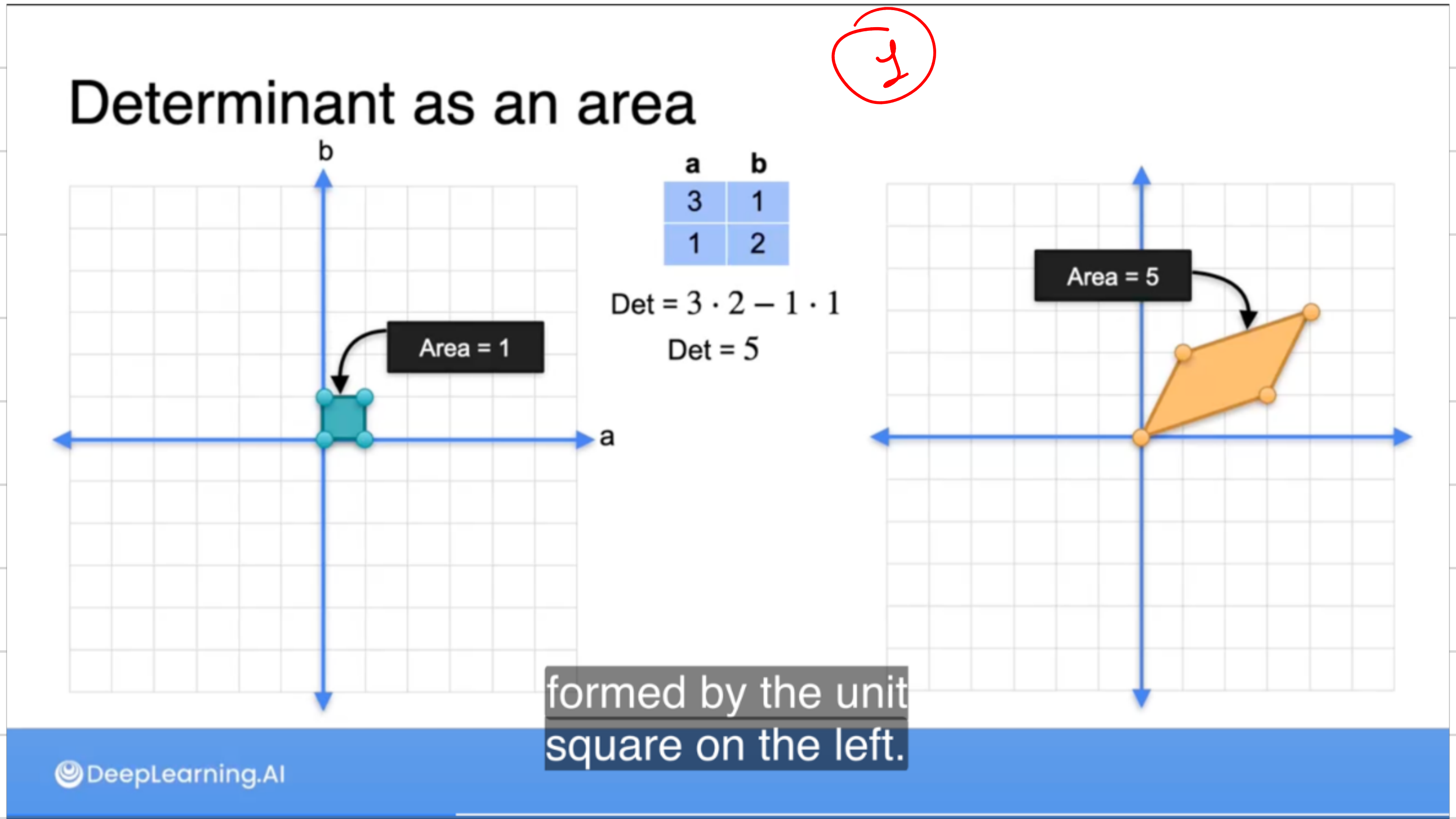
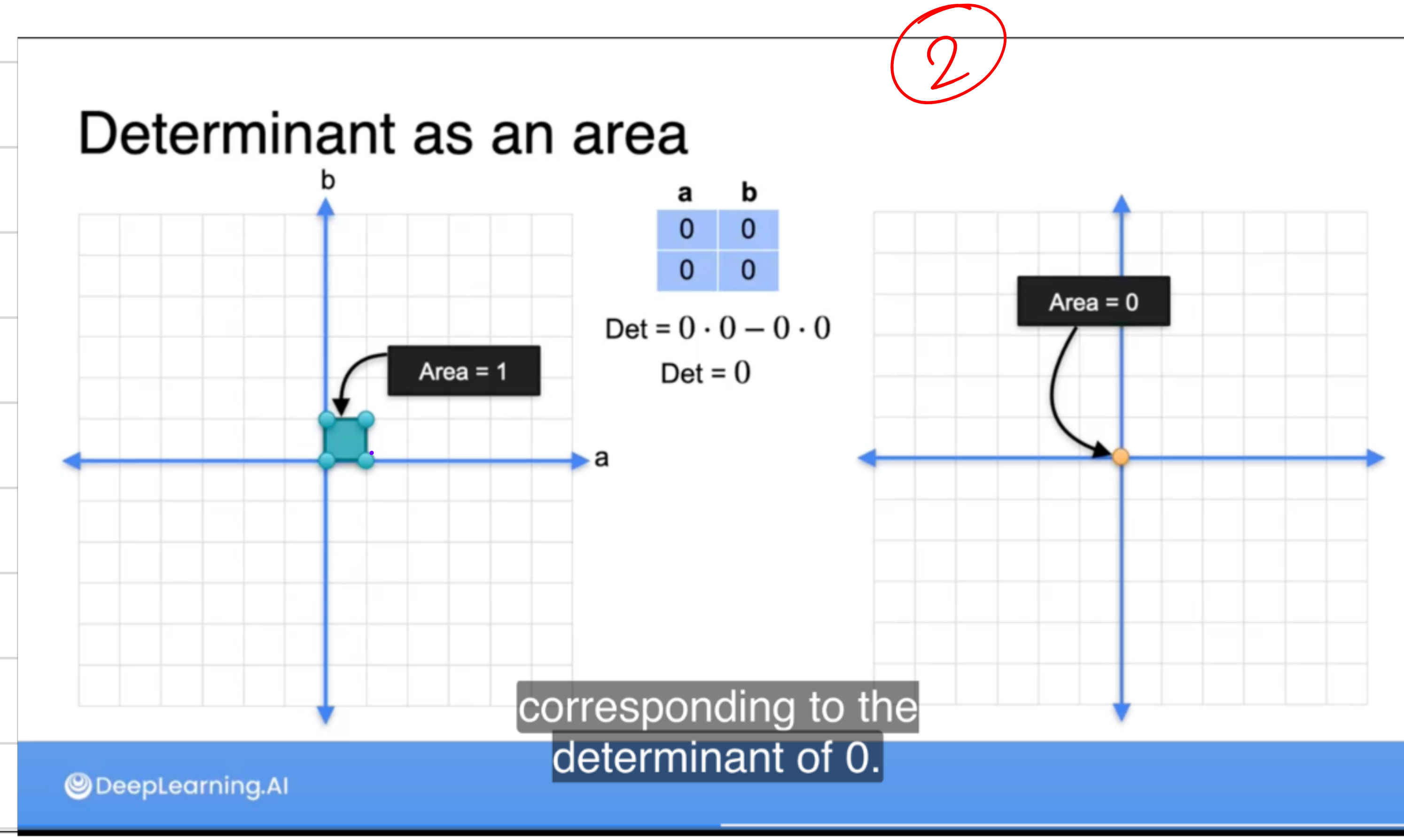
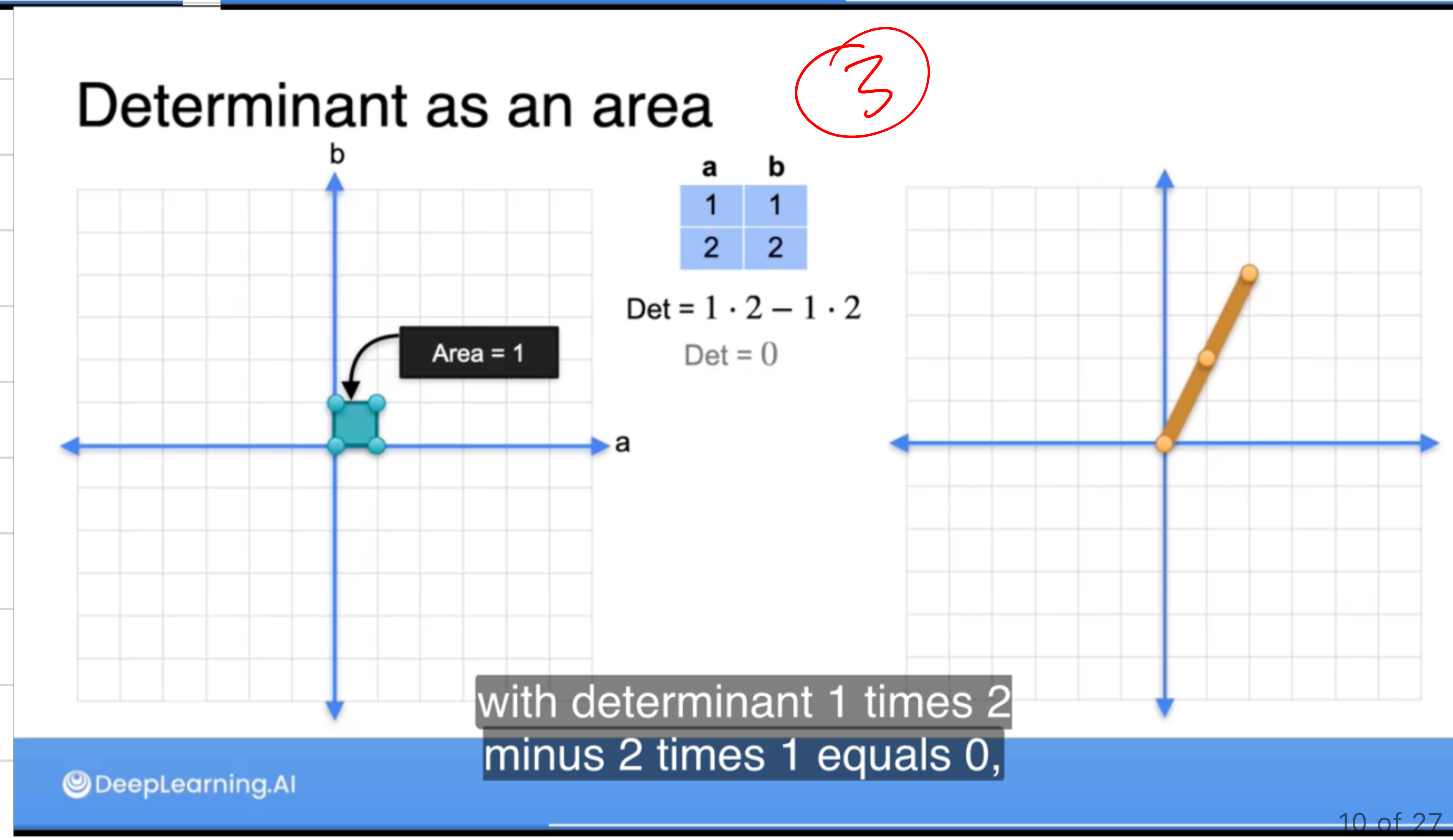




↳ |D| acting as an area.

↳ -ve Determinant = ?

Doesn't Affect the Singularity of the Matrix.



Determinant as an area

Non-singular

a	b
3	1
1	2

Determinant = 5

Area = 5

Singular

a	b
1	1
2	2

Determinant = 0

Area = 0

Singular

a	b
0	0
0	0

Determinant = 0

Area = 0

Now, here's something that
you may be wondering,

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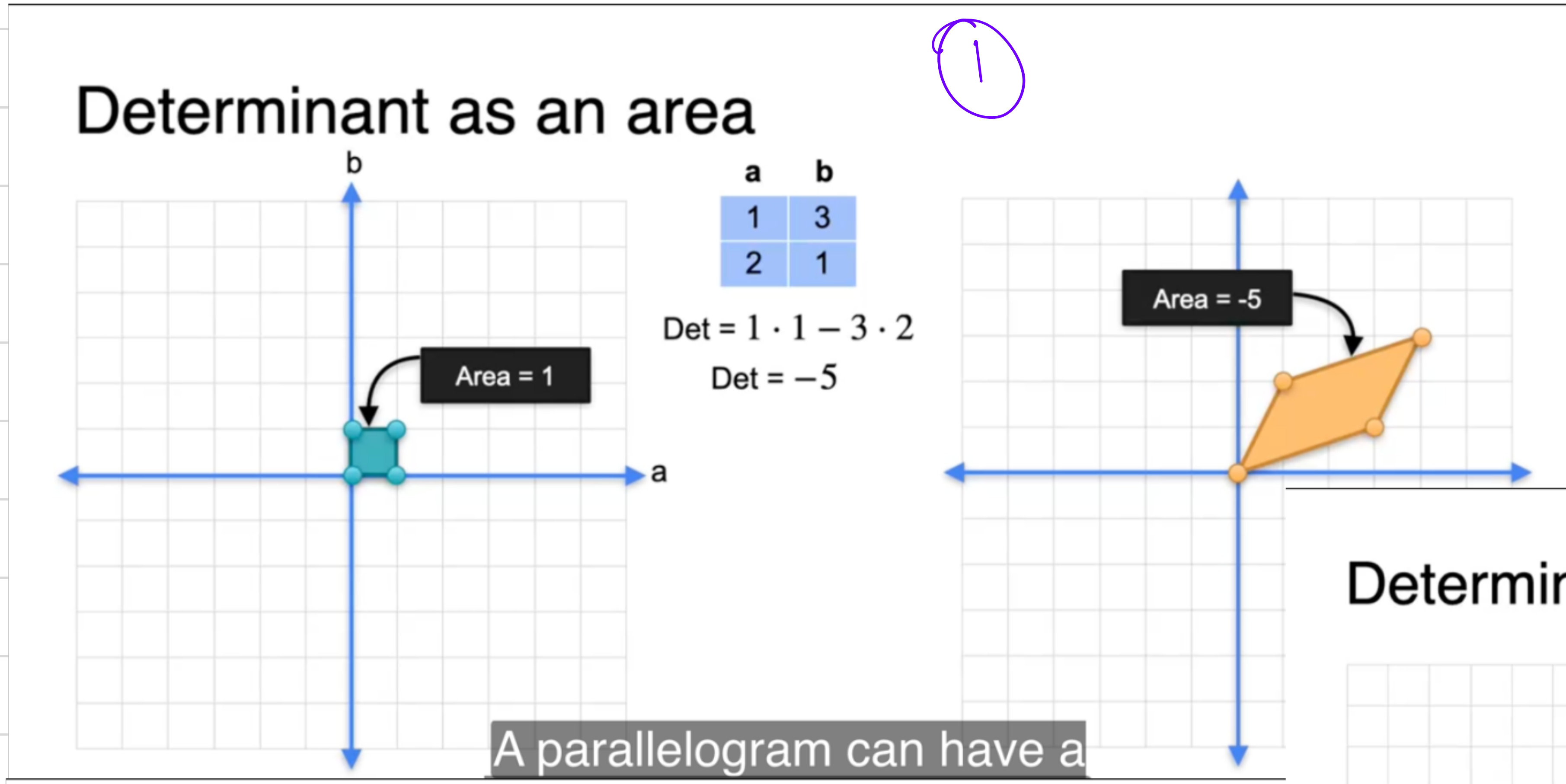
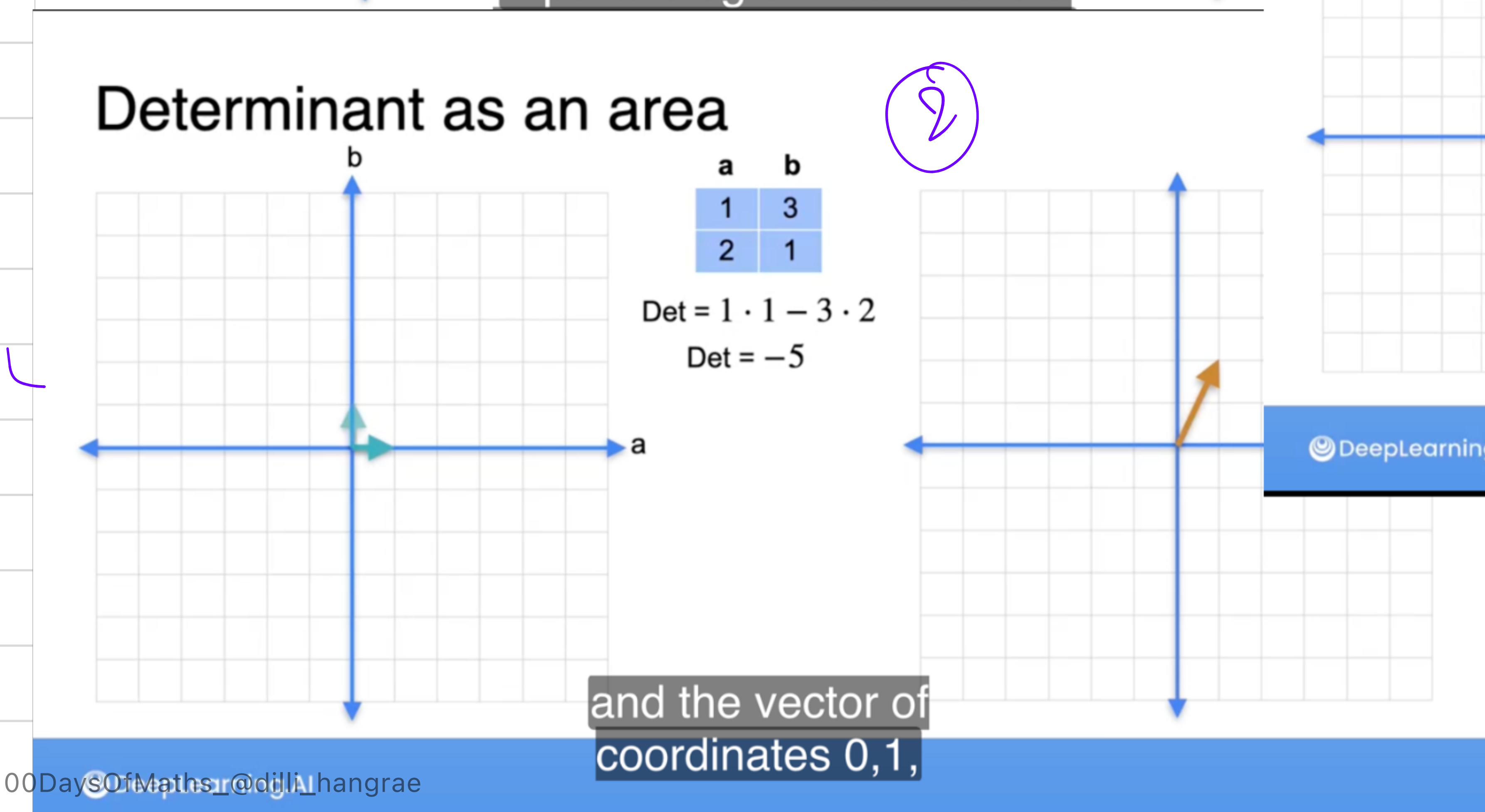
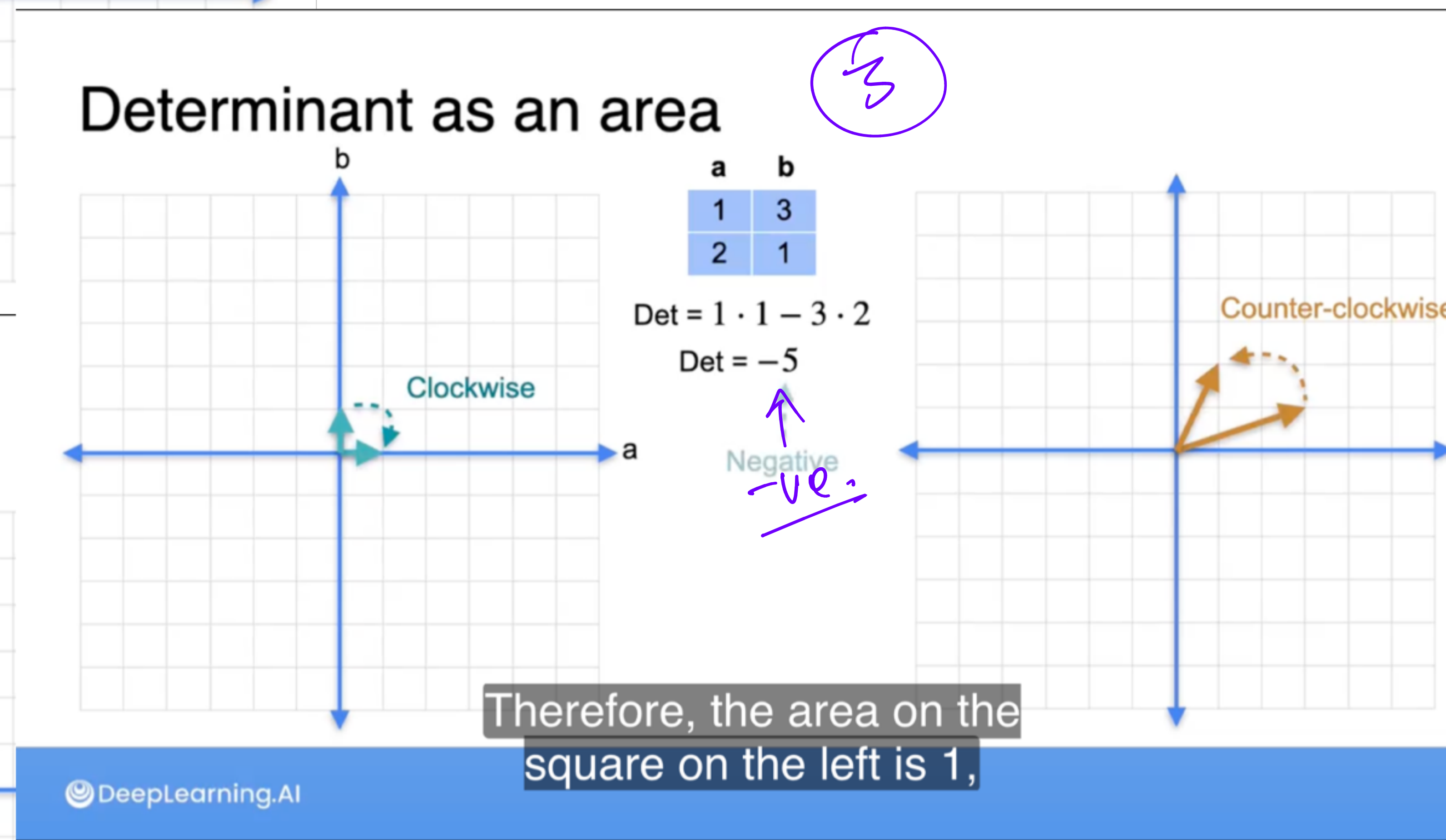


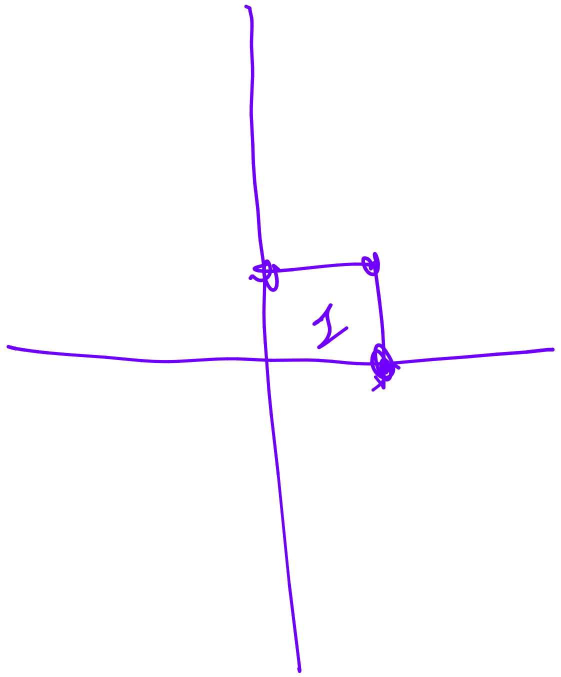




Next is |D| of product.

Determinant of a product

$\det(AB) = \det(A) \cdot \det(B)$



Before

$$\begin{bmatrix}3&1\\1&2\end{bmatrix}\begin{bmatrix}1&1\\-2&1\end{bmatrix}=\begin{bmatrix}1&4\\-3&3\end{bmatrix}$$

$\det = 5$

$(3 \cdot 2 - 1 \cdot 1)$

$\det = 3$

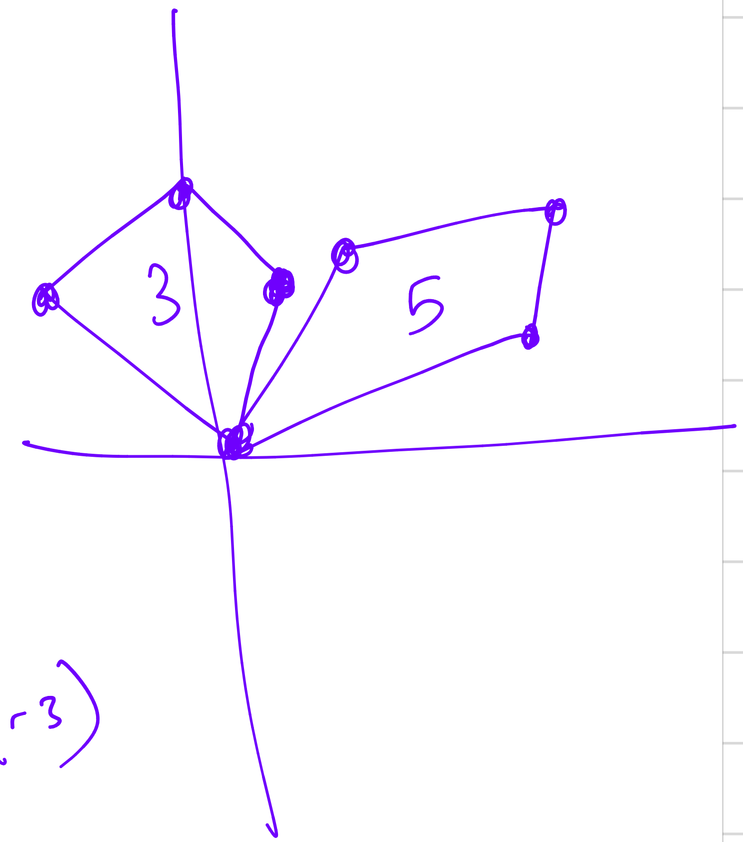
$(1 \cdot 1 - 1 \cdot (-2))$

$\det = 15$

$1 \cdot 3 - 4 \cdot (-3)$

$\Rightarrow 3 + 12$

$\Rightarrow 15$



Area goes up by 3

Now what are the determinant of the matrix?

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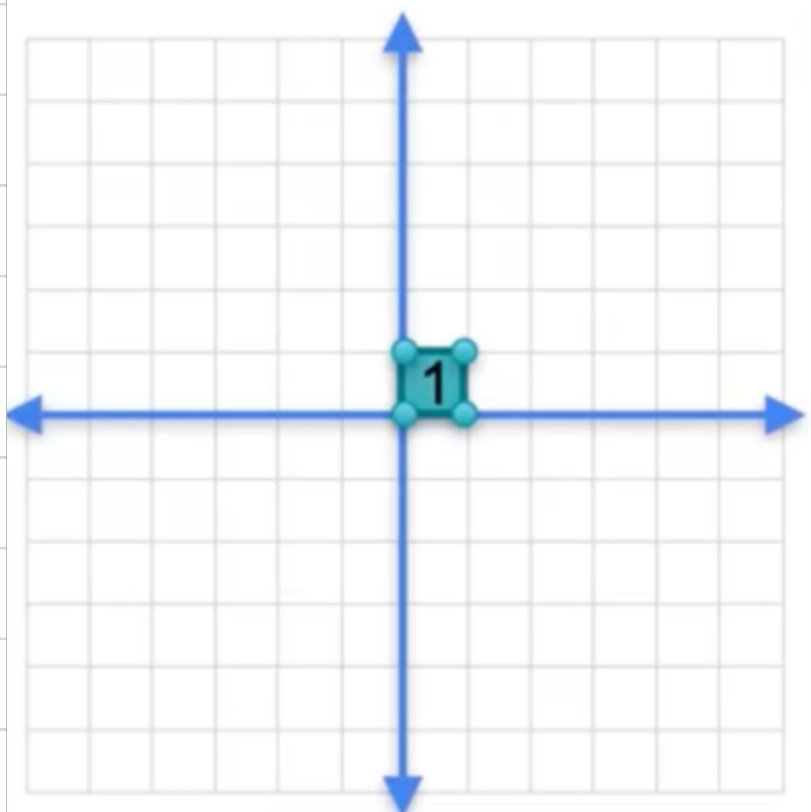
$\det(AB) = \det(A) \cdot \det(B) \quad \therefore \det(AB) = 0$

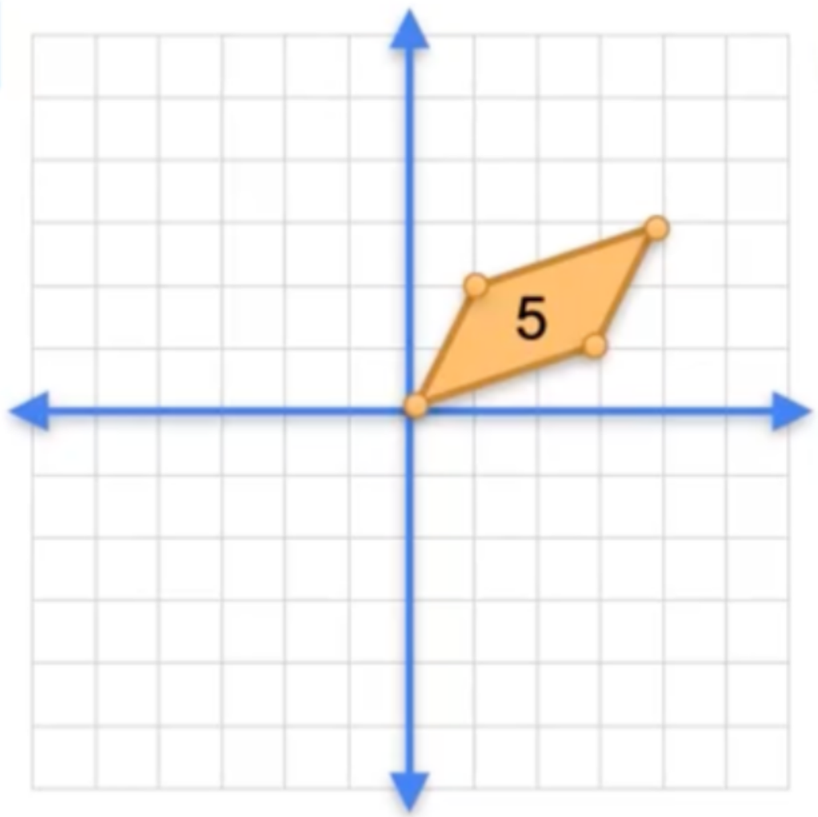
Product of singular and a non-singular matrix (in any order) $\Rightarrow$ singular

Determinant of a product



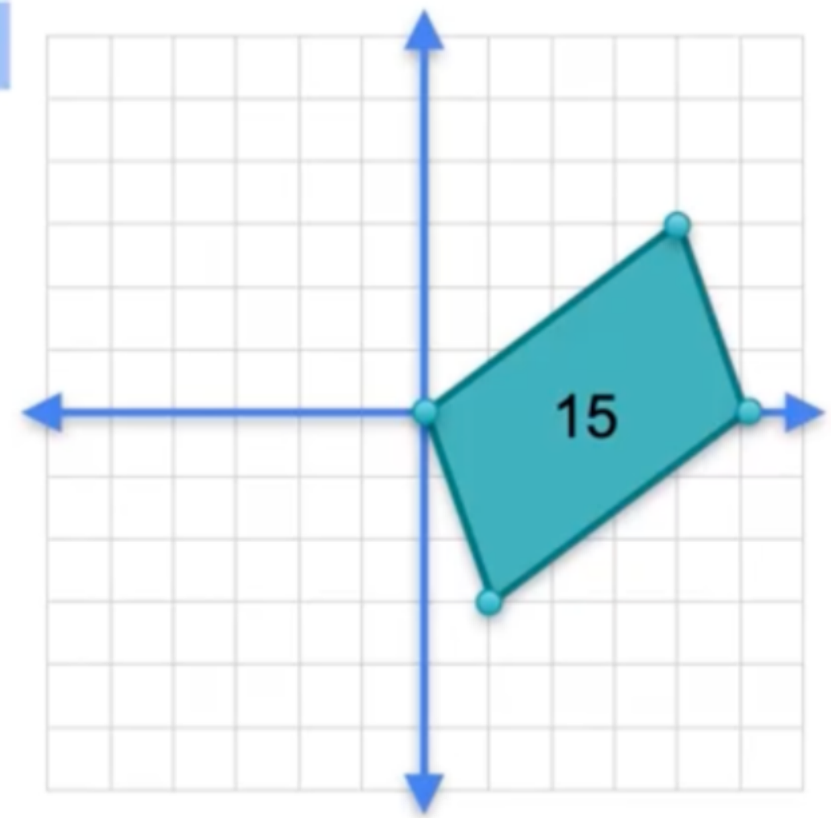
3	1
1	2

Det = 5



1	1
-2	1

Det = 3



$\det(AB) = \det(A) \det(B) = 0$.

OR is Singular .

first you're blowing
up the areas by five,

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When one factor is singular…

Non-singular		Singular		Singular	
3	1	1	2	4	8
1	2	1	2	3	6
Det = 5		Det = 0		Det = 0	

Transformation's of 1 to Area

See how 1 → 5 → 0.

because w
its determ

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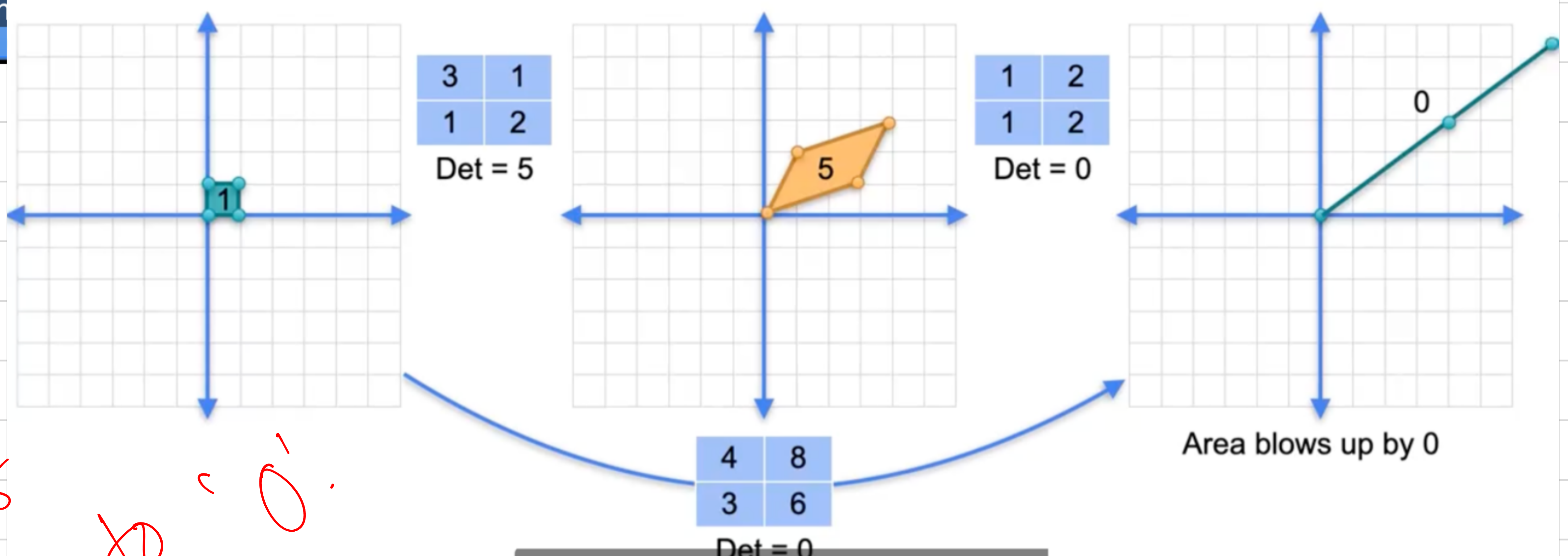
If one factor is singular…



In other words, whatever
area you have left,

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This means if no Non-Singular then it will blow up the Area s to '0'.

Determinants of Inverses:

Why? $\det(A^{-1}) = \frac{1}{\det(A)}$

$\det(AB) = \det(A)\,\det(B)$

$\det(AA^{-1}) = \det(A)\cdot\det(A^{-1})$

$\det(I) = \det(A)\,\det(A^{-1})$

$\Rightarrow \frac{1}{\det(A)}$

Determinant of an inverse

$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$

det = 5 det = 0.2

$5^{-1} = 0.2$

$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$

det = 8 det = 0.125

$8^{-1} = 0.125$

$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$

det = 0 det = ??

And it's actually a singular matrix of determinant 0,

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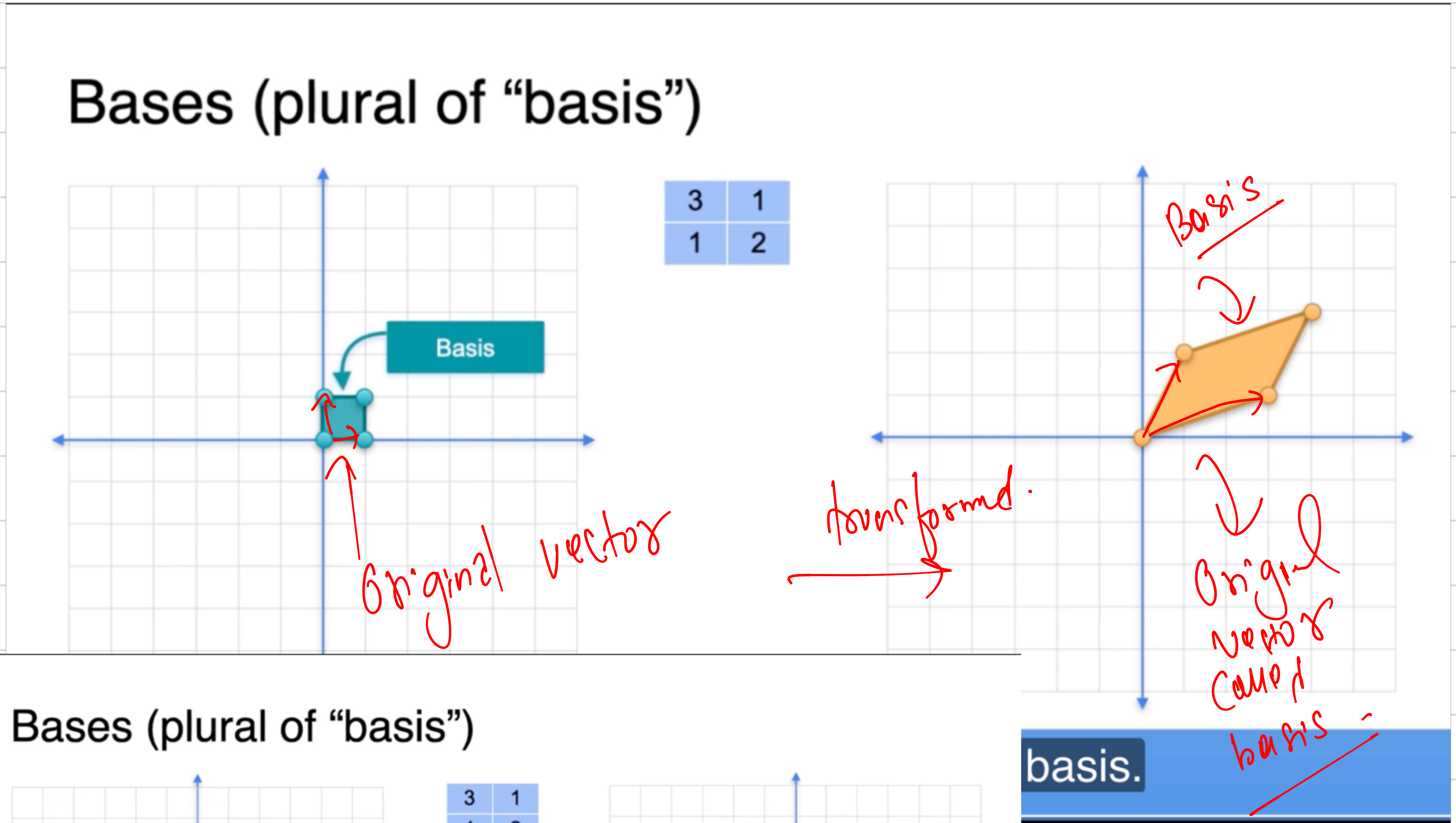
$\det(A^{-1}) = \frac{1}{\det(A)}$

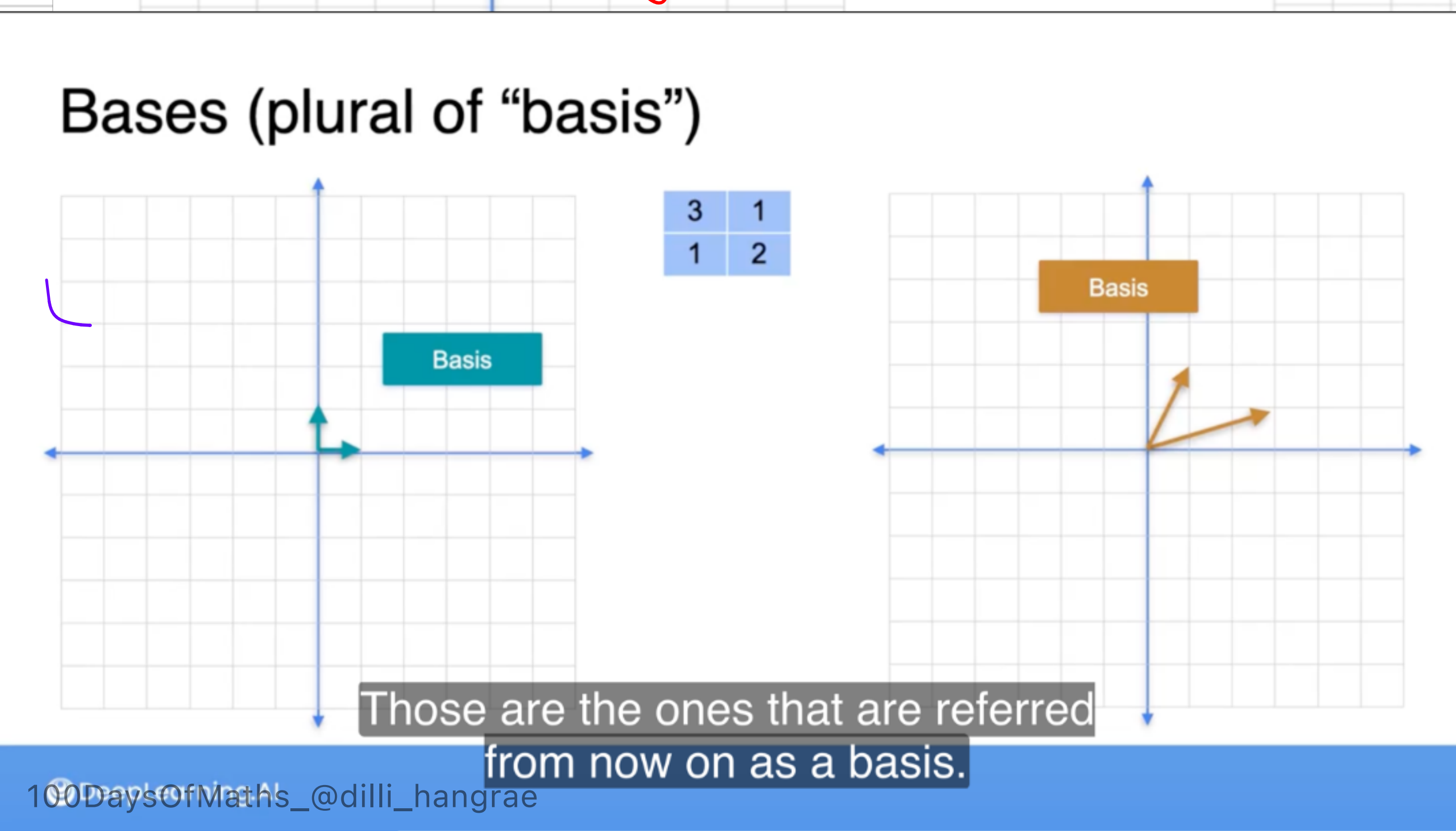
$\det(I) = \det(A)\cdot\frac{1}{\det(A)}$

$1 = \det(A)\cdot 1/\det(A)$

$\det(A^{-1}) = 1 \cdot 1/\det(A)$

Bases:





Property of basis:

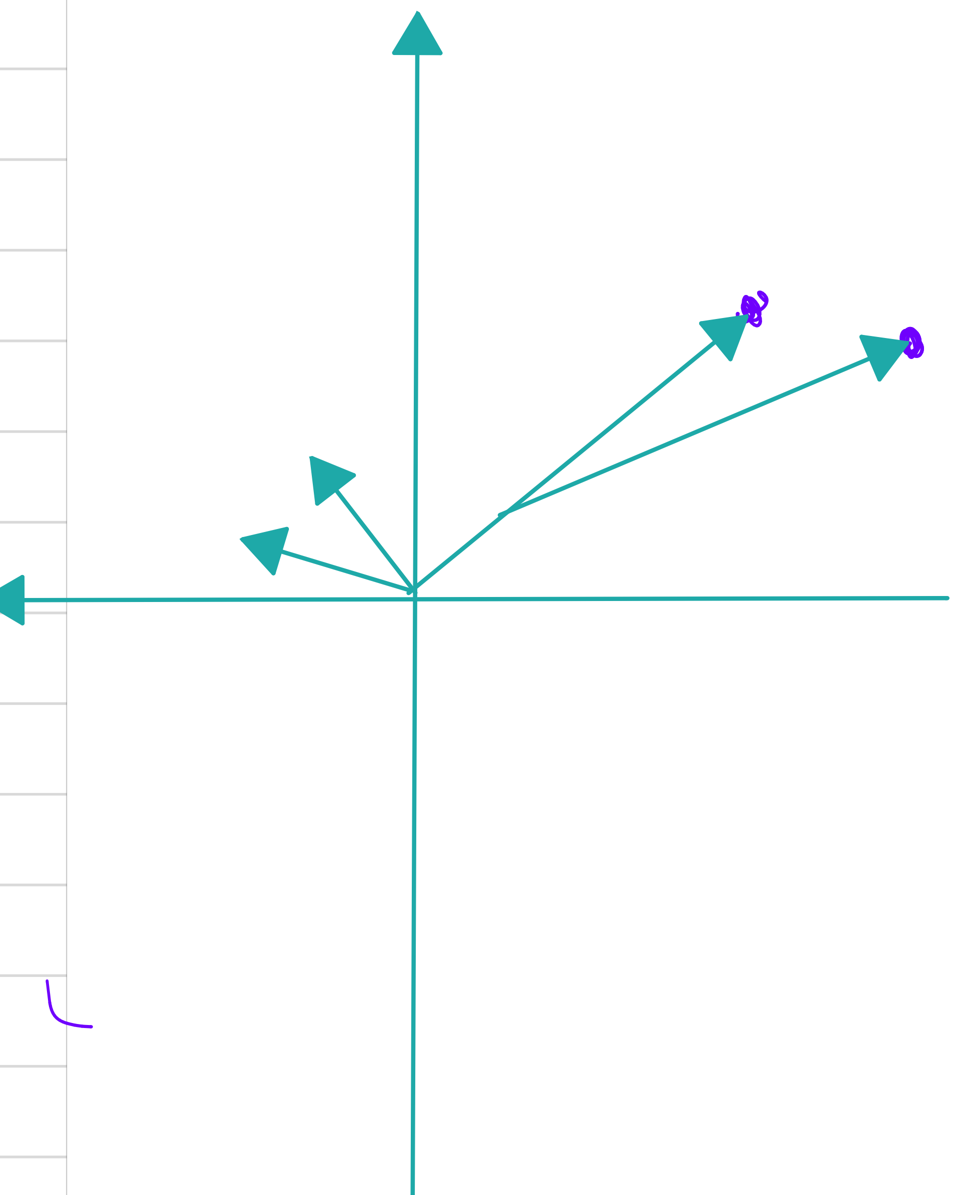
(1) Every point in the space can be expressed as a linear combination of elements in the basis.

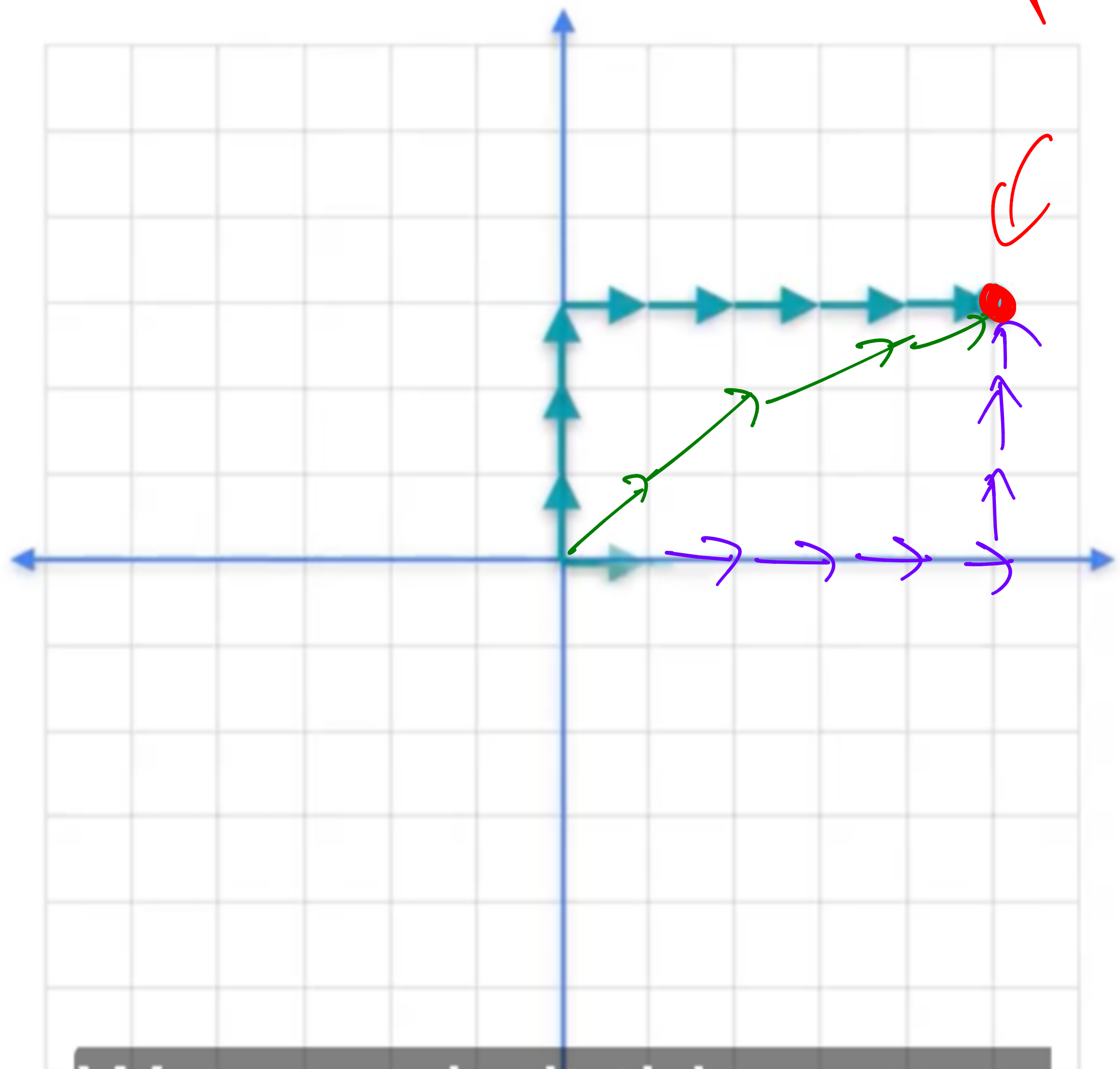
Look example below:

Almost Any two vector form → basis.

Bases (plural of “basis”)

Many ways to Reach Here.





Bases

We can do it this way or this way or this other way.

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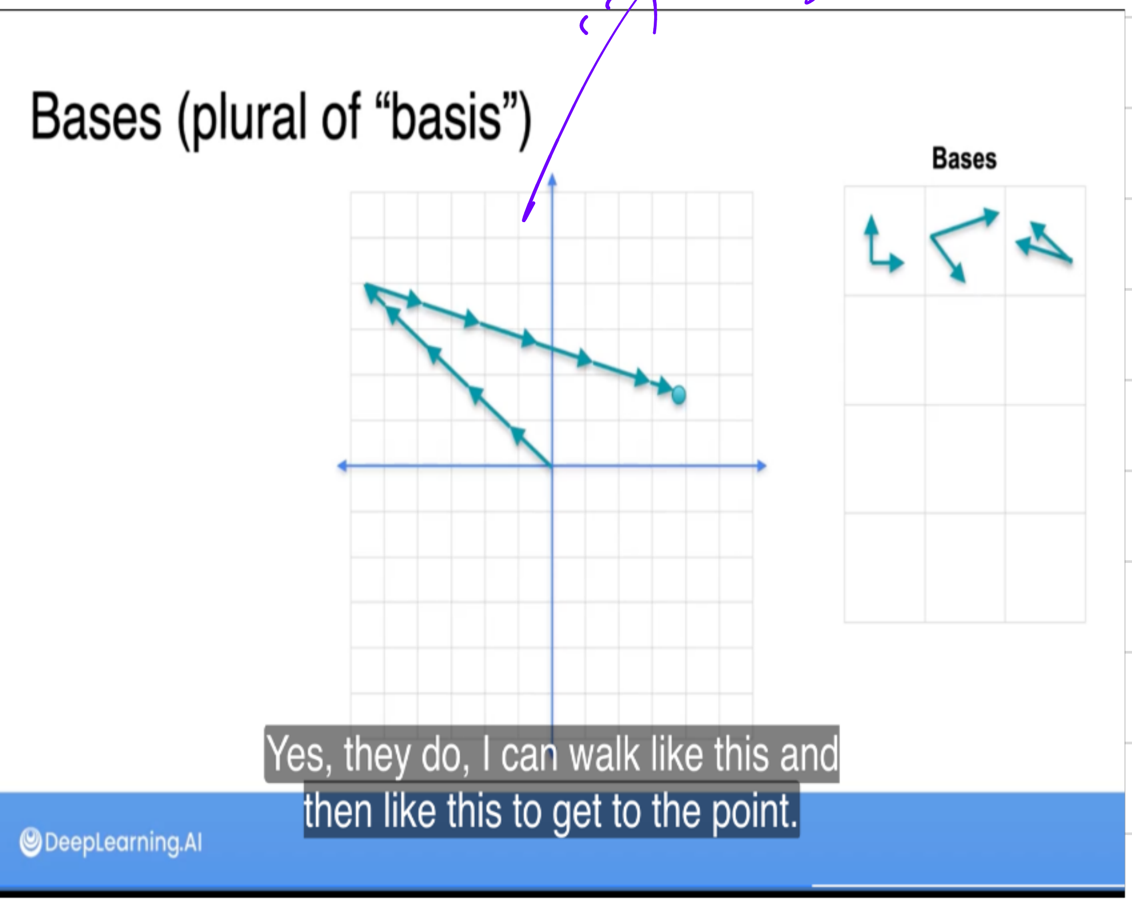
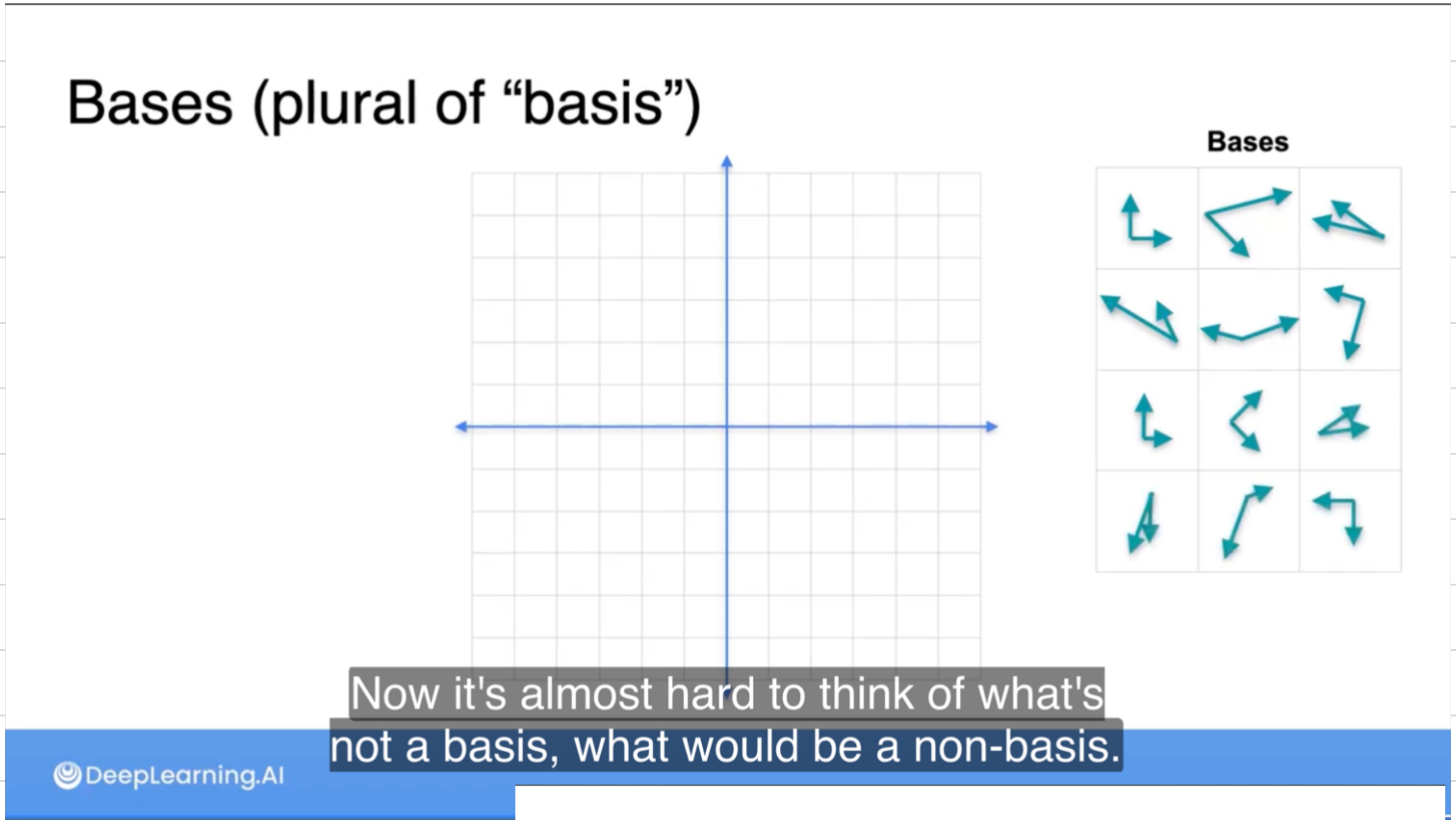
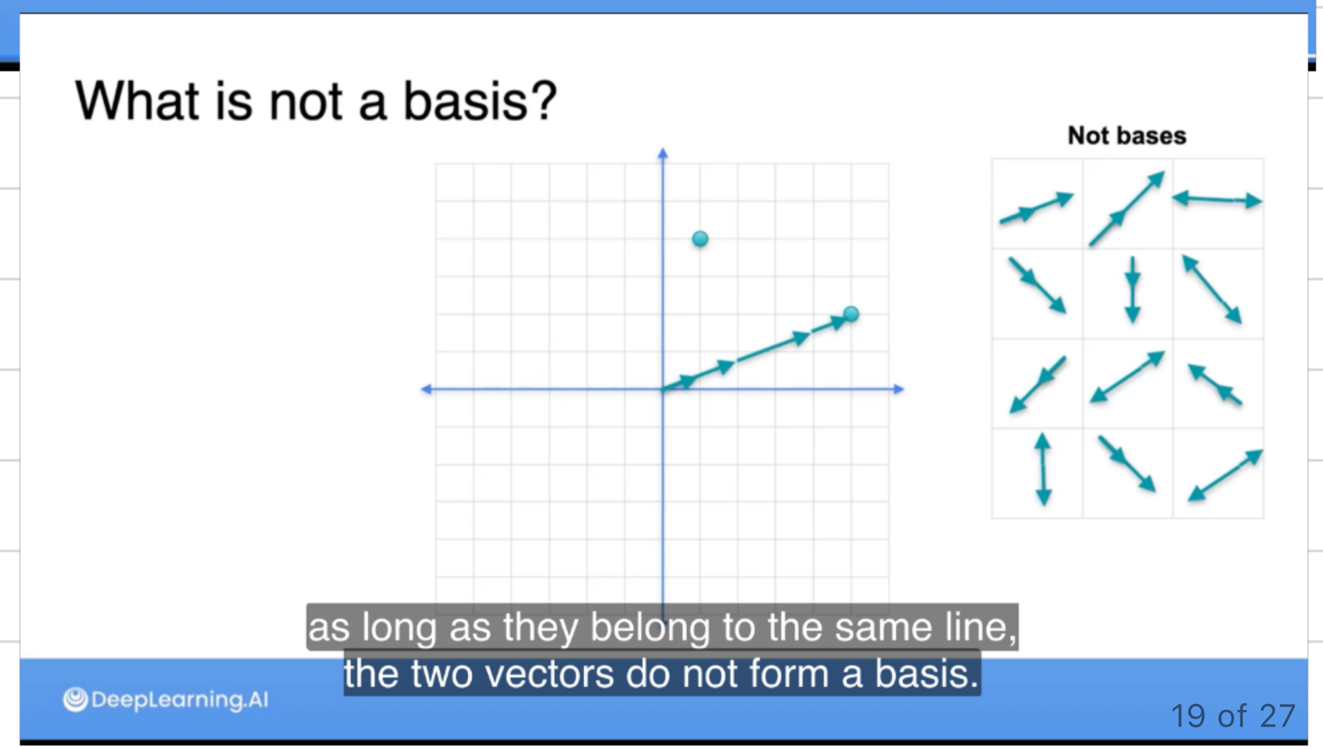
Backward and forward





Condition Not to be Basis.

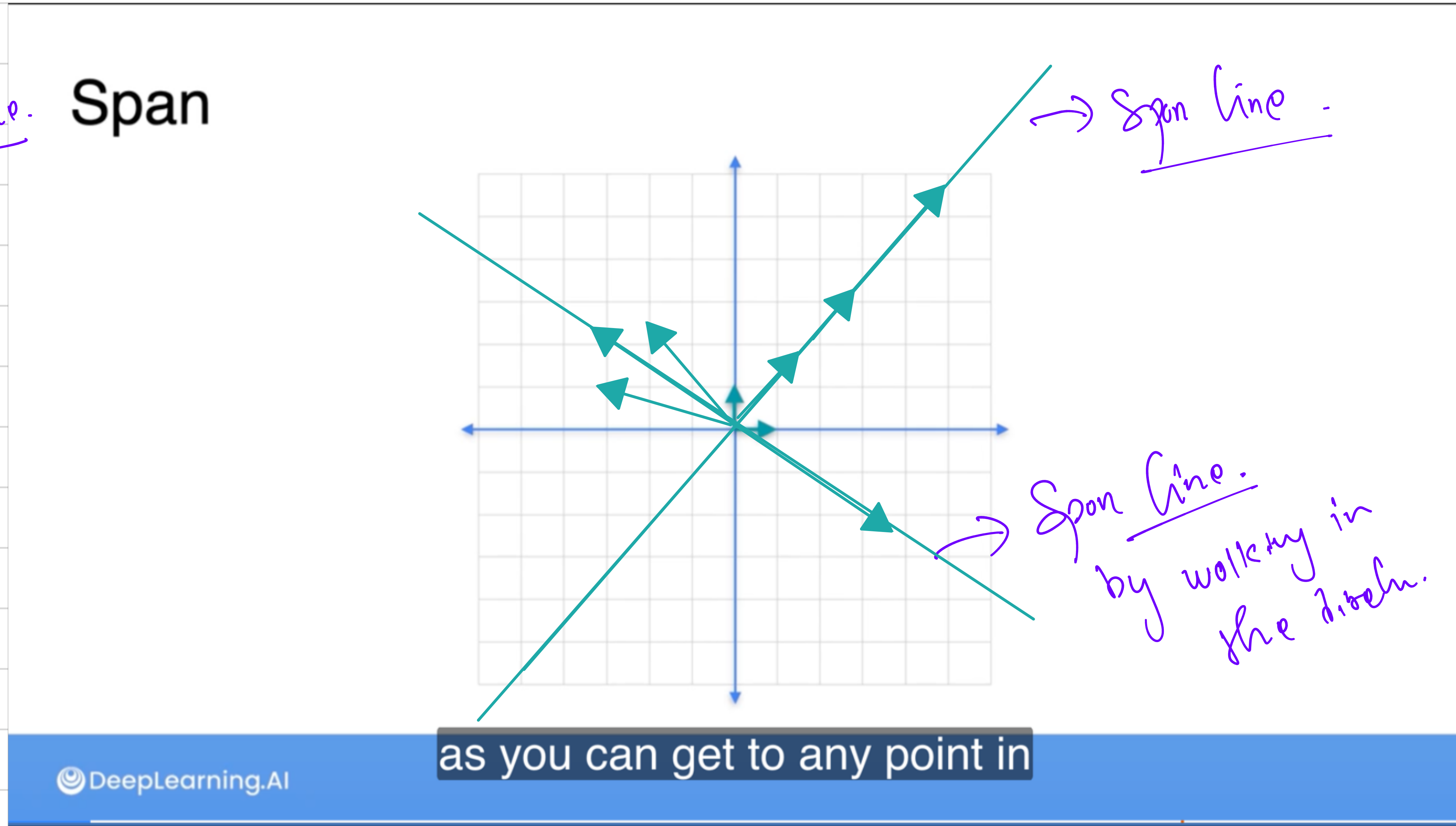
2 vectors having Same direction.



Span in Linear Algebra:

The span of a set of vectors is simply the set of points that can be reached by walking in the direction of these vectors in any combination.

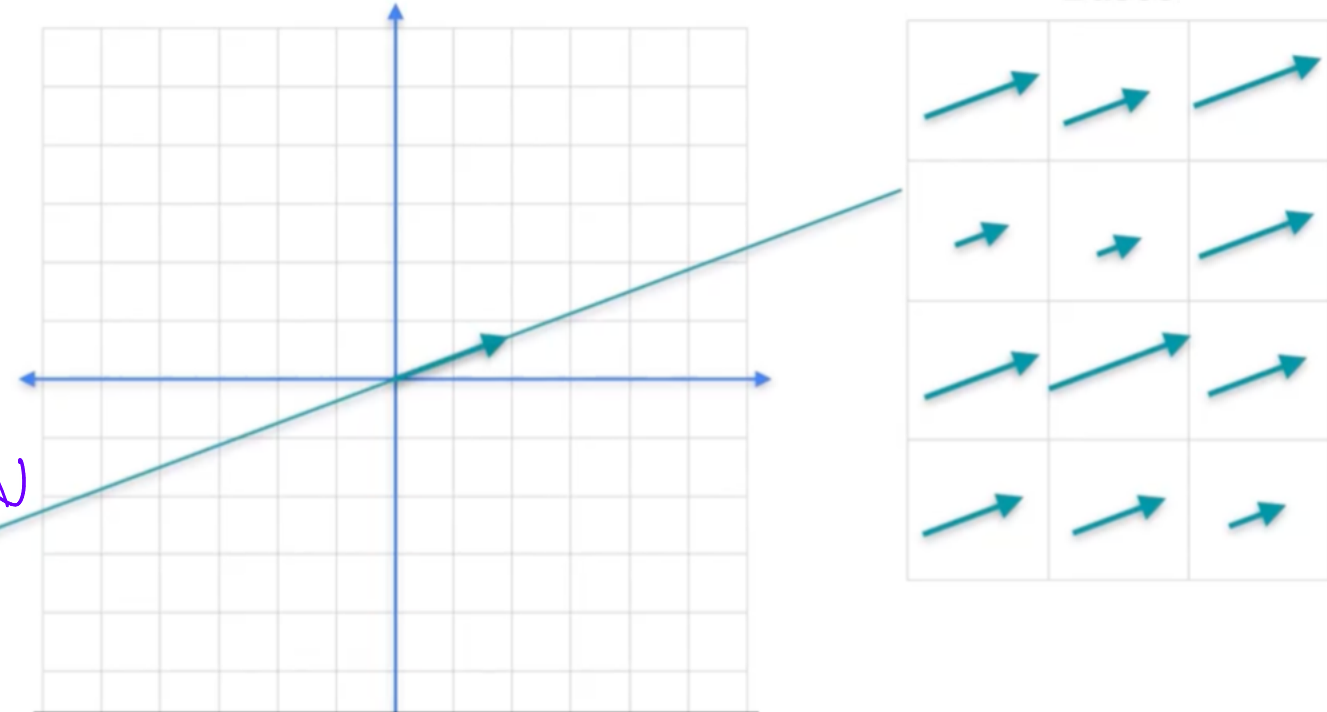
Set of points → Span line.



→ Start with origin vector are basis.

→ A basis is a minimal spanning set

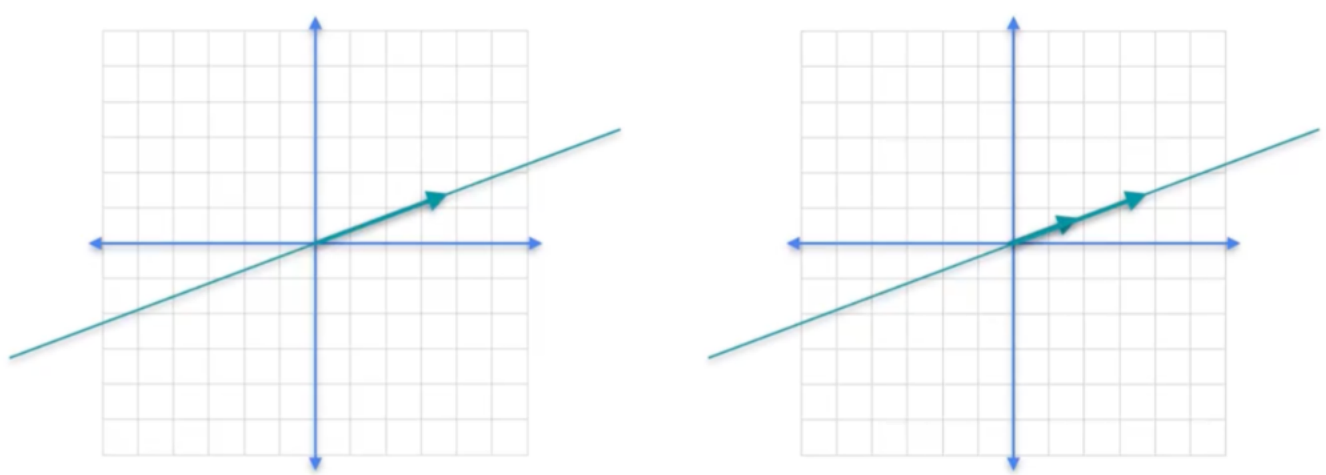
Is this a basis for something?



forms Basis of line.

that same direction is also
a basis for that line

A basis is a minimal spanning set



Basis

Not a basis

This happens also for

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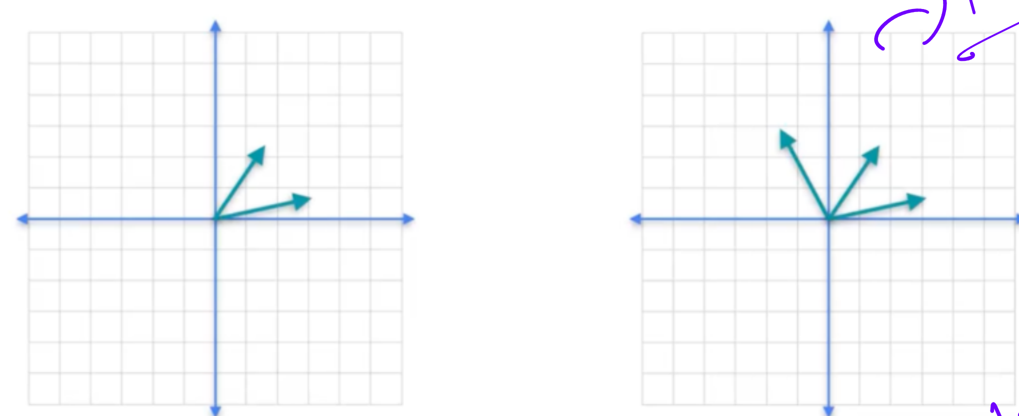
A basis is a minimal spanning set



→ Redundant

Basis

Too Big to be Basis

here do not form a
basis of the plane.

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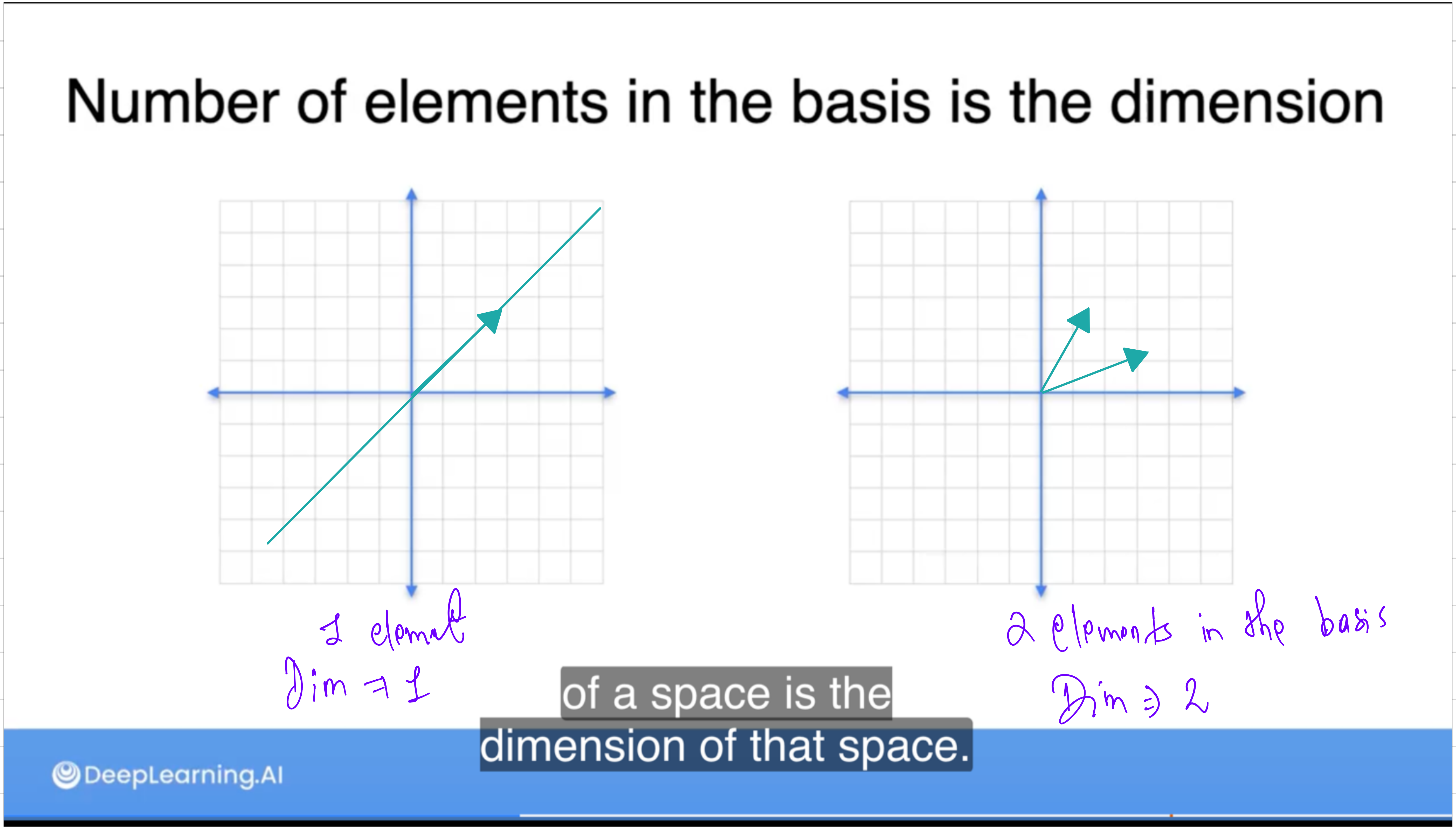
Number of elements in the basis is the dimension
1 element
Dim => 1
2 elements in the basis
Dim => 2
of a space is the
dimension of that space.
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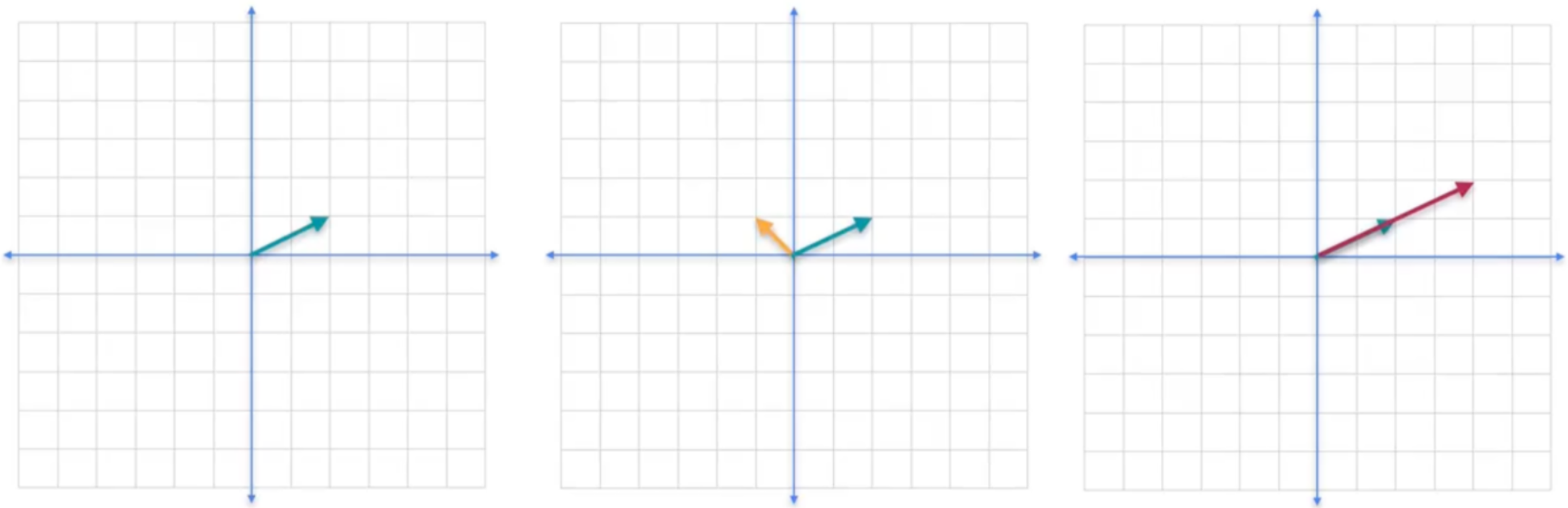
Linearly independent and linearly dependent vectors



Remained in Straight line

Notice also that even though we

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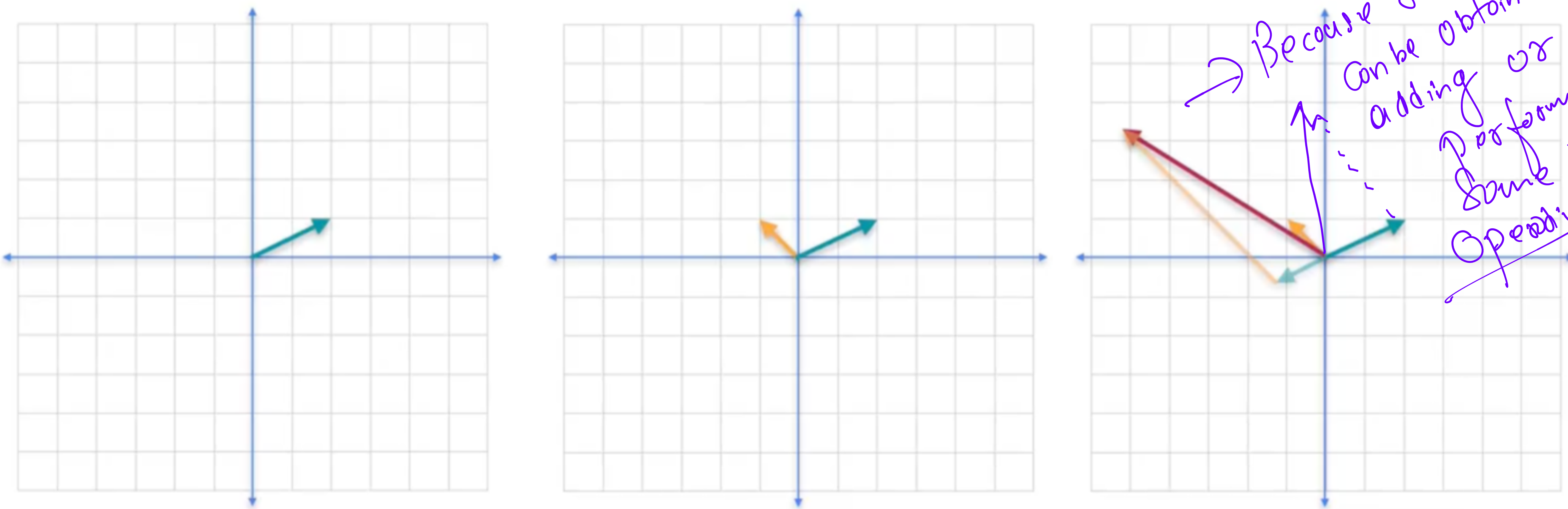
$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \Rightarrow 1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} - 1\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Linearly independent and linearly dependent vectors



Linearly independent | Linearly independent | Linearly dependent

dimension of the space
you're trying to span,

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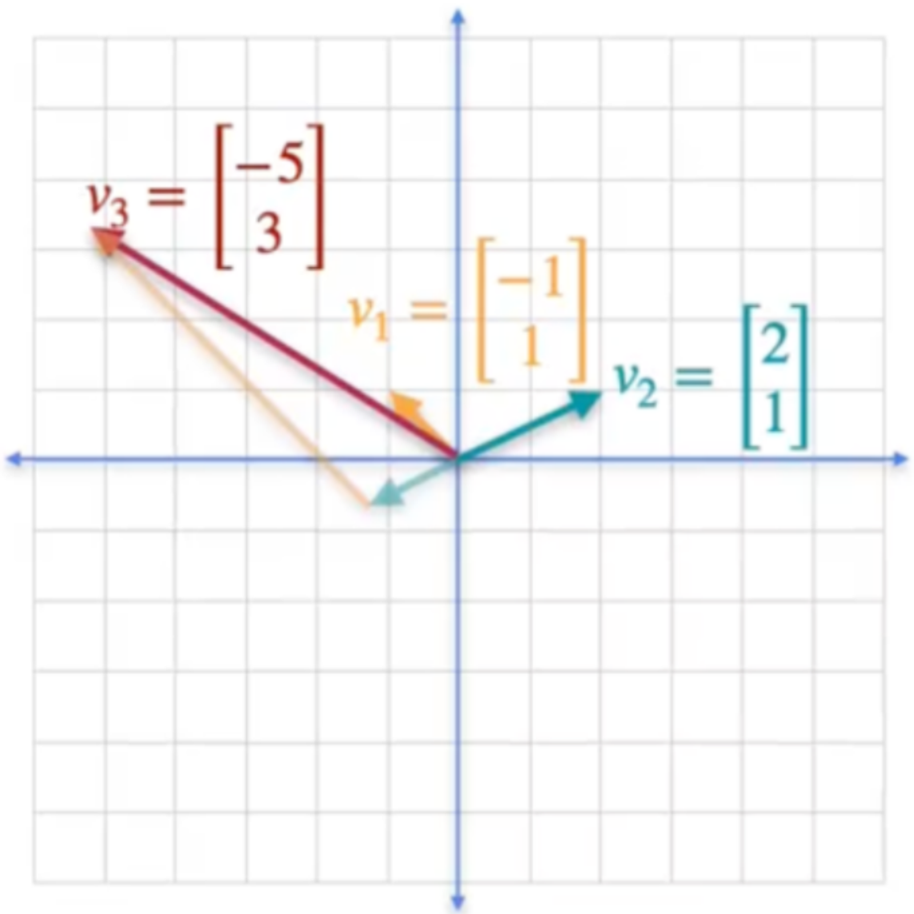
Let's see how to check for linear dependence



Linearly dependent

$$\alpha v_1 + \beta v_2 = v_3$$

$$\alpha \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

$$-\alpha + 2\beta = -5$$
$$\alpha + \beta = 3$$

$$3\beta = -2$$
$$\boxed{\beta = -2/3}$$

$$\alpha - 2/3 = 3$$
$$\boxed{\alpha = 11/3}$$

V_3 is a linear comb of v_1 and v_2

If solution exists linearly Dependent

In other words, you are looking for

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Formal Definition

Basis: A form.

- Spans a vector space
- is linearly Independent.

Solution

Are these vectors linearly independent?

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Not a basis!

Linearly independent

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

Linearly independent

$$\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Linearly independent

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

But since each linear
independent set only

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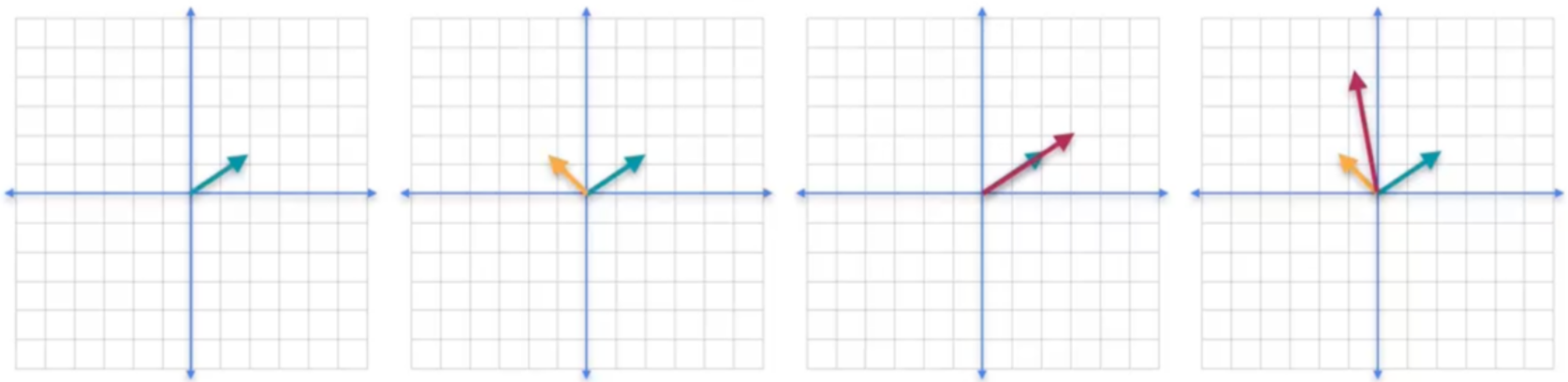
Basis: a formal definition

A basis is a set of vectors that:

- Spans a vector space
- Is linearly independent

Not all sets of N vectors are a basis for N-dimensional space



Spans a line
Linearly independent
Is a basis

Spans the plane
Linearly independent

Spans a line
Linearly dependent

Spans the plane
Linearly dependent
Not a basis

n vectors will form a basis for an n-dimensional space.

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Source: Linear Algebra for Machine Learning and Data Science offered by Coursera.