Day · 76, Feb 14, 2025 (Falgun 2, 2081 B.S.)

Probability Distributions:

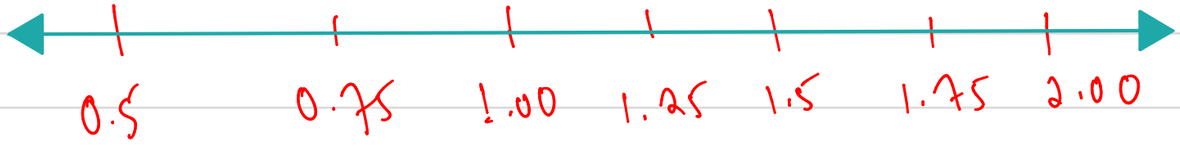
→ Describe the likelihood of the possible outcomes of a random event. Eg: probability of succesful clinical trials.

Random Variables. Represents the values for the possible outcomes of a random event.

Discrete Random Variable → has a countable possible values of P

Continuous Random variable → Takes all the possible values in some range of numbers. It measure the outcome (continuous).

Continuous values.



↳ Discrete Distributions represent discrete random variables.

↳ Continuous Distributions represent Continuous Random variables.

Sample Space:

- The set of all possible values for a random variable.
  - Sample space for single coin toss = {Heads, Tails}.
  - Sample space for single Die Roll = {1, 2, 3, 4, 5, 6}

Single Die Roll

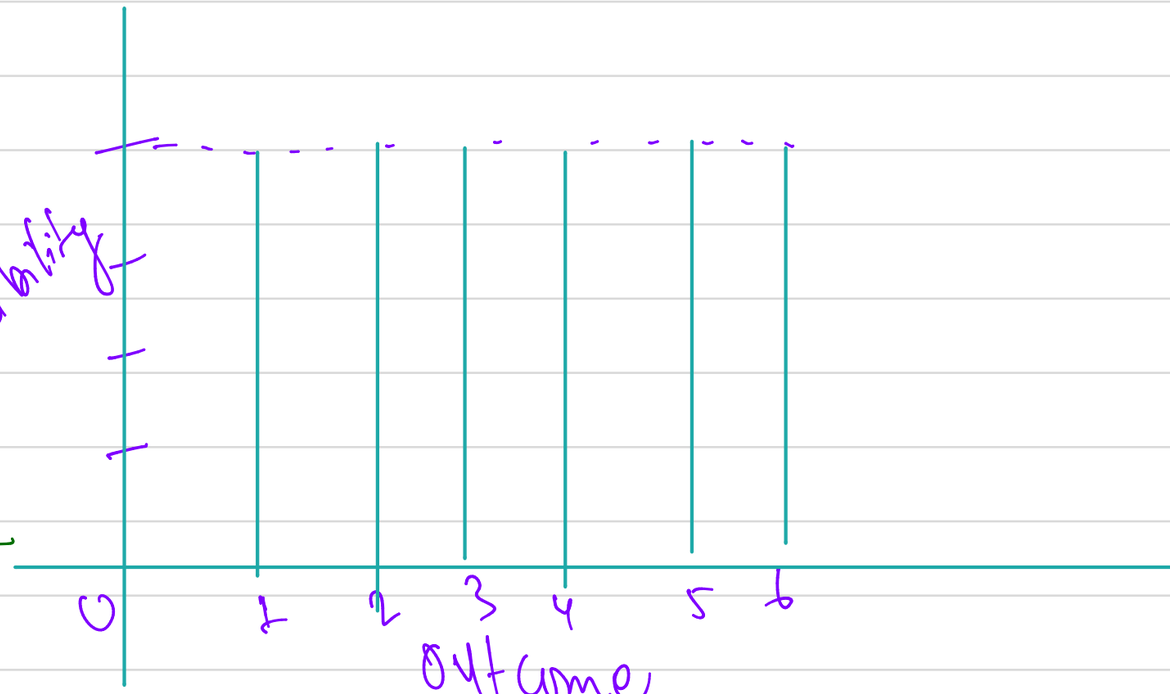
- Sample space = {1, 2, 3, 4, 5, 6}

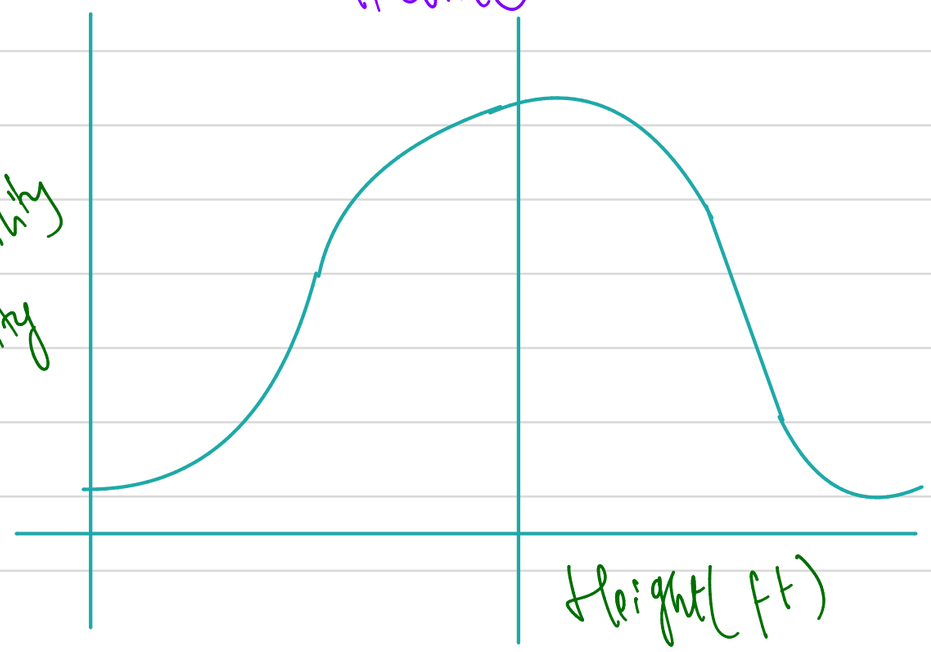
- probability of each outcome = 16.7%

Infinite number of values = Continuous Random Variable.

eg. 5.67 + 5.66 + 5.15 + ... + ∞

So, we need Continuous Probability Distribution using Probability Density Function —





Binomial Distribution

A discrete distribution that models the probability of events with only two possible outcomes, success or failure.

- Each event is independent
- the probability of success is the same for each event.
- Eg: tossing the coin for 10 times where each toss is 'F or 'S'.

Mutually Exclusive

Two outcomes are mutually exclusive if they cannot occur at the same time.

↳ Binomial Distributions used in ML, Banking, medicine and investing.

Random Experiment

→ A process whose outcome cannot be predicted with certainty.

→ More than one possible outcome and each outcome of the experiment depends on chance.

Binomial Experiment

↳ Consists of a number of repeated trials

↳ Each trial has only two possible outcomes and the 'p' is Success is the same for each trial.

↳ Each trial is independant.

Binomial Experiment Examples:

- 10 repeated coin toss
  - 2 possible outcomes: heads or tails
  - 'p' of success for each toss is the same: 50%.
  - the outcome of one coin toss does not affect the outcome of any other coin toss.

- 100 customer visits with two possible outcomes: return or not return
  - p of success for each customer visit is the same = 10%.
  - the outcome of one customer visit does n't affect the outcome of any other customer visit.

Binomial Distribution formula

$$P(X=k) = \frac{n!}{k!\,(n-k)!}\, p^k (1-p)^{n-k}$$

- k refers to no. of Success
- n refers to the number of trials
- p refers to the probability of Success in a given trial.

$$P(X=k) = \frac{n!}{k!\,(n-k)!} * p^k * (1-p)^{n-k}$$

$\frac{n!}{k!\,(n-k)!} \rightarrow$ ${}^{n}C_{k}$ (n-choose k).

$C\binom{n}{k} \rightarrow$

$P(X=0) = 0.729$

$P(X=2) = 0.027$

$P(X=3) = 0.001$.

Normal Distribution:

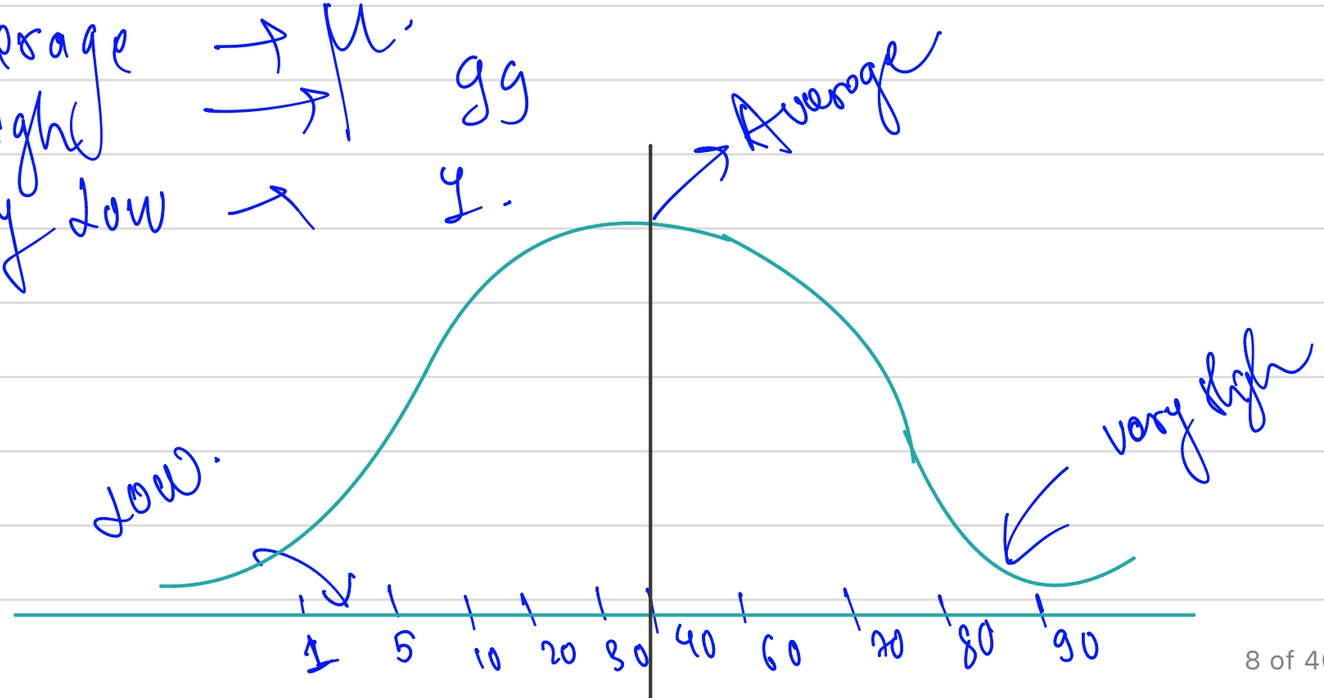
↳ While Discrete probability Distributions like Binomial & poisson Distribution can only take the discrete value into an account.

Normal Distribution:

→ A continuous probability distribution that is symmetrical on both sides of mean and bell-shaped.

Eg: Maximum number score Average → μ.
very less " " high → gg
very " " " very low → 1.

↳ used in Advanced Statistics like Hypothesis testing. (In terms of score.)



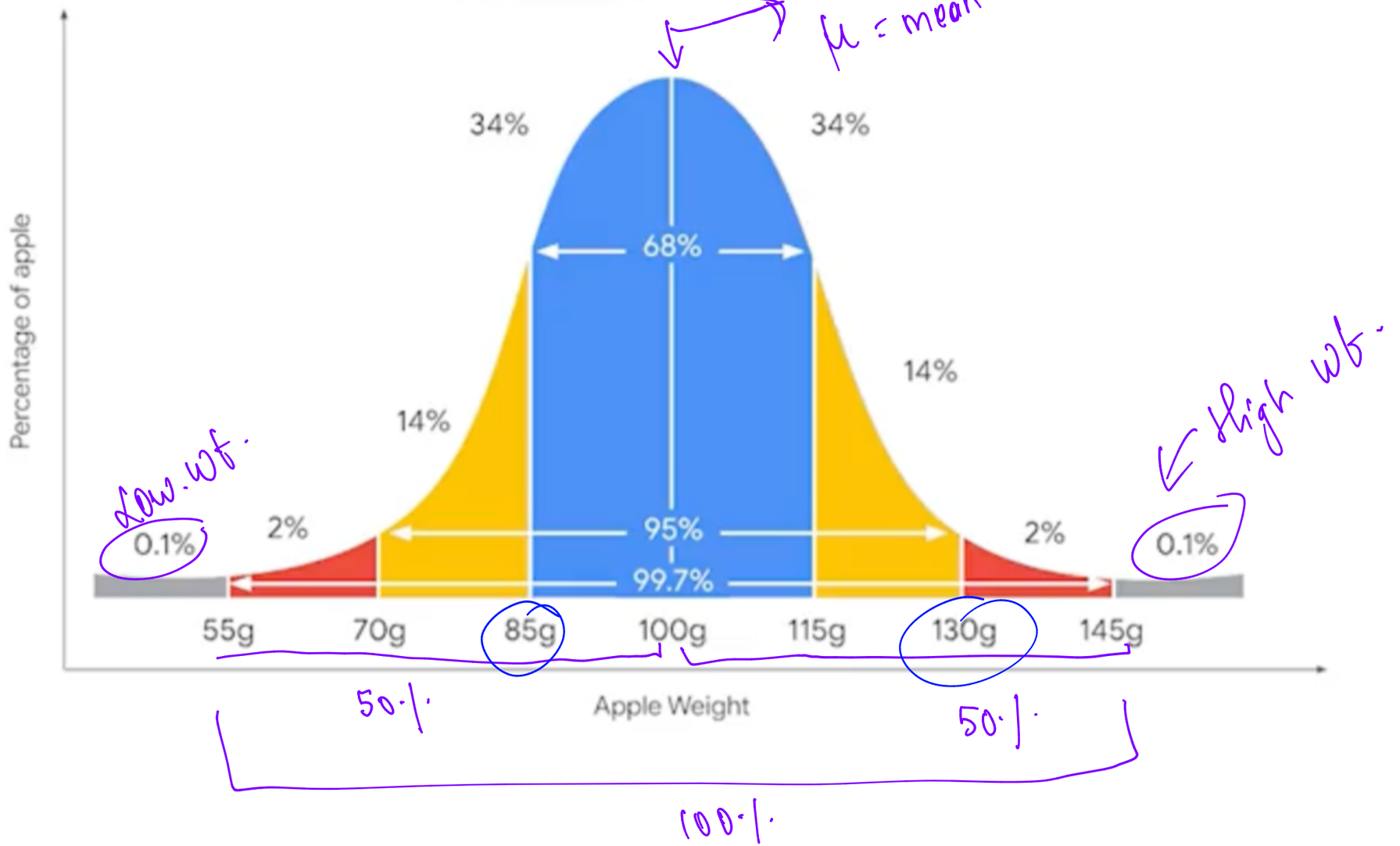
Features of Gaussian Distribution:

1. Shape is a bell curve.
2. Mean is located at the center of the curve
3. Curve is symmetrical on both sides of the center
4. the total area under the curve equals 1.

Standard Deviation: Calculates the typical distance of a data point from the mean of your dataset.

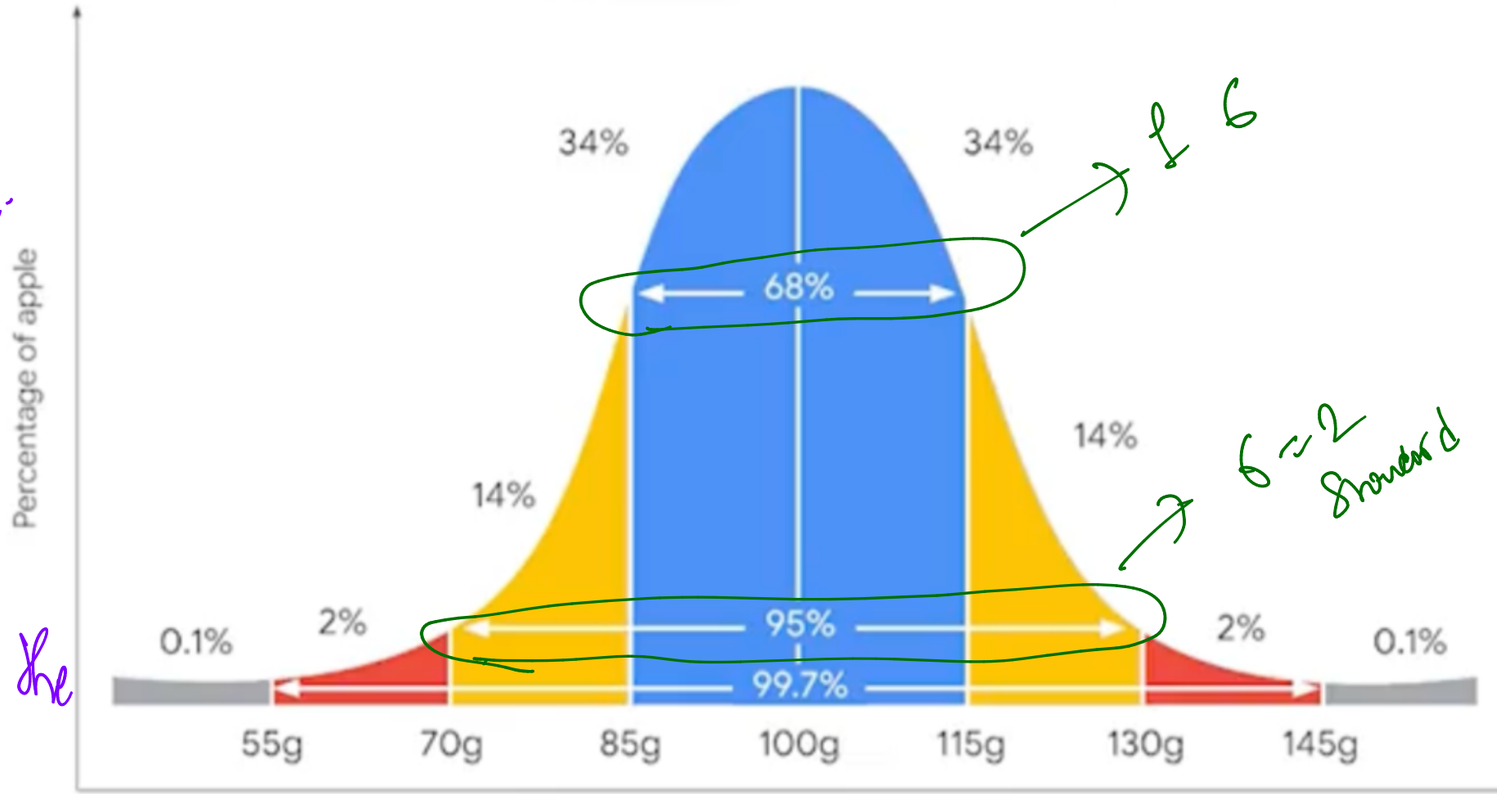
mean → center

σ ⇒ Spread of data from the center or mean.


Apple Weight Distribution
μ = mean = mode = median
34%
34%
68%
14%
14%
95%
2%
2%
99.7%
0.1%
0.1%
low wt.
high wt.
55g
70g
85g
100g
115g
130g
145g
Percentage of apple
Apple Weight
50%
50%
100%

Empirical Rule:

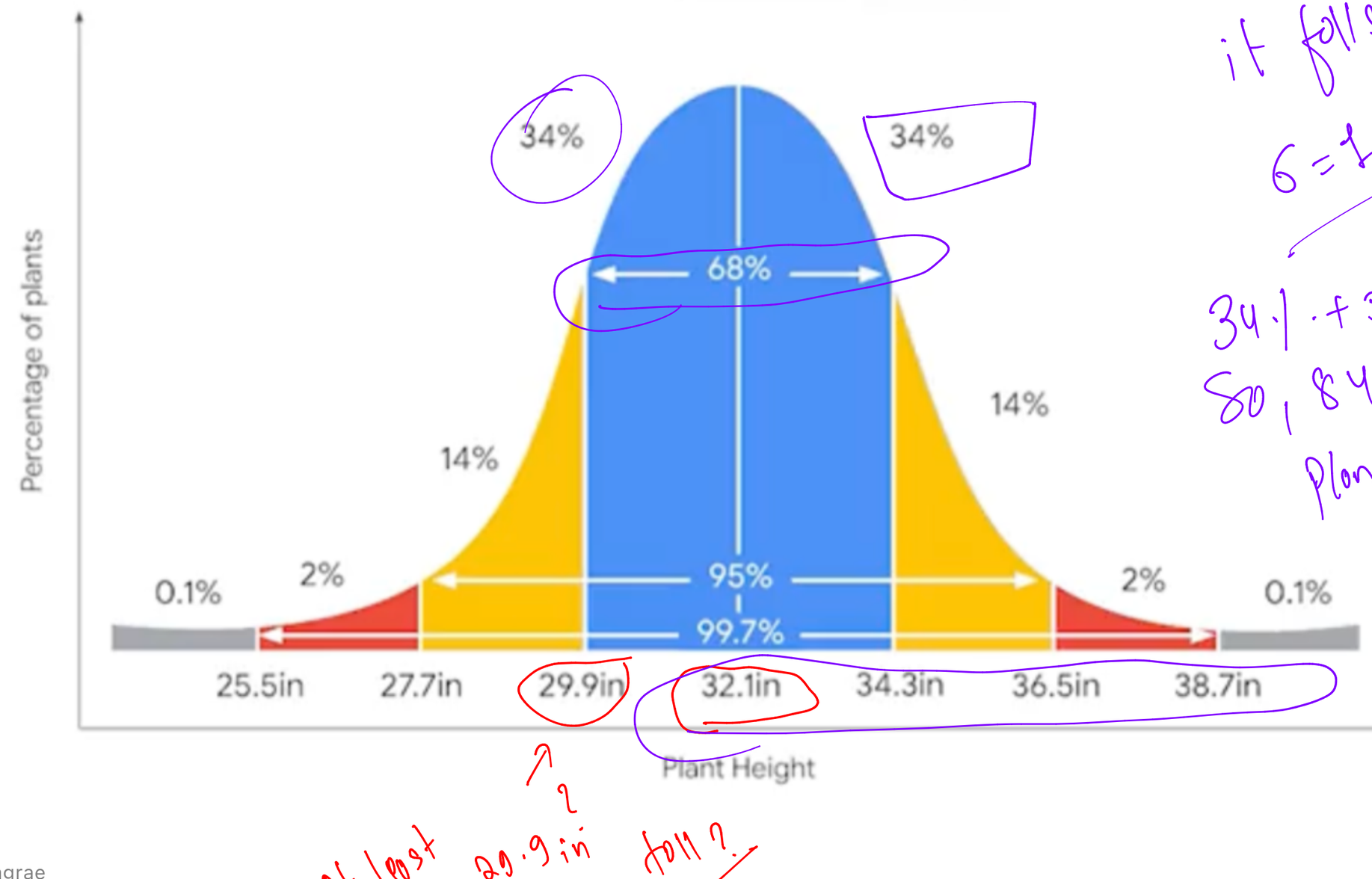
- 68.1% of values fall within 1 standard deviation of the μ.
- 95.1% of values ... 2 standard deviation of the μ.
- 99.7% of values fall under the 3 standard deviation of μ.



→ 6 = 1 means 68% which means 85g and 115g between the ranges of apple weight ranges.

→ 6 = 2 means of ranges between 70g and 130g.

So, 34% from the centre of mean for plant to be atleast 29.9 inch tall because if falls under the $\sigma = 1$ standard.



$34\% + 50\% \Rightarrow 84\%$

So, 84% of plants are taller than 29.9 in.

At least 29.9 in tall?

Explanation:

If you want to determine the percentage of plants taller than 29.9 inches for your backyard landscape design, you can use the Empirical Rule of normal distribution.

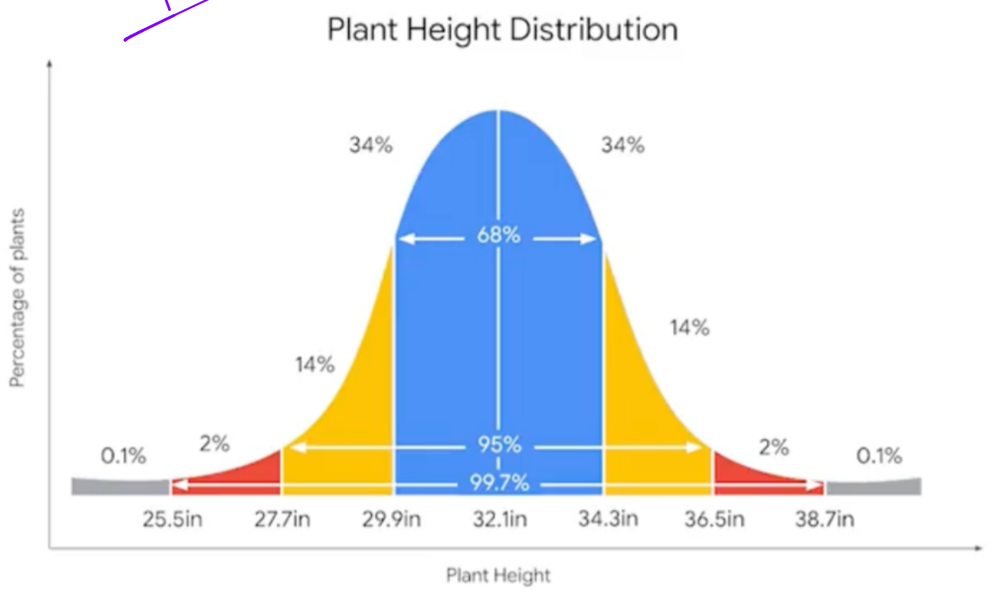
Since 29.9 inches is one standard deviation below the mean, the Empirical Rule states that 68% of values fall within one standard deviation of the mean. This means 34% of values lie between 29.9 and the mean. Additionally, in a normal distribution, 50% of values are above the mean.

To find the percentage of plants taller than 29.9 inches, add these two values:
34% (between 29.9 and the mean) + 50% (above the mean) = 84%.

Thus, 84% of your plants meet the height requirement. This quick estimation method helps analyze data patterns efficiently. As a future data professional, understanding the normal distribution will be key to interpreting and making data-driven decisions.

Source: The power of Statistics Coursera Course provided by Google.

Explanation:



Standardize Data using Z-scores:

→ Z-score is a measure of how many σ below or above the population mean a data point is.

→ Z-score gives idea on how far the data is from the μ.

→ Z-score is 0 if the value is equal to the mean.

→ Z-Score is +ve if the value is greater than the μ.

→ Z-Score is -ve if the value is less than the μ.

Standardization the process of putting different variables in the same scale.

Z-score helps to understand the relationship between different Dataset.

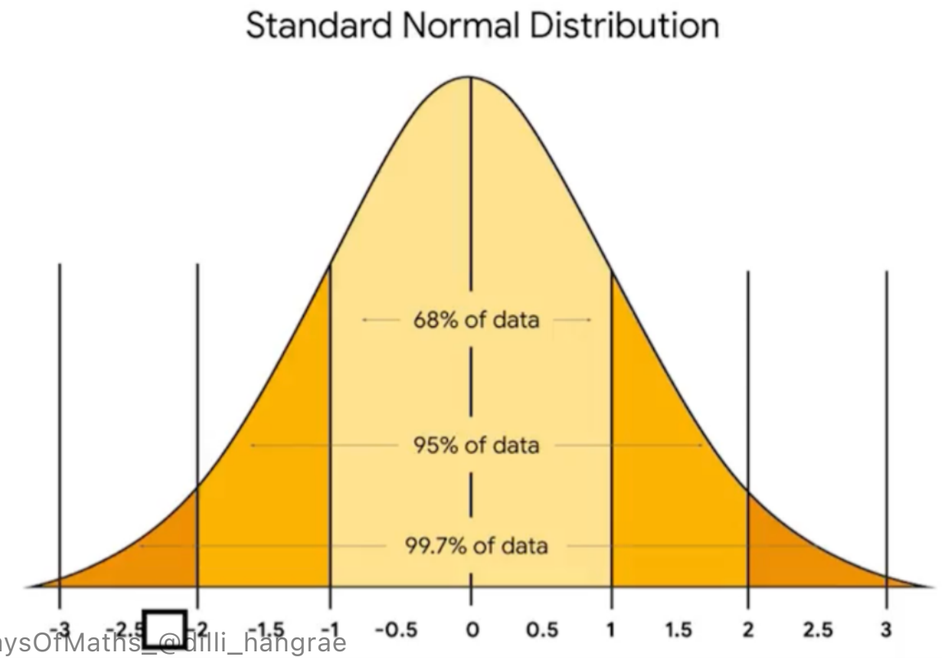
Z-Score Application in Anomaly Detection Application

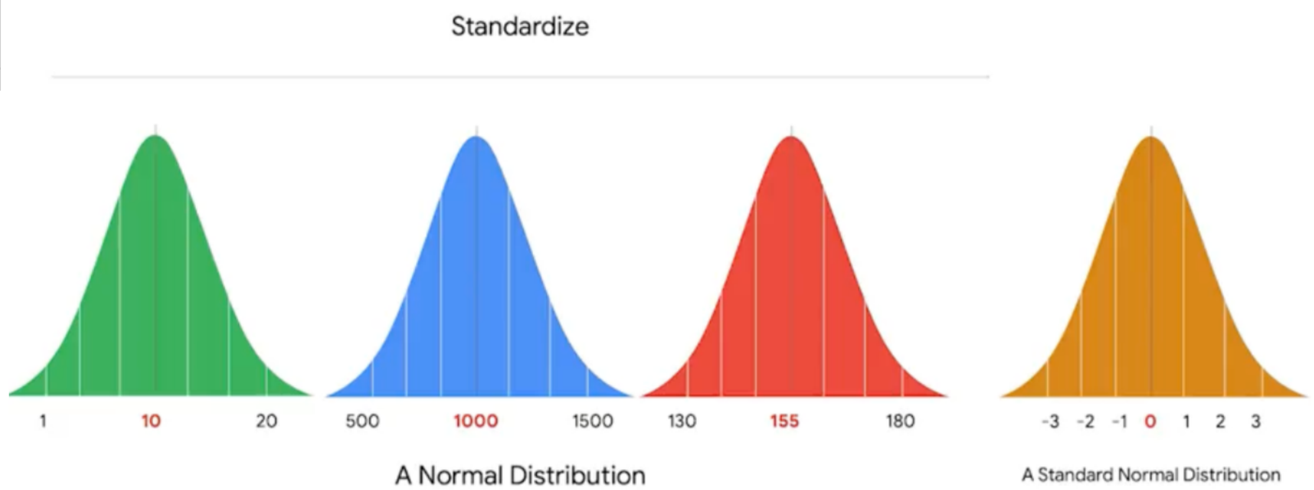
(1) Fraud in financial Transactions

(2) flaws in manufacturing products

(3) Intrusions in Computer Networks.

Standardize





↳ Standardization helps to perform and analyze the distribution statistically efficient & smooth.

score → X , mean → μ

$$z = \frac{X - \mu}{\sigma}$$

σ → SD

$$z = \frac{(133 - 100)}{15}$$

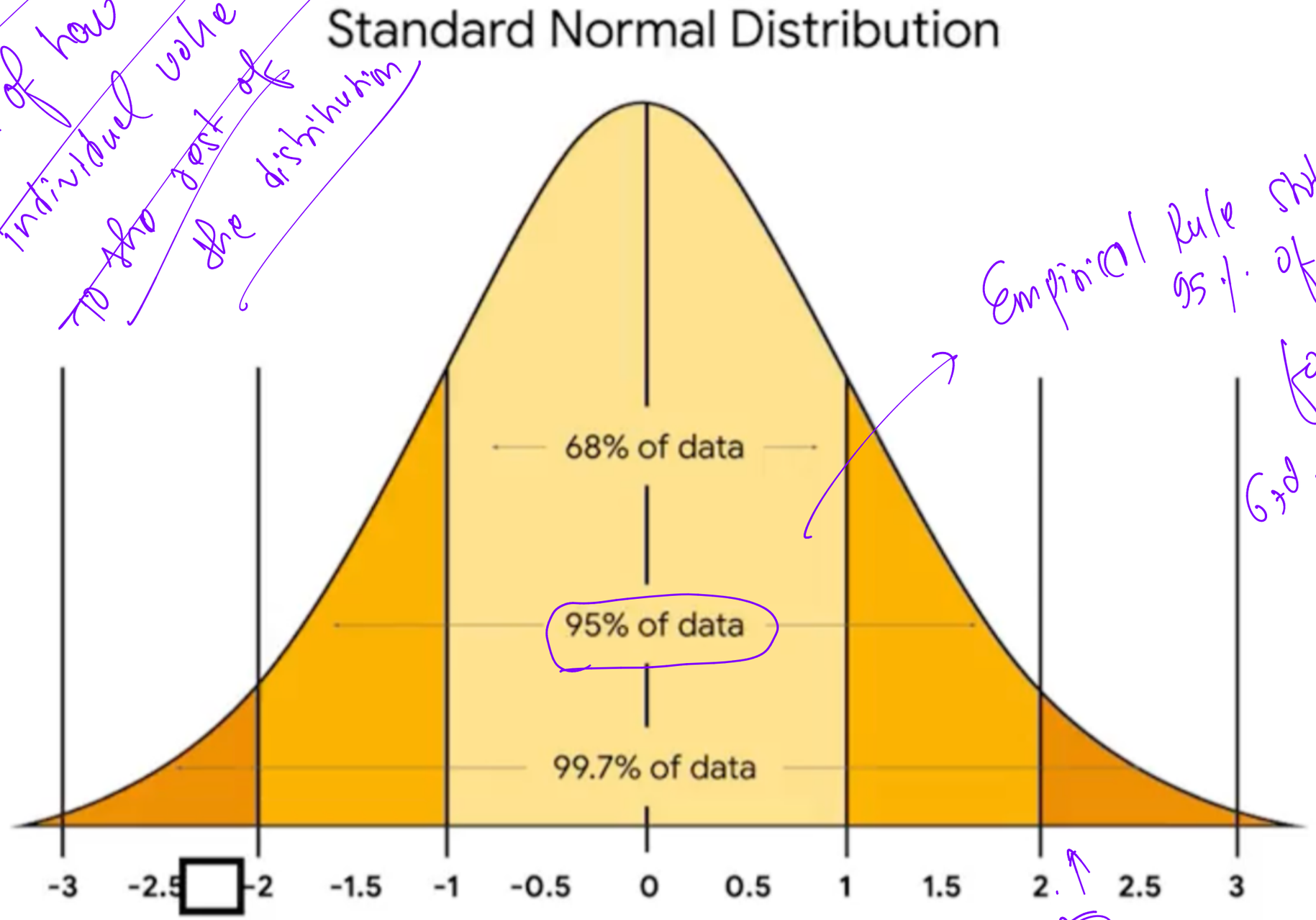
$$z = 2.2$$

2.2 tells us that your test score is 2.2 standard deviations above the mean or average score.

So, the z-score of data value equal to the μ. when z-score $= \mu$.

A z-score is a measure of how many standard deviations below or above the population mean or data point is-

→ Z-score gives idea of how an individual value to the rest of the compares the distribution



→ Empirical Rule states that 95% of the values falls under the 2nd standard deviation

2.2

Example:

Say you score in 85, you want to find out if that's a good score relative to the rest of the class.

Whether or not it's a good score depends on the mean and standard deviation of all exam scores.
Suppose the exam scores are normally distributed with a mean score of 90 and a standard deviation of 4, you can use the formula to calculate the z-score of a raw score of 85.

Your z-score is yours raw score, 85 minus the mean score 90, divided by the standard deviation 4.
This is 85 minus 90 divided by 4 equals -5, divided by 4 equals 1.25. Your z-score of -1.25 tells you that your exam score of 85 is 1.25 standard deviations below the mean or average exam score. Z-scores give you an idea of how individual values compared to the mean.

As a data professional, you'll use z-scores to help you better understand the relationship between specific values in your data set.

Introduction to Sampling:

Types of statistics → Descriptive statistics summarize the main features of a dataset.
→ Inferential statistics use sample data to draw conclusions about a larger population.

#Sampling - process of drawing a subset of data from a population.

Questions Answered by Sampling:

A representative sample accurately reflects the characteristics of a population. If a sample doesn't accurately reflect the characteristics of a population, then the inferences'll be likely be unreliable and inacurate.

This subset is your sample, then you can

Questions answered by sampling

- How many products in an app store do we need to test to feel confident that all the products are secure from malware?
- How do we select a sample of users to run an effective A/B test for an online retail store?
- How do we select a sample of customers of a video streaming service to get reliable feedback on the shows they watch?

↳ Sampling should be drawn randomly or unbiased.

↳ finding the Avenge Height of People in Nepal we use Sampling?

Day - 77, Feb 15, 2025 (Falgun 3, 2081 BS).

Sampling process:

Step 1: Identify the target population

↓

Step 2: Select the sampling frame

↓

Step 3: Choose the sampling method

↓

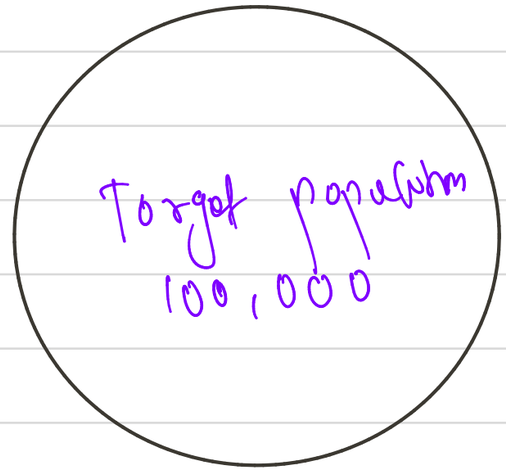
Step 4: Determine the sample size

↓

Step 5: Collect the sample data

Target population:

The complete set of elements that you're interested in knowing more about

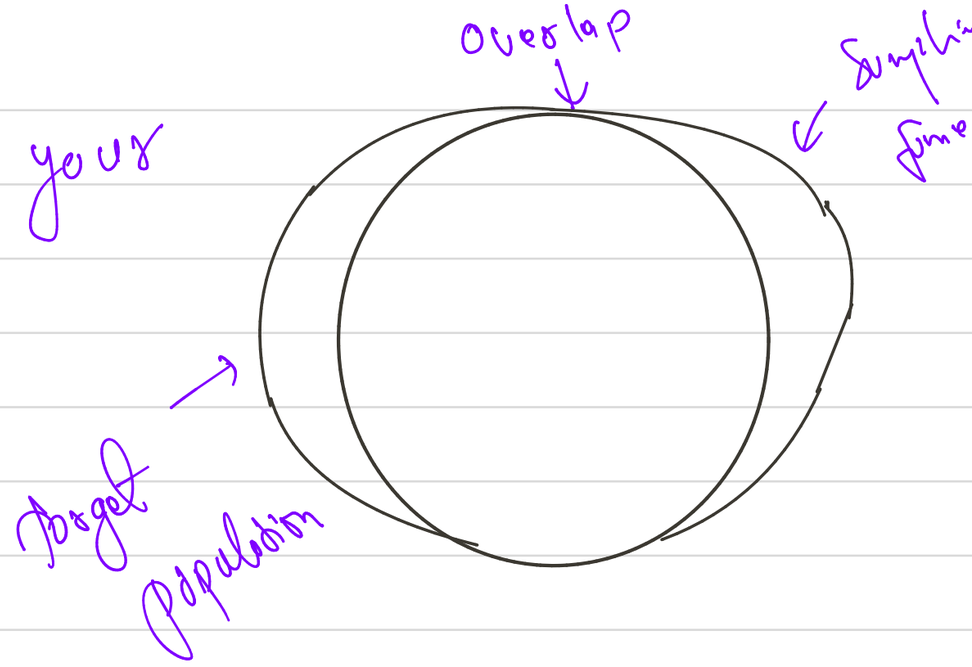


Sampling frame: A list of all the items in your target population.

Sampling

- Target population is general
- Sampling frame is specific

Sampling frame is the accessible part of your target population.



Two types of Sampling Methods:

1) Probability Sampling
2) Non-probability Sampling

Sampling Methods

- Probability Sampling: Uses random selection to generate a sample.
- Non-probability Sampling: Based on convenience or personal experience.

Sample Size:

The number of individuals or items chosen for a study or experiment.

Identifying the target population is the first step in the sampling process. The sampling process helps determine whether a sample is representative of the population and if it is unbiased.

Different Methods of Probability Sampling.

- Simple Random Sampling
- Stratified Random Sampling
- Cluster Random Sampling
- Systematic Random Sampling

Simple Random Sample

Every member of a population is selected randomly and has an equal chance of being chosen.

- Representative
- Avoid Bias

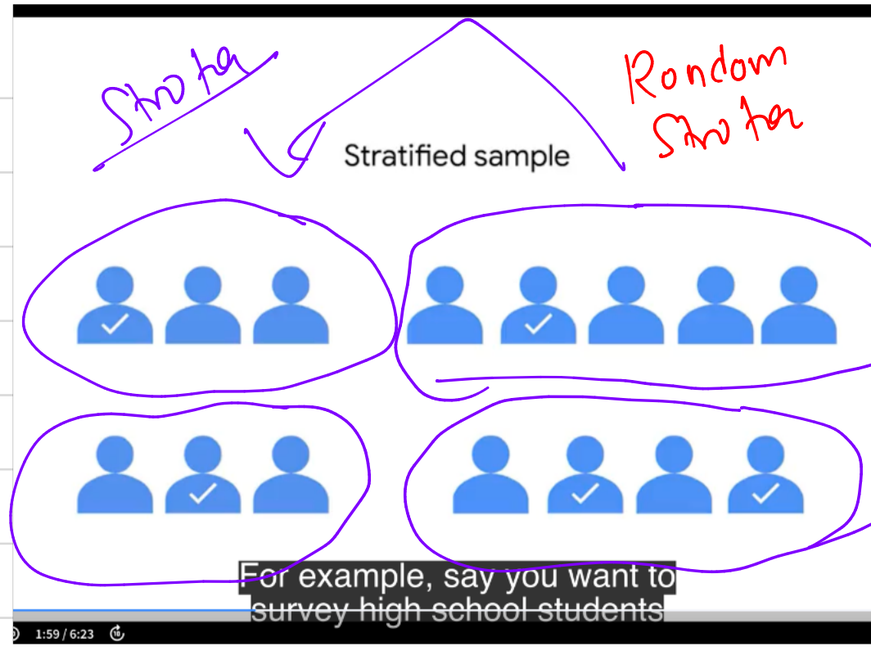
Stratified Random Sampling

Divide a population into groups and randomly select some members from each group to be in the sample.

- Members from each group are included.
- Strata can be included by gender, age and other appropriate group.

Cluster Random Sample:

Divide a population into clusters, randomly select certain clusters, and include all members from the chosen clusters in the sample



→ Difficulty in creating cluster that accurately represent population.

Helpful when dealing with large and diverse populations that have clearly defined subgroups.

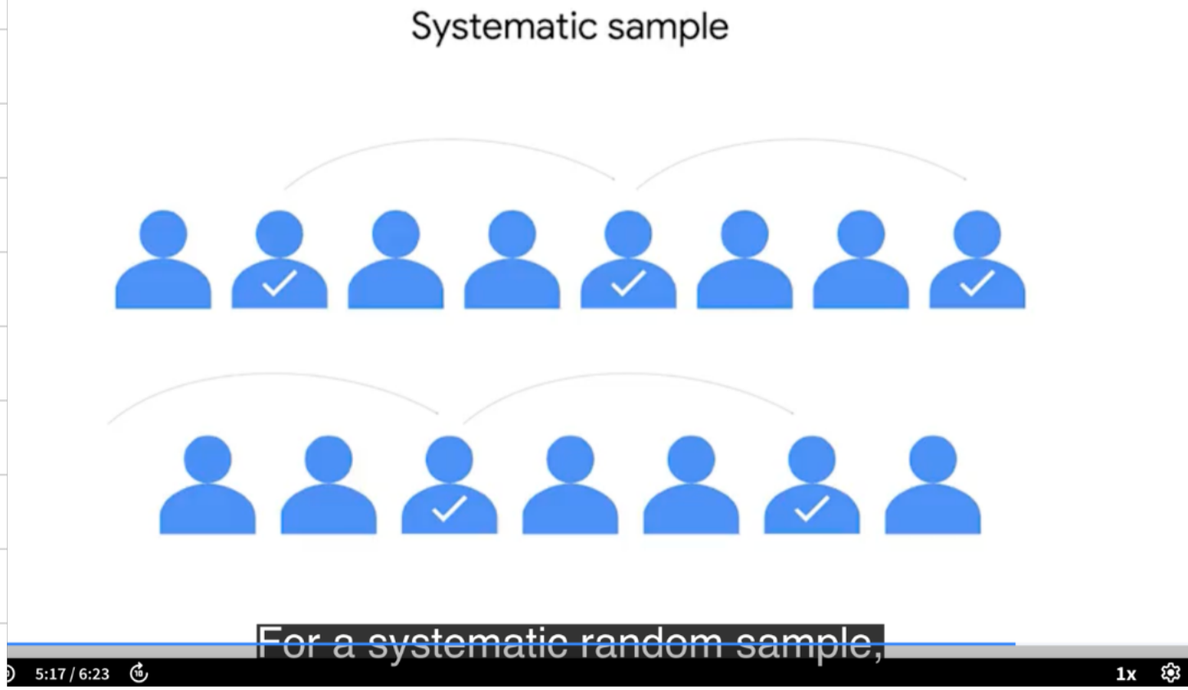
Systematic Random Sample

put every member of a population into an ordered sequence. then, you choose a random starting point in the sequence and select members for your sample at regular intervals.

- Quick and Convenient.
- Representative.

- Simple
- Stratified
- Cluster
- Systematic

} - those are all Random selection based probability sampling methods.



The Impact of Bias in Sampling:

Since Sampling plays very Important in analysis -

① Sampling Bias

When a sample is not representative of the population as a whole

② Non-probability sampling methods: Don't use Random selection or they typically do not represent the samples. It is often less expensive and more Convenient for researchers to conduct.

Non-probability Sampling Methods

- Convenience Sampling
- Voluntary Response Sampling
- Snowball Sampling
- Purposive Sampling

Convenience Sample

→ Choose members of a population that are easy to contact or reach.

→ For example polling in a high school.

Undercoverage Bias

↳ When some members of a population are inadequately represented in the sample. Who don't attend school are excluded.

Voluntary Response Sample:

→ Consists of members of population who volunteer to participate in a study.

↳ For example, review of online survey in Restaurant -

↳ Voluntary Response Sample suffers from Non-Response Bias (when certain groups of people are less likely to provide responses). They have stronger opinion that makes it unrepresentative.

Snowball Sample:

Researchers recruit initial participants to be in a study and then ask them to recruit other people to participate in the study.

Example - Investigating the Cheating among students.

Often lead to the bias called Sampling bias because students favors similar characteristics Students, which might be unrepresentative of the Sampling.

Purposive Sample:

↳ Researchers select participants based on the purpose of their study.

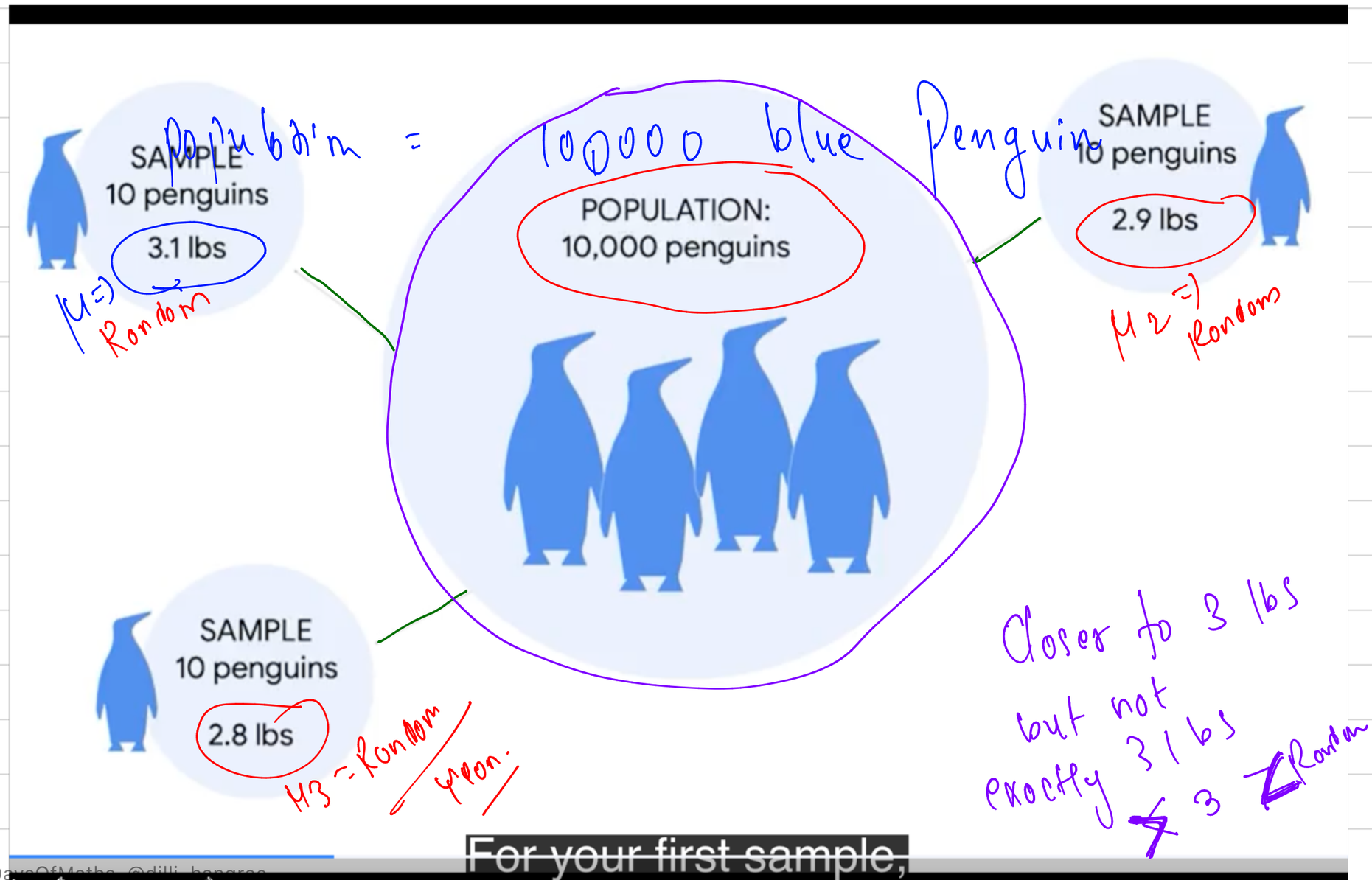
↳ participants whose profile doesn't fit are rejected, example researchers trying to evalute the effectiveness of university.

How Sampling Affects Data:

Statistic vs. Parameter

- the mean weight of a random sample of 100 penguins is a statistic.
- the mean weight of the total population of 10,000 penguins is a parameter.

Sampling Distribution : A probability distribution of a sample statistics. Eg: Coin toss, Die rolls, etc.



Central Limit Theorem

Concept:

- Sample of 100 penguins. Mean How accurately represents the mean of the Population μ Height.

As μ varies from Sample to Sample

- True μ = 3 lbs

Estimate $\mu_1 = 3$ lb

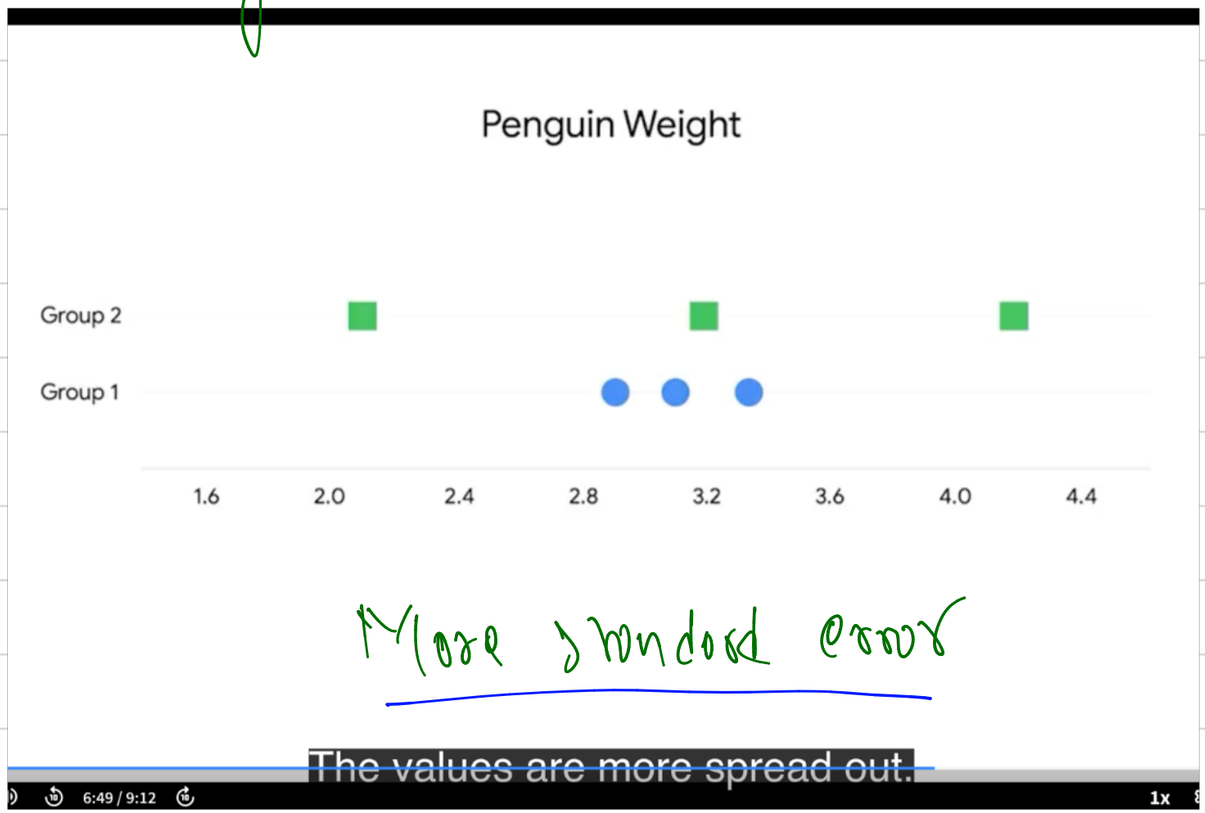
" μ_2 3.2

Forms the Gaussian Distribution

Standard Error: Measures the variability among all the Sample means.

- Larger Standard error = Sample means are more spread out
- Smaller Standard error = Sample means are closer together

the less Standard Error → More likely accurate mean.



# Standard Error of the mean = $\frac{S}{\sqrt{n}}$ Where S = Std. dev
n => Sample Size.

$$\Rightarrow \frac{1}{\sqrt{100}}$$

$$= 0.175$$

This means that your best estimate for the true population mean weight of all penguins is 3 pounds, but you should expect that the mean weight from one sample to the next will vary with a standard deviation of about 0.1 pounds.

As your Sample Size gets larger the Standard Error gets Smaller.

Sample Size = 10,000

$\sigma = 0.1$

(Assume) and $\mu = 3.0$

Sample 100,000

$\sigma \longrightarrow 0.1$

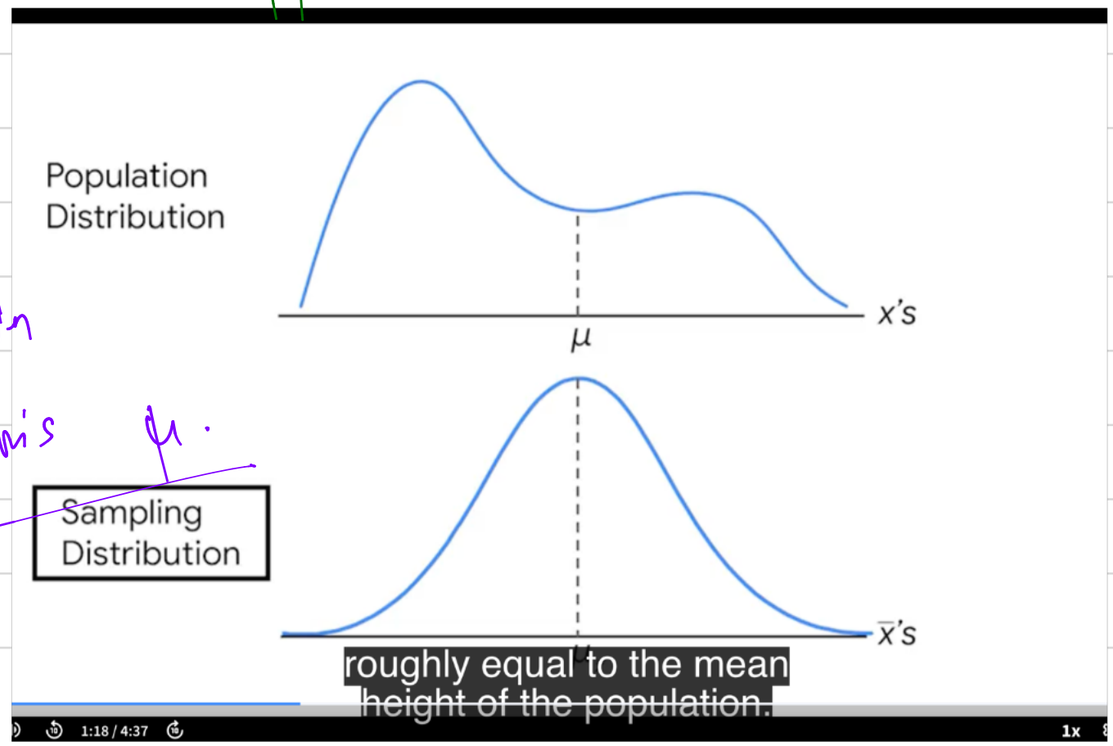
Central limit Theorem: can be used to estimate

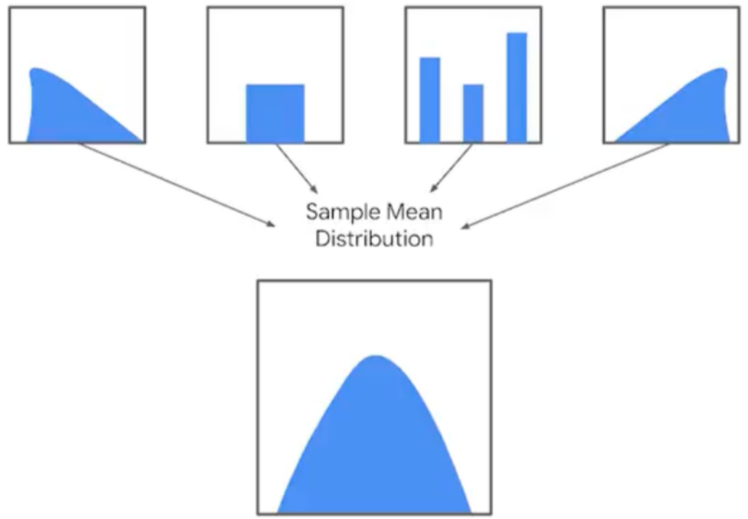
- The mean annual household income for an entire city or Country.
- the mean height and weight for an entire animal or plant population.

Central limit theorem

→ the sampling distribution of the mean approaches a normal distribution as the sample size increases.

Sampling Distribution's $\mu \simeq$ population Distribution's μ.

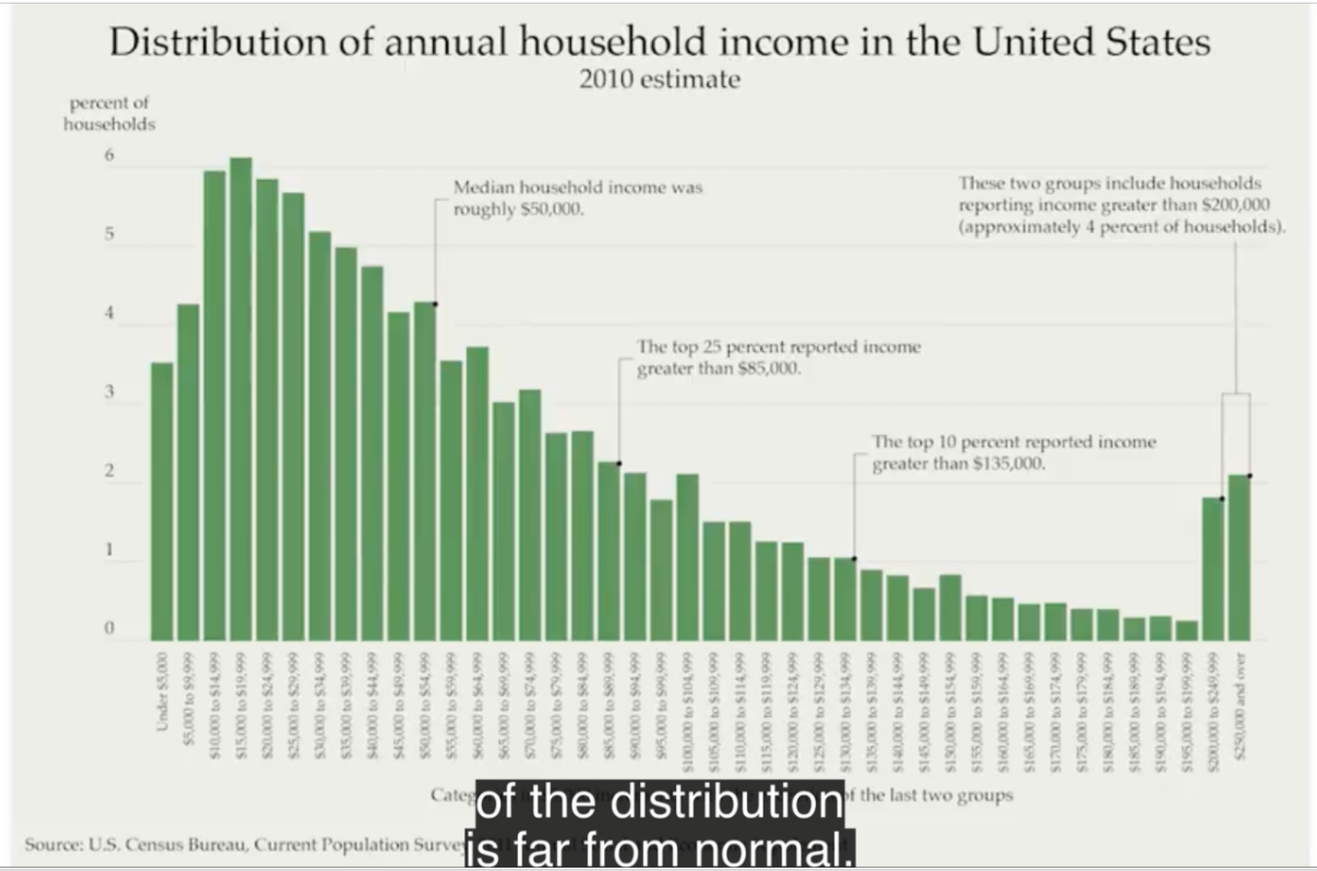




Normal distribution

If you collect a large enough sample,

1:52 / 4:37



of the distribution is far from normal.

① Even skewed Distribution of population gives Normal Distribution when sample mean is taken and plotted in a graph.

The central limit theorem says that as your sample size increases, the shape of your sampling distribution will increasingly resemble a bell curve.

If you take a large enough sample from the population, the mean of your sampling distribution will equal the population mean.

The Sampling distribution of the proportion

The percentage of individuals or elements in a population that share a certain characteristic.

Sampling distribution of the proportion can be used to estimate the proportion of

- Visitors to a website who make a purchase before leaving
- Assembly line products that meet quality control standards.
- voters who supports a candidate in an upcoming election.

Q. Example in Sample 100 teenagers like or don't like Sneakers.

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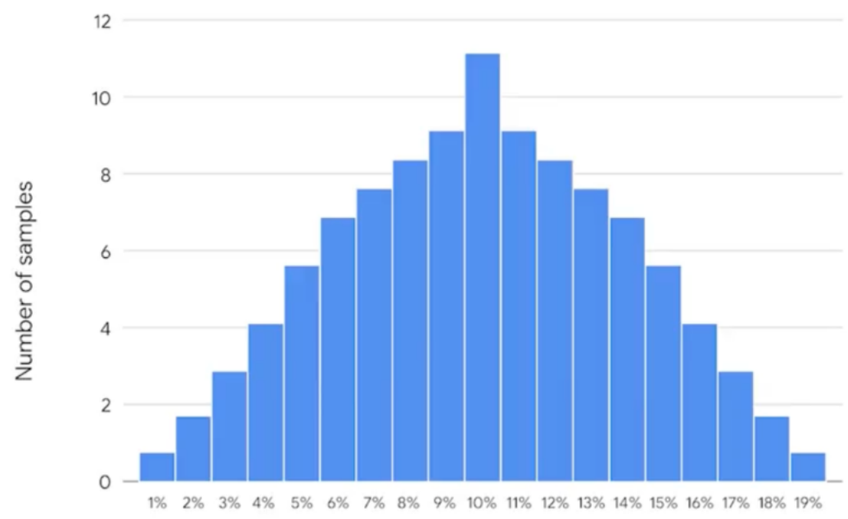
Occasionally, a sample may turn out to have a proportion that's very small or



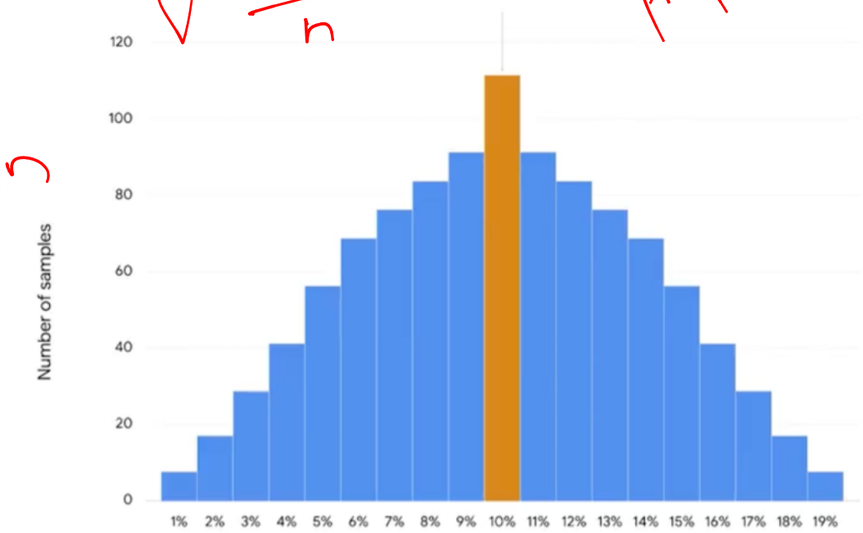
you can show the sampling distribution of the proportion in a histogram

$SE(\hat{p}) \approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

↑ More variability in sample proportion

↓ less Accuracy

Use SE to identify the variability between samples.



proportion will be an accurate estimate

In statistics, you say hat when you refer to the carrot symbol above the letter p. The formula is the square root of p hat multiplied by one minus p hat divided by n.

For example, suppose you survey 100 teenagers about their sneaker preferences and find that your estimate for the population proportion of teens who prefer slip-on sneakers is 10% or 0.1.

In this case, p hat is 0.1 and n is 100. When you plug in the numbers into the formula for standard error of the proportion, it = 0.03.
As your sample size gets larger, your standard error gets smaller. Because standard error measures the difference between your sample proportion and the true population proportion.

As your sample gets larger, your sample proportion gets closer to the true population proportion.
The more accurate the estimate of the population proportion, the smaller the standard error.

Your estimate will help stakeholders at the Sneaker Company make decisions about product development.
Based on your results, they may want to put less money into developing slip-on sneakers.

Typically, the next step for a data professional would be to use the standard error to construct a confidence interval.
This describes the uncertainty of your estimate and gives your stakeholders more detailed information about your results.
Later on in this course, you'll learn how to calculate and interpret confidence intervals to more accurately predict preferences of a population.

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