Day-8, Nov-23, 2024 (Mangshir 08, 2081 BS)

Limit:

Q. How close to 3 does 'x' have to be so that $f(x)$ differs from 5 by less than 0.1?

Here, the distance from x to 3 is $|x-3|$

and the distance from $f(x)$ to 5 is $|f(x)-5|$

So, our problem is to find a number δ such that $|f(x)-5| < 0.1$ if $|x-3| < \delta$ but $x \neq 3$.

if $|x-3|>0$, then $x\neq 3$, so an equivalent formulation to our problem is to find a number δ such that $|f(x)-5|<0.1$ if $0<|x-3|<\delta$.

So, here we can observe if $0<|x-3|<\frac{0.1}{2}=0.05$

then $|f(x)-5| \Rightarrow |2x-1-5|$

$\Rightarrow |2x-6|$

$\Rightarrow 2|x-3|$

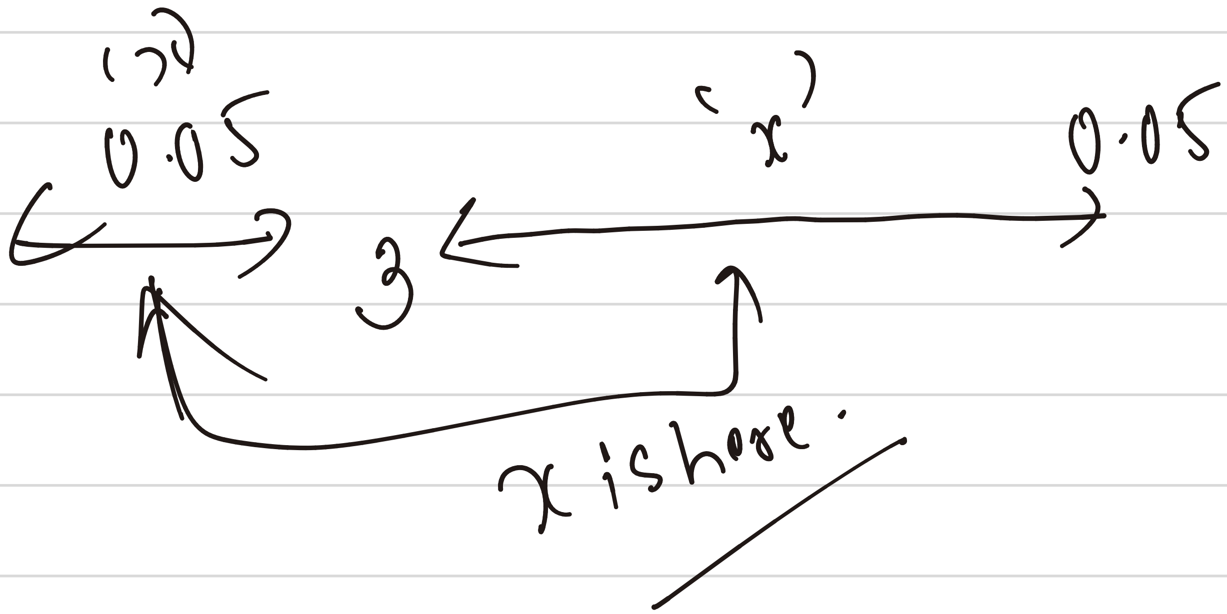
$\Rightarrow 2|x-3|<2\times 0.05$

[if $\delta>0.05$ then it doesn't work for $\varepsilon=0.1$]

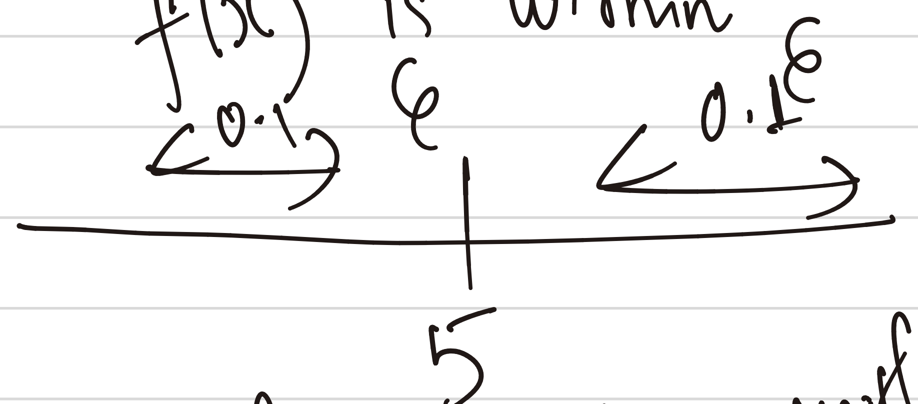
i.e $|f(x)-5|<0.1$ if $0<|x-3|<0.05$

i.e. $|f(x) - 5| < 0.1$ if $0 < |x-3| < 0.05$

Answer is. if 'x' is within a distance of $0.05 = \delta$ from 3, then $f(x)$ will be within a distance of $0.1 = \varepsilon$, from 5



then $f(x)$ is within ε



$f(x)$ will be within distance of $d = 0.1 = \varepsilon$.

So, the problem was talking about the two entities differently.

If we change the number 0.1 in our problem to the smallest number 0.01, then by using the same method we find that $f(x)$ will differ from 5 by less than 0.01 provided that 'x' differs from 3 by less than $\frac{0.01}{2} = 0.05$

[if $\delta > 0.05$ then it does not work for $\epsilon = 0.01$, if $\delta = 0.05$ for $\epsilon = 0.01$ then $|f(x) - 5| = |2x - 1 - 5|$

$$\Rightarrow 2|x - 3|$$

$$\Rightarrow 2|x - 3| < 2 \times 0.05$$

$$0.1 \geq 0.01 = \epsilon$$

$$\therefore |f(x) - 5| < 0.01 \text{ if } 0 < |x-3| < 0.05$$

Similarly,

$$|f(x) - 5| < 0.001 \text{ if } 0 < |x-3| < 0.0005$$

The numbers 0.1, 0.01, and 0.001 that we have considered are error tolerances that we might allow.

for 5 to be precise limit of $f(x)$ as 'x' approaches 3, we must not only be able to bring the difference between $f(x)$ and 5 below each of these 3 numbers; we must also be able to bring it below any +ve number. And, by the same reasoning if we write ε for any arbitrary positive number, then we find δ such as,

$$|f(x) - 5| < \varepsilon \text{ if } 0 < |x-3| < \delta = \frac{\varepsilon}{2} \quad \text{--- eqn (i)}.$$

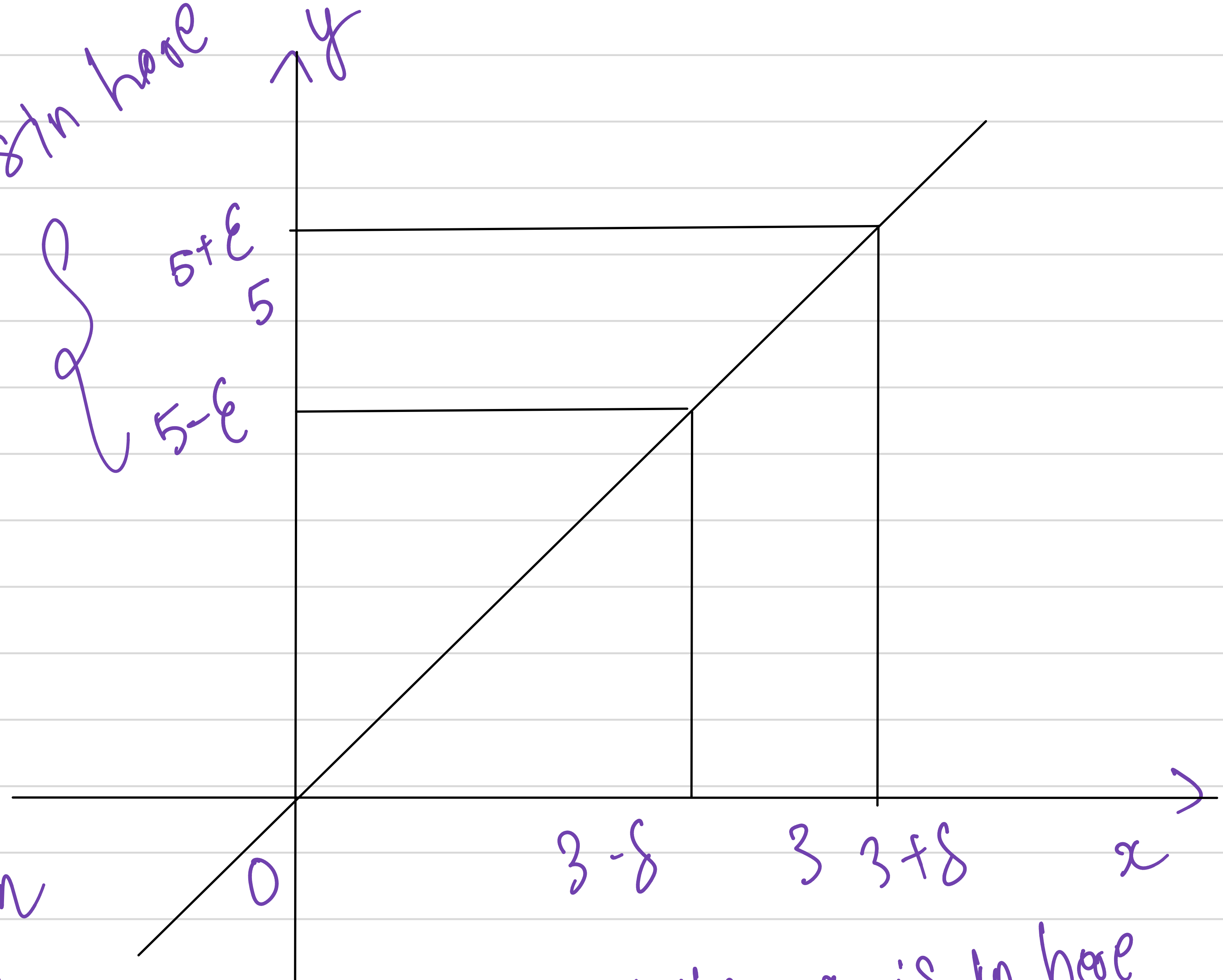
which is precise way of saying that $f(x)$ is close to 5 when x is close to 3. Precisely, we can make the values of $f(x)$ within an arbitrary distance ε from 5 by taking the values of x within a distance $\frac{\varepsilon}{2}$ from 3 but $(x \neq 3)$.

if $3-\delta < x < 3+\delta$ $(x \neq 3)$ then

$5-\varepsilon < f(x) < 5+\varepsilon$

This is implemented by figure. By looking the values $x \neq 3$ to lie in the interval $(3-\delta, 3+\delta)$

$f(x)$ is in here



when x is in here $(x \neq 3)$

we can make the value of $f(x)$ lie in the interval $(5-\varepsilon, 5+\varepsilon)$ with this concept, a precise definition of limit.