Day-55, Jan-24, 2025 (Magh 11, 2081)

#Probability ? probability Distribution ?

→ probability is the likelihood of an event occuring. Example throwing a coin or dice. and occurance of Head or Tail or dice numbered Six.

Example: In a school there are 50 Kids and 3 Kids play Soccer an f Kids don't play Soccer. What is the probability that a kid picked at random plays Soccer?

→ Answer is 30 %.

formula.

$$P(\text{Soccer}) = \frac{\text{Soccer}}{\text{total}}$$

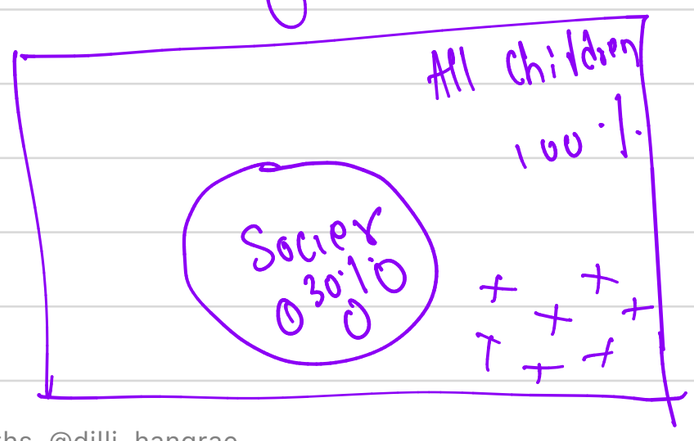
[Since 3 → Soccer play
7 → No Soccer play]

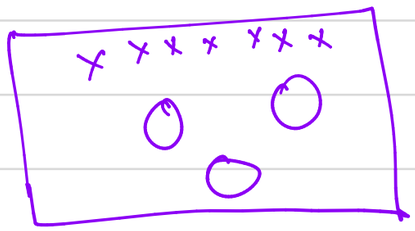
$$\Rightarrow \frac{3}{10}$$

$$\Rightarrow 30\%$$

Whose Soccer is 'event'
and total is 'Sample Space'.

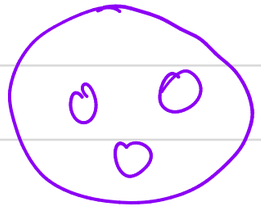
Venn-Diagram





Sample Space

$$P(S) \Rightarrow \frac{\text{event}}{\text{total}}$$



Event

Coin Example 1:

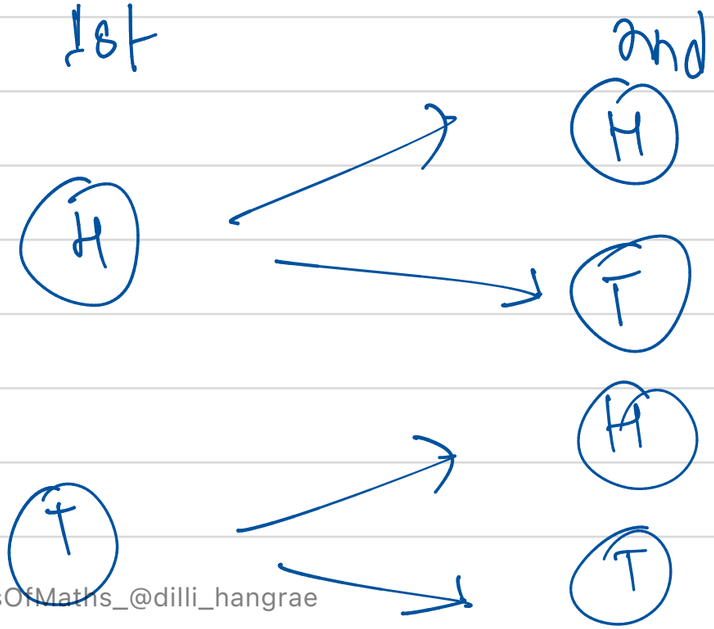
Experiment can be defined as the process that produces output.

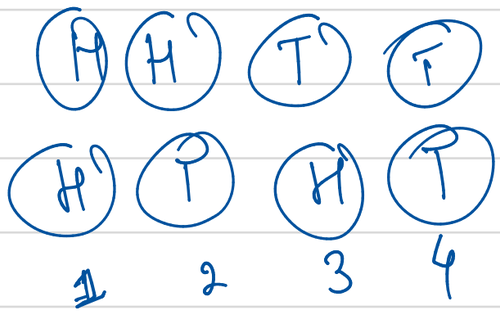
H T

50% 50%

$$p(\text{heads}) = \frac{1 \text{ Head}}{|\text{Head} + |\text{tail}|}$$

$$= \frac{1}{2} \Rightarrow 0.5$$





Q. What is the probability of getting two heads when flipping two coins?

Answer: 4 possible outcomes

So the answer is 1/4.

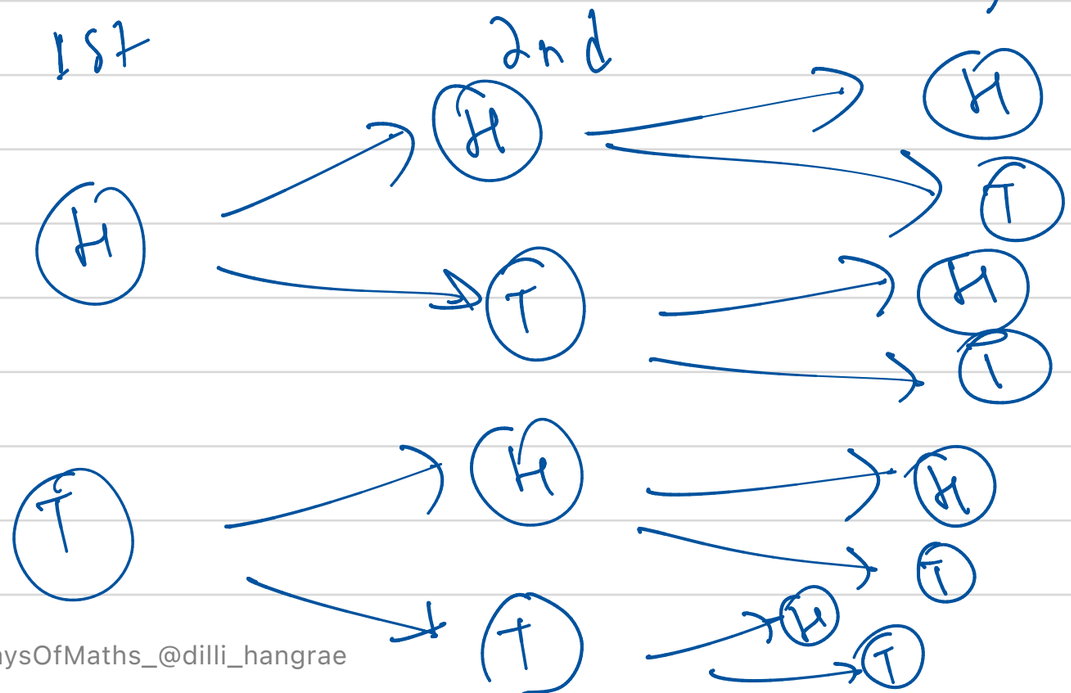
$P(HH) \Rightarrow$ (H) (H) / 4

$\Rightarrow$ 1/4

$\Rightarrow$ 1/4

Q. What is the probability of a coin landing on heads 3 times?

→ 1/8



So,

$P(HHH) =$ (H) (H) (H) / 8

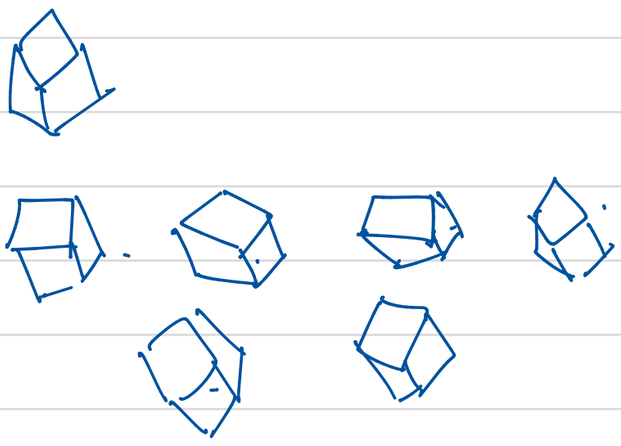
(H T H T)
(H T H T)
(H T H T)
(H T H T)

$\Rightarrow$ 1/8

Dice Example:

Q. What is the probability of obtaining 6?

Ans: 1/6



When rolling two 6-sided dice, what is the probability of getting 6, 6?

⇒ 1/36

Complement of an Event and It's Probability:

D O O X X X X X X X

30 % 70 %

What is the probability of a child NOT playing Soccer?

⇒ 7/10 ⇒ P(not soccer) = 7/10!

Q. When flipping a coin 3 times, what is the probability of not landing on heads 3 times ?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$= 1 - 1/8$$

$$= 7/8$$

Q. When throwing a 6-sided dice, what is the probability of obtaining anything other than 6 ?

Answer $P(\text{not } 6) = 1 - 1/6$

$$= 5/6$$

Sum of Probabilities

$P(S) = 0.3$

$P(B) = 0.4$

→ At a school, kids can only play one sport.

→ What is the probability that a kid plays soccer or basketball?

→ Hint: If they were only 10 kids?

Ans → $P(S) + P(B)$

$\Rightarrow 0.7$

S S S B B B B x x x

1-30% → ← 40% →

← 70% →

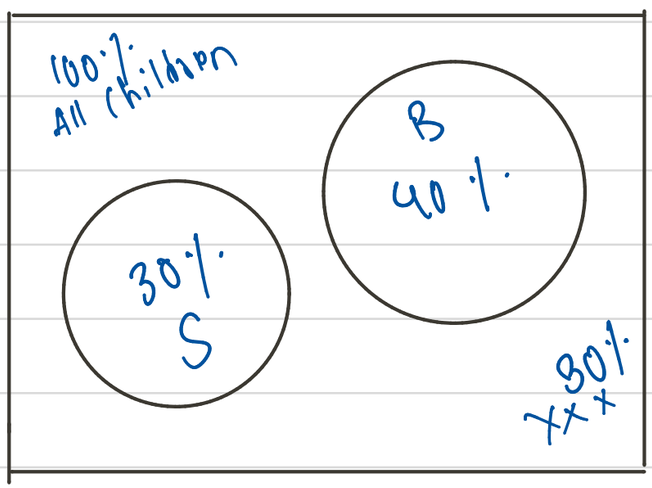
$P(\text{Soccer or basketball}) = \frac{\text{Soccer or basketball}}{\text{total}}$

$\Rightarrow \frac{3+4}{10}$

$= 0.7$

$P(\text{Soccer or basketball}) = P(\text{soccer}) + P(\text{basketball})$

Venn - Diagram:



$P(\text{Soccer} \cup \text{basketball}) = P(\text{Soccer}) + P(\text{basketball})$

$P(A \cup B) \Rightarrow P(A) + P(B)$

Q. When throwing a 6-sided dice what is the probability of obtaining an even number or a 5?

$P(\text{even number or } 5) = P(\text{even number}) + P(5) \Rightarrow \frac{3}{6} + \frac{1}{6} \Rightarrow \frac{2}{3}.$

Q. If you roll two fair dice, what is the probability of obtaining a sum of 7 or Sum of 10 ?

Here, 9/36

Sum of 7

(1,6), (6,1), (5,2), (2,5)

(4,3), (3,4)

Sum of 10

(4,6) (6,4), (5,5)

Sum of 7 or Sum of 10 $\Rightarrow \frac{6}{36} + 3/36$ $\Rightarrow 9/36$

If you roll two fair dice, what is the probability of difference of 2 or difference of 1 ?

Note: Remember that you must consider the differences in both ways, ie. (dice 1 - dice 2) and (dice 2 - dice 1).

A

$P(\text{diff} = 2)$

(1,3) (2,4) (3,5) (4,6)

(3,1) (4,2) (5,3) (6,4)

$\Rightarrow \frac{8}{36} + \frac{10}{36}$

$\Rightarrow 18/36$

$\Rightarrow 1/2$

B

$P(\text{diff} = 1)$

(1,2) (2,1) (2,3)

(3,2) (3,4) (4,3)

(4,5) (5,4) (5,6)

(6,5)

Sum of Probabilities (Joint Events):

Probability of Rain = 80 %.
Probability of Sunny = 50 %.

] - Disjoint But in the Real world we have $P = 1$ and joint probability

Total kid = 10

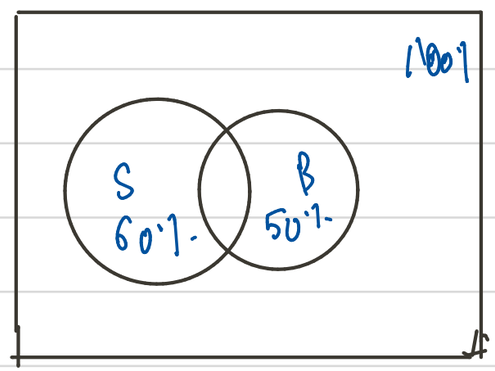
$P(S) = 0.6$

$P(B) = 0.5$

S = Soccer

P(B) = Basketball.

$P(S \cup B) = ?$



$P(S \cap B)$

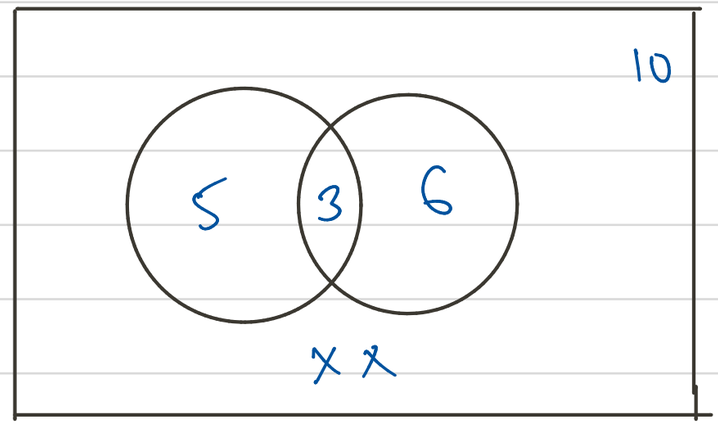
$P(S \cup B) = P(S) + P(B)$

Q. In the school there are 6 kids playing Soccer, 5 kids playing basketball and 3 kids playing both.

Answer:

$$5 + 6 - 3 = 8$$

$$P(S \cup B) = P(S) + P(B) - P(S \cap B)$$



Q. In a school, we know that the probability that a kid plays soccer is 0.6, the probability that a kid plays basketball is 0.5 and the probability that a kid plays both soccer and basketball is 0.3. What p that child plays S or B?

Here,

$P(S) = 0.6$ $\quad P(B) = 0.5$ $\quad P(S \cap B) = 0.3$

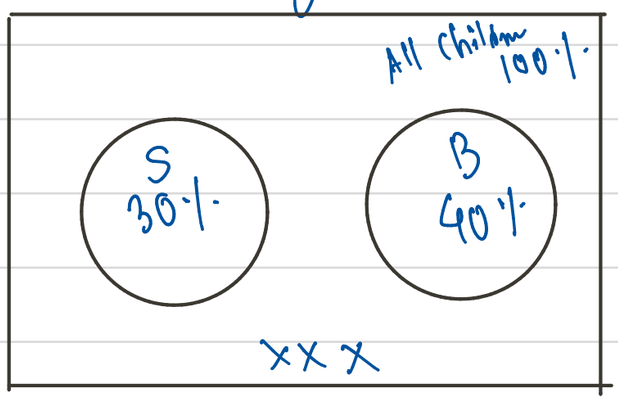
$$P(S \cup B) = P(S) + P(B) - P(S \cap B)$$

$$\Rightarrow 0.6 + 0.5 - 0.3$$

$$\Rightarrow 0.8$$

Disjoint Events vs Joint Events:

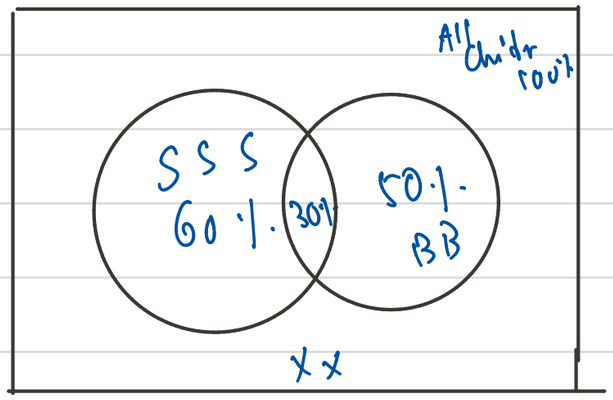
Disjoint



Mutually Exclusive

$P(S \cup B) = P(S) + P(B)$

Joint



Non-Mutually Exclusive

$P(S \cup B) = P(S) + P(B) - P(S \cap B)$

Joint Events- Dice

What is the probability of obtaining a sum of 7 or difference of 2?

Ans — 14/36 Answer

A

Sum = 7

(5,2) (2,5) (3,4) (4,3)

(6,1) (1,6)

∪

B

diff = 1.

(2,1) (1,2) (2,3) (3,2)

(4,3) (3,4) (4,5)

(5,4) (6,5) (5,6)

A or B

We need to minus the matched pairs between Sum and diff -

$P(S=7 \text{ or } d=1) \Rightarrow \frac{6}{36} + \frac{10}{36} - \frac{2}{36}$

$\Rightarrow \frac{14}{36}$

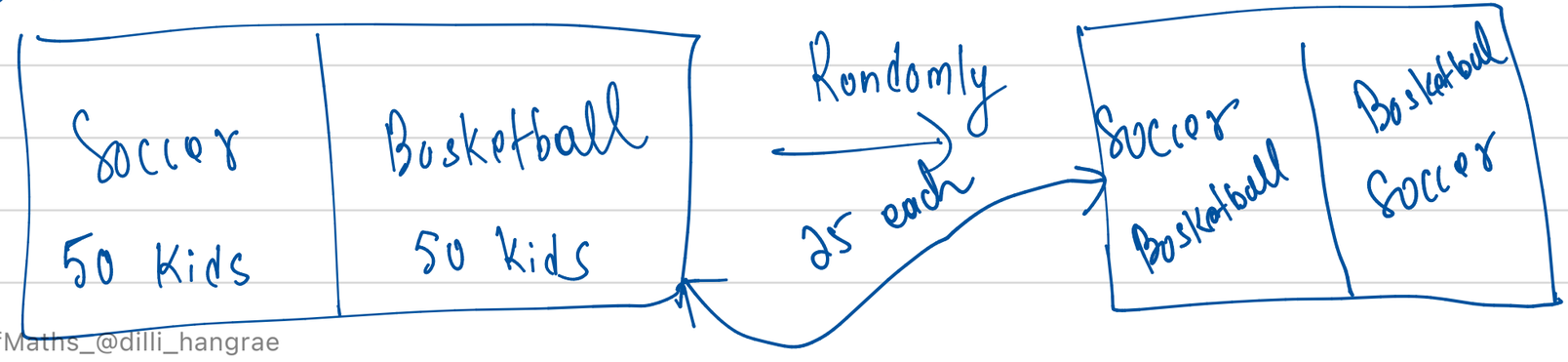
$\Rightarrow \frac{7}{18}$

Independence: → Occurrence of one event doesnot affect other event.

→ In chess the moves are independent whereas Coin toss is dependent

Q. In a school with 100 kids, where 50 like Soccer and 50 don't, if we randomly split the kids in two rooms with 50 kids each, what is your best estimate on the number of kids in room 1 that like Soccer?

Ans ⇒ 25



Q. In a school with 100 kids, where 40 like soccer and 60 don't, if we randomly split the kids in two rooms with 30 kids in one room and 70 in other, what is your best estimate on the number of kids in room 1 that like soccer?

Answer: 12 because 60% — 40% so 40% of 30 → 40% of 30 Toh ⇒ 12.

20% — 30% so, 60% of 70

$\Rightarrow \frac{60}{100} \times 70 \Rightarrow 42$

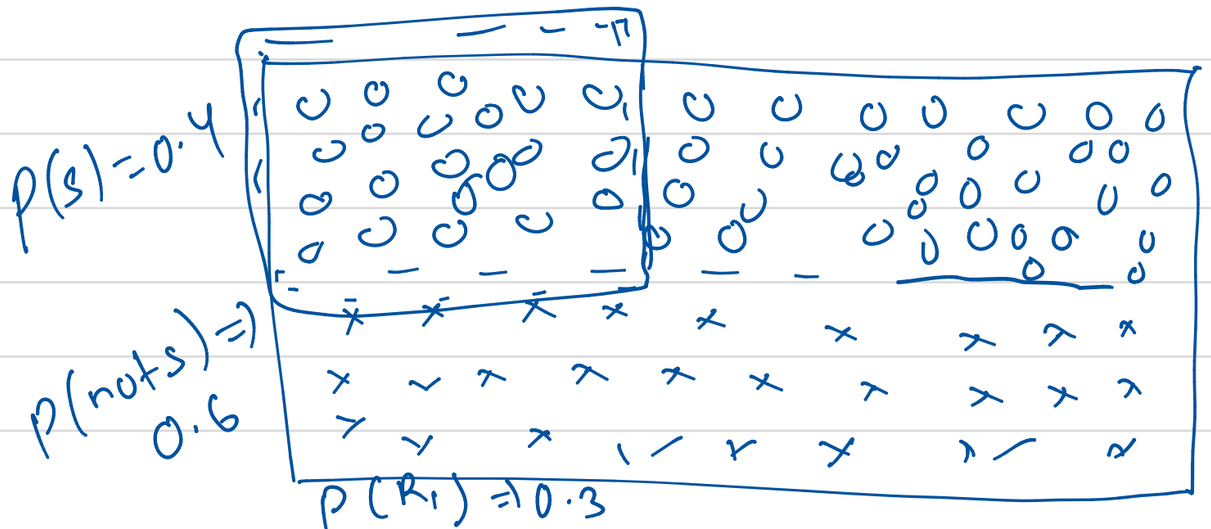
Independent Events:

P (Socies and Room 1)

$P(S \cap R_1) = P(S) \cdot P(R_1)$

$\Rightarrow 0.4 \cdot 0.3$

$\Rightarrow 0.12$



$P(\text{Soccer and Room I})$ means the intersection of Soccer and Room ie.

$$P(S \cap R) \Rightarrow p(S) \cdot p(R)$$

Coin Example: What is the p of landing heads 5 times?

$$p(5 \text{ Heads}) \Rightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$\Rightarrow \frac{1}{32} \quad [\text{since the } p \text{ is independent}]$$

Dice Example:

→ Two dice. rolled, what is the p of 6?

$$\Rightarrow \frac{1}{6} \times \frac{1}{6} \qquad \Rightarrow \frac{1}{36}$$

Independant Events.

If you roll a die 10 times, what is the p of getting 10 sixes?

$$p(10 \text{ sixes}) = \left(\frac{1}{6}\right)^{10}$$

$$\Rightarrow \left(\frac{1}{6}\right)^{10}$$

Birthday Problem

When you are with 'n' number of friends what is the p that likely you and those n have same birthday date?

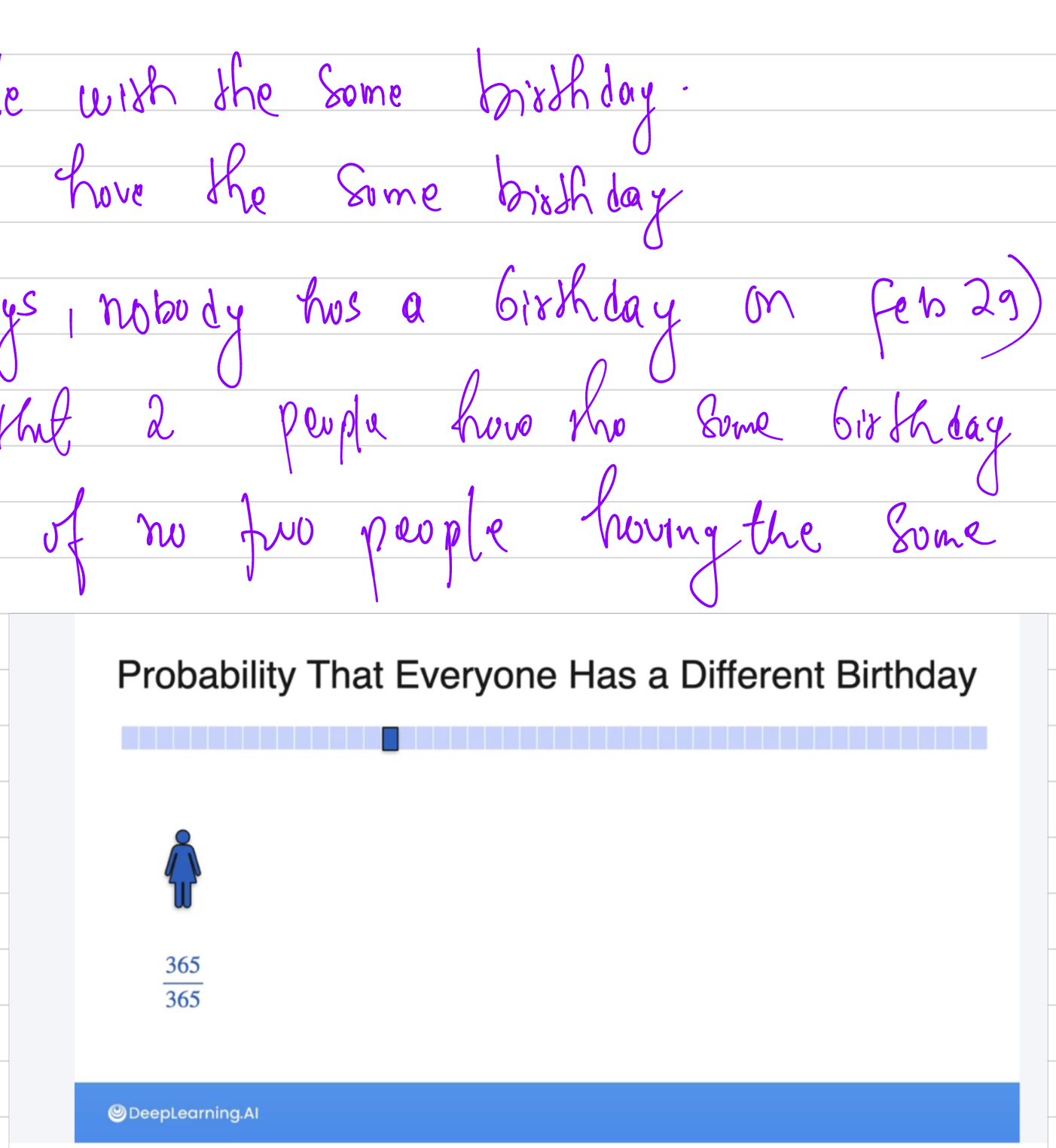
Intuitive: you have 30 friends at a party. Who do you think is more likely:

- that there exist two people with the same birthday.
- That no two of them have the same birthday

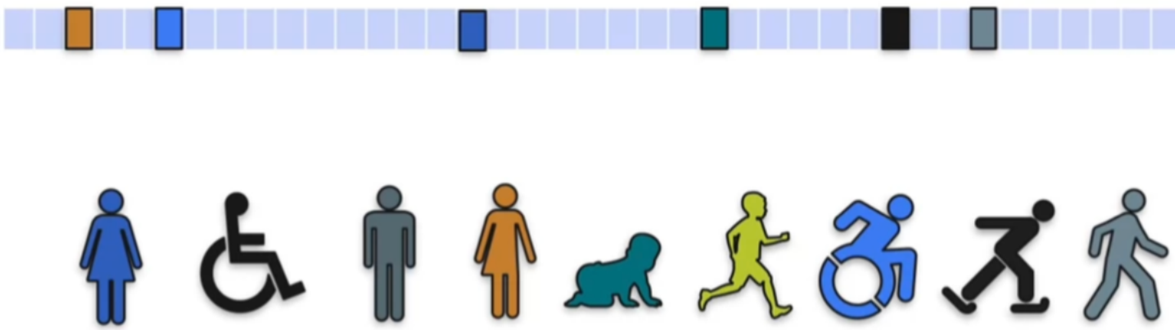
- (Assume the year has 365 days, nobody has a birthday on Feb 29)

Ans: It's more likely that 2 people have the same birthday

Infact, the probability of no two people having the same birthday is around 0.3.



Probability That Everyone Has a Different Birthday



$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} = 0.905$$

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$\frac{365}{365}$ means 1 people have different birthday -

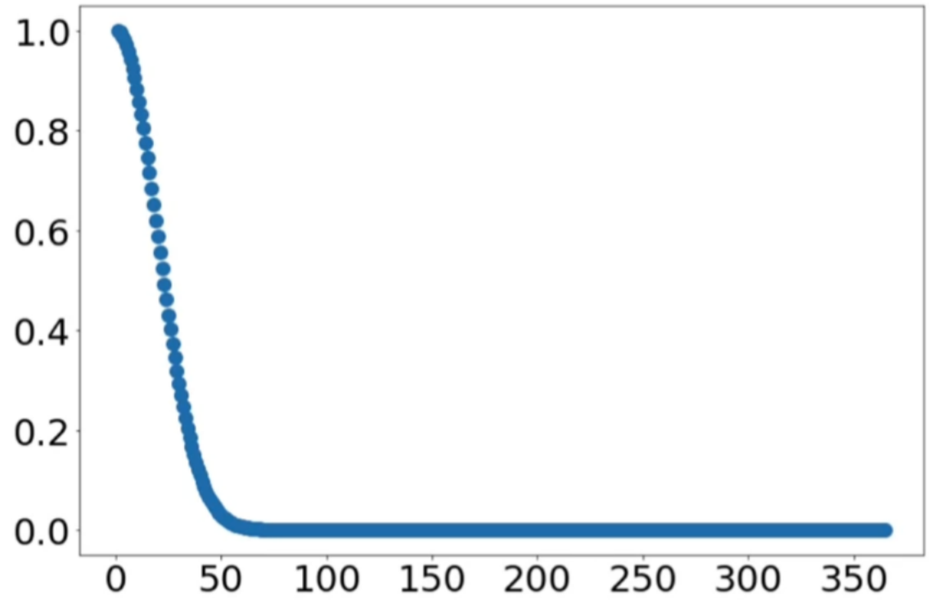
$\frac{363}{365}$ means 3 have different Probability Birthday.

As the 'n' of sample increases, the p gets decreased that having same birthday.

Among 366
Almost '0' or
Zero probability
that they have the
same birthday:

Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
366 people: 0



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Source: Coursera Probability & Statistics for Machine Learning.