#Day-10, Nov-25, 2024 (Mangshir 10, 2081 B.S.)

Example 2: Use the given graph of 'f' to find a number δ such that if $|x-1| < \delta$ then $|f(x) - 1| < 0.2$.

Here given,

$a = 1$

$\epsilon = 0.2$

$L = 1$

$\delta = ?$

$\rightarrow$ Since, if $|x-1| < \delta$ then $|f(x) - 1| < 0.2$

Since $|f(x) - 1| < 0.2$

or, $0.8 < f(x) < 1.2$

$\therefore$ $f(x)$ varies from 0.8 to 1.2

from the figure for $x = 0.7$, $f(x) = 1.2$
and for $x = 1.1$, $f(x) = 0.8$

$\because$ x varies from 0.7 to 1.1 which are not at symmetric from $x = 1$.
The distance from $x = 1$ and $x = 0.7$ is 0.3 and distance from 1 to 1.1 is 0.1

So, $\delta = 0.1$.

if $|x-1| < \delta$ then
$|f(x) - 1| < 0.2$

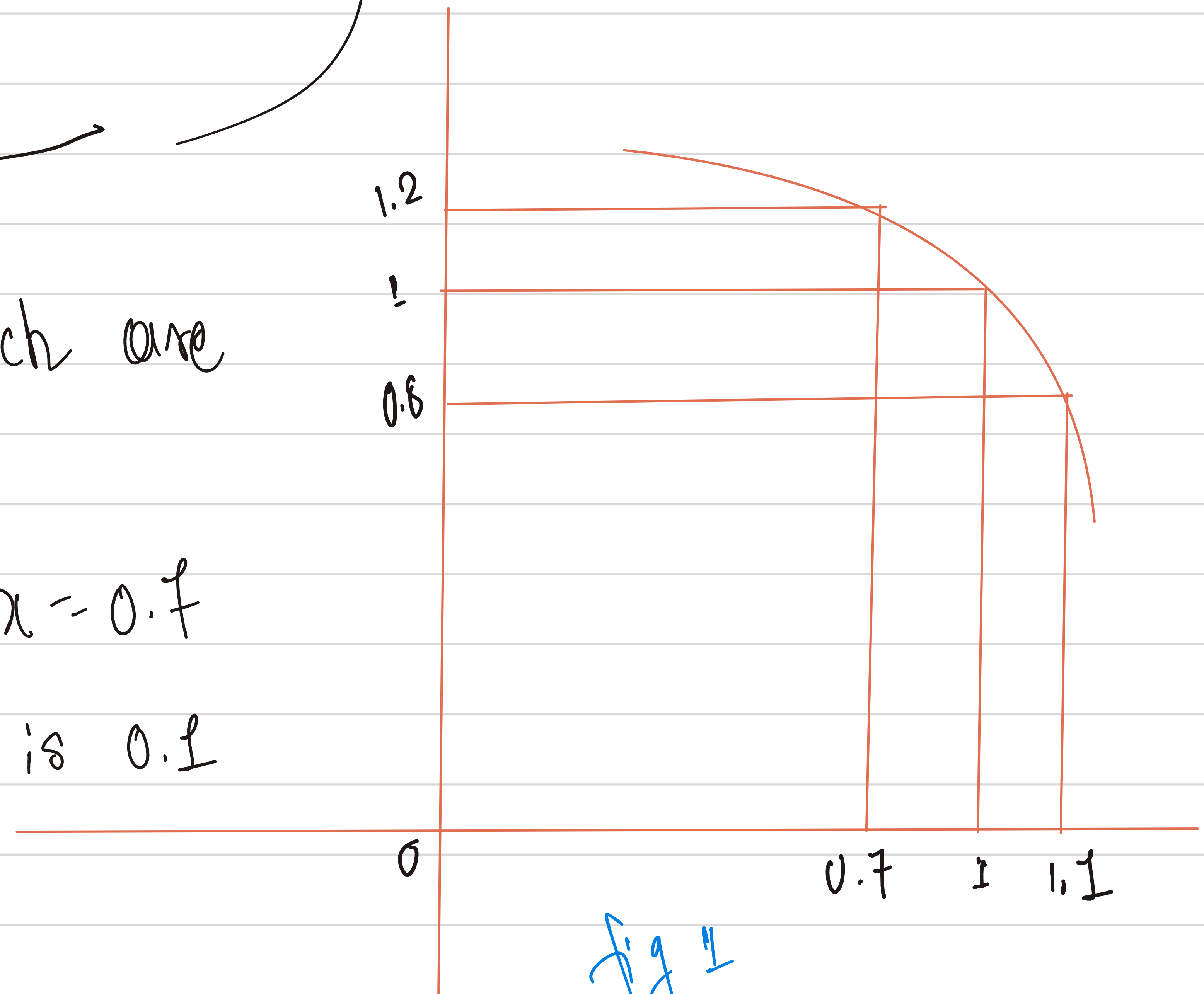


fig 1

for a given $a = 1$, $\epsilon = 0.2$, $l = 1$,
$\delta = ?$

Example: Use the graph of $f(x) = \sqrt{x}$ to find a number δ such that if $|x-4| < \delta$ and $|\sqrt{x} - 2| < 0.4$.

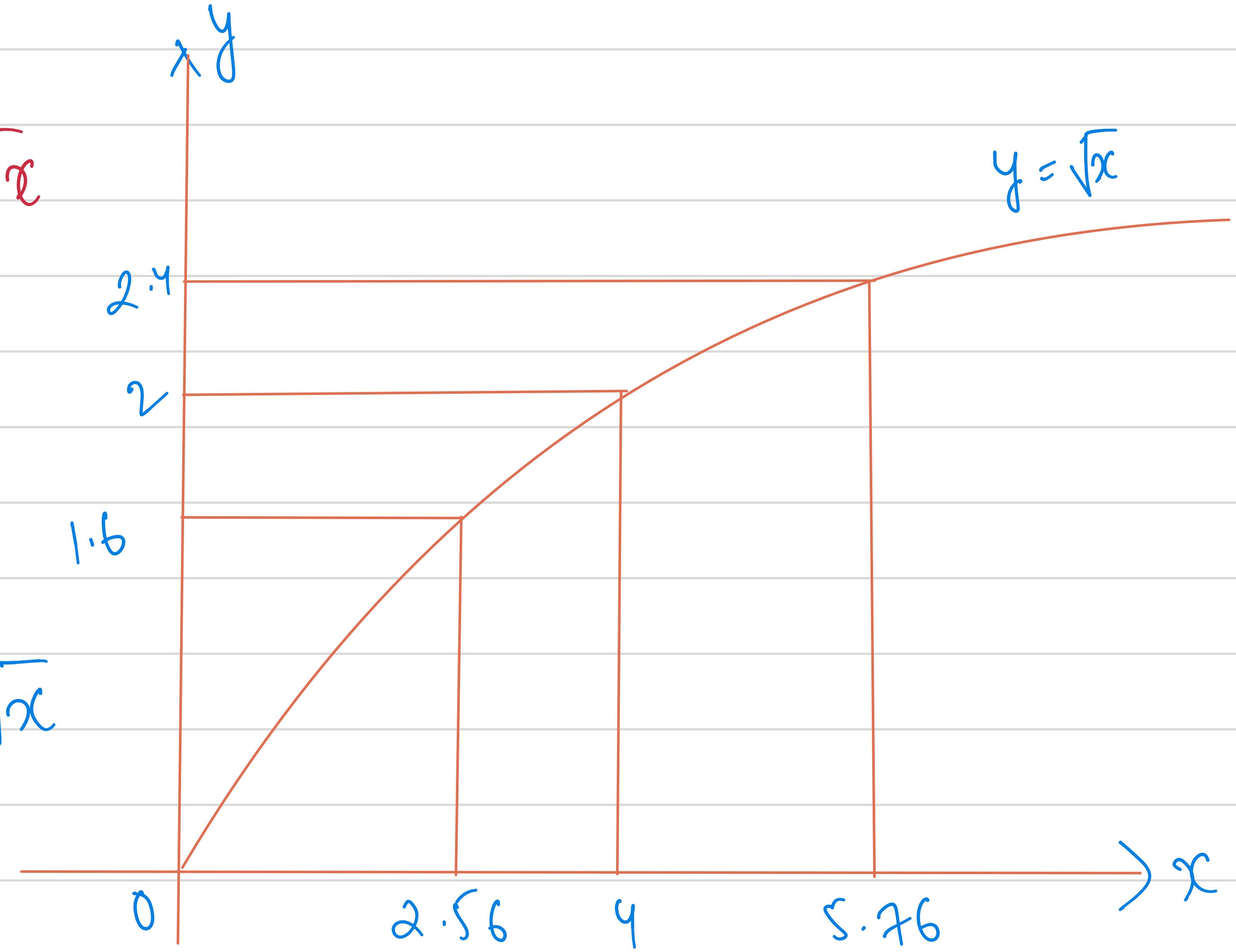


fig 2

Here, limit of the function $f(x) = \sqrt{x}$ at $x = 4$ is 2

i.e. $L = 2$ and $a = 4$, $\epsilon = 0.4$

Now, $-0.4 < \sqrt{x} - 2 < 0.4$

$\Rightarrow \underline{1.6 < \sqrt{x} < 2.4 \text{ whenever } |x-4| < \delta}$

We need to concentrate near the region of the point $(4, 2)$.

We need to find the values of x for which the curve $y=\sqrt{x}$ lies between the horizontal line $y=1.6$ and $y=2.4$.

$\therefore$ The curve $y=\sqrt{x}$, $y=1.6$ and $y=2.4$ near the point $(4,2)$.

Now, the point of intersection of $y=\sqrt{x}$, $y=1.6 \Rightarrow x=2.56$.

$$y=\sqrt{x}$$
$$y=1.6$$
$$\Rightarrow x=2.56$$

Again the point of intersection of $y=\sqrt{x}$ and $y=2.4$ is $x=5.76$.

Now, rounding to be safe we have it $2.56 < x < 5.76$ then $1.6 < \sqrt{x} < 2.4$.

This interval $(2.56, 5.76)$ is not symmetric about $x=4$. The

distance from $x=4$ to the left end point is $4-2.56 = 1.44$
and the distance of $x=4$ from the right end point $x = 5.76$ is $5.76 - 4$
$\Rightarrow 1.76$

$\therefore \delta =$ smallest of two numbers $= 1.44$ (or any +ve smaller number)

$\therefore$ Hence, we can write $|x-4| < 1.44$ then $|\sqrt{x} - 2| < 0.4$.

Generalized (Simpler form)

$f(x) = \sqrt{x}$, $a = 4$, $\epsilon = 0.4$

limit $(L) = 2$ where limit at $x = 4$ so, $f(x) = \sqrt{x}$

Then, $f(x) = \sqrt{x}$

① Setting up the inequality for $f(x)$

$$|\sqrt{x} - 2| < 0.4$$

means

$1.6 < \sqrt{x} < 2.4$ (Concept of L.H. Limit & R.H. Limit).

$2 - 0.4 < \sqrt{x} < 2 + 0.4$.

$$\left[1.6 < \sqrt{x} < 2.4.\right]$$

Squaring both sides –

$$(1.6)^2 < (\sqrt{x})^2 < (2.4)^2$$

$$2.56 < x < 5.76$$

Now, we need δ inequality it seems both are not symmetric -

$4 - 2.56 = 1.44$ to the left of 4.

$5.76 - 4 \Rightarrow 1.76$ to the right of 4.

Choosing the smaller value we get,

$$\boxed{\delta = 1.44}$$

Conclusion:

if $|x-4| < 1.44$

then $|\sqrt{x} - 2| < 0.4$.

Example 2:

$f(x)$ and $a = 1$, $\epsilon = 0.2$

This follows the same idea.

1. Set bounds for $f(x)$

$$|f(x) - 1| < 0.2$$

$$\Rightarrow \quad 0.8 < f(x) < 1.2$$

2. Graph $f(x)$

· The graph shows that for $f(x)$ to stay within the range, x must lie in between $[0.7, 1.1] \rightarrow$ See above graph. (fig 1)

3. find δ

Distance from $x = 1$ is:

• Left: $1 - 0.7 \Rightarrow 0.3$

• Right: $|0.1 - 1| \Rightarrow 0.1$

Conclusion: if $|x-1| < 0.1$ then $|f(x) - 1| < 0.2$

Example 4: A function $f(x)$ approaches a limit 'L' as 'x' approaches a point 'a', if for every small $\epsilon > 0$, there exists a 'δ' > 0 such that:

$$\left(|x-a| < \delta \Rightarrow |f(x) - L| < \epsilon . \right)$$

The goal is to find a δ value that ensures the function stays within a given range (controlled by ϵ) near a specific point.

Example 1: $f(x) = x^3 - 5x + 6$

Find δ such that if $|x-1| < \delta$ then $|f(x) - 2| < 0.2$.

1 Inequality for $f(x)$.

$$|f(x) - 2| = |x^3 - 5x + 6 - 2|$$

$$\Rightarrow |x^3 - 5x + 4| < 0.2$$

2 Graphical Approach: (from Fig 2)

$\therefore$ The curve $y = x^3 - 5x + 6$ is plotted.

- Horizontal lines $y = 1.8$ and $y = 2.2$ (Since $2 \pm 0.2 = [1.8, 2.2]$) intersect the curve.

- using the graph, the intersections occur at $x \approx 0.92$ and $x \approx 1.12$.

3. find δ:

The distance from $x = 1$ to these points is:

- Left: $1 - 0.92 \Rightarrow 0.08$
- Right: $1.12 - 1 \Rightarrow 0.12$

Choose the smaller value: $\delta = 0.08$

So, if $|x-1| < 0.08$ then $|f(x) - 2| < 0.2$

What is an inequality?

An inequality compares two values or expressions using symbols like:

- $<$: less than
  Example: $3 < 5$ (3 is less than 5)
- $>$: Greater than
  Example: $7 > 4$ (7 is greater than 4)
- $\leq$ less than or equal ($x \leq 10$)
- $\geq$ Greater than or equal to $y \geq 2$ (y can be 2 or larger)
- $\neq$: Not Equal to:
  Example: $x \neq 4$ (x cannot be 4)

Solving Inequalities:

finding the values of a variable that make the inequality true.

Examples: ① $x + 3 > 5$

$\boxed{x > 2}$

② $2x \leq 10$

$\boxed{x \leq 5}$

③ $-2x < 6$

$\boxed{x > -3}$

Combining terms: $3x - 4 < 2x + 5$

$\rightarrow 3x - 4 - 2x < \cancel{2x} + 5 - \cancel{2x}$

$\rightarrow x - 4 < 5$

$3x - 4 < 2x + 5$

$\Rightarrow 3x - 4 - 2x < \cancel{2x} + 5 - \cancel{2x}$

$\Rightarrow x - 4 < 5$

$\Rightarrow x < 9$

Types of Inequalities:

① Linear Inequalities:

$$\left[\begin{array}{l} 3x + 5 > 14 \\ 3x > 9 \\ x > 3 \end{array}\right]$$

② Quadratic Inequalities:

eg: x^2.

$$x^2 - 4 \geq 0$$

$$(x-2)(x+2) \geq 0$$

Testing → $x < -2$

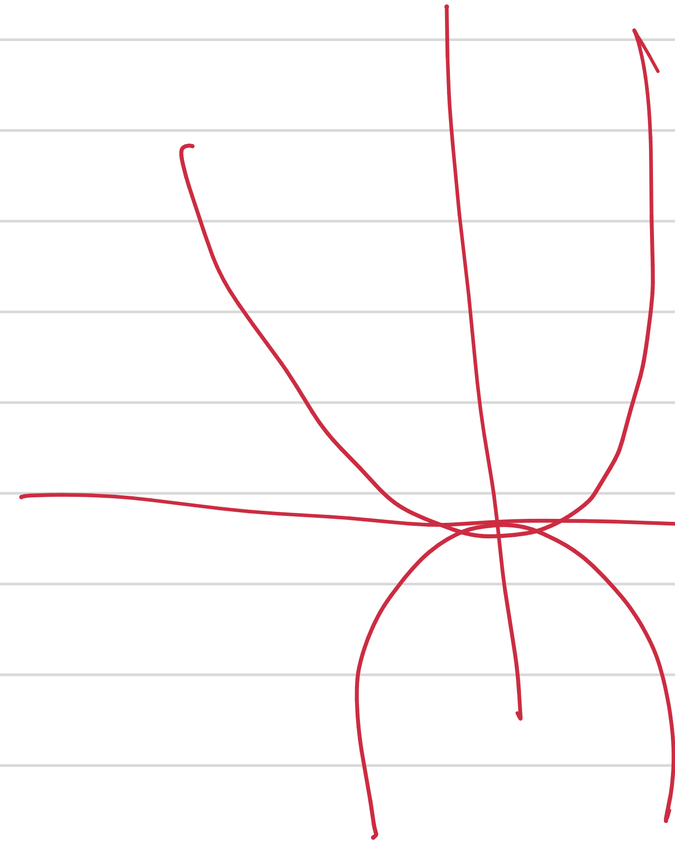
→ $x \geq 2$

So, $-2 \leq x \leq 2$

for $x \geq 2$

$$x^2 \geq 4$$

$$(x \leq -2 \text{ or } x \geq 2)$$



$y = x^2$

if $y = x^2$ is flipped upside down

~~Graphing Inequalities:~~

$x > 2$ or, $x > 2$ dashed at $x = 2$.

$y \leq x + 3$ $y = x + 3$

Compound Inequalities: Involve more than one inequality

Solve: $3 < 2x + 1 \leq 7$

$3 < 2x + 1$

$2 < 2x$

$1 < x$

- $3 < 2x + 1$
- $2x + 1 \leq 7$
- $2x \leq 6$

$x \leq 3$

So, $x \in (1, 3]$.

Absolute Value Inequalities: Involve the value function, $|x|$

Rule:

for $|x| < a$:

$$-a < x < a$$

for $|x| > a$:

$$x < -a \quad \text{or} \quad x > a$$

Example: Solve $|x-2| \leq 3$

$$-3 \leq x - 2 \leq 3$$ (Removing the absolute values).

Add '2' to all sides:

$$-1 \leq x \leq 5$$