Day-30, Dec-18, 2024 (Poush-3, 2081 B.S.)

Limits and Continuity

Function and Types of Function

Infinity, Limit as a Infinity function, Limit Theorem

Left-Hand and Right-Hand Limit

Limits of Trigonometric functions

Continuity and Discontinuity Types

Asymptote → Horizontal & Vertical

Composite function, Intermediate Mean Value Theorem,

Linear Mathematical Model, Even & Odd Symmetry

Derivative, Tangent & Velocity

Differentiation, Differential, Instantaneous Velocity,

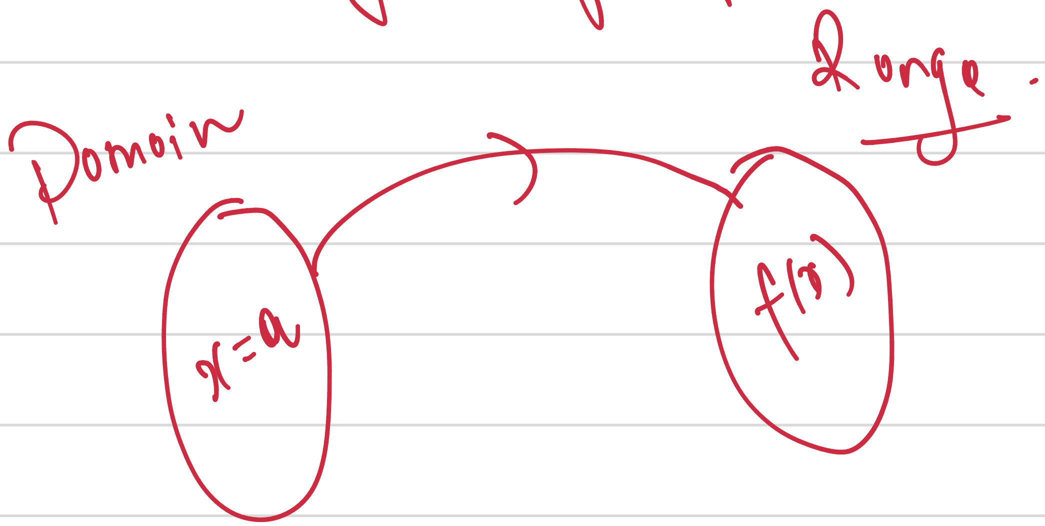
Fundamental Formula on Differentiation

Derivative of Inverse Circular Function, Trigonometric functions

Hyperbola and Rate of Change,

Rolle's, MVT, Extreme Mean Value Theorem,

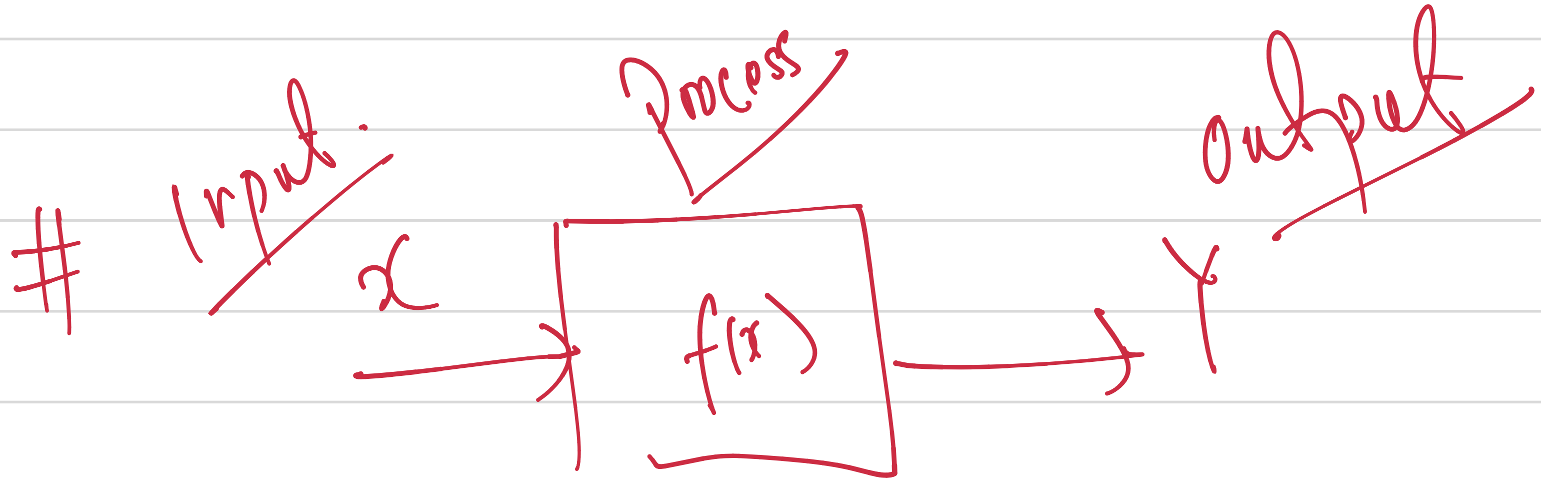
$f(x)$ is a function that takes non-empty set or input and map the $f(x) \longrightarrow Y$ such that x is the pre-image and Y is the Range of $f(x)$.



Sometimes function takes this form

$\frac{0}{0} \longrightarrow$ Indeterminate Form

for $\frac{x}{x^2}$ when $x = 0$

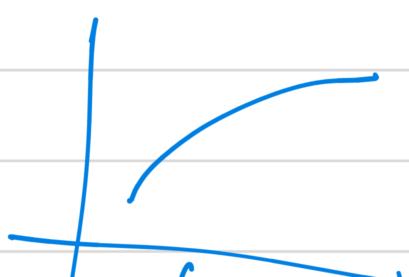


Continuity and Limit

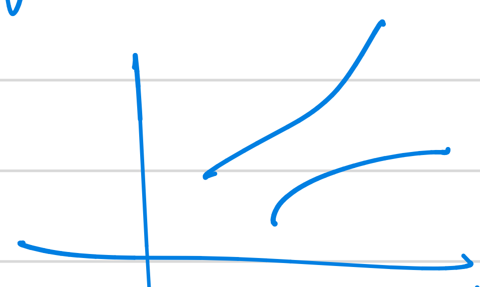
Limit is a concept that states a number 'x' becomes very close to a particular number say 'a' but never reaches to that number such that $x \to a$ $\Rightarrow$ x tends to a. $x = 0.99, 0.999, 0.9999$ and $a = 1$.

Continuity

In the given interval $[a,b]$ a graph of function is said to be continuous if it is smooth and doesnot break the curve.



Continuity.



Discontinuity

$\rightarrow$ Discontinuity

Continuity of function exists when

$$f(x_a) = f(x_b)$$

$$\lim_{x \to x_0^+} f(x_a) = \lim_{x \to x_0^-} f(x_b) \text{ exists.}$$

Types of Discontinuities:

1. Removable discontinuity $\lim_{x \to x_0^-} f(x) \neq \lim_{x \to x_0^+} f(x)$
2. Ordinary Discontinuity $\lim_{x \to x_0} f(x) \neq f(x_0)$ at $x = x_0$.
3. Infinite Discontinuity $\lim_{x \to x_0} f(x) = +\infty$ or $f(x) = -\infty$ at $x = x_0$.

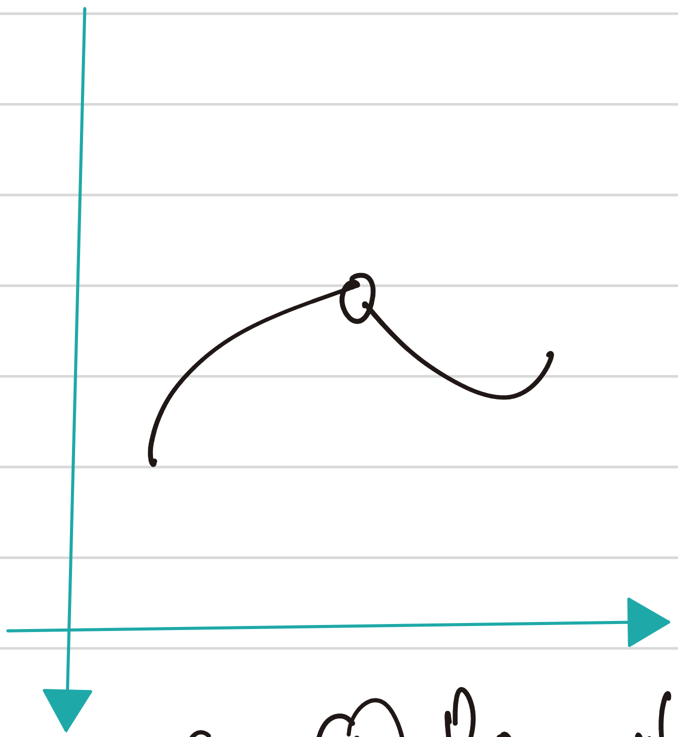


Fig ① Removable

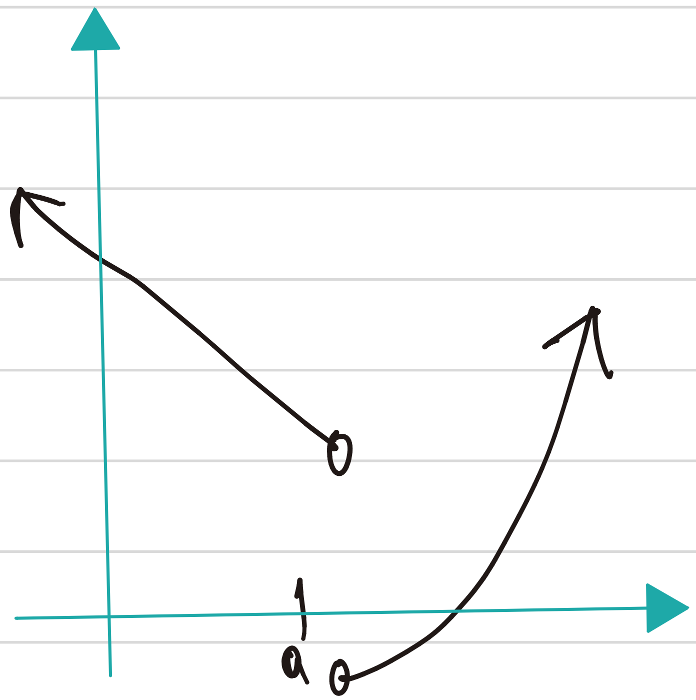


Fig ②

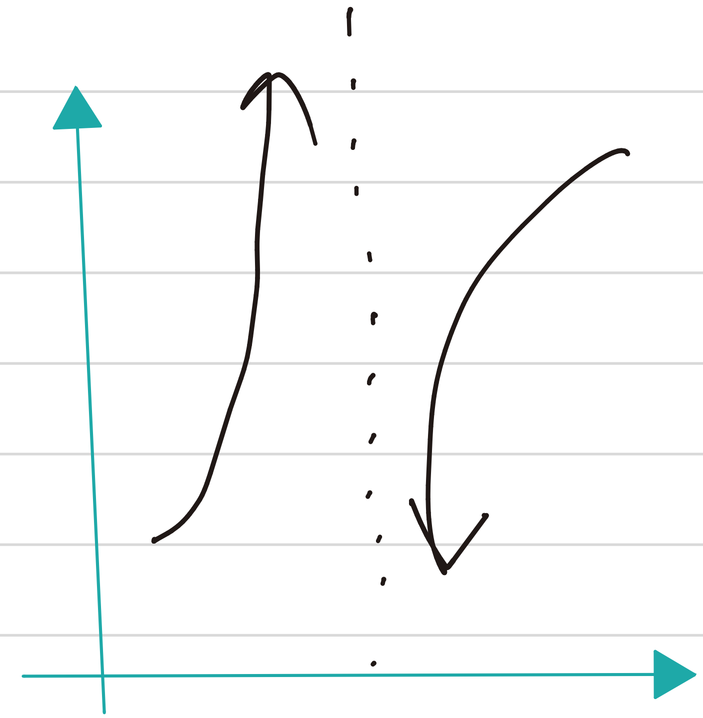


Fig ③ Infinite.

Limit of Trigonometric function

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \qquad \lim_{\theta \to 0} \frac{\cos\theta}{\theta} = 0.$$

$$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$$

A continuous function can be defined as those class of function whose curve can be drawn without lifting pen or those properties following are the continuous function.

1. $f(x) \pm g(x)$ is continuous at $x = a$.
2. $f(x) \cdot g(x)$ is " " "
3. $\frac{f(x)}{g(x)}$ at $x \neq 0$
4. $\sqrt[n]{f(x)}$ at $f(x) \geq 0$

Important theorem on limit:

① For all rational

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$$

② $$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$$

③ $$\lim_{x \to c} [f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$$

④ $$\lim_{x \to c} [f(x) \cdot g(x)] = \left(\lim_{x \to c} f(x)\right) \cdot \left(\lim_{x \to c} g(x)\right)$$

⑤ quotient rule:

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$$

if $\lim_{x \to c} g(x) \neq 0$.

Composite function $[(f o g)(x) = f(g(x))]$

If 'g' is continuous function at 'a' and 'f' is continuous at g(a) then the composite function fog given by $(f o g)(x) = f(g(x))$.

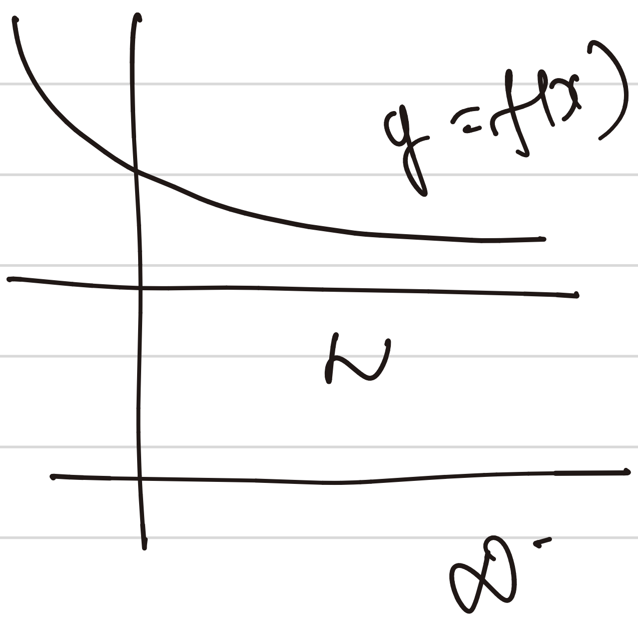
Intermediate Value Theorem:

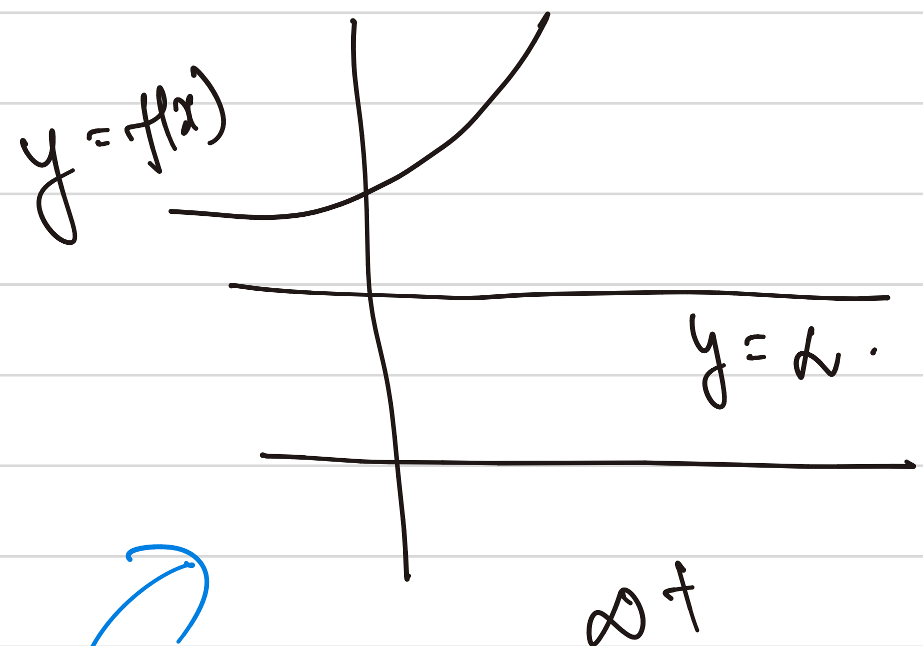
A continuous function takes on every intermediate values between the function values. f in [a,b] and N be a number in between f such that 'c' exists $f(c) = N$.

Limit at Infinity : Horizontal Asymptote:

$\lim\limits_{x\to\infty} f(x) = L$ or $f(x) = L$ or $f(x)$ tends to ∞.

$\left[\lim\limits_{x\to\infty^-} f(x) = \lim\limits_{x\to\infty^+} f(x) \right]$ So, larger and larger -ve or +ve.





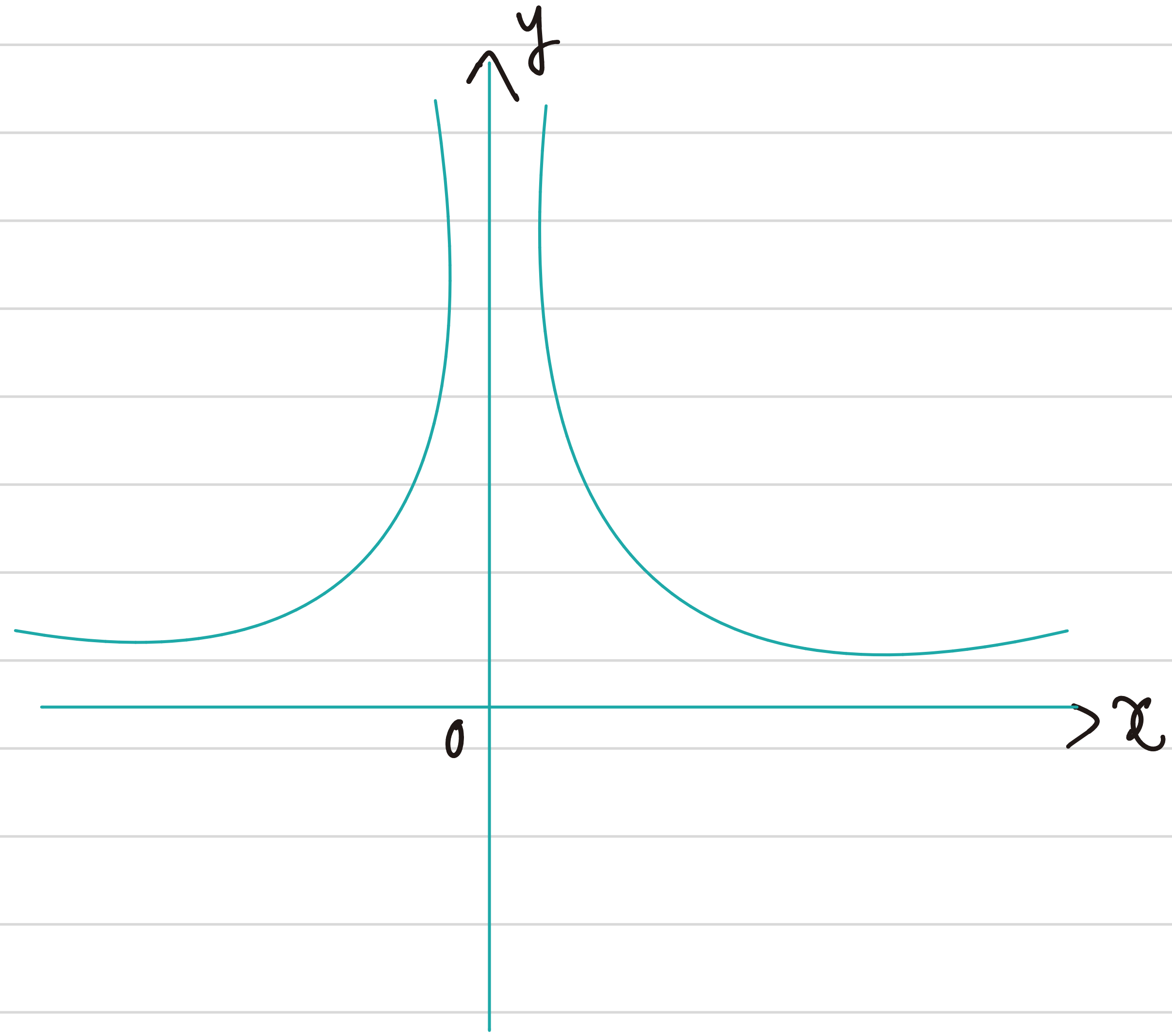
Horizontal Asymptote: $\lim\limits_{x\to\infty^+} f(x) = L$ or $\lim\limits_{x\to\infty^-} f(x) = L$

Vertical Asymptote:

i) $\lim_{x \to a} f(x) = \infty$ or $-\infty$

ii) $\lim_{x \to a^-} f(x) = \infty$ or $-\infty$

iii) $\lim_{x \to a^+} f(x) = \infty$ or $-\infty$



Slant (Oblique) Asymptote:

Let $f(x) = \dfrac{p(x)}{d(x)}$ be a rational function if degree of numerator is more than denominator then it is Slant i.e. deg of $p(x) >$ $d(x)$

Eg:

$$y = \frac{x^2 - \frac{3x}{2}}{2x + 1}.$$

function.

\# Even Symmetry: → Symmetrical about y-axis

\# Odd " → Symmetrical about origin.

\# Sum → $(f+g)(x) \Rightarrow f(x) + g(x)$

\# Difference → $(f-g)(x) \Rightarrow f(x) - g(x)$

\# Product → $(f \cdot g)(x) \Rightarrow f(x) \cdot g(x)$

\# Quotient → $(f/g)(x) \Rightarrow \frac{f(x)}{g(x)}$

Fundamental formula on Differentiation

1) Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

2) Sum Rule: $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

3) Product Rule:

$$\frac{d}{dx}(u \times v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

4) General Power Rule:

$$\frac{d}{dx}(u^n) = nu^{n-1} \cdot \frac{du}{dx}$$

5) The Quotient Rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} \cdot u \cdot \frac{dv}{dx}}{v^2}$$

6) The Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

7) $\frac{d}{dx}(K) = 0$ where K is constant

8) $\frac{d}{dx}(Ku) = K \cdot \frac{du}{dx}$ where K is constant and u is a function of x.

9) $f(x, y) = 0$ is called implicit function.

Derivatives of Inverse Circular functions.

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

iii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

iv) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

v) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

vi) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Extreme Value Theorem → If 'f' is continuous on a closed interval [a, b], then 'f' attains an absolute maximum f(c) and an absolute minimum f(d) at some points 'c' and 'd' in [a, b].

Fermat's Theorem - If 'f' has a local maximum (or minimum) at 'c' and if 'f' is differentiable at c then $f'(c) = 0$.

Rolle's Theorem: If f be a function that satisfies:

i) f is continuous on the closed interval [a, b].
ii) f is differentiable on the open interval (a, b).
iii) $f(a) = f(b)$