Day-85, Feb 25, 2025 (Falgun 13, 2081)

① Classification with a Neural Network: Motivation

② Classification with a Neural Network: Minimizing log-loss

③ Gradient Descent and back propagation with figures

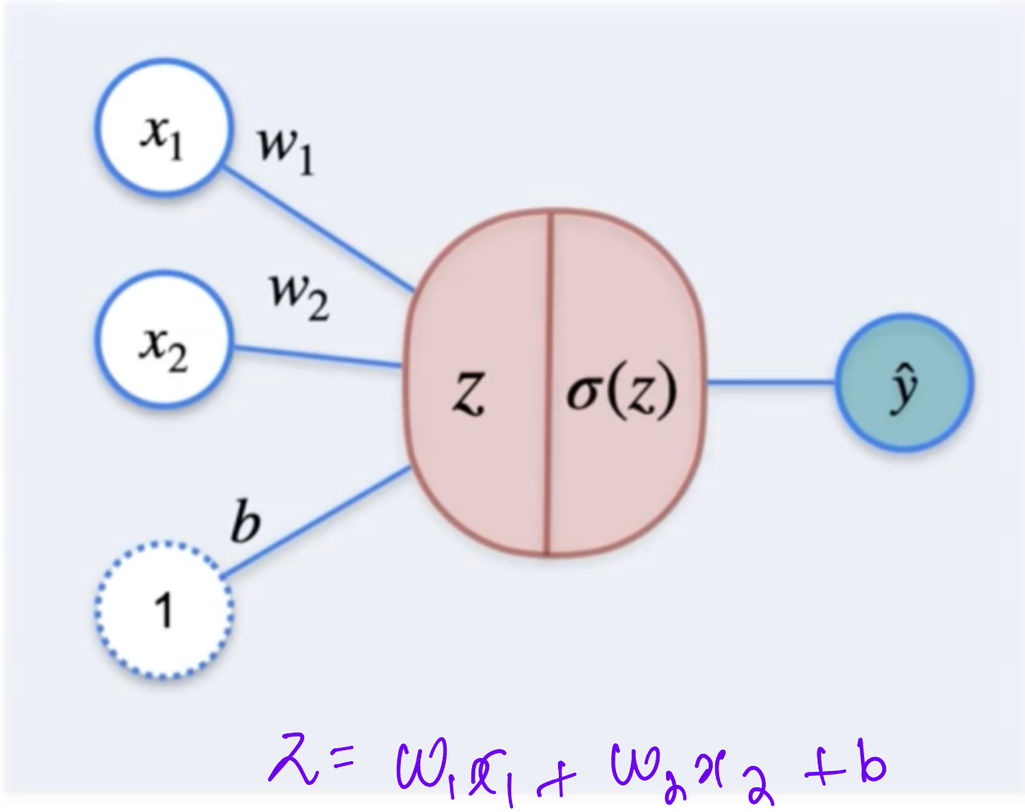
④ Newton's Method

⑤ An Example in Newton's Method.

Source: Calculus for Machine Learning and Data Science: Coursera offered by Deeplearning.AI.

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	*3*	*0*	*Happy* 😊
Beep beep!	*0*	*2*	*Sad* 😔
Aack beep beep beep!	*1*	*3*	*Sad* 😔
Aack beep aack!	*2*	*1*	*Happy* 😊



of the previous variables time
multiplied by the weights

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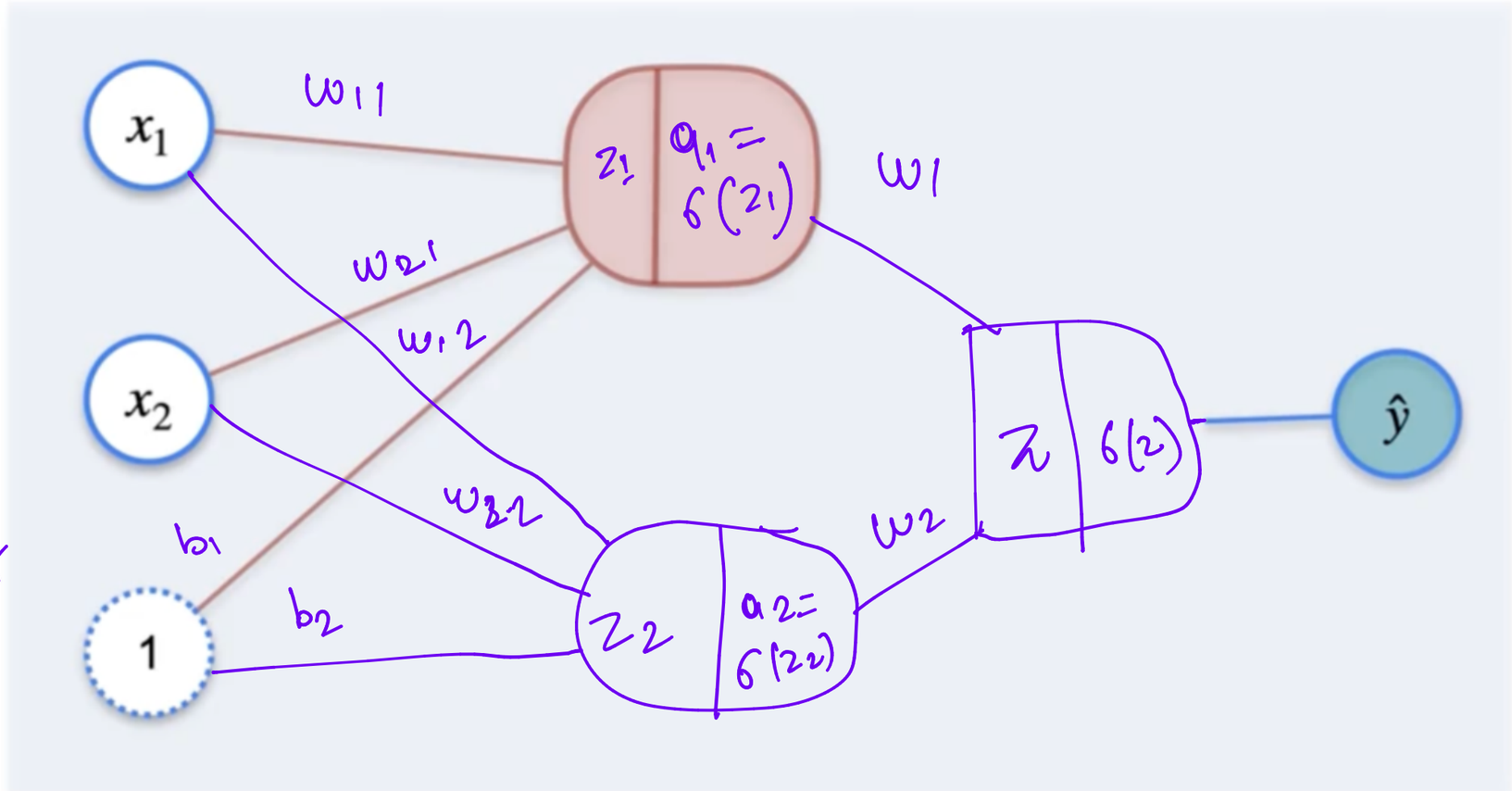
2,2,1 Neural Network

- Neural network of depth 2
  - One input layer
  - One hidden layer
  - One output layer



Here we have one perceptron,

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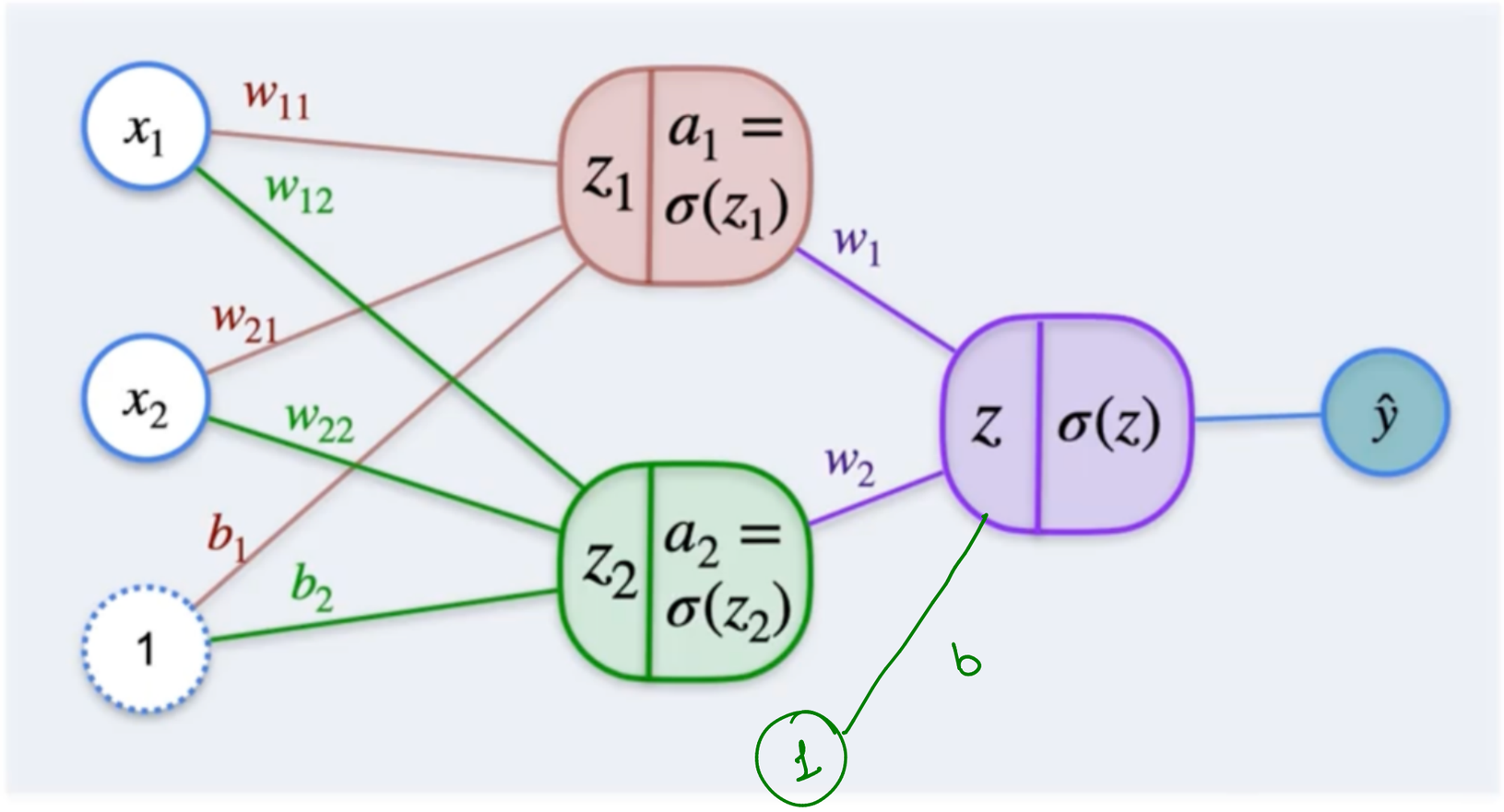
2,2,1 Neural Network

Neural network of depth 2

- one input layer
- one hidden layer
- one output layer



Perceptrons normally have a bias

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2,2,1 Neural Network

$a_1 = \sigma(z_1)$

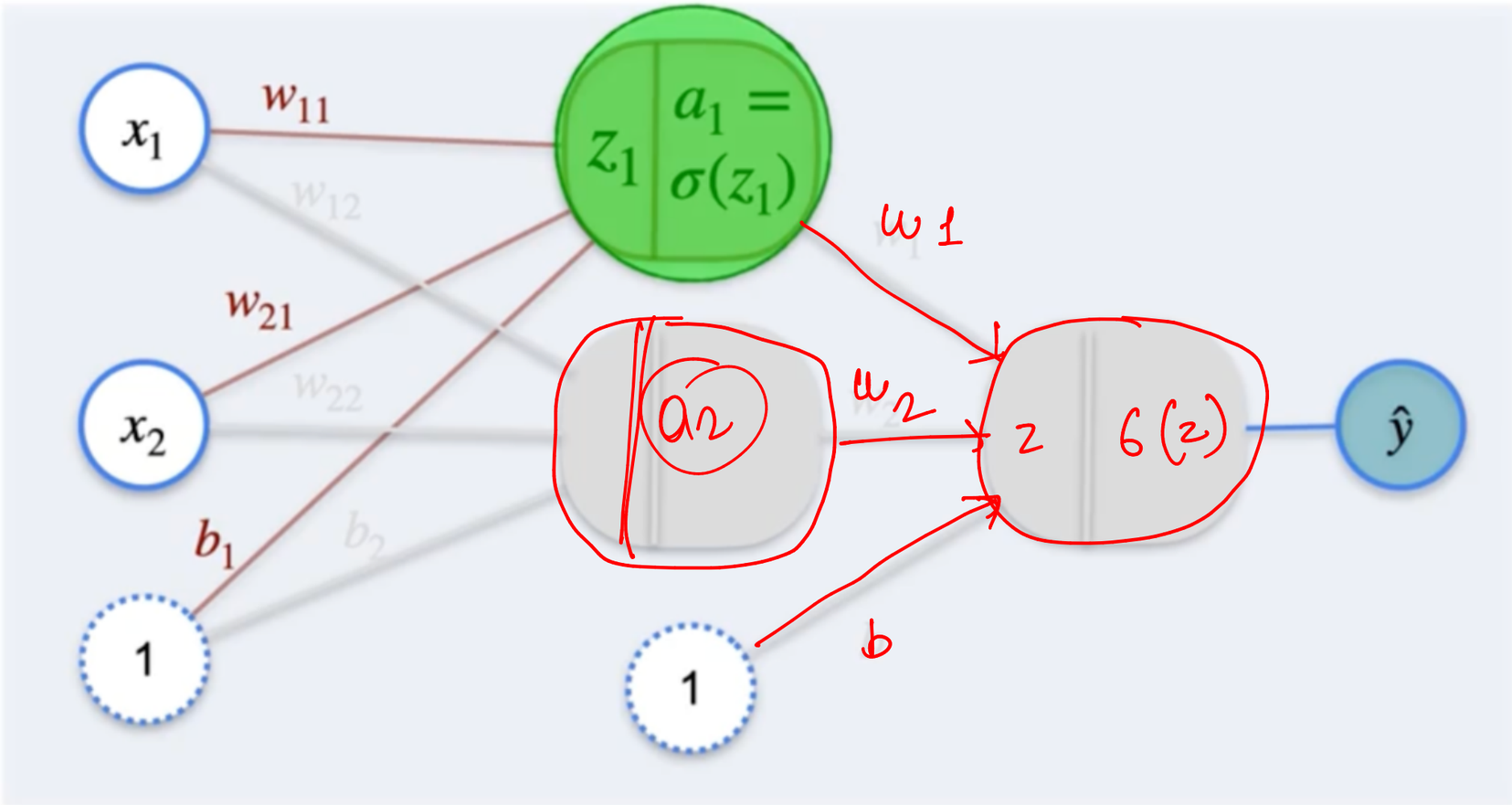
$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$

$a_2 = \sigma(z_2)$

$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$

$\hat{y} = \sigma(z)$

$z = a_1 w_1 + a_2 w_2 + b$



Focusing on this node alone,

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

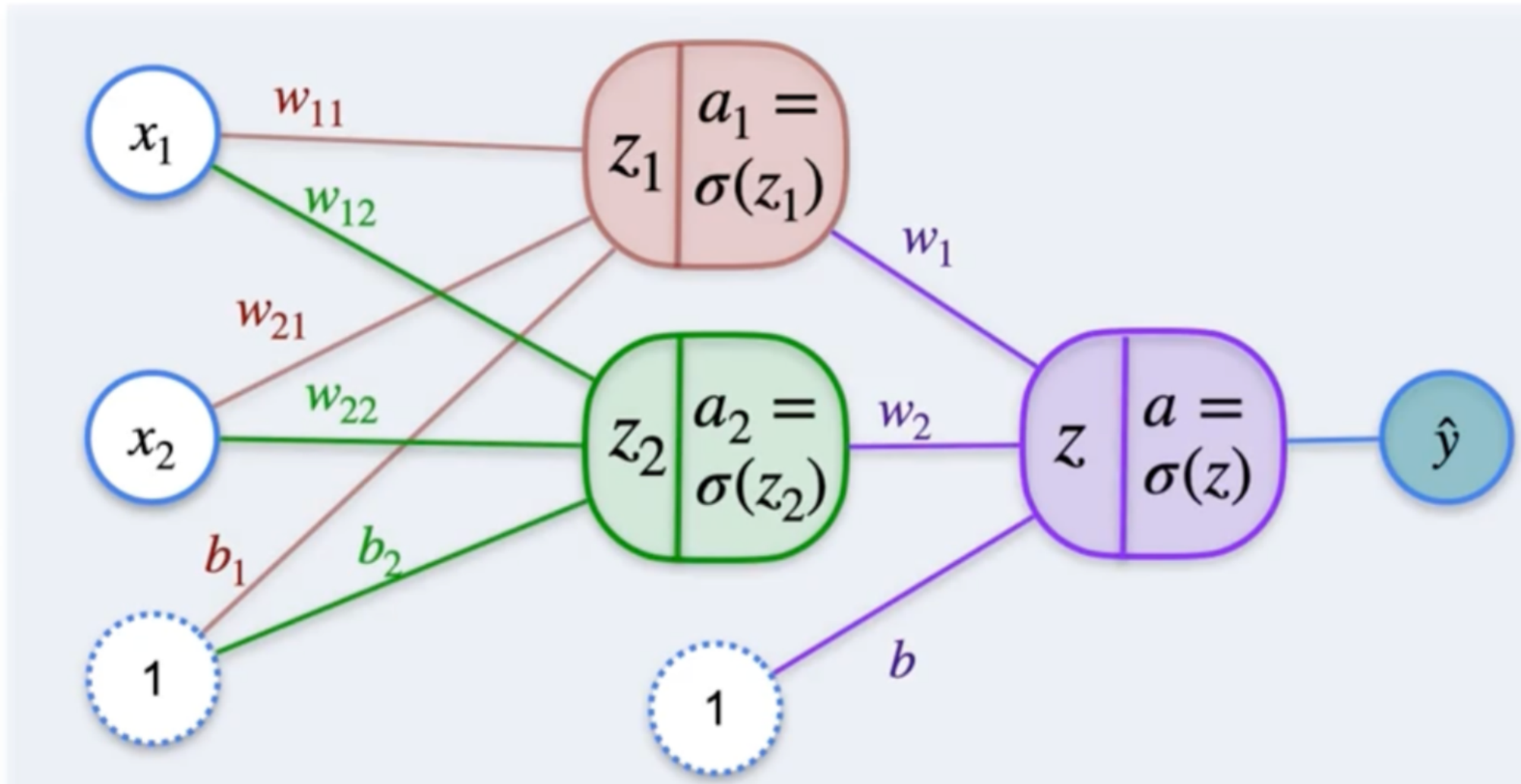
$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$



As a reminder, here are all the variables.

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The values for z_1 z_1 and z_2 is:

$$z_1 = w_{11}x_1 + w_{21}x_2 + b_1$$

$$z_2 = w_{12}x_1 + w_{22}x_2 + b_2$$

$$z = w_1x_1 + w_2x_2 + b$$

Loss function

$$\rightarrow L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1w_{11} + x_2w_{21} + b_1$$

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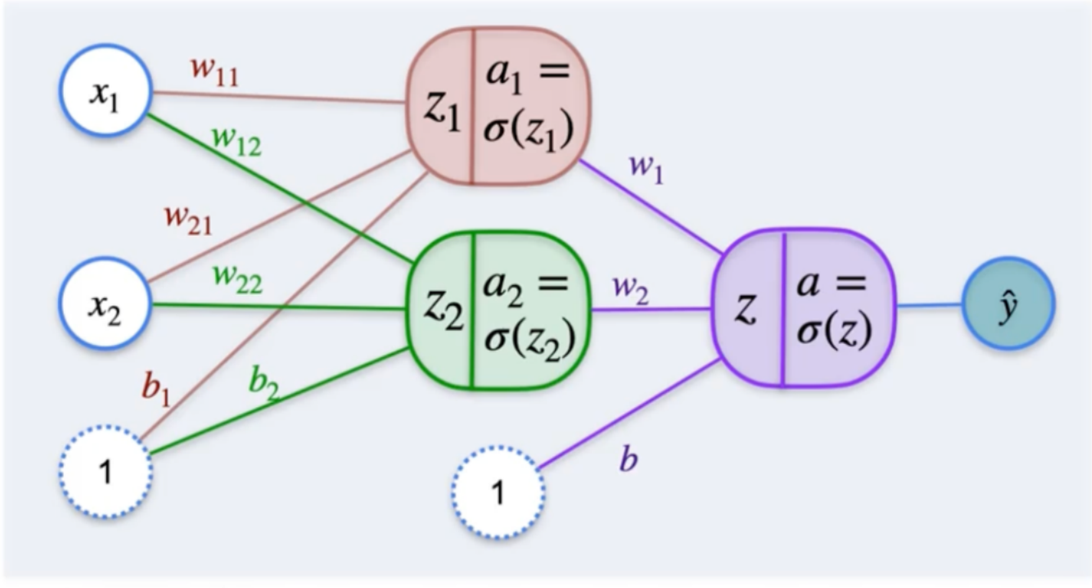
$$z_2 = x_1w_{12} + x_2w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$

$$z = a_1w_1 + a_2w_2 + b$$

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$



where y hat is the prediction

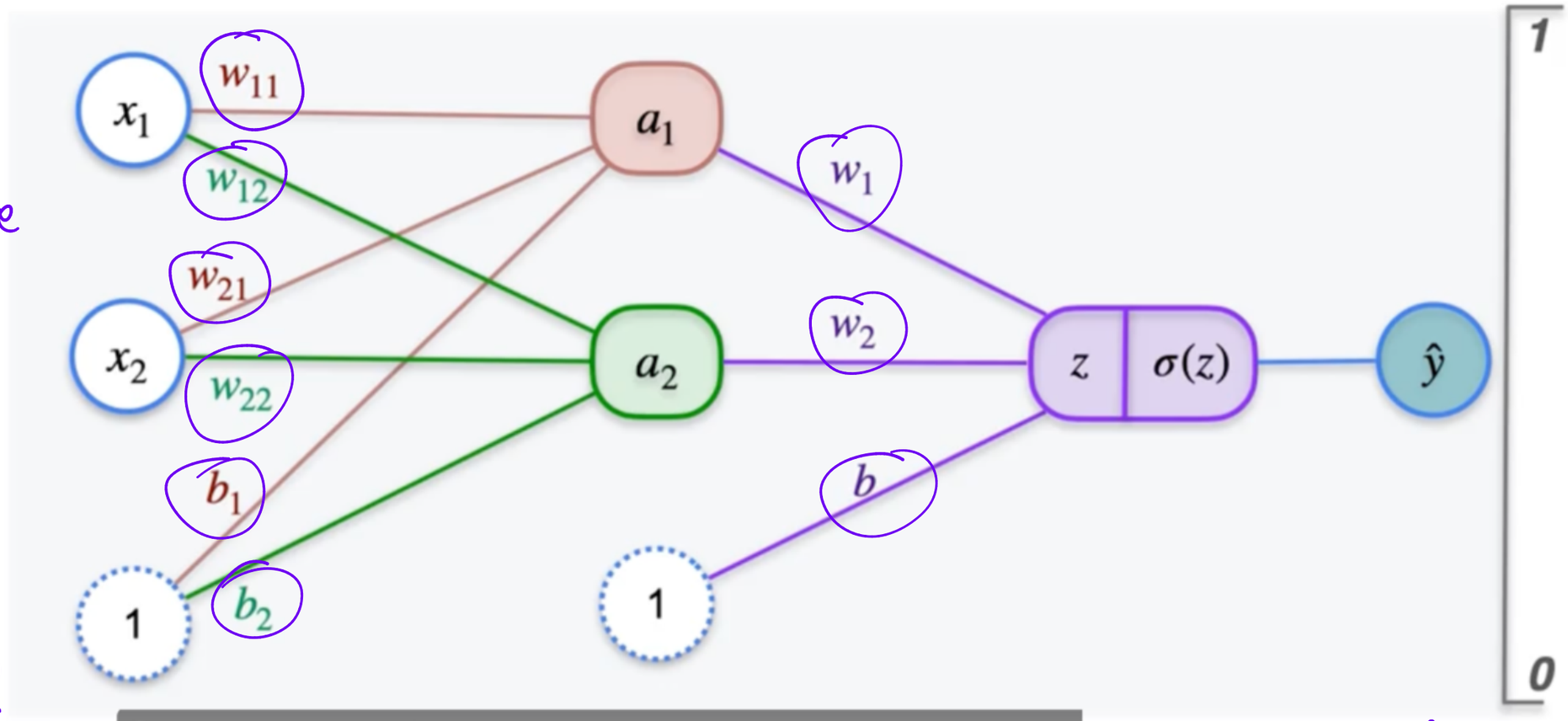
Goal:

2,2,1 Neural Network

Goal

Adjust each of the highlighted weights and biases.

#To Reduce the loss function?



So recall that the goal is to adjust each one of the highlighted weights and

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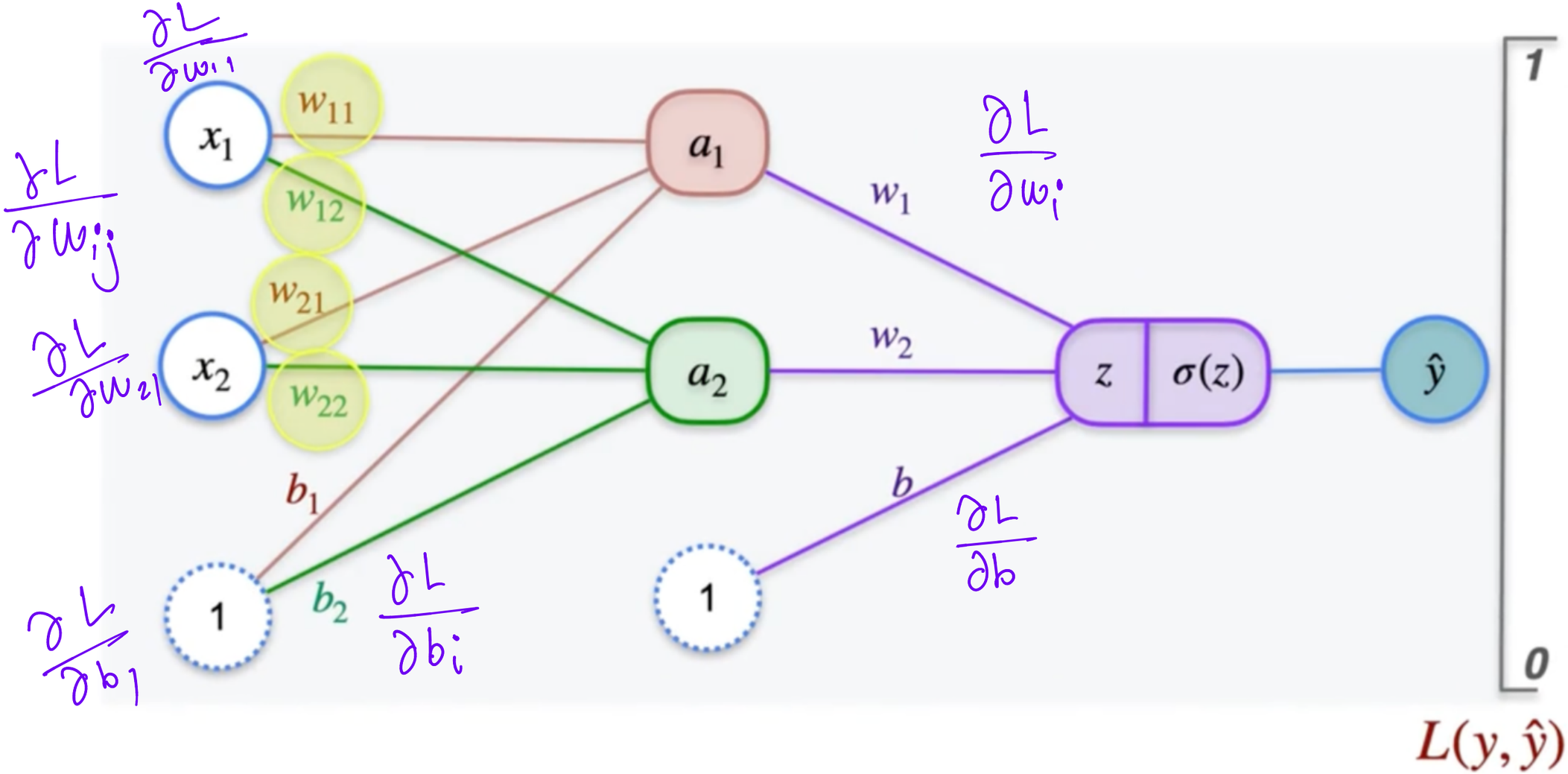
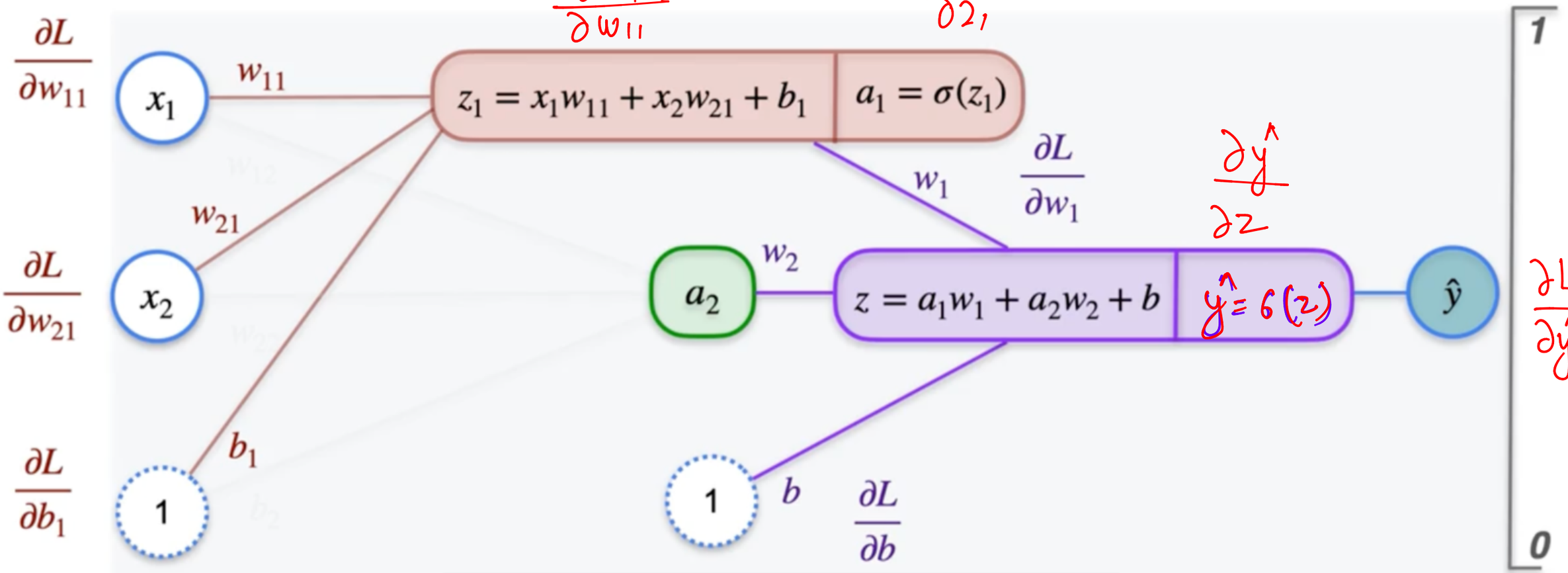
2,2,1 Neural Network



Reduce log loss.

2,2,1 Neural Network



a2 times the weights W1 and W2 and the bias b.

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$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_{11}} = x_1 \cdot a_1(1-a_1)\, w_1\, \hat{y}(1-\hat{y}) \cdot \frac{-(y-\hat{y})}{\hat{y}(1-\hat{y})}$$

$$\left[\frac{\partial L}{\partial w_{11}} \Rightarrow -x_1 a_1 w_1 (1-a_1)(y-\hat{y})\right]$$

Perform gradient descent with

$$w_{11} \rightarrow w_{11} - \alpha \cdot x_1 \cdot w_1 a_1(1-a_1)(y-\hat{y})$$

to find optimal value of w_{11} that gives the least error.

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_{11}} = x_1 \cdot a_1(1-a_1) \cdot w_1 \cdot \cancel{\hat{y}(1-\hat{y})} \cdot \frac{-(y - \hat{y})}{\cancel{\hat{y}(1-\hat{y})}}$$

$$= -x_1 w_1 a_1 (1-a_1)(y - \hat{y})$$

Perform gradient descent with

$$w_{11} \rightarrow w_{11} - \alpha \text{-} x_1 w_1 a_1 (1-a_1)(y - \hat{y})$$

to find optimal value of w_{11} that gives the least error

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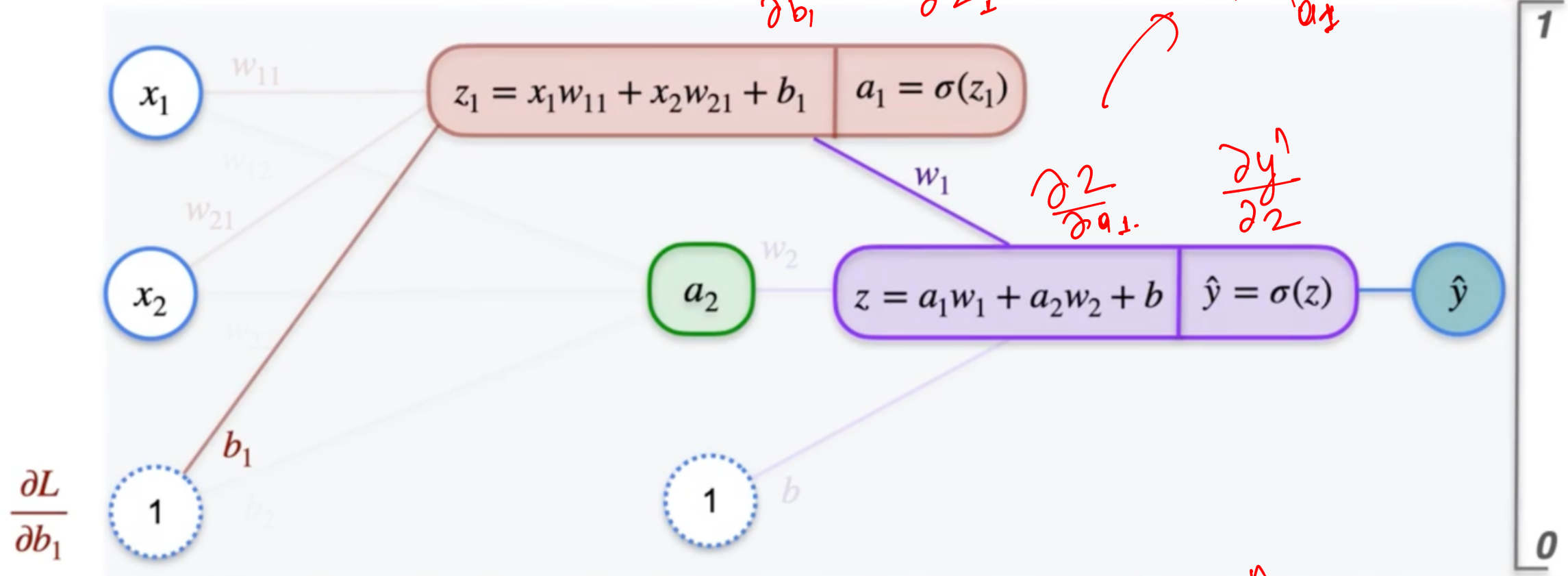
2,2,1 Neural Network



$$\frac{\partial L}{\partial b_1} \Rightarrow \frac{\partial z_1}{\partial b_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

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2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial b_1} = \frac{\partial z_1}{\partial b_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial b_1} = 1 \cdot a_1(1-a_1)\, w_1\, \hat{y}(1-\hat{y}) \cdot \frac{-(y-\hat{y})}{\hat{y}(1-\hat{y})}$$

Sigmoid f'

$$\Rightarrow -w_1 a_1 (1-a_1)(y-\hat{y})$$

Perform Gradient Descent with

$$b_1 \rightarrow b_1 - \alpha\left(-w_1 a_1 (1-a_1)(y-\hat{y})\right)$$

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2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

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$$\frac{\partial L}{\partial b_1} = 1 \cdot a_1(1-a_1) \cdot w_1 \cdot \cancel{\hat{y}(1-\hat{y})} \cdot \frac{-(y - \hat{y})}{\cancel{\hat{y}(1-\hat{y})}}$$

$$= -w_1 a_1 (1-a_1)(y - \hat{y})$$

Perform gradient descent with

$$b_1 \rightarrow b_1 - \alpha(-w_1 a_1 (1-a_1)(y - \hat{y}))$$

to find optimal value of b_1 that gives the least error

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2,2,1 Neural Network

→ Weight updates.

$$w_{11} \rightarrow w_{11} + \alpha x_1 w_1 a_1 (1 - a_1)(y - \hat{y})$$

$$w_{12} \rightarrow w_{11} + \alpha x_2 w_1 a_1 (1 - a_1)(y - \hat{y})$$

$$b_1 \rightarrow b_1 + \alpha w_1 a_1 (1 - a_1)(y - \hat{y})$$

$$w_{21} \rightarrow w_{21} + \alpha x_1 w_2 a_2 (1 - a_2)(y - \hat{y})$$

$$w_{22} \rightarrow w_{22} + \alpha x_2 w_2 a_2 (1 - a_2)(y - \hat{y})$$

$$b_2 \rightarrow b_2 + \alpha w_2 a_2 (1 - a_2)(y - \hat{y})$$

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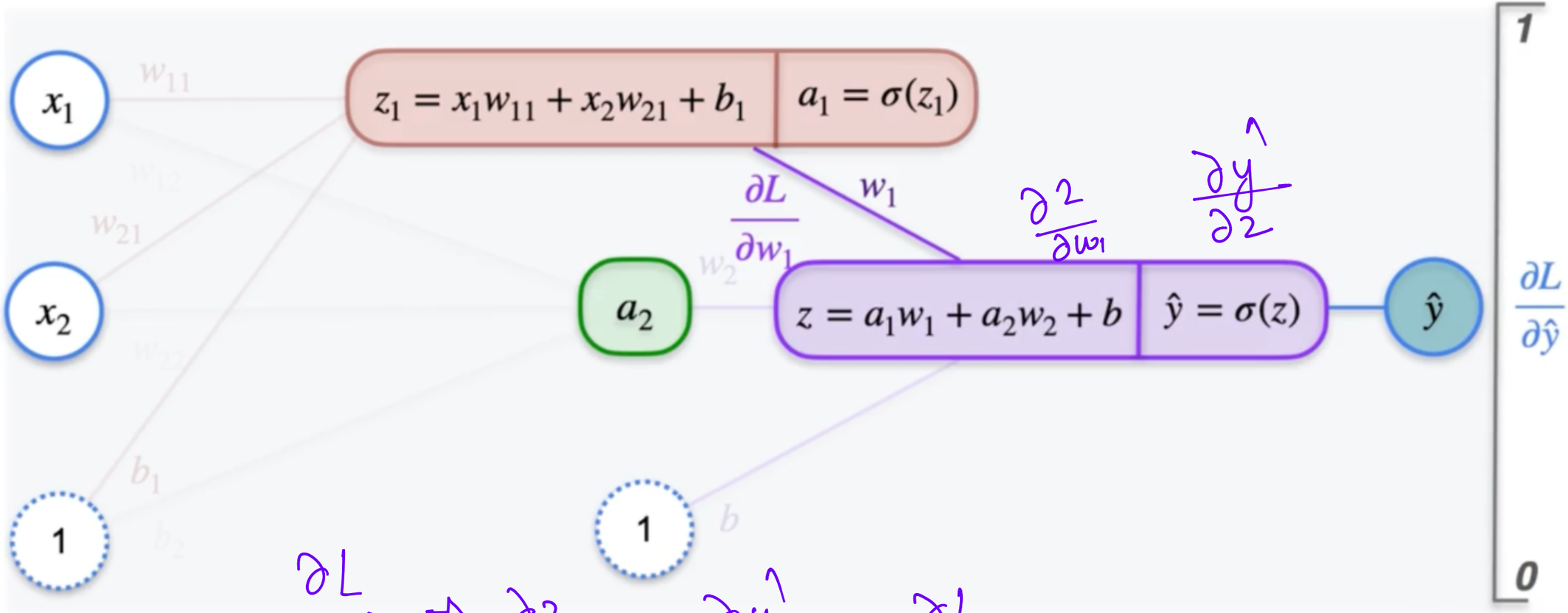
2,2,1 Neural Network



$$\frac{\partial L}{\partial w_1} \Rightarrow \frac{\partial z}{\partial w_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_1} \Rightarrow a_1 \cdot \hat{y}(1-\hat{y}) \cdot \frac{-(y-\hat{y})}{\hat{y}(1-\hat{y})}$$

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2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial z}{\partial w_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_1} = a_1 \cdot \cancel{\hat{y}(1-\hat{y})} \cdot \frac{-(y - \hat{y})}{\cancel{\hat{y}(1-\hat{y})}}$$

$$= -a_1(y - \hat{y})$$

Perform gradient descent with

$$w_1 \rightarrow w_1 - \alpha(-a_1(y - \hat{y}))$$

to find optimal value of w_1 that gives the least error

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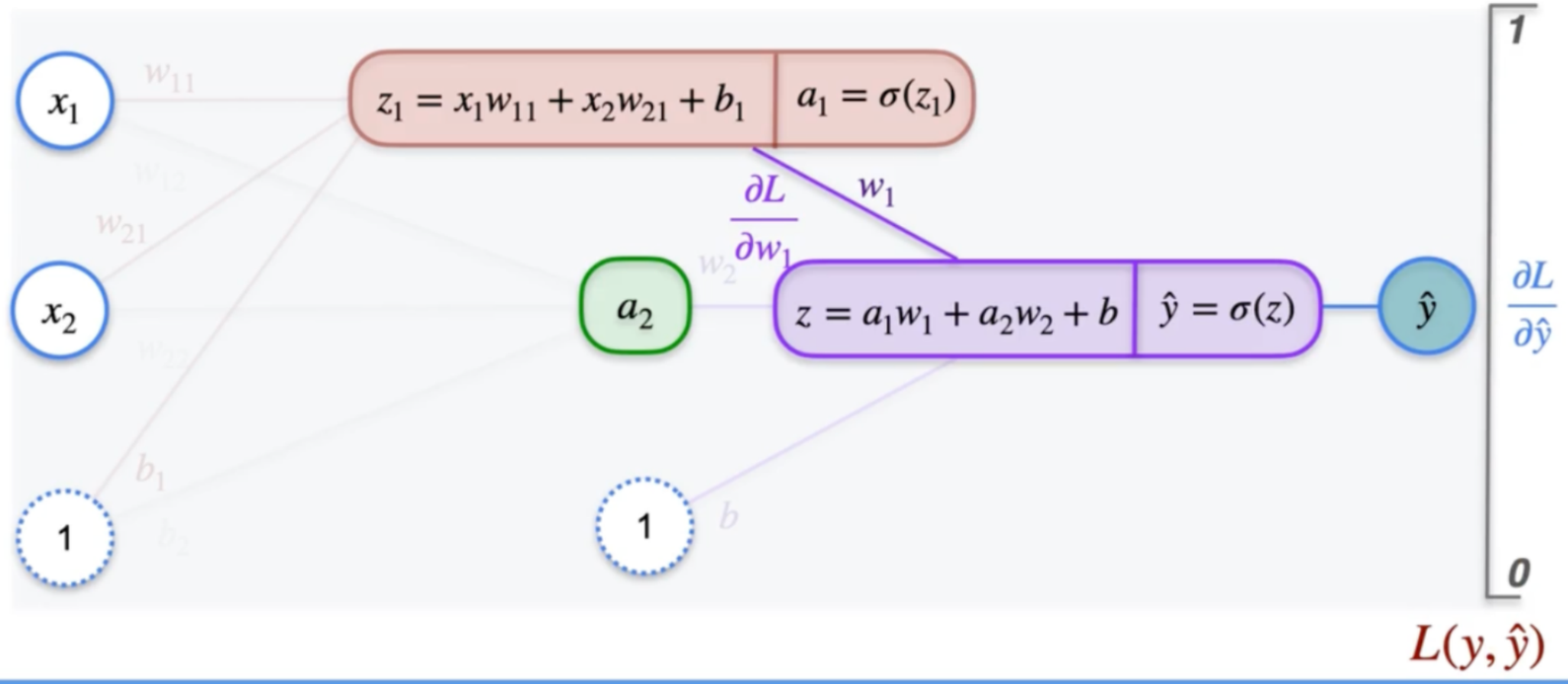
2,2,1 Neural Network



#.

2,1 Neural Network

$$w_1 \rightarrow w_1 + \alpha a_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 + \alpha a_2(y - \hat{y}))$$

$$b \rightarrow b + \alpha(y - \hat{y}))$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1w_1 + a_2w_2 + b$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial z}{\partial w_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_1} = a_1 \cdot \cancel{\hat{y}(1-\hat{y})} \cdot \frac{-(y - \hat{y})}{\cancel{\hat{y}(1-\hat{y})}}$$

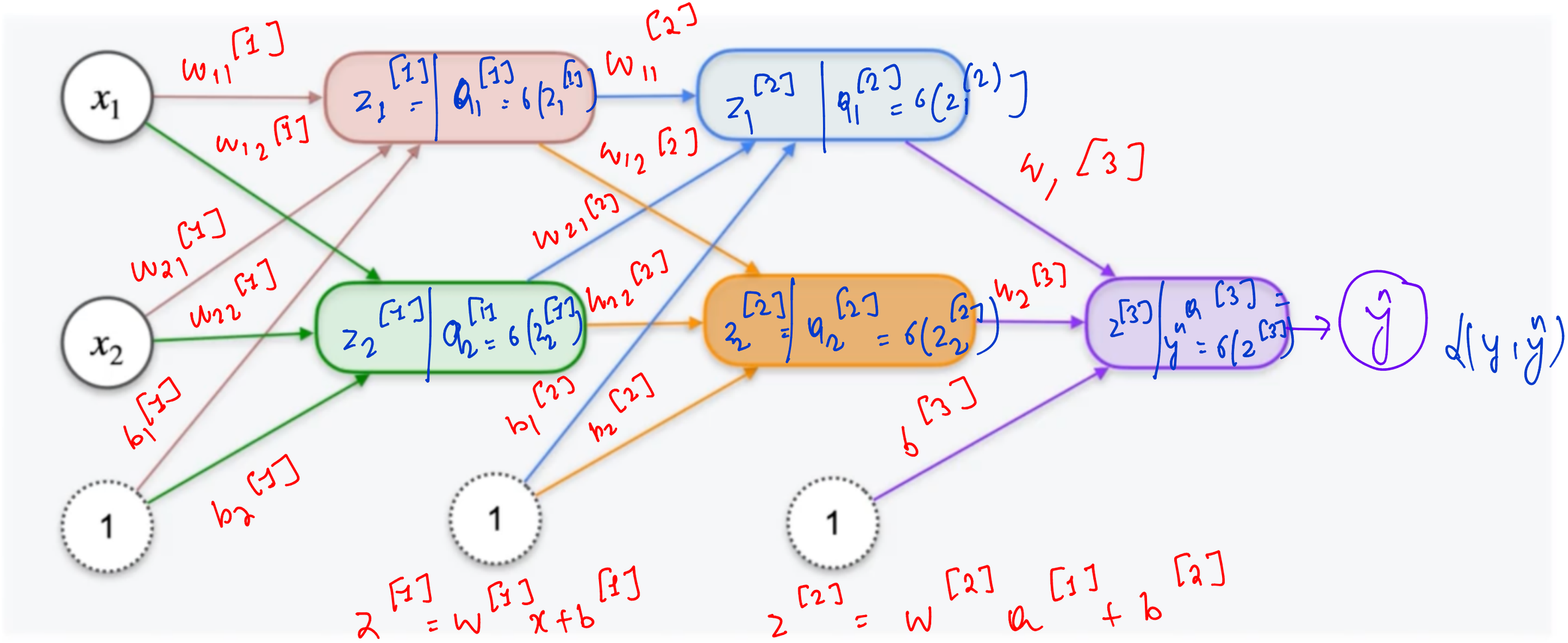
$$= -a_1(y - \hat{y})$$

Perform gradient descent with

$$w_1 \rightarrow w_1 - \alpha(-a_1(y - \hat{y}))$$

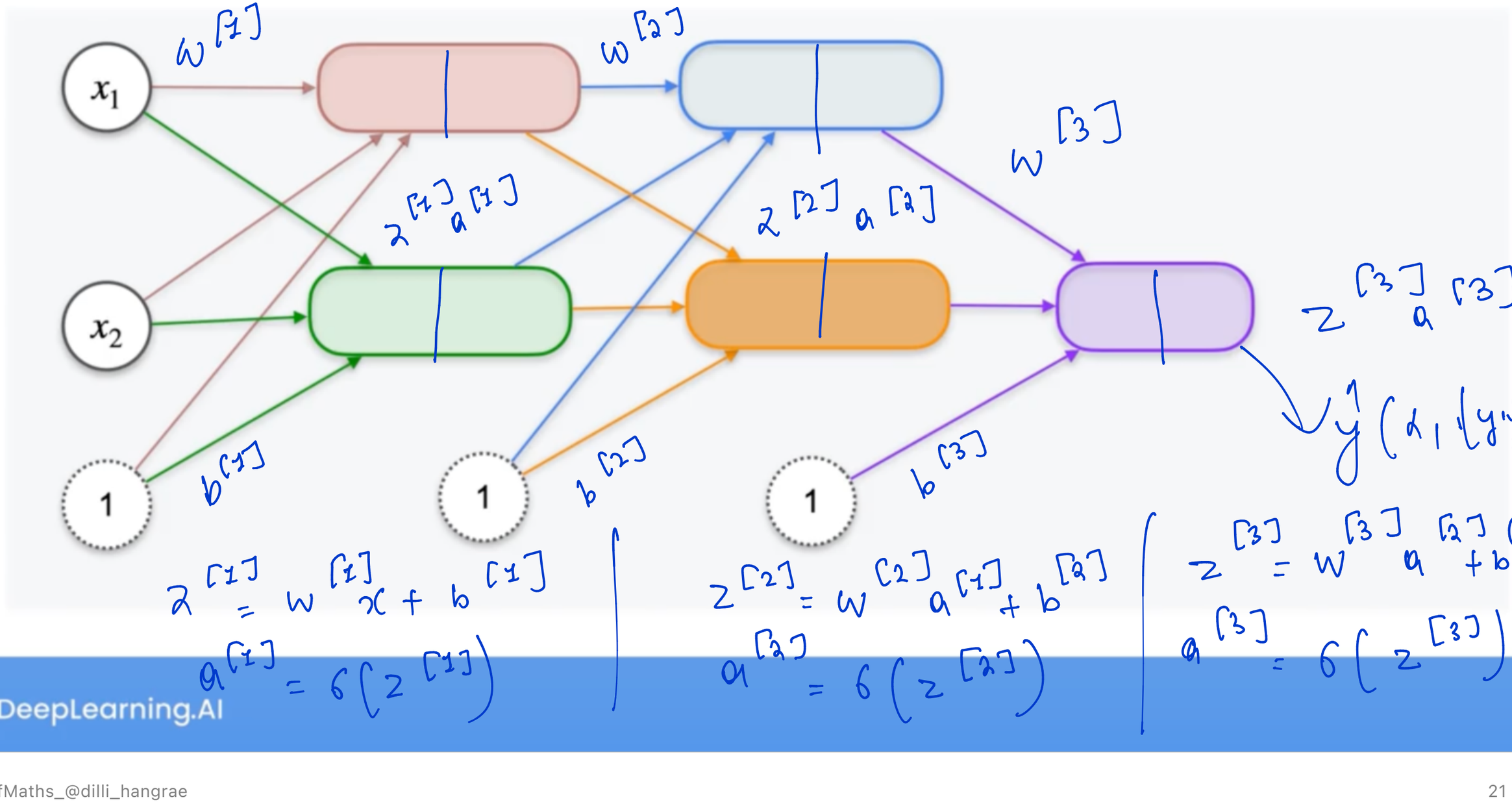
to find optimal value of w_1 that gives the least error

Back Propagation Introduction



$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y})$$

Back Propagation Introduction



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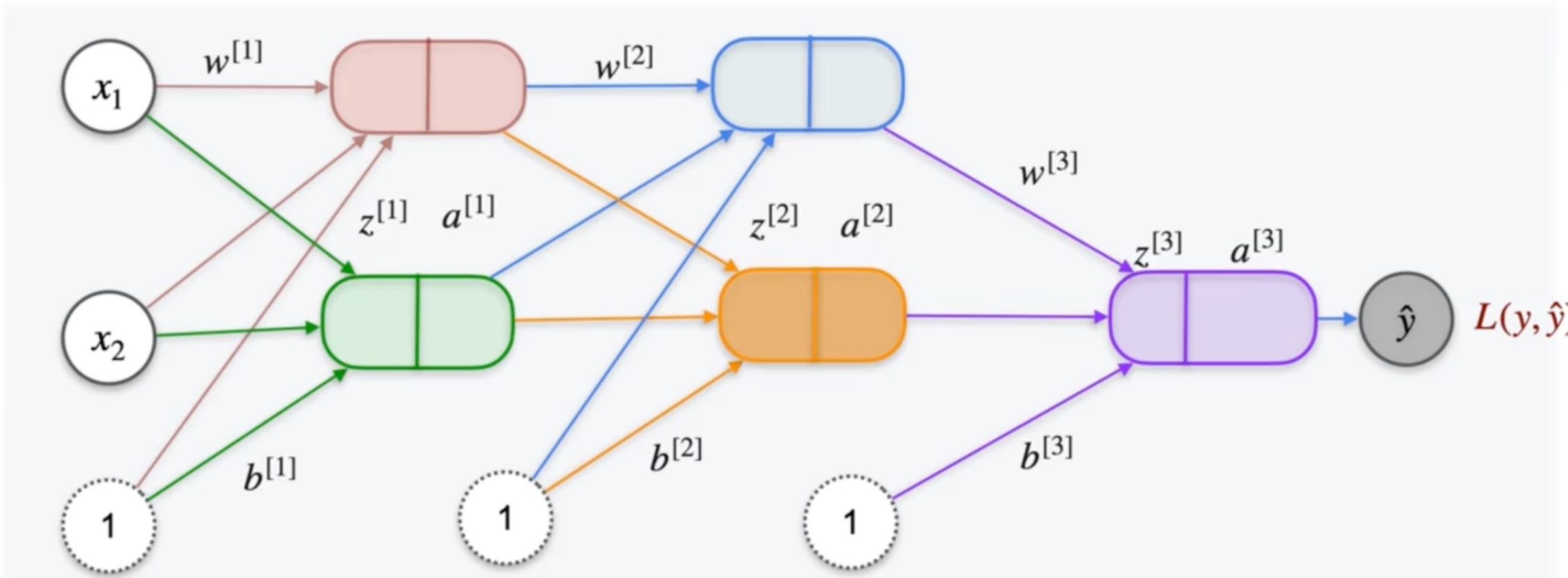

$$z^{[1]} = w^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$

$$z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$

$$z^{[3]} = w^{[3]}a^{[2]} + b^{[3]}$$
$$a^{[3]} = \sigma(z^{[3]})$$

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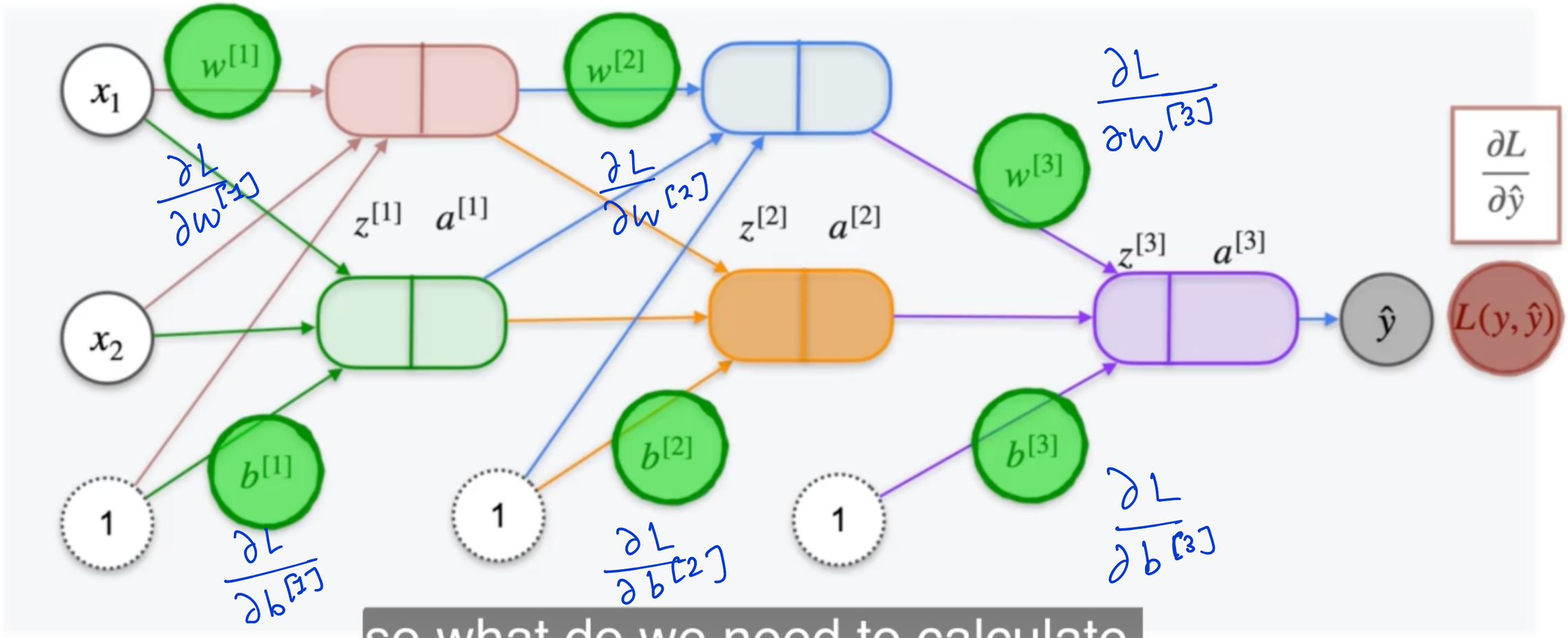

so what do we need to calculate,
we need to calculate DL over DY hat and

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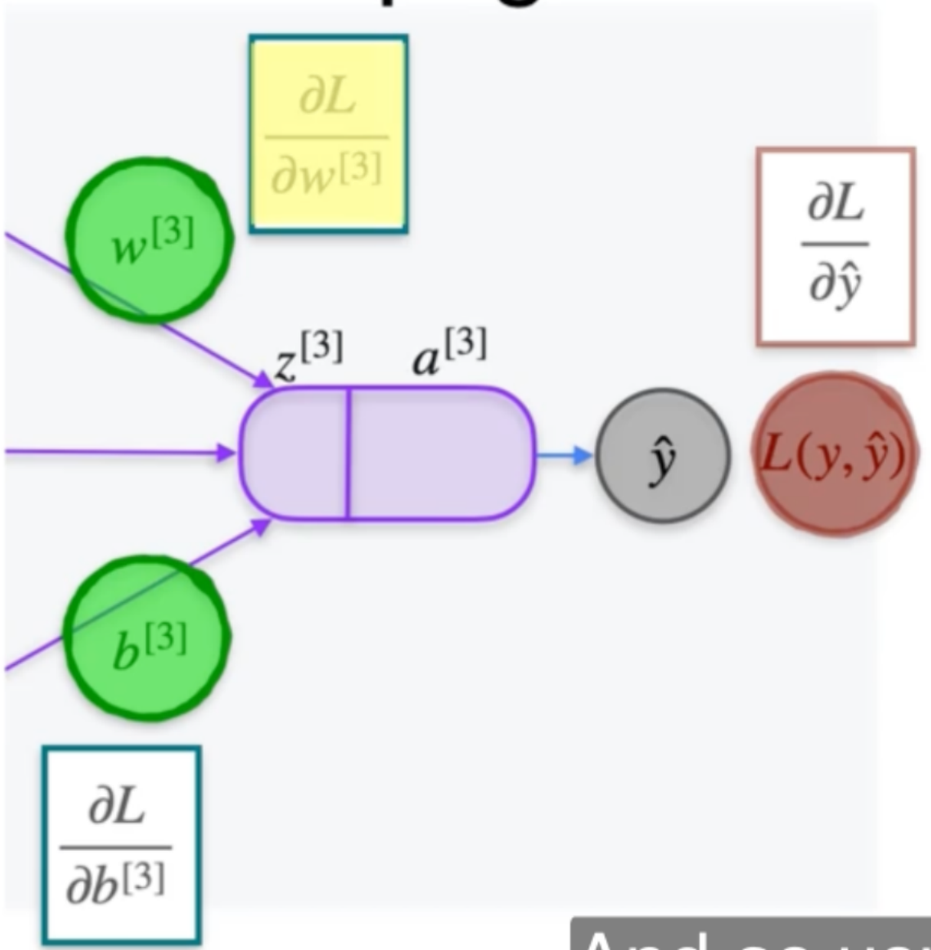

$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial L}{\partial b^{[3]}} = \frac{\partial z^{[3]}}{\partial b^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

And as usual we need to build a chain because we need

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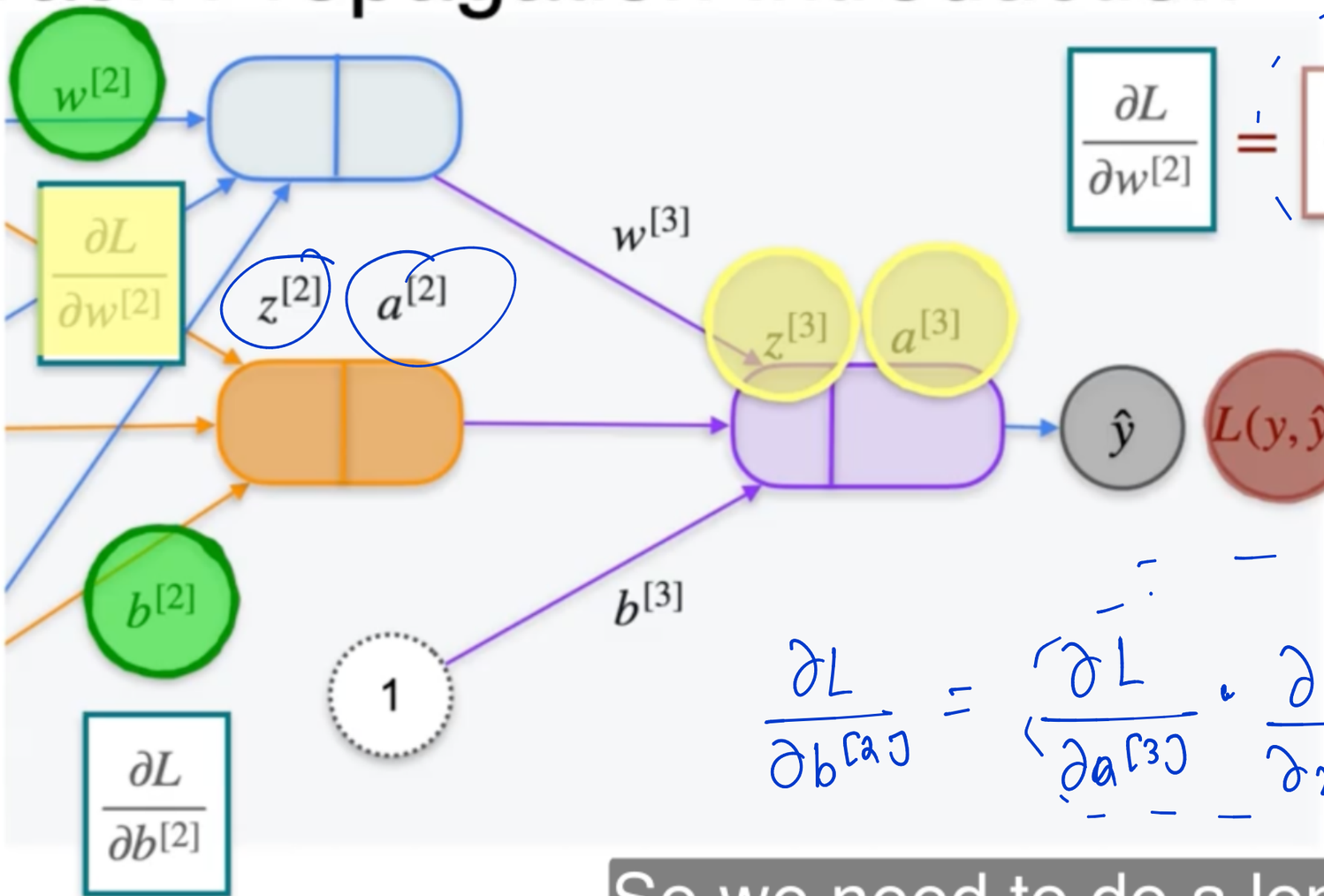

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \left(\frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \right) \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

So we need to do a long,
long chain rule of variables,

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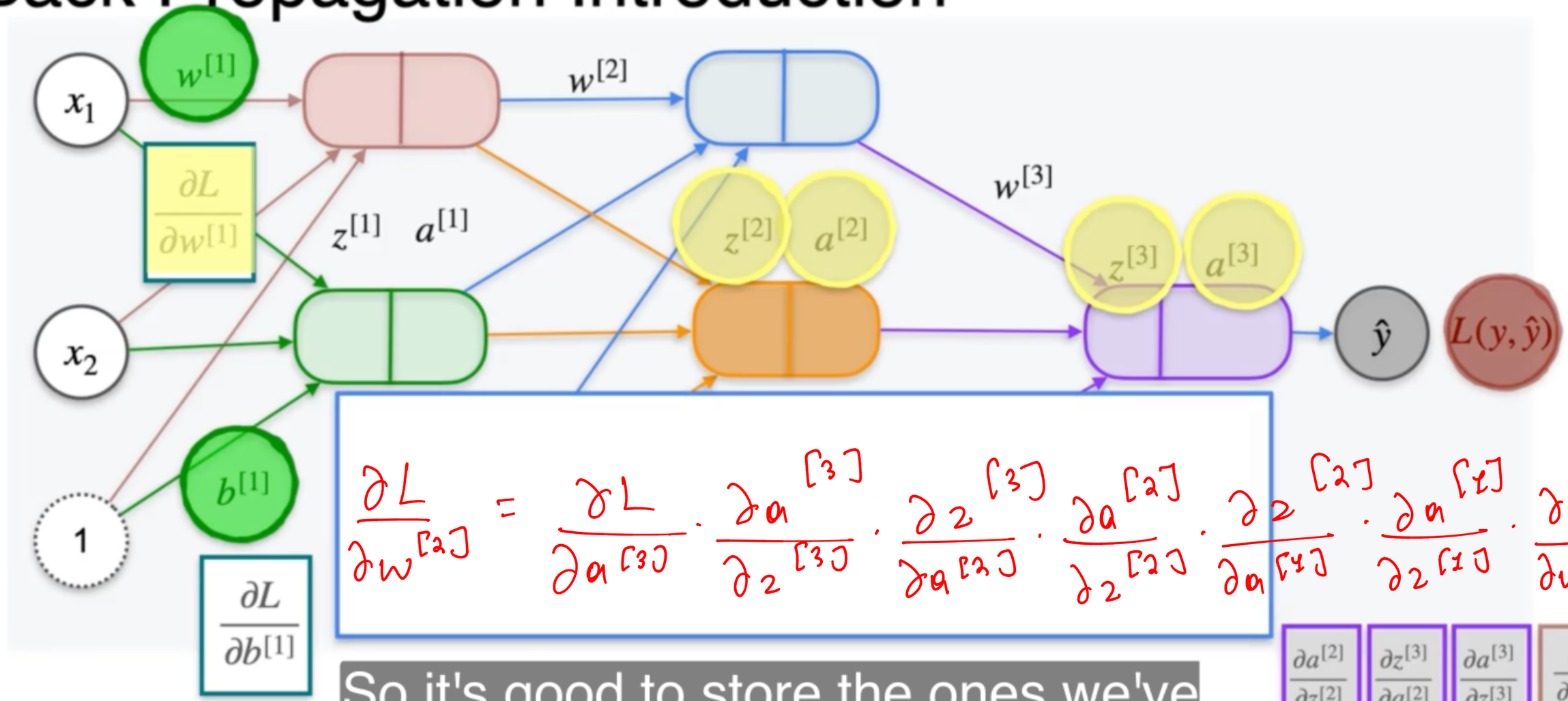

So it's good to store the ones we've already calculated and finally for

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Newton's Method (Alternate Method of Gradient Descent):

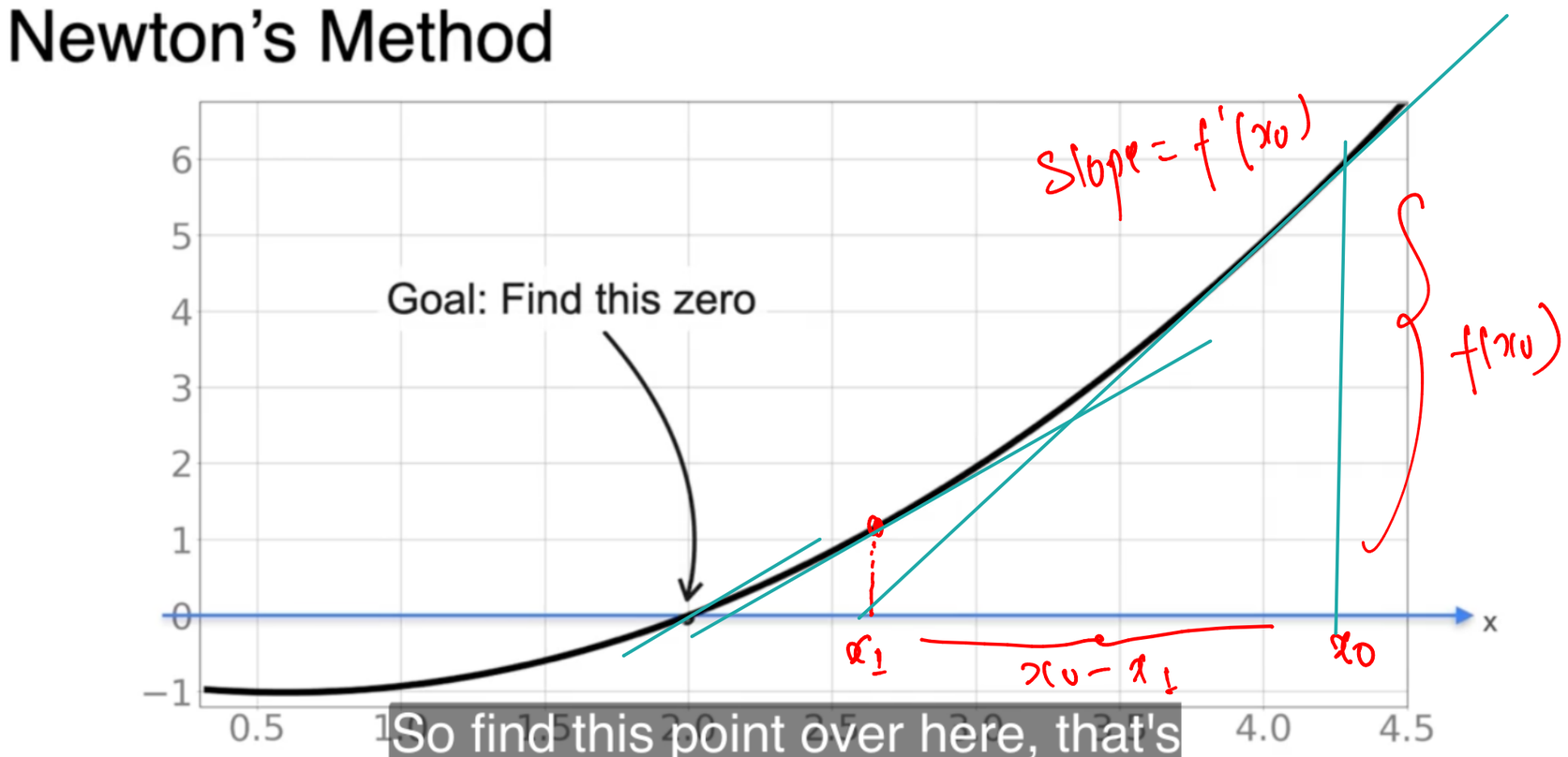
$$\frac{f(x_0)}{x_0 - x_1} = f'(x_0)$$

$$\frac{\text{rise}}{\text{run}} = \text{Slope}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Also

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$



So find this point over here, that's the point where F of X is equal to zero.

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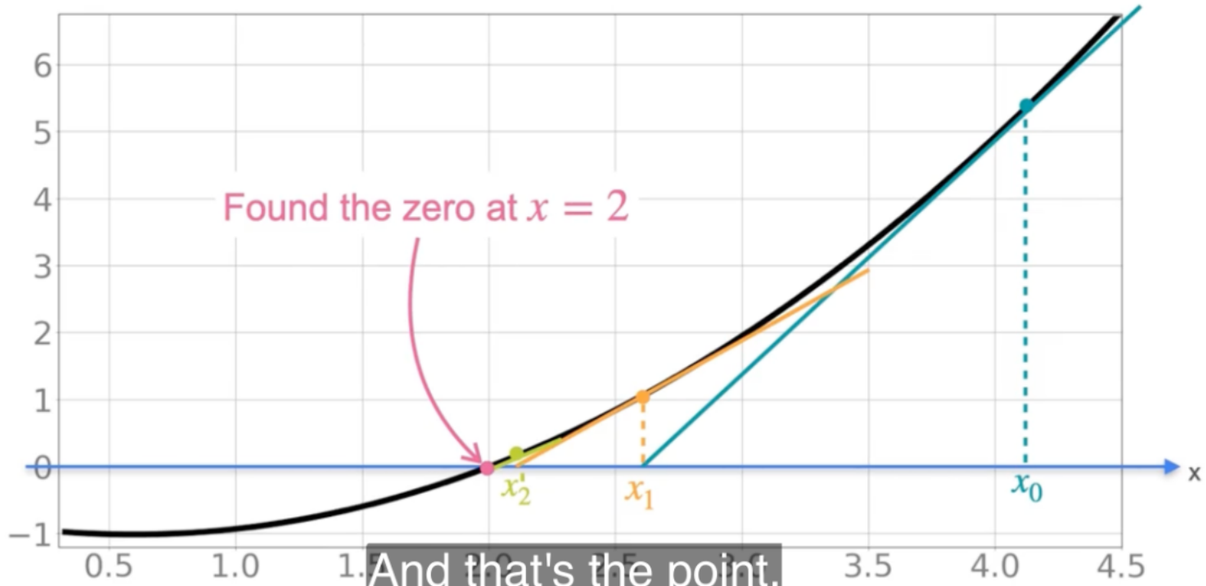
Newton's Method



And that's the point,
to be able to approximate the zero,

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Newton's Method for Optimization.

Goal: find a zero of $f(x)$.

minimize $g(x)$ → find zeros of $g'(x)$

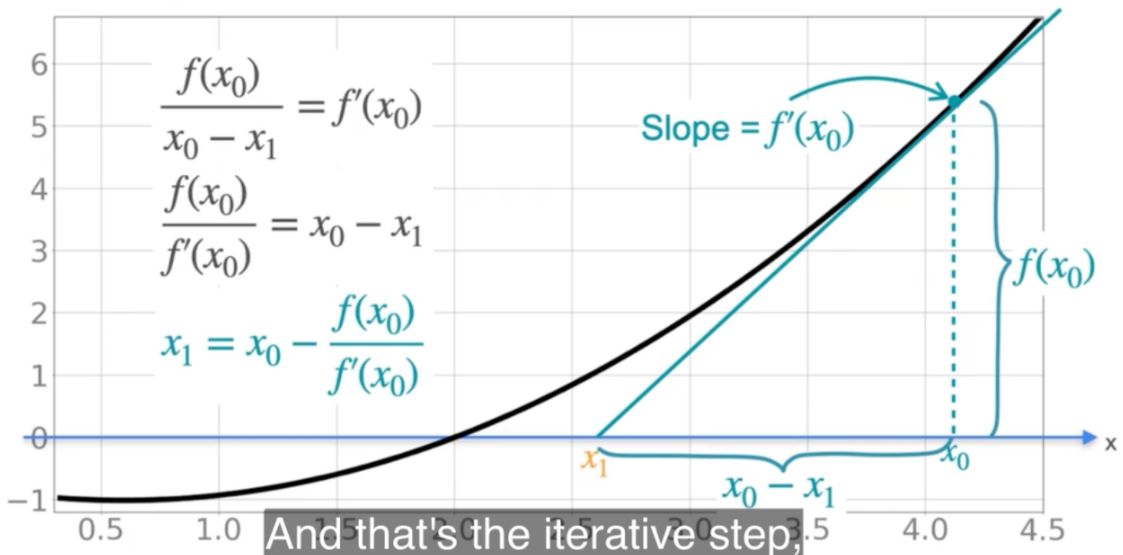
$f(x) \mapsto g'(x)$ $\quad f'(x) \mapsto (g'(x))'$

1) Start with some x_0.

2) Update:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

Update Approximation



And that's the iterative step,
you have F zero,

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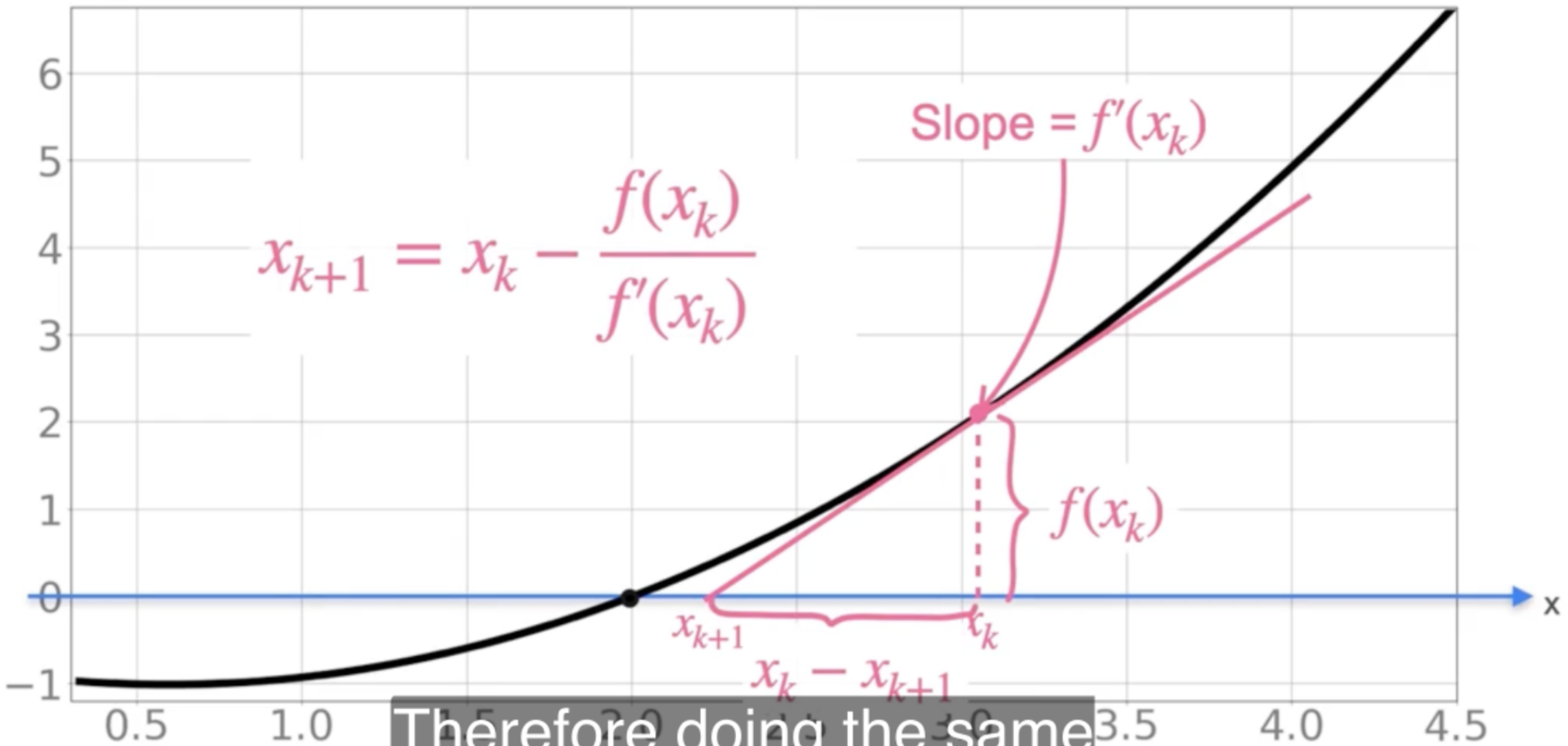
Update Approximation

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$



Therefore doing the same methods before escape plus one

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Summarize Algorithm:

Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

1) Start with some x_0

2) Update:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

3) Repeat 2) until you find the root.

NM for Optimization

Goal: minimize $g(x)$ ⇒ find zeros of $g'(x)$

$f(x) \mapsto g'(x)$ $\quad$ $f'(x) \mapsto (g'(x))'$

1) Start with some x_0

2) Update:

$$x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$$

3) Repeat 2) until you find the candidate for minimum.

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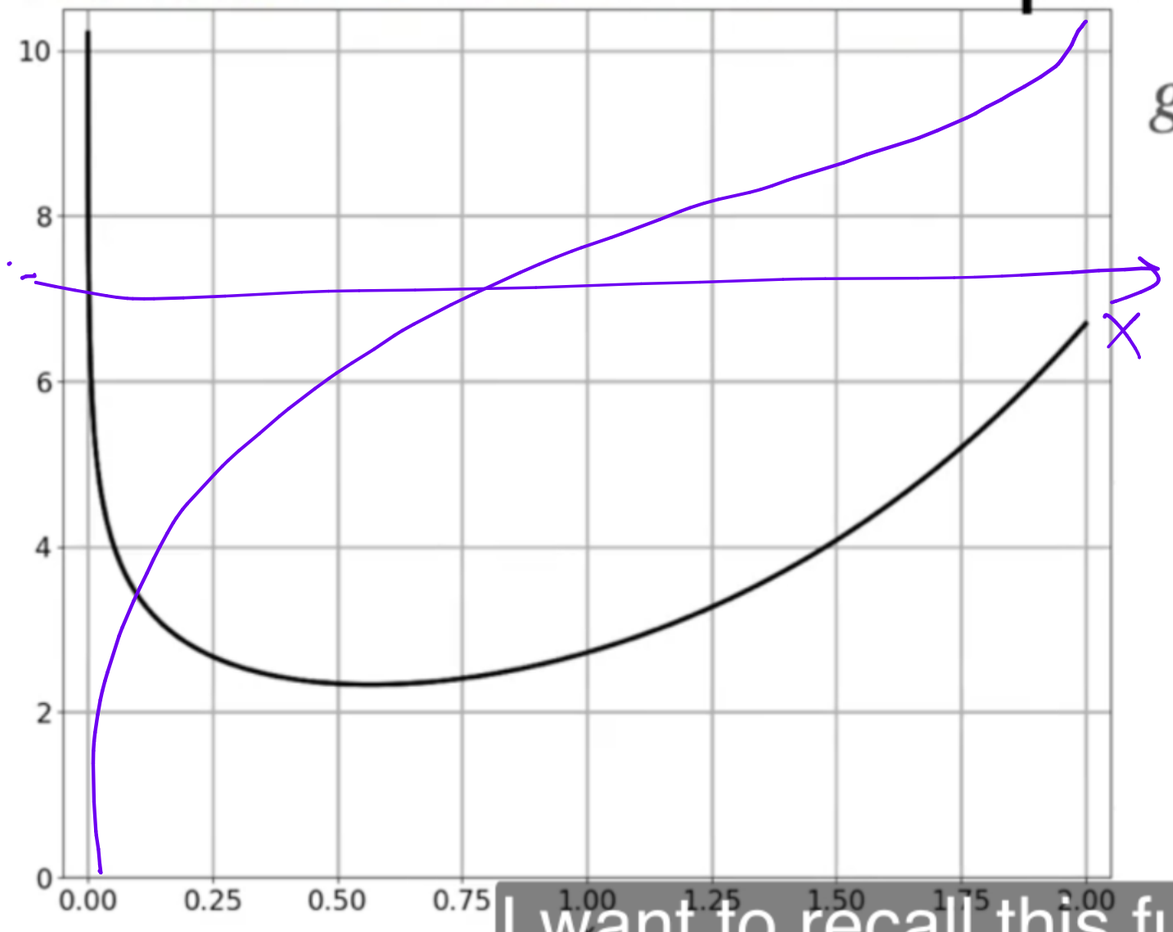
Newton's Method for Optimization

Newton's Method for Optimization



$f(x)$

$g(x) = e^x - \log(x)$ | $g'(x) = e^x - 1/x$

Minimum: $x^* = 0.5671$

$(g'(x))' = e^x + \frac{1}{x^2}$

$f'(x)$

I want to recall this function you saw
in Week 2 when you learn about gradient

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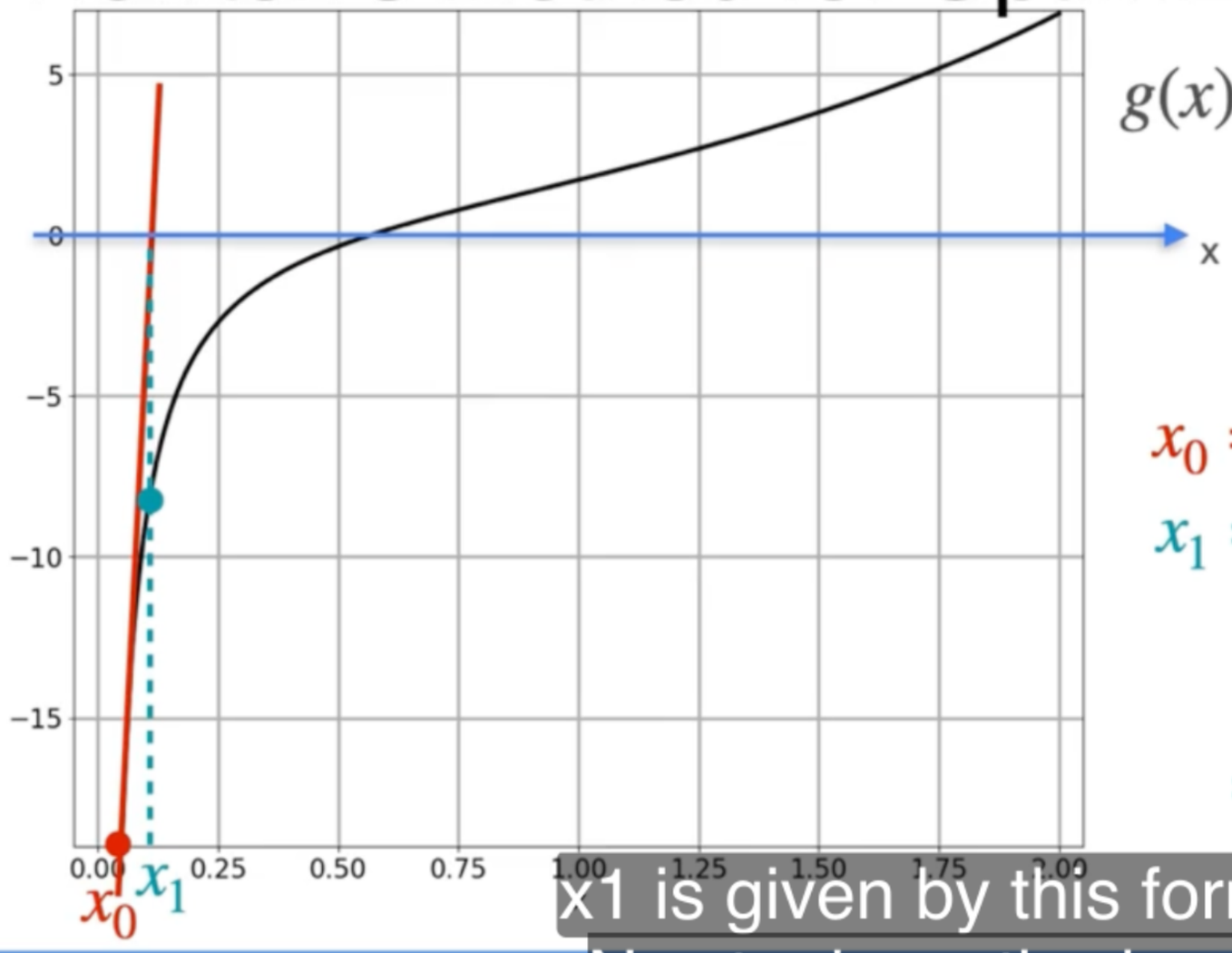
Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_0 = 0.05 \qquad \overbrace{\quad}^{f'(x)}$$

$$x_1 = x_0 - \frac{g'(x_0)}{(g'(x_0))'}$$

$$= 0.05 - \frac{\left(e^{0.05} - \frac{1}{0.05}\right)}{\left(e^{0.05} + \frac{1}{0.05^2}\right)} \Rightarrow 0.097$$

x1 is given by this formula using Newton's method we evaluate

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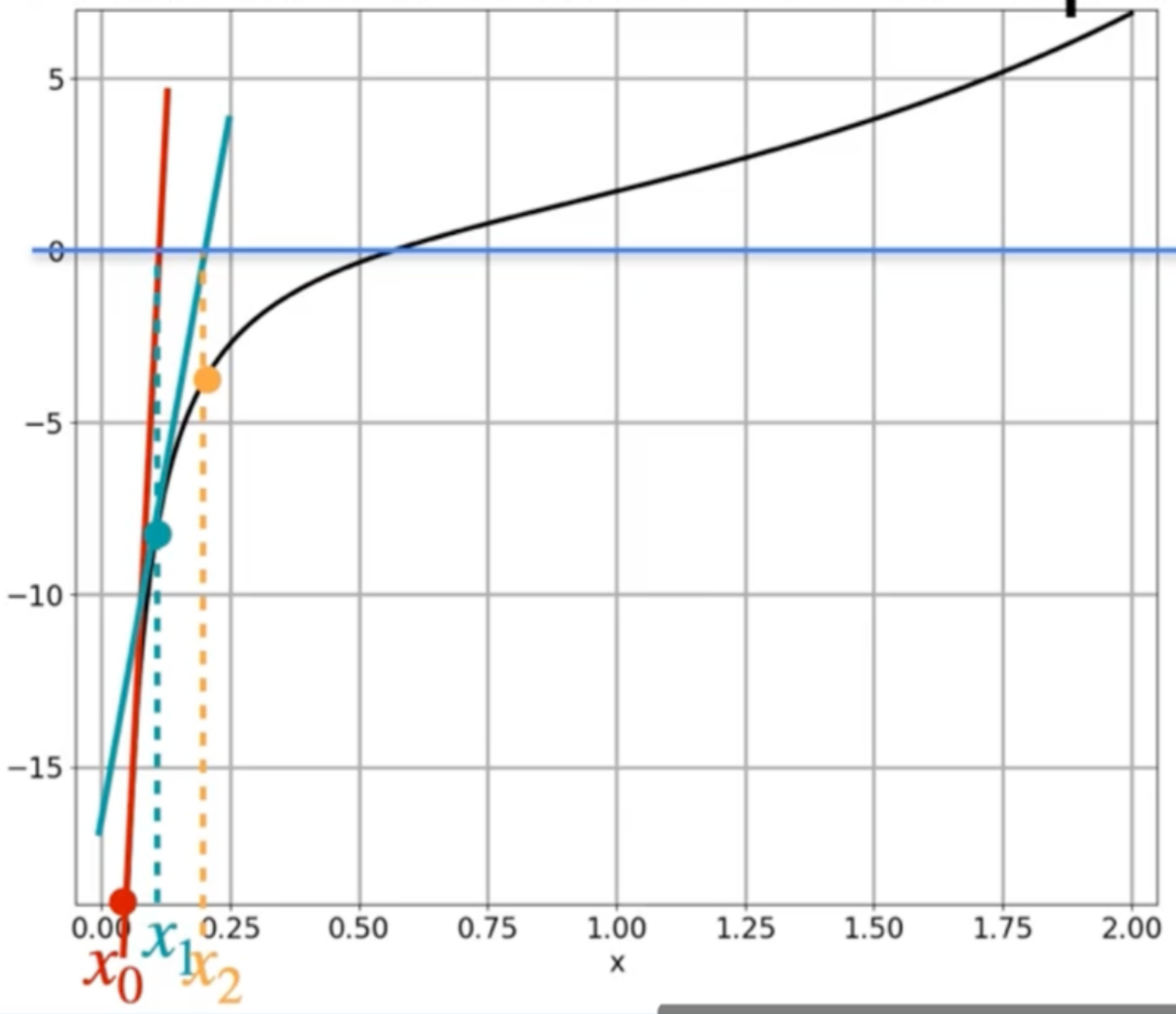
Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_1 = 0.097$$

$$x_2 = x_1 - \frac{g'(x_1)}{(g'(x_1))'}$$

$$= 0.097 - \frac{\left(e^{0.097} - \frac{1}{0.097}\right)}{\left(e^{0.097} + \frac{1}{0.097^2}\right)} = 0.183$$

By plugging in these values.

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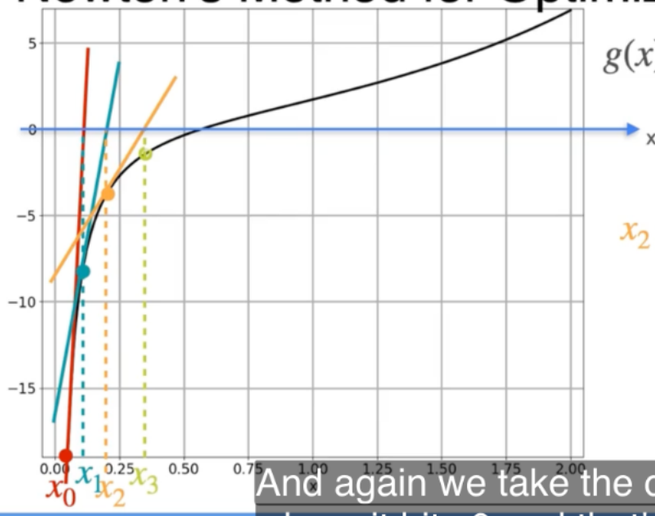
Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_2 = 0.183$

And again we take the derivative, see where it hits 0 and that's our point x3.

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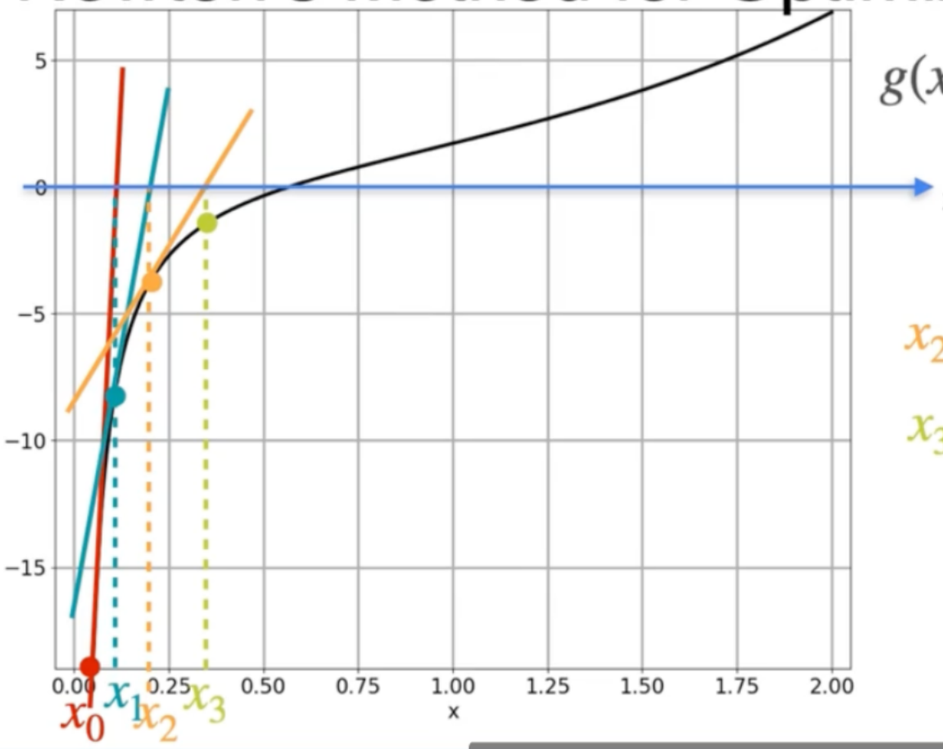
Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

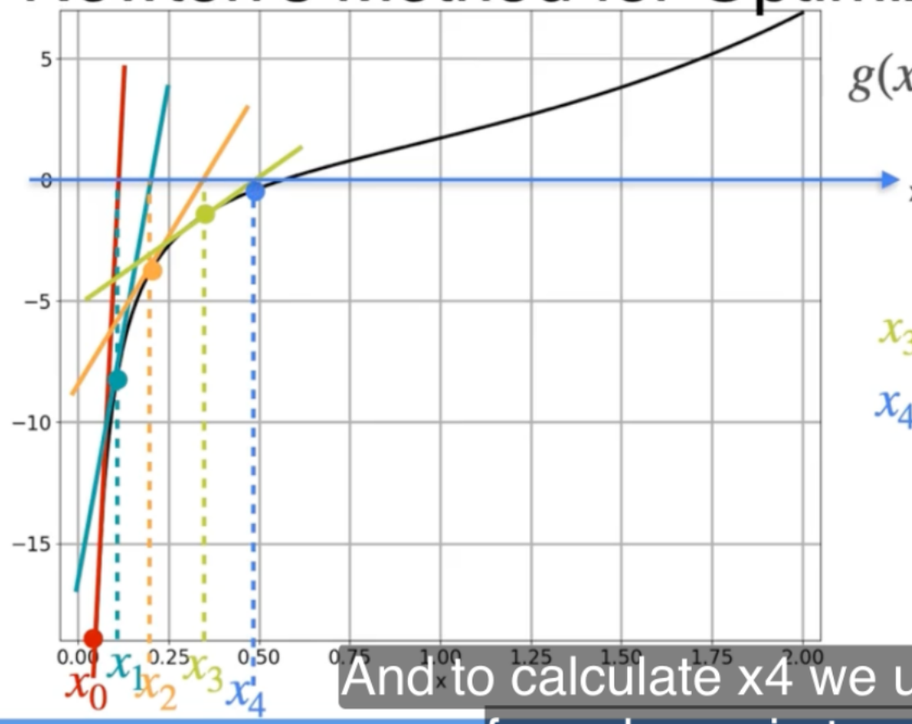
$x_2 = 0.183$

$$x_3 = x_2 - \frac{g'(x_2)}{(g'(x_2))'}$$

$$= 0.183 - \frac{\left(e^{0.183} - \frac{1}{0.183}\right)}{\left(e^{0.183} + \frac{1}{0.183^2}\right)} = 0.320$$

And the value for x3 is 0.320.

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

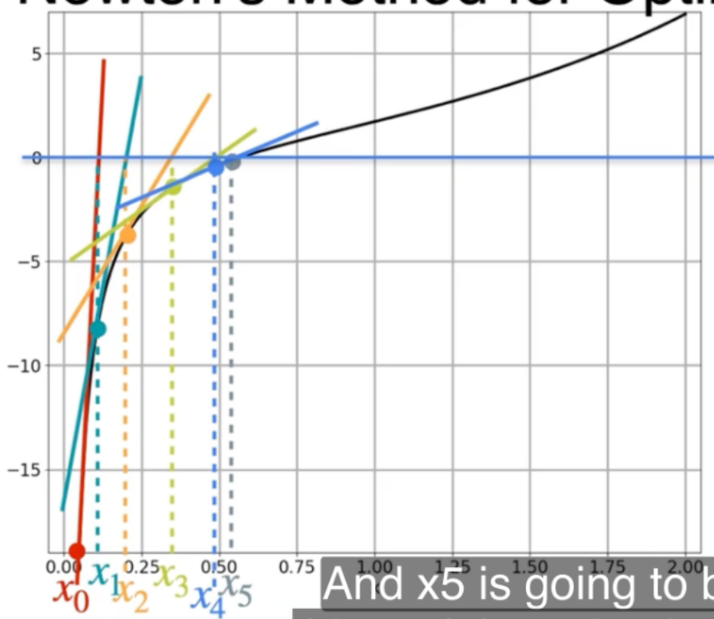
$x_3 = 0.320$

$$x_4 = x_3 - \frac{g'(x_3)}{(g'(x_3))'}$$

$$= 0.320 - \frac{\left(e^{0.320} - \frac{1}{0.320}\right)}{\left(e^{0.320} + \frac{1}{0.320^2}\right)} = 0.477$$

And to calculate x4 we use the iterative formula again to get 0.477

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

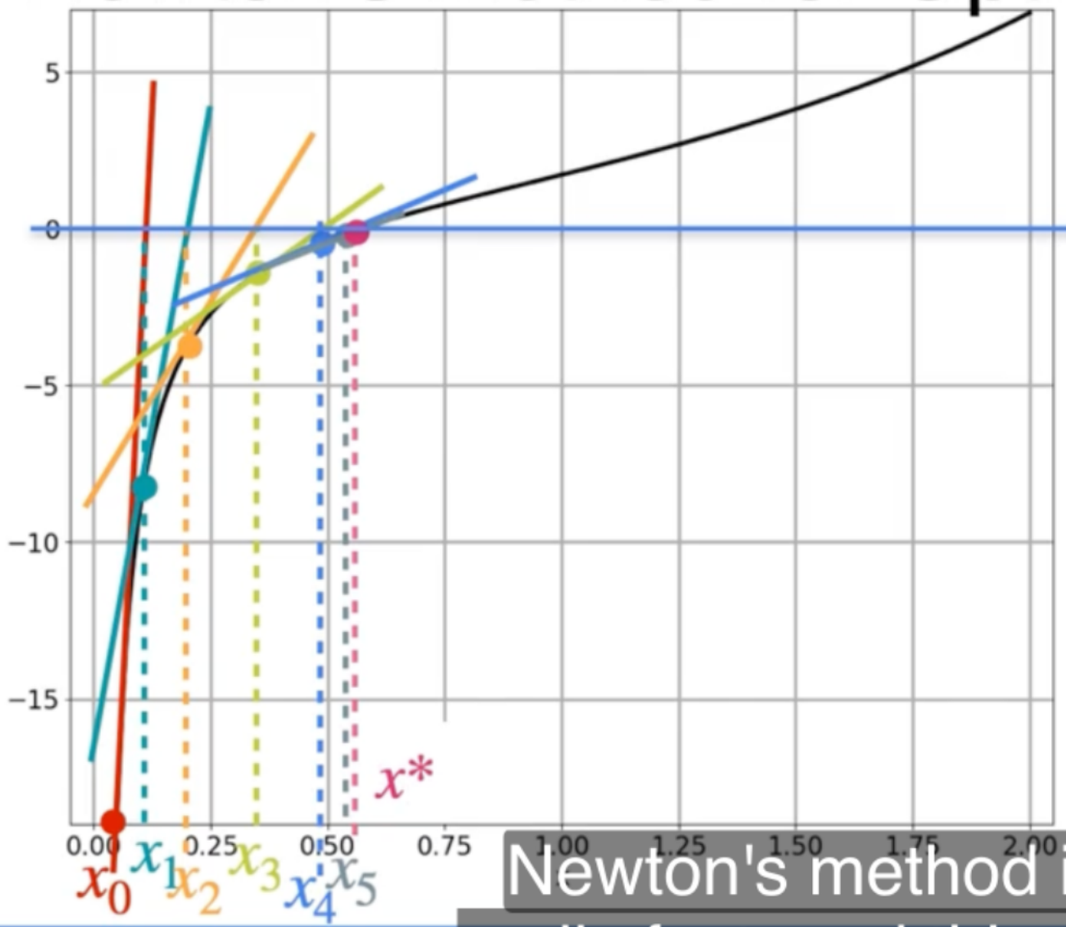
$x_4 = 0.477$

$$x_5 = x_4 - \frac{g'(x_4)}{(g'(x_4))'}$$

$$= 0.447 - \frac{\left(e^{0.447} - \frac{1}{0.447}\right)}{\left(e^{0.447} + \frac{1}{0.447^2}\right)} = 0.558$$

And x5 is going to be calculated like this and the value is going to be 0.558.

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_5 = 0.558$$

$$x^* = x_5 - \frac{g'(x_5)}{(g'(x_5))'}$$

$$= 0.558 - \frac{\left(e^{0.558} - \frac{1}{0.558}\right)}{\left(e^{0.558} + \frac{1}{0.558^2}\right)} = 0.567$$

Newton's method is actually really, really fast, and this example shows it.

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