Day-20, Dec-5, 2024 (Mangshir-20, 2081 BS)

Derivative:

Differential (Changes) calculus is a theory which has its origin in the solution of two old problems.

① Drawing a tangent line to a curve

② Calculating the velocity of non-uniform motion of a particle.

Both problems are continuous functions. But Derivatives can be interpreted as slopes and rate of change. Developing rules for finding derivatives and these differentiation rules enables us to solve different types of functions/problems.

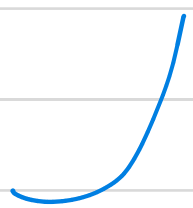
Tangent and Velocity :

→ tangent word originates from latin 'tangens' which means touching.

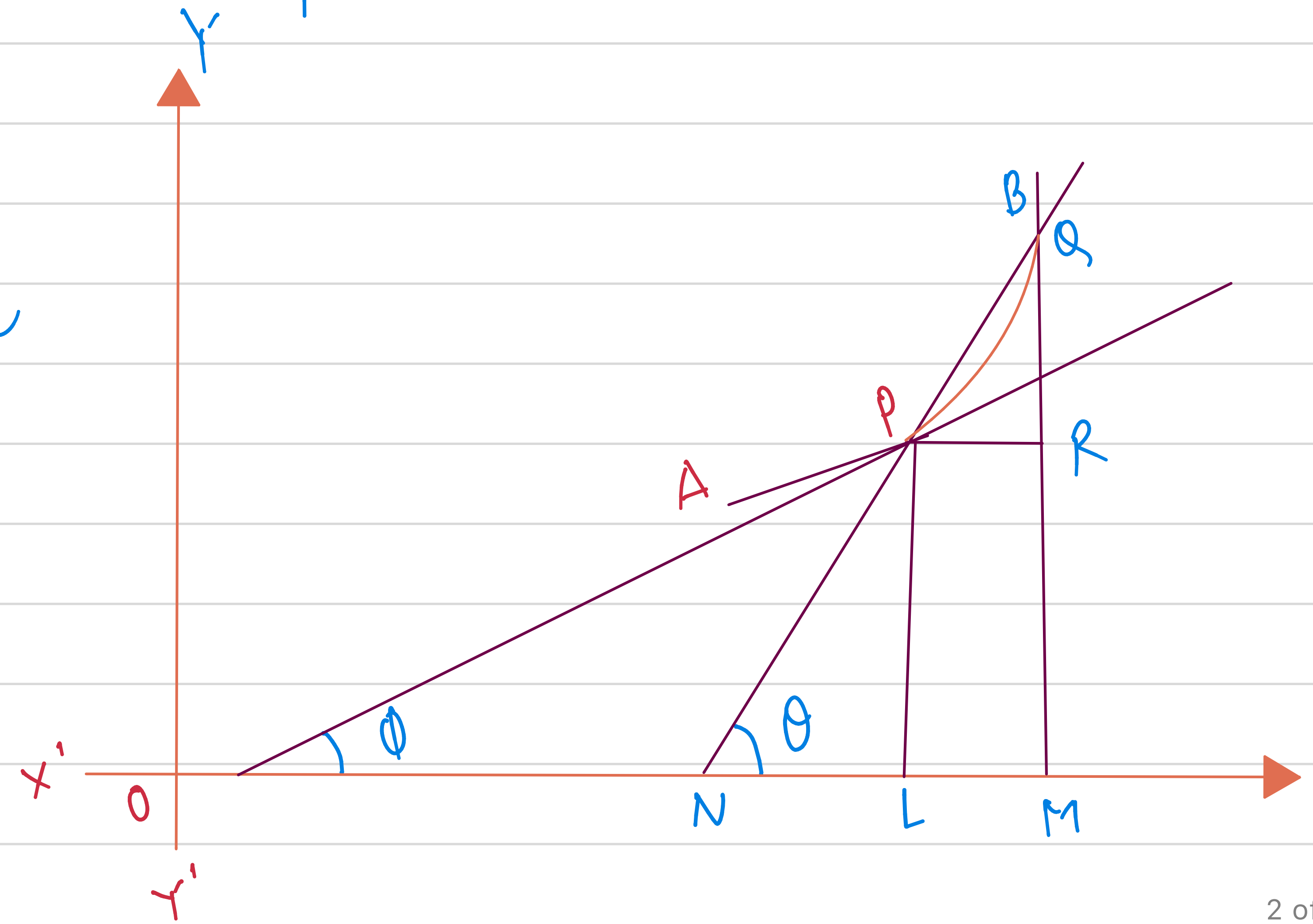
Tangent: A line that touches the curve at a point.

Velocity Rate of change in Speed/motion with respect to time.

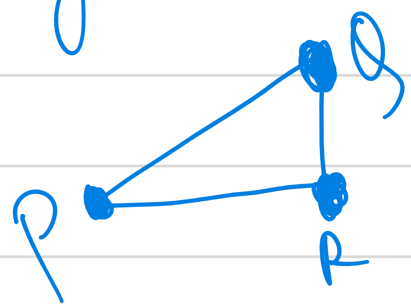
Tangent line to a Curve

Let AB  be a Continuous curve given by $y = f(x)$ and P, Q be any two points in it.

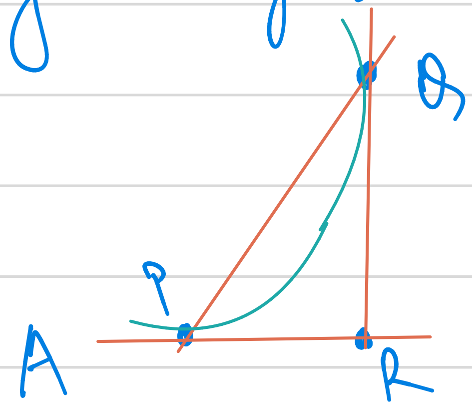
Let the co-ordinates of P and Q be (x, y) and (x', y'). When a point



moves along the curve from the point P to the point Q, it moves horizontally through the distance PR and vertically through the distance RQ.



$(P, Q) = (x, y)$



So, $PR = LM$

$\Rightarrow OM - OL$

$$\boxed{PR = x' - x}$$

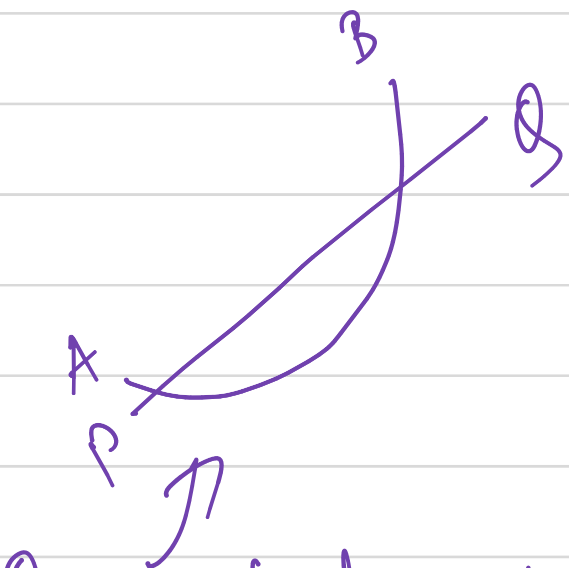
$RQ = QM - RM$

$\Rightarrow y' - PL$

$$\boxed{RQ = y' - y}$$

These quantities $x'-x$ and $y'-y$ are called the increments in x and y respectively and are denoted by Δx and Δy ie.

$$\Delta x = x'-x \qquad \text{and} \qquad \Delta y = y'-y$$



Also, $\Delta y = f(x') - f(x)$

$$\Rightarrow f(x+\Delta x) - f(x).$$

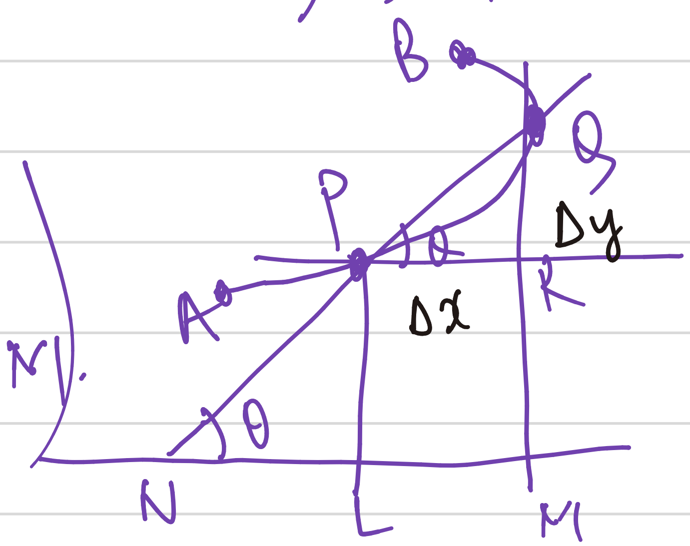
If we join the points P and Q, we get Secant PQ which makes an angle θ with the x-axis ie. $\angle QNM = \theta$, so, $\angle QPR = \angle QNM = \theta$

$$\tan\theta = \frac{QR}{PR}$$

$$\Rightarrow \frac{\Delta y}{\Delta x}$$

($\angle QNM = \theta$

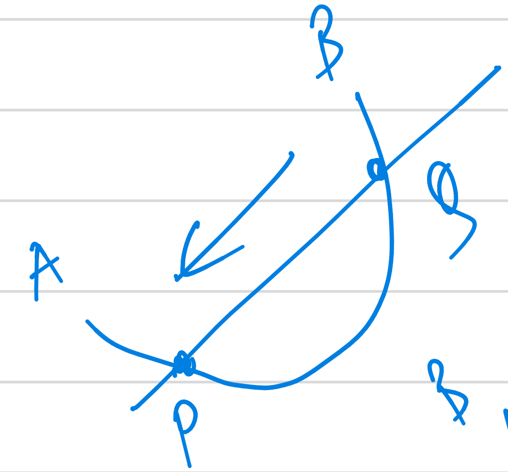
$\angle QPR = \angle QNM$.)



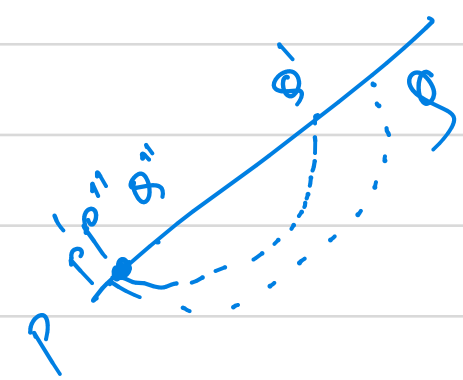
$$\tan\theta = \frac{QR}{PR}$$

$$\Rightarrow \frac{\Delta y}{\Delta x}$$

which is the slope of the secant PQ. As Q moves along the curve and approaches P, the secant rotates about P.



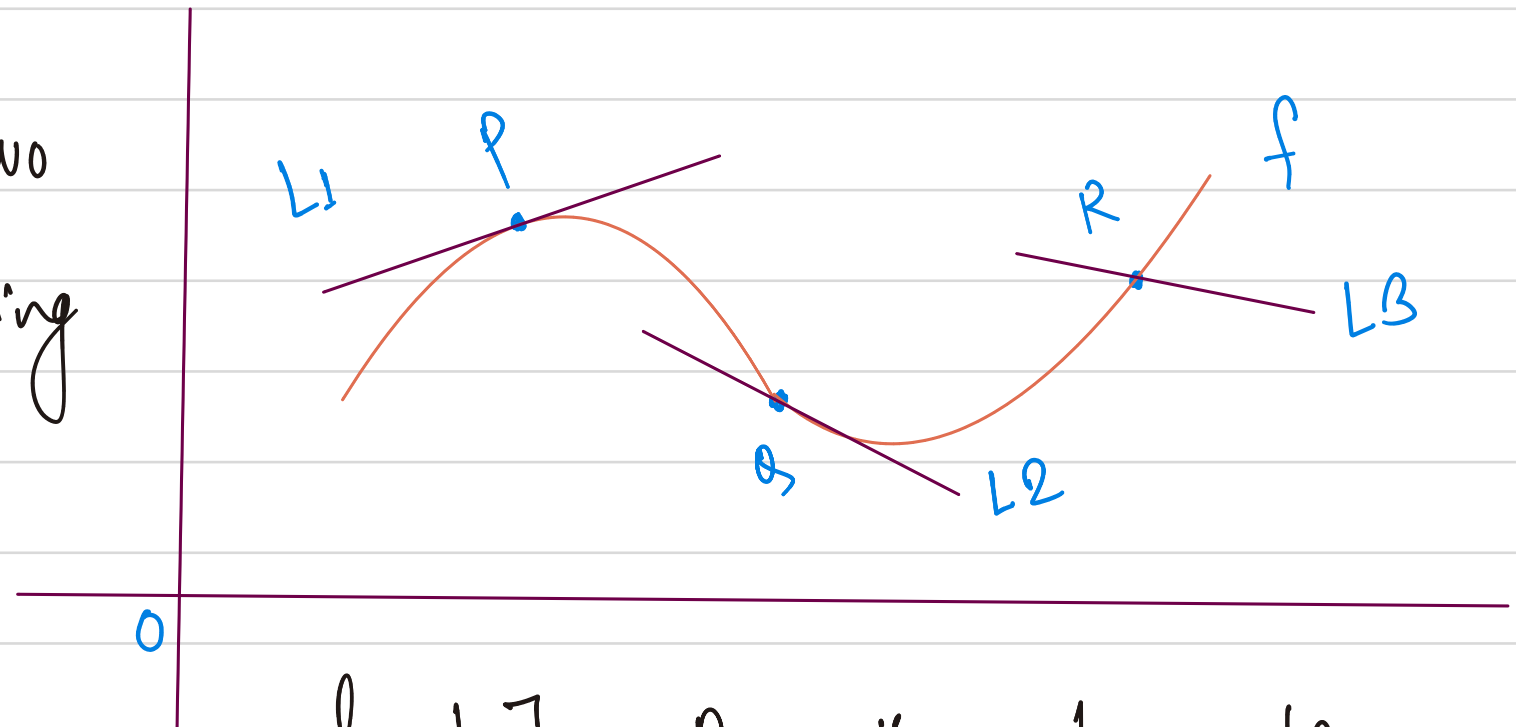
The limiting position of the secant, when Q ultimately coincides with P, is the tangent of P, making the angle of θ with the x-axis. In the situation $\Delta x, \Delta y$ tend to zero. So,



$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \tan\theta = \tan\phi$$

or, $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \tan\phi$

Thus, $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x}$ or $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ gives the slope of a tangent to the curve given by the function f.



In the above figure $L1$ and $L2$ are two straight lines that are passing by touching the curve f at a single point 'P' and 'Q' respectively, therefore $L1$ and $L2$ are tangents to f [Straight line, measure we get 1]. But the line $L3$ intersects to f at R, so it is not a tangent to f.