Day-81, Feb-19, 2025 (Falgun 08, 2081)

1. Derivative as trigonometric functions
2. Meaning of exponential (e).
3. Derivative of $\log(x)$
4. Existence of the derivative (differentiable vs non-differentiable function)
5. Properties of the dx: The Sum Rule
6. Product Rule
7. Chain Rule.

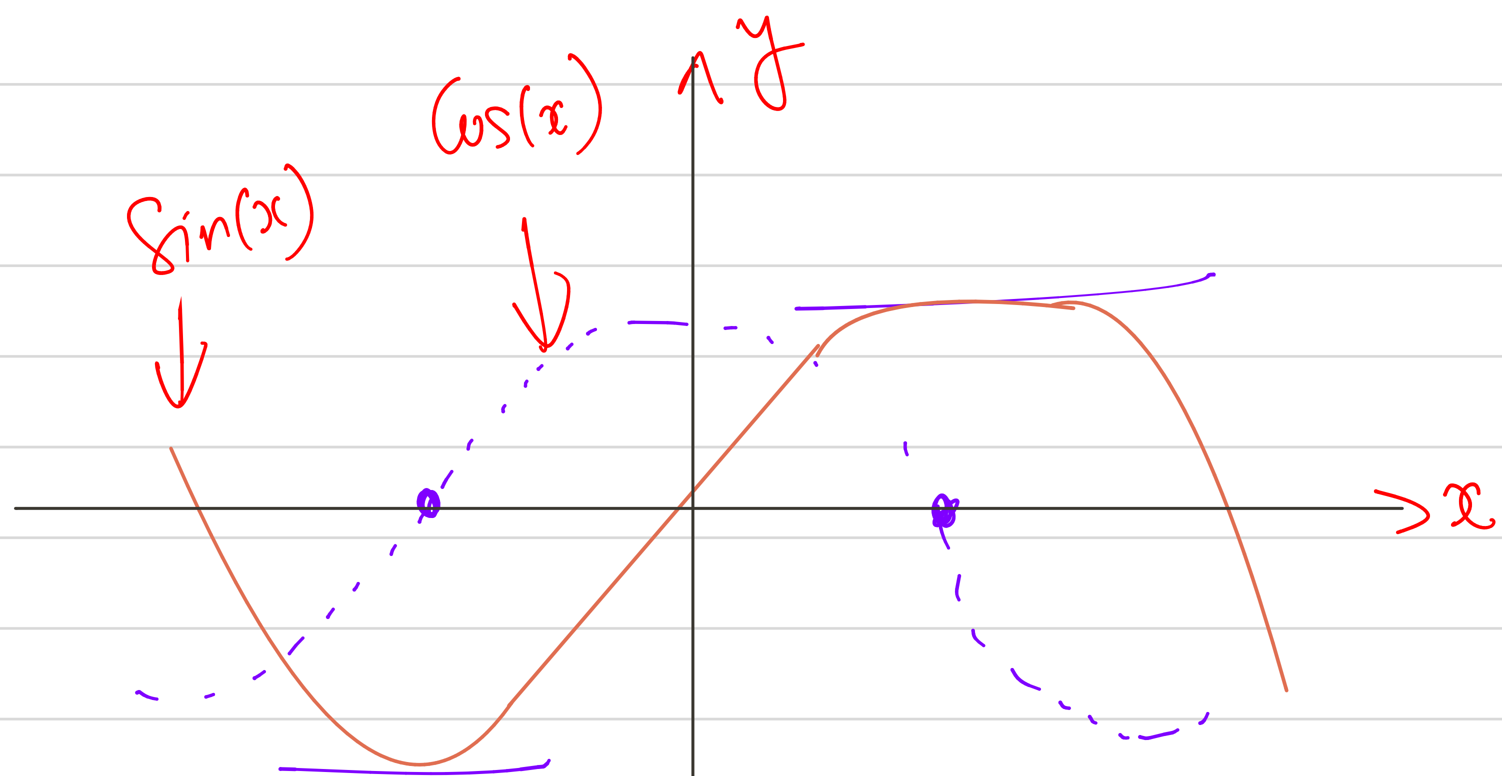
Day-81, Feb-19, 2025 (Falgun 8, 2081)

Derivative of Trigonometric Functions

Sine $y = f(x)$

$= \sin(x)$

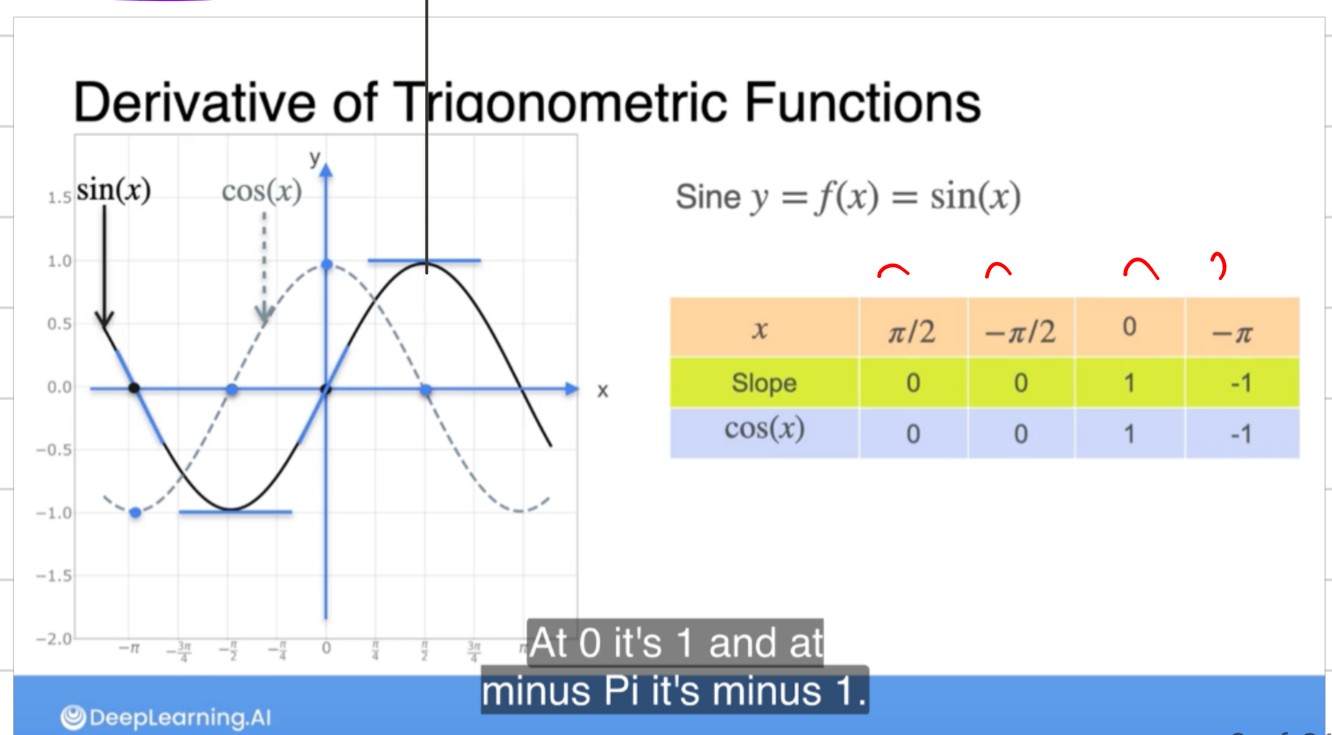
$$\begin{bmatrix} f(x) = \sin(x) \\ f'(x) = \cos(x) \end{bmatrix}$$



$y = \cos x$.

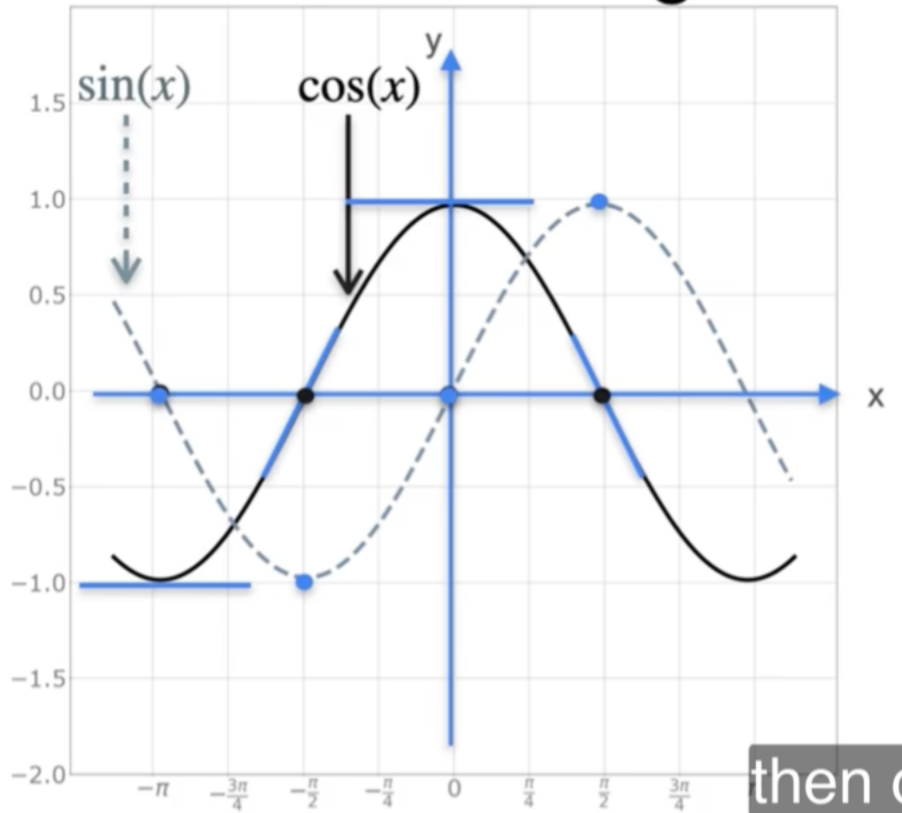
$$\begin{bmatrix} f'(x) = -\sin x \end{bmatrix}$$

$$\begin{bmatrix} f(x) = \cos(x) \\ f'(x) = -\sin(x) \end{bmatrix}$$



x	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0	0	1	-1
$\cos(x)$	0	0	1	-1

Derivative of Trigonometric Functions



Cosine $y = f(x) = \cos(x)$

x	0	$-\pi$	$\pi/2$	$-\pi/2$
Slope	0	0	-1	1
$\sin(x)$	0	0	1	-1

$$f(x) = \cos(x) \Rightarrow f'(x) = -\sin(x)$$

then derivative
negative sine of

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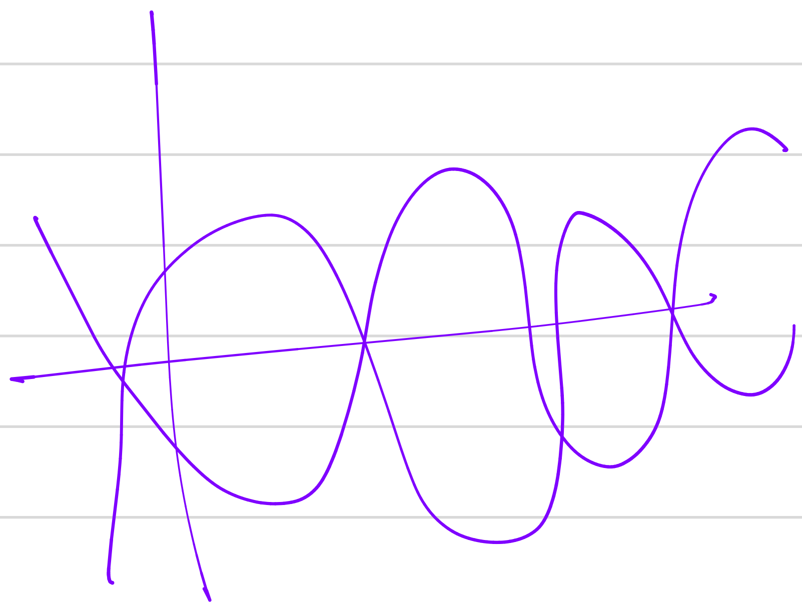
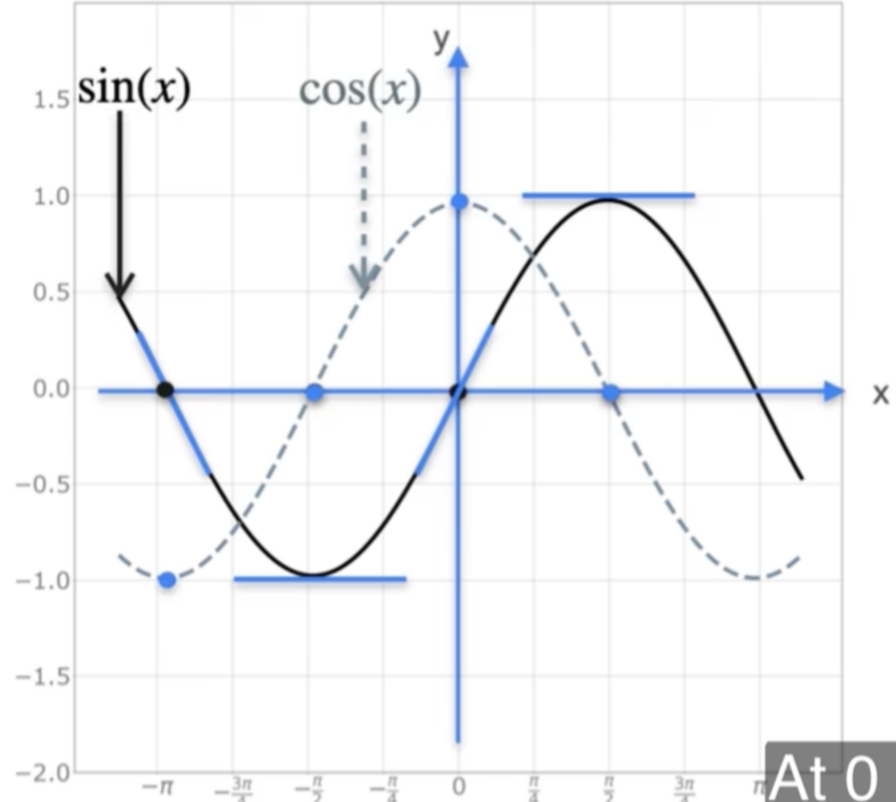


Derivative of Trigonometric Functions



Sine $y = f(x) = \sin(x)$

x	$\pi/2$	$-\pi/2$	0	$-\pi$
Slope	0	0	1	-1
$\cos(x)$	0	0	1	-1

At 0 it's 1 and at
minus Pi it's minus 1.

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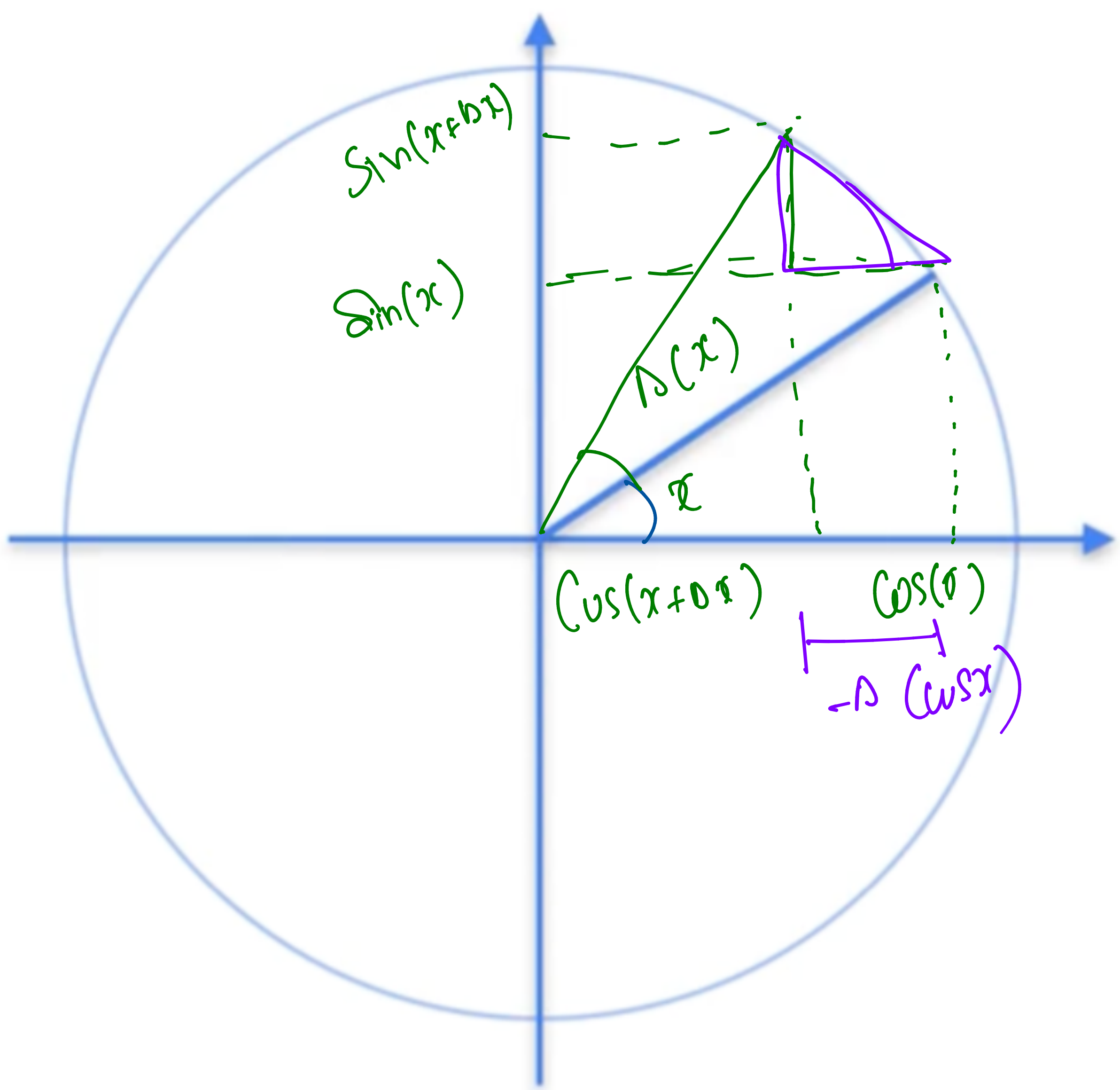
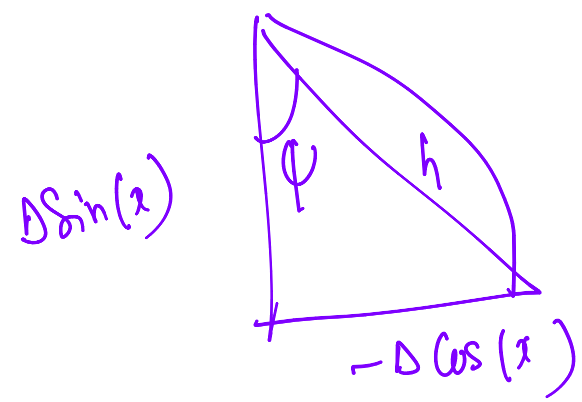
Derivative of Trigonometric Functions



$$\Delta(\sin x) = h\cos(\psi)$$
$$-\Delta(\cos x) = h\sin(\psi)$$



$$\cos\psi = \frac{adj}{hyp} \Rightarrow \frac{\Delta \sin(x)}{h}$$

$$\sin(\psi) = \frac{opp}{hyp} \Rightarrow \frac{-\Delta(\cos x)}{h}$$

any radial line
and x is the angle

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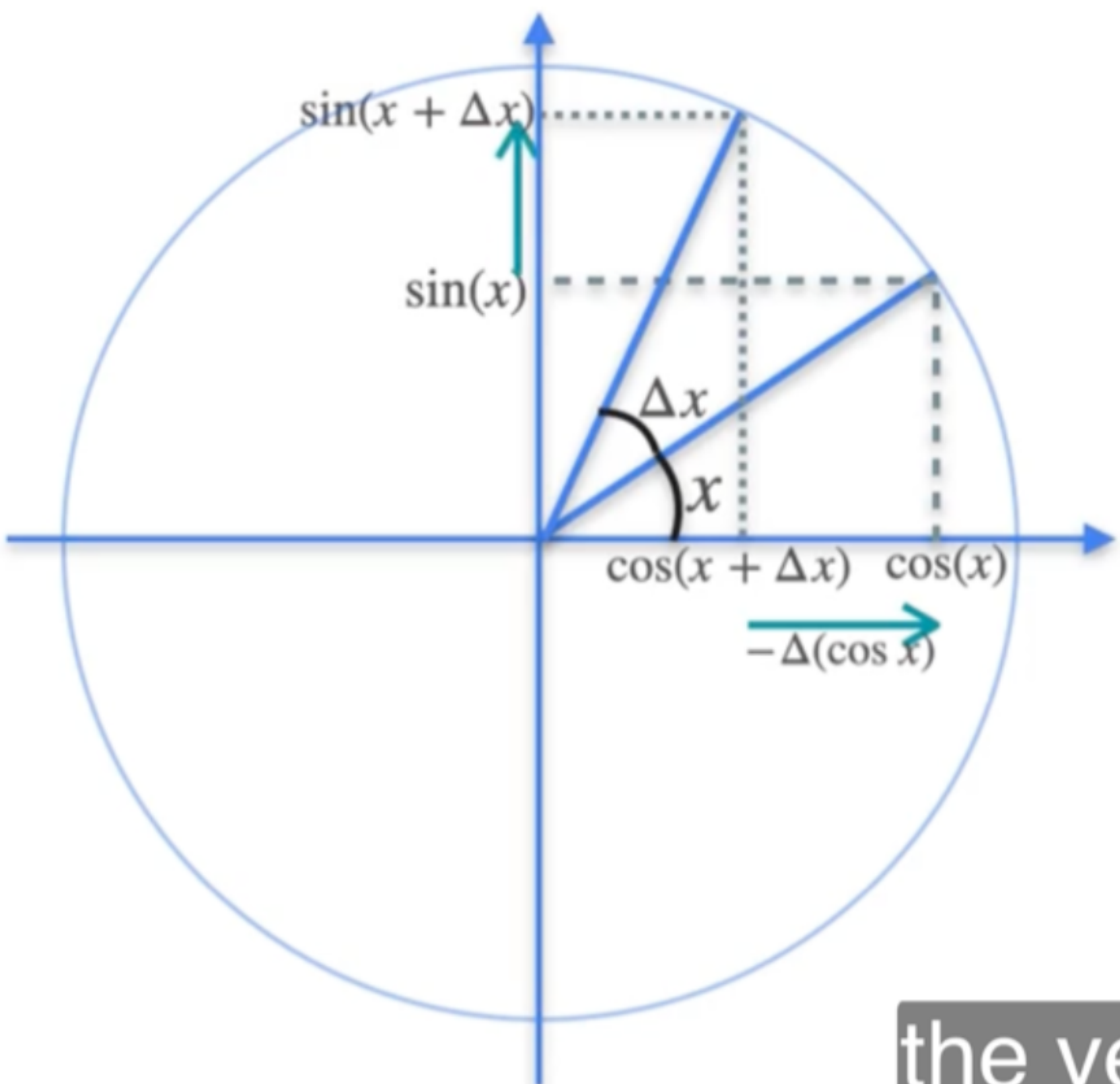
Derivative of Trigonometric Functions



the vertical axis is going to
be sine of x plus Delta x.

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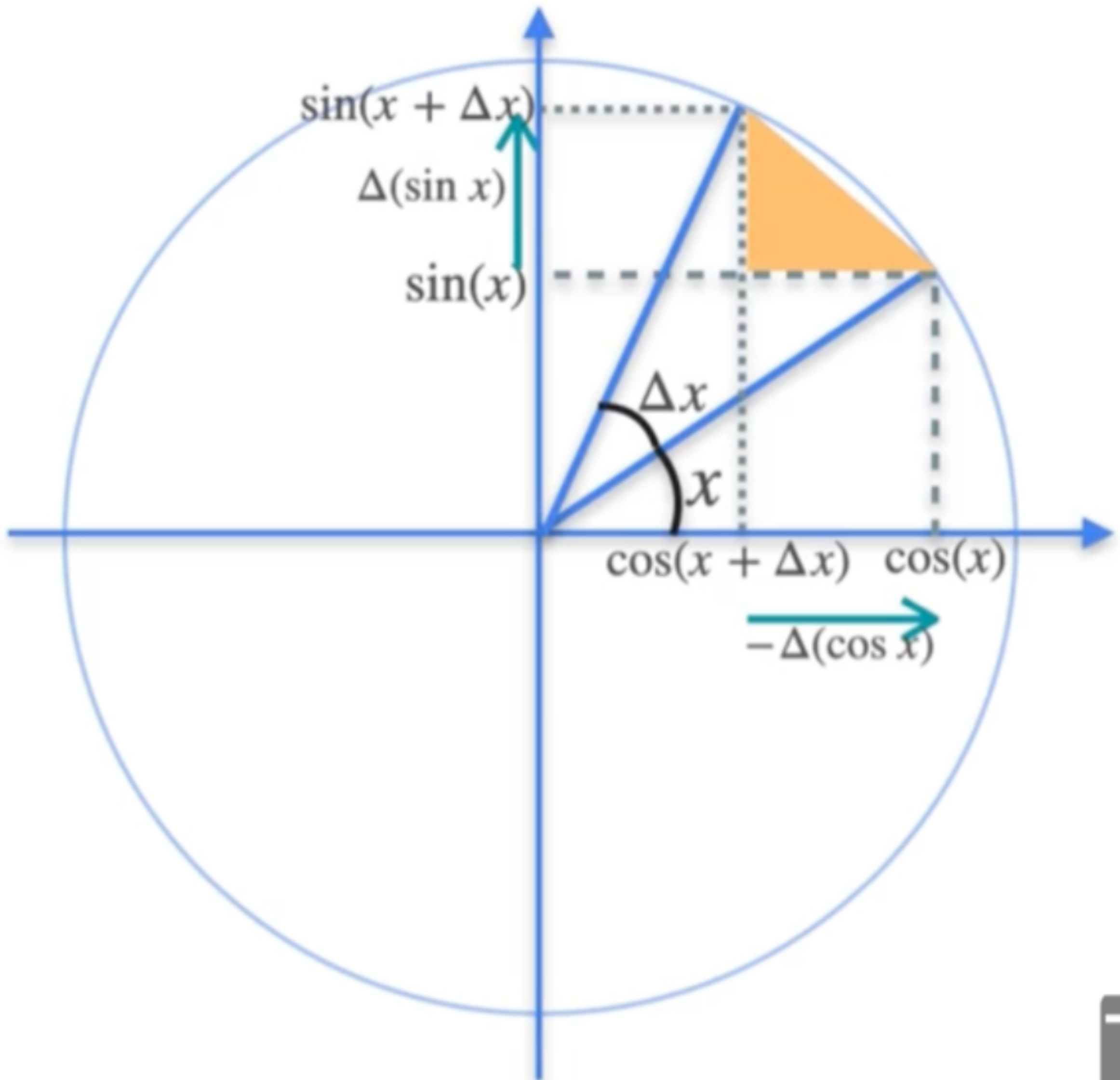
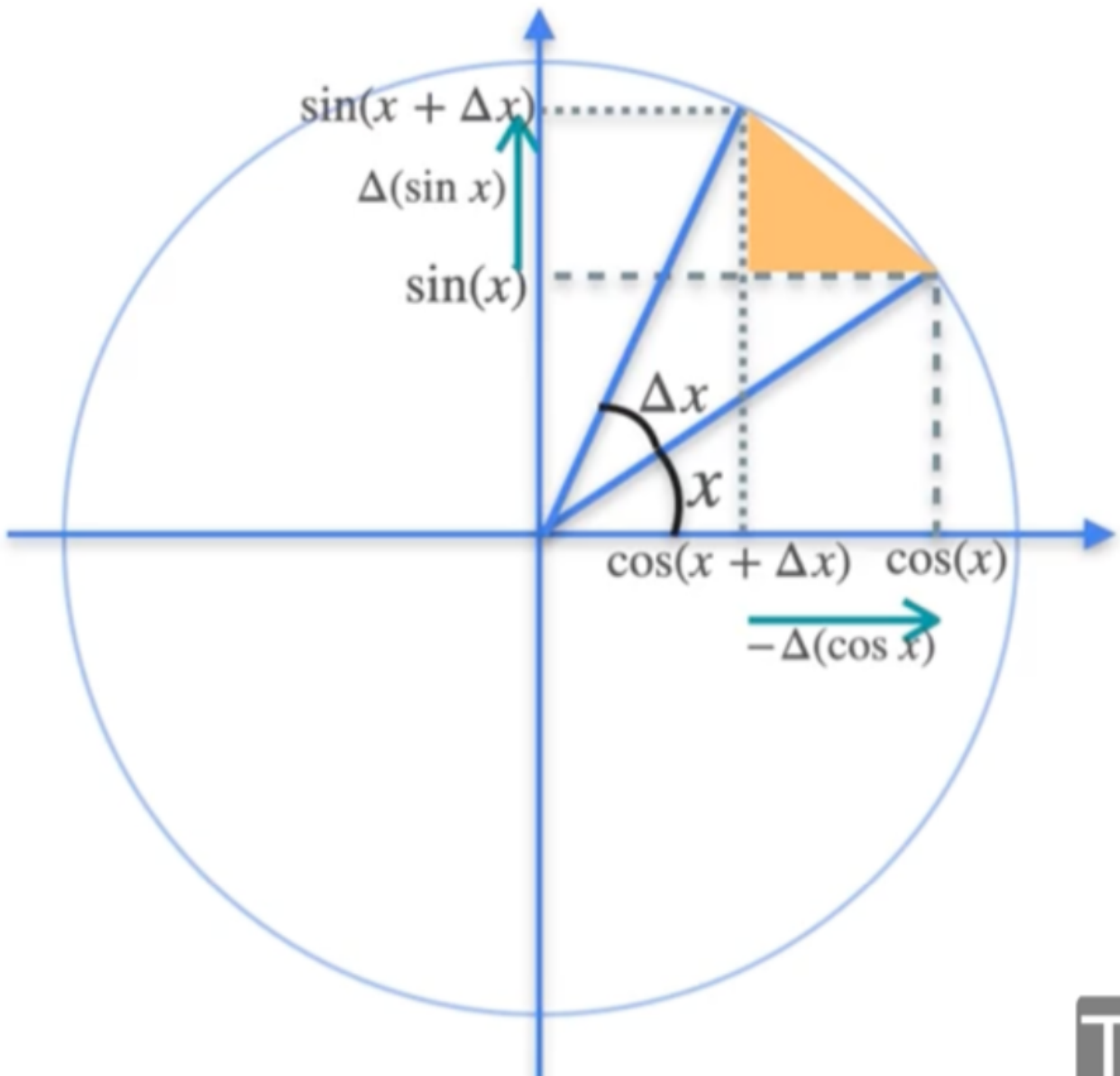
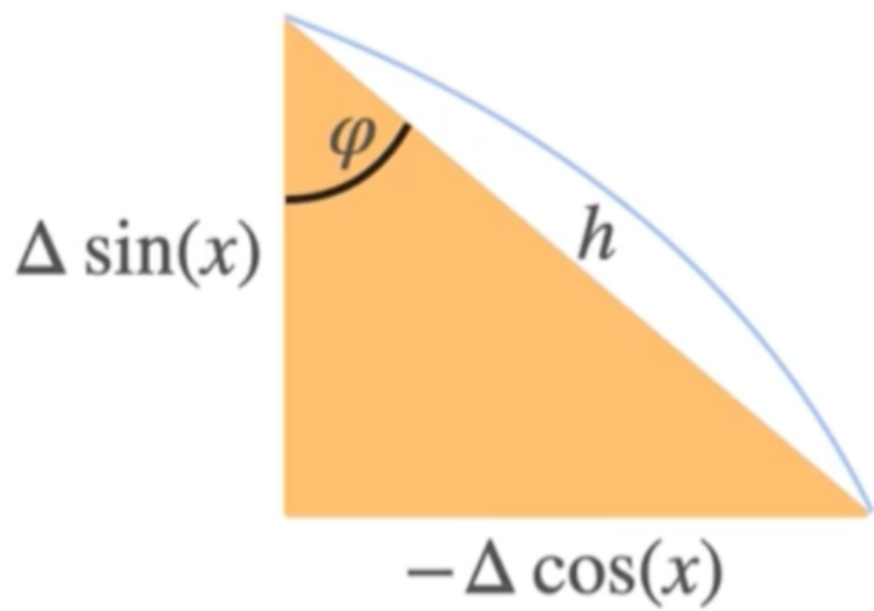
Derivative of Trigonometric Functions



This triangle has
one base equal to

Derivative of Trigonometric Functions





$$\cos(\varphi) = \frac{adj}{hyp} = \frac{\Delta(\sin x)}{h}$$

The adjacent side
here is Delta sine x,

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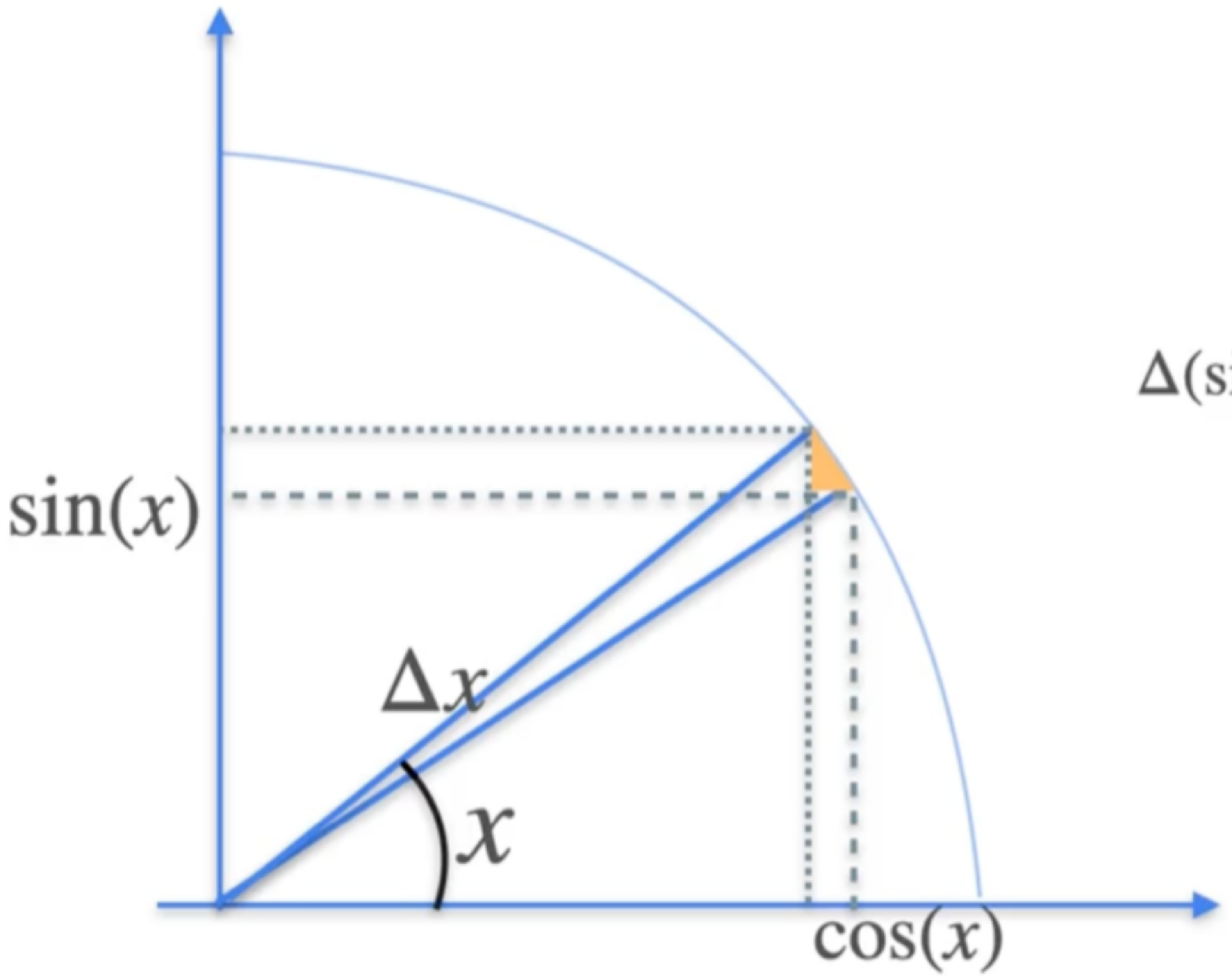
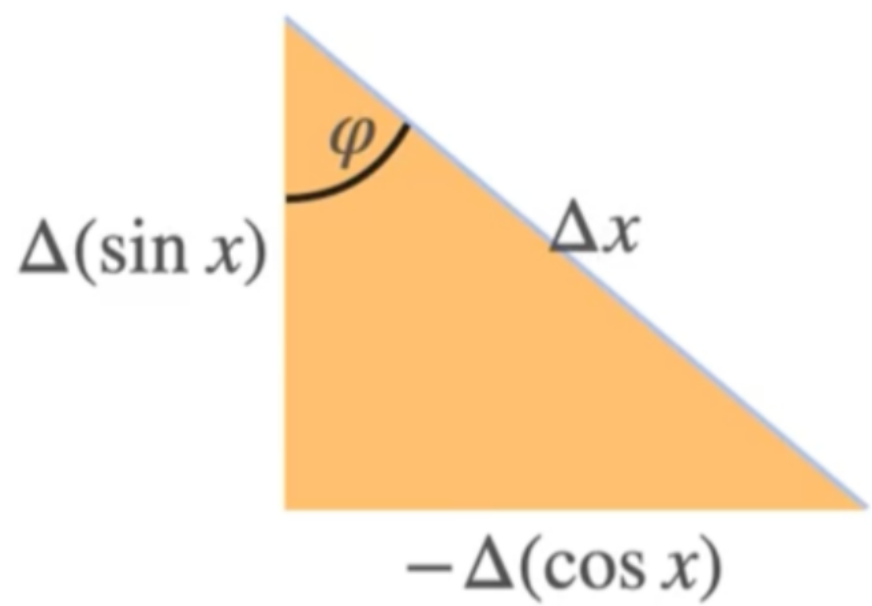
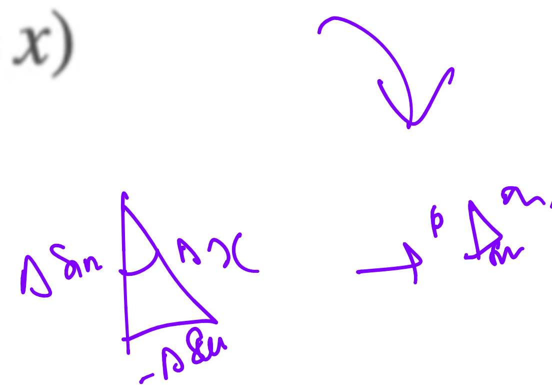
Derivative of Trigonometric Functions





$$\Delta(\sin x) = h\cos(\varphi)$$
$$-\Delta(\cos x) = h\sin(\varphi)$$



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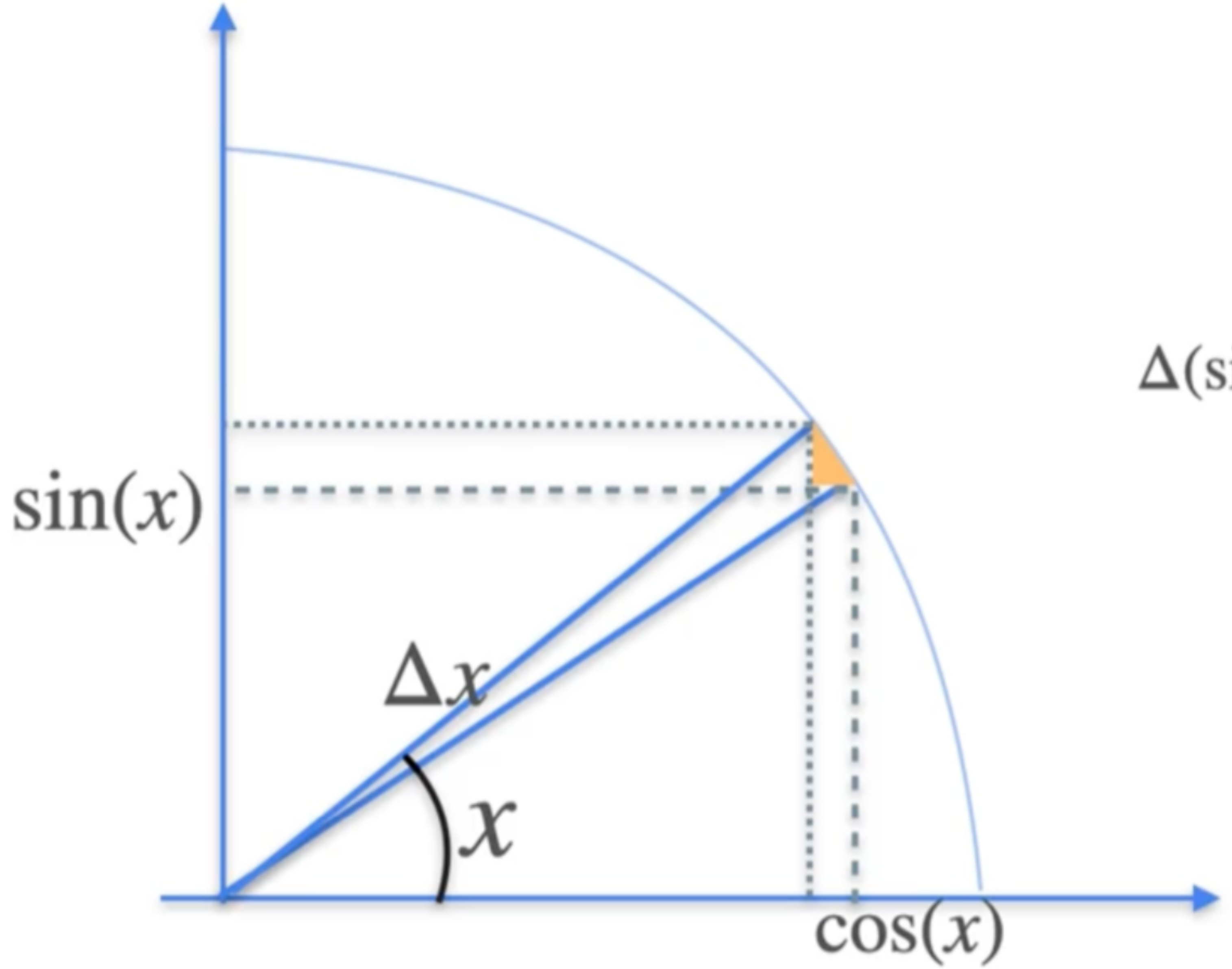
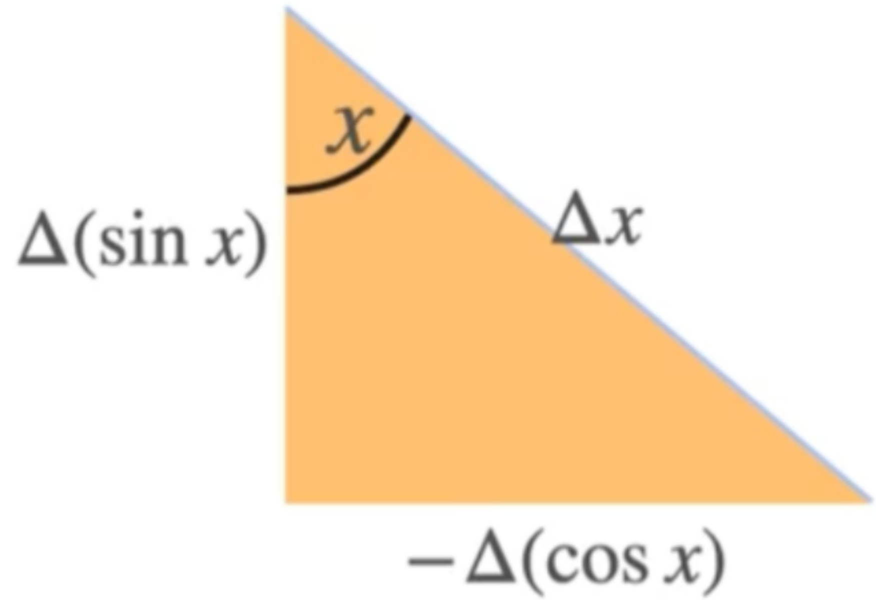
and approaches Delta of x.

Derivative of Trigonometric Functions





$$\Delta(\sin x) = \Delta x \cos(x)$$
$$-\Delta(\cos x) = \Delta x \sin(x)$$

$$f(x) = \sin(x) \Rightarrow f'(x) = \cos(x)$$
$$g(x) = \cos(x) \Rightarrow g'(x) = -\sin(x)$$

Sine prime of x is cos x

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Meaning of the Exponential (e)

$(e = 2.71828182\ldots\ldots)$ → Irrational number

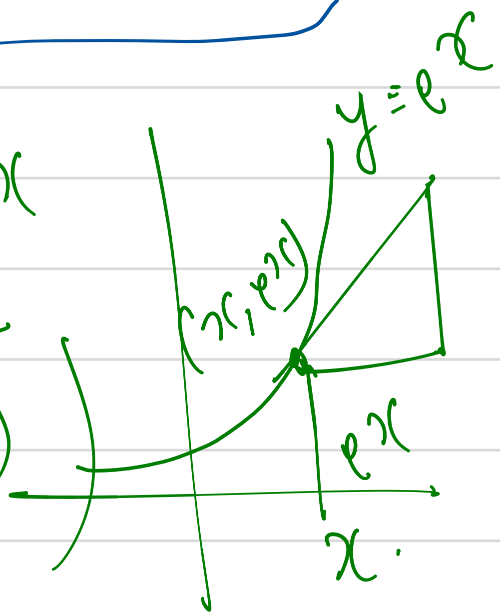
n	1	10	100	1000	.....	∞
$\left(1+\frac{1}{n}\right)^n$	2	2.594	2.705	2.717	.....	e

$e = 2.71828128$

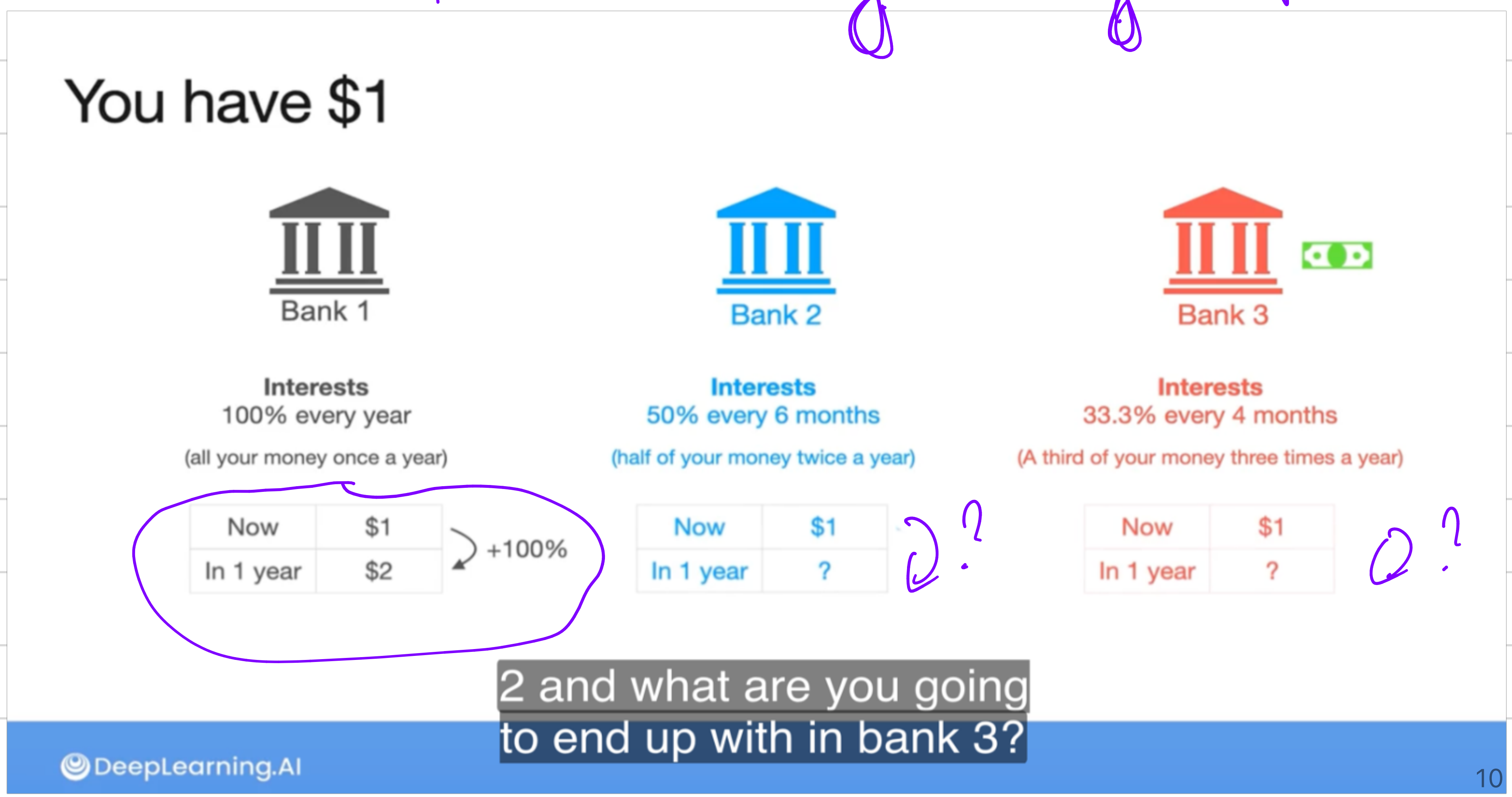
$$\boxed{\begin{aligned} f(x) &= e^x \\ \frac{d}{dx} f(x) &= e^x \end{aligned}}$$

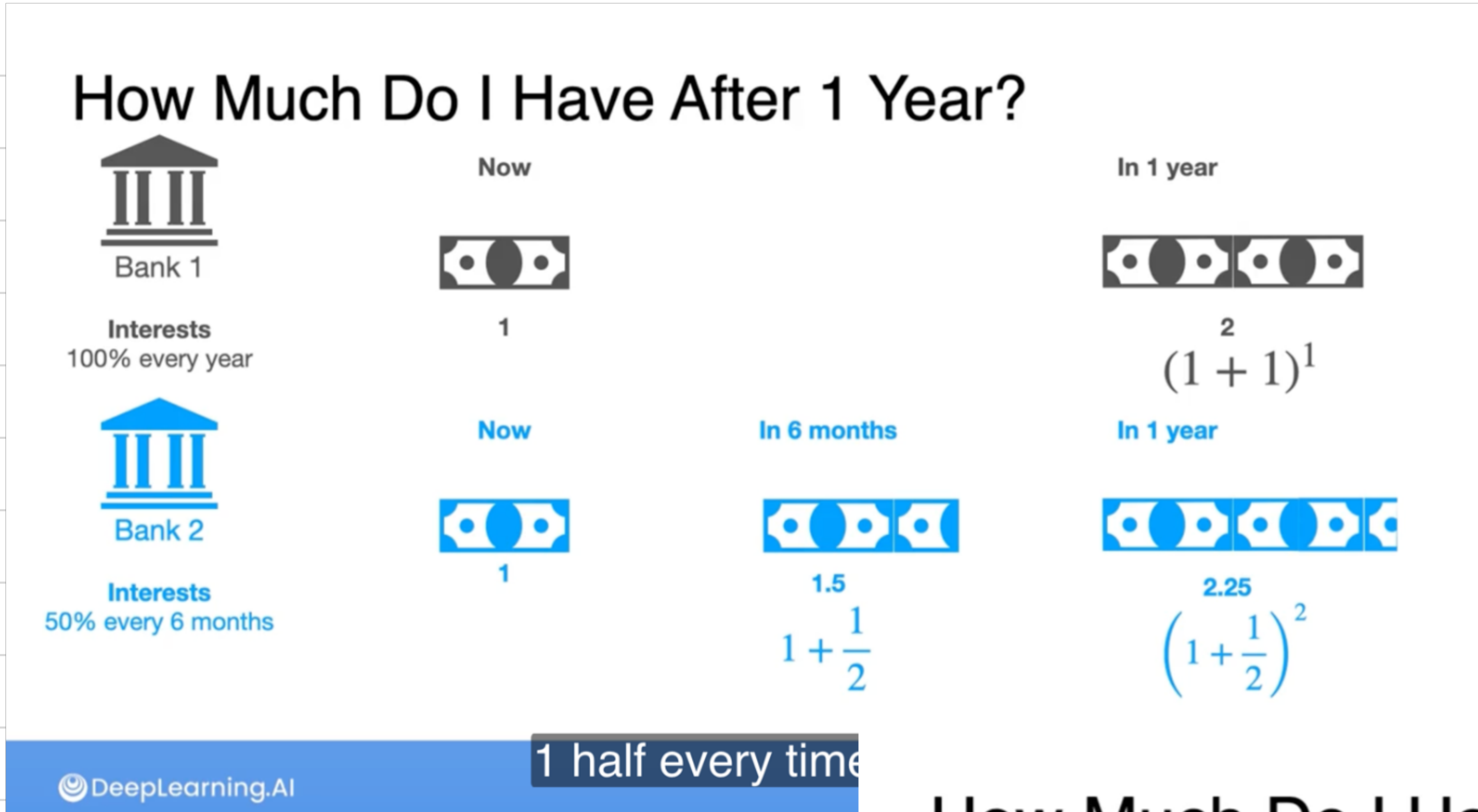
Slope $= \frac{e^x}{1}$

$(f(x) = f'(x))$



Let's understand it using banking Interests:





← 1 year →

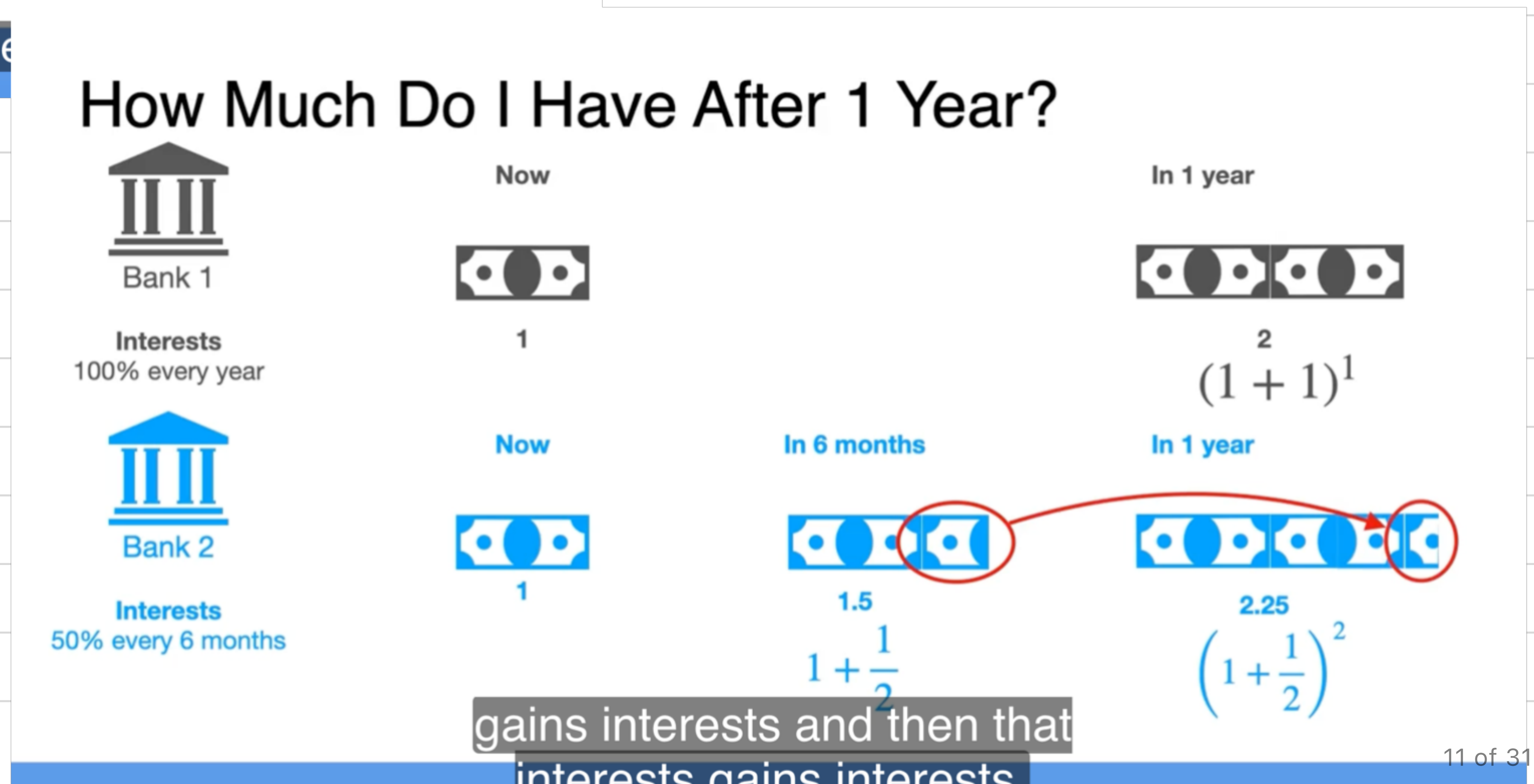
| 1 | 2 | 3 | . . . . . | k | . . . | n-2 | n-1 | n |

$1 \quad 1+1/n \quad (1+1/n)^2$

$(1+1/n) + \frac{1}{n}(1+1/n)$

Money

Interest



How Much Do I Have After 1 Year?

Bank 12

Interests
1/12 every month

Now	In 1 month	In 2 months	...	In 12 months
1	$1+\frac{1}{12}$	$\left(1+\frac{1}{12}\right)^2$	$\cdots$	$\left(1+\frac{1}{12}\right)^{12}$
Original money	Add 1/12 of the money	Add 1/12 of the money		Add 1/12 of the money

At the end of the year: 2.61

In General

Bank n

Interests
1/n every interval

1 year

1	2	3	.	.	.	.	.	.	k	.	.	.	.	.	.	n-2	n-1	n

Now	In 1 interval	In 2 intervals	In 3 intervals	...	In k intervals	...	In 1 year (n intervals)
1	$1+\frac{1}{n}$	$\left(1+\frac{1}{n}\right)^2$	$\left(1+\frac{1}{n}\right)^3$		$\left(1+\frac{1}{n}\right)^k$		$\left(1+\frac{1}{n}\right)^n$

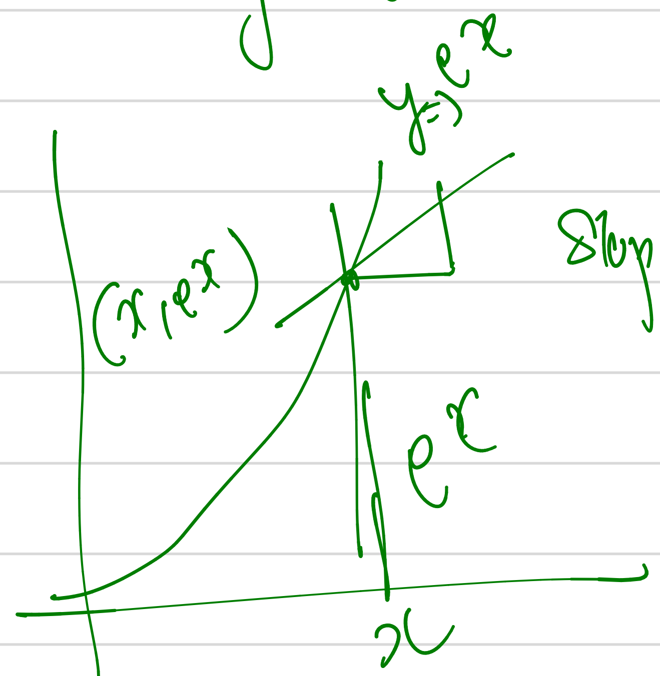
$$\underbrace{\left(1+\frac{1}{n}\right)}_{\text{Money}} + \underbrace{\frac{1}{n}\left(1+\frac{1}{n}\right)}_{\text{Interest}}$$

money every second or every millisecond.

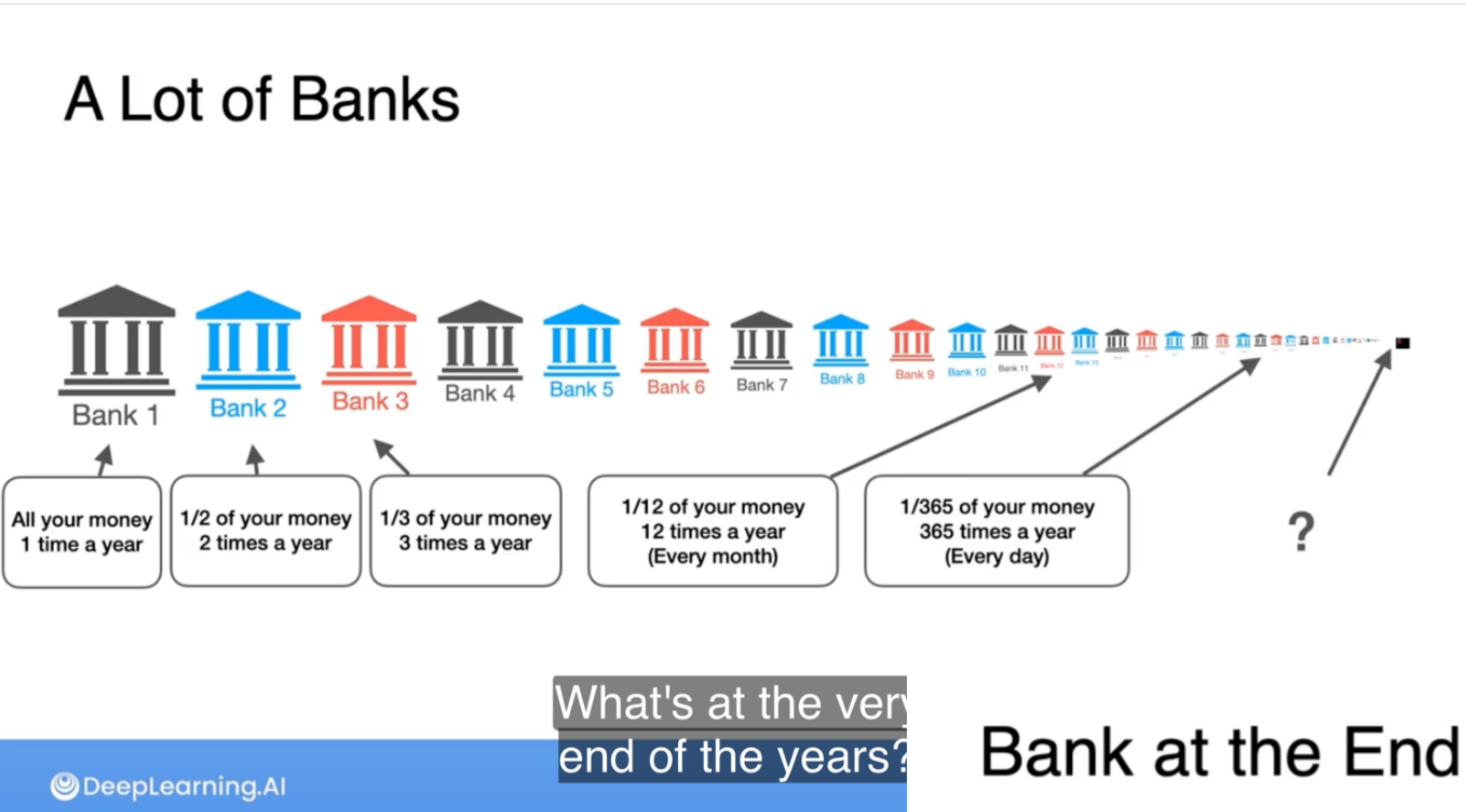
Derivative of e^x is also e^x.

(1) Property one is

$y = e^x$.



$$\text{Slope} \Rightarrow \frac{e^x}{1} \Rightarrow e^x$$



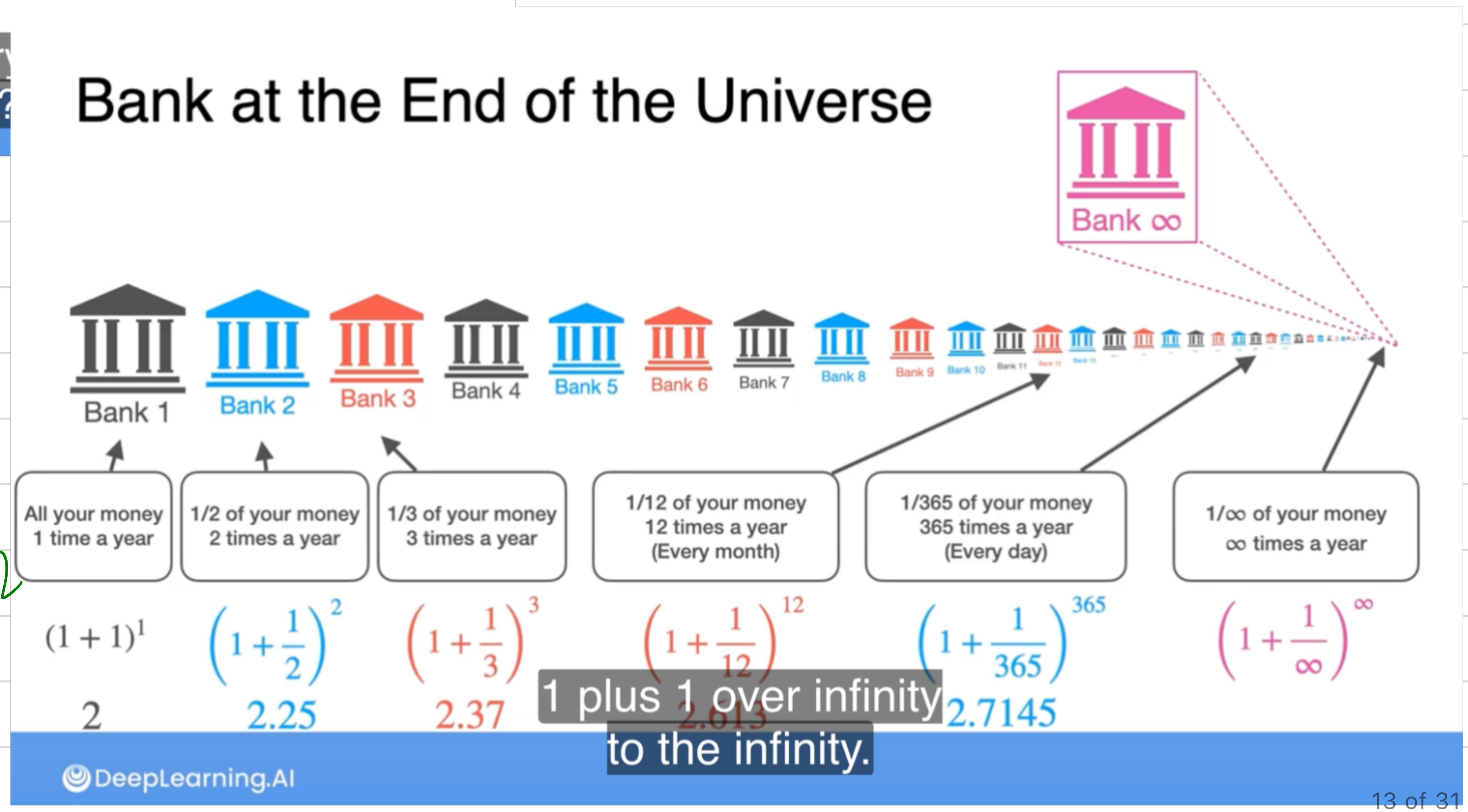
e closer to 2.7182812 . .

So, $\left(1+\frac{1}{\infty}\right)^{\infty}$

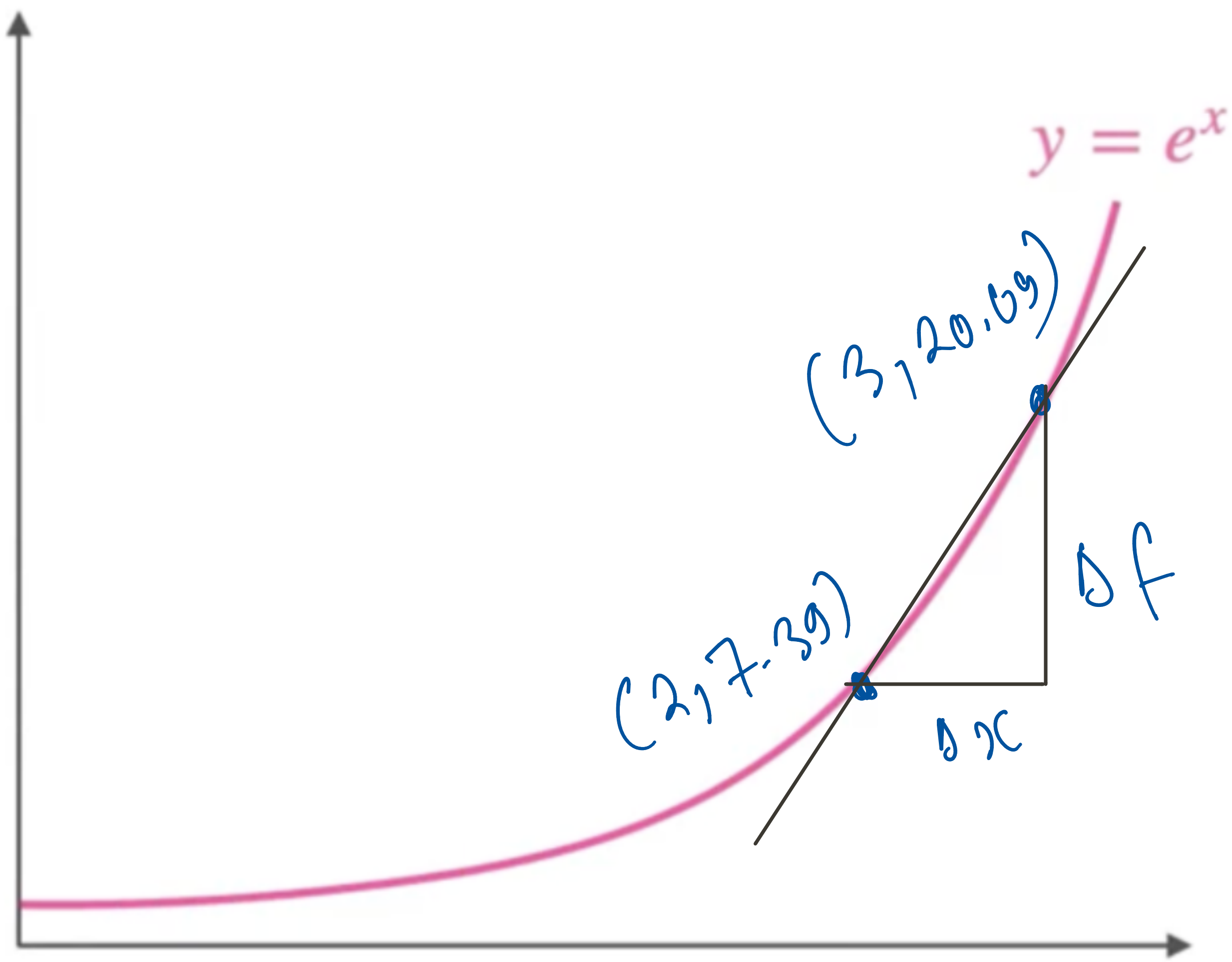
is some number

We keep it 2.71828182

$e = 2.7182812$



Derivative of e^x



Exponential: $y = f(x) = e^x$

Slope $\frac{\Delta f}{\Delta x} \Rightarrow \frac{e^{x+\Delta x} - e^x}{\Delta x}$.

Here is y equals e^x again.

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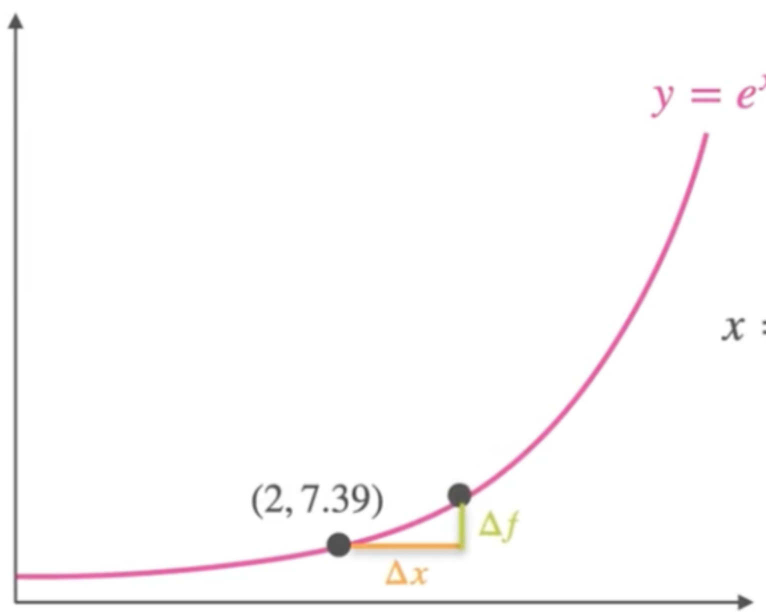
Derivative of e^x



Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$x = 2$

Δx	1.0
Δf	12.70
Slope	12.70

Now let's reduce
interval to length on

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$e^{2+1} - e^{2} \Rightarrow 20.09 - 7.39$

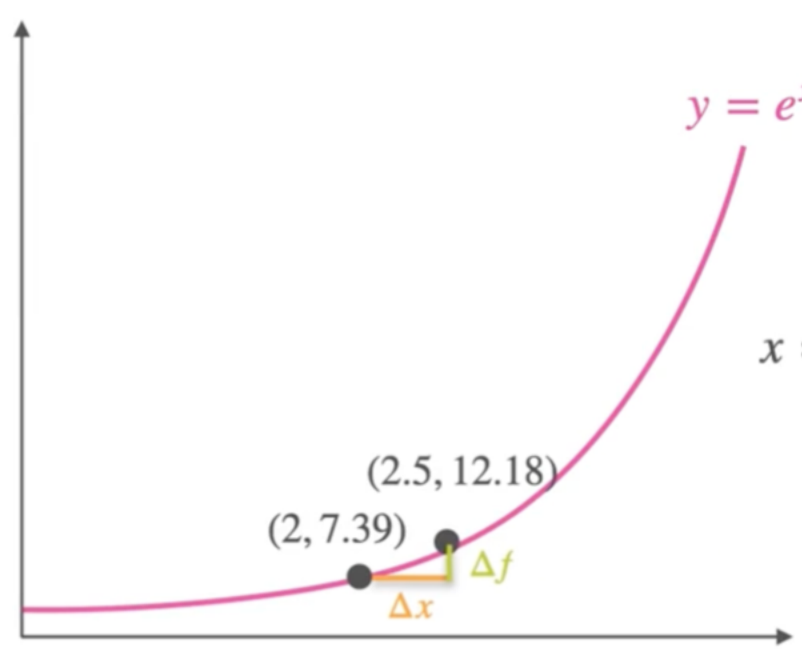
$\Rightarrow 12.70$

$e^{2+0.5} - e^{2} \Rightarrow 12.18 - 7.39$

$\Rightarrow 4.79$

Now $\frac{4.79}{2} \Rightarrow 9.59$

Derivative of e^x



Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$x = 2$

Δx	1.0	1/2
Δf	12.70	4.79
Slope	12.70	9.59

Now let's reduce the
interval to length one-half.

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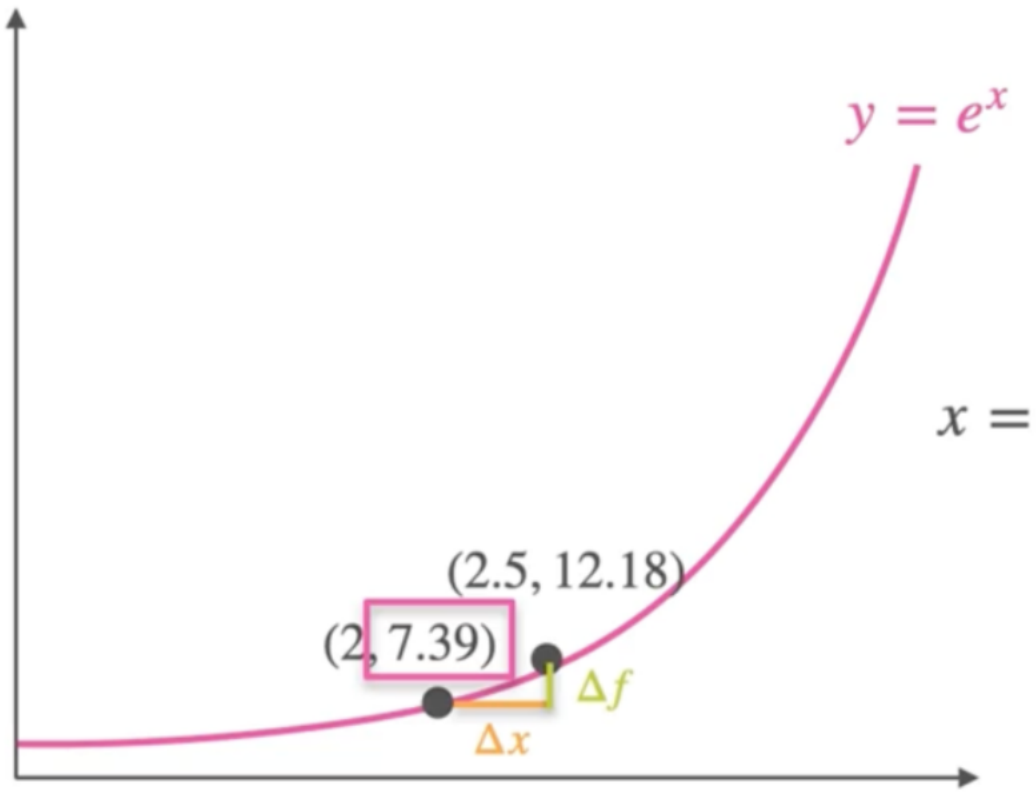
Derivative of e^x



Exponential: $y = f(x) = e^x$

Slope: $\frac{\Delta f}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x}$

$x = 2$

Δx	1.0	1/2	1/4	1/8	1/16	1/1000
Δf	12.70	4.79	2.10	0.98	0.48	0.007
Slope	12.70	9.59	8.39	7.87	7.62	$\boxed{7.39}$

$\boxed{e^2}$

the tangent line at the point

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Tangent at the point x, e^x has slope e^x.

One of the interesting property of function e^x.

the slope of the tangent line at the point to $e^2 \Rightarrow e^2$ that what happens at every point.

Derivative of $\log(x)$:

$\log(3)$

$e^{?} \Rightarrow 3$

what ? is $e^{?} \Rightarrow 3$ Here ? is the $\log(x)$

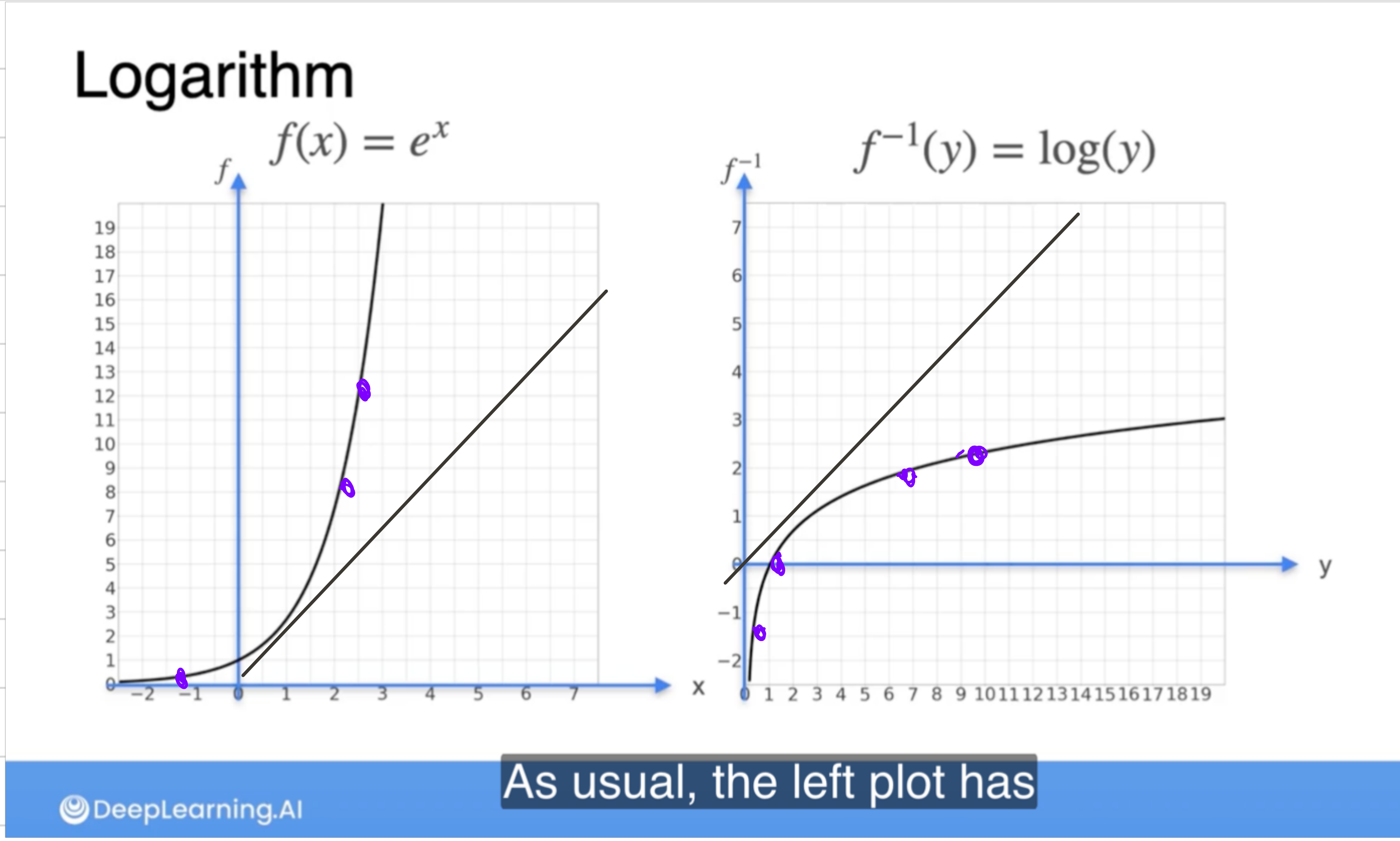
$f(x) \Rightarrow e^{x}$

$f^{-1}(y) \Rightarrow \log(y)$

$\Rightarrow e^{\log(x)} \Rightarrow x$

$\Rightarrow \log(e^{y}) \Rightarrow y$

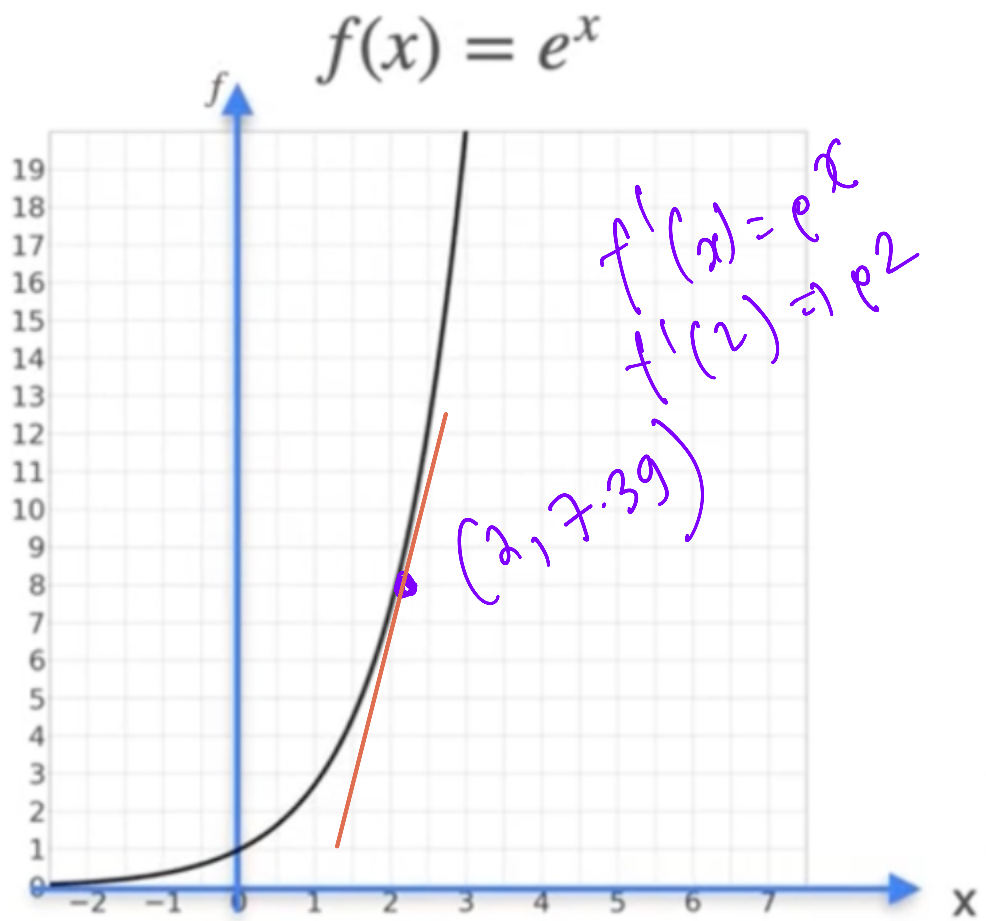
$\log(x)$ is the inverse of e^{x}.

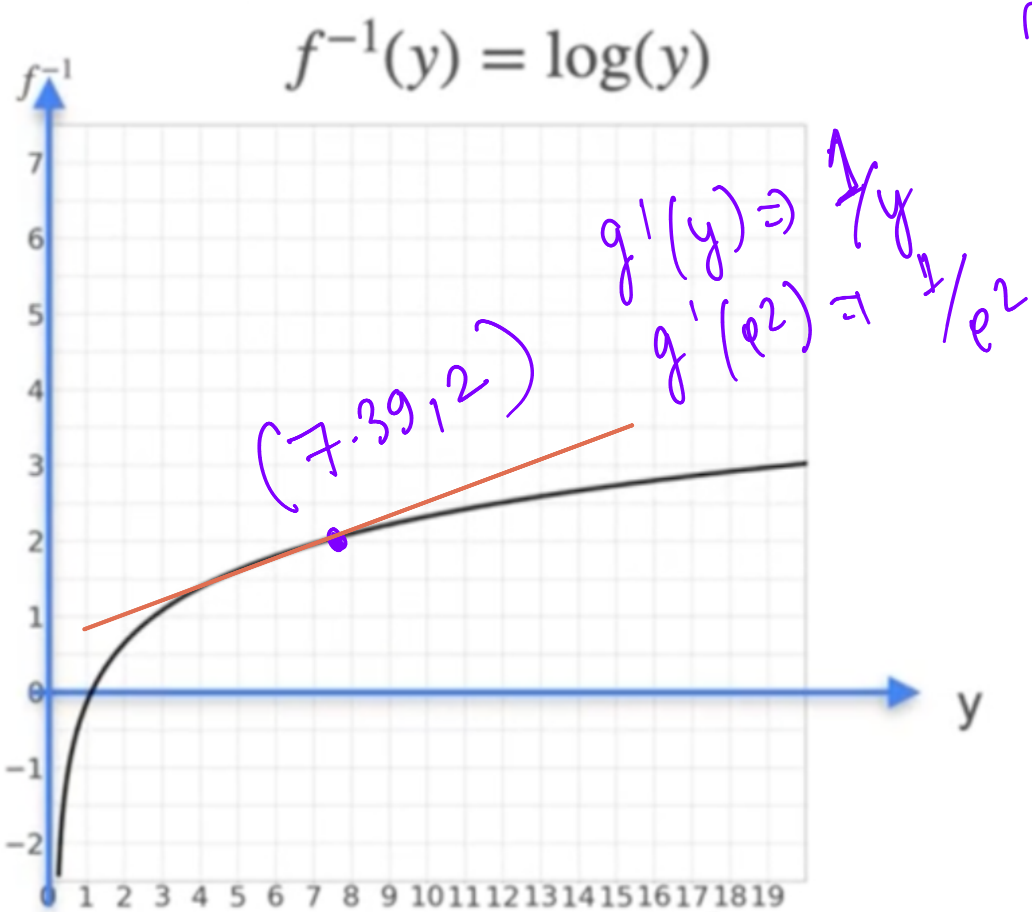


Logarithm

$$\left[\frac{d}{dy} f^{-1}(y) \Rightarrow \frac{1}{f'(x)}\right]$$

↳ Using the result for inverses





Derivative of $\log(y) \Rightarrow 1/y$

$$\frac{d}{dy} f^{-1}(y) \Rightarrow \frac{1}{f'(f^{-1}(y))}$$

$$\frac{d}{dy} \log(y) \Rightarrow \frac{1}{e^{\log(y)}} \Rightarrow 1/y$$

$$\frac{d}{dy} \log(y) \Rightarrow 1/y$$

We're going to use the result for inverses in order to

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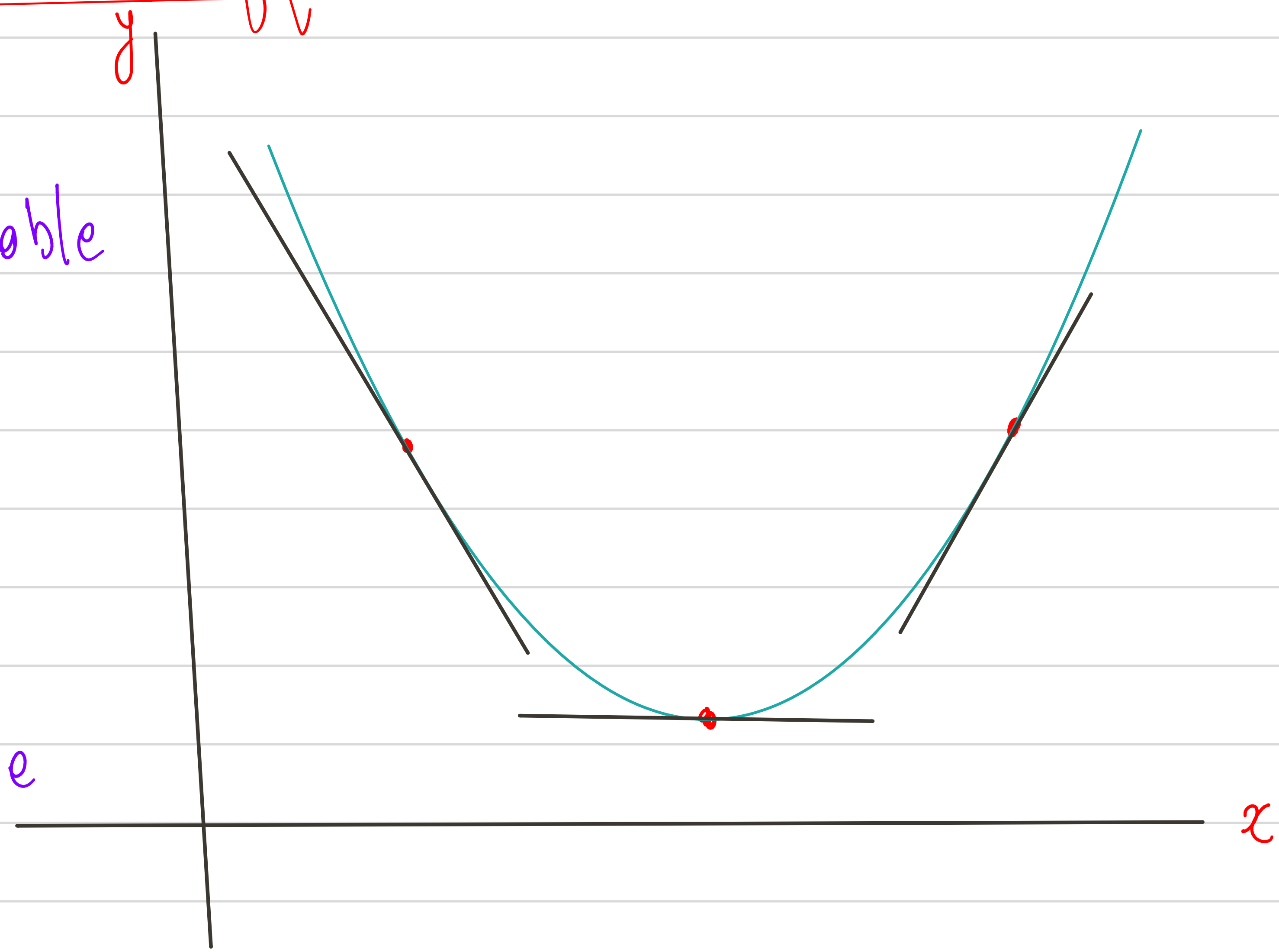
Existence of Derivative and Non-Differentiable Function

① for a function to be differentiable at a point

- The derivative has to exist for that point.

- For a function to be differentiable at an interval:

  the derivative has to exist for every point in the interval

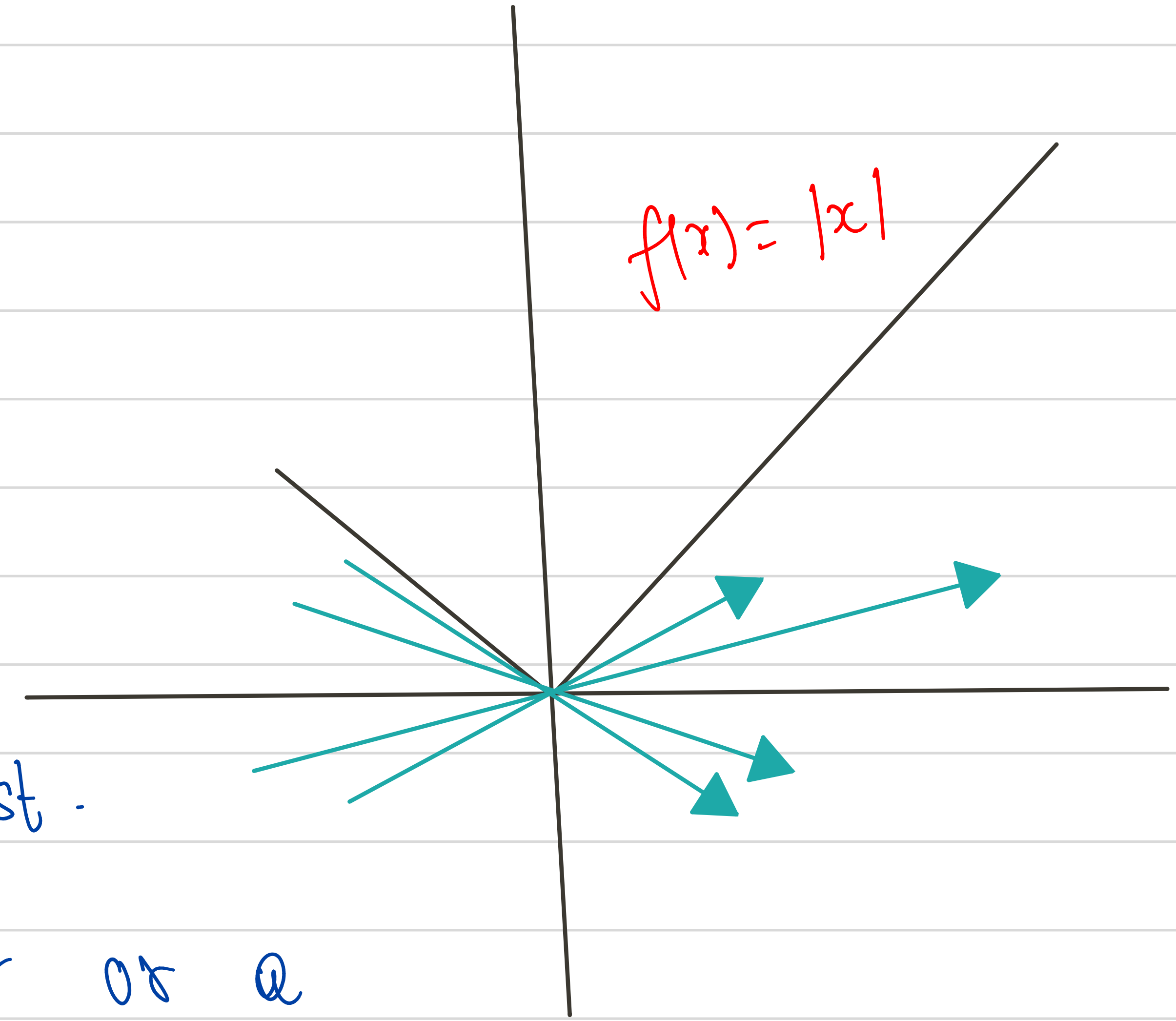


Non-Differentiable Functions

The absolute value function.

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

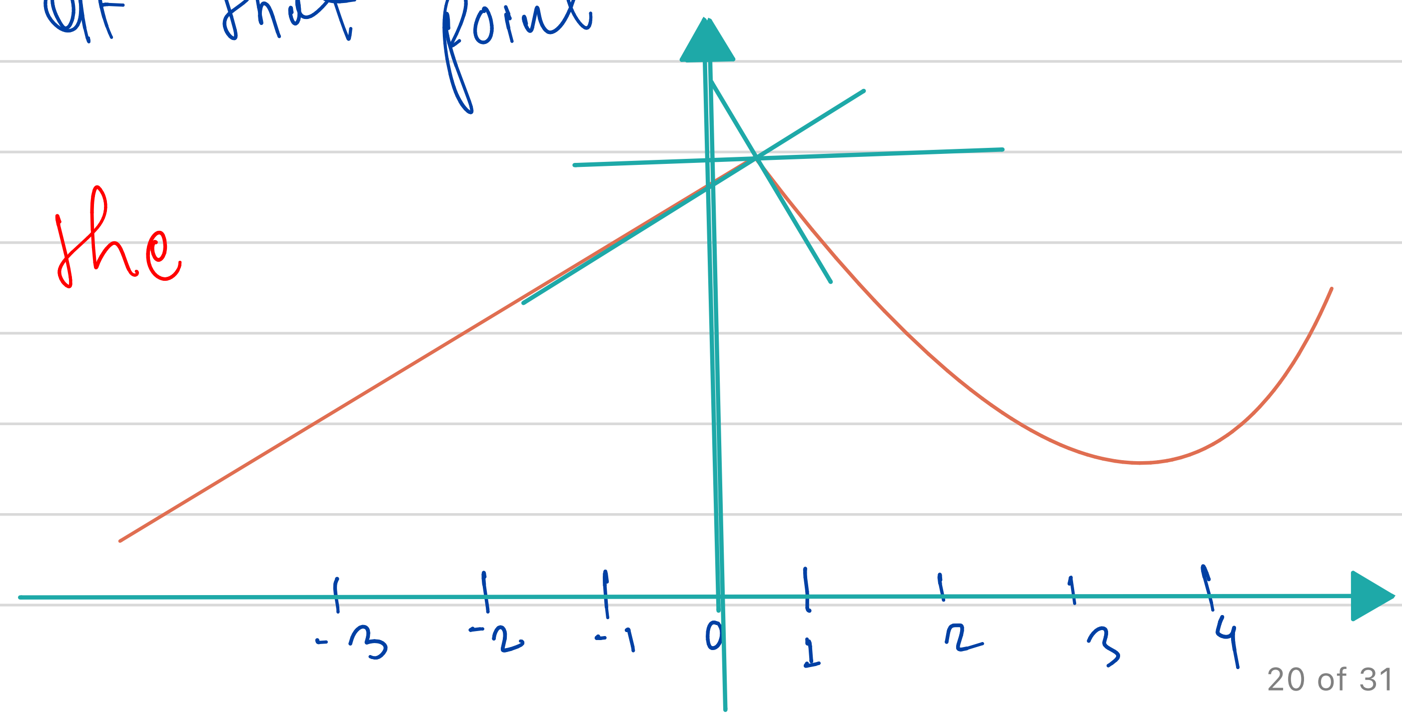
At $x = 0$, the derivative doesn't exist.



Generally when a function has a corner or a cusp, the function is not differentiable at that point.

At which point in this function does the derivative not exist?

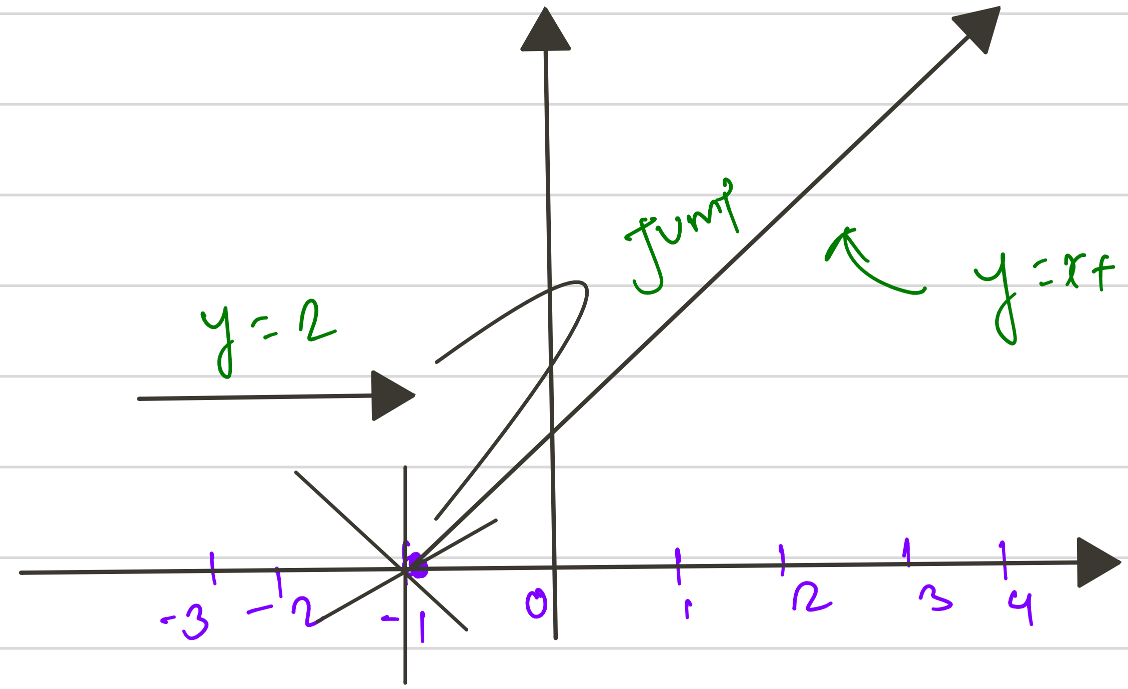
⇒ At $x = 1$, derivative doesn't exist



$$f(x) = \begin{cases} f(x) = 2 & \text{for } x < -1 \\ f(x) = x + 1 & \text{for } x \geq -1 \end{cases}$$

Called as a step function or a piecewise function. It has a jump discontinuity at x equals -1.

Over here, as I mentioned is defind as $y=2$ and $y=x+1$.

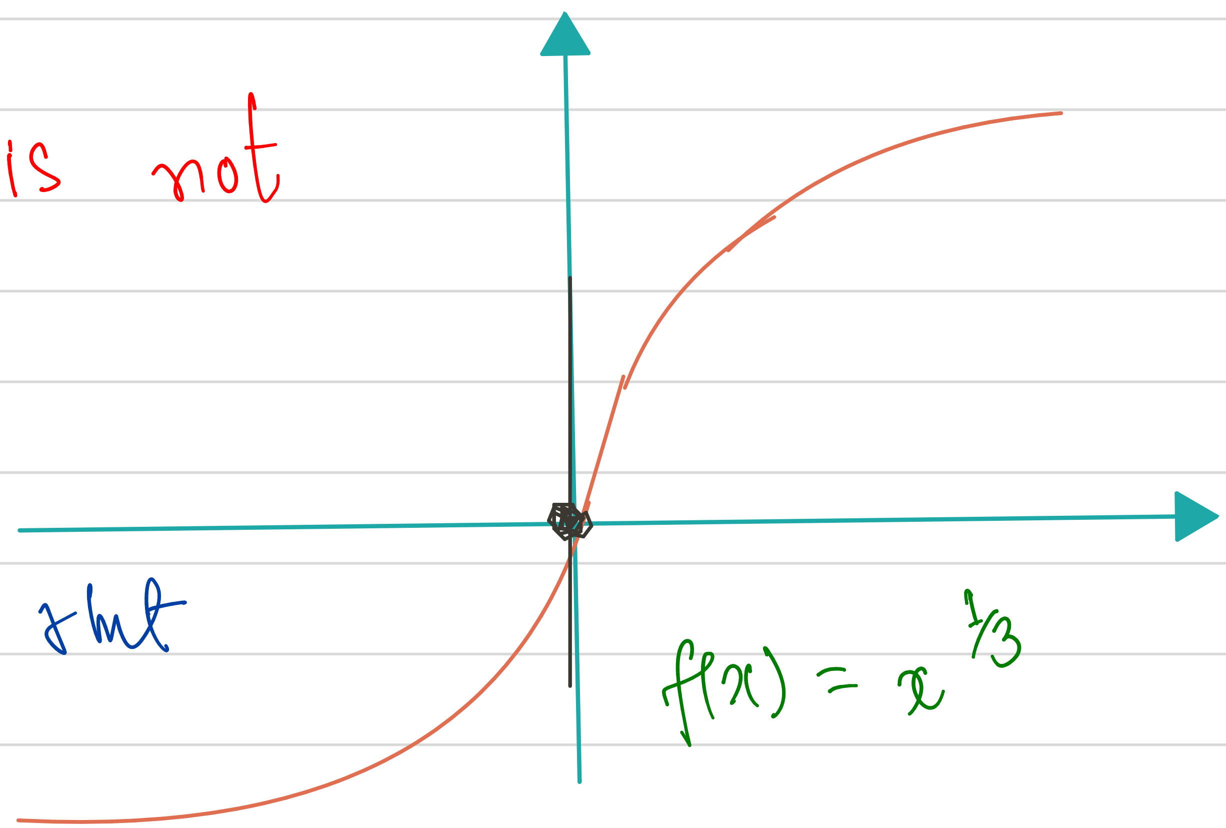


Example Jump Discontinuity

Note: A function with a vertical tangent is not differentiable.

Vertical tangents

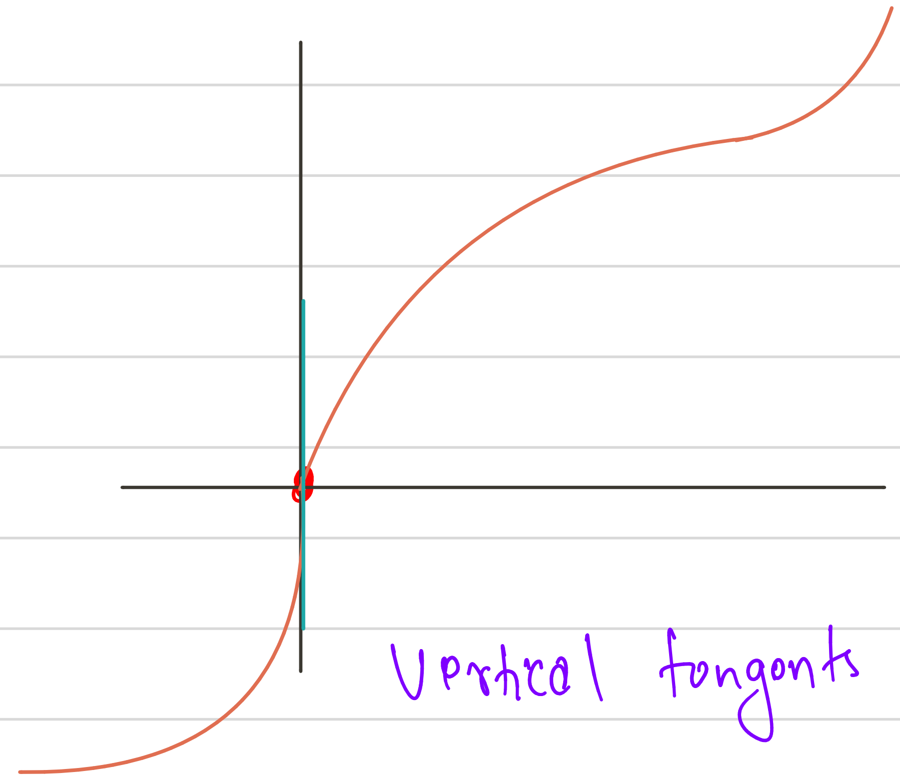
At $x=0$, this graph has a tangent line that runs straight up parallel to the y-axis



Recap. Non-Differentiable functions

Corners/ Cusps

Jump Discontinuity



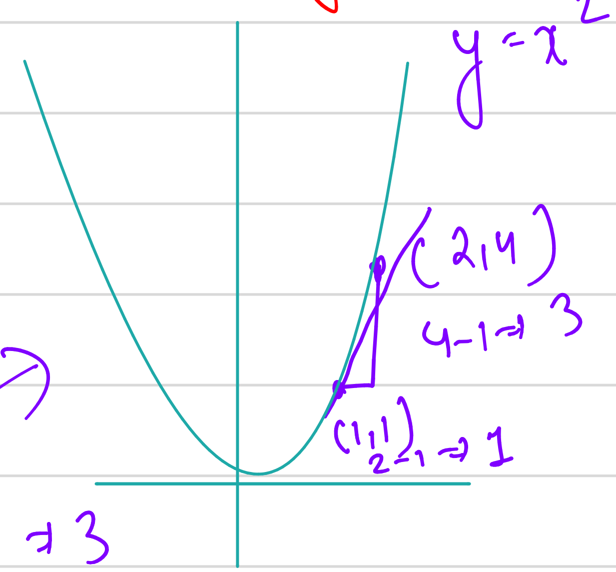
Vertical tangents

Properties of the derivative: Multiplication by Scalars

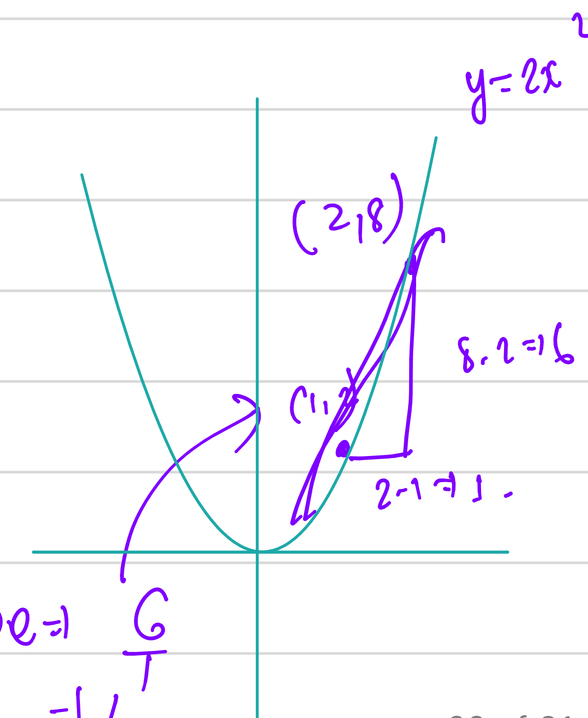
→ $f = 4g$

→ $f' = 4g'$

→ $f' = cg'$



Slope => $\frac{3}{1}$ => 3



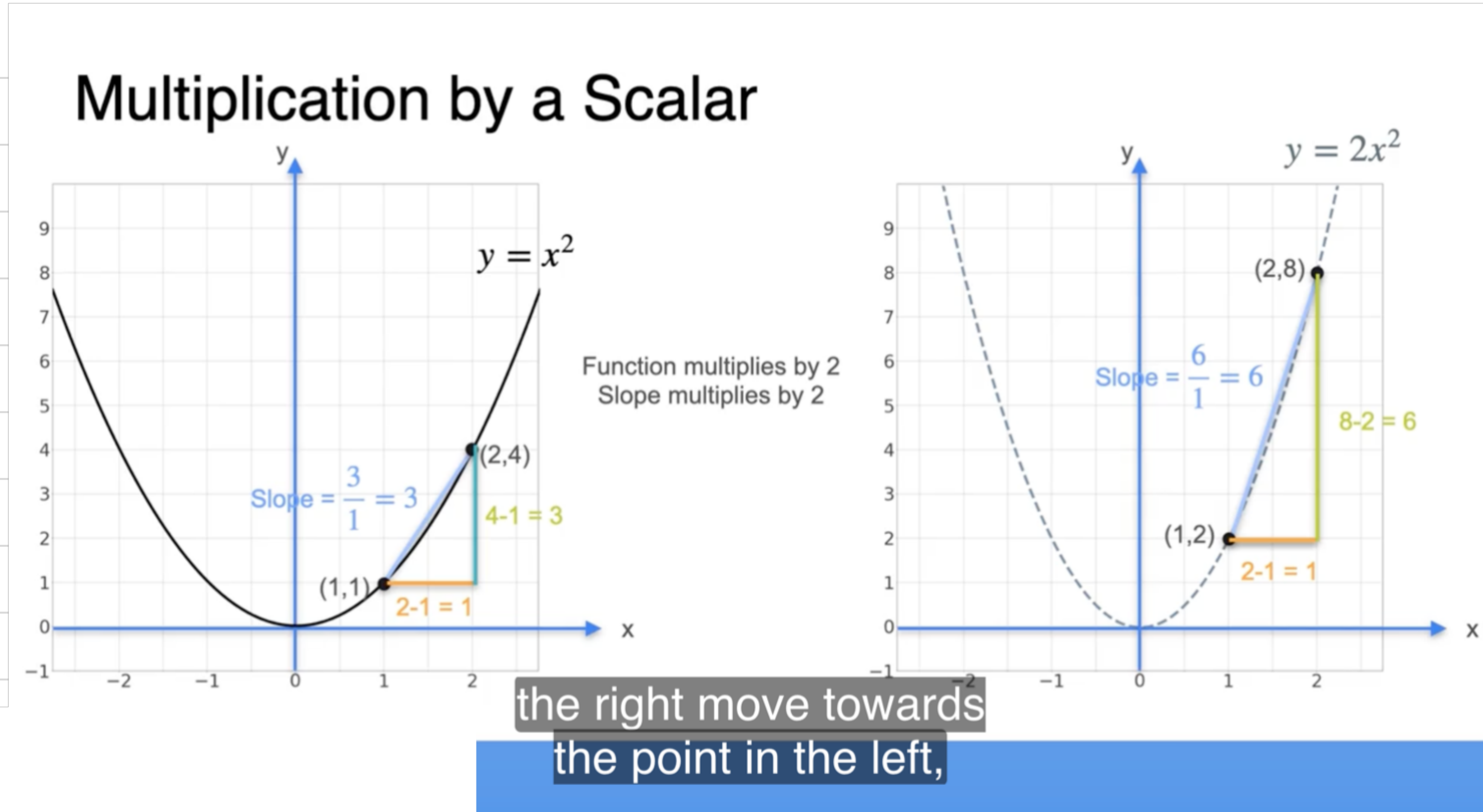
Slope => $\frac{6}{1}$ => 6

→ $y = x^2$

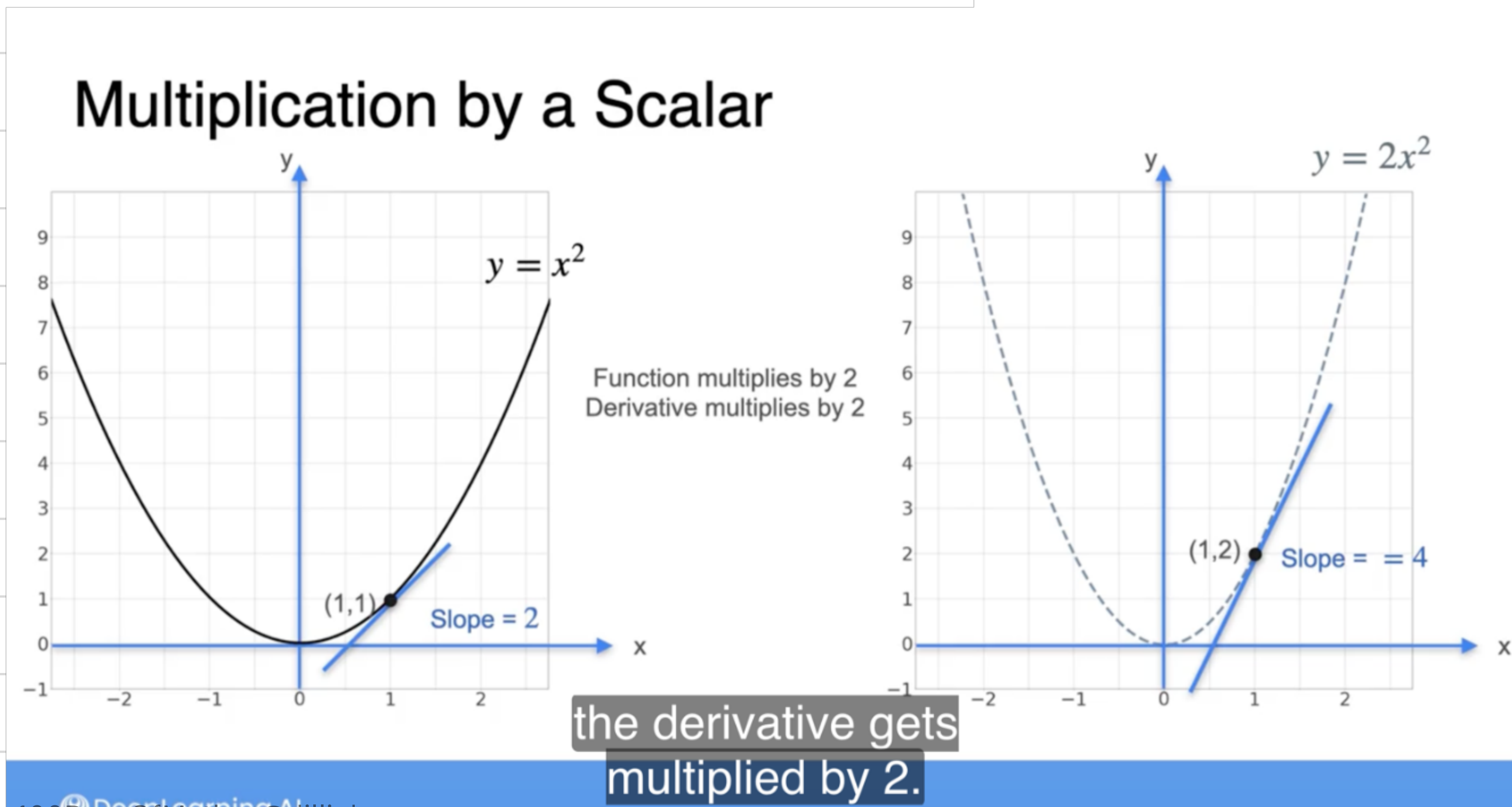
→ $y = 2x^2$

$y = (x_1, x_2) \Rightarrow (1,1)$

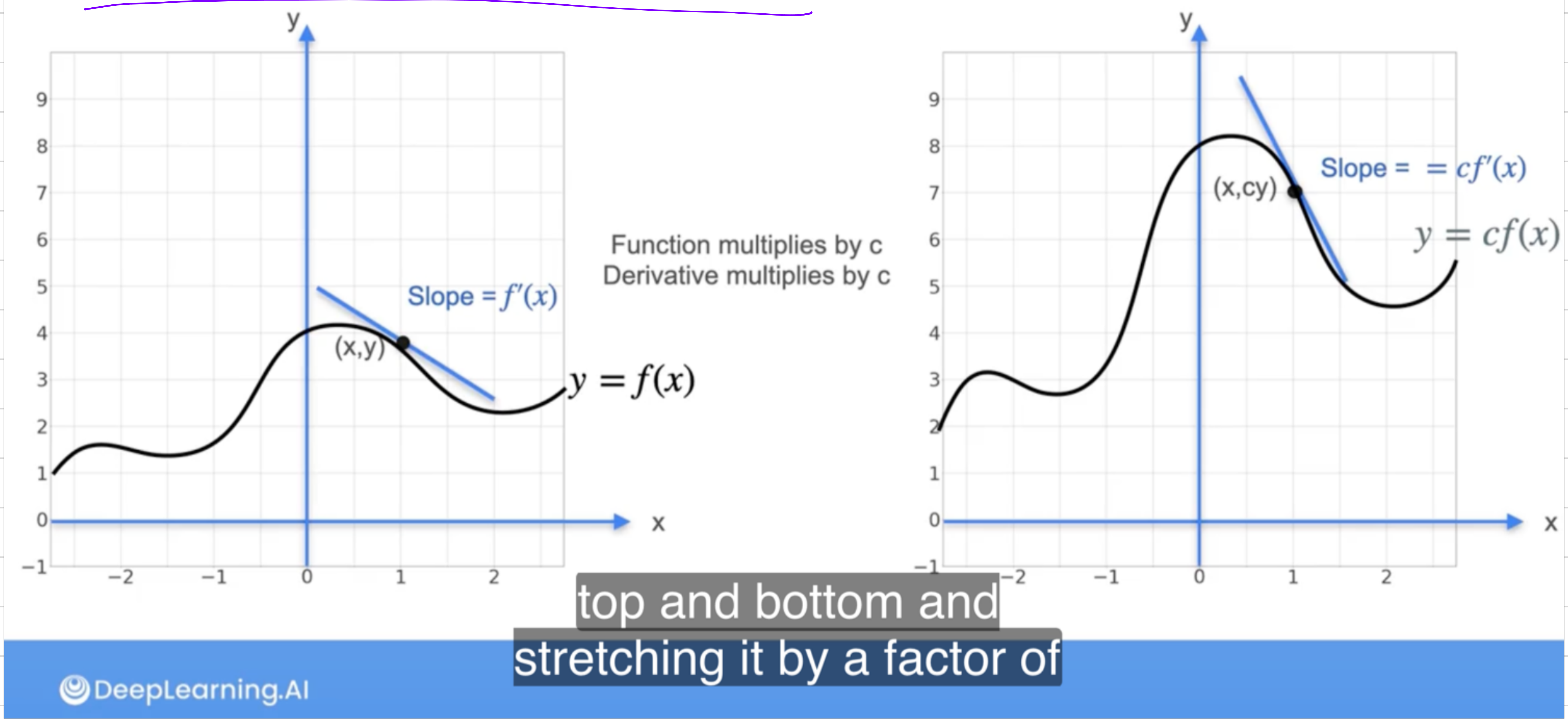
$y' = (x_1', x_2') \Rightarrow (1,2)$



Source: Calculus for Machine Learning and Data Science offered by DeepLearning.AI in Coursera.



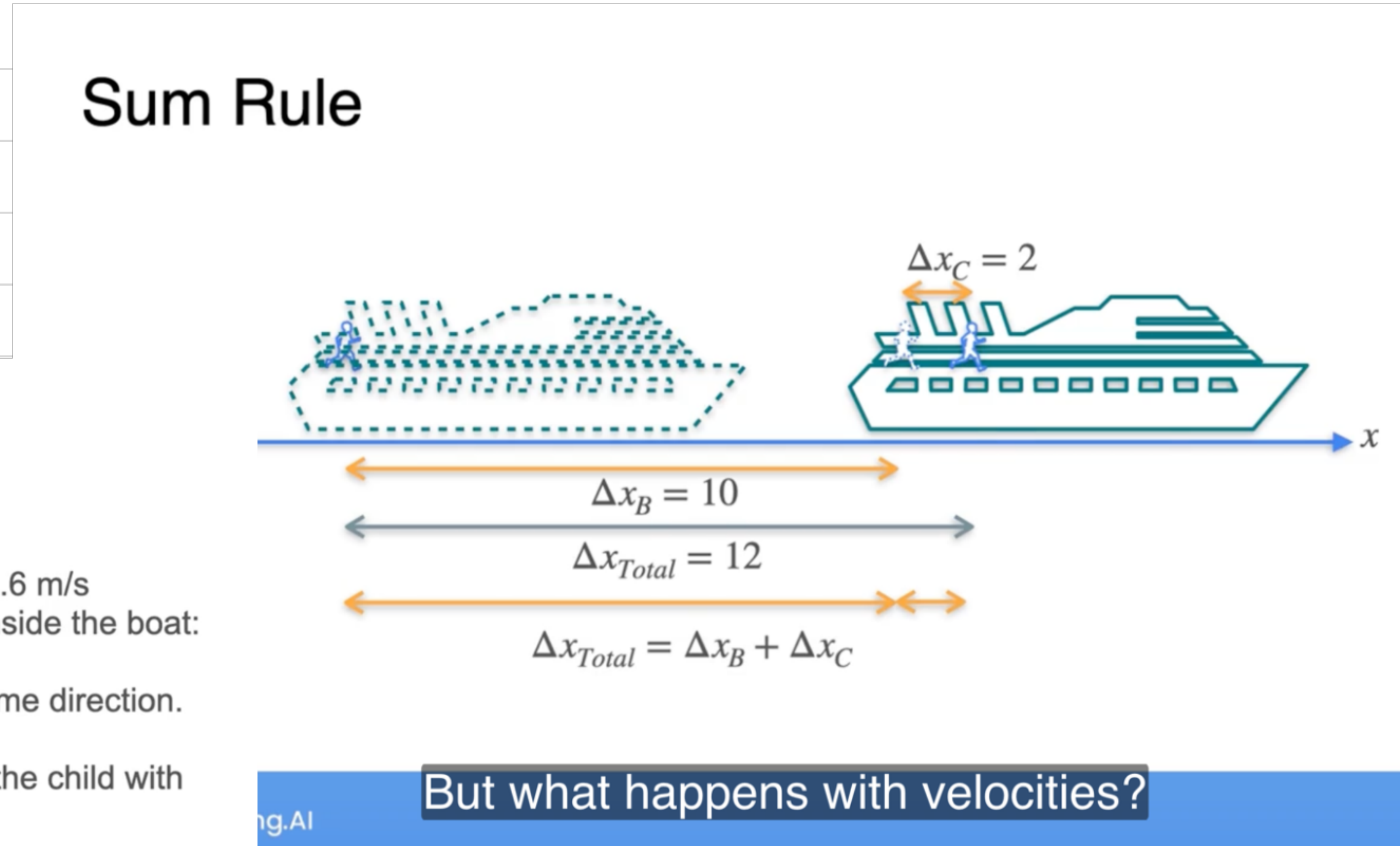
Multiplication by a Scalar

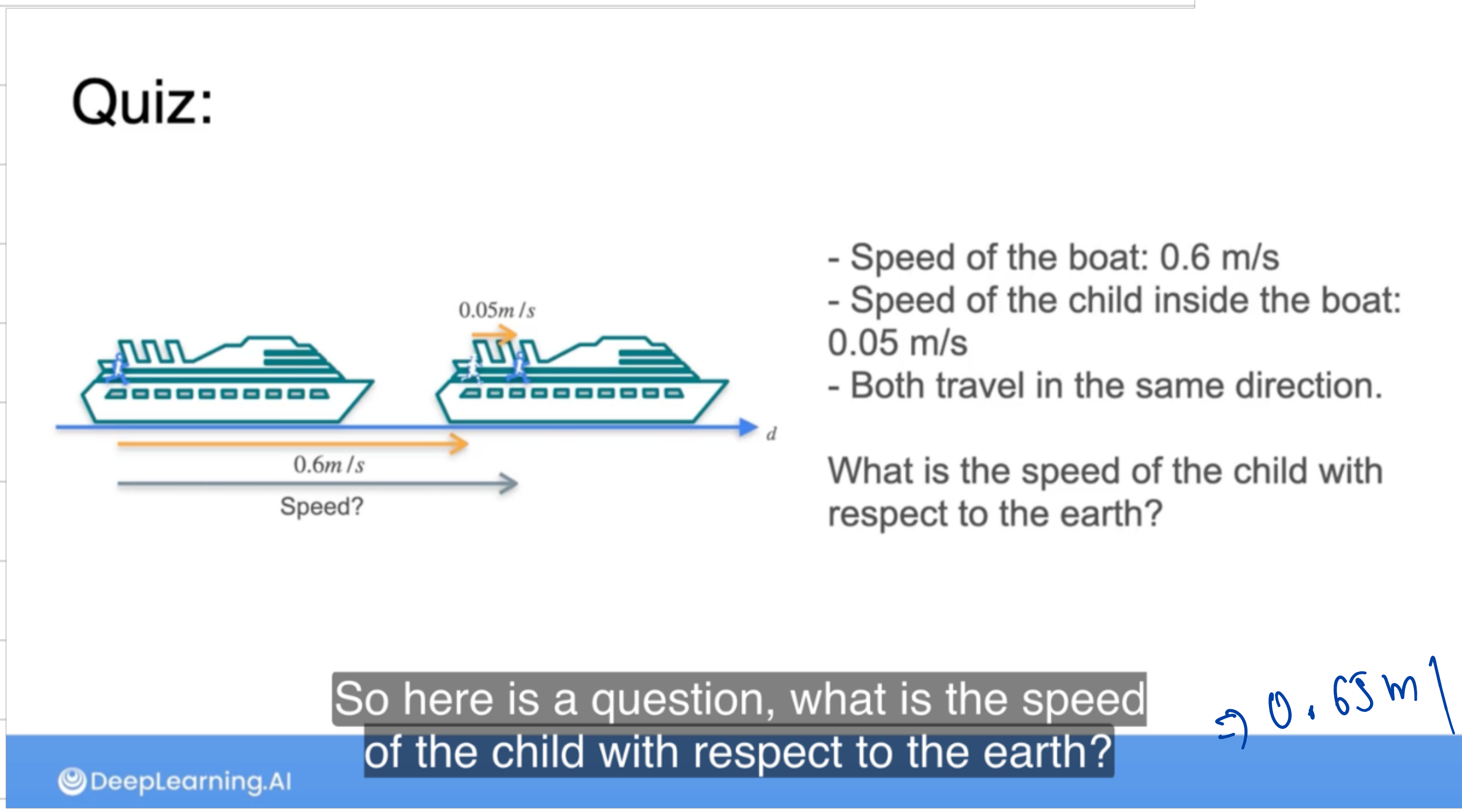


Properties of the dx : Sum Rule

$$f = g + h$$

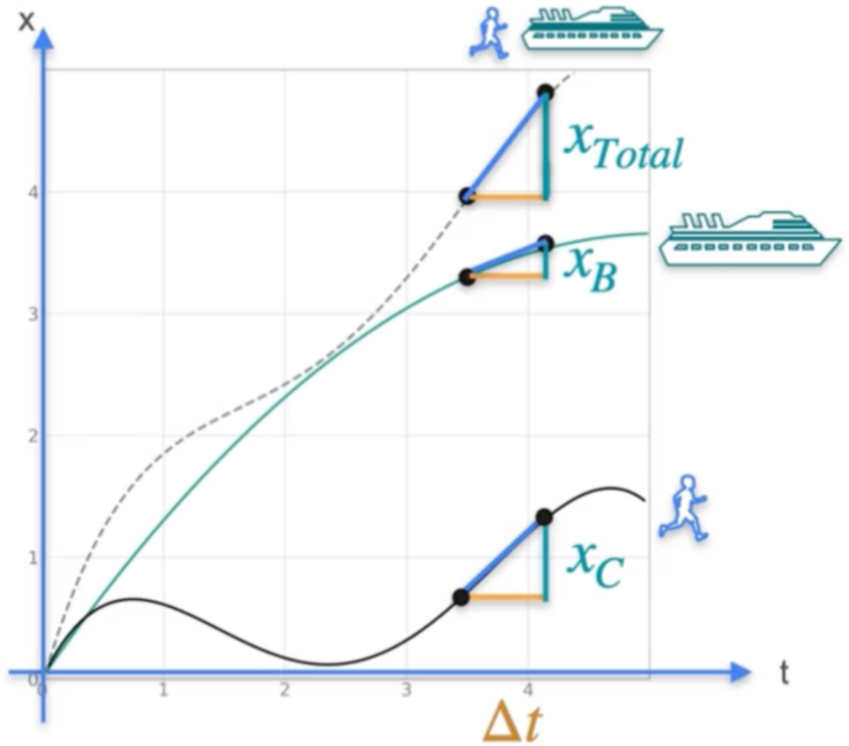
$$\rightarrow f' = g' + h'$$





$\Rightarrow 0.65\,m/s$ Because both (speed) units are in the same direction.

Sum Rule



$$x_{Total} = x_B + x_C$$

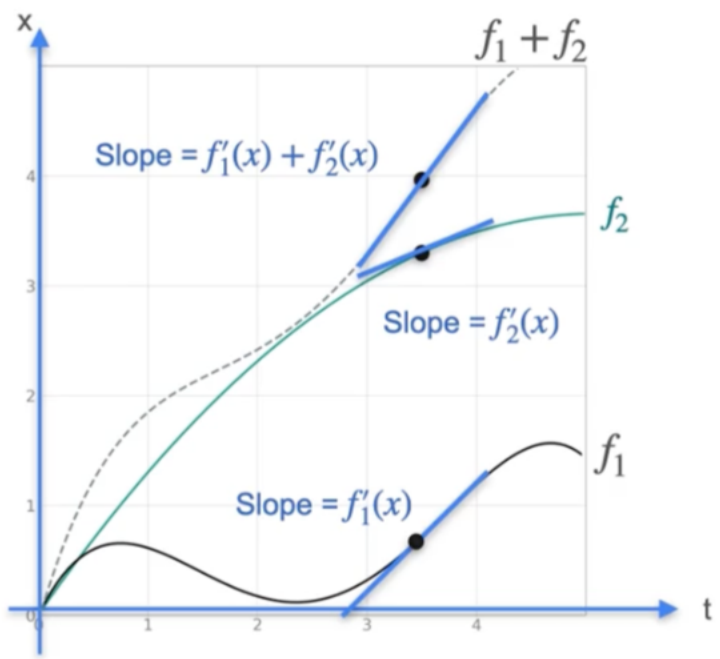
$$\Downarrow$$

$$\frac{x_{Total}}{\Delta t} = \frac{x_B}{\Delta t} + \frac{x_C}{\Delta t}$$

$$\Downarrow$$

$$v_{Total} = v_B + v_C$$

Sum Rule



$$f = f_1 + f_2$$

$$\Downarrow$$

$$f' = f_1' + f_2'$$

And the slope over here is going to be the sum of the slopes.

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If $g(x) = x^2$ and $f(x) = 2x$, then

Answer is $2 + 2x$

Because $g(x) = x^2$

$g'(x) = 2x$

$f(x) = 2x$

$f'(x) \Rightarrow 2$

So $f' + g' \Rightarrow 2 + 2x$

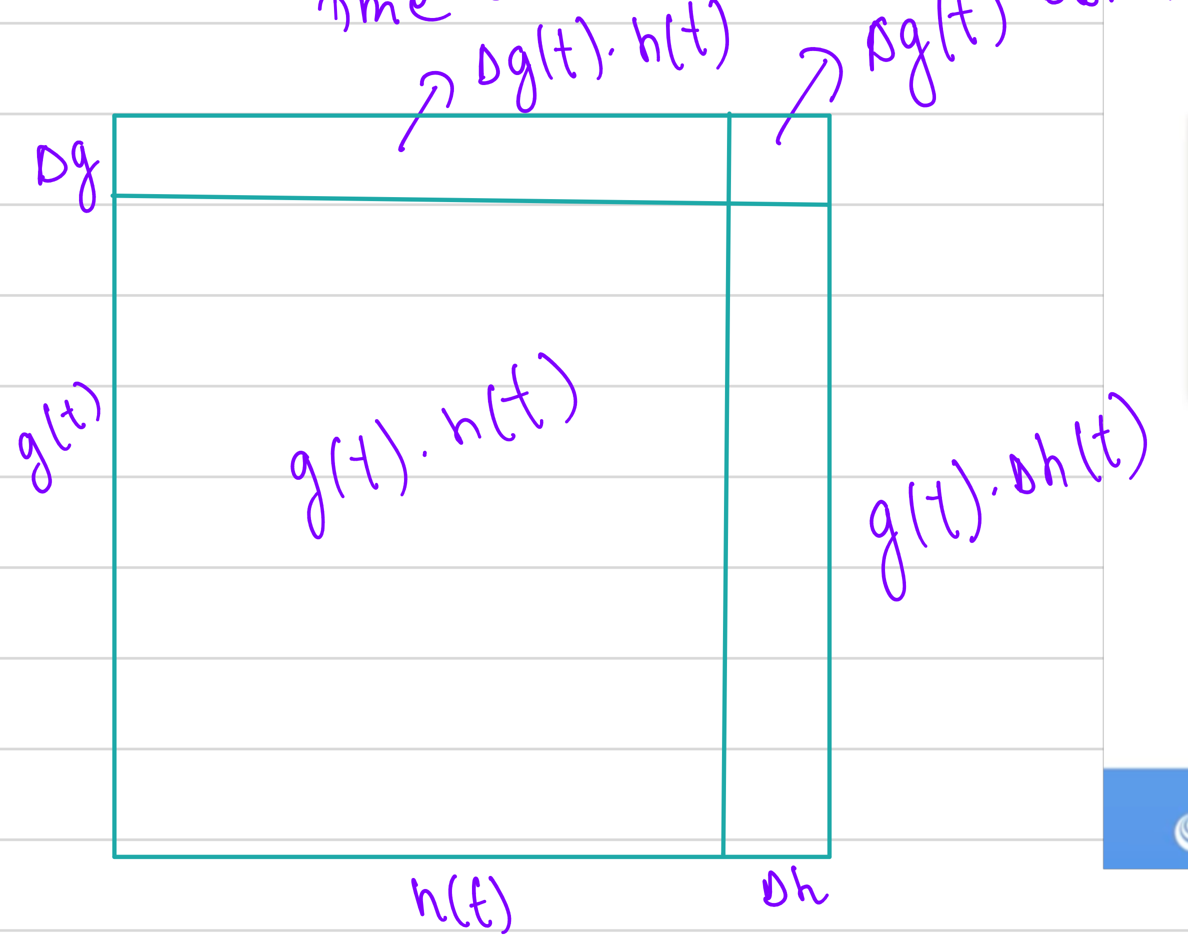
Product Rule

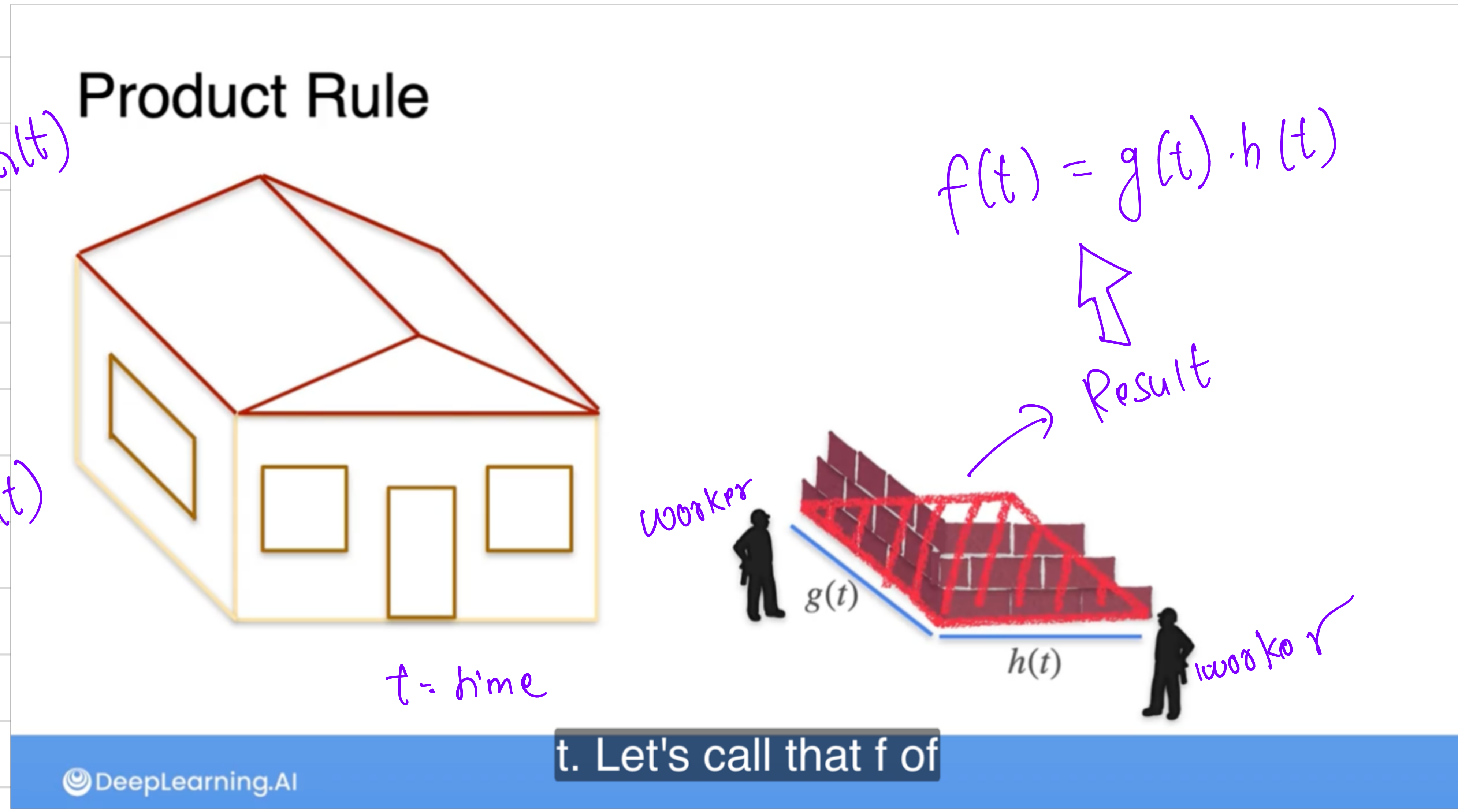
$\rightarrow f = gh$

$\rightarrow f' = g'h + gh'$

$y = f(t)$
$= g(t) \cdot h(t)$

⇨ Rate of change of Area / time





$$f'(t) = g'(t)\,h(t) + g(t)\,h'(t)$$

Given that

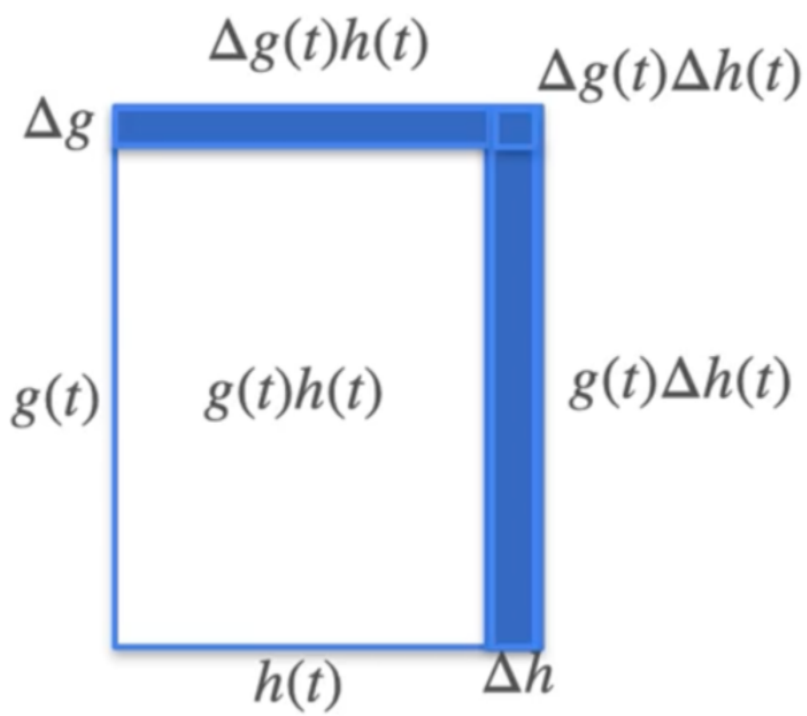
$$f(x) = xe^{x}$$

↳ $f'(x)$ is

$$e^{x} + x\,e^{x}$$

using $\boxed{f'(t) = g'(t)h(t) + g(t)h'(t)}$

Product Rule



$$y = f(t) = g(t)h(t)$$

$$\overbrace{\frac{\Delta f(t)}{\Delta t}}^{f'(t)} = \frac{\overbrace{\Delta g(t)h(t)}^{g'(t)h(t)} + \overbrace{g(t)\Delta h(t)}^{g(t)h'(t)} + \Delta g(t)\Delta h(t)}{\Delta t}$$

$$\Rightarrow \frac{\Delta g(t)}{\Delta t}\cdot h(t) + g(t)\frac{\Delta h(t)}{\Delta t} + \overbrace{\frac{\Delta g(t)\cdot h(t)}{\Delta t}}^{0}$$

as $\Delta t \to 0$

Delta t. We can rewrite this in the following way.

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The Chain Rule:

$\rightarrow \frac{d}{dt} g(h(t))$

function 1 function 2

if $\frac{dg}{dh}$. $\frac{dh}{dt}$ are Known then $\frac{dg}{dh} \cdot \frac{dh}{dt}$ is chain Rule.

$$\frac{d}{dt} f(g(h(t))) = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt}$$

$$\Rightarrow \frac{d}{dt}(g(h(t)) \Rightarrow \frac{dg}{dh} \cdot \frac{dh}{dt}$$

Two Variables

$$\Rightarrow g'(h(t)) \cdot h'(t)$$

Third / Three Variables:

$$\frac{d}{dt} f(g(h(t))) \Rightarrow \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dt}$$

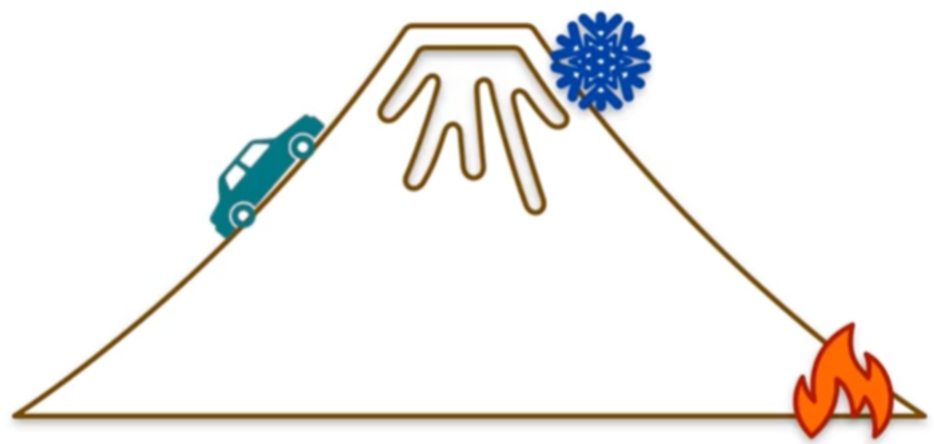
$$\left[\frac{d}{dt} f(g(h(t))) \Rightarrow f'(g(h(t))) \cdot g'(h(t)) \cdot h'(t) \right]$$

if we can find third one

$\frac{dh}{dt}$ and $\frac{dT}{dh}$

then $\frac{dT}{dt} = \frac{dT}{dh} \cdot \frac{dh}{dt}$

Idea of Chain Rule



Temperature changes w.r.t. height $\frac{dT}{dh}$

height changes w.r.t. time $\frac{dh}{dt}$

temperature changes w.r.t. time $\frac{dT}{dt}$.

$$\frac{dT}{dt} \Rightarrow \frac{dT}{dh} \cdot \frac{dh}{dt}$$

and t keeps track of time.

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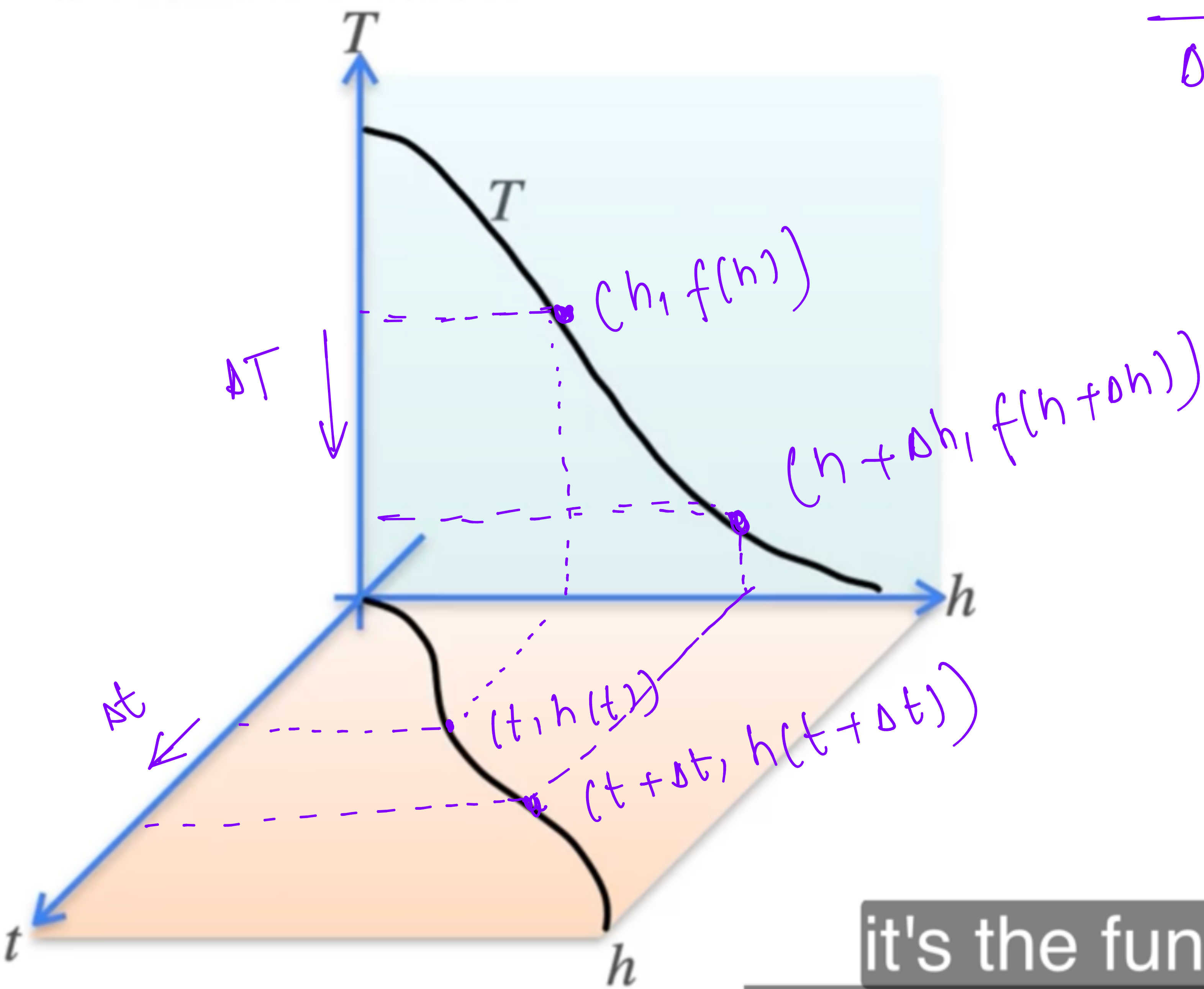
Chain Rule



$\Delta t \to \Delta h \to \Delta T$

$$\frac{\Delta T}{\Delta t} = \frac{\Delta T}{\Delta h} \cdot \frac{\Delta h}{\Delta t}$$

$$\frac{dT}{dt} = \frac{dT}{dh} \cdot \frac{dh}{dt}$$

$f(x) = e^{2x}$. $f'(x) = ?$

$2x \Rightarrow 2$

$f(x) = g(2x) \to g' \Rightarrow 2.$

$g(x) \Rightarrow e^x \to g'(x) = e^x$

So, $f'(x)$ for $f(x) = e^{2x}$

then, $\left[f'(x) = 2e^{2x} \right]$

it's the function t
that keeps track of how

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