Day-32, Dec-20, 2024 (Poush 5, 2081 BS)

Echelon Matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Row-Echelon form:

$$\begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

Row-Reduced Echelon form

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Pivot Elements:

$$\begin{bmatrix} \overset{P}{1} & 0 & 0 \\ 0 & \overset{P}{1} & 0 \\ 0 & 0 & 1^{P} \end{bmatrix}$$

Row-Reduced Echelon form

$$\begin{bmatrix} 1 & \underline{0} & 5 \\ 0 & 1 & \underline{0} \\ 0 & 0 & 1 \end{bmatrix}$$

Row Reduction Algorithm (1st phase - forword, 2nd phase Backward)

1st phase- forword ⇒ Convert matrix to row echelon form creates zero below the leading entries of each row.

2nd phase- Backward ⇒ produces unique reduced echelon form

Solution of linear System:

If we have Augmented Matrix then we can

associated system of equation

$$x_1 - 5x_3 = 1$$
$$x_2 + x_3 = 4$$
$$0 = 0$$

equivalent $\Rightarrow$ $\left[\begin{array}{ccc:c} 1 & 0 & -5 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Hence, general solution is.

$$\left\{ \begin{array}{l} x_1 = 1 + 5x_3 \\ x_2 = 4 - x_3 \\ x_3 \text{ is free} \end{array} \right\}.$$

$\left[\begin{array}{ccccc:c} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right] \rightarrow$ General Solution is –

$$x_1 = -24 + 2x_3 - 3x_4$$
$$x_2 = -7 + 2x_3 - 2x_4$$
$$x_3 = \text{free} \quad x_4 = \text{free} \quad x_5 = 4$$

Theorem: Existence and Uniqueness Theorem:

A linear system is consistent if and only if an echelon form of Augmented Matrix has no row of the form $[0 \; 0 \; \ldots \; b]$ with $b \neq 0$.

If the system is consistent then the solution set contains either unique solution when there is no free variable or infinitely many solutions when there is at least one free variable.

Example: $x + y = 5$ and $x + y = 3$

$$\begin{bmatrix} 1 & 1 & 5 \\ 1 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$$ which shows system is inconsistence

because last row $\begin{bmatrix} 0 & 0 & b \end{bmatrix}$ i.e. $b \neq 0$ so it has no solution.

If $\begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -2 \end{bmatrix}$

$x_1 + x_2 = 3$

$-2x_2 = -2$

Consistence because the last row is not form of $\begin{bmatrix} 0 & 0 & b \end{bmatrix}$ where $b \neq 0$ and x_2 are basic variable but no free variables so has exactly one solution.

Example: If one row in echelon form of an augmented matrix is $[0 \ 0 \ 0 \ \ 5]$ then the associated linear system is inconsistent

Given that an Augmented matrix has a row of the form $[0 \ 0 \ 0 \ 5]$, so the associated eqn -

$0.x_1 + 0.x_2 + 0.x_3 = 5$ $\quad [0 \ 0 \ b] \ \underline{b \neq 0}.$

$[0 = 5]$ $\quad \underline{[0 \ b:] \ z}$

This is not possible this means the system has no solution.

Vector Equations:

The position of notation plays a vital role as it can affect the solution so the idea of vector is useful in the study of linear system.

Vectors in R^2

Row & Column Vector: A matrix with only one column is called a column vector and A matrix with only one row is called a column vector

$$R = [x_1, x_2 \; . \; . \; . \; x_n] \qquad C = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Equal Vectors

Two vectors in R^2 are equal if and only if their corresponding entries are equal. Otherwise, the vectors are not equal.

$$R_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad R_2 = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

yes R_1 and R_2 are equal vectors.

$$R_1 = \begin{bmatrix} 2 & 3 \end{bmatrix} \qquad R_2 = \begin{bmatrix} 2 & 3 & 5 \end{bmatrix}$$ Not Equal Vector.

Sum of two vectors:

The addition of two vectors in R^2 means the sum of corresponding entries of the vectors

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$u + v = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}$$

Scalar Multiple of Vector

If $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ in R^2 and c be scalar then the multiplication of u by c is denoted by cu and is defined

$$cu = c \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\left[cu = \begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix} \right]$$

$u = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ $c = 11$

$$cu = \begin{bmatrix} -33 \\ 44 \end{bmatrix}$$