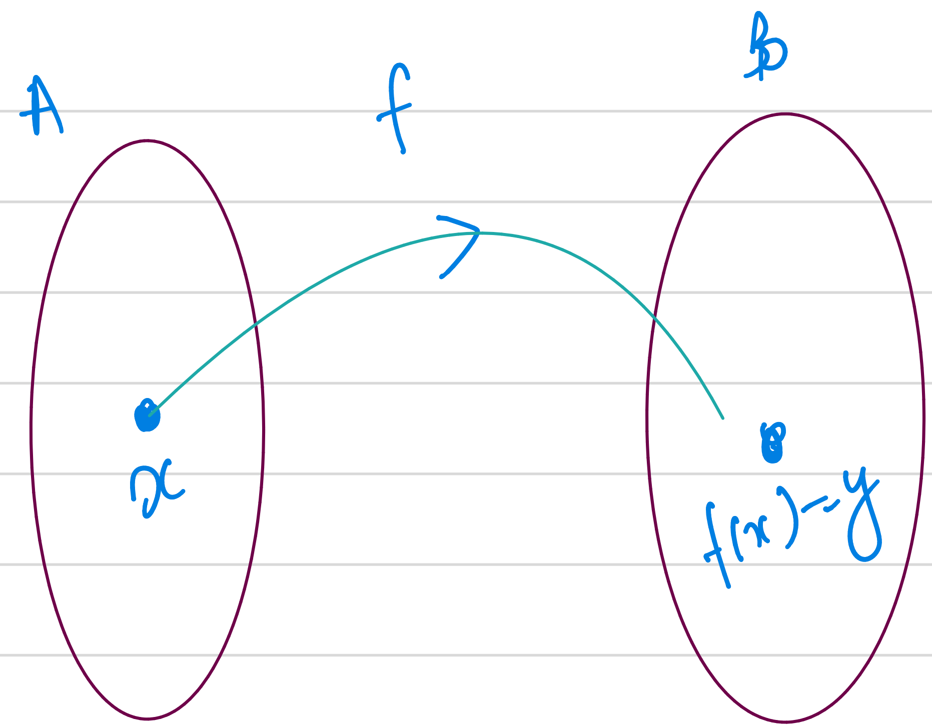
Day-17, Dec-2, 2024 (Mangshir 17, 2081).

→ Function? Ways to define the function?
→ Mathematical Model of function acting in a real world.
→ How functions are combined and transformed into graphs.

Functions And its Representation

Let A and B be the non-empty sets then a rule from A to B is called function if every element of domain A has unique association with elements of co-domain B.



Let $f: A \to B$ be a function.

i) Set A is called domain

ii) Set B is called Co-domain

iii) y is the image of x under f i.e. $y = f(x)$

iv) x is called Pre-image of y

v) Range of f is denoted by $f(A)$ and defined by

$$f(A) = \{ f(x) : \forall x \in A \}$$

The domain of function is set of all possible input values x, which plug in the function formula to produce the output values y

(OR $f(x)$). While finding the domain note that:

- The denominator of fraction cannot be zero.
- The sign of variables under the even roots (square roots, fourth root and so on) must be positive.

The range of function is set of all possible output values y (or $f(x)$) which result from plugging all the input values of x in the function formula.

While finding the Range:

Transfer the functional formula $y = f(x)$ in to $x = g(y)$ and perform as above for domain (here for y).

Examples: Identify the domain and range of function.

(1) $y = x^2$

Here, Given $y = x^2$

for domain, for all real values of x, y is exist. So, domain is set of all real number i.e. domain is $(-\infty, \infty)$.

for Range, $y = x^2$ or $x = \sqrt{y}$. for all $y \geq 0$, x is defined, thus y is set of all non-negative real number.

Hence, range is $y \geq 0$ or $[0, \infty)$.

ii) Given $y = 1/x$

For domain, when $x = 0$, y doesn't exist, it means y is exist for all real number except 0. The domain is set of all real number except 0 i.e. $(-\infty, 0) \cup (0, \infty)$.

$\rightarrow (-\infty, 0, \infty)$

For range $y = 1/x$ or $x = 1/y$. Here, x is exist for all real number except $y = 0$. Thus, the range is set of all real number except 0 ie $(-\infty, 0) \cup (0, \infty)$.

iii) Given $y = \sqrt{4-x}$

for domain y is exist only when $4-x \geq 0$ i.e. $x-4 \leq 0$
i.e. $x \leq 4$
Thus, domain is $x \leq 4$ i.e. $(-\infty, 4]$.

for Range,

$$y = \sqrt{4-x}$$

$$\text{or, } x = 4 - y^2$$

Here, x is exist for all real value of y. But given $y = \sqrt{4-x}$ which is non-negative (because for $x \leq 4$, y is non-negative). Thus, range is set of all non-negative real number $[0, \infty)$.

iv) Given, $y = \sqrt{9 - x^2}$

for domain, y is exist for $9 - x^2 \geq 0$.

i.e. $x^2 - 9 \leq 0$

i.e. $(x-3)(x+3) \leq 0$

i.e. $-3 \leq x \leq 3$

Hence, domain is $-3 \leq x \leq 3$ i.e. $[-3, 3]$

for Range,

$$y = \sqrt{9 - x^2}$$

$$\Rightarrow y^2 = 9 - x^2$$

$$\Rightarrow x = \sqrt{9 - y^2}$$

Here, x is exist for $9-y^2 \geq 0$.

i.e. $y^2-9 \leq 0$

i.e. $-3 \leq y \leq 3$.

But given $y=\sqrt{9-x^2}$ which is non-negative because (for $-3 \leq x \leq 3$, y is non-negative) Hence

Range is $0 \leq y \leq 3$ i.e. $[0,3]$.

$\boxed{y=3}$ $\Rightarrow$ $y=\sqrt{9-0}$

$y=\sqrt{9-(1)^2}$

$y=\sqrt{9-(2)^2}$

& $y=\sqrt{9-(3)^2}$

$\boxed{y \Rightarrow 3}$

$y=\sqrt{9-(4)^2}$

$y=\sqrt{7}$

functions are represented by four ways which are verbally

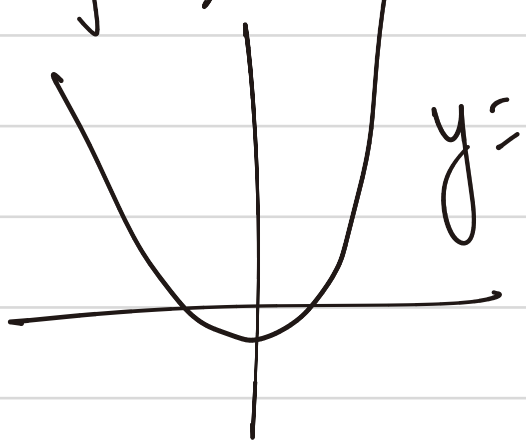
→ (it means by a description in words).

→ Numerically (It means by table of values)

→ Visually (it means by a graph)

→ Algebraically (It means by an explicit formula).

→ $y = f(x)$

→ 

$y = x^2$.

→ $5x + 3y = 6$

→ A function $f(x)$ is said to be parabola if it exhibits the concave property having polynomial degree or more than one degree.

Vertical Line Test for function

→ Not every curve in xy plane can be the graph of a function

→ So, the vertical line test that which curves in xy plane are graph of functions

→ Vertical line test state that a curve in the XY plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

Note: In general, the equation where power of y is even then such curve is not function of x.

Example: Identify graph of $x^2+y^2=4$ is a function or not.

Given equation,

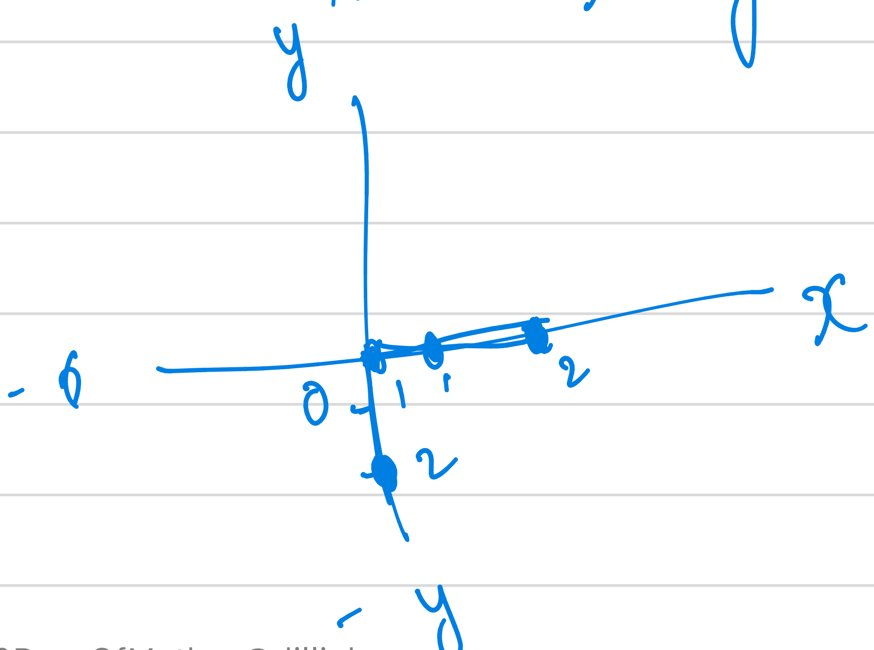
$$x^2+y^2=4$$

put $x=0$, then

$$y=\pm 2$$

Here vertical line $x=0$ meet curve at two points $y=\pm 2$.

Thus $x^2+y^2=4$ is not function (According to the vertical line test).



Example:

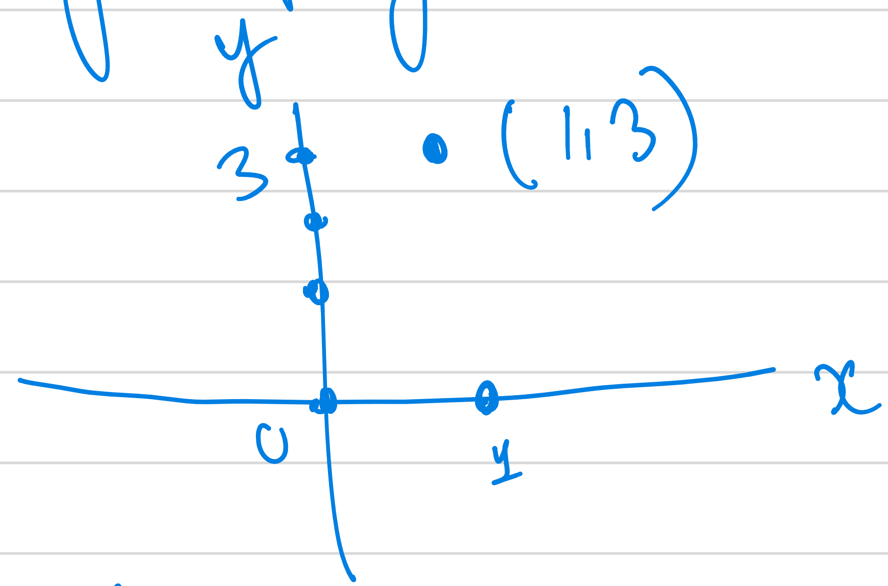
$y = x^3 + 2$ is function or not.

Given $y = x^3 + 2$

for any vertical line say $x = 1$.

$y = 1^3 + 2$

$= 3$ Curve meet single point $y = 3$

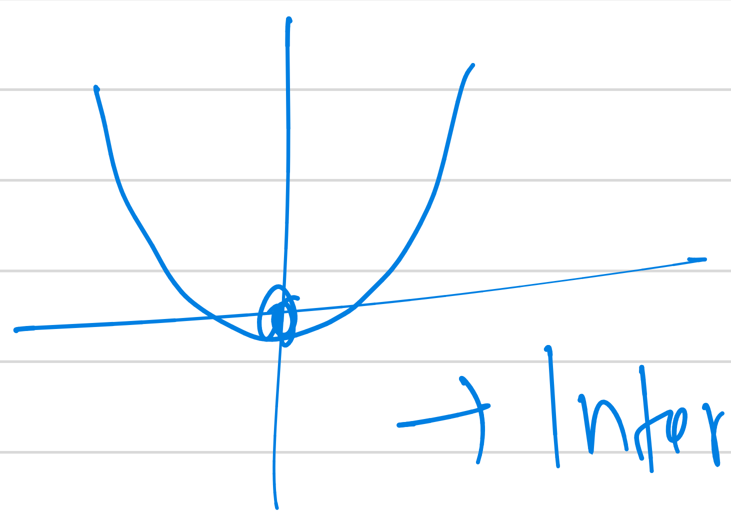


Thus, $y = x^3 + 2$ is a function.

(1) $y = x^2$

$y = (1)^2$

$y = (2)^2$



→ Intersects only at one point when $x = 1, x = 2 \ldots$