

Day- 91, Feb 21, 2025 (Folgan 17, 2081 BC)

- 1) Vectors and linear Transformations (Machine Learning Motivation)
- 2) Vectors and their properties
- 3) Vector Operations, Multiplying a Matrix by a vector
- 4) The dot Product & Geometric Dot Product
- 5) Matrices as linear transformations and linear transformations as matrices | Matrix Multiplication
- 6) Identity Matrix, Matrix Inverse, Condition for Matrices to be Inverse?
- 7) Neural Networks and Matrices.

# Linear Regression

$$\begin{array}{c} \text{linear Model.} \\ \hline w_0 x + b = \hat{y} \end{array}$$



$$w_1x_1^{(1)} + w_2x_2^{(1)} + \dots + w_nx_n^{(1)} + b = y^{(1)}$$

$$w_1 x_1^{(2)} + w_2 x_2^{(2)} + \dots + w_n x_n^{(2)} + b = y^{(2)}$$

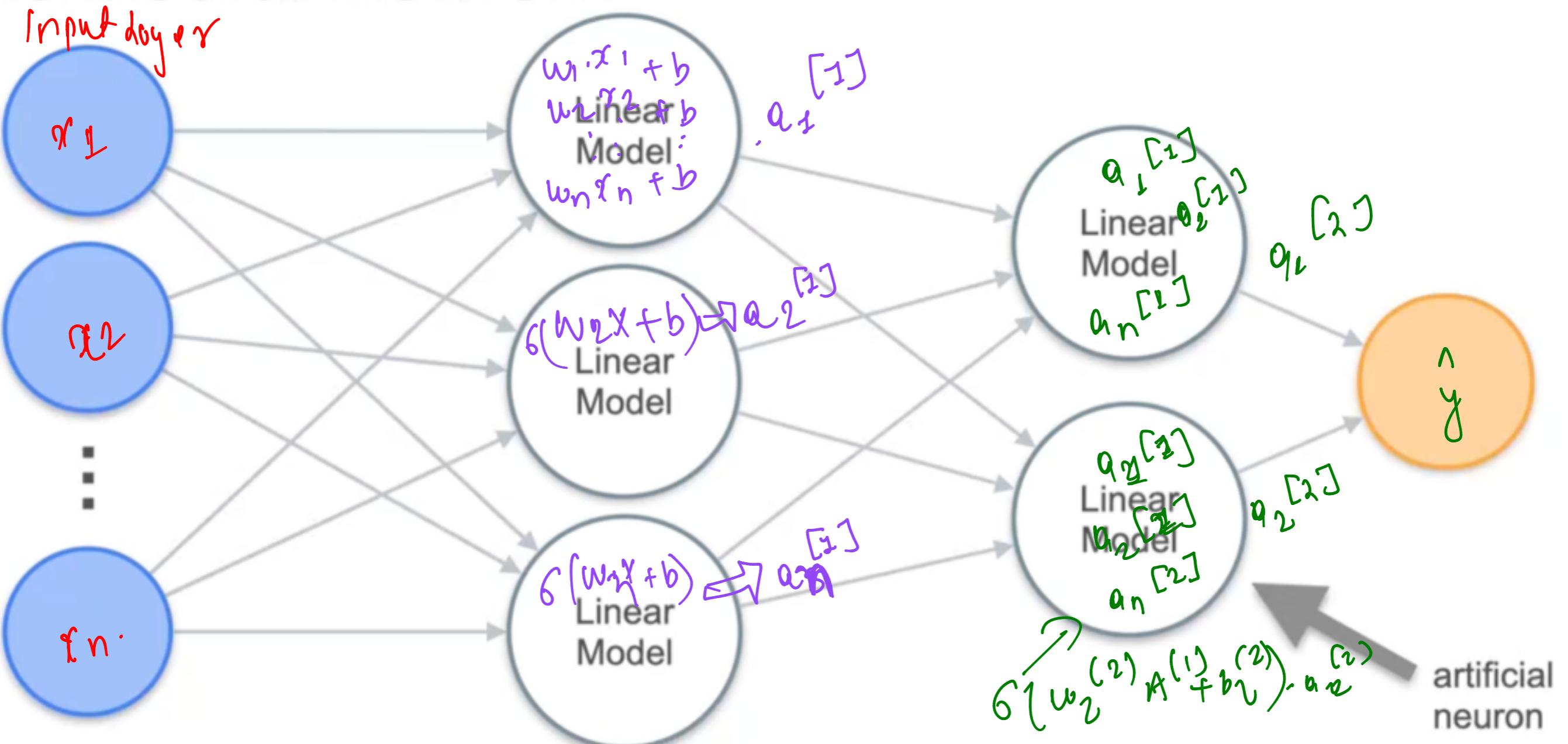
$$w_1 x_1^{(m)} + w_2 x_2^{(m)} + \dots + w_n x_n^{(m)} + b = y^{(m)}$$

## System of Linear Equations

**Equations** As a reminder, the way that worked was that you have

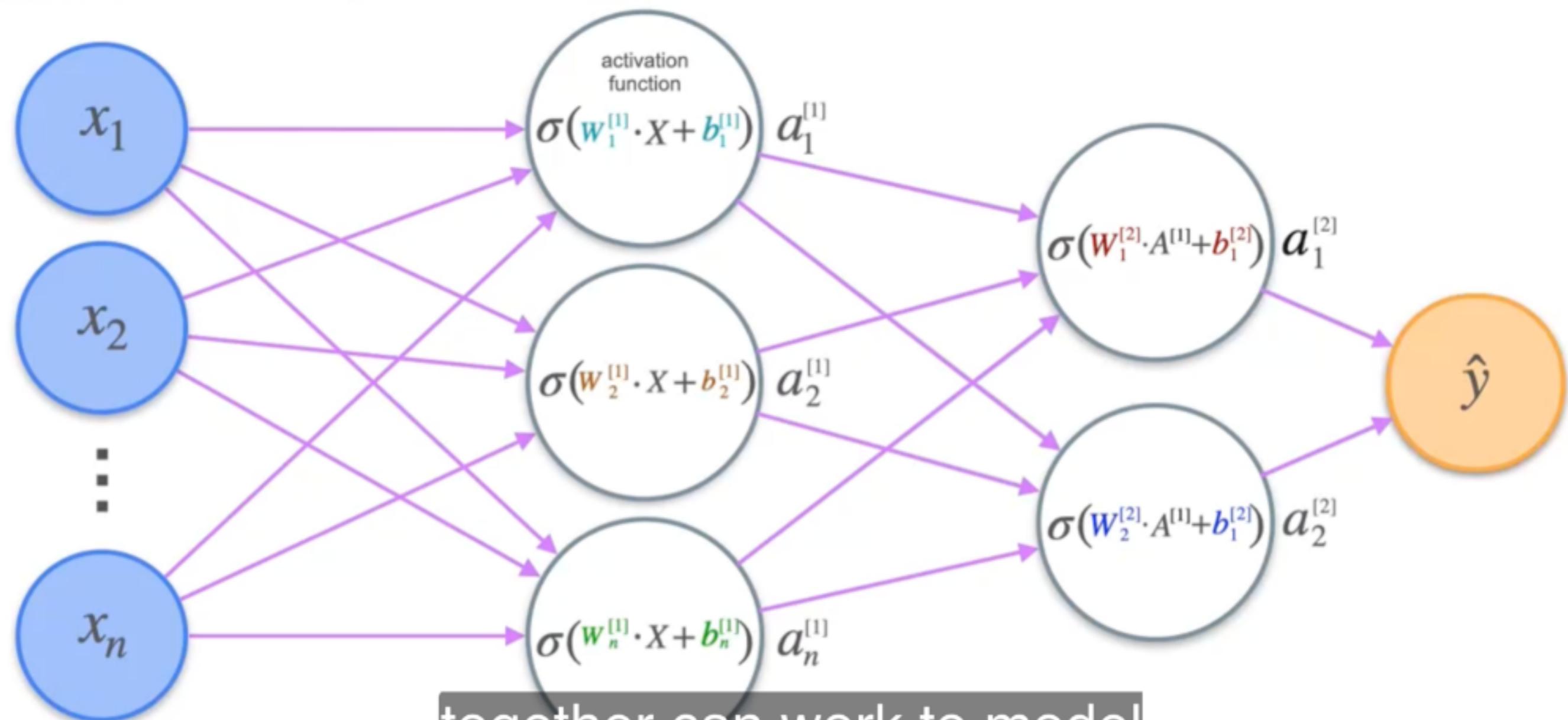
They are 'just' linear Models stacked and backed by Activation functions (6)

# The Neural Network



so-called artificial neurons,

# The Neural Network

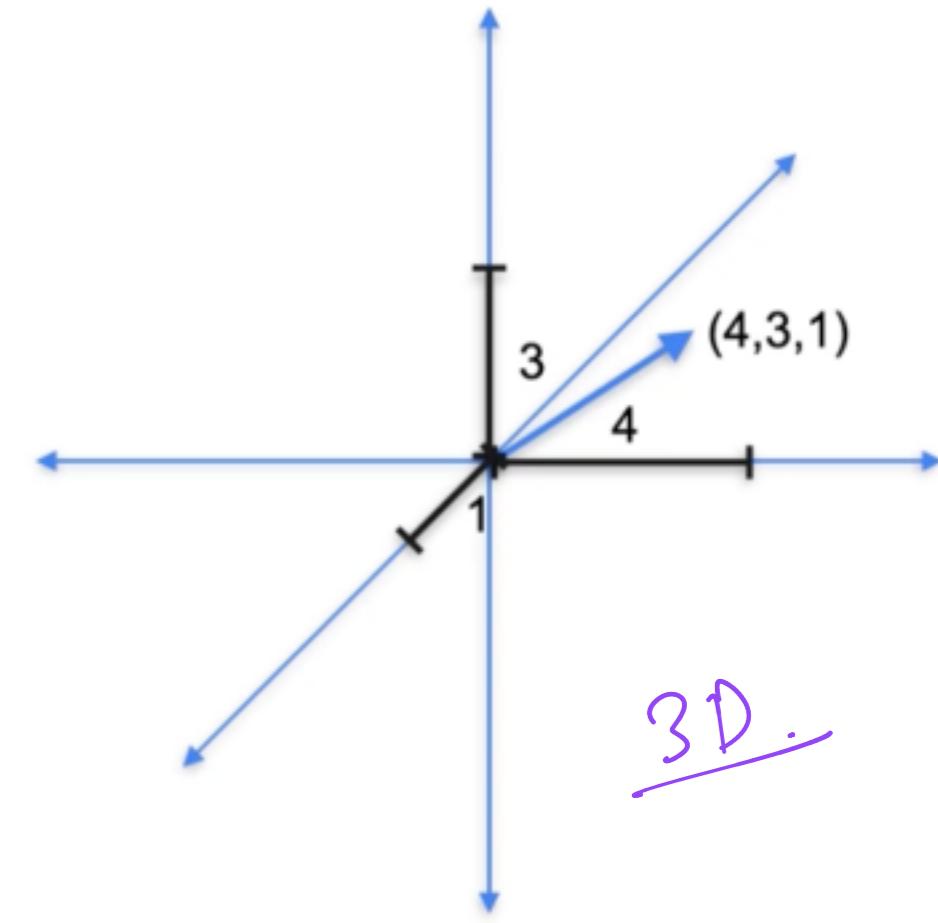
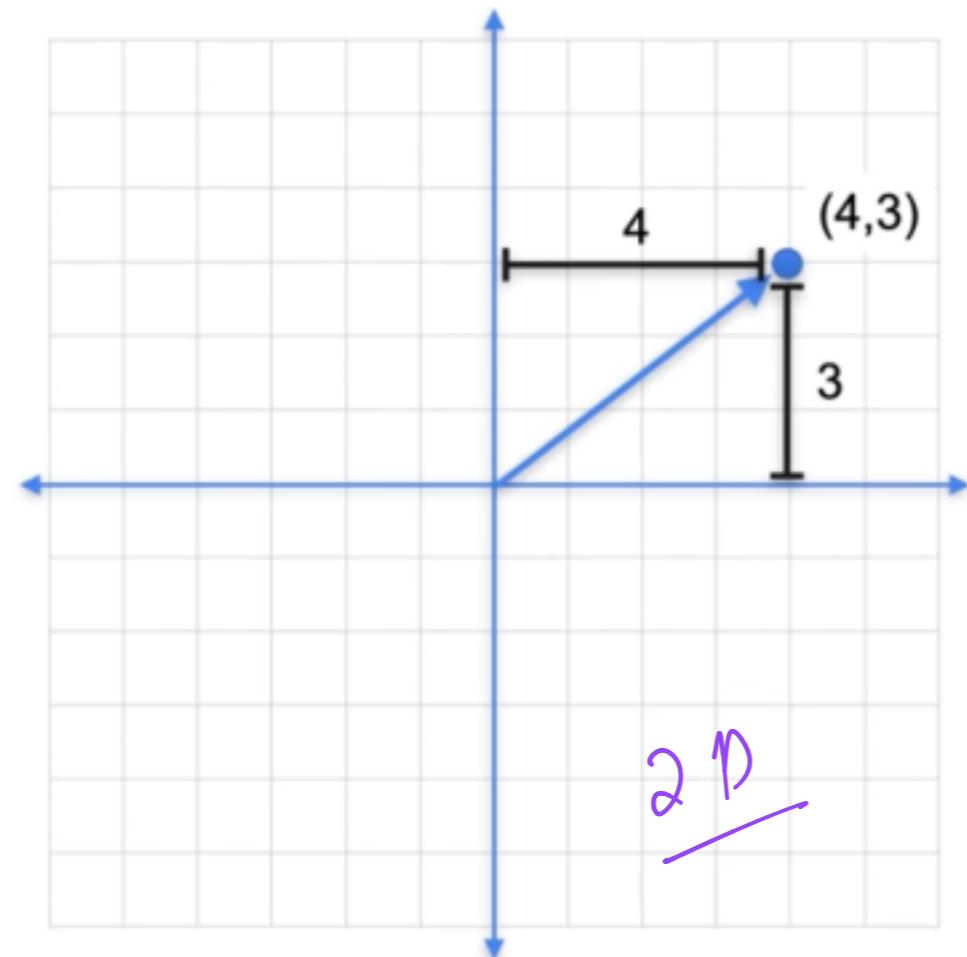
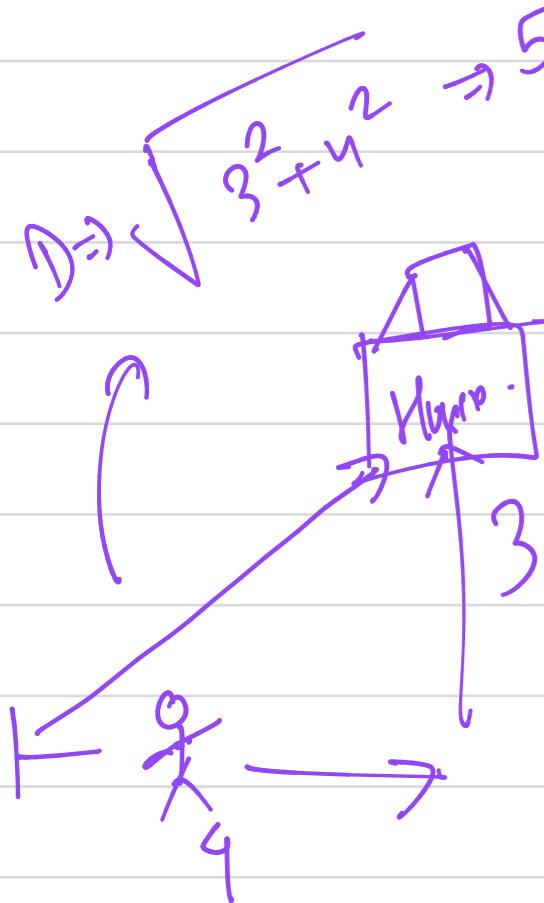


together can work to model  
highly nonlinear systems.

## # Vectors & Their Properties:

→ Vectors can be seen as arrows in the plane, or in a higher dimensional space.

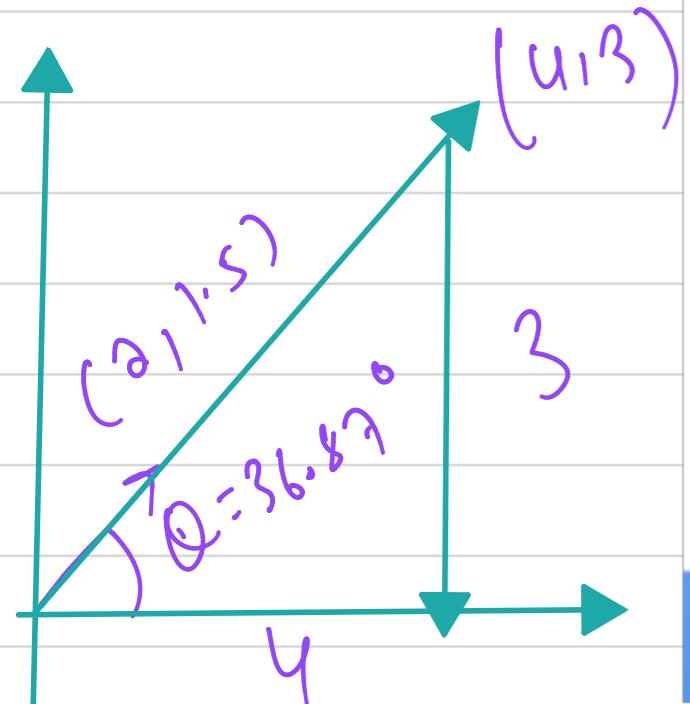
### Vectors



the magnitude or size,

Where to use

$L_2$  or  $L_1$ ?

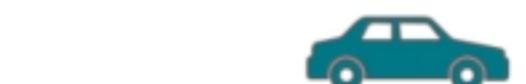


## Norms

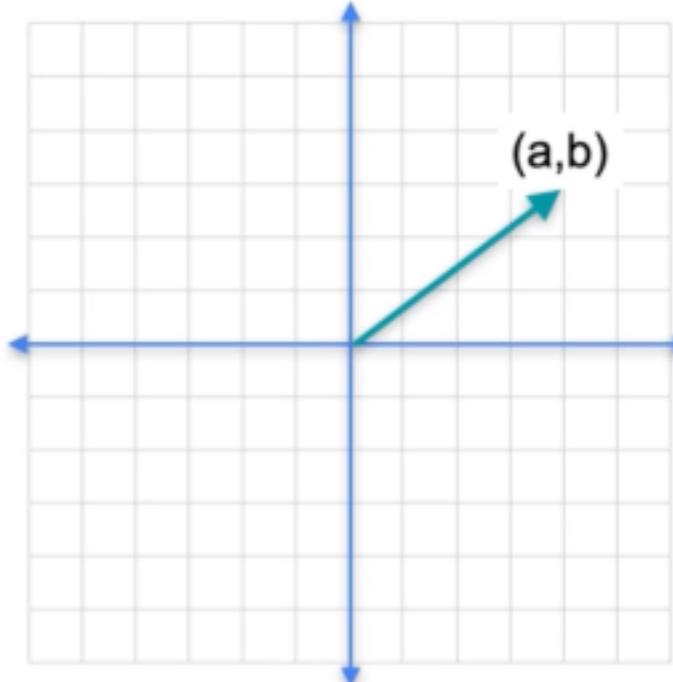
$$L_1\text{-Norm} = |(a, b)|_1$$

$$\Rightarrow |a| + |b|$$

Absolute  
Distance



$L_1\text{-norm} = |(a, b)|_1 = |a| + |b|$  but the distance to walk  
is always positive.



$$L_2\text{-Norm} = |(a, b)|_2$$

$$\Rightarrow \sqrt{a^2 + b^2}$$

Coefficient of  
Square:

- 1) Co-ordinates can help to deduce the Vector direction too [  $\tan\theta = 3/4$  ]
- 2) Smaller or larger depends upon the Vector Norms-

## Vector notation

Row vector  
 $x = (x_1 \ x_2 \ \dots \ x_n)$

Column vector  
 $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

Numbered Components

Other Vector Notation

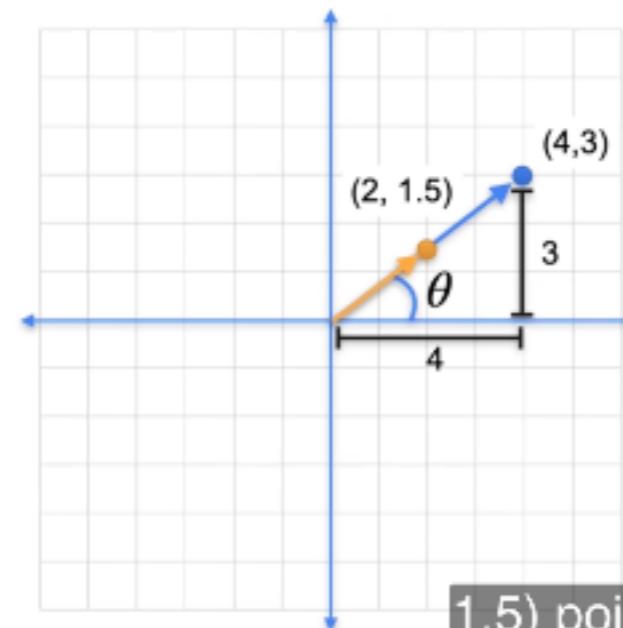
$\vec{x}$  May appear in outside resources

$[x_1 \ x_2 \ \dots \ x_n]$  Will use in this course, helps highlight vector is part of a matrix

$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

a vector is part of

## Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$
$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$

1.5) points in the same direction as the vector (4,

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$$x = (x_1 \ x_2 \ \dots \ x_n)$$

$$\|x\|_1 = |x_1| + |x_2| + \dots + |x_n|$$

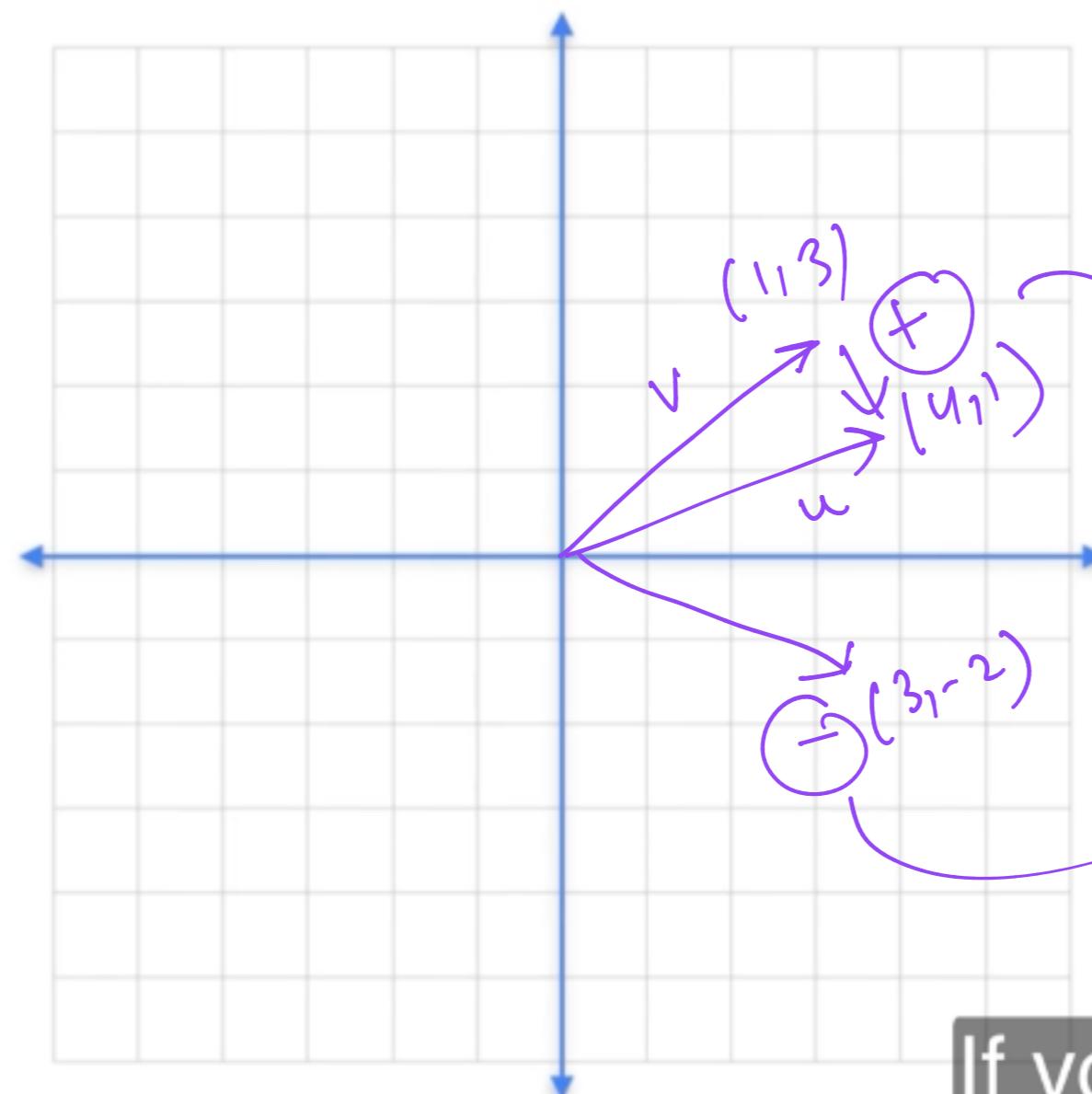
L<sub>1</sub> Norm.

L<sub>2</sub> norm.

Common practice to use.

$$\|x\|_2 = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

# Sum of vectors



Sum and Difference  
Element-wise operation  $\oplus$ .

$$(u+v) = (u+1, 1+3) \Rightarrow (5, 4)$$

$$(u-v) = (u-1, 1-3) \Rightarrow (3, -2)$$

If you'd like to  
add two vectors,

# General definition: sum and difference

Some no. of Components

$$x = (x_1 \ x_2 \ \dots \ x_n)$$

$$y = (y_1 \ y_2 \ \dots \ y_n)$$

## Sum

$$x + y = (x_1 + y_1 \ x_2 + y_2 \ \dots \ x_n + y_n)$$

Sum component by component

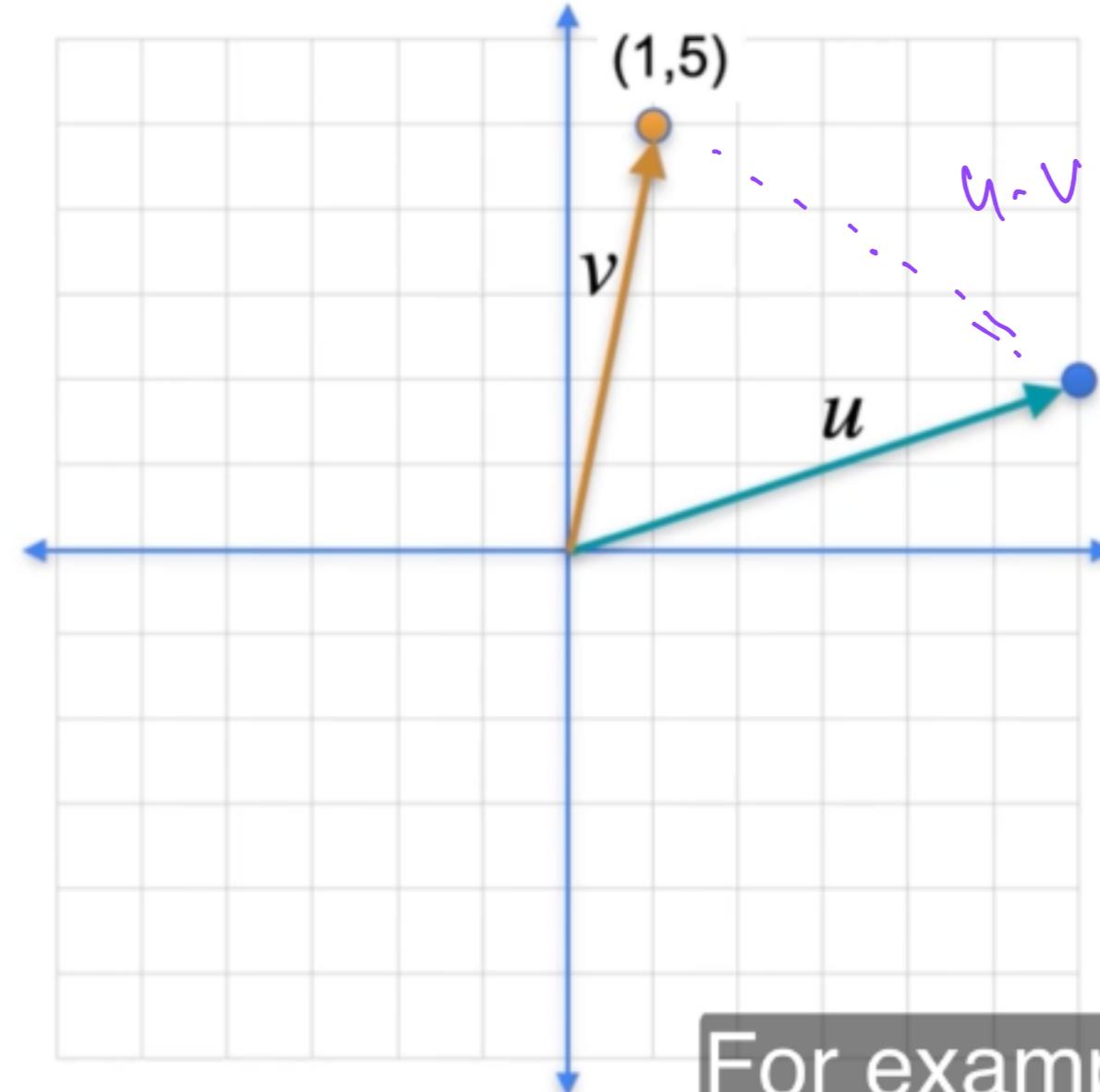
$$x - y = (x_1 - y_1, x_2 - y_2, \dots, x_n - y_n)$$

Subtract Component by Component

Note: Same number of Components -

the sum component by component.

# Distances



(How far apart 2 vectors are?)

use L<sub>1</sub> Distance -

$$\begin{aligned} L_1 &\Rightarrow (u-v) \\ &= (5, -3) \end{aligned}$$

use L<sub>2</sub>

$$\begin{aligned} \|u-v\|_2 &\Rightarrow \sqrt{5^2 + 3^2} \\ &\Rightarrow 5.83 \end{aligned}$$

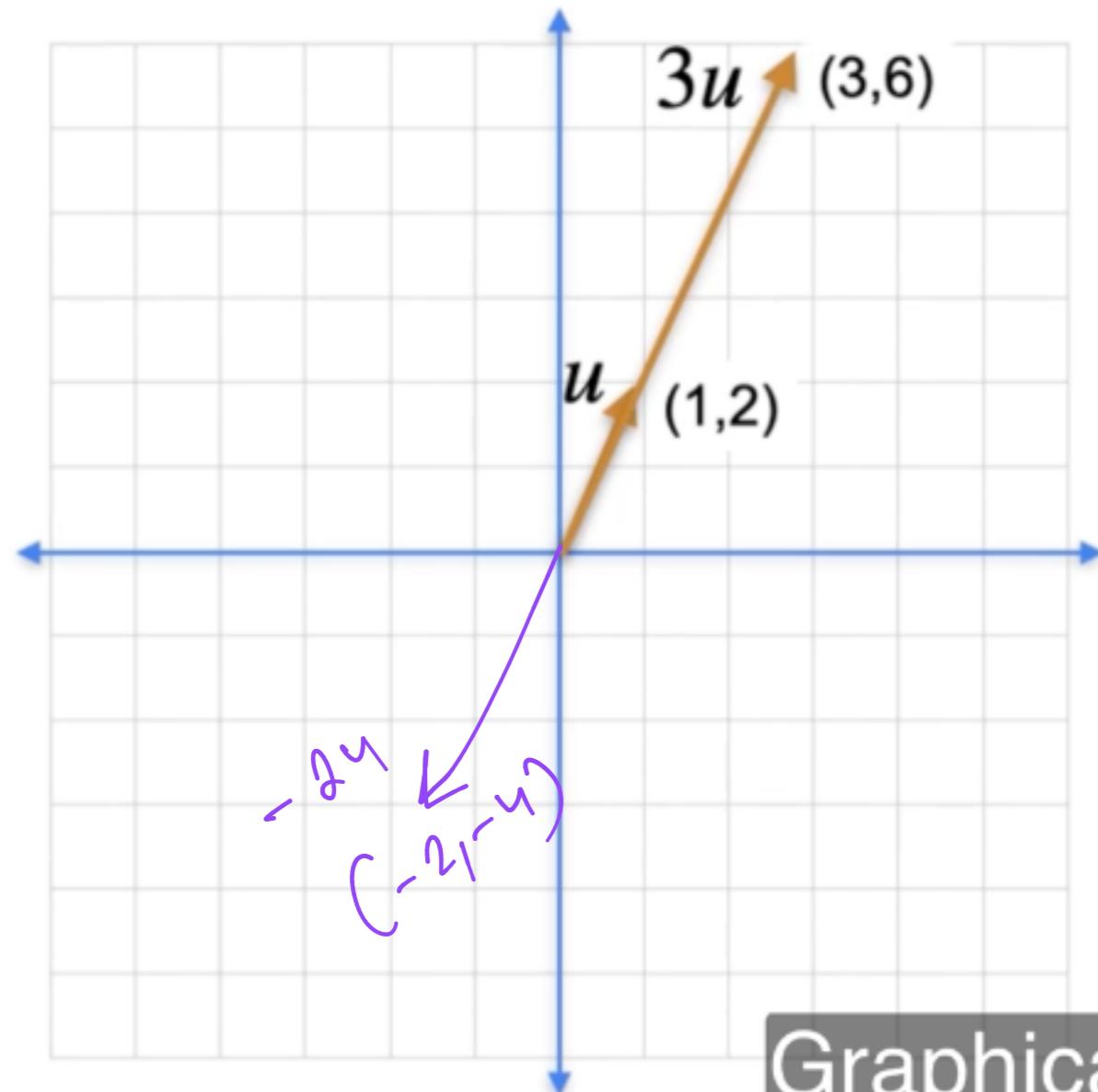
L<sub>1</sub>

$$\|u-v\|_1 \Rightarrow |5| + |-3|$$

$\Rightarrow 8$

For example, how different  
is the vector 1,

# Multiplying a vector by a scalar (+ve) .



$$u = (1, 2)$$

$$\lambda = 3$$

$$\lambda u = (3, 6)$$

$$u = (1, 2) \text{ -ve ?}$$

$$\lambda = -2$$

$$\lambda(u) = (-2, -4)$$

$$x = (x_1, x_2, \dots, x_n)$$

$$\lambda x = (\lambda x_1, \lambda x_2, \dots, \lambda x_n)$$

Multiplication by  
a scalar

Graphically means you  
stretch the vector 1,

Dot product  $\rightarrow$  Express linear equation using Matrices and Vectors

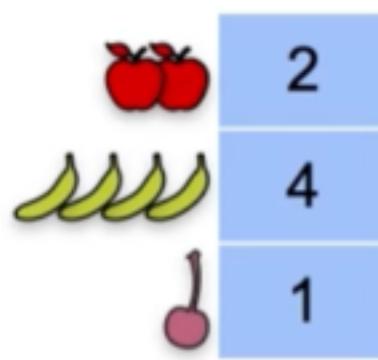
## A shortcut for linear operations

### Quantities

2 apples

4 bananas

1 cherry

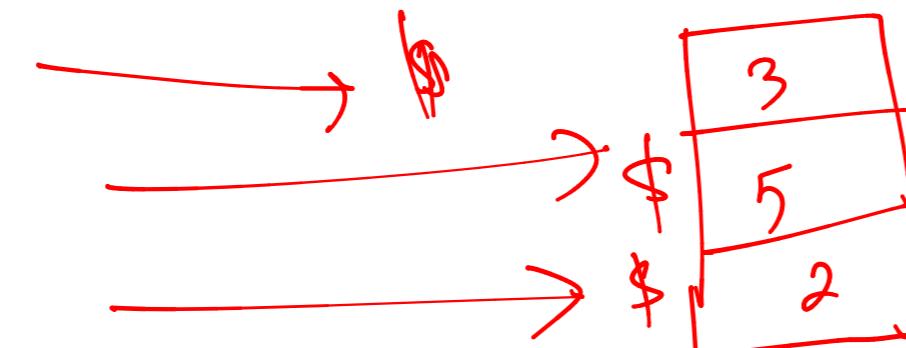


### Prices

apples: \$3

bananas: \$5

cherries: \$2



### Total price

$$2 \times 3 = 6$$

$$4 \times 5 = 20$$

$$1 \times 2 = 2$$

$$6 + 20 + 2 = 28$$

$$[2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = \$28]$$

2, 4, and 1.

# The dot product

Connection between  
Dot product and  
Norm?

$$\begin{matrix} 2 & 4 & 1 \end{matrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

$$(3 \sqrt{9}) (4 \sqrt{16}) = 28$$

and then forget about the  
fruits and get the dot product.

# Norm of a vector using dot product

Vector Transpose

$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

$$(2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2) = 28$$

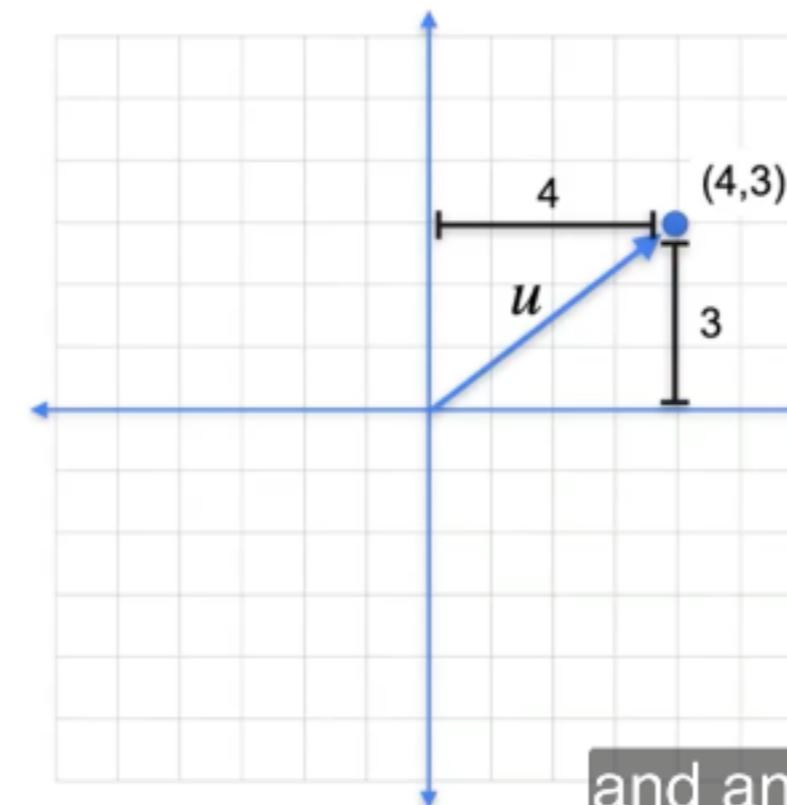
Transpose T

$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

↗

Convert columns to rows.

$$\begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}^T \Rightarrow \begin{bmatrix} 5 & 6 & 7 \end{bmatrix}$$



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} = 25$$

$$L2\text{-norm} = \sqrt{\text{dot product}(u, u)}$$

$$\|u\|_2 = \sqrt{\langle u, u \rangle}$$

and an angled bracket  
on the right.

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$$\begin{bmatrix} 2 & 6 \\ 3 & 8 \\ 5 & 9 \end{bmatrix}^T \Rightarrow \begin{bmatrix} 2 & 3 & 5 \\ 6 & 8 & 9 \end{bmatrix}$$

Columns  $\rightarrow$  Rows OR Rows  $\rightarrow$  Columns

# General definition: dot product

Same number  
of components

$$x = (x_1 \ x_2 \ \dots \ x_n) \qquad \qquad y = (y_1 \ y_2 \ \dots \ y_n)$$

$$x \cdot y = (x_1 \times y_1) + (x_2 \times y_2) + \dots + (x_n \times y_n)$$

$\langle x, y \rangle$  is another notation for the dot product

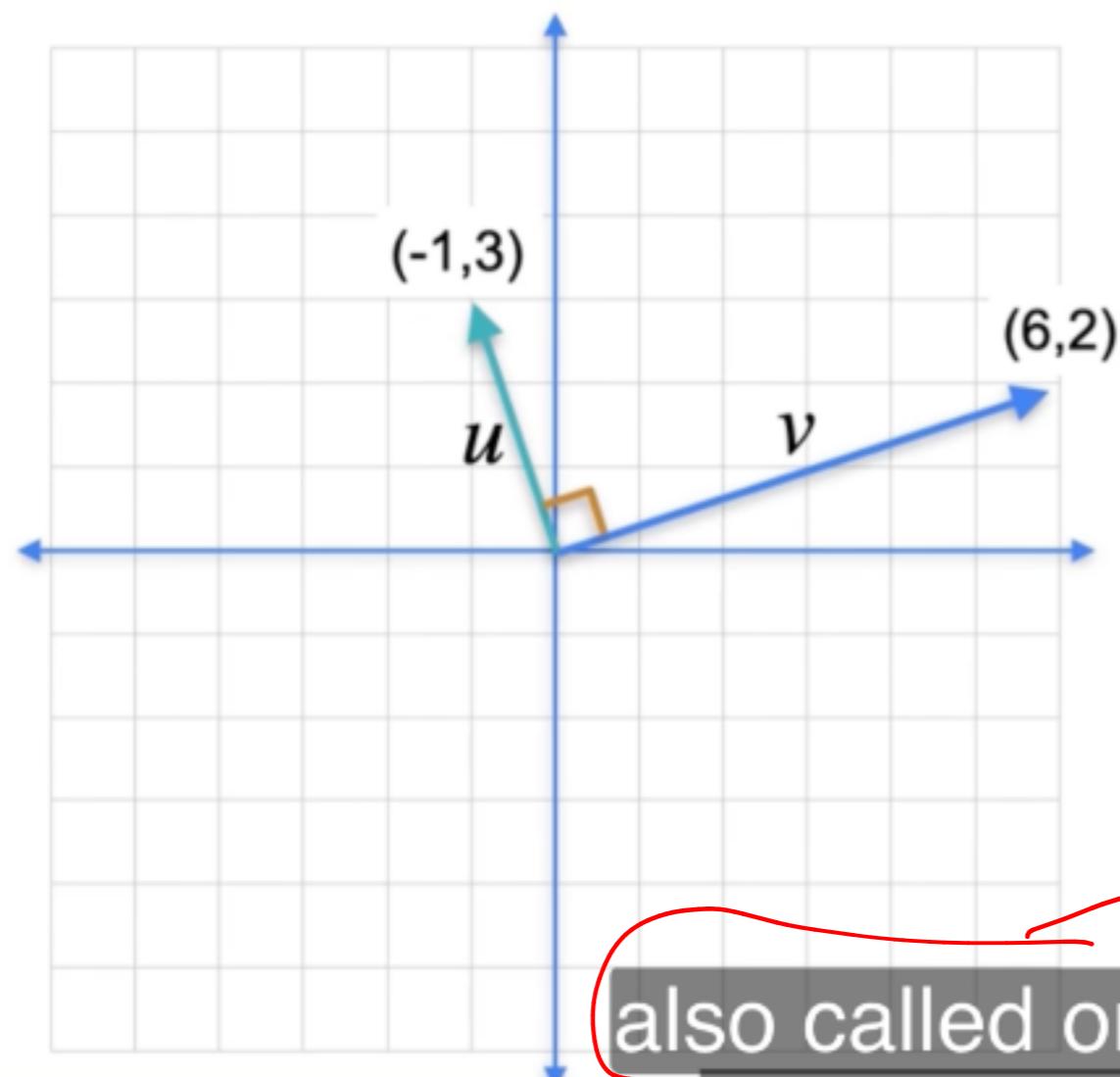
$$x \cdot y^T = (x_1 \ x_2 \ \dots \ x_n) \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \rightarrow \text{Column Vector on the Right}$$

Row Vector on the Left

another notation for the dot product.

Geometric Dot Product | Angle between two vectors is very important |

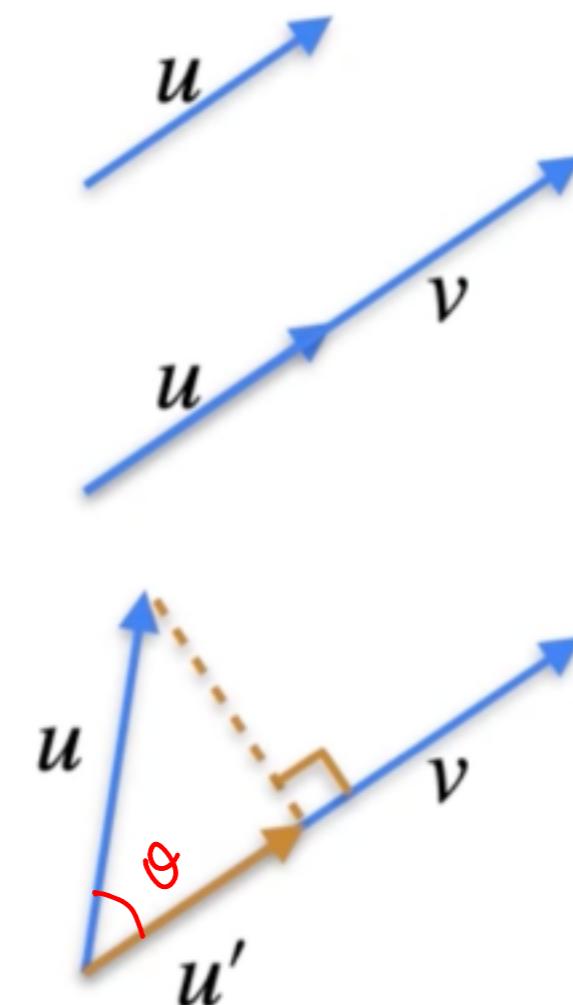
## Orthogonal vectors have dot product 0



also called orthogonal vectors,  
with entries -1, 3 and 6, 2.

$$\begin{aligned} & \text{Unidirection} \\ & \langle u, v \rangle = |u|^2 \\ & [6 \ 2] \begin{bmatrix} -1 \\ 3 \end{bmatrix} \Rightarrow [0] \\ & \langle u, v \rangle = 0 \\ & \text{the dot product between} \\ & \text{two 1 or orthogonal} \\ & \text{vectors is always 0.} \end{aligned}$$

# The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

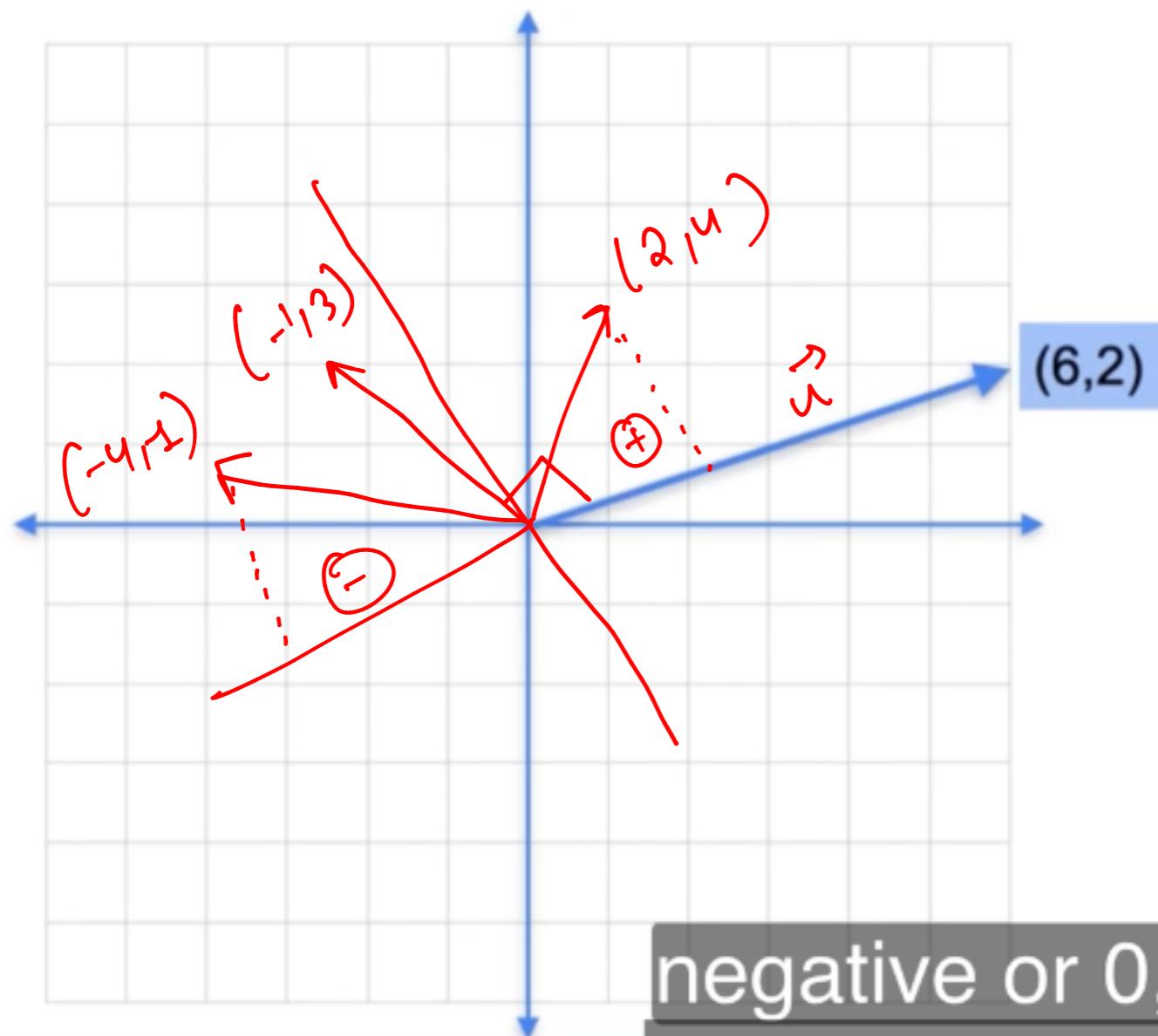
$$\langle u, v \rangle = |u| \cdot |v|$$

$$\langle u, v \rangle = |u'| \cdot |v|$$

$$\Rightarrow |u| \cdot |v| \cdot \cos \theta$$

v is the product of the magnitude  
of u prime and the magnitude of v.

# Geometric dot product



negative or 0, for example,  
take a look at a vector 6, 2.

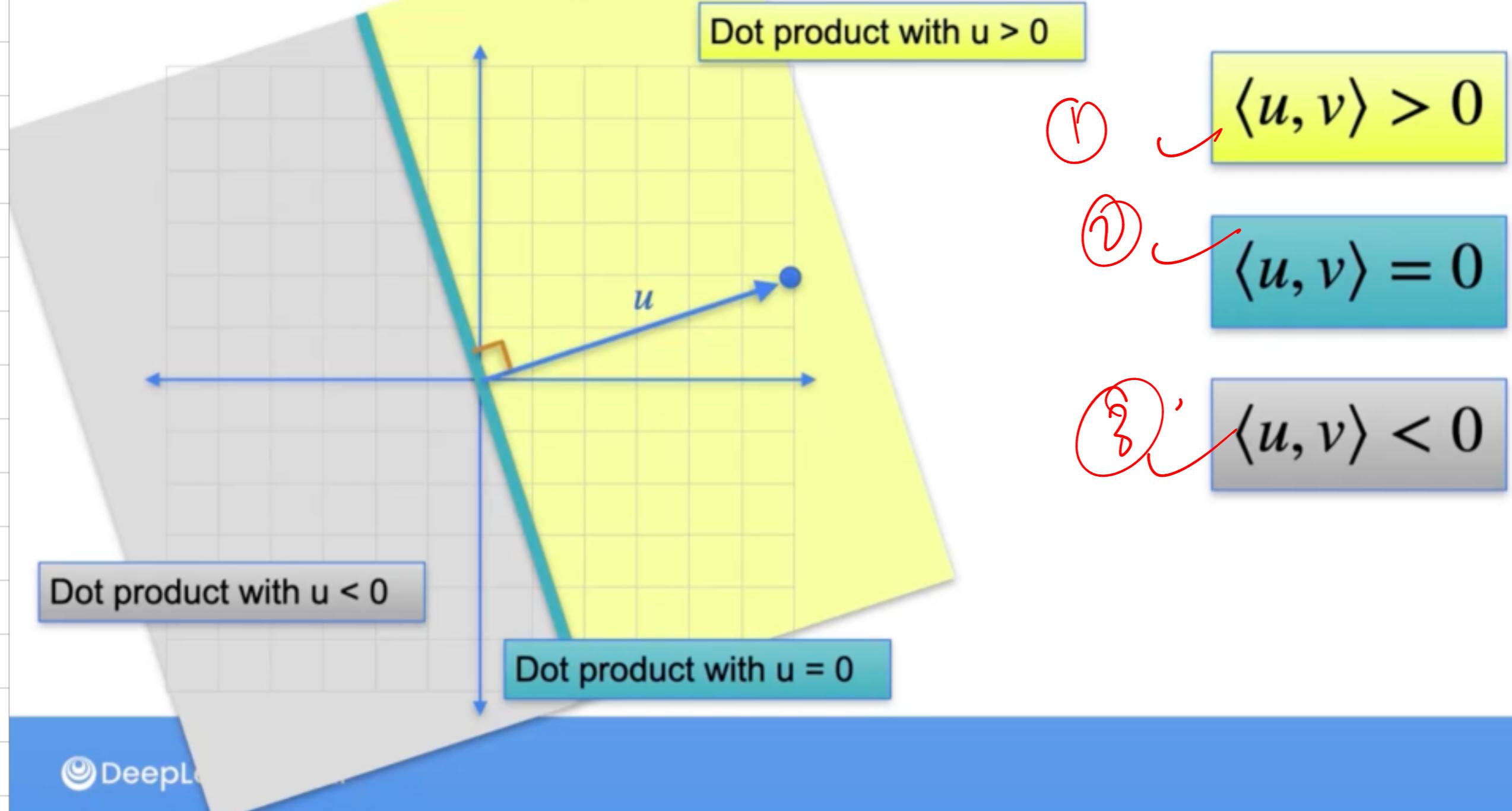
$$[6 \ 2] \begin{bmatrix} -4 \\ 1 \end{bmatrix} \Rightarrow -22 \quad (-ve)$$

$$[6 \ 2] \begin{bmatrix} 1 \\ 3 \end{bmatrix} \Rightarrow [0]$$

$$[6 \ 2] \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20 \quad (+ve)$$

# Glimpse of Classification for ML ??

## Geometric dot product



# Multiplying a Matrix by a Vector

$$a+b+c=10 \quad | \quad a+2b+c=15 \quad | \quad a+b+2c=12$$

$\approx 10$

## Equations as dot product

$$2a + 4b + c = 28$$

$2a + 4b + c = \$28$

## Equations as dot product

### System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

### Matrix product

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix} \cdot \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 10 \\ 15 \\ 12 \end{matrix}$$

$3 \times 3$        $3$

If you have more equations in your system,

An equation with a known variable, such as, for example,

If columns = length of vector  
can be unstacked

# Equations as dot product

## Matrix product

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{matrix} \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 10 \\ 15 \\ 12 \\ 13 \end{matrix}$$

4 × 3   Length 3



Matrix can be rectangular

the vector containing the three components, a, b,

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4 × 3      3 × 3      → 3 × 3  
3 :  
4 × 3      3 × 1      4 × 1

# Equations as dot product

## Matrix product

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{matrix} \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 10 \\ 15 \\ 12 \\ 13 \end{matrix}$$

4 × 3   Length 3   Length 4



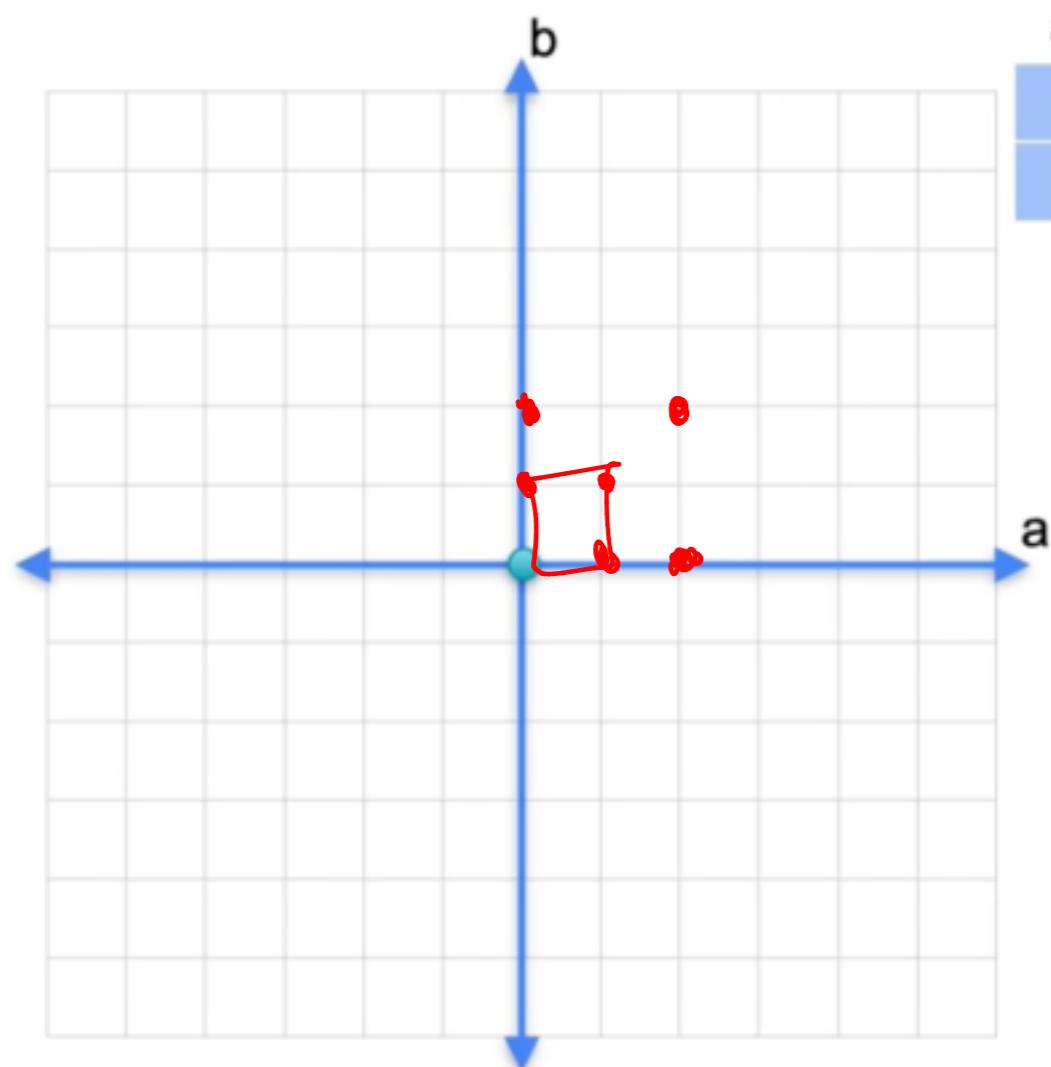
Matrix can be rectangular

prepare you for the graded quiz at the end of the week.

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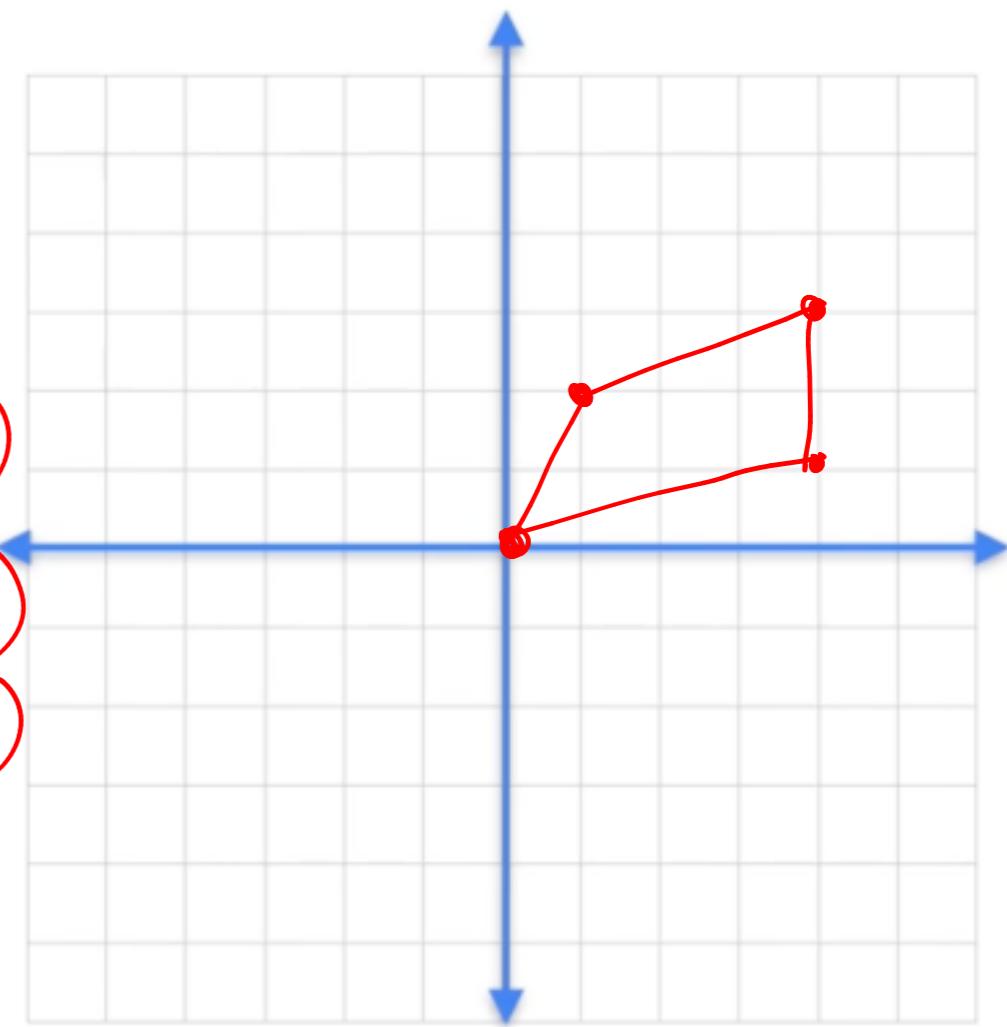
# linear Transformations: transform A → B in a standard way.

## Matrices as linear transformations



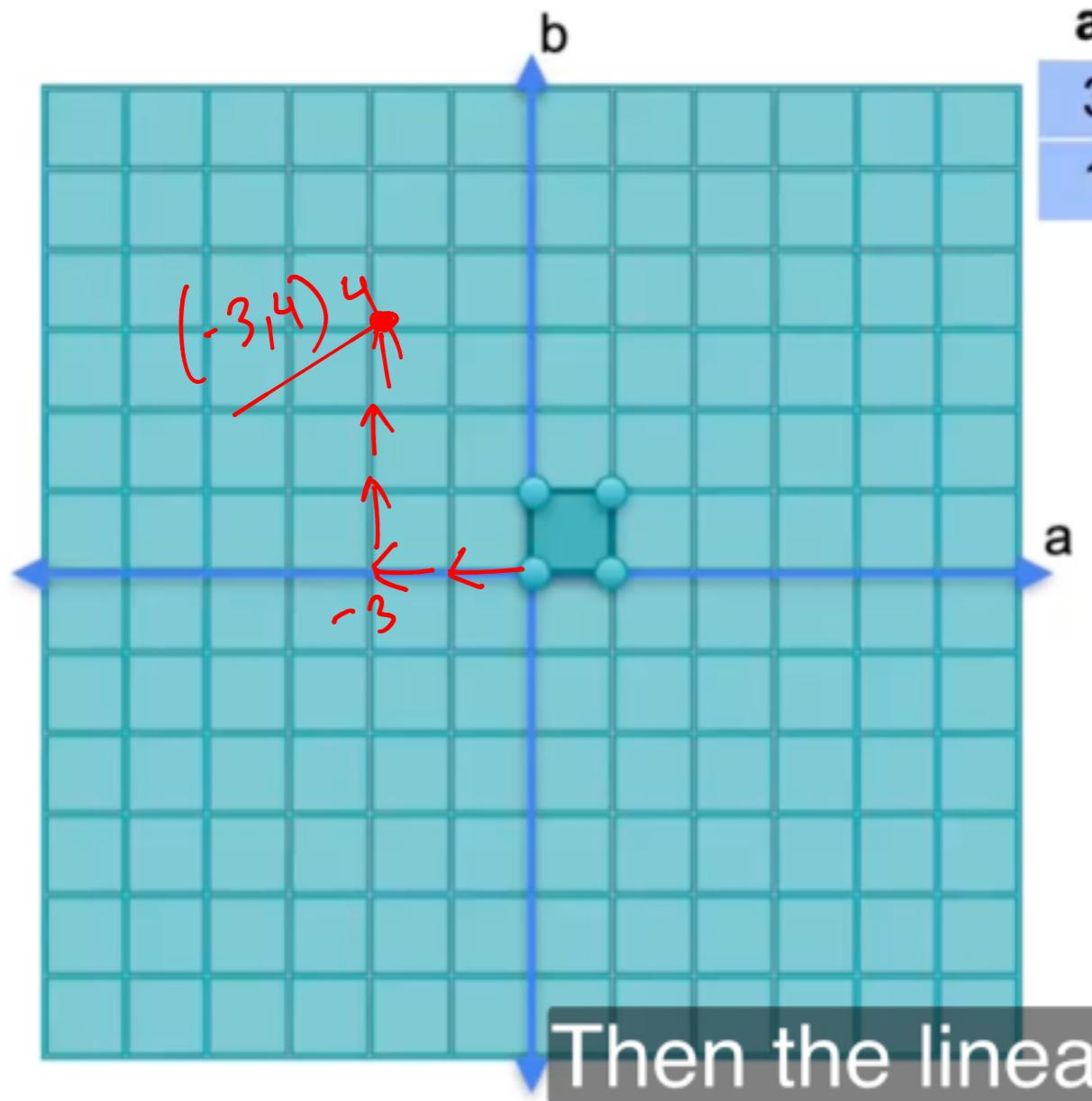
a	b	
3	1	0
1	2	0

$$\begin{aligned}(0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \\ (1,1) &\rightarrow (4,3)\end{aligned}$$

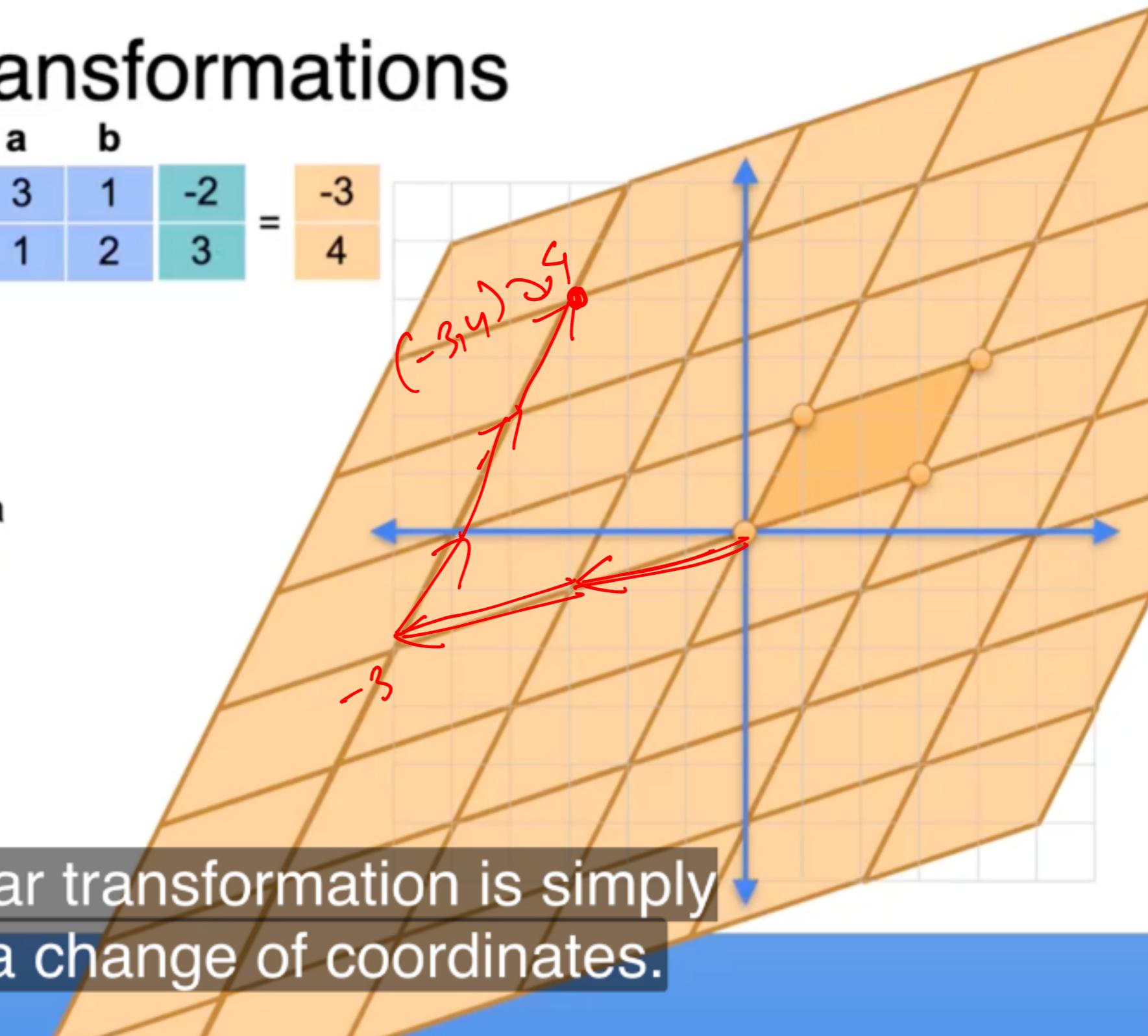


The 0 0 that becomes the vector 0 0,

# Matrices as linear transformations



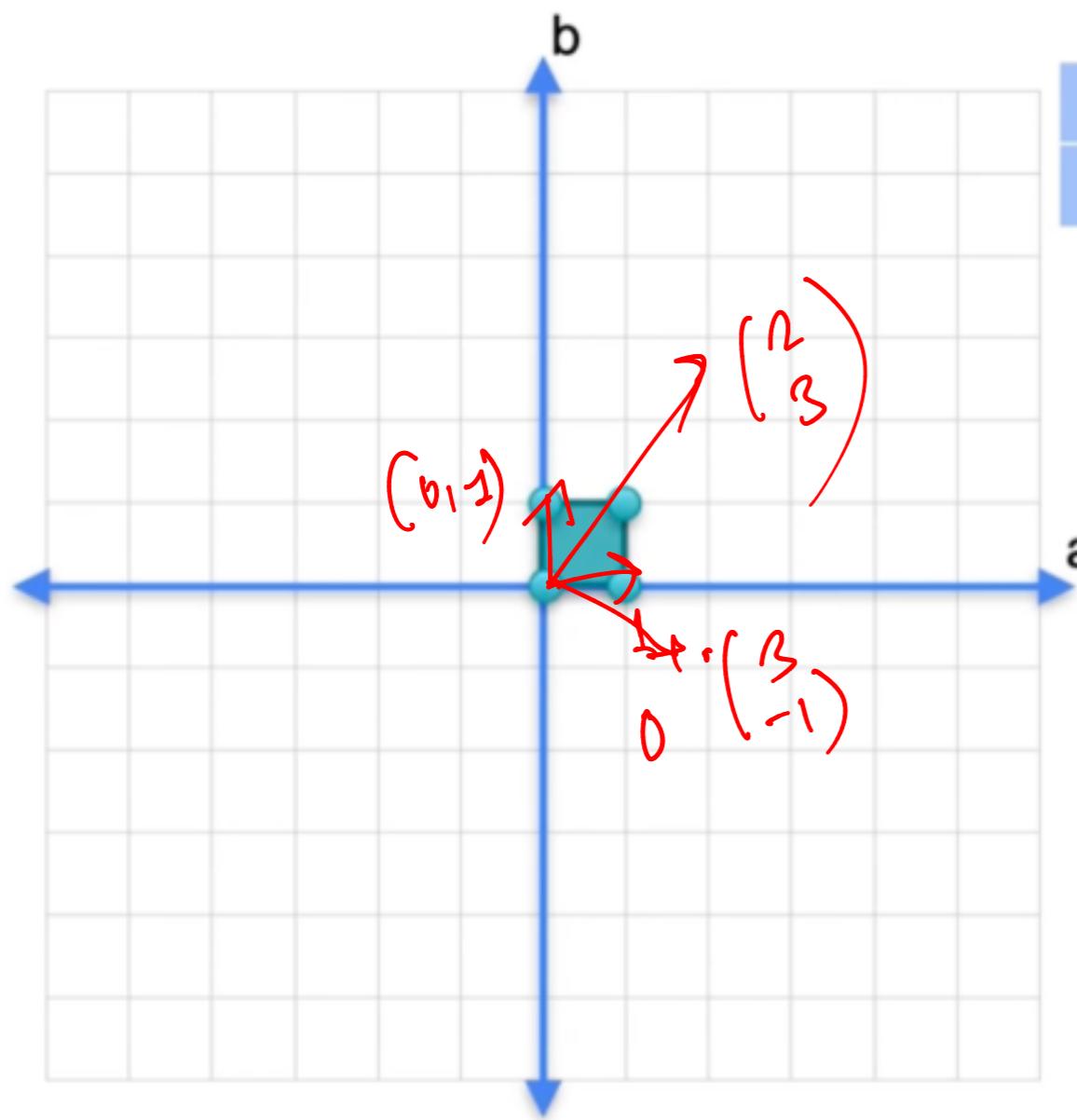
$$\begin{array}{cc} \mathbf{a} & \mathbf{b} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} -2 \\ 3 \end{matrix} = \begin{matrix} -3 \\ 4 \end{matrix} \end{array}$$



Then the linear transformation is simply defined as a change of coordinates.

# Linear Transformations as Matrices:

## Linear transformations as matrices



$a$	$b$
?	?
?	?

$\Rightarrow \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$a \quad b$

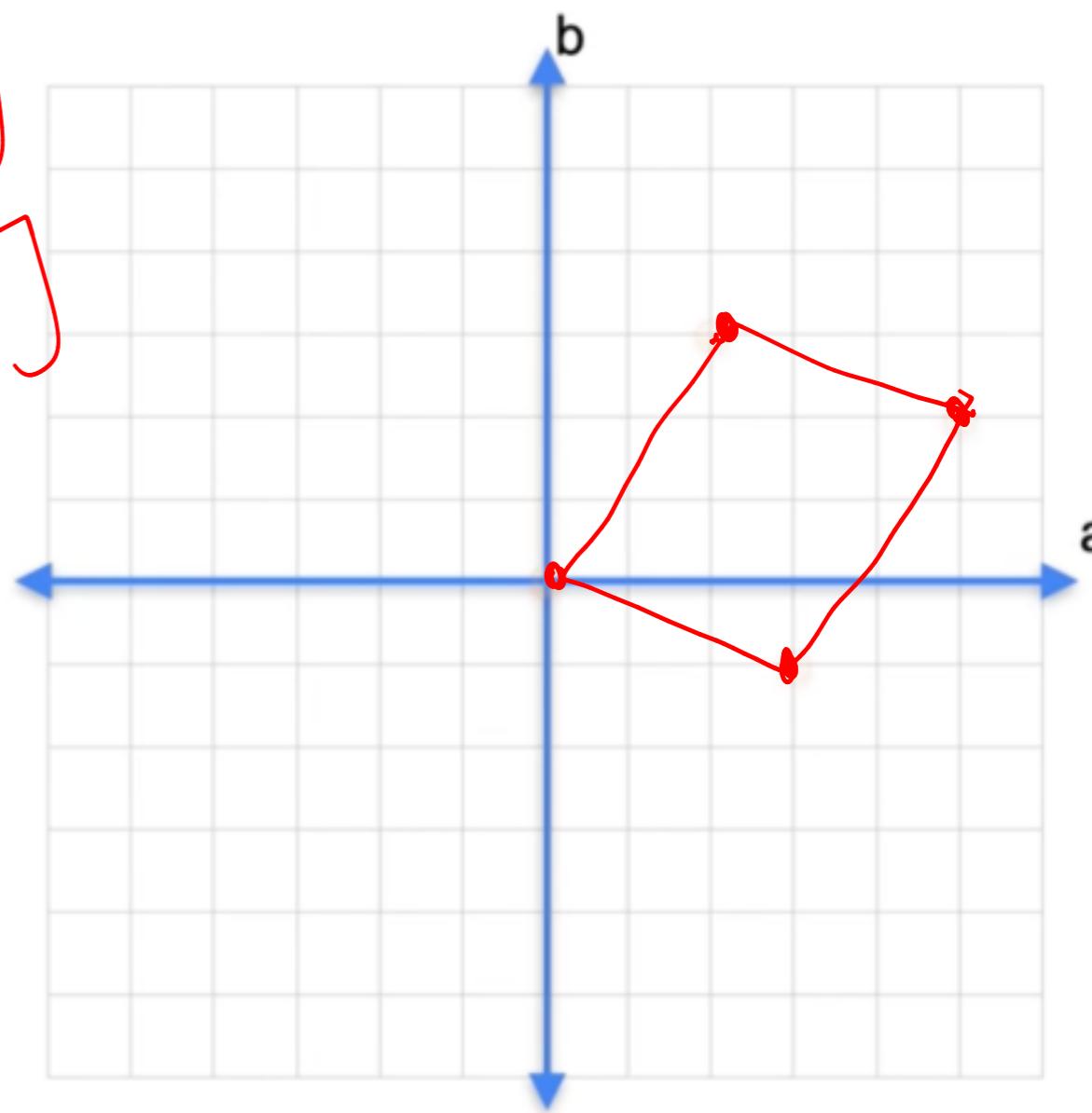
$? \quad ?$

$(0,1) \rightarrow (0,0)$

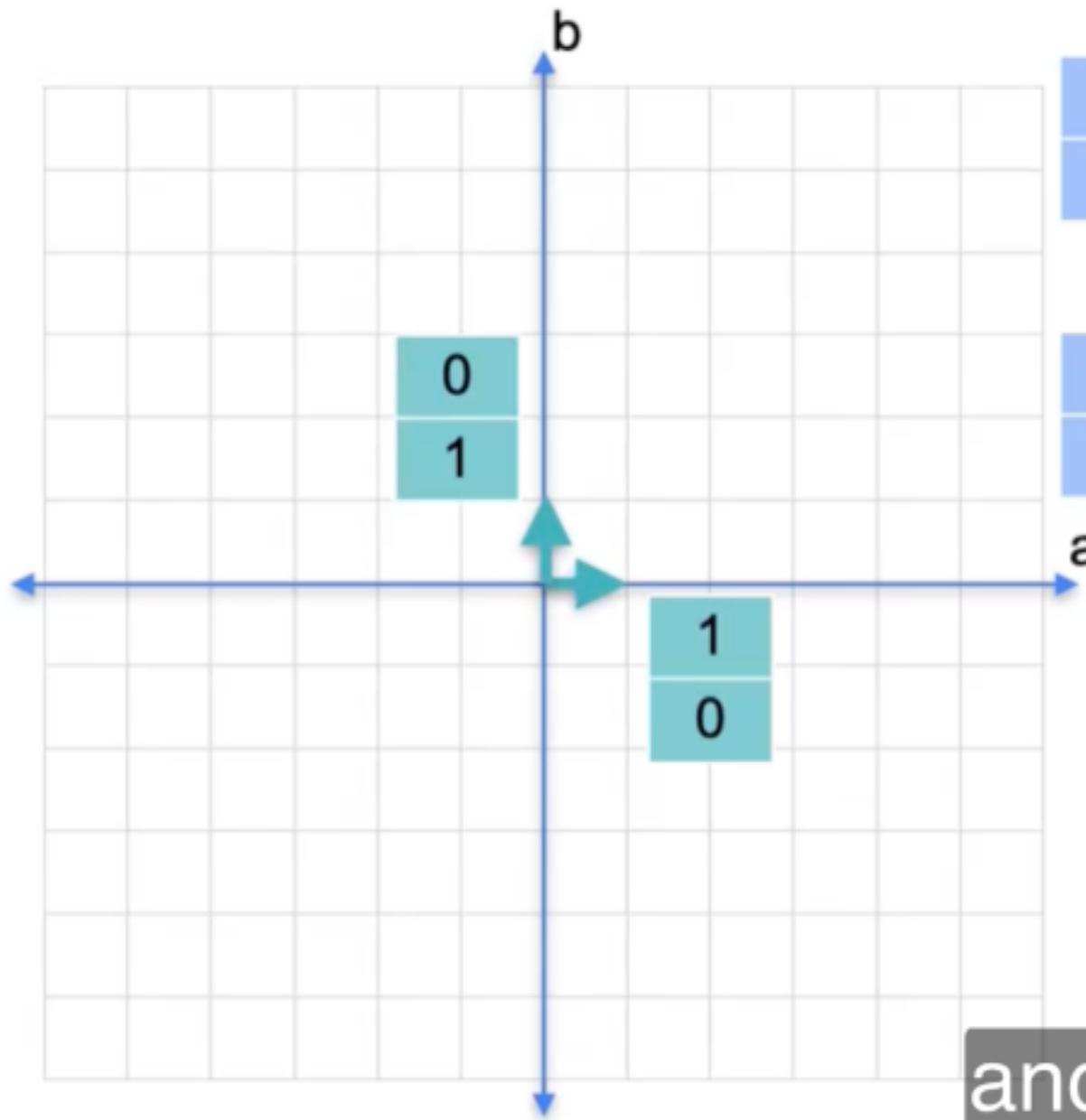
$(1,0) \rightarrow (3,-1)$

$(1,0) \rightarrow (3,-1)$

$(0,1) \rightarrow (2,3)$



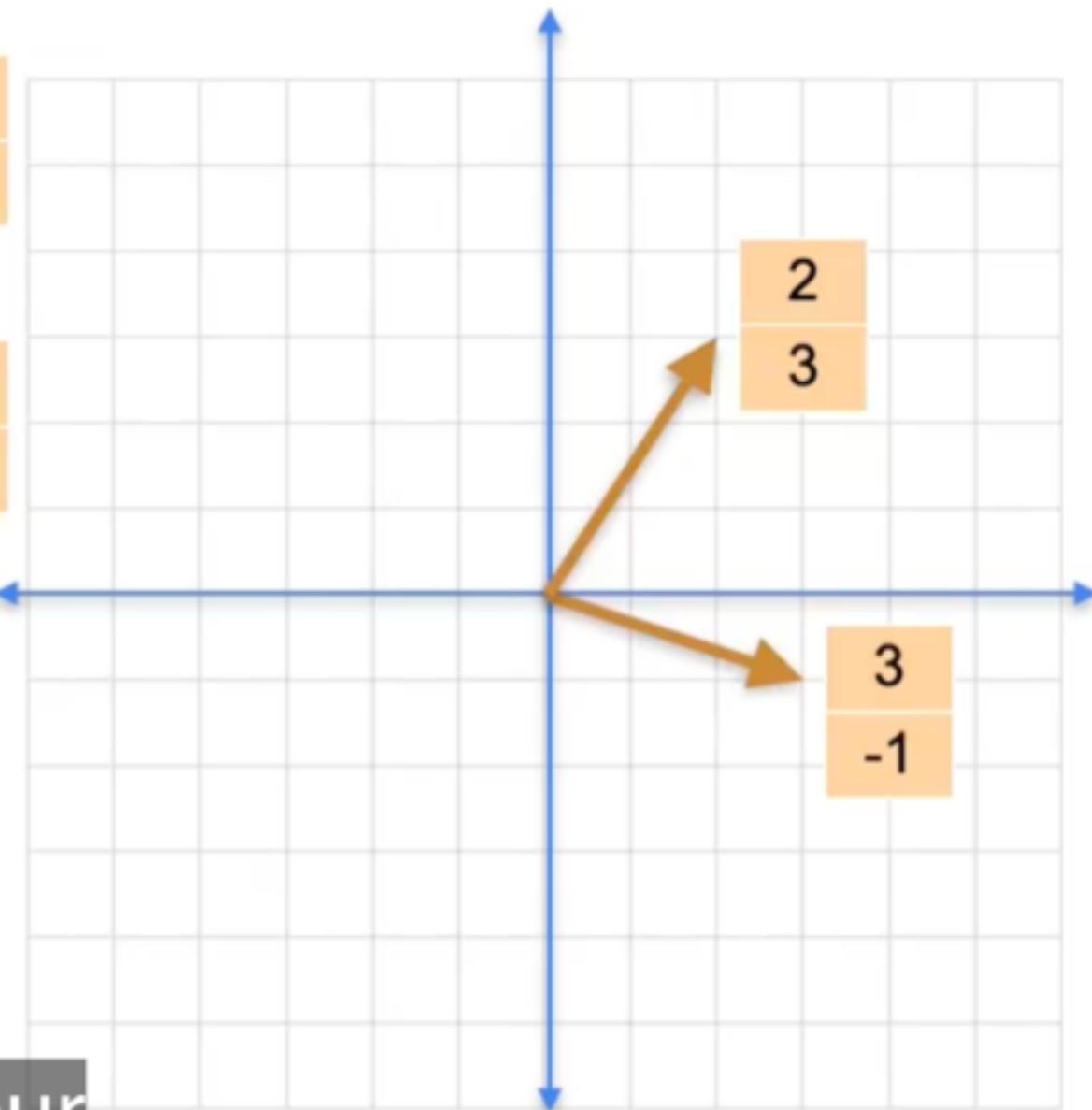
# Linear transformations as matrices



$$\begin{array}{c|c|c|c} & \mathbf{a} & \mathbf{b} & \\ \hline & 3 & 2 & 0 \\ & -1 & 3 & 1 \end{array} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{array}{c|c|c|c} & \mathbf{a} & \mathbf{b} & \\ \hline & 3 & 2 & 1 \\ & -1 & 3 & 0 \end{array} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

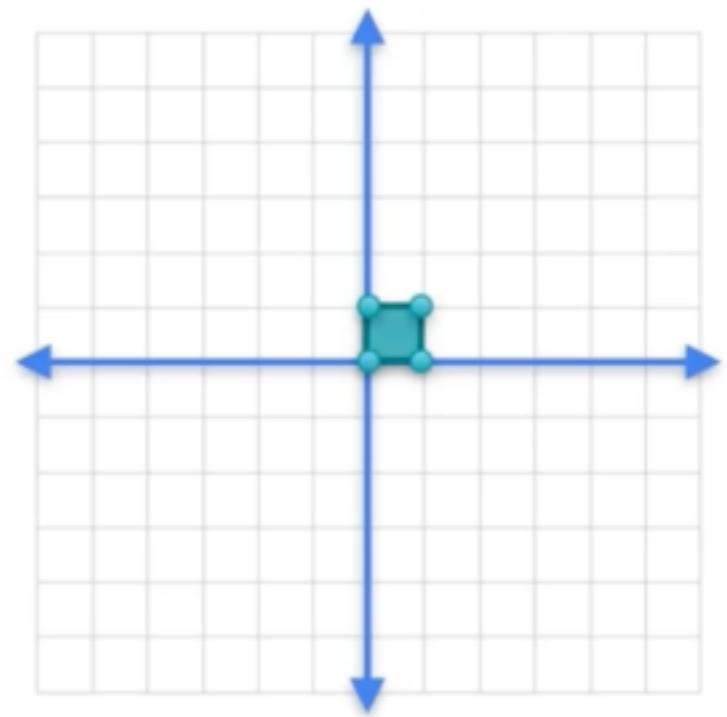
$$\begin{aligned} (1,0) &\rightarrow (3, -1) \\ (0,1) &\rightarrow (2, 3) \end{aligned}$$



and those are your  
columns of the matrix.

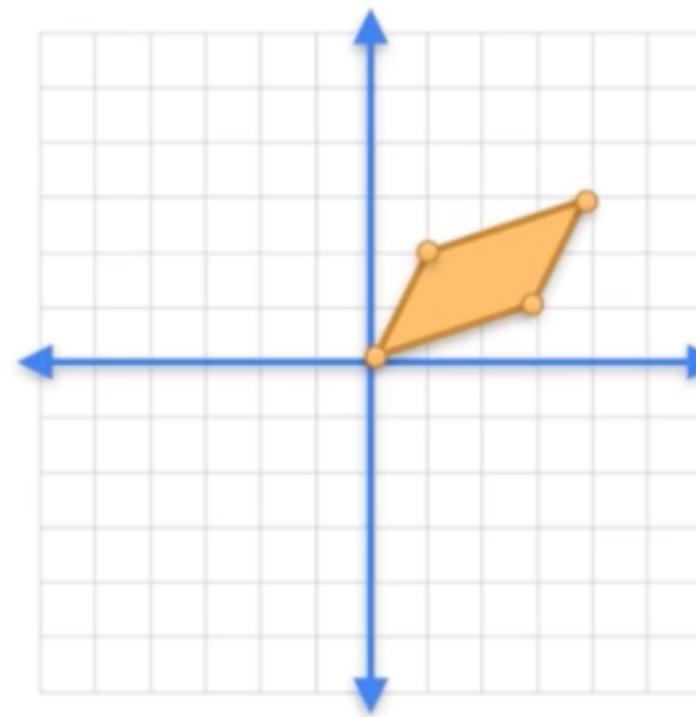
# # Multiplication of Matrices

## Combining linear transformations

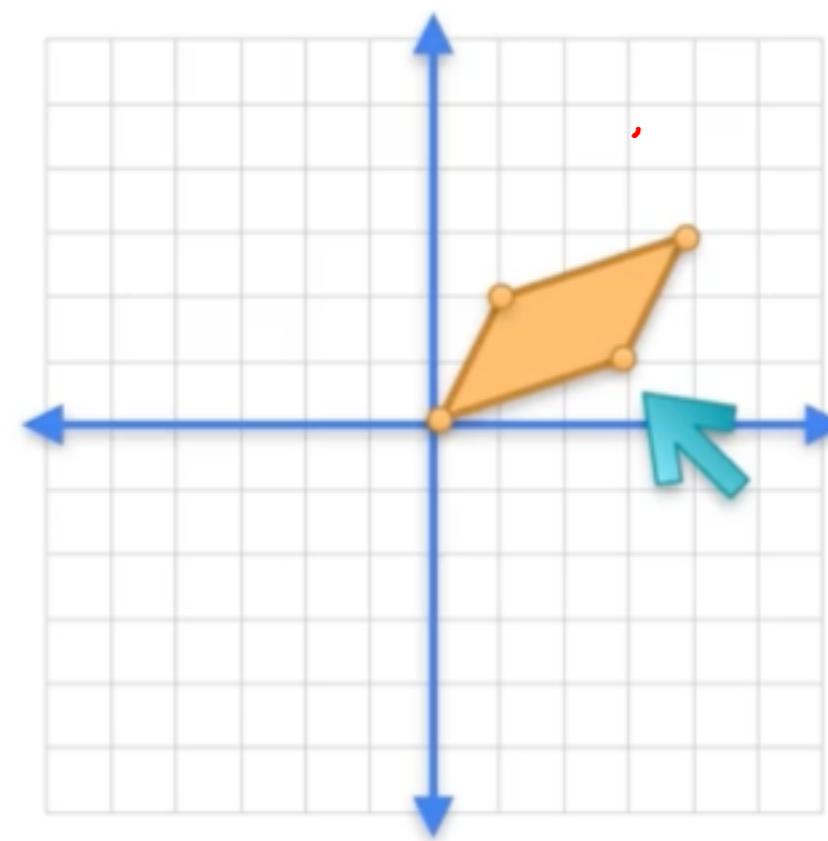


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

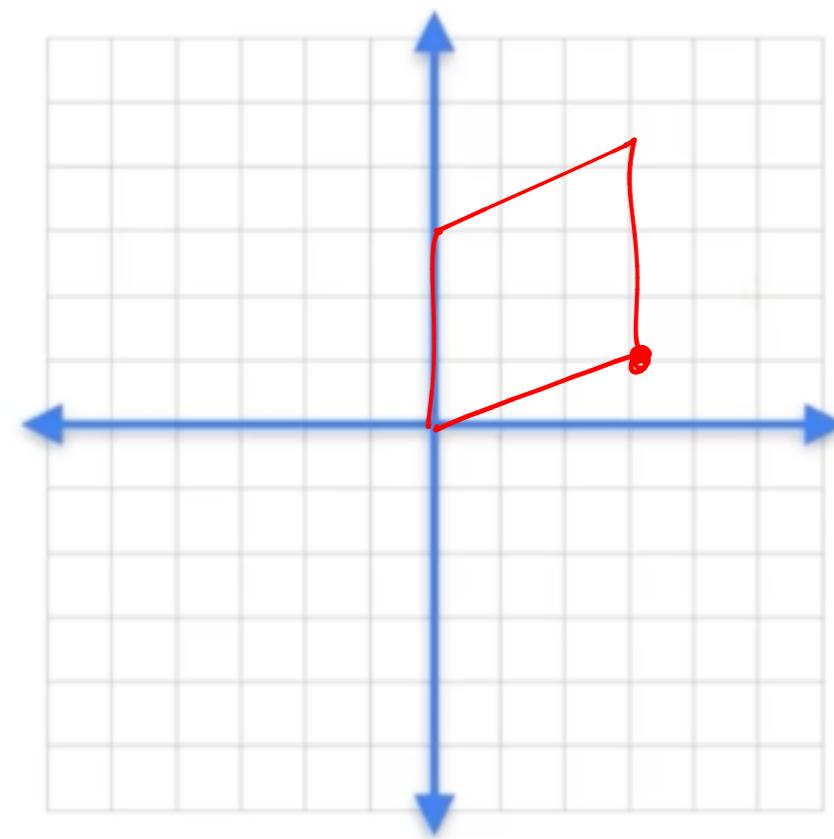


# Combining linear transformations



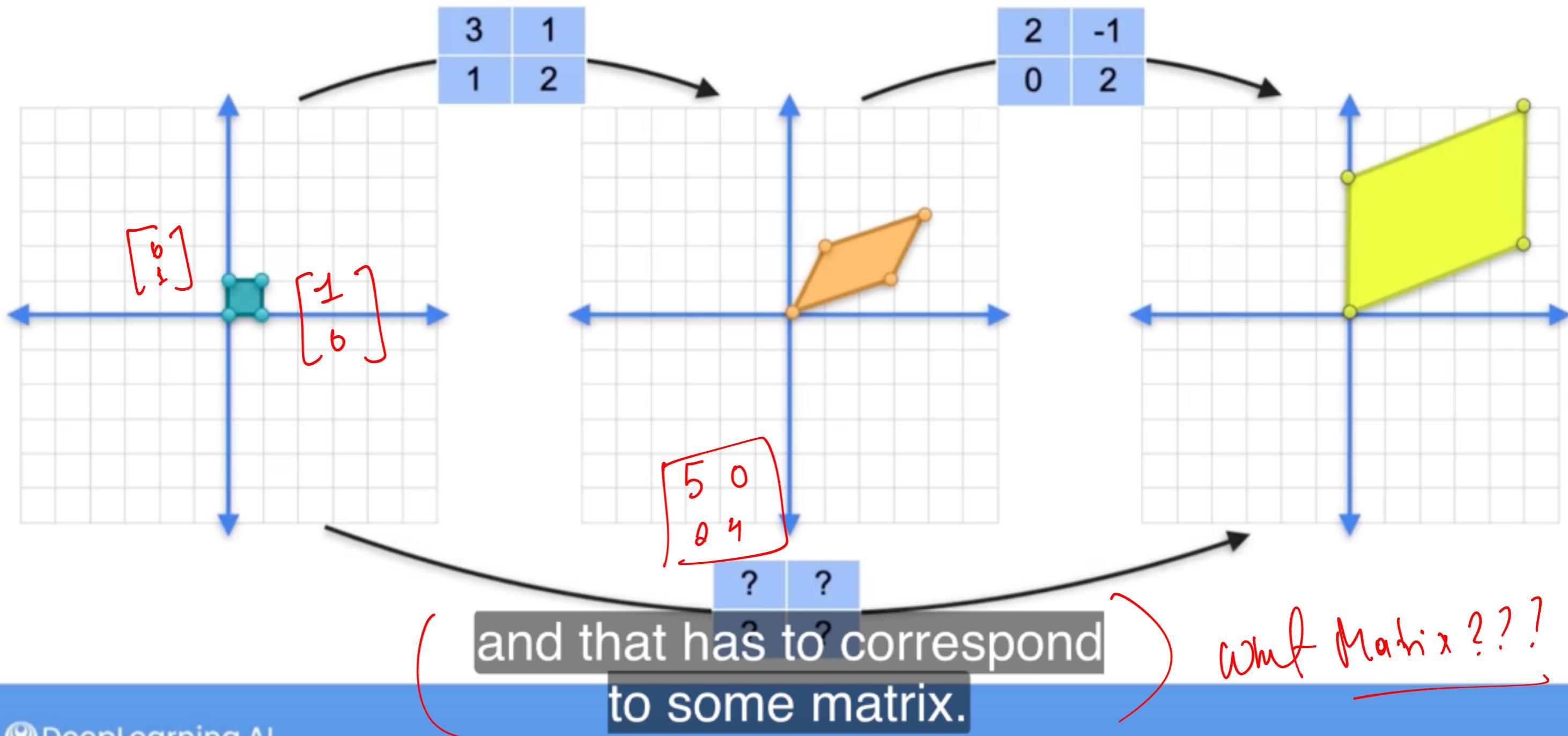
$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 12 & 2 \end{matrix} \begin{matrix} 0 \\ 9 \end{matrix}$$

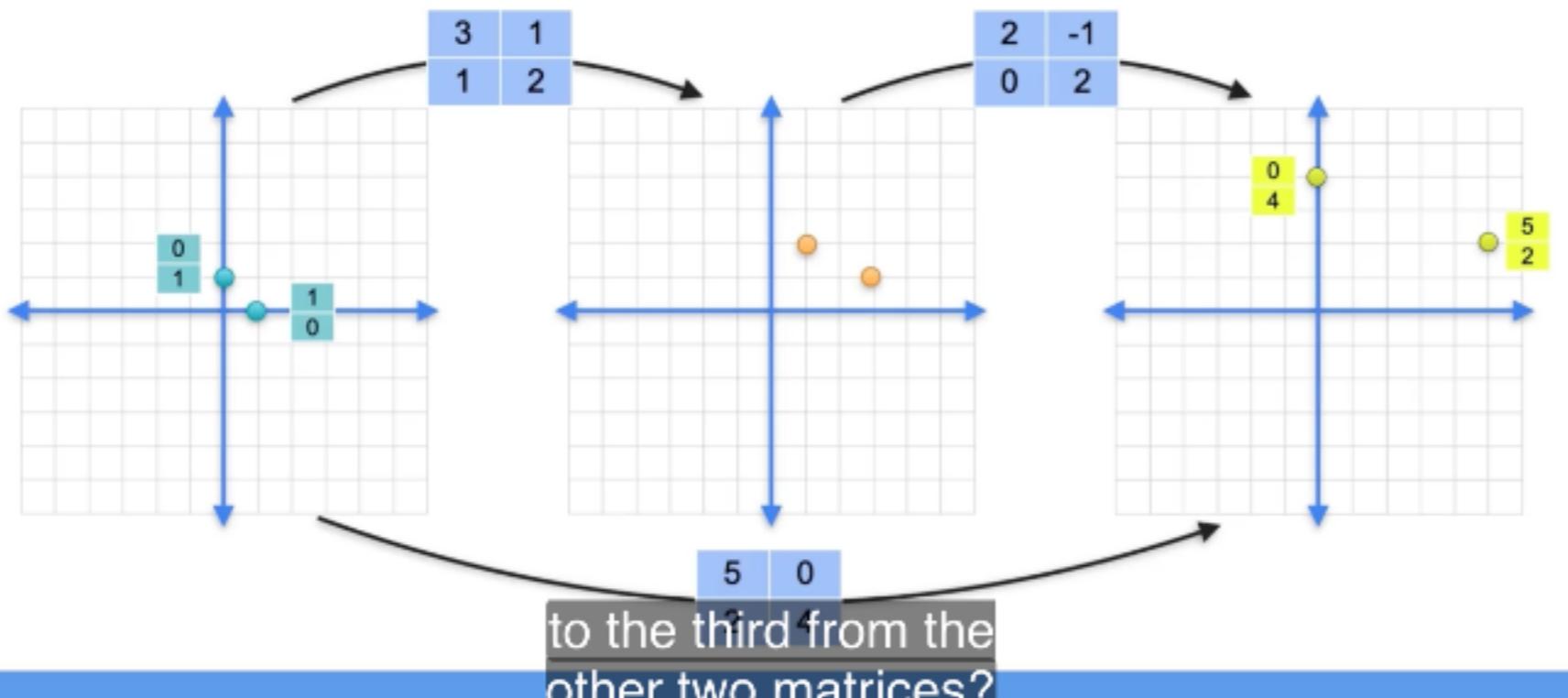


goes to point 2, 5 on the right.

# Combining linear transformations



## Combining linear transformations



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## Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 0 & 2 & 1 & 2 \end{bmatrix}$$

$2 \times 2 \quad 2 \times 2$

$2 \times 2 \quad 2 \times 2$

go in the top left corner,

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$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} s & t \\ u & v \end{bmatrix}$$

$$\begin{bmatrix} a \cdot e + b \cdot g & a \cdot f + b \cdot h \\ c \cdot e + d \cdot g & c \cdot f + d \cdot h \end{bmatrix}$$

$$= \begin{bmatrix} 2 \cdot 3 + (-1) \cdot 1 & 2 \cdot 1 + (-1) \cdot 2 \\ 0 \cdot 3 + 2 \cdot 1 & 0 \cdot 1 + 2 \cdot 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

$(2 \times 3) \quad (3 \times 4) \Rightarrow 2 \times 4 \text{ Matrix Result}$

$$\begin{bmatrix} 3 & 1 & 4 \\ 2 & -1 & 2 \end{bmatrix}$$

$$\begin{array}{|cccc|} \hline 3 & 0 & 1 & -2 \\ \hline 1 & 5 & 2 & 0 \\ \hline -2 & 1 & 4 & 0 \\ \hline \end{array}$$

$$R_1 \quad \begin{array}{|cccc|} \hline c_1 & c_2 & c_3 & c_4 \\ \hline 2 & 9 & 21 & -6 \\ \hline 1 & -3 & 8 & -4 \\ \hline \end{array}$$

$$\begin{bmatrix} 3 & 1 & 4 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

$$\Rightarrow (3 \times 3 + 1 \times 1 + (-8)) =$$

$$\begin{bmatrix} 2 & -1 & 2 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 4 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} = (0 + 5 + 4) = 9$$

$$\Rightarrow 6 - 1 - 4 \\ \Rightarrow 5 - 4$$

$$\boxed{2 \times 3} \quad \boxed{3 \times 4}$$

$$\begin{bmatrix} 3 & 1 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow 3 + 2 + 0 | 6 \cdot \Rightarrow$$

$$\begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$\boxed{2 \times 2} \times \boxed{1 \times 2}$$

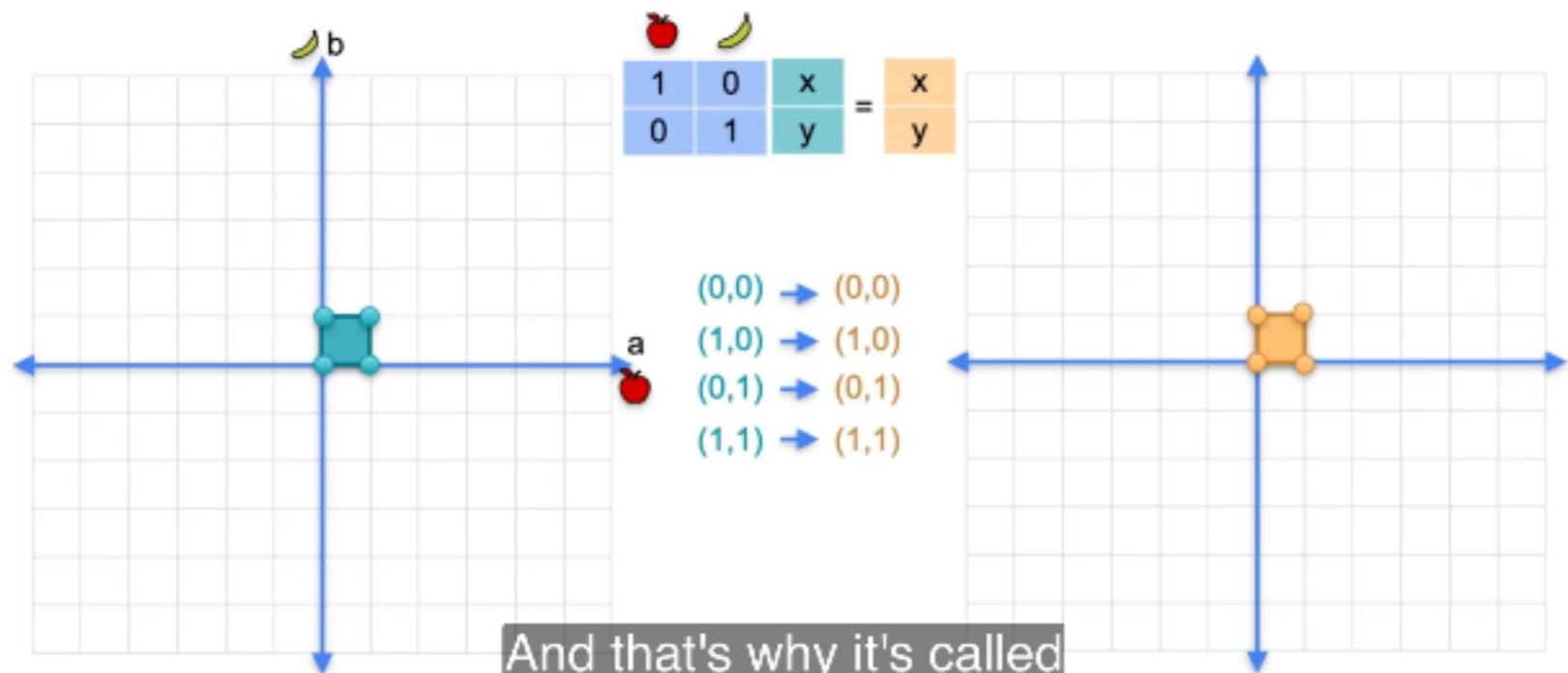
# Dimension of the matrices

$$\begin{matrix} \begin{matrix} 3 & 1 & 4 \\ 2 & -1 & 2 \end{matrix} & \cdot & \begin{matrix} 3 & 0 & 1 & -2 \\ 1 & 5 & 2 & 0 \\ -2 & 1 & 4 & 0 \end{matrix} & = & \begin{matrix} 2 & 9 & 21 & -6 \\ 1 & -3 & 8 & -4 \end{matrix} \\ \begin{matrix} 2 \times 3 \\ 2 \times 3 \end{matrix} & \cdot & \begin{matrix} 3 \times 4 \\ 3 \times 4 \end{matrix} & & \begin{matrix} 2 \times 4 \\ 2 \times 4 \end{matrix} \end{matrix}$$

- Columns of first matrix must match rows of second (**numbers in red match**)
- Result takes number of rows from first matrix (**numbers in blue match**)
- Result takes number of columns from second matrix (**numbers in purple match**)

# The Identity Matrix?

## The identity matrix



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## The identity matrix

$$\begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix} \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} = \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix}$$

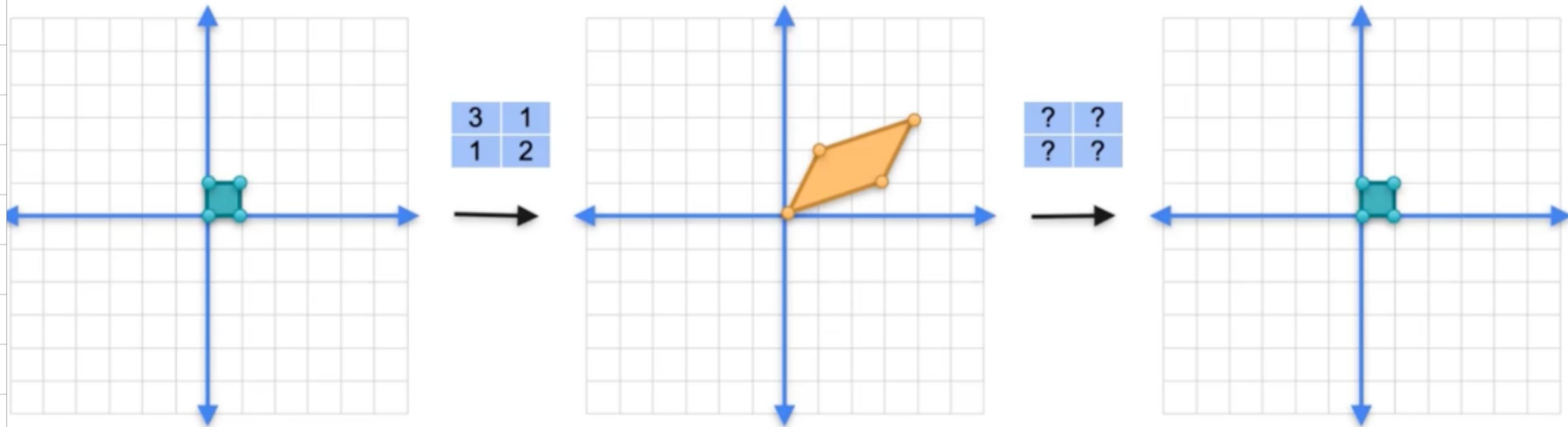
And the identity matrix in a linear transformation is very simple,

In linear transformation the inverse matrix is the one that undoes the job of the original matrix, namely the one that returns the place to where it was the beginning.

# Matrix Inverse (Only if  $|D| \neq 0$ ) (Inverse of -5 is  $-\frac{1}{5}$ )

## Matrix inverses

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \rightarrow \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$



transformations is the one corresponding to the identity matrix,

$$\begin{bmatrix} 5 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ c \end{bmatrix} \rightarrow 0$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$5a + 2c = 1$$

$$5b + 2d = 0$$

$$1a + 1c = 0$$

$$1b + 1d = 1$$

## How to find an inverse?

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix}$$

Now how do we find the entries  
in this inverse matrix?

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$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad 3a + 1c = 1 \quad a = \frac{2}{5}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad 3b + 1d = 0 \quad b = -\frac{1}{5}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad 1a + 2c = 0 \quad c = -\frac{1}{5}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad 1b + 2d = 1 \quad d = \frac{3}{5}$$

$b = -1/5, c = -1/5$  and  $D = 3/5$ .

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$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|D| = 2 - 2 \Rightarrow 0.$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a \\ c \end{bmatrix} \Rightarrow 1$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} b \\ a \end{bmatrix} \Rightarrow 0$$

$$\begin{bmatrix} 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a \\ c \end{bmatrix} \Rightarrow 0$$

$$\begin{bmatrix} 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} b \\ c \end{bmatrix} = 1$$

## Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{array}{|cc|} \hline 1 & 1 \\ 2 & 2 \\ \hline \end{array} \quad \begin{array}{|cc|} \hline a & b \\ c & d \\ \hline \end{array} = \begin{array}{|cc|} \hline 1 & 0 \\ 0 & 1 \\ \hline \end{array}$$

$$a + c = 1$$

$$2b + 2d = 1$$

$$2a + 2c = 0$$

$$b + d = 0$$

This is clearly a contradiction, since equation 1 says  $a+c=1$ , and equation 3 says  $2a+2c=0$ .

So therefore this matrix  
doesn't have an inverse.

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Rule for Matrix to have Inverse!

$$5^{-1} = 0 \cdot 2$$

$$8^{-1} = 0 \cdot 125$$

$0^{-1} \Rightarrow ???$  Not Defined

$$a + c = 1$$

$$b + d = 0$$

$$2a + 2c = 0$$

$$2b + 2d = 1.$$

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$\text{Det} = 5$   
Invertible

$(D) \neq 0$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$\text{Det} = 8$   
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

$\text{Det} = 0$   
Non-Invertible  
 $(D) = 0$

# Noun| Networks And Matrices:

~~Spam Classification~~

## Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Lottery: 3 pts.  
Win: 2 pts.

Scopes:  
Lottery: — points  
Win: — points.

Solution  
1 point → L  
1 point → W  
threshold 1.5 points

Joe Proposition

but of course the  
appearance doesn't

# Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

## Scores:

Lottery: \_\_\_\_ points

Win: \_\_\_\_ points

## Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

## Rule:

If the number of points of the sentence is bigger than \_\_\_\_,  
then the email is spam.

## Goal: Find the best points and threshold

Lottery: \_\_\_\_ point

Win: \_\_\_\_ point

Threshold: \_\_\_\_ points

the classifier are as close as

# Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

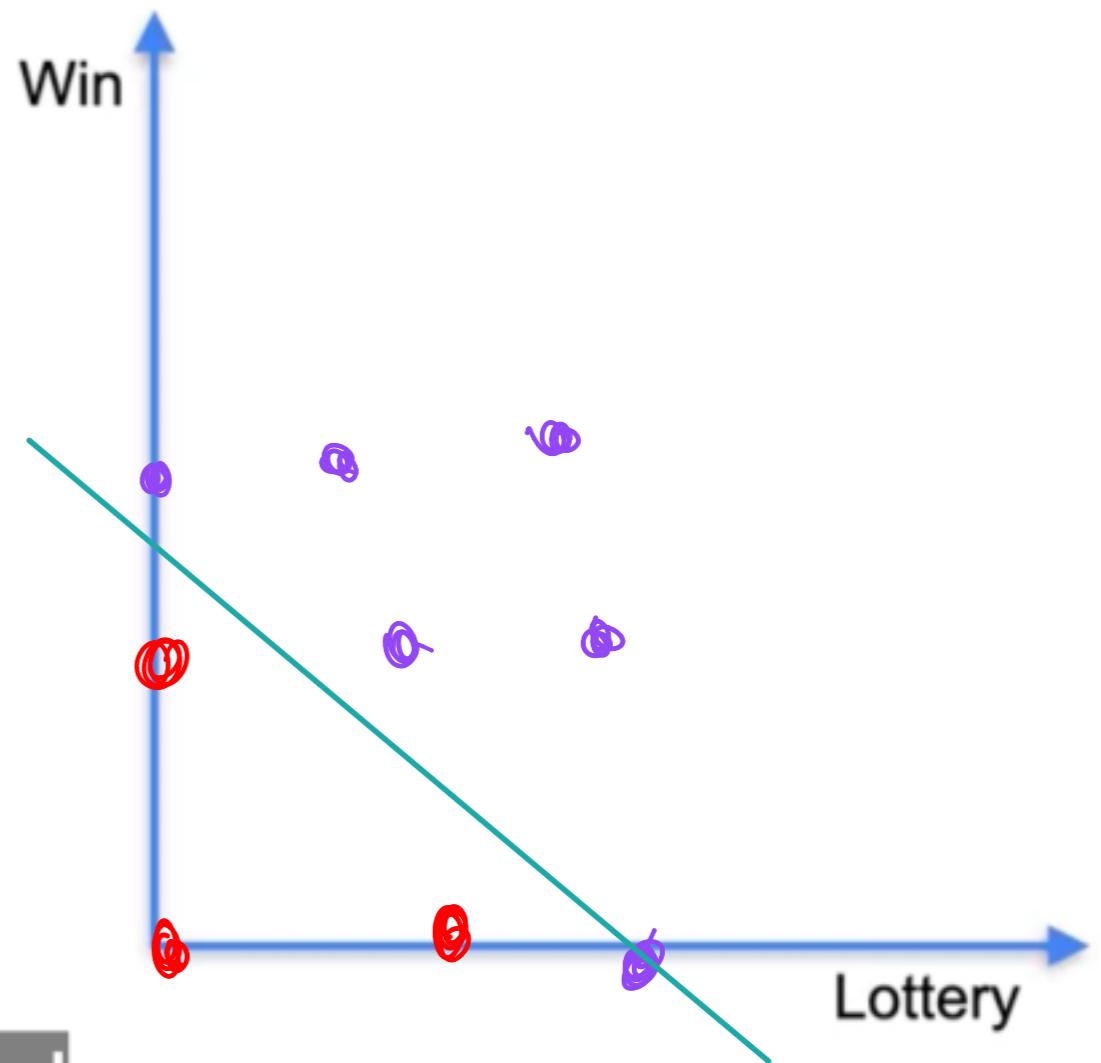
Score	> 1.5?
2	Yes
3	Yes
0	No
2	Yes
1	No
1	No
4	Yes
2	Yes
3	Yes

**Solution:**  
Lottery: 1 point  
Win: 1 point  
Threshold: 1.5 points

# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

the horizontal  
axis is the number



# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Line:

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$

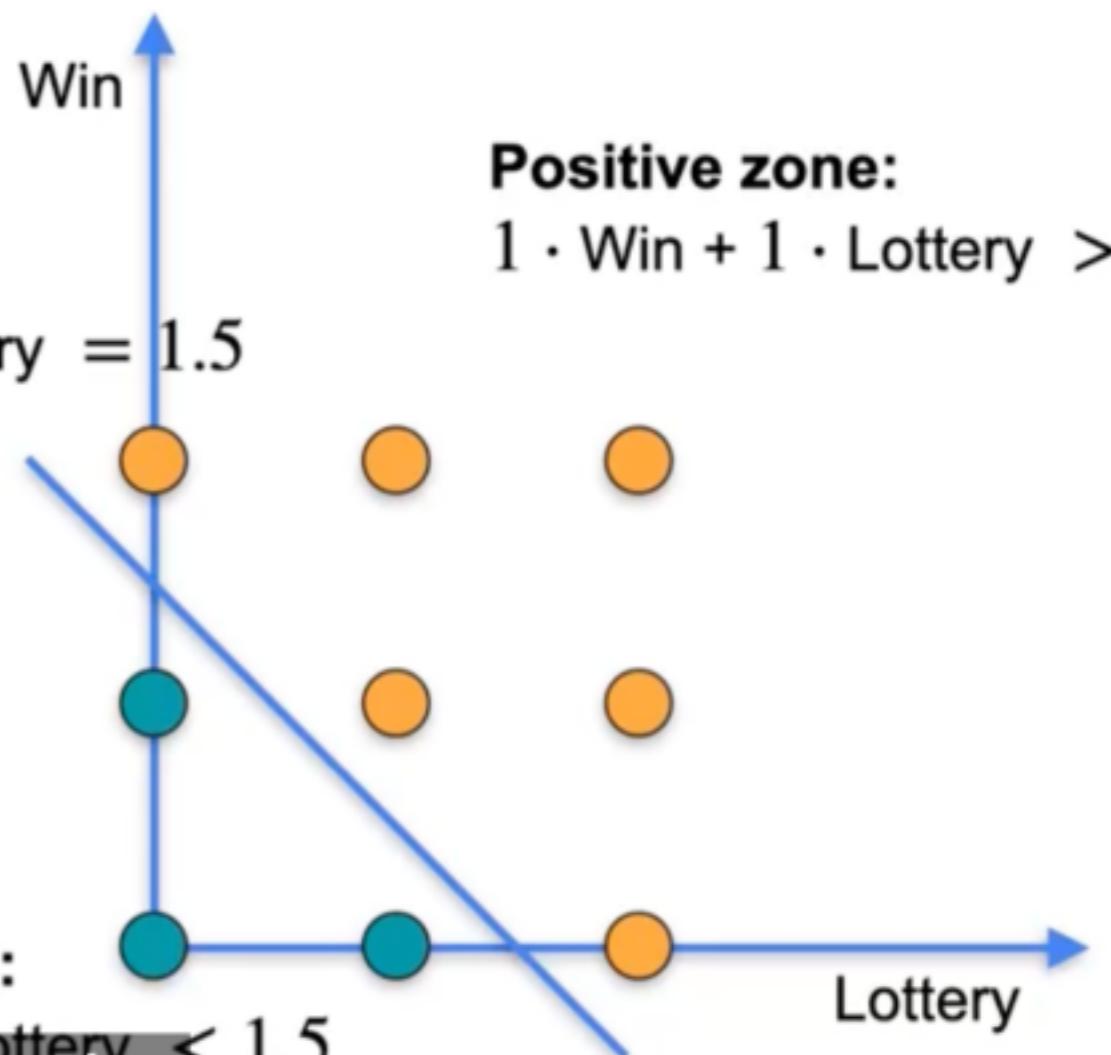
**Positive zone:**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} > 1.5$$

**Negative zone:**

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} < 1.5$$

and it's actually the  
simplest neural network.



# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	?

$$0.1 + 1.0 = 1.1 > 1.5 \text{ Yes Spam}$$

0 1

Model
1
1

Check:  $> 1.5$ ?

$$y = 2.1 + 1.1 = 3 \rightarrow \text{So, it is Spam because } 3 > 1.5$$

the word lottery and one appearance of the word win.

# Matrix multiplication

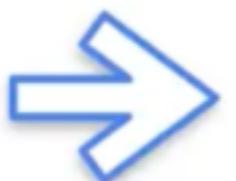
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

$\xrightarrow{2}$   
 $\xrightarrow{3}$   
 $\xrightarrow{0}$   $W$

Model
1
1

Prod
2
3
0
2
1
1
4
2
3

$\checkmark$  Check:  $> 1.5?$



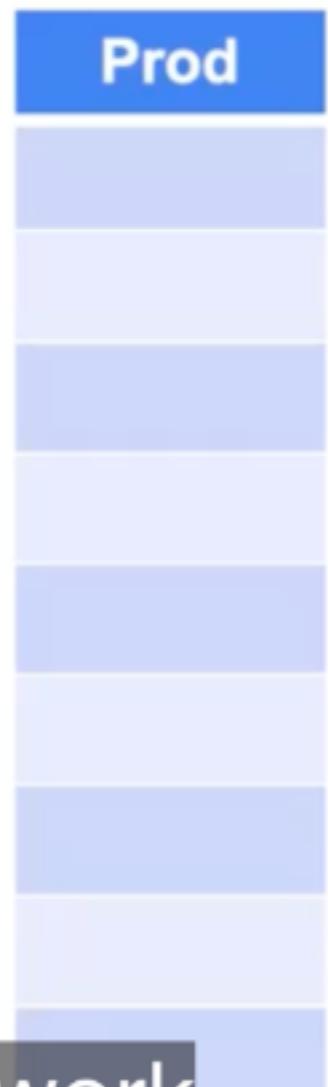
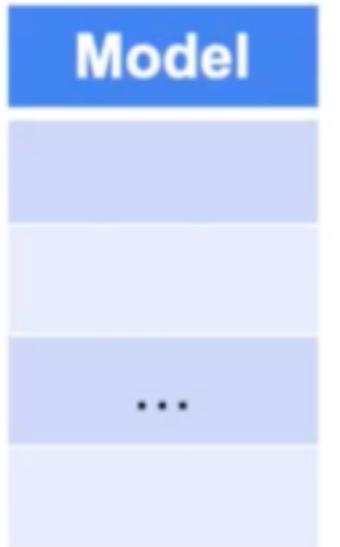
Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

in this vector to  
get our predictions.

# Perceptrons

( Works Similar Above Example)

Spam	Word1	Word2	...	WordN
Yes				
Yes				
No				
Yes				
No				
No				
Yes				
Yes				
Yes				



Check:  
→

Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

but this would work  
exactly the same.

# Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

the results is larger than  
the threshold of 1.5,

## Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} > 1.5$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check:  $> 0$ ?

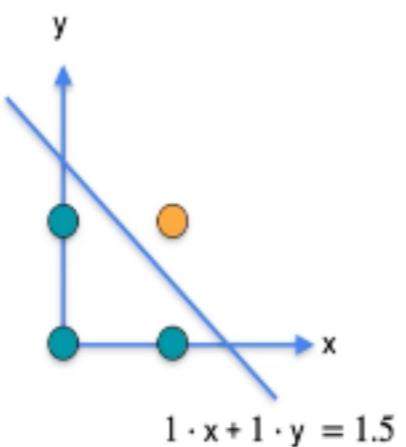
Model
1
1
-1.5

Bias

## The AND operator

### The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1



except that now there's only  
a subset of the points.

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model	=	Dot prod
1	=	0
1	=	1
1	=	1
2	=	2

Check: >1.5?



Check
No
No
No
Yes

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you get predictions that look

Source: Linear Algebra for Machine Learning and

Dom Simeon Course offered by

DeepLearning.AI

# The perceptron

