Day-100, Mar 10-2025, Falgun 26, 2081 BS.

① Bell Man Equation (12min)

② Making Decisions (2min)

③ State-Action function Definition & Example (15min)

④ Learning the State-value function (16min)

⑤ Algorithm Refinement ε-greedy policy

⑥ The State of Reinforcement Learning.

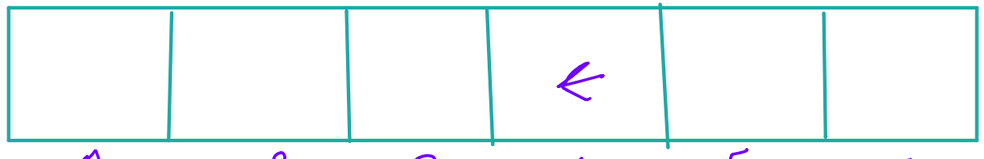
⑦ Acknowledgements:

⑧ What's Next?

Bellman Equation # Q(s,a) → Help to compute Q s of a.

Bellman Equation

$Q(s, a)$ = Return if you
- start in state s.
- take action a (once).
- then behave optimally after that.



R(1) = 100 R(2) = 0 ... R(6) = 40

R(s) = reward of current state

s : current state

a: Current action

s': state you get to after taking action a

a': action that you take in state s'

$$Q(s,a) = R(s) + \gamma \max_{a'} Q(s', a')$$

Bellman Equation

$$Q(s,a) = R(s) + \gamma \max_{a'} Q(s',a')$$

100	100	50	12.5	25	6.25	12.5	10	6.25	20	40	40
100		0		0		0		0		40	
1		2		3		4		5		6	

$S = 2$
$a = \rightarrow$
$s' = 3$

$S = 4$
$a = \leftarrow$
$s' = 3$

$Q(2, \rightarrow) = R(2) + 0.5 \max_{a'} Q(3, a')$

$\Rightarrow 0 + 0.5(25)$

$[\, Q(2, \rightarrow) \Rightarrow 12.5 \,]$

$Q(4, \leftarrow) \Rightarrow R(4) + 0.5 \max_{a'} (3, a')$

$\Rightarrow 0 + (0.5) \times 25$

$[\, Q(4, \leftarrow) \Rightarrow 12.5 \,]$

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Explanation of Bellman Equation

$Q(s, a)$ = Return if you
- start in state s.
- take action a (once).
- then behave optimally after that.

$S \rightarrow S'$

The best possible return from state s' is $\max_{a'} Q(s', a')$

$Q(S, a) = R(S) + \gamma \max_{a'} Q(S', a')$

$R_1 + \gamma R_2 + \gamma^2 R_3 + \gamma^3 R_4 + \cdots\cdots$

Broken down into Components

$\Rightarrow R_1 + \gamma [R_2 + \gamma R_3 + \gamma^2 R_4 + \cdots]$

$\max_{a'} Q(S', a')$ Best possible Return

$R(S) \rightarrow$ Immediate Reward

$\max_{a'} Q(S', a') \rightarrow$ Return from behaving optimaly Starting from state S'

$Q(S, a) = R_1(S) + \gamma \max_{a'} Q(S', a')$

3. Bellman Equation Explanation

The Bellman equation is:

$$Q(S, A) = R(S) + \gamma \max_{A'} Q(S', A')$$

Where:

- S' is the **next state** after taking action A from state S.
- A' is the **next action** to take from S'.
- $\max_{A'} Q(S', A')$ represents the **best possible future return** from state S'.

Breaking It Down:

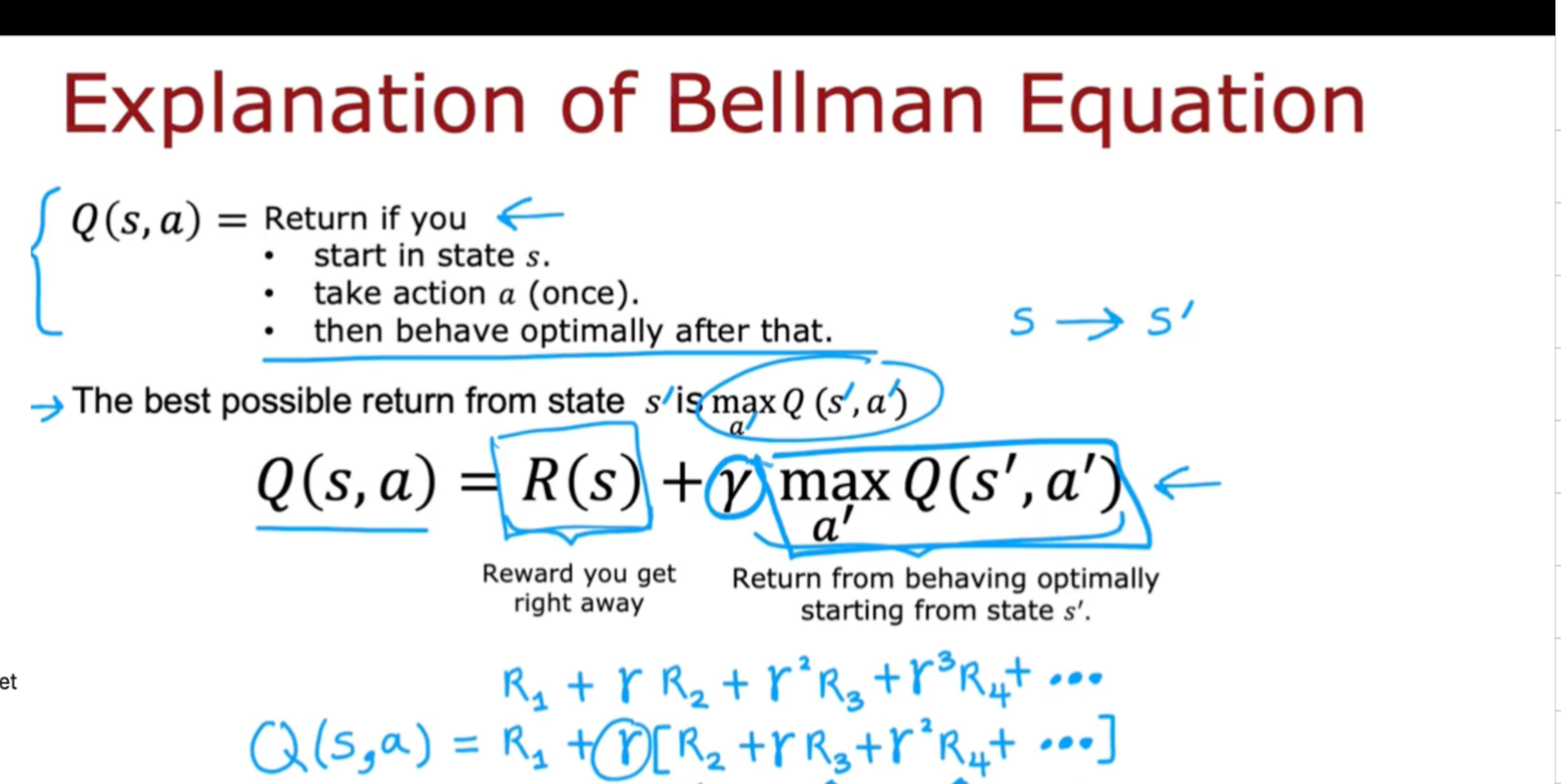
1. **First part:** $R(S)$ is the **immediate reward** for being in state S.
2. **Second part:** $\gamma \max_{A'} Q(S', A')$ represents the **discounted future rewards**—what you will get **starting from S' and behaving optimally**.

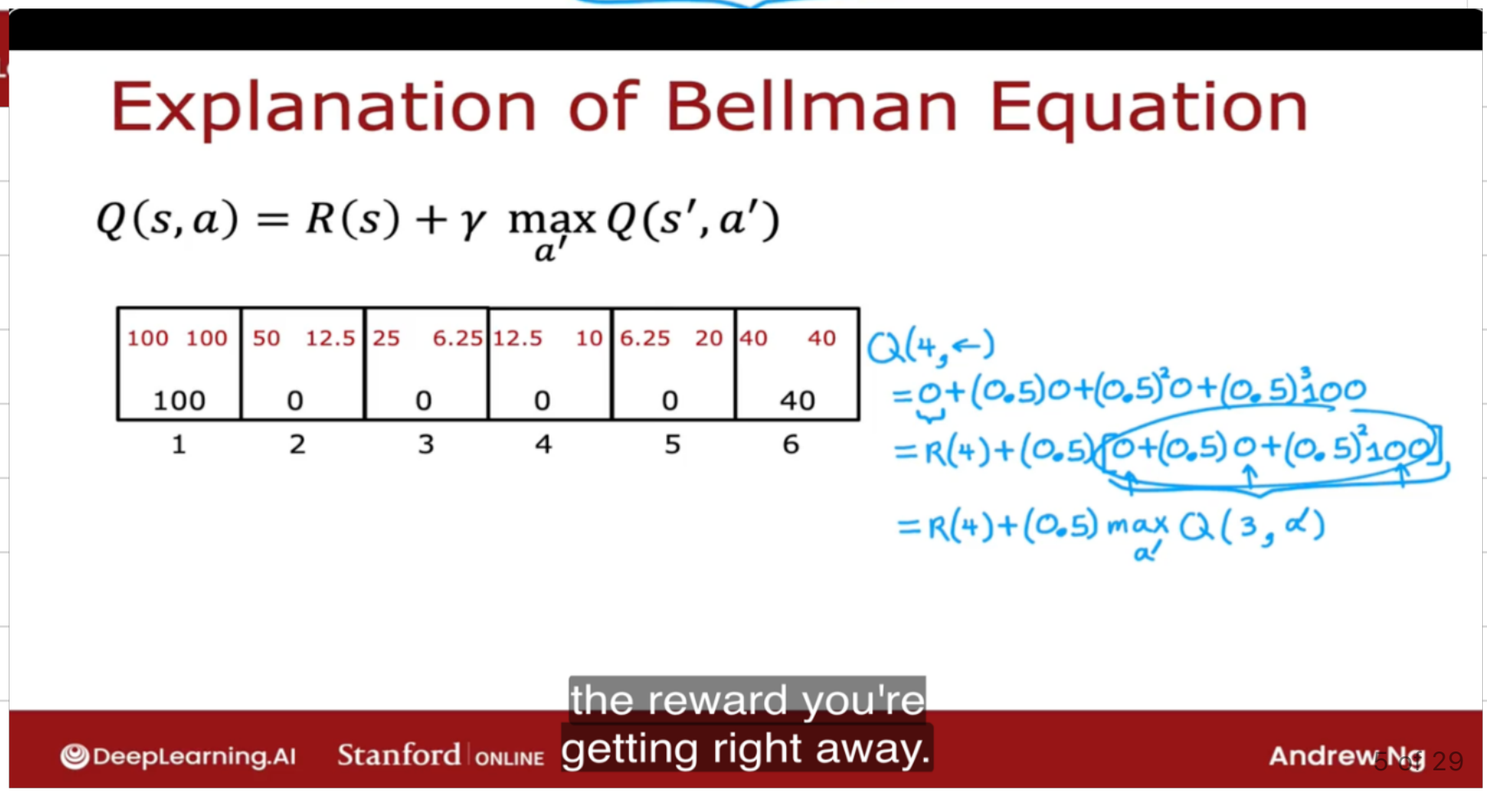
4. Example Calculation

Consider the **Markov Decision Process (MDP)** example in the lecture:

- The reward values for states:
  - $R(1) = 100$
  - $R(2) = 0$
  - $R(3) = 0$
  - $R(4) = 0$
  - $R(5) = 0$
  - $R(6) = 40$
- Suppose the **discount factor** is $\gamma = 0.5$.

Computing $Q(2, \text{right})$:





Computing $Q(2, \text{right})$:

- If you're in **State 2** and go **right**, you move to **State 3**.
- The Bellman equation gives:

$$Q(2, \text{right}) = R(2) + \gamma \max_{A'} Q(3, A')$$

- Since $R(2) = 0$, and the two possible $Q(3, A')$ values are **25** and **6.25**:

$$Q(2, \text{right}) = 0 + 0.5 \times 25 = 12.5$$

Computing $Q(4, \text{left})$:

- If you're in **State 4** and go **left**, you move to **State 3**.

$$Q(4, \text{left}) = R(4) + \gamma \max_{A'} Q(3, A')$$

- Since $R(4) = 0$ and the best possible value from **State 3** is **25**:

$$Q(4, \text{left}) = 0 + 0.5 \times 25 = 12.5$$

Terminal States:

For **terminal states**, the Bellman equation simplifies to:

$$Q(S, A) = R(S)$$

because there's no future state (S') to consider.

For example:

- If $S = 1$, then $Q(1, A) = 100$.
- If $S = 6$, then $Q(6, A) = 40$.

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5. The Intuition Behind the Bellman Equation

The Bellman equation **breaks down the total expected return into two parts**:

1. **Immediate reward** $R(S)$ (what you get right now).

5. The Intuition Behind the Bellman Equation

The Bellman equation **breaks down the total expected return into two parts**:

1. **Immediate reward** $R(S)$ (what you get right now).
2. **Future rewards**, discounted by γ, assuming optimal actions thereafter.

Mathematically, this breakdown follows from the way rewards accumulate over time:

$$\text{Total Return} = R_1 + \gamma R_2 + \gamma^2 R_3 + \dots$$

Rearranging, we get:

$$\text{Total Return} = R(S) + \gamma(\text{Future Return})$$

This leads to the **Bellman equation**.

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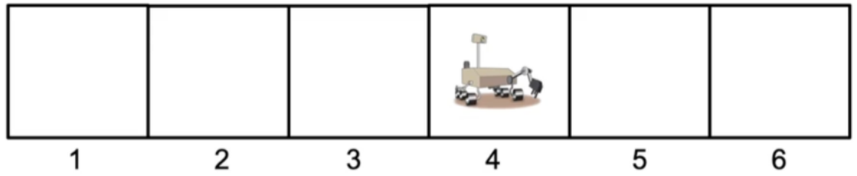
6. Key Takeaways

- **Bellman equation recursively defines $Q(S, A)$.**
- **To find the best action in a state, choose the one that maximizes $Q(S, A)$.**
- **The equation helps compute optimal policies in RL.**
- **If you understand the basic intuition, you can apply the equation even if the details seem complex.**

Discrete vs Continuous State

Discrete State:



Continuous State:

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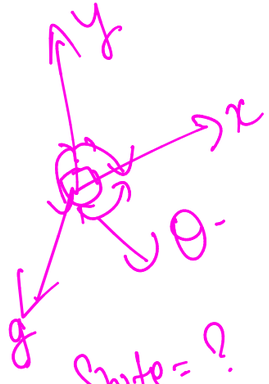
Autonomous Helicopter (3D)



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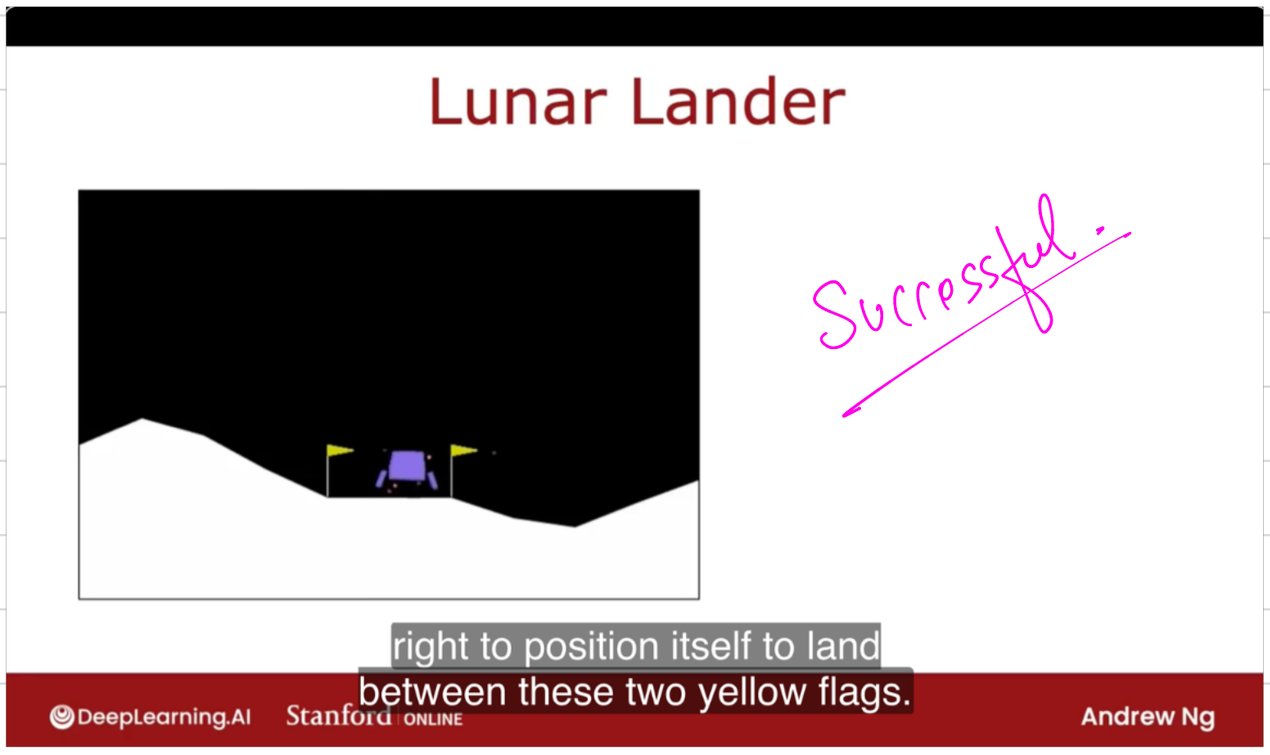
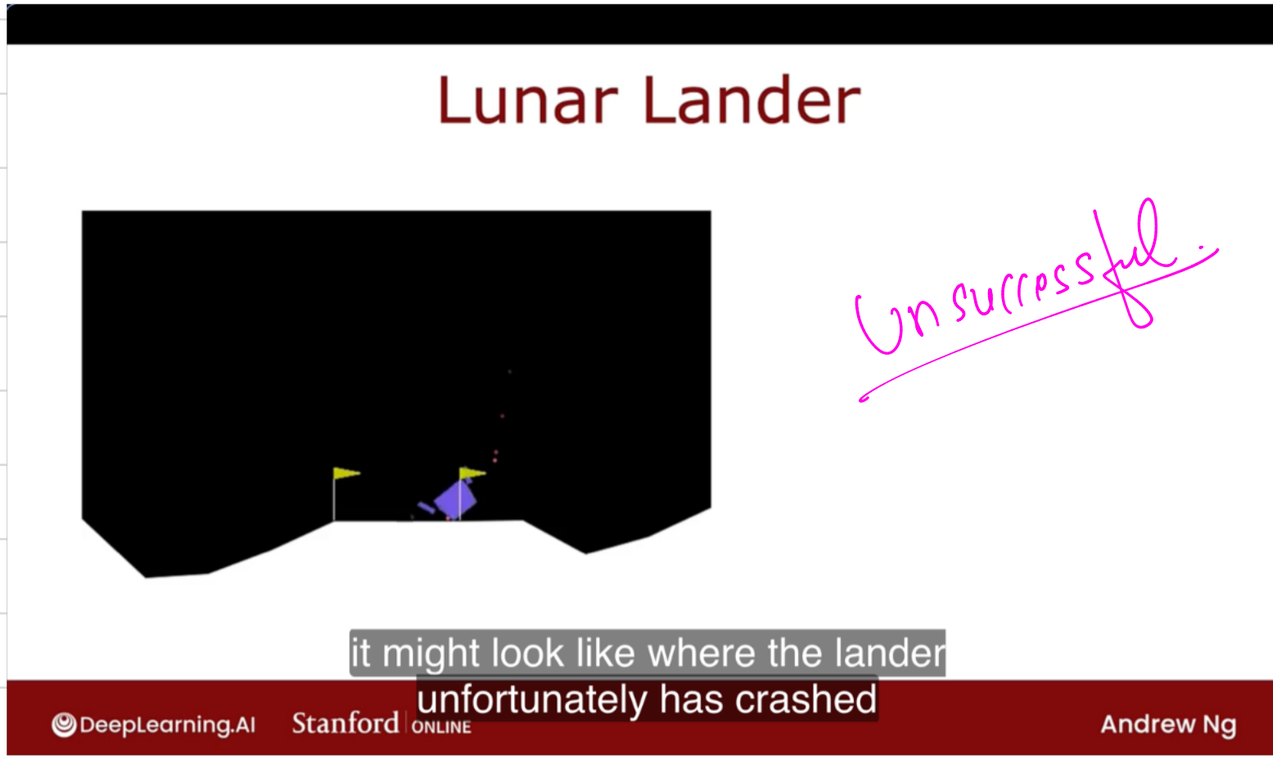
#1 Lunar Lander (Simulated).



Successful.



Unsuccessful.

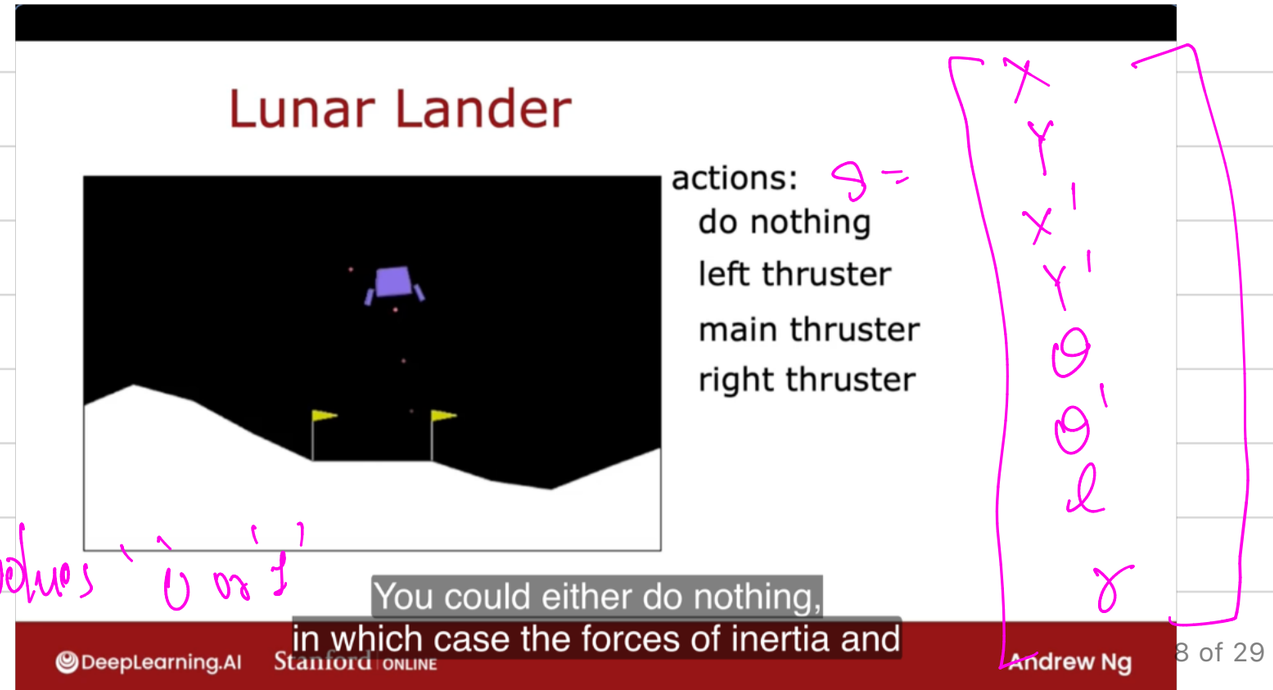
→ Actions that make Safe landing.

→ Means various Correct actions must be triggered at a sequence of time

left → l leg.

r → r right leg

x = y = binary values '0 or 1'



$\vartheta = \begin{bmatrix} x \\ y \\ x' \\ y' \\ \theta \\ \theta' \\ l \\ r \end{bmatrix}$

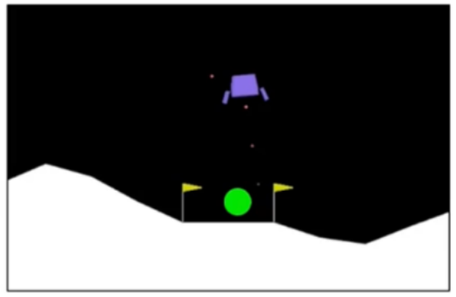
Lunar Lander and Reinforcement Learning

The **Lunar Lander** problem is a reinforcement learning environment where an agent (the lander) learns to land safely on a pad by optimizing its actions.

1. **State Space**:
   - The lander's state is represented by:
     - Position: x, y
     - Velocity: x', y' (horizontal and vertical speeds)
     - Angle: θ (tilt)
     - Angular velocity: θ'
     - Leg contact indicators: L and R (binary values indicating whether the left and right legs are touching the ground).
2. **Action Space**:
   - The agent can take one of four discrete actions:
     - **Do nothing**: Gravity pulls the lander downward.
     - **Fire left thruster**: Pushes the lander to the right.
     - **Fire right thruster**: Pushes the lander to the left.
     - **Fire main thruster**: Slows the descent.
3. **Reward Function**:
   - The goal is to reward good landings and penalize bad behavior:
     - Landing on the pad: **+100 to +140**.
     - Moving toward the pad: **positive reward**.
     - Moving away: **negative reward**.
     - Crashing: **-100**.
     - Landing softly: **+100 per leg**.
     - Landing with left or right leg: **+10 per leg**.
     - Fuel usage penalty:
       - Main engine: **-0.3 per use**.

Reward Function

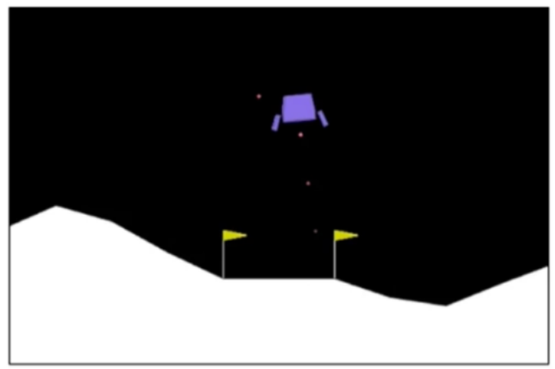
- Getting to landing pad: 100 – 140
- Additional reward for moving toward/away from pad.
- Crash: -100
- Soft landing: +100
- Leg grounded: +10
- Fire main engine: -0.3
- Fire side thruster: -0.03



Lunar Lander Problem

Learn a policy π that, given $s = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \\ \theta \\ \dot{\theta} \\ l \\ r \end{bmatrix}$

picks action $a = \pi(s)$ so as to maximize the return.



use large value
$\gamma = 0.985$

Our goal is to learn a policy pi.

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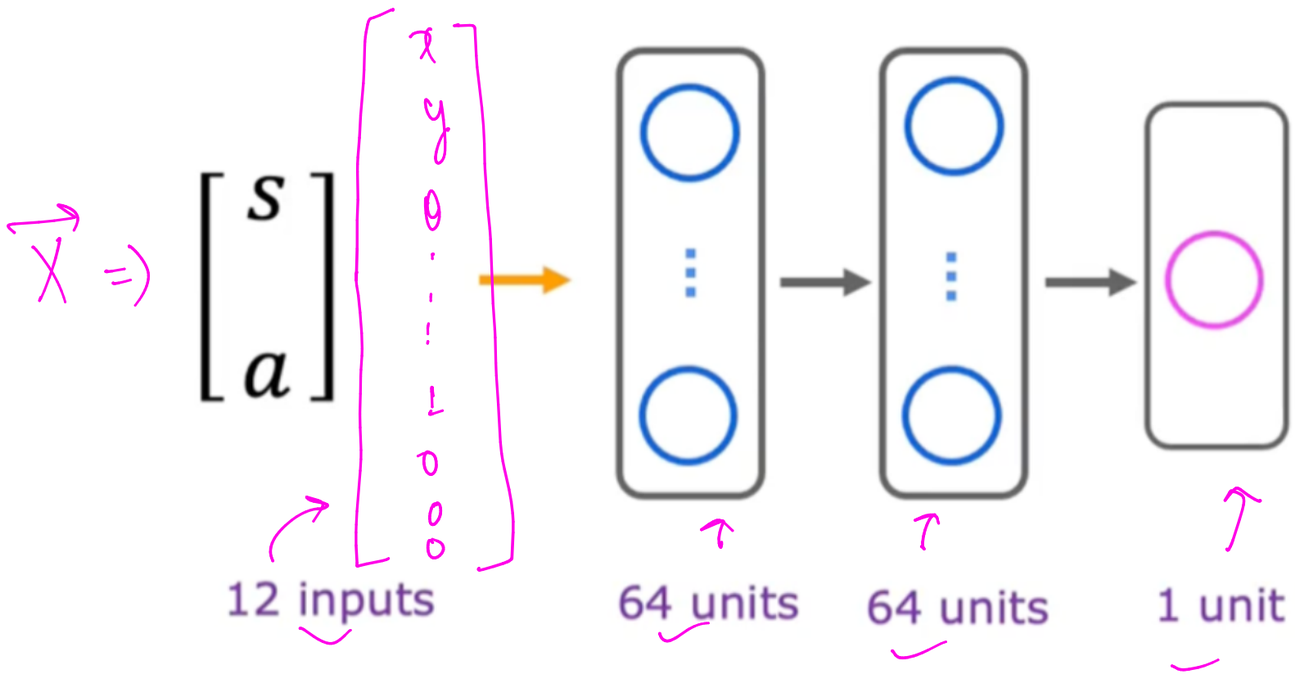
Deep learning (State-Value Q^* function) ⇨ train NN to approximate $Q(s,a)$.

Deep Reinforcement Learning



Q^*

(Input given in pair)

In a state s, use neural network to compute

$Q(s,\text{nothing})$, $Q(s,\text{left})$, $Q(s,\text{main})$, $Q(s,\text{right})$ → One hot feature vector: [0 1 0 0]

Pick the action a that maximizes $Q(s,a)$

↑ decides which action

any action a and put them together.

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Bellman Equation

$$Q(s, a) = R(s) + \gamma \max_{a'} Q(s', a')$$

x — y

$f_{w,b}(x) \approx y$

$(S, a, R(S), S')$

$(S^{(2)}, a^{(1)}, R(S^{(1)}), S'^{(1)})$ ←

$(S^{(2)}, a^{(2)}, R(S^{(2)}), S'^{(2)})$ ←

$(S^{(3)}, a^{(3)}, R(S^{(3)}), S'^{(3)})$ ←

Input

No policy then
Take Random Actions.

Result of Action gives, S'

Initially it's Random

$y^{(1)} = R(S^{(1)}) + \gamma \max_{a'} Q(S'^{(1)}, a')$

$y^{(2)} = R(S^{(2)}) + \gamma \max_{a'} Q(S'^{(2)}, a')$

x	y
$x^{(1)} = (S^{(1)}, a^{(1)})$ ↑ $(8, 4)$	$y^{(1)}$
$x^{(2)} = (S^{(2)}, a^{(2)})$	$y^{(2)}$

First Training Example

a equals R of s plus Gamma,

Bellman Equation (A piece RL Algorithm)

$$Q(\underbrace{s, a}_{x}) = \underbrace{R(s) + \gamma \max_{a'} Q(s', a')}_{y}$$

$f_{W,B}(x) \approx y$

$(s, a, R(s), s')$

$(s^{(1)}, a^{(1)}, R(s^{(1)}), s'^{(1)})$ ←

$(s^{(2)}, a^{(2)}, R(s^{(2)}), s'^{(2)})$ ←

$(s^{(3)}, a^{(3)}, R(s^{(3)}), s'^{(3)})$ ←

$$y^{(1)} = R(s^{(1)}) + \gamma \max_{a'} Q(s'^{(1)}, a')$$

$$y^{(2)} = R(s^{(2)}) + \gamma \max_{a'} Q(s'^{(2)}, a')$$

x	y
$x^{(1)} = (s^{(1)}, a^{(1)})$ (8, 4)	$y^{(1)}$
$x^{(2)} = (s^{(2)}, a^{(2)})$	$y^{(2)}$
$x^{(10,000)}$	$y^{(10,000)}$

What I describe here is

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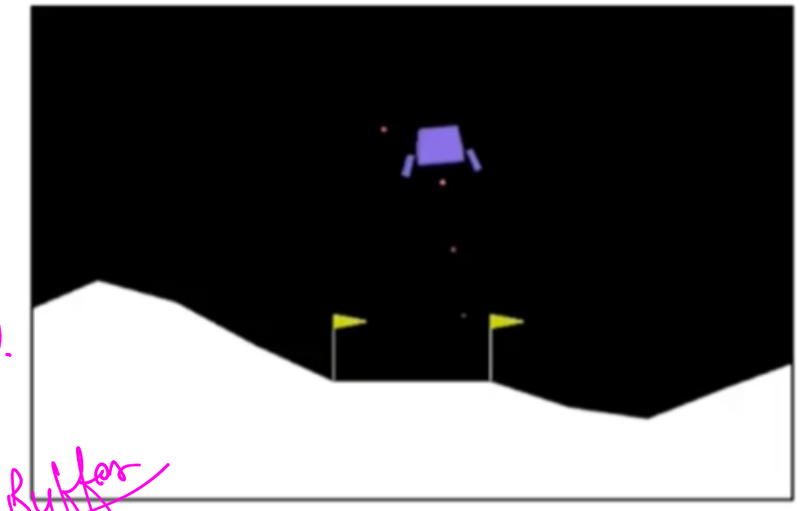
Learning Algorithm

Initialize neural network randomly as guess of $Q(s, a)$.



Repeat {

Take actions in the lunar lander. Get $(s, a, R(s), s')$.

Store 10,000 most recent $(s, a, R(s), s')$ tuples ← Replay Buffer

Train Neural Network.

Create training set of 10,000 examples using

$x = (s, a)$ and $y = R(s) + \max_{a'} Q(s', a')$

Train Q_{new} such that $Q_{new}(s, a) \approx y$

Set $Q = Q_{new}$.

}

$x_1 \quad y \quad x^{(1)}, y^{(1)}$

$\vdots$

$x^{(10000)}, y^{(10000)}$

$f_{w,b}(x) \approx y$

training linear regression and you

Learning Algorithm

Initialize neural network randomly as guess of $Q(s,a)$.

Repeat {

Take actions in the lunar lander. Get $(s,a,R(s),s')$.

Store 10,000 most recent $(s,a,R(s),s')$ tuples.

Replay Buffer

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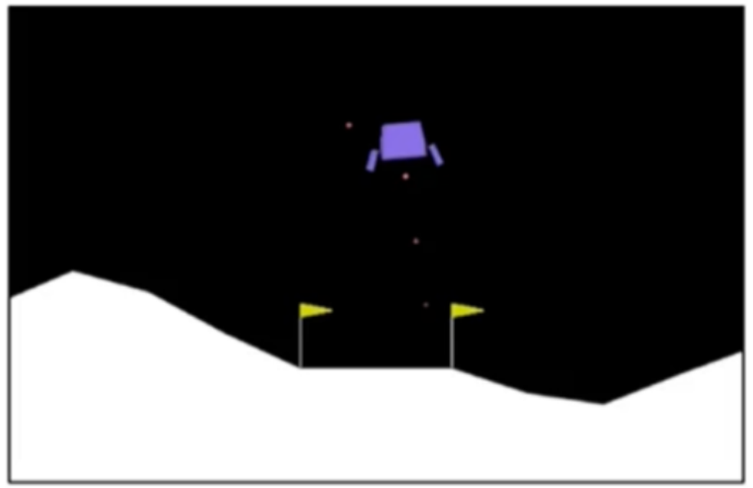

x, y

$x^{(1)}, y^{(1)}$

$x^{(10000)}, y^{(10000)}$

Then for the next time you train

Deep Q-Network (DQN) Algorithm for Learning the Q-Function

1. Problem Setup

The goal of reinforcement learning is to approximate the **state-action value function** $Q(s, a)$, which helps in selecting optimal actions.

For **Lunar Lander**, the state s consists of:

- (x, y) position
- $(\dot{x}, \dot{y})$ velocity
- $(\theta, \dot{\theta})$ orientation
- Leg contact indicators (Left, Right)

The actions a are **one-hot encoded**:

- **Nothing**: $[1, 0, 0, 0]$
- **Left thruster**: $[0, 1, 0, 0]$
- **Main thruster**: $[0, 0, 1, 0]$
- **Right thruster**: $[0, 0, 0, 1]$

Need to be Rewarded

2. Q-Function Approximation with a Neural Network

A neural network is used to approximate $Q(s, a)$, where:

- **Input**: $X = [s, a]$ (12 features: 8 for state, 4 for action)
- **Architecture**:
  - Hidden layer 1: 64 neurons
  - Hidden layer 2: 64 neurons
  - Output: Single value $Q(s, a)$

2. Q-Function Approximation with a Neural Network

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- **Architecture**:
  - Hidden layer 1: 64 neurons
  - Hidden layer 2: 64 neurons
  - Output: Single value $Q(s, a)$

3. Algorithm: Deep Q-Network (DQN)

Step 1: Initialize Neural Network

- Initialize all parameters **randomly**
- This serves as an initial guess for $Q(s, a)$

Step 2: Collect Experience Tuples

- Run the **Lunar Lander** simulator by taking random actions
- Store experience tuples:

$$(s, a, R(s), s')$$

- Maintain a **Replay Buffer** storing the last 10,00[illegible] tuples

2. Q-Function Approximation with a Neural Network

A neural network is used to approximate $Q(s, a)$, where:

- **Input**: $X = [s, a]$ (12 features: 8 for state, 4 for action)
- **Architecture**:
  - Hidden layer 1: 64 neurons
  - Hidden layer 2: 64 neurons
  - Output: Single value $Q(s, a)$

We train the network to predict $Q(s, a)$ using the **Bellman equation**:

$$Q(s, a) = R(s) + \gamma \max_{a'} Q(s', a')$$

where:

- $R(s)$ is the reward at state s
- s' is the next state after taking action a
- γ is the discount factor (typically 0.99)

$$y^i = R(s^i) + \gamma \max_{a'} Q(s'^i, a')$$

Step 4: Train Neural Network

- Use **Mean Squared Error (MSE) Loss**:

$$\mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} (Q(s^i, a^i) - y^i)^2$$

- Train using **Gradient Descent**

Step 5: Update Q-function

- Replace the old Q-function with the newly trained model

Step 6: Repeat

- Continue collecting new experiences, updating the neural network, and refining $Q(s, a)$
- Over time, $Q(s, a)$ improves, leading to better actions

4. Summary of Key Equations

1. **Bellman Equation:**

$$Q(s, a) = R(s) + \gamma \max_{a'} Q(s', a')$$

2. **Target for Supervised Learning:**

$$y^i = R(s^i) + \gamma \max_{a'} Q(s'^i, a')$$

3. **Loss Function (MSE):**

$$\mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} (Q(s^i, a^i) - y^i)^2$$

This **DQN Algorithm** allows reinforcement learning to efficiently learn optimal control strategies for the **Lunar Lander** problem. 🚀

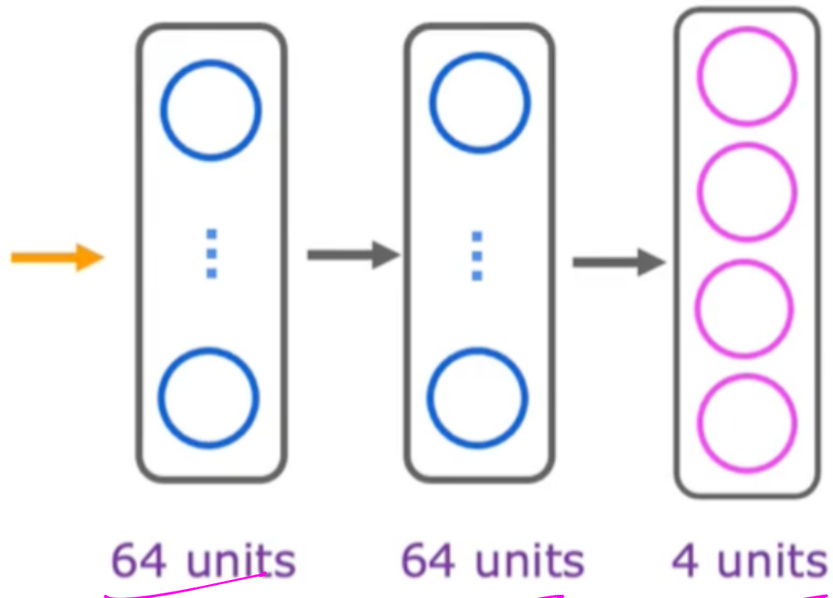
Algorithm Refinement:

Deep Reinforcement Learning

(Modified / Better).

$$s = \begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \\ l \\ r \end{bmatrix}$$

8 Inputs



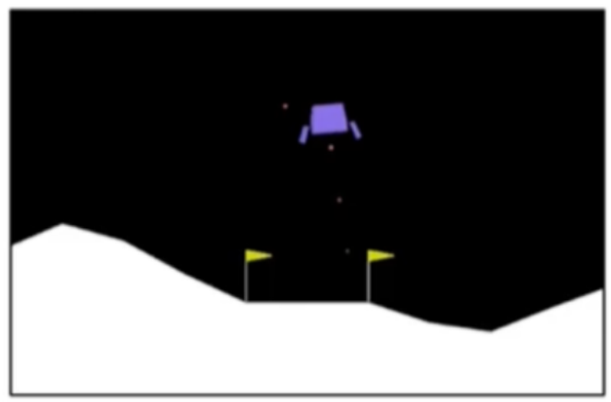
$Q(s, \text{nothing})$
$Q(s, \text{left})$
$Q(s, \text{main})$
$Q(s, \text{right})$

In a state s, input s to neural network.

Pick the action a that maximizes $Q(s, a)$.

Train S' at a time.

$R(S) + \gamma \max_{a'} Q(s', a')$



Here's a modified neural network

Algorithm Refinement ε-greedy policy:

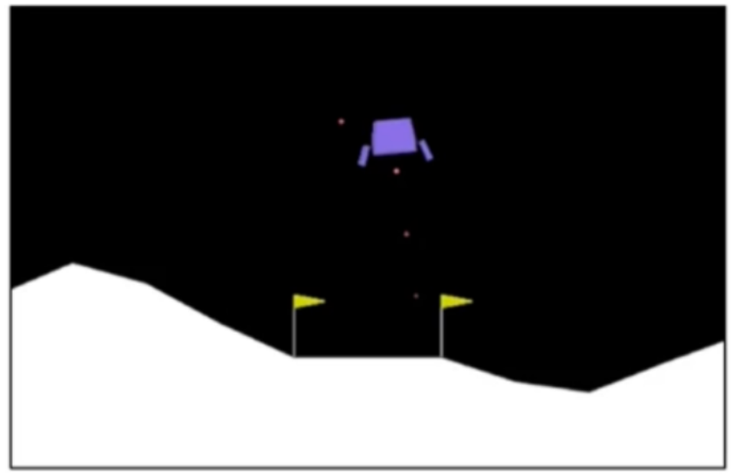
Learning Algorithm

Initialize neural network randomly as guess of $Q(s,a)$.

Repeat {

Take actions in the lunar lander. Get $(s, a, R(s), s')$.

Store 10,000 most recent $(s, a, R(s), s')$ tuples.



Train model:

Create training set of 10,000 examples using

$x = (s, a)$ and $y = R(s) + \gamma \max_{a'} Q(s', a')$.

Train Q_{new} such that $Q_{new}(s, a) \approx y$. $f_{w,b}(x) \approx y$

Set $Q = Q_{new}$.

→ NN Algorithm to approximate $Q(s,a)$

Here's the algorithm that you saw earlier.

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How to choose actions while still learning?

In some state s

Option 1:

Pick the action a that maximizes $Q(s,a)$.

Option 2:

→ With probability 0.95, pick the action a that maximizes $Q(s,a)$.

→ With probability 0.05, pick an action a randomly.

When you're in some state s,

How to choose actions while still learning?

In some state s

Q(s,main) is low

↑

a

Option 1:

Pick the action a that maximizes $Q(s,a)$.

Option 2:

With probability 0.95, pick the action a that maximizes $Q(s,a)$. Greedy, "Exploitation"

With probability 0.05, pick an action a randomly. "Exploration"

ϵ → greedy policy ($\epsilon = 0.05$)

0.95 ⇒ ϵ

0.05 ⇒

→ Start ϵ high 1.0 ~ 0.01

→ Gradually decrease

versus trying to maximize your return by say,

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RL is quite flexible in terms parameters compared to Supervised Learning-

Mini-Batch and Soft Updates → (Helps RL Algorithm efficiency)

How to choose actions while still learning?

x	y
2104	400
1416	232
1534	315
852	178
...	...
3210	870

$$J(w,b) = \frac{1}{2m}\sum_{i=1}^{m}\left(f_{w,b}(x^{(i)}) - y^{(i)}\right)^2$$

$m = 100,000,000$

House Sizes

repeat {

$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} J(w, b)$$

}

Gradient Descent · becomes slower for larger m.

Problem ?
↳ Computing Intensive. Σ, Average and Do Backpropgath and so on.

way back in the first course of this specialization

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How to choose actions while still learning?

x	y
2104	400
1416	232
1534	315
852	178
...	...
3210	870

100,000,000

$$J(w,b) = \frac{1}{2m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)^2$$

m = 100,000,000

m' = 1,000

repeat {

$$w = w - \alpha\frac{\partial}{\partial w}\frac{1}{2m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)^2$$

$$b = b - \alpha\frac{\partial}{\partial b}\frac{1}{2m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)^2$$

}

Look at 1,000, rather than all.

(Mini-Batch)

we would pick some subset of 1,000 or m prime examples.

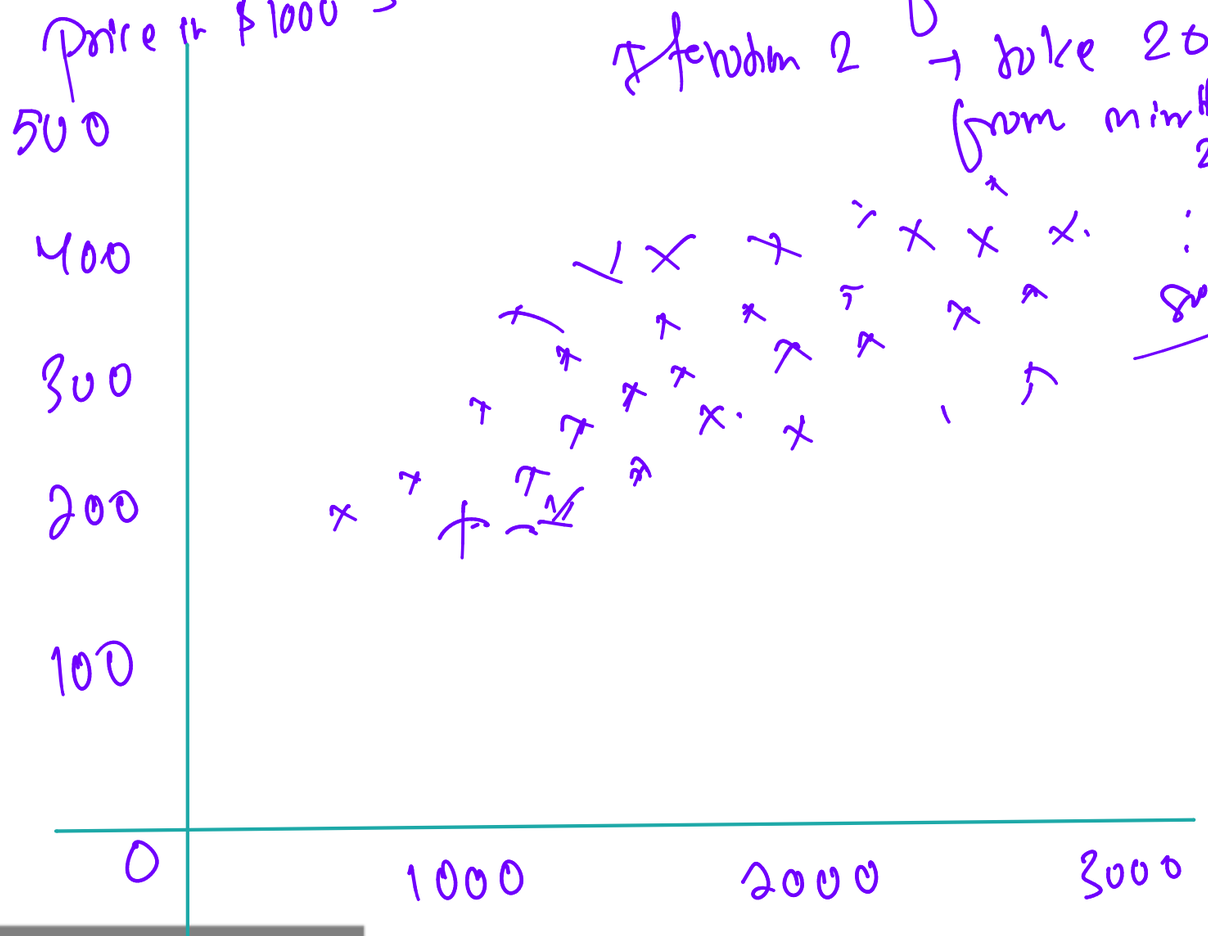
Mini-batch

x	y
2104	400
1416	232
1534	315
852	178
...	...
3210	870

mini-batch 1

mini-batch 2

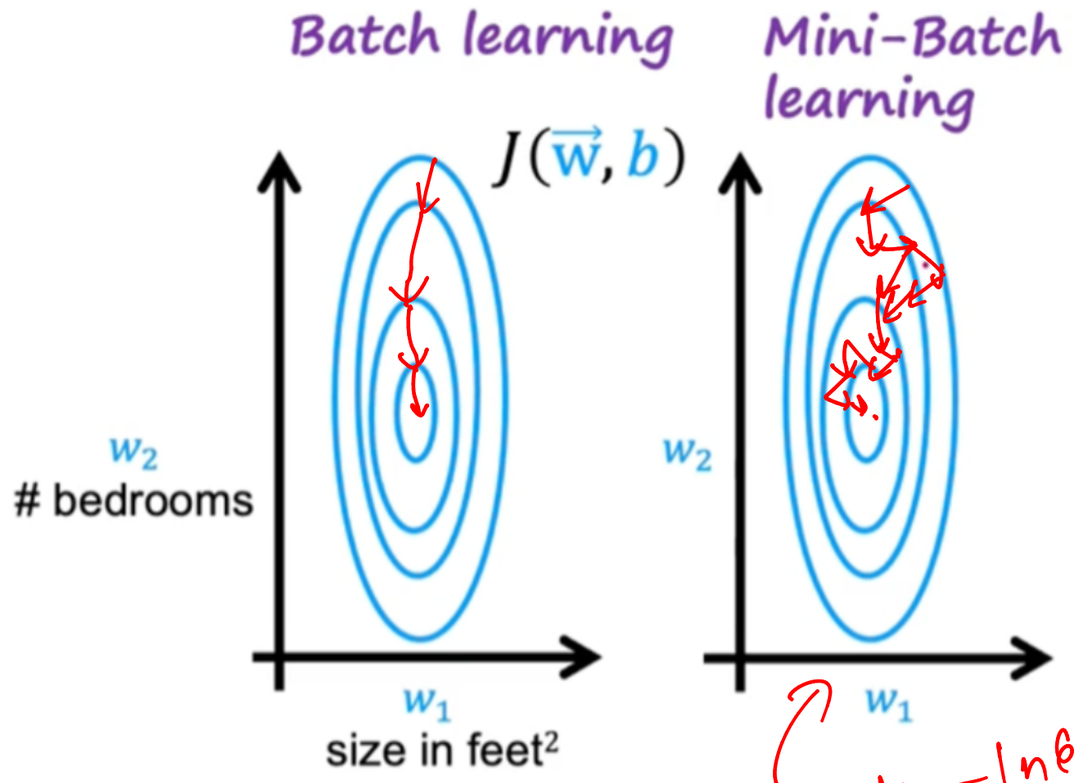
mini-batch 3



so that every iteration
is looking at

Mini-batch

x	y
2104	400
1416	232
1534	315
852	178
...	...
3210	870



Computation – Inexpensive
But may not find Global min as Batch Gradient.

You might remember with batch gradient descent,

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Learning Algorithm

Initialize neural network randomly as guess of $Q(s,a)$.

Repeat {

Take actions in the lunar lander. Get $(s,a,R(s),s')$.

Store 10,000 most recent $(s,a,R(s),s')$ tuples.

Train model:

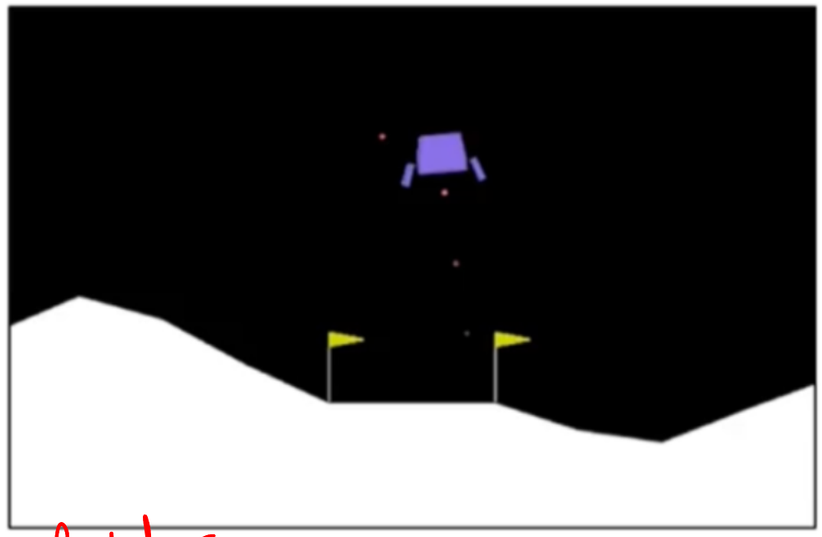
Create training set of 10,000 examples using

$$x = (s,a) \text{ and } y = R(s) + \gamma \max_{a'} Q(s',a').$$

Train Q_{new} such that $Q_{new}(s,a) \approx y$.

Set $Q = Q_{new}$.

}



recent tuples in the replay buffer,

Helps to Converge faster

Soft Update

↳ Accept only a little bit value

Set $Q = Q_{new}$.

W, B ← Q

W_{new}, B_{new} ← Q_{new}

$W = 0.01 W_{new} + 0.99 W$

$B = B_{new}$

$W = 0.01 W_{new} + 0.99 W$

$B = 0.01 B_{new} + 0.99 B$

$W = \gamma W_{new} + (1-\gamma) W$

$Q(s,a)$ → works well applying Soft update

For Lunar Simulation

When you train the new neural network,

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Stats of RL

Limitations of Reinforcement Learning

- Much easier to get work in a simulation than a real robot!
- for fewer applications than SL and USL. (Unsupervised Learning)
- But .. exciting Research direction with potential for future applications.

RL is sometimes Hype but still long way to go (2025-03-03)

One of the reasons for some of the hype about reinforcement learning is, it turns

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This lecture introduces **two key refinements** to the reinforcement learning algorithm for **Deep Q-Networks (DQN)**:

1. **Mini-batch Gradient Descent** – Speeds up training by using a subset of data.
2. **Soft Updates** – Stabilizes learning by gradually updating the Q-network.

---

1. Mini-Batch Gradient Descent in RL

Why Mini-Batches?

- **Batch Gradient Descent**: Computes gradients using **all** training examples.
- **Issue**: If the dataset is very large (e.g., 100M examples), every step is computationally expensive.
- **Mini-Batch Gradient Descent**: Uses a subset m' of training examples (e.g., 1,000).
  - Each step is much **faster** and updates the model with less computation.
  - Works well for **supervised learning** (e.g., linear regression, neural networks) and **reinforcement learning**.

How It Works in Reinforcement Learning?

- Store **10,000 tuples** $(s, a, R(s), s')$ in a **Replay Buffer**.
- Instead of training on all **10,000** experiences, use a mini-batch of **1,000** samples.
- Update the Q-function using:

$$y^i = R(s^i) + \gamma \max_{a'} Q(s'^i, a')$$

- This introduces some noise but significantly **speeds up learning**.

---

2. Soft Updates for Stable Learning

Why Soft Updates?

- In the original **Q-update step**, we set:

$$Q \leftarrow Q_{\text{new}}$$

- If the new network Q_{new} is **worse** due to a bad update, it can degrade learning.
- Instead, **soft updates** gradually adjust Q using:

$$W \leftarrow \tau W_{\text{new}} + (1 - \tau) W$$

$$B \leftarrow \tau B_{\text{new}} + (1 - \tau) B$$

where τ is a small value (e.g., 0.01).

Effect of Soft Updates

- Reduces **oscillations** and prevents instability.
- Allows **gradual improvements** instead of abrupt changes.
- Ensures **smooth convergence** in reinforcement learning.

Needs to be Reviewed

Conclusion

- **Mini-batch learning** improves speed by training on small subsets.
- **Soft updates** make training more stable.
- These refinements **enhance DQN performance** and help the **Lunar Lander** land safely! 🚀

2. Soft Updates for Stable Learning

Why Soft Updates?

- In the original **Q-update step**, we set:

1. Mini-Batch Gradient Descent in Reinforcement Learning

Problem with Full-Batch Gradient Descent

- In **supervised learning**, batch gradient descent updates parameters using all m training examples:

$$J(w,b) = \frac{1}{2m}\sum_{i=1}^{m}(h(x^{(i)}) - y^{(i)})^2$$

  - Where $h(x^{(i)}) = wx^{(i)} + b$.
  - The gradient descent updates are:

$$w := w - \alpha\frac{\partial J}{\partial w}$$

$$b := b - \alpha\frac{\partial J}{\partial b}$$

- **Issue**: For large datasets (e.g., $m = 100M$), computing gradients for all examples in every step is computationally expensive.

Mini-Batch Gradient Descent Solution

- Instead of using all m samples, pick a **mini-batch** of m' (e.g., 1,000).
- The gradient updates become:

$$J_{\text{mini}}(w,b) = \frac{1}{2m'}\sum_{i=1}^{m'}(h(x^{(i)}) - y^{(i)})^2$$

  - New update rules:

$$w := w - \alpha\frac{\partial J_{\text{mini}}}{\partial w}$$

$$b := b - \alpha\frac{\partial J_{\text{mini}}}{\partial b}$$

- **Advantage**: Faster iterations with some noise, but still converges to the minimum.

Mini-Batch Learning in Reinforcement Learning

- In **Deep Q-Networks (DQN)**, store past experiences (s, a, r, s') in a **Replay Buffer**.
- Instead of training on the entire buffer, sample a **mini-batch** of m' experiences.
- Use **Q-learning update rule** for each mini-batch sample:

$$y^i = r^i + \gamma \max_{a'} Q(s'^i, a')$$

  - Update the **Q-function** using gradient descent:

$$\theta := \theta - \alpha\nabla_\theta \sum_{i=1}^{m'}(y^i - Q(s^i, a^i; \theta))^2$$

  - Reduces computational cost while still improving the policy.

2. Soft Updates for Stable Learning

Problem with Direct Q-Function Updates

- Original **Q-learning update** directly replaces old weights:

$$W \leftarrow W_{\text{new}}, \quad B \leftarrow B_{\text{new}}$$

- **Issue**: If W_{new} is poorly trained, it may degrade performance.

Soft Update Solution

- Instead of full replacement, use a weighted update:

$$W \leftarrow \tau W_{\text{new}} + (1-\tau)W$$

$$B \leftarrow \tau B_{\text{new}} + (1-\tau)B$$

  - Where τ (e.g., 0.01) controls how much of the new parameters are used.
  - Ensures **smooth transitions** and prevents instability.

Acknowledgement: I want to thank Coursera (finorcial aid) for providing valuable materials.

Date of 100Days of Maths Completion: 2025/03/10 (2081/11/26 B.S)

Resources: ① B.Sc CSIT 1st and 2nd Semester (KEC publication Maths Books) 2078 B.S.

② Coursera - Deeplearning.AI Courses
(Statistics, Linear Algebra, Calculus, Machine Learning)

③ ChatGPT, DeepSeek, Claude, Gemini (Coding & Additional Information)

Completed Courses: Coursera → Mathematics for Machine Learning and Data Science (3 Courses)

Coursera → Machine Learning (3 Courses)

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More Info? Visit: github.com/Dilli822/100DaysOfMaths/