Day- 19, Dec-4, 2024 (Mangshir-19, 2081 B.S.)

Combination of functions.

→ Functions can be +, -, x, / (except where denominator is zero in division)

Let f and g are two functions then $f+g$, $f-g$, $f \cdot g$ and f/g are new function which are defined by

① Sum: $(f+g)(x) = f(x) + g(x)$;

② Difference: $(f-g)(x) = f(x) - g(x)$;

③ Product: $(f \cdot g)(x) = f(x) \cdot g(x)$ [∵ Not Composite]

(iv) Quotient : $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ as long as $g(x) \neq 0$.

The domain of each of these combination is intersection of the domain of f and the domain of g.

Example: If functions are defined by the formulas

$$f(x) = \sqrt{x} \quad \text{and} \quad g(x) = \sqrt{2-x}$$

Here we have to find the formulas for functions $f+g$, $f-g$, $f \cdot g$, f/g and g/f.

↳ $(f+g)(x) = f(x) + g(x) \Rightarrow \sqrt{x} + \sqrt{2-x}$

The domain for $f(x) = \sqrt{x}$ is $x \geq 0$ i.e. $A = [0, \infty)$

The domain for $g(x) = \sqrt{2-x}$

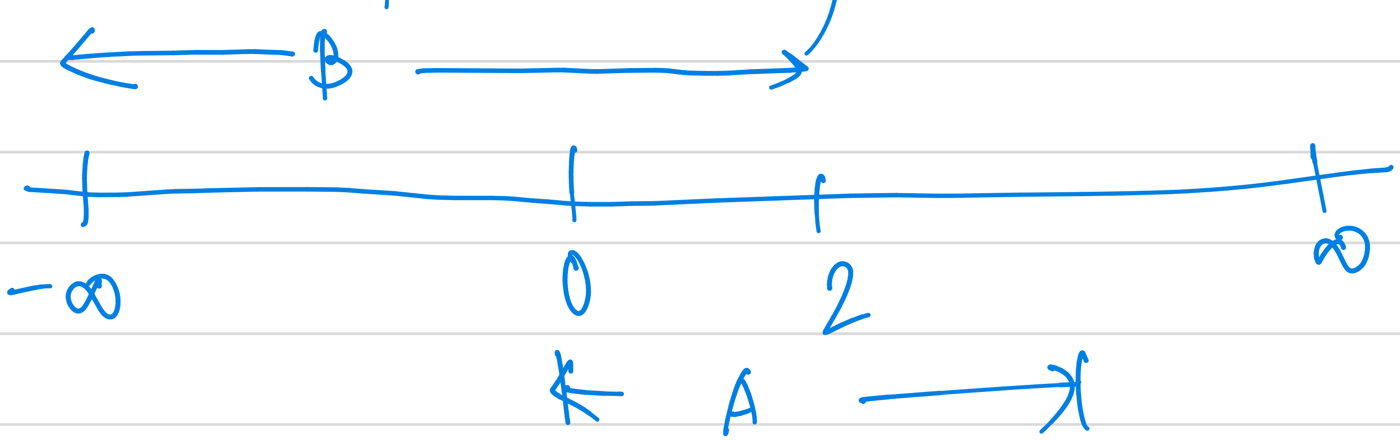
$$2 - x \geq 0$$

$$x - 2 \leq 0$$

$$x \leq 2 \quad \text{i.e. } B = (-\infty, 2]$$

Domain for $(f+g)(x)$ is $A \cap B$.

Here, $A = [0, \infty)$ $B = (-\infty, 2]$ $\therefore A \cap B = [0, 2]$



(2) Again $(f-g)(x) = f(x) - g(x)$

$$\Rightarrow \sqrt{x} - \sqrt{2-x}$$

and domain is $A \cap B = [0, 2]$.

$(f \cdot g)(x) = f(x) \cdot g(x)$

$$\Rightarrow \sqrt{x} \cdot \sqrt{2-x}$$

$$\Rightarrow \sqrt{2x - x^2}$$

Thus domain for $f \cdot g = [0, 2]$.

$f/g\,(x) = \dfrac{f(x)}{g(x)} \Rightarrow \dfrac{\sqrt{x}}{\sqrt{2-x}}$ [for $x \neq 2$]

Domain for f/g is $[0, 2)$

and $\frac{g}{f}(x) = \frac{g(x)}{f(x)}$

$\Rightarrow \frac{\sqrt{2-x}}{\sqrt{x}}$ [for $x \neq 0$]

Domain for g/f is $(0, 2]$.

Composite functions:

If f and g are functions, then composite function fog is defined by

$$(fog)(x) = f(g(x))$$

Note: the domain of $f \circ g(x)$ is intersection of domain of $g(x)$ and $f(g(x))$.

Example: If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$ find the each function and it's domain.

a) $f \circ g$ b) $g \circ f$ c) $f \circ f$ d) $g \circ g$

a) $f \circ g(x) = f(g(x))$

$\Rightarrow f\left(\sqrt{2-x}\right)$

$\Rightarrow \sqrt{\sqrt{2-x}}$ $\Rightarrow (2-x)^{1/4}$

To find domain of fog

first we find domain of $g(x) = \sqrt{2-x}$

Domain of $g(x)$ is

$$2 - x \geq 0$$

$$\left[\text{i.e.} \quad x \leq 2 \quad \text{i.e.} \quad A = (-\infty, 2]\right]$$

and domain of $f(g(x)) = (2-x)^{1/4}$

$$2 - x \geq 0.$$

$$x \leq 2 \qquad \text{i.e.} \quad B = (-\infty, 2].$$

Here $A \cap B = (-\infty, 2]$ is domain of fog.

(b) $gof(x) = g(f(x))$

$\Rightarrow g(\sqrt{x}) \Rightarrow \sqrt{2-\sqrt{x}}$

for the domain of gof

The domain of $f(x) = \sqrt{x}$ is $x \geq 0$ ie. $A = [0, \infty)$

the domain of $g(f(x)) \Rightarrow \sqrt{2-\sqrt{x}}$

is $2-\sqrt{x} \geq 0$

ie. $\sqrt{x} \leq 2$

ie $0 \leq x \leq 4$

$B = [0,4]$

Domain of gof is $A \cap B = [0,4]$.

i) $f \cdot f(x)$

$f(f(x)) \Rightarrow f(\sqrt{x})$

$\Rightarrow \sqrt{\sqrt{x}}$

$\Rightarrow x^{1/4}$

$\therefore$ The domain of $f(x)$ is $A = [0, \infty)$

$\therefore$ The " " $f(x)$ is $B = [0, \infty)$

$\therefore$ Domain of $f \circ f$ is $A \cap B = [0, \infty)$

d) $g \circ g(x) = g\left(\sqrt{2-x}\right)$

$\Rightarrow \sqrt{2-\sqrt{2-x}}$

Domain of $g(x)$ is A $\Rightarrow (-\infty, 2]$

Domain of $g(g(x)) = 2-\sqrt{2-x} \geq 0$

i.e. $\sqrt{2-x} \leq 2$

i.e. $0 \leq 2-x \leq 4$

i.e. $-2 \leq -x \leq 2$

i.e. $2 \geq x \geq -2$

$\rightarrow$ Domain of $g(g(x))$ $B = [-2, 2]$

$\rightarrow$ Domain of $g \circ g$ is $A \cap B = [-2, 2]$

Example: If $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{x}{x-2}$. find formula for fog and also find its domain.

Here $fog(x) = f(g(x))$

$$\Rightarrow f\left(\frac{x}{x-2}\right)$$

$$\Rightarrow \frac{1}{\frac{x}{x-2}+3}$$

$$\Rightarrow \frac{1}{\frac{x+3x-6}{x-2}}$$

$$\Rightarrow \frac{x-2}{4x-6}.$$

$$\left[\therefore fog(x) = \frac{x-2}{2(2x-3)}\right]$$

Here, domain of $g(x)$ is $\mathbb{R} - \{2\} = A$

and domain of $f(g(x))$ is $\mathbb{R} - \{\frac{3}{2}\} = B$

$\therefore$ Thus, domain of $f \circ g(x)$ is $A \cap B = \mathbb{R} - \{2, \frac{3}{2}\}$

Example! Given $f(x) = \sqrt{x+1}$. find function f, g such that $F = f \circ g$.

Given $F(x) = \sqrt{x+1}$

$\Rightarrow t^2 \quad (x+1 = t)$

$\Rightarrow f(t)$, where $f(t) = t^{1/2}$

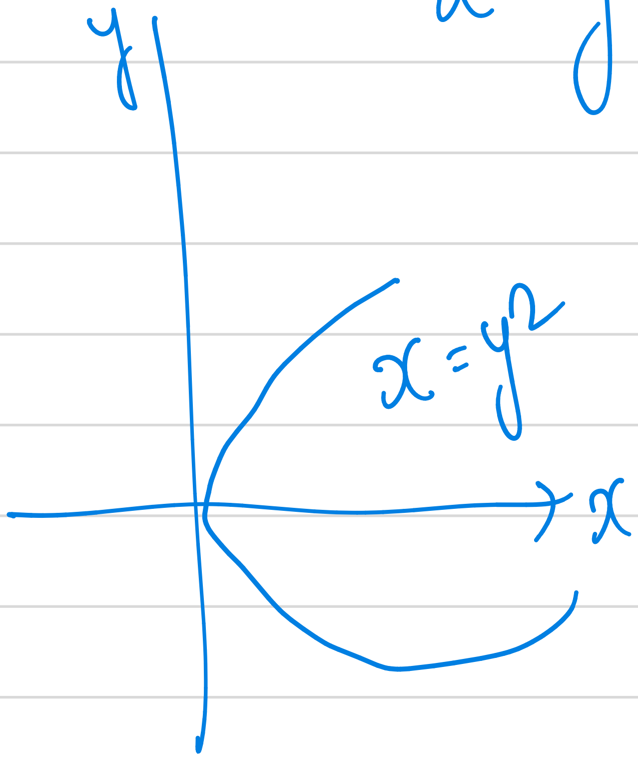
$\Rightarrow f(x+1)$ so, $f(x) = x^{1/2}$ and $g(x) = (x+1)$.

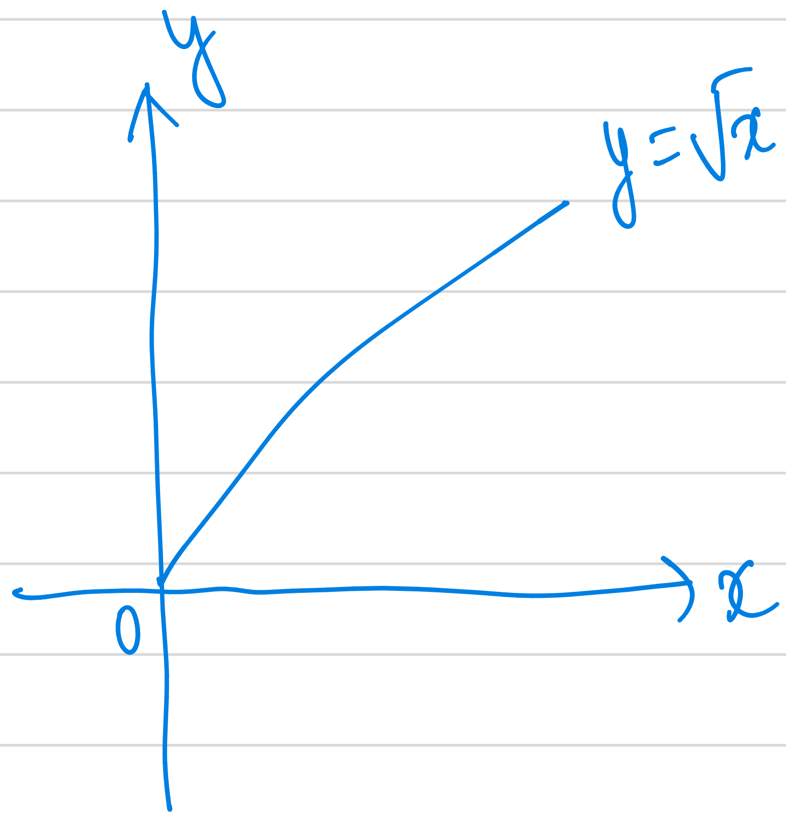
Some Well Known Functions and their Graph

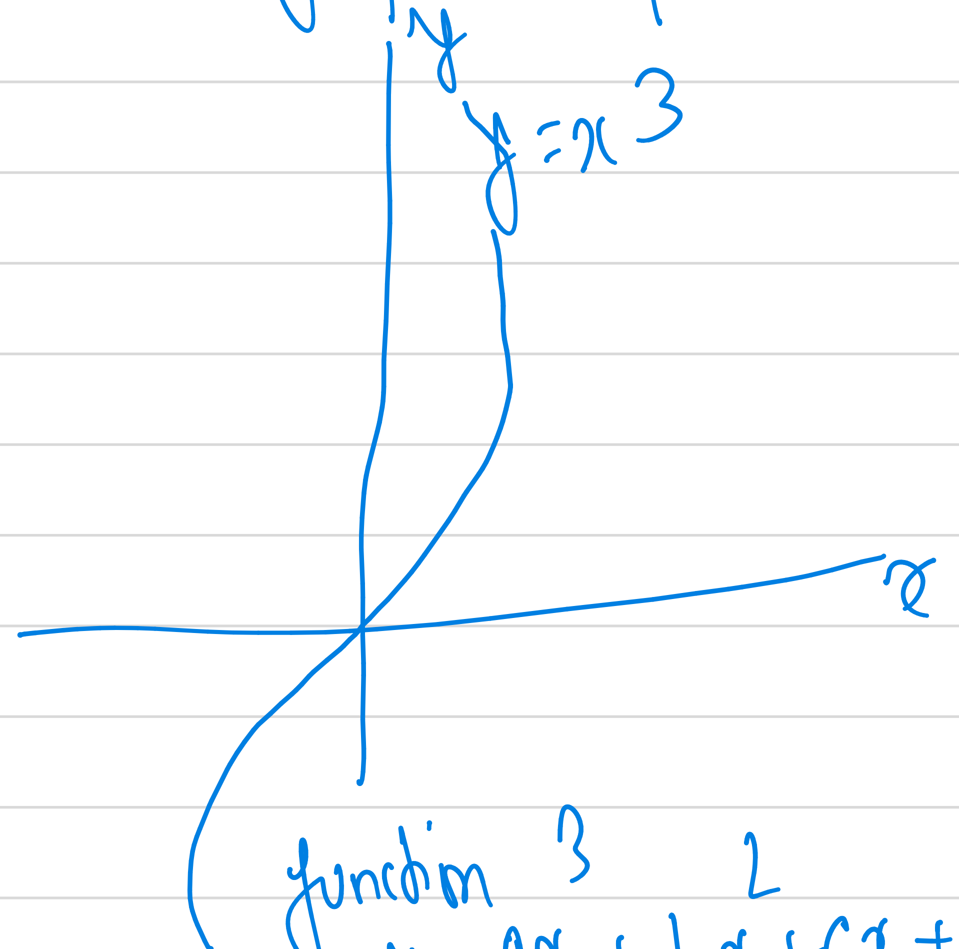
i) The function $y = mx + c$ is linear function

ii) $y = ax^2 + bx + c$, $a \neq 0$ is quadratic function (graph is Parabola)

$x = y^2$





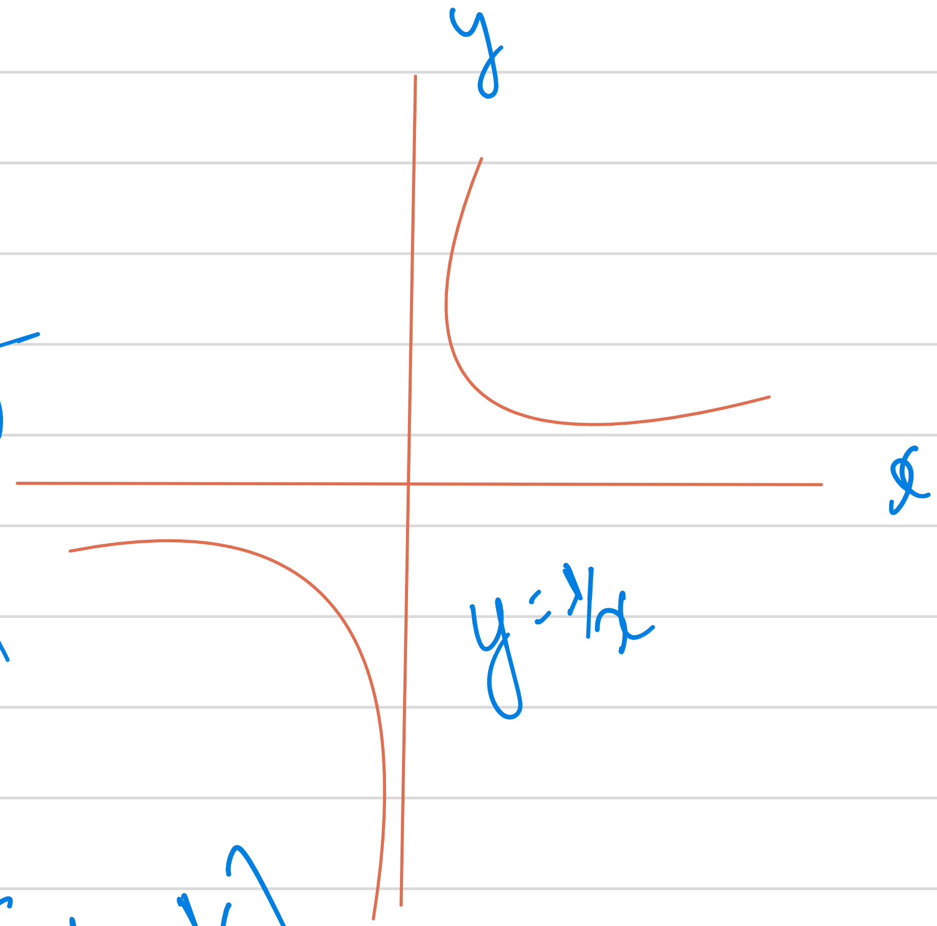


function $y = ax^3 + bx^2 + cx + d$

is $a \neq 0$. Cubic function

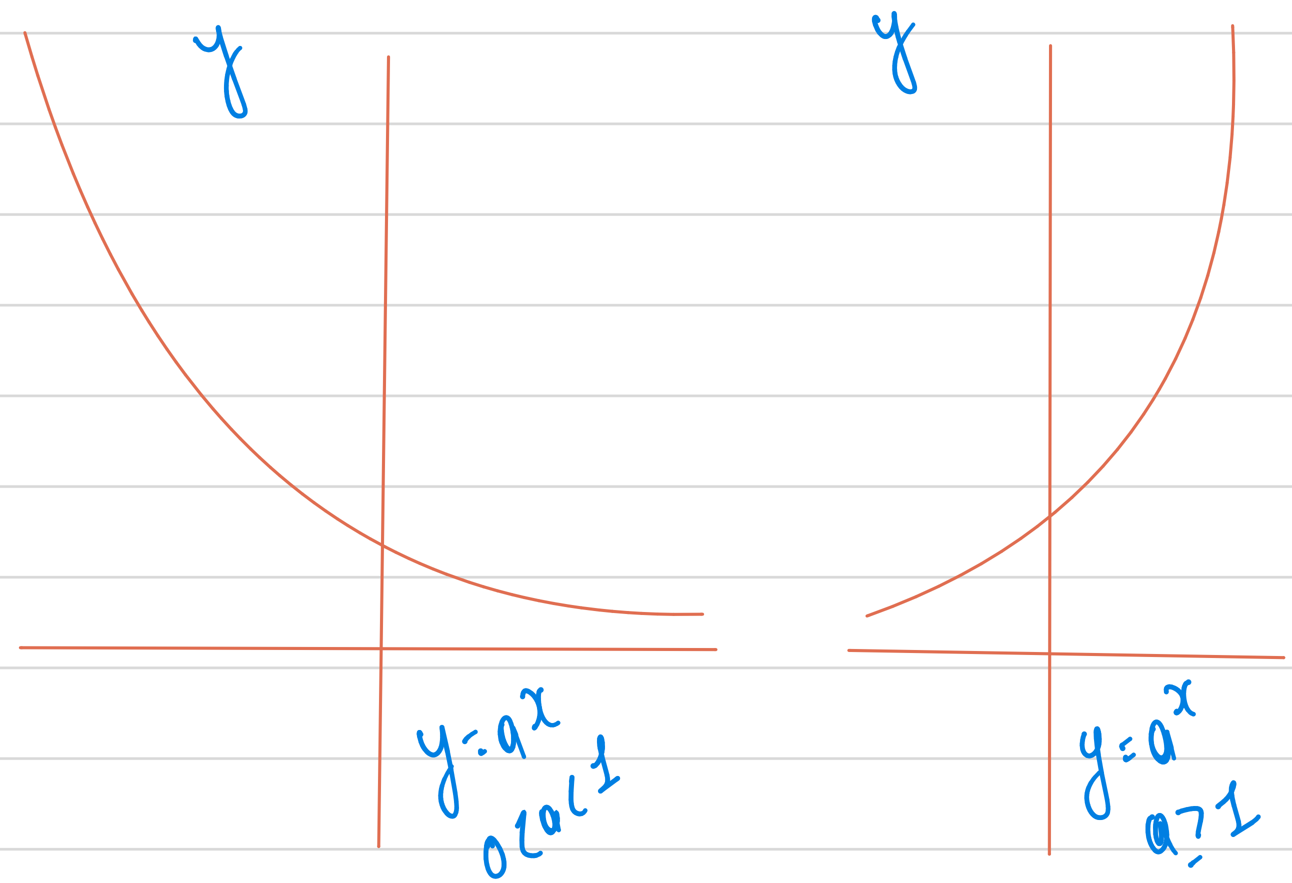
$y = x^3$

(iv) The function $y = \frac{f(x)}{g(x)}$, $g(x) \neq 0$ is rational function



$\left[y = \frac{1}{x} \right]$

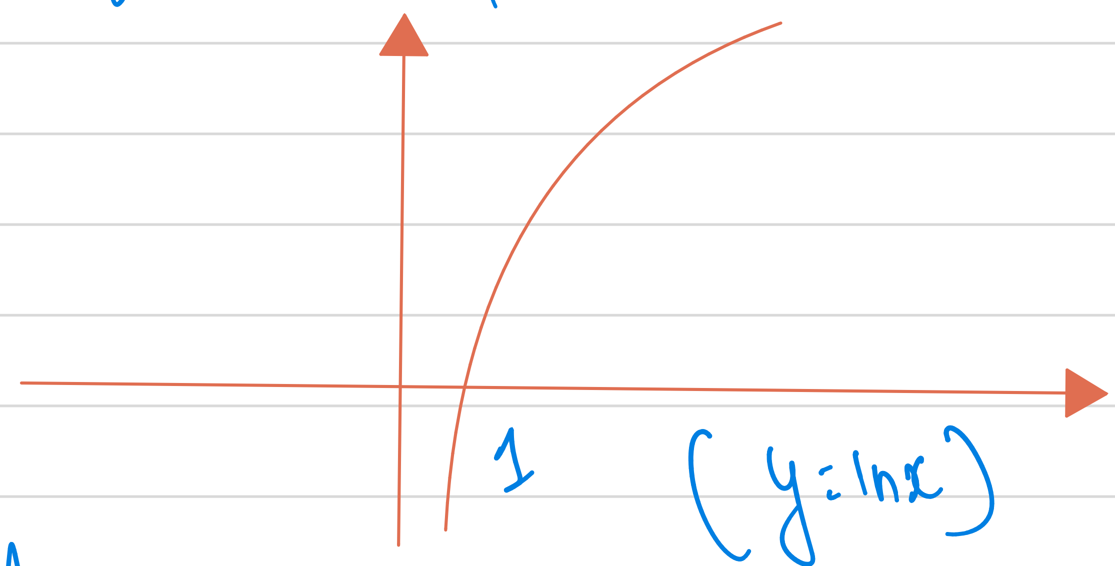
$\left[y = \frac{1}{x} \right.$ is Rational function.
Ratio-values. $\left. \right]$



The function $y = a^x$, $a > 0$ is exponential function.

Graph of $y = a^x$, $a > 1$ and Graph of $y = a^x$, $0 < a < 1$ are shown in figure.

→ The function of $y = \log_a x$ is logarithmic function. Graph of $y = \ln x$ as shown in figure.



vii) The function $y = \sin x$, $y = \cos x$ are trigonometric functions.

Graph of $y = \sin x$

Graph of $y = \cos x$ are shown in figure —