Day-79, Feb 17, 2025 (Falgun 5, 2081)

Introduction to Hypothesis Testing

A statistical procedure that uses sample data to evaluate an assumption about a population parameter.

Steps for Performing a Hypothesis test

1. State the null Hypothesis and the alternative Hypothesis
2. Choose a significance level or α
3. find the P-value
4. Reject or fail to reject the null Hypothesis

↳ Probability of landing on tails for any given toss

- fair Coin = 50 %
- Rigged Coin = (90 - 100) %

Probability

- 2 tails in a row = 0.5×0.5
  $\Rightarrow 0.25$
- 4 tails in a row $\Rightarrow 0.5 \times 0.5 \times 0.5 \times 0.5$
  $\Rightarrow 6.25$

Let's increase the confidence that the outcome is not due to chance.

$\alpha = 5\%$ (threshold) → due to chance

- 6 tails in a row = $0.5 * 0.5 * 0.5 * 0.5 * 0.5 * 0.5$

$\Rightarrow 0.00156$

$\Rightarrow 1.56\%$

Null Hypothesis

A statement that is assumed to be true unless there is convincing evidence to the contrary.

Alternative Hypothesis

A statement that contradicts the null hypothesis, and is accepted as true only if there is convincing evidence for it.

Hypotheses

- Null : The coin is fair
- Alternative: The coin is not fair

Significance Level (α)

The probability of rejecting the null Hypothesis when it is true

Significance level (α) = 5%

P-value.

The probability of observing results as or more extreme than those observed when the null Hypothesis is true.

A lower p-value means there is stronger evidence for the alternative Hypothesis

Decide

Reject or fail to Reject the null Hypothesis.

Because of Probability (Certainty)

Drawing a Conclusion:

- if p-value < Significance level: Reject the Null Hypothesis.
- if p-value > Significance level. fail to Reject the Null Hypothesis.

→ p-value: 1.56 %

→ Significance level: 5 %

→ 1.56 % < 5 %

Conclusion: Reject the null Hypothesis.

→ p-value: 1.56 %

→ Significance level: 1 %

→ 1.56 % > 1 %

Conclusion: Fail to Reject the Null Hypothesis.

The null Hypothesis is a statement that is assumed to be true unless there is convincing evidence to the contrary. The null Hypothesis typically assumes that observed data occurs by chance.

Types of errors in Hypothesis Testing

- Type I error (false positive)
- Type II error

Type I error (false positive)

The rejection of a null Hypothesis that is actually true.

Type II error (false negative)

The failure to reject a null Hypothesis which is actually false.

One-Sample Test for Means:

There are two types of Sample test:
- One-Sample Test
- Two-Sample Test

Two-Sample test:

Determines whether or not two population parameters such as two means or two proportions are equal to eachother.

One-Sample Hypothesis test

- A company's average sales revenue is equal to a target value.
- A medical treatment's average rate of success is equal to a set goal.
- A stock portfolio's average rate of return is equal to a market benchmark.

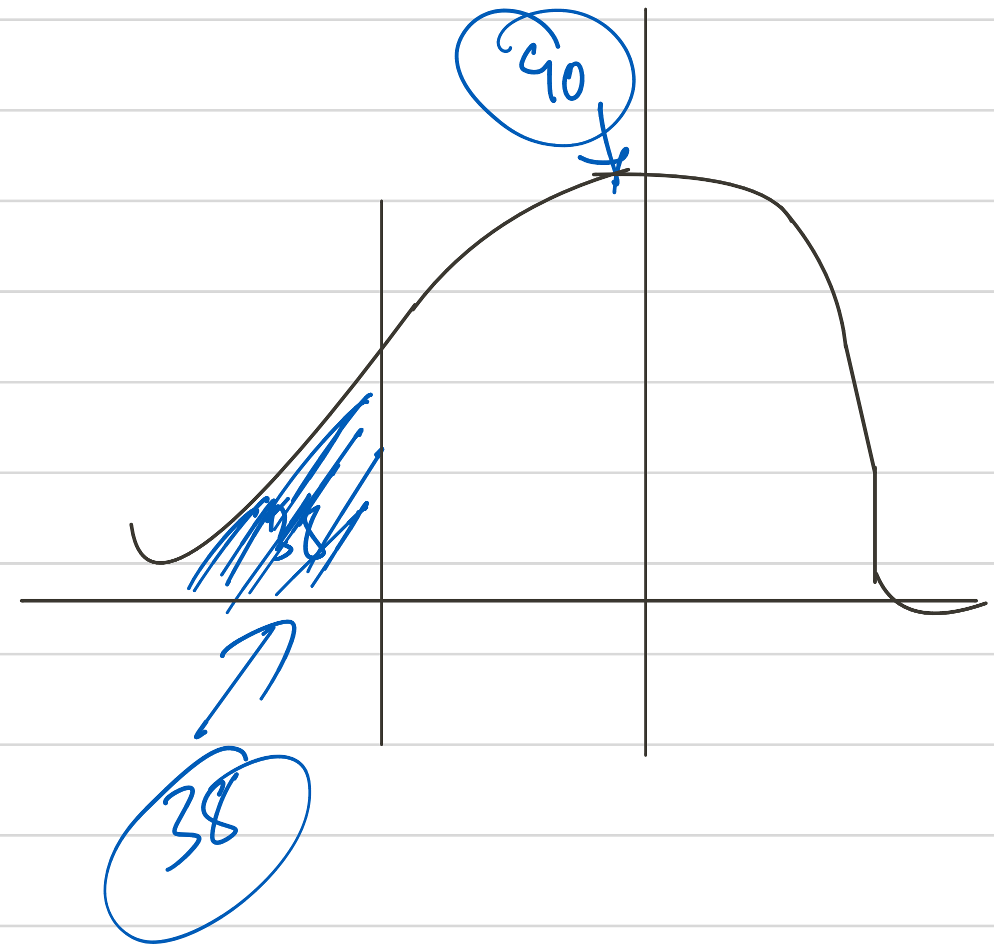
Z-test and T-test

One Sample Z-test Assumptions

- The data is a random sample of normally distributed population.
- The population standard deviation is known.

Online Delivery Data

- Population
  - μ = 40min
  - Standard Deviation = 5 min
- Sample
  - μ = 38 min
  - Standard Deviation = 5 min



Steps for Conducting a Hypothesis Test

- State the H_0 and H_1
- Choose a α-level
- find the p-value
- Reject or fail to reject the H_0.

Suppose:: $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$\boxed{z = -2.82}$

> So, One-Sample Hypothesis test the population mean is equal to an observed value.

> A data professional conducts a hypothesis if p-value $< \alpha$ then Reject the null Hypothesis.

Two-Sample Tests: Means:

> Test Types
- One-Sample
- Two-Sample

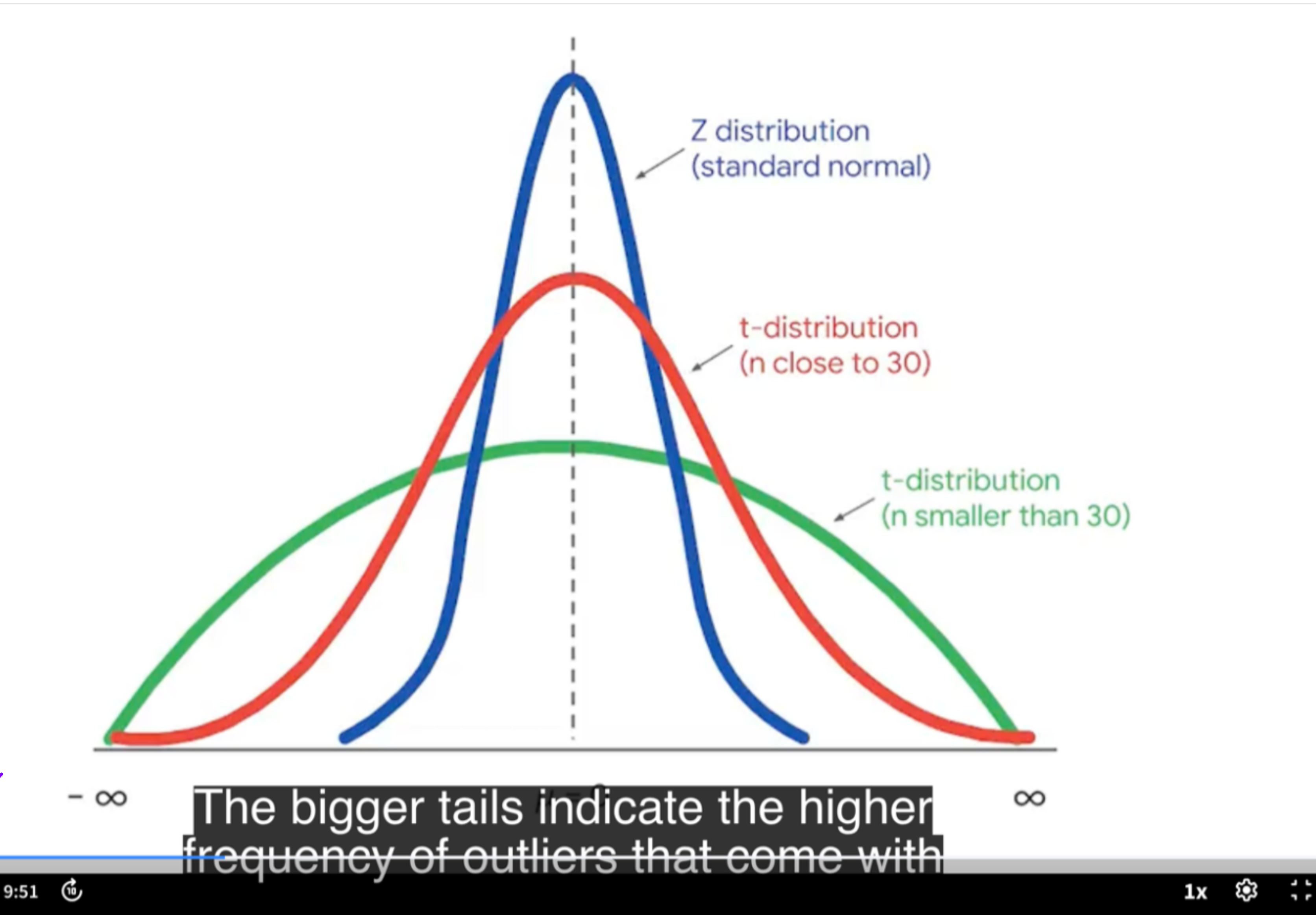
Two-Sample t-test for means assumptions

- The two samples are independent of each other.
- For each sample, the data is drawn randomly from a normally distributed population.

Example: A/B Testing.

Version A

- Sample size = 40
- Sample mean = 300 sec
- Sample Standard Deviation = 18.5 sec.



Version B

- Sample Size = 38
- Sample mean = 305 sec
- Sample standard deviation = 16.7 sec

> Use, T- test two Sample Test:

→ • Null: No Difference • Alternative: Yes, difference

→ $\alpha = 5\%$. → Choose P-value.

→ if p-value < 5% then Reject the Null Hypothesis.

→ else Fail to Reject

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$

$$t = \frac{300 - 305}{\sqrt{\left(\frac{18.5^2}{40} + \frac{16.7^2}{38}\right)}}$$

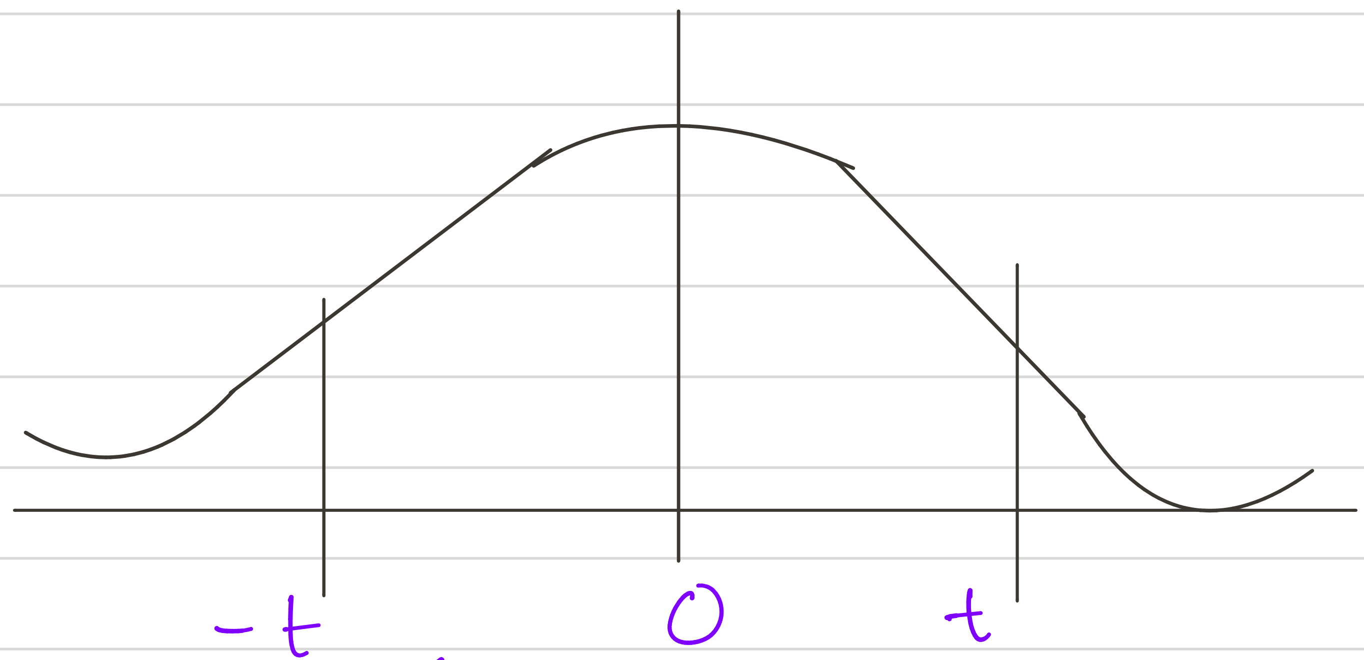
$$\boxed{t \Rightarrow -1.2508}$$

$$\boxed{p\text{-value} = 21.48\,\%}$$

→ $p < \alpha$ Reject H_0.

→ $p > \alpha$ Fail to Reject H_0.

p-value.



$[-1.2508]$ $[1.2508]$

$[-1.2508, 1.2508]$

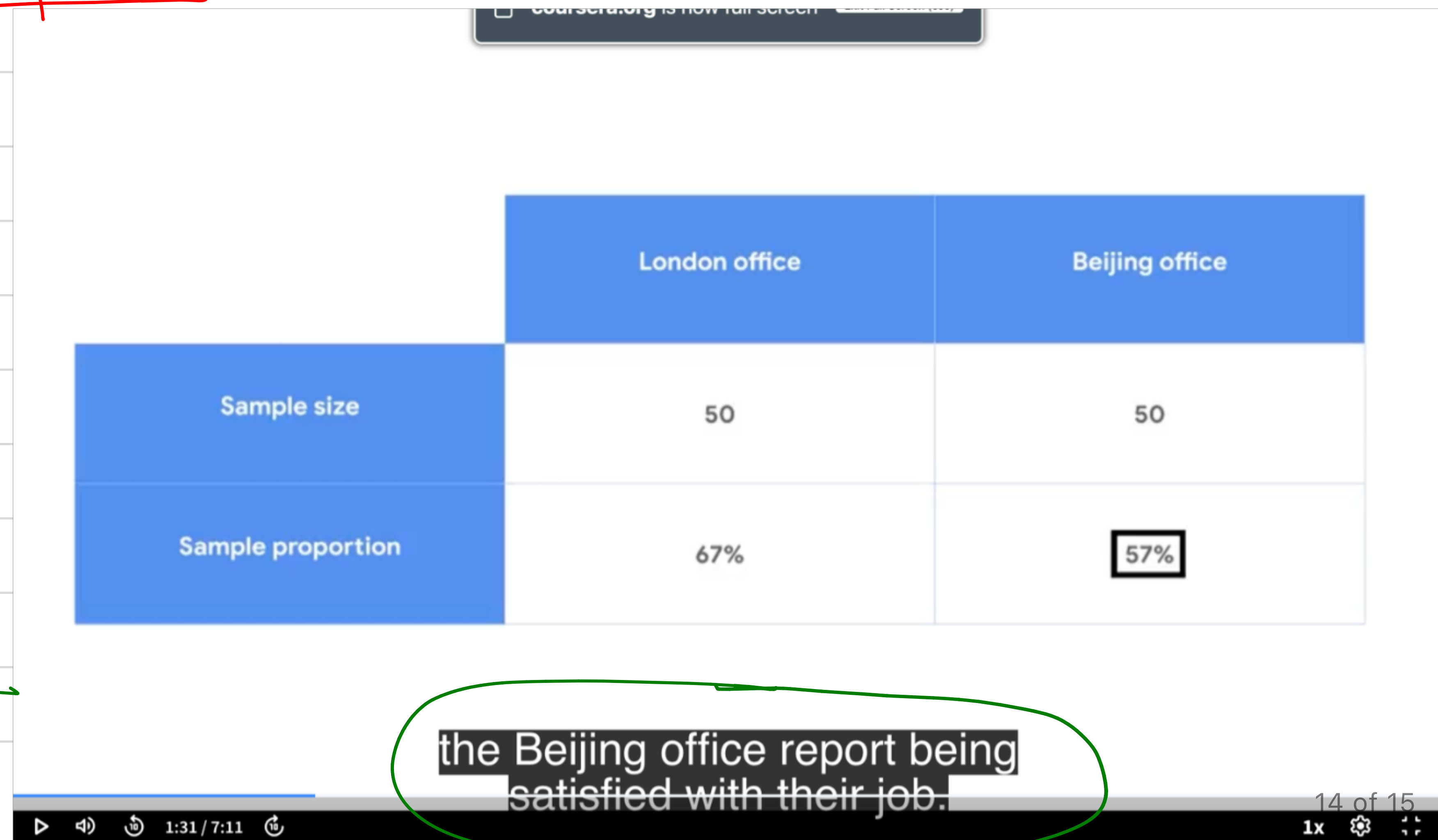
Use a two-Sample Z test to Compare the Proportion of

- Defects among manufacturing products on two assembly lines
- Side Effect to a new medicine for two trial groups.

t-tests donot apply to Proportions:

$$z \Rightarrow \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_0(1-\hat{p}_0)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$\Rightarrow \frac{0.67 - 0.57}{\sqrt{0.62(1-0.62)\left(\frac{1}{50}+\frac{1}{50}\right)}}$$



	London office	Beijing office
Sample size	50	50
Sample proportion	67%	$\boxed{57\%}$

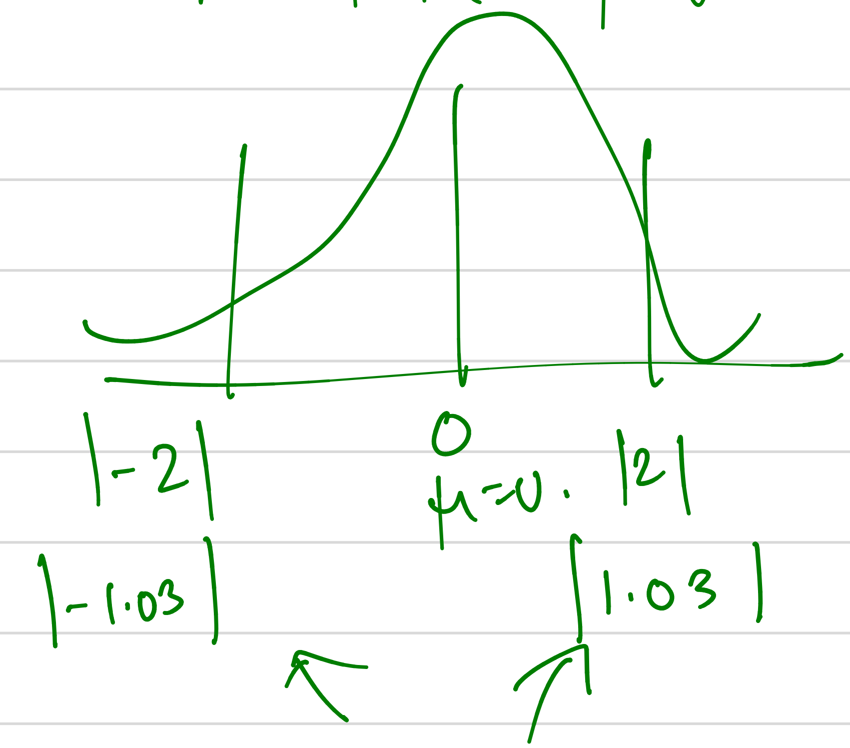
$$\boxed{Z_{score} = 1.03}$$

$p\text{-value} = 0.303$

$= 30.3\%$ of p

that the $|d_L - d_c|$

Two-tailed test.



Source: The power of statistics offered by Google at Coursera.