Tribhuvan University

Institute of Science and Technology

2066

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Bachelor Level/First Year/ Second Semester/ Science — Full Marks: 80

Computer Science and Information Technology — Pass Marks: 32

(MTH.155 – Linear Algebra) — Time: 3hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A **(10 x 2 = 20)**

1. When is system of linear equation consistent or inconsistent?

2. Write numerical importance of partitioning matrices.

3. How do you distinguish singular and non-singular matrices?

4. If A and B are n x n matrices, then verify with an example that det(AB) = det(A)det(B).

5. Calculate the area of the parallelogram determined by the columns of
$$A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix}.$$

6. Determine if $w = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is Nul(A), where, $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.

7. Determine if $\{v_1, v_2, v_3\}$ is a basis for λ^3, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

8. Find the characteristic polynomial for the eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$.

9. Let $\vec{v} = (1, -2, 2, 0)$. Find a unit vector $\vec{u}$ in the same direction as $\vec{v}$.

10. Let $\{u_1, \cdots\cdots\cdots, u_p\}$ be an orthogonal basis for a subspace W of R^n. Then prove that for each $y \epsilon W$, the weights in $y = c_1u_1 + \cdots\cdots\cdots + c_pu_p$ are given by
$$c_j = \frac{y.u_j}{u_j.u_j} \qquad (j = 1, \cdots\cdots\cdots, p)$$

Group B **(5 x 4 = 20)**

11. Prove that any set $\{v_1, \cdots\cdots\cdots, v_p\}$ in R^n is linearly dependent if $p > n$.

12. Consider the Leontief input – output model equation $x = cx + d$, where the consumption matrix is

$$C = \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}.$$

Suppose the final demand is 50 units of manufacturing, 30 units of agriculture, 20 units for services. Find the production level x that will satisfy the demand.

13. What do you mean by basis of a vector space? Find the basis for the row space of

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

OR

State and prove the unique representation theorem for coordinate systems.

14. What do you mean by eigen values, eigen vectors and characteristic polynomial of a matrix? Explain with suitable examples.

15. Define the Gram-Schmidt process. Let W=span$\{x_1, x_2\}$, where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Then construct an orthogonal basis $\{v_1, v_2\}$ for w.

Group C **(5 x 8 = 40)**

16. Given the matrix

$$\begin{bmatrix} 0 & 3 & -6 & 6 & -5 \\ 3 & -7 & 8 & -5 & 9 \\ 3 & -9 & 12 & -9 & 15 \end{bmatrix},$$

discuss the for word phase and backward phase of the row reduction algorithm.

17. Find the inverse of $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$, if it exists, by using elementary row reduce the augmented matrix.

18. What do you mean by change of basis in R^n? Let $b_1 = \begin{bmatrix} -9 \\ 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$, $c_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $c_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, and consider the bases for R^2 given by B={b_1, b_2} and C={c_1, c_2}. Find the change of coordinates matrix from B to C.

19. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$, if possible

OR

Find the eigen value of $A = \begin{bmatrix} 0.50 & -0.60 \\ 0.75 & 1.1 \end{bmatrix}$, and find a basis for each eigen space.

20. Find a least-square solution for Ax = b with $A = \begin{bmatrix} 1 & -6 \\ 1 & -2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix}$, $b = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}$. What do you mean by least squares problems?

OR

Define a least-squares solution of Ax = b, prove that the set of least squares solutions of Ax = b coincides with the non-empty set of solutions of the normal equations $A^TAx = A^Tb$.