# Year 2068

## Attempt all question:

### Group A (10 × 2 = 20)

1. Write down the conditions for consistent of non-homogenous system of linear equations.
2. What is meant by independent of vectors?
3. What is normal form of a matrix?
4. Define nonsingular linear transformation with suitable example.
5. Consider the matrix $A = \begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$ as a linear mapping. Write the corresponding co-ordinate equations.
6. State the numerical importance of determinant calculation by row operation.
7. Show that $\{(1,1), (-1,0)\}$ form a bias for $R^2$.
8. Let $T: R^2 \to R^2$ be a linear transformation defined by
   $$T(x, y) = (x + y, y).$$ Find Ker T.
9. If $\lambda$ is an eigen values of matrix A, find the eigen values of $A^{-1}$.
10. Let $u = (1,2,-1,3)$ and $v = (3,0,2,-2)$. Compute the inner product $(u, u + v)$.



### Group B (5 × 4 = 20)

11. Determine whether the following vectors in $R^3$ are linearly dependent:
    a. $(1,0,1), (1,1,0), (-1,0,-1)$,
    b. $(2,1,1), (3,-2,2), (-1,2,-1)$.
12. Investigate and interpret geometrically the transformation of the unit square whose vertices are $O(0,0,1), A(1,0,1), B(0,1,1)$ *and* $C(1,1,1)$ effected by the $3 \times 3$ matrix:

$$\begin{bmatrix} 1 & 1 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

OR

Is the set of vectors $\{(1,0,1), (0,1,0), (-1,0,1)\}$ orthrogonal? Obtain the corresponding orthonormal set in $R^3$.

13. In the vector space $R^2$, express the given vector (1,2) as a linear combination of the vectors $(1,-1) and\ (0,1)$
14. Find the matrix representation of the linear transformation $T: R^2 \rightarrow R^2$ defined by $T(x,y) = (x, x+2y)$ relative to the basis (1,0) and (1,1)
15. Let u and v be nonzero vector in $R^n$ and the angle between them be $\theta$. Then prove that

$$u.v = ||u||\ ||v||\ cos\theta$$

Where the symbol have their usual meanings.

## Group C (5×8=40)

16. Test for consistency and solve:

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 32$$

17. Let U and V be vector spaces over a field and assume that dim U=dim V. If $T: U \rightarrow V$ is a linear transformation, then prove that the following are equivalent;
    i. T is invertable
    ii. T is one-one and onto, and
    iii. T is non-singular

OR

Verify that the set of matrices of the form $\begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$ is a subspace of the vector space of 3×3 matrices.

18. Verify Cayley-Hamilton Theorem for matrix:

$$A = \begin{bmatrix} 6 & 2 & -1 \\ -6 & -1 & 2 \\ 7 & 2 & -2 \end{bmatrix}$$

19. Diagonalize the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

OR

Compute the multiplication of partitioned matrices for

$$A = \begin{bmatrix} 1 & 2 & 4 & 6 & 7 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 2 & 3 & 6 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -1 & 2 \\ 2 & 3 & 1 \\ 1 & 4 & 5 \\ 2 & 2 & 0 \\ 0 & 7 & 6 \end{bmatrix}$$

20. Find the equation $y = \beta_0 + \beta_1 x$ for the least squares line that best fits the data points (2, 0), (3, 4), (4, 10), (5, 16).