Unit Vector:
Null Space of Vector
Determinant
Inverse of matrix
Integral Domain
Field
Ring
Groups
Binary operation
Spanning Tree
MST
Poset
Recursive Algorithm, b^n
Ceiling and Floor Function
Applications of conditional probability or graph coloring theorem
Inclusion–Exclusion principle

Division algorithm or Euclidean division.

Define reflexive closure and symmetric closure. with example.
reflexivity, symmetry, and transitivity.
Show that the relation R = {(a, b): |a – b| is even} is an equivalence relation in the set of integers.
What is tautology with an example?

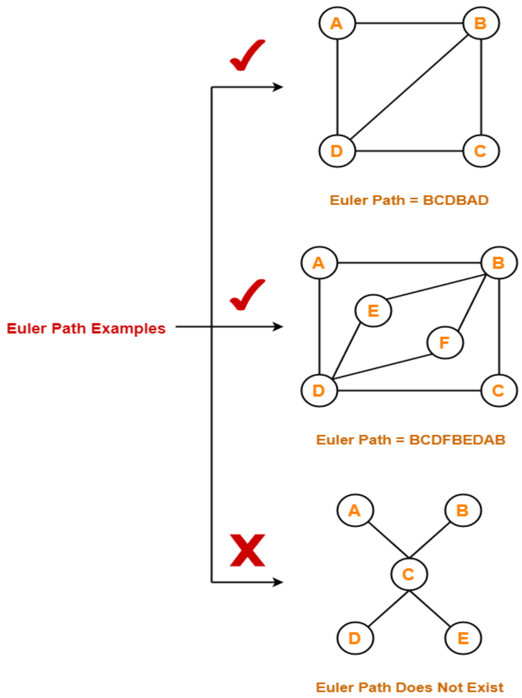
1. Find the min no of teachers in a college to be sure that four of them are born in the same month.
2. Find the min no of students in a class so that three of them are born in the same month?
3. How many ways can the letter of the word "arrange" can be arranged?
4. In how many ways can the letter be arranged so that two r's never come together?
5. A man has ten friends of whom six are relatives.How many ways can he invite 5 guests such that two of them may be relatives?
6. Expand $(x+y)^4$ using binomial theorem.
7. Find the coefficient of the term, containing y8 in the binomial expansion of $(x+3y)^{17}$
8. Find the general term in the expansion of $(x^2 + \frac{a2}{x})^5$
9. Find the seventh term of $(x + \frac{1}{x})^{10}$
10. Find the middle term of the expansion of (2a+3x)30

11. If there are 24 boys and 18 girls in a class, find the number of ways of selecting one student as a class representative.
12. Sum rule principle, permutation, and combination, product rule principle
13. Let E be the event of choosing a prime number less than 10, and F be the event of choosing an even number less than 10, and find the number of ways that E or F can occur.
14. A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. How many possible projects are there to choose from?
15. An office building contains 27 floors and has 37 offices on each floor.How many offices are there in the building?
16. How many different three-letter initials with none of the letters can be repeated can people have?
17. Three persons enter a car, where there are 5 vacant seats.In how many ways can they take up their seats?
18. Suppose a license plate contains two letters followed by three digits with the first digit, not zero. How many different license plate can be printed?
19. There are 32 microcomputers in a computer center. Each microcomputer has 24 ports.How many different ports to a microcomputer in the center are there?
20. In how many ways can an organization containing 26 members elect a president, treasurer, and secretary(assuming no person is elected to more than one position)?
21. How many strings are there four lowercase letters that have the letter x in them?

22. How many functions are there from the set {1,2,3……n} where n is a positive integer, to the set {0,1}.
23. Find the minimum number of students required in a class to be sure that three of them are born in the same month.
24. Show that if any 15 people are selected, then we may choose a subset of 3 so that all 3 were born on the same day of the week.
25. Suppose a laundry bag contains many red, white, and blue socks.Find the minimum number of socks that one needs to choose in order to get two pairs (four socks) of the same colour.
26. How many numbers must be selected from the set {1,3,5,7,9,11,13,15} to guarantee that at least one pair of these numbers add up to 16?
27. Find the least number of cables required to connect eight computers to four printers to guarantee that four computers can directly access four different printers.Justify your answer.

Eular Path:

- Cover every edges of the Graph only once
- Starts and ends with different vertices
- At most have 2 vertices with odd degrees

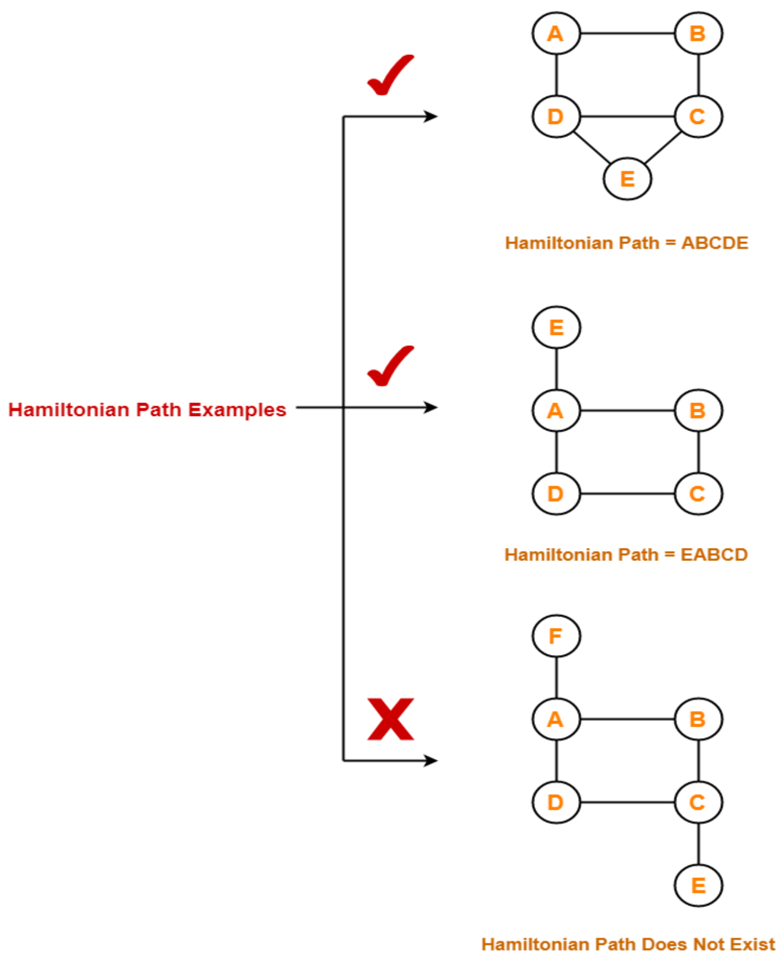


Eular Circuit

- Starts and end at the same vertex
- Closed path that visits every edge exactly once
- Every vertex must have even degree

Hamilton Path:

- Cover every vertices exactly once
- Start and end with different vertex



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Hamilton Circuit.

-Start and end with same vertex, covering every vertices once

SD Flow

1. In graph theory, SD flow stands for "source to destination flow."
2. It is a concept used to determine the maximum flow that can pass through a network from a source node to a destination node.
3. The source node is the node where the flow originates, and the destination node is the node where the flow terminates.
4. It refers to the amount of flow that can be sent through graph
5. This flow should not violate any capacity constraints
6. It should not exceed the capacity of any edge

7. SD Flow is also known as the "maximum flow" or "maximum capacity" of a graph.

ST Cut Flow

In discrete mathematics, a cut is a partition of the vertices of a graph into two disjoint sets. A cut in a graph is said to be an "ST cut" if the two sets of vertices created by the cut contain the source vertex "S" and the destination vertex "T" of a specified edge (S, T). In other words, an ST cut is a cut that separates the source vertex S from the destination vertex T in a graph.

The capacity of an ST cut is the sum of the capacities of all edges crossing the cut, i.e., the sum of the weights of all edges that connect a vertex in the source set to a vertex in the destination set.

The concept of ST cuts is used in the Max-Flow Min-Cut theorem, which states that the maximum flow in a network is equal to the minimum capacity of an ST cut in the network. This theorem is a fundamental result in network flow theory and has important applications in various fields, including transportation, communication, and computer science.

- A cut is a partition of the vertices of a graph into two disjoint sets.
- **An ST cut is cut in a graph that separates the source vertex S from the destination vertex T.**
- The capacity of an ST cut is the s***um of the weights of all edges that cross the cut and connect a vertex in the source set to a vertex in the destination set.***
- **The Max-Flow Min-Cut theorem states that the maximum flow in a network is equal to the minimum capacity of an ST cut in the network.**
- The Max-Flow Min-Cut theorem has important applications in various fields, including transportation, communication, and computer science.

Premise

A premise, on the other hand, is a proposition that is used as the basis for a logical argument or deduction. A premise is a statement or assumption that is accepted as true in order to reach a conclusion. In a logical argument, the premises are the statements or assumptions that are used to support the conclusion.

For example, consider the following logical argument:

> Premise 1: All men are mortal.
>
> Premise 2: Socrates is a man.

Conclusion: Therefore, Socrates is mortal.

In this argument, "All men are mortal" and "Socrates is a man" are the premises, and "Socrates is mortal" is the conclusion. The premises are accepted as true in order to support the conclusion.

Existential Quantifier and Universal Quantifier

In discrete mathematics, quantifiers are used to describe the quantity of elements in a set that satisfy a certain property or condition. The two most common quantifiers are the existential quantifier and the universal quantifier.

The existential quantifier, denoted by $\exists$ (read as "there exists"), is used to indicate that at least one element in a set satisfies a certain property. For example, the statement "There exists an even integer" can be written as $\exists x$(x is even).

The universal quantifier, denoted by $\forall$ (read as "for all"), is used to indicate that every element in a set satisfies a certain property. For example, the statement "All even integers are divisible by 2" can be written as $\forall x$(x is even $\rightarrow$ x is divisible by 2).

In both cases, the variable x is used to represent an arbitrary element in the set. The quantifiers allow us to make general statements about the elements in the set without having to list them all out individually.

The Law of Detachment and Modus Ponens are related, but they are not exactly the same.

Modus Ponens is a specific form of the Law of Detachment, which states that if a conditional statement $p \rightarrow q$ is true, and p is true, then q must also be true. In other words, Modus Ponens is the rule of inference that allows us to infer the consequent q from the antecedent p and the conditional statement $p \rightarrow q$.

For example, if we know that "If it is raining, then the ground is wet" ($p \rightarrow q$) and we observe that it is indeed raining (p), then we can use Modus Ponens to conclude that "The ground is wet" (q).

On the other hand, the Law of Detachment is a more general rule of inference that applies to any valid argument in the form of "If $p \rightarrow q$ and p, then q". This means that if we have a valid argument of this form, we can use the Law of Detachment to conclude that q follows logically from the premises $p \rightarrow q$ and p.

In summary, Modus Ponens is a specific application of the Law of Detachment, where we use a conditional statement and its antecedent to infer its consequent, while the Law of Detachment is a more general principle that can be applied to any valid argument in this form.

Rules of inference are used to derive new logical statements or propositions from existing ones. Here are some common rules of inference:

Modus Ponens (MP): If p implies q and p are true, then q is true.

1. Example: If it is raining, then the ground is wet. It is raining. Therefore, the ground is wet.

Modus Tollens (MT): If p implies q and q is false, then p is false.

2. Example: If it is raining, then the ground is wet. The ground is not wet. Therefore, it is not raining.

Dijkstra's algorithm

is a popular algorithm in computer science used for finding the shortest path between two nodes in a graph with non-negative edge weights.

Dijkstra's algorithm is a way to find the shortest path between two points in a network. It starts at one point and looks at all the neighboring points, picking the one closest to the starting point. Then it looks at the neighbors of that point, and so on. It keeps track of the distances to each point and updates them if it finds a shorter path. Finally, it reaches the destination point and returns the shortest path. It's used in many applications, like finding the fastest route between two cities, but only works for networks where distances are never negative.

How does Dijkstra's algorithm works? Steps in the Simplest language.

Dijkstra's algorithm is used to find the shortest path between two points in a graph. Here are the steps in simple language:

1. Start at the initial node and set its distance to 0. Set the distances of all other nodes to infinity.
2. Mark the initial node as visited.

3. For each neighbor of the initial node, calculate the distance from the initial node to that neighbor and update its distance if it's shorter than the previous distance.
4. Select the unvisited node with the smallest distance and mark it as visited.
5. For each neighbor of the current node, calculate the distance from the current node to that neighbor and update its distance if it's shorter than the previous distance.
6. Repeat steps 4 and 5 until the destination node is marked as visited.
7. The shortest path is the sum of the distances from the initial node to the destination node, which is the value of the destination node's distance.

To keep track of the path, you can store the predecessor of each node (the previous node on the shortest path). Then, once the destination node is reached, you can follow the predecessors back to the initial node to obtain the shortest path.

Ceiling and Floor Function

The Ceiling and Floor Functions, on the other hand, are functions that are used in calculus and analysis to **round real numbers to the nearest integer.** The floor function, **denoted by $\lfloor x \rfloor$, gives the largest integer that is less than or equal to x,** while the ceiling function, **denoted by $\lceil x \rceil$, gives the smallest integer that is greater than or equal to x**. For example, $\lfloor 3.7 \rfloor = 3$ and $\lceil 3.7 \rceil = 4$. These functions are useful in many applications, such as computing limits, derivatives, and integrals of functions involving real numbers.

Pascal’s Triangle

It is the triangular arrangement of the numbers that give coefficients in the expansion of any binomial expression, such as that $(x+y)^n$.

It is named for the 17th-century French mathematician Blaise Pascal.

Formula

- n is the number of rows
- m is the number of elements
- and n is non-negative integers and 0 ≤m ≤n.

$\binom{n}{m}$= $\binom{n+1}{m+1}$ + $\binom{n-1}{m}$

Compute the following:

a. 3 mod 4 = 3

b. 7 mod 5 = 2

c. -5 mod 3 = 1

d. 11 mod 5 = 1

e. -8 mod 6 = 4

a. 3 mod 4:

To find the remainder when 3 is divided by 4, we can use the division algorithm. We need to find the quotient and remainder when 3 is divided by 4.

3 = 4 * 0 + 3

So the quotient is 0 and the remainder is 3. Therefore, 3 mod 4 = 3.

b. 7 mod 5:

To find the remainder when 7 is divided by 5, we can again use the division algorithm.

7 = 5 * 1 + 2

So the quotient is 1 and the remainder is 2. Therefore, 7 mod 5 = 2.

c. -5 mod 3:

To find the remainder when -5 is divided by 3, we first need to find the greatest multiple of 3 that is less than or equal to -5. This is -6. Then, we can use the formula:

remainder = number - divisor * quotient

where the number is -5, the divisor is 3, and the quotient is the integer part of -5/3, which is -2.

remainder = -5 - 3 * (-2)

= -5 + 6

= 1

Therefore, -5 mod 3 = 1.

d. 11 mod 5:

To find the remainder when 11 is divided by 5, we can use the division algorithm.

11 = 5 * 2 + 1

So the quotient is 2 and the remainder is 1. Therefore, 11 mod 5 = 1.

e. -8 mod 6:

To find the remainder when -8 is divided by 6, we first need to find the greatest multiple of 6 that is less than or equal to -8. This is -12. Then, we can use the formula:

remainder = number - divisor * quotient

where the number is -8, the divisor is 6, and the quotient is the integer part of -8/6, which is -2.

remainder = -8 - 6 * (-2)

= -8 + 12

= 4

Therefore, -8 mod 6 = 4.

Show that the relation R = {(a, b): |a – b| is even} is an equivalence relation in the set of integers.

To show that R is an equivalence relation, we need to show that it satisfies the three properties of reflexivity, symmetry, and transitivity:

1. Reflexivity: For all integers a, a - a = 0, which is even. Therefore, (a, a) ∈ R for all integers a, so R is reflexive.
2. Symmetry: Let (a, b) ∈ R. Then |a - b| is even. But since the absolute value of a number is always non-negative, we have |b - a| = |a - b|, which is also even. Therefore, (b, a) ∈ R, and R is symmetric.
3. Transitivity: Let (a, b) and (b, c) ∈ R. Then |a - b| and |b - c| are even. Since the sum or difference of even numbers is always even, we have:

- (a - b) + (b - c) = a - c is even
- (a - b) - (b - c) = a - 2b + c is even

In either case, $|a - c|$ is even, so $(a, c) \in R$. Therefore, R is transitive.Since R satisfies all three properties, it is an equivalence relation in the set of integers.

What are reflexivity, symmetry, and transitivity?

Reflexivity, symmetry, and transitivity are properties of binary relations between elements in a set.

1. Reflexivity: A relation R on a set A is reflexive if every element in A is related to itself by R. In other words, $(a, a) \in R$ for all $a \in A$.
2. Symmetry: A relation R on a set A is symmetric if for any elements a and b in A, if $(a, b) \in R$, then $(b, a) \in R$. In other words, if the relation holds for one direction, it also holds for the opposite direction.
3. Transitivity: A relation R on a set A is transitive if for any elements a, b, and c in A, if $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$. In other words, if there is a path between a and b and a path between b and c, then there is a path between a and c.

Show that the relation $R = \{(a, b): |a - b| \text{ is even}\}$ is an equivalence relation in the set of integers.
To show that the relation $R = \{(a, b): |a - b| \text{ is even}\}$ is an equivalence relation on the set of integers, we need to show that it satisfies the three properties of reflexivity, symmetry, and transitivity.

1. Reflexivity: For any integer a, $|a - a| = 0$, which is an even number. Therefore, $(a, a) \in R$ for all integers a, and R is reflexive.
2. Symmetry: Let $(a, b) \in R$, which means $|a - b|$ is even. Then, $|b - a| = |-(a - b)| = |a - b|$ is also even. Therefore, $(b, a) \in R$, and R is symmetric.
3. Transitivity: Let (a, b) and $(b, c) \in R$, which means $|a - b|$ and $|b - c|$ are even. Then, their sum or difference is also even. Specifically:

- $(a - b) + (b - c) = a - c$ is even
- $(a - b) - (b - c) = a - 2b + c$ is even

In either case, $|a - c|$ is even, which means $(a, c) \in R$. Therefore, R is transitive.

Since R is reflexive, symmetric, and transitive, it is an equivalence relation on the set of integers.

List any one example of tautology. Represent the following sentences into predicate logic.

Not all employees are loyal
All students having good attitude are lovable.

One example of a tautology is the statement "A or not A," where A is any proposition. This statement is always true, regardless of the truth value of A.

Using predicate logic, we can represent the following sentences as:

1. Not all employees are loyal
   - Let $E(x)$ be "x is an employee" and $L(x)$ be "x is loyal"
   - The given sentence can be represented as $\neg\forall x(E(x) \rightarrow L(x))$ or equivalently, $\exists x(E(x) \land \neg L(x))$
2. All students having a good attitude are lovable.
   - Let $S(x)$ be "x is a student", $G(x)$ be "x has a good attitude", and $L(x)$ be "x is lovable"
   - The given sentence can be represented as $\forall x(S(x) \land G(x) \rightarrow L(x))$

What is tautology with an example?

In logic, a tautology is a statement that is always true, regardless of the truth values of its individual components. In other words, a tautology is a logical expression that is true under all possible interpretations of its propositional variables.

One example of a tautology is the statement "Either it will rain tomorrow, or it will not rain tomorrow." This statement is always true, regardless of whether it actually rains tomorrow or not, because it covers all possible scenarios.

Another example of a tautology is the logical expression "$P \land (P \rightarrow Q) \rightarrow Q$", also known as the **law of detachment or modus ponens.** This expression is always true because it represents a valid logical inference rule, which states that if P implies Q and P is true, then Q must also be true.

Prove that "If the product of two integers a and b are even then either a is even or b is even", using the contradiction method.

To prove the statement "If the product of two integers a and b are even then either a is even or b is even" using the contradiction method, we assume the opposite of the statement, namely that the product of two integers a and b is even, but neither a nor b is even.

Assume that a and b are odd integers such that their product ab is even. Since a is odd, we can write it as $a = 2k + 1$ for some integer k. Similarly, b can be written as $b = 2m + 1$ for some integer m.

Then, their product ab can be expanded as:

ab = (2k + 1)(2m + 1)

= 4km + 2k + 2m + 1

= 2(2km + k + m) + 1

Since ab is even, it must be of the form 2n for some integer n. Therefore, we have:

2n = 2(2km + k + m) + 1

Rearranging this equation, we get:

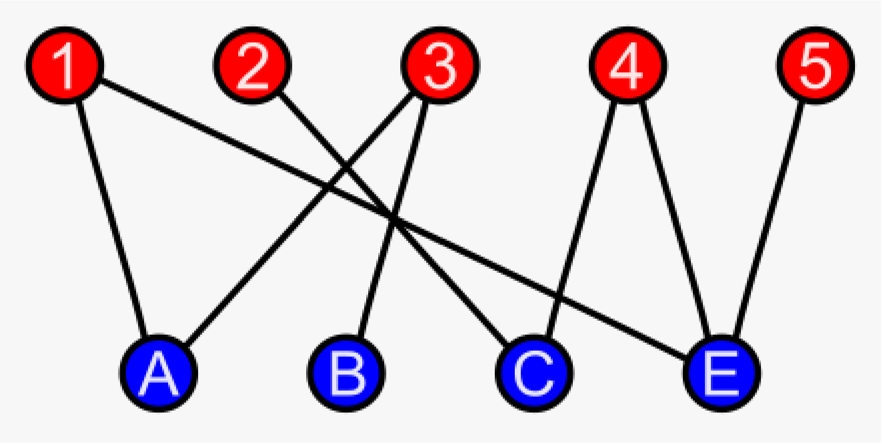
2n - 1 = 2(2km + k + m)

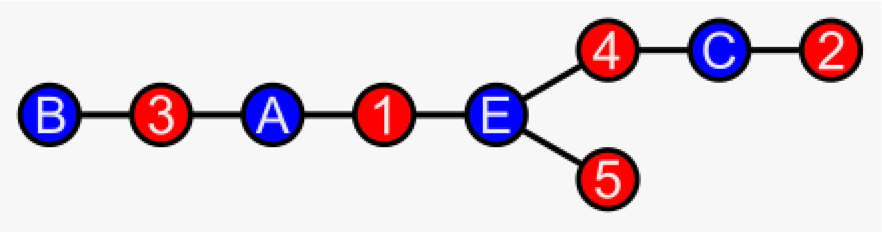
The left-hand side of this equation is odd, while the right-hand side is even, which is a contradiction. Therefore, our assumption that both a and b are odd is false.

Hence, we can conclude that if the product of two integers a and b is even, then either a is even or b is even, which completes the proof.

Define a bipartite graph with an example. State the necessary conditions for the graphs to be isomorphic.

A bipartite graph is a type of graph in which the vertices can be partitioned into two disjoint sets such that every edge of the graph connects a vertex from one set to a vertex in the other set. That is, a bipartite graph is a graph whose vertices can be colored with two colors so that no two adjacent vertices have the same color.





The necessary conditions for two graphs to be isomorphic can be summarized in the following 5 points:

1. They must have the same number of vertices.
2. They must have the same number of edges.
3. They must have the same degree sequence (i.e., the degrees of the vertices in each graph must be the same).
4. They must have the same number of connected components.
5. They must have the same number of cycles.

PigeonHole vs Generalized Pigeonhole Principle:

The generalized pigeonhole principle and the pigeonhole principle are both related to the idea of counting objects and assigning them to containers, but they differ in the number of objects and containers involved.

The pigeonhole principle states that if n items are placed into m containers, where n is greater than m, then at least one container must contain more than one item. This principle applies to the case where there are only a few containers and a large number of objects.

The generalized pigeonhole principle, on the other hand, applies to the case where there are many containers and a large number of objects. Specifically, it states that if n objects are placed into k containers, where k is less than n, then there is at least one container that contains ceil(n/k) objects.

In other words, the generalized pigeonhole principle is a more general version of the pigeonhole principle that applies to cases where there are more containers than in the original principle.

Both principles are important tools in combinatorics and discrete mathematics, and they have numerous applications in various fields such as computer science, cryptography, and probability theory.

The pigeonholes principle states that if n items are put into m containers, with $n > m$, then at least one container must contain more than one item.

The generalized pigeonhole principle states that if n objects are placed into k boxes, then there is at least one box containing $\lceil n/k \rceil$ objects, where $\lceil n/k \rceil$ is the ceiling function, which rounds n/k up to the nearest integer. This principle generalizes the original pigeonhole principle, which states that if n + 1 objects are placed into n boxes, then at least one box must contain two or more objects.

Given the premises “If it rains or strike holds then the exam will be cancelled. If the doesn’t rain then it will be sunny day. The exam was not cancelled. show that it is sunny day”.

Using modus tollens, we can reason as follows:

- "If it rains or strike holds then the exam will be cancelled." This can be written as: $(r \lor s) \rightarrow \neg e$, where r stands for "it rains", s stands for "strike holds", and e stands for "the exam is cancelled."
- "If the doesn’t rain then it will be sunny day." This can be written as: $\neg r \rightarrow s$.
- "The exam was not cancelled." This can be written as: $\neg e$.

From the premises, we can write the following logical statements:

p: It rains

q: Strike holds

r: Exam is cancelled

s: It is sunny

Using these symbols, we can rewrite the premises as follows:

1. $(p \vee q) \rightarrow r$
2. $\neg p \rightarrow s$
3. $\neg r$

We need to show that s is true, i.e., the conclusion is:

4. s

Using the truth table method, we can show that the premises logically imply the conclusion:

p	**q**	**r**	**s**	**$(p \vee q) \rightarrow r$**	**$\neg p$**	**$\neg r$**	**$\neg p \rightarrow s$**
T	T	T	T	T	F	F	T
T	T	F	T	T	F	T	T

T	F	T	T	T	F	F	T
T	F	F	T	F	F	T	T
F	T	T	T	T	T	F	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	T	T

The only row where all premises are true is the first row, and in that row, the conclusion s is also true. Therefore, we can conclude that it is a sunny day.

Find the value of -2 MOD 3 and 315 MOD 5. Illustrate an example to show the join operation between any two boolean matrixes.

To find the value of -2 MOD 3, we need to find the remainder when -2 is divided by 3.

We can use the division algorithm to do this:

$-2 = (3)(-1) + 1$

Since the remainder is 1, we have -2 MOD 3 = 1.

To find the value of 315 MOD 5, we need to find the remainder when 315 is divided by 5.

We can use the division algorithm again:

315 = (5)(63) + 0

Since the remainder is 0, we have 315 MOD 5 = 0.

The value of -2 MOD 3 is 1, since -2 is equivalent to 1 modulo 3.

The value of 315 MOD 5 is 0, since 315 is equivalent to 0 modulo 5.

Here's an example to illustrate the join operation between two boolean matrices:

Suppose we have two matrices A and B:

A = | 1 0 1 |

| 0 1 0 |

| 1 1 0 |

B = | 1 1 0 |

| 0 1 1 |

| 1 0 0 |

To join A and B, we perform an element-wise OR operation between the corresponding entries in A and B:

A ∨ B = | 1 1 1 |

| 0 1 1 |

| 1 1 0 |

In other words, the entry in the i-th row and j-th column of A OR B is 1 if and only if at least one of the entries in the i-th row and j-th column of A or B is 1.

Given an example of fallacy. State the necessary and sufficient conditions for a graph to have an Euler path and an Euler circuit.

A fallacy is a type of error in reasoning that occurs when an argument is logically flawed. It is a mistake or failure in an argument or line of reasoning that renders it invalid. Here's an example of a fallacy:

Example: All cats have fur. My dog has fur. Therefore, my dog must be a cat.

This argument is an example of the fallacy of affirming the consequent, where the truth of a conclusion is assumed based on the fact that the statement appears to be true. In this case, even though both cats and dogs may have fur, the presence of fur alone does not make a dog a cat. Therefore, the argument is invalid and flawed.

Conditions for Euler Path

A graph must be connected in order to have an Euler path

- In a graph with an Euler path, there are at most two vertices with odd degrees. It has exactly zero or two vertices with odd degrees.
- visits every edge of a graph exactly once.
- Starts and end with different vertex

Necessary and sufficient conditions for Euler circuit:

- A graph has an Euler circuit if and only if it is connected and every vertex has an even degree.
- closed path in a graph that visits every edge exactly once and starts and ends at the same vertex. In other words, an Euler circuit is a cycle that covers every edge of the graph exactly once

Give a recursive defined function to find the factorial of any given positive integer. in discrete mathematics language.
In discrete mathematics language, the factorial of a positive integer n is defined

recursively as follows:

- Base case: If n = 0 or n = 1, then n! = 1.
- Recursive case: If n > 1, then n! = n * (n-1)!, where (n-1)! denotes the factorial of (n-1).

Compute the following values. a) 3 mod 4 b)7 mod 5. c) -5 mod 3 d)11 mod 5be)-8 mod 6
To compute the following values:

a) 3 mod 4

- Divide 3 by 4: 3 = 0*4 + 3
- The remainder is 3, therefore 3 mod 4 = 3.

b) 7 mod 5

- Divide 7 by 5: 7 = 1*5 + 2
- The remainder is 2, therefore 7 mod 5 = 2.

c) -5 mod 3

- Divide -5 by 3: -5 = (-2)*3 + 1
- The remainder is 1, therefore -5 mod 3 = 1.

d) 11 mod 5

- Divide 11 by 5: 11 = 2*5 + 1
- The remainder is 1, therefore 11 mod 5 = 1.

e) -8 mod 6

- Divide -8 by 6: -8 = (-2)*6 + 4
- The remainder is 4, therefore -8 mod 6 = 4.

Recursive Algorithm

An algorithm is called the recursive algorithm if it solves a problem by reducing it to an instance of the same problem with smaller input.

```
power(b, n):
  if n == 0:
    return 1
  else:
    return b * power(b, n-1)
```

Steps:

1. Procedure powerN (int base, int n)
2. if(n < 0) then
3. return (“illegal power argument”)
4. if(n == 0) then
5. return 1
6. Else
7. return base * powerN(base, n - 1)
8. end powerN

List any two applications of conditional probability.

Conditional probability is an important concept in probability theory and has many practical applications in various fields. Here are two examples:

1. Medical diagnosis: Conditional probability is commonly used in medical diagnosis to determine the probability of a particular disease or condition given a certain set of symptoms. For example, if a patient presents with a particular set of symptoms, a doctor can use conditional probability to calculate the probability of the patient having a certain disease or condition.
2. Fraud detection: Conditional probability is also used in fraud detection in various industries such as banking and insurance. By analyzing patterns and transactions, companies can calculate the probability of a particular transaction being fraudulent given certain conditions. For example, a bank may use conditional probability to identify potentially fraudulent credit card transactions by looking at the spending patterns of a customer and comparing them to their usual behavior.

You have 9 families you would like to invite to a wedding. Unfortunately, you can only invite 6 families. How many different sets of invitations could you write?

To find the number of different sets of invitations you could write, we can use the combination formula, which is:

nCr = n! / r!(n - r)!

where n is the total number of items, r is the number of items we want to choose, and ! denotes the factorial function (i.e., the product of all positive integers up to a given number).

In this case, we have n = 9 (the total number of families) and r = 6 (the number of families we want to invite). So the number of different sets of invitations we can write is

9C6 = 9! / 6!(9 - 6)! = (9 x 8 x 7) / (3 x 2 x 1) = 84

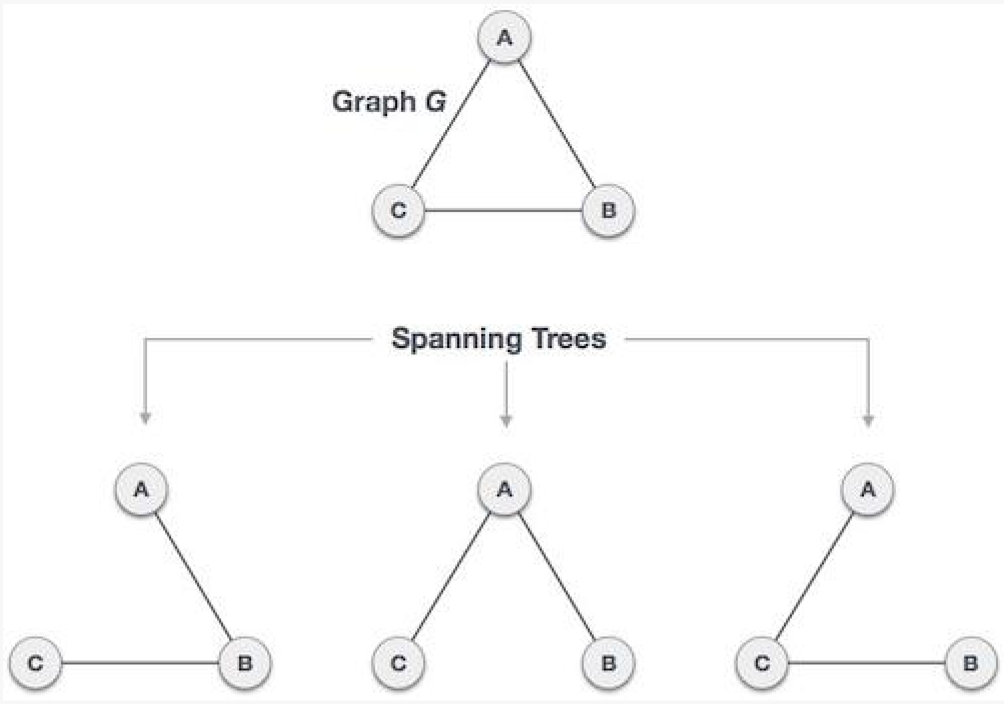
Therefore, we could write 84 different sets of invitations.

Define the spanning tree and the minimum spanning tree(MST).

Let G be a simple graph.

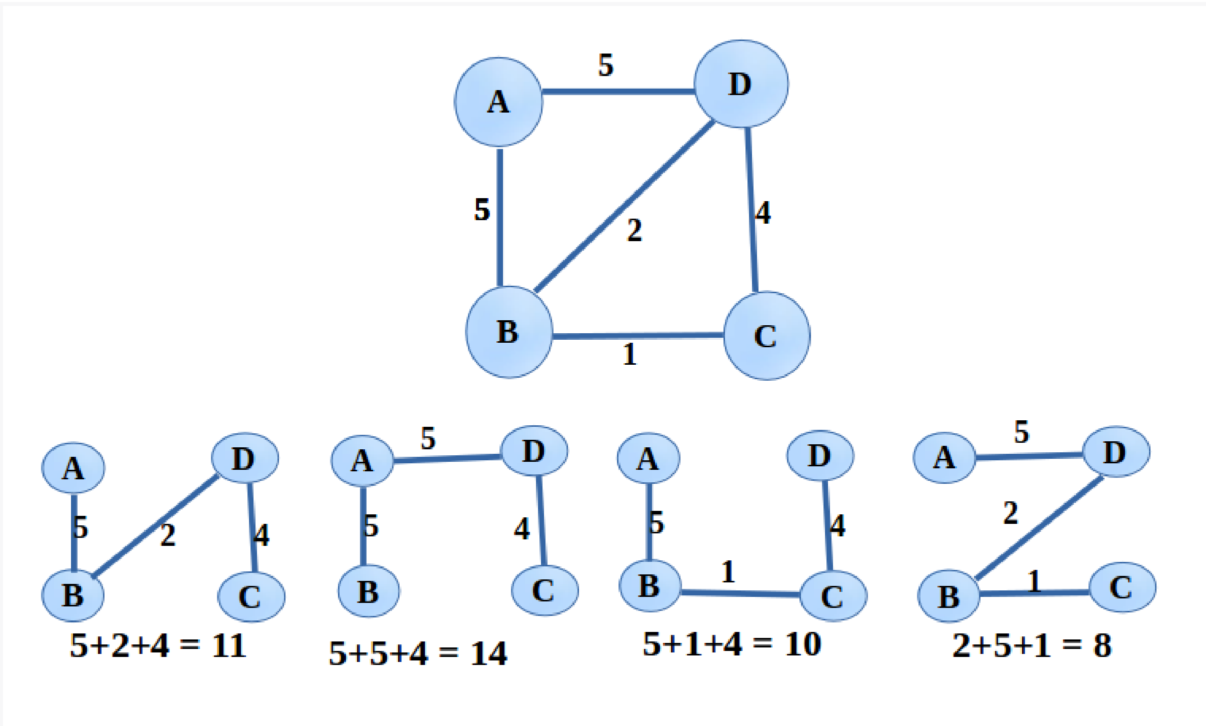
A spanning tree of G is a sub-graph of G that is a tree containing every vertex of G. It must be connected and have a path between any two vertices.

Every connected simple graph has a spanning tree.



MST

MST in a connected weighted graph is a spanning tree that has the smallest possible sum of the weights of its edges.



Which of the following are possets?

- (Z, =)
- (Z, ≠)
- (Z, ⊆)

A partially ordered set (poset) is a set together with a binary relation that satisfies certain properties. Specifically, a posset is a set P together with a binary relation ≤ such that for any elements a, b, and c in P:

1. a ≤ a (reflexivity)
2. if a ≤ b and b ≤ a, then a = b (antisymmetry)
3. if a ≤ b and b ≤ c, then a ≤ c (transitivity)

The first property is reflexivity, which says that every element in the posset must be related to itself. The second property is antisymmetry, which says that if two elements are related to each other, they must be the same element. The third property is transitivity, which says that if one element is related to a second element, and the second element is related to a third element, then the first element must be related to the third element.

The set of integers Z, together with the "equal to" relation =, is a poset.

To show that (Z, =) is a poset, we need to verify the three axioms of a poset:

1. Reflexivity: For any integer a in Z, we have a = a. This is true because every integer is equal to itself.
2. Antisymmetry: For any integers a,b in Z, if a = b and b = a, then a = b. This is also true because if a and b are both integers and they are equal to each other, then a and b must be the same integer.
3. Transitivity: For any integers a,b,c in Z, if a = b and b = c, then a = c. This is true because the "equal to" relation is transitive: if a is equal to b and b is equal to c, then a is equal to c.

Therefore, (Z, =) is a poset.

On the other hand, the set of integers Z, together with the "not equal to" relation ≠, is not a poset, because the "not equal to" relation is not reflexive. For any integer a in Z, we have a ≠ a, which violates the reflexivity property of a poset.

Finally, the set of integers Z cannot form a poset under the subset relation ⊆, because the subset relation is not defined between two elements of Z. Instead, we could talk about the poset (P(Z), ⊆), where P(Z) denotes the power set of Z, that is, the set of all subsets of Z. In this poset, the relation ⊆ represents the inclusion relationship between subsets, as explained in the previous answer.

The relation (Z, ≠) is not a poset because it violates the reflexivity property of a poset. Specifically, the "not equal to" relation is not reflexive, since for any integer a in Z, we have a ≠ a. Therefore, (Z, ≠) is not a poset.

Define reflexive closure and symmetric closure. with example. Find the remainder when 4x2 – x + 3 is divided by x + 2 using remainder theorem.

Reflexive closure:

The reflexive closure of a relation R on a set A is obtained by adding to R all the pairs (a,a) for every element a in A that is not already related to itself by R. In other words, the reflexive closure of R ensures that every element in A is related to itself.

For example, suppose we have a relation R = {(1,2), (3,3), (4,5)} on the set A = {1,2,3,4,5}. The reflexive closure of R, denoted by R*, is obtained by adding (1,1), (2,2), and (5,5) to R. So, R* = {(1,2), (1,1), (2,2), (3,3), (4,5), (5,5)}.

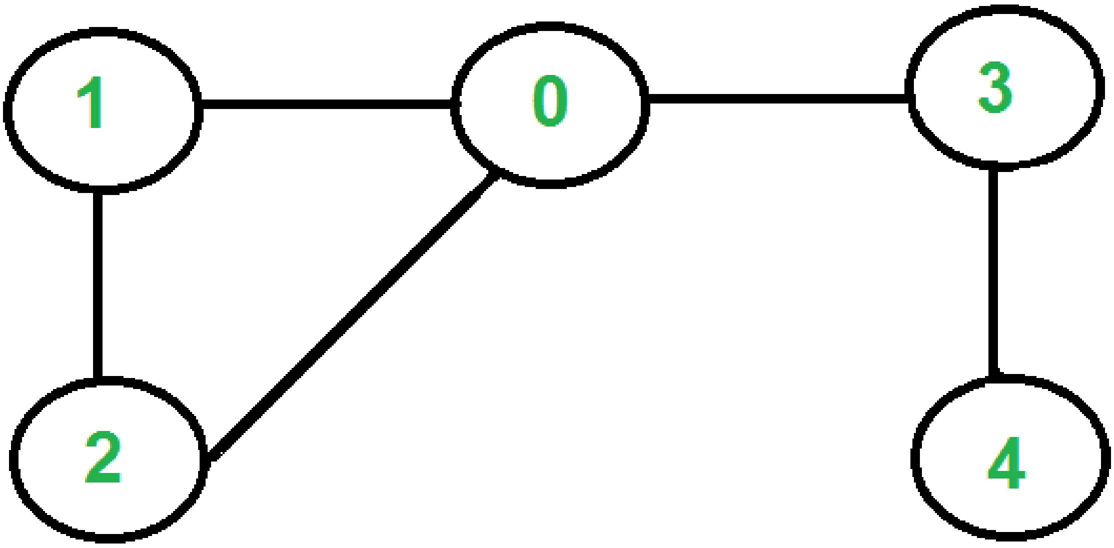
Symmetric closure:

The symmetric closure of a relation R on a set A is obtained by adding to R all the pairs (b,a) for every pair (a,b) in R, where a and b are distinct elements of A. In other words, the symmetric closure of R ensures that every pair of related elements is "reversible."

For example, suppose we have a relation R = {(1,2), (3,3), (4,5)} on the set A = {1,2,3,4,5}. The symmetric closure of R, denoted by R#, is obtained by adding (2,1), (5,4), and (3,3) to R. So, R# = {(1,2), (2,1), (3,3), (4,5), (5,4)}. Note that (3,3) is already symmetric, so it is not duplicated in R#.

Define the Euler path and the Hamilton path. Give examples of both Euler and Hamilton path.

Euler path: An Euler path is a path in a graph that passes through every edge exactly once. In other words, it is a path that starts and ends at different vertices and traverses each edge of the graph exactly once. A graph has an Euler path if and only if it has exactly two vertices with odd degree, or if it is a connected graph with all vertices having even degree.



The graph has Eulerian Paths, for example "4 3 0 1 2 0", but no Eulerian Cycle. Note that there are two vertices with odd degree (4 and 0)

Hamilton path: A Hamilton path is a path in a graph that passes through every vertex exactly once. In other words, it is a path that starts and ends at different vertices and visits each vertex of the graph exactly once. A graph has a Hamilton path if and only if there is a path between every pair of distinct vertices in the graph.

List any two applications of the graph coloring theorem. Prove that “A tree with n vertices has n-1 edges”

Two applications of the graph coloring theorem are:

1. Scheduling: Graph coloring can be used to schedule tasks or events, where each task or event is represented by a vertex in the graph and the edges

represent constraints or dependencies between the tasks or events. By coloring the vertices with different colors, we can assign a time slot or resource to each task or event without violating any constraints or dependencies.

2. Map coloring: Graph coloring can be used to color a map so that no two adjacent regions have the same color. This problem is equivalent to coloring the vertices of a graph so that no two adjacent vertices have the same color, where the vertices represent the regions of the map and the edges represent the boundaries between the regions.

How many 3 digits numbers can be formed from the digits 1,2,3,4 and 5 assuming that: repetitions of digits are allowed and repetitions of digits are not allowed

To find the total number of 3-digit numbers that can be formed using the digits 1, 2, 3, 4, and 5 with repetitions allowed, we can use the multiplication principle. For each of the three digits, we have five choices (since repetitions are allowed), so the total number of 3-digit numbers is:

5 x 5 x 5 = 125

Therefore, there are 125 different 3-digit numbers that can be formed using the digits 1, 2, 3, 4, and 5, with repetitions allowed.

Assuming that repetitions of digits are not allowed:

To find the total number of 3-digit numbers that can be formed using the digits 1, 2, 3, 4, and 5 with repetitions not allowed, we can use the permutation formula. Since we are choosing three digits from a set of five, the total number of possible arrangements is

P(5,3) = 5! / (5-3)! = 60

Therefore, there are 60 different 3-digit numbers that can be formed using the digits 1, 2, 3, 4, and 5, with repetitions not allowed.

Define ceiling and floor function. Why do we need Inclusion–Exclusion principle? Make it clear with a suitable example.

The ceiling function and floor function are mathematical functions that are used to round a number to the nearest integer. The floor function rounds down to the nearest integer, while the ceiling function rounds up to the nearest integer.

The floor function, denoted by ⌊x⌋, gives the largest integer that is less than or equal to x. For example, ⌊3.8⌋ = 3, ⌊5⌋ = 5, and ⌊-2.3⌋ = -3.

The ceiling function, denoted by ⌈x⌉, gives the smallest integer that is greater than or equal to x. For example, ⌈3.8⌉ = 4, ⌈5⌉ = 5, and ⌈-2.3⌉ = -2.

The Inclusion-Exclusion principle is a counting technique used in combinatorics to find the size of the union of sets. It allows us to count the number of elements that belong to at least one of several sets, without overcounting or undercounting any element.

Maths

Integral Domain:

A commutative ring with identity which has no zero divisor is called integral domain.Example: Q,Z ,R and Z5 are integral domain because they are commutative ring with identity which has no zero divisor . But z12 is not integral domain because it has commutative ring with identity has zero divisor ie.2.5 = 0, 5.6=0, 8.5=0.

Field:

A Commutative ring with identity is called field if its every non-zero elements are unit elements.

Binary Operation:

A binary operation * on a set S, is a function mapping S x S into S.

For each (a,b)E S x S, then we will denote the element *((a,b)) of S by a * b.

Group:

Let (G, *) be a binary structure then G is said to be a group with the binary operation if the following conditions are satisfied.* can be multiplicative and additive

1. Closure
2. Associativity
3. Existence of identity element

4. Existence of Inverse element

Ring:

A non-empty set R together with two binary operators + and • denoted by <R, +, • > is called Ring. If the following conditions are satisfied:

1. Closed
2. Associativity
3. Commutative
4. Distributive
5. Existence of Identity element
6. Existence of Inverse element

Unit Vector:

A vector having length 1 is called a unit vector. i.e. $\frac{v}{||v||}$

Null Space of Vector:

Let A be m x n matrix then null-space of matrix A is denoted by NulA and defined by

Nul A = $\{x : x \in R^n ; Ax = 0\}$

Determinant:

For n 2, the determinant of an n x n matrix =[ ] of n terms of the form

$$\det(A) = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det(A_{ij})$$

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What is posset in discrete mathematics?

In discrete mathematics, a poset (partially ordered set) is a set equipped with a binary relation that satisfies certain properties. Specifically, a poset is a set P together with a binary relation ≤ (often denoted by ⩽, ≤, or ⩿) that satisfies the following three axioms:

1. Reflexivity: $\forall a \in P, a \leq a$
2. Antisymmetry: $\forall a,b \in P$, if $a \leq b$ and $b \leq a$, then $a=b$

3. Transitivity: $\forall a,b,c \in P$, if $a \leq b$ and $b \leq c$, then $a \leq c$

The relation $\leq$ is often used to represent a notion of "precedence" or "ordering" among the elements of P. For example, if P is the set of all tasks that need to be completed in a project, and $a \leq b$ means "task a must be completed before task b", then $(P, \leq)$ forms a poset.

Posets are useful in many areas of mathematics, computer science, and other fields. They are particularly important in the study of order theory, which is the branch of mathematics that deals with the general study of order relations.

state division and remainder algorithm in discrete mathematics in simplest.

The division and remainder algorithm, also known as the division algorithm or Euclidean division, is a fundamental concept in discrete mathematics that allows us to divide one integer by another and obtain both a quotient and a remainder.

The algorithm states that given two integers, a and b (where b is non-zero), there exist unique integers q and r such that:

$a = bq + r$

where q is the quotient and r is the remainder.

The quotient is the number of times that b can be subtracted from a, while the remainder is the amount left over after all the possible subtractions have been made.

For example, if we want to divide 17 by 5, we can use the division algorithm as follows:

$17 = 5(3) + 2$

Here, 3 is the quotient and 2 is the remainder.

Note that the remainder r is always non-negative and less than the divisor b. This can be proven using the well-ordering principle of the natural numbers, which states that every non-empty set of natural numbers has a least element.

The division and remainder algorithm is a crucial tool in many areas of discrete mathematics, including number theory, algebra, and cryptography.