Tribhuvan University

Institute of Science and Technology

2065

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Bachelor Level/First Year/ Second Semester/ Science — Full Marks: 80

Computer Science and Information Technology — Pass Marks: 32

(MTH.155 – Linear Algebra) — Time: 3hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A **(10 x 2 = 20)**

1. Illustrate by an example that a system of linear equations has either equations has either exactly one solution or infinitely many solutions.

2. When is a linear transformation invertible?

3. Solve the system
$$3x_1 + 4x_2 = 3, \qquad 5x_1 + 6x_2 = 7$$
by using the inverse of the matrix $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$.

4. State the numerical importance of determinant calculation by row operation.

5. State Cramer's rule for an invertible n x n matrix A and vector $b \epsilon R^n$ to solve the system $Ax = b$. Is this method efficient from computational point of view?

6. Determine if $\{v_1, v_2 v_3\}$ is basis for R^3, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

7. Determine if $W = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is a Nul(A) for $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.

8. Show that 7 is an eigen value of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$.

9. If $S = \{u_1, \cdots\cdots\cdots, u_p\}$ is an orthogonal set of nonzero vectors in R^2, show S is linearly independent and hence is a basis for the subspace spanned by S.

10. Let $W = span\{x_1, x_2\}$ where $x_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$. Their construct orthogonal basis for W.

Group B **(5 x 4 = 20)**

11. Determine if the given set is linearly dependent:

a) $\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$ b) $\begin{bmatrix} -2 \\ 4 \\ 6 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -9 \\ 15 \end{bmatrix}$

12. Find the 3 x 3 matrix that corresponds to the composite transformation of a scaling by 0.3, a rotation of 90^0, and finally a translation that adds (-0.5, 2) to each point of a figure.

OR

Describe the Leontief Input-Output model for certain economy and derive formula for $(I-C)^{-1}$, where symbols have their usual meanings.

13. Find the coordinate vector $[X]_B$ of a x relative to the given basis $B = \{b_1, b_2\}$, where

$$b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}, \qquad x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

14. Let $A = \begin{bmatrix} 4 & -8 \\ 4 & 8 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and basis $B = \{b_1, b_2\}$. Find the B-matrix for the transformation $x \rightarrow Ax$ with $P = \{b_1, b_2\}$.

15. Let u and v be non-zero vectors in R^3 and the angle between them be ϕ. Then prove that

$$u.v = \|u\|\|v\| \cos \emptyset,$$

where the symbols have their usual meanings.

Group C **(5 x 8 = 40)**

16. Let $T: R^n \rightarrow R^m$ be a linear transformation. Then T is one-to-one if and only if the equation T(x) = 0 has only the trivial solution, prove the statement.

OR

Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ and define $T: R^2 \rightarrow R^3$ by $T(x) = Ax$. Then

a) Find T(u)
b) Find an $x \in R^2$ whose image under T is b.
c) Is there more than one x whose image under T is b?
d) Determine if c is the range of T.

17. Compute the multiplication of partitioned matrices for

$$A = \left[\begin{array}{ccc|cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hline 0 & 4 & -2 & 7 & -1 \end{array}\right] \quad and \quad B = \left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hline -1 & 3 \\ 5 & 2 \end{array}\right].$$

18. What do you mean by change of basis in R^n? Let $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, $c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}$, $c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$, and consider the bases for R^2 given by $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$.
    a) Find the change of coordinate matrix from C to B.
    b) Find the change of coordinate matrix from B to C.

OR

Define vector spaces, subspaces, basis of vector space with suitable examples. What do you mean by linearly independent set and linearly dependent set of vectors?

19. Diagonalize the matrix $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$, if possible.

20. Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the data points (2, 1), (5, 2), (7, 3), (8, 3). What do you mean by least squares lines?