

Tribhuvan University
Institute of Science and Technology
2075
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Bachelor Level / First Year / Second Semester / Science
Computer Science and Information Technology (MTH. 163)
(Mathematics II)
(NEW COURSE)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Group A

Attempt any three questions:

(3×10=30)

1. When a system of linear equation is consistent and inconsistent? Give an example for each. Test the consistency and solve: $x + y + z = 4$, $x + 2y + 2z = 2$, $2x + 2y + z = 5$. (2+1+7)
2. What is the condition of a matrix to have an inverse? Find the inverse of the matrix $A = \begin{pmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{pmatrix}$ in exists. (2+8)
3. Define linearly independent set of vectors with an example. Show that the vectors $(1, 4, 3)$, $(0, 3, 1)$ and $(3, -5, 4)$ are linearly independent. Do they form a basis? Justify. (2+5+3)
4. Find the least-square solution of $Ax = b$ for $A = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ 5 \\ 7 \\ 3 \end{pmatrix}$. (10)

Group B

Attempt any ten questions:

(10×5=50)

5. Change into reduce echelon form of the matrix $\begin{pmatrix} 0 & 3 & -6 \\ 3 & -7 & 8 \\ 3 & -9 & 12 \end{pmatrix}$. (5)
6. Define linear transformation with an example. Is a transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(x, y) = (3x + y, 5x + 7y, x + 3y)$ linear? Justify. (2+3)
7. Let $A = \begin{pmatrix} -1 & -2 \\ 5 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & 2 \\ k & -1 \end{pmatrix}$. What value (s) of k if any will make $AB = BA$? (5)

8. Define determinant. Evaluate without expanding $\begin{vmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{vmatrix}$.

9. Define subspace of a vector space. Let $H = \left\{ \begin{pmatrix} s \\ t \\ 0 \end{pmatrix} : s, t \in \mathbb{R} \right\}$. Show that H is a subspace of \mathbb{R}^3 .

10. Find the dimension of the null space and column space of $A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$.

11. Find the eigenvalues of the matrix $\begin{pmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{pmatrix}$.

12. Find LU factorization of the matrix $\begin{pmatrix} 2 & 5 \\ 6 & -7 \end{pmatrix}$.

13. Define group. Show that the set of all integers \mathbb{Z} forms group under addition operation.

14. Define ring with an example. Compute the product in the given ring $(-3, 5) (2, -4)$ in $\mathbb{Z}_4 \times \mathbb{Z}_{11}$.

15. State and prove the Pythagorean theorem of two vectors and verify this for $u = (1, -1)$ and $v = (1, 1)$.