Tribhuvan University

Institute of Science and Technology

2071

Bachelor Level / First Year /Second Semester/Science
Computer Science and Information Technology
(MTH. 155 – Linear Algebra)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A (10×2=20)

1. What is a system of linear equations? When the system is consistent and inconsistent?

2. Define linearly dependent and independent vectors. If (1, 2) and (3, 6) are vectors then the vectors are linearly dependent or independent?

3. Define invertible matrix transformation.

4. Let S be the parallelogram determined by the vectors $b_1 = (1, 3)$ and $b_2 = (5, 1)$ and let $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$. Compute the area of the image S under the mapping $x \to Ax$.

5. Show that the matrices $A = \begin{pmatrix} 5 & 1 \\ 3 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 4 & 3 \end{pmatrix}$ do not commute.

6. Define vector space.

7. Determine if w = (1, 3, -4) is in Nul A, where $\begin{pmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{pmatrix}$.

8. Is u = (3, -2) is an eigen value of $\begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$?

9. Find the inner product of (2, -5, -1) and (3, 2, -3).

10. Find the norm between the vectors u = (1, 2, 3, 4) and v = (0, 1, 2, 3).

Group B (5×4=20)

11. Let $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, u = (1, 0, -3) and v = (5, -1, 4). If $T : R^3 \to R^3$ defined by T (x) = Ax, find T (u) and T (v).

12. Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, show that det (A + B) = det A + det B iff a + d = 0.

13. If v_1 and v_2 are the vectors of a vector space V and H = span $\{v_1, v_2\}$, then show that H is a subspace of V.

14. Find the eigen values of $A = \begin{pmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{pmatrix}$.

15. Show that (v_1, v_2, v_3) is an orthogonal basis of R^3, where

$$v_1 = \left(3/\sqrt{11}, 1/\sqrt{11}, 1/\sqrt{11}\right),\ v_2 = \left(-1/\sqrt{6}, 2/\sqrt{6}, 1/\sqrt{6}\right), v_3 = \left(-1/\sqrt{66}, -4/\sqrt{66}, 7/\sqrt{66}\right).$$

OR

Find an orthogonal projection of y onto u, where y = (7, 6), u = (4, 2).

Group C (5×8=40)

16. Determine if the following system is inconsistent.

$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_2 + 2x_3 = 1$$
$$5x_1 - 8x_2 + 7x_3 = 1$$

OR

Let $a_1 = (1, -2, -5)$, $a_2 = (2, 5, 6)$ and b = (7, 4, -3), are the vectors. Determine whether b can be generated as a linear combination of a_1 and a_2. That is determine whether x_1 and x_2 exist such that $x_1 a_1 + x_2 a_2 = b$ has solution, find it.

17. If the consumption matrix C is

$$C = \begin{pmatrix} 0.5 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{pmatrix}$$

and the final demand is 50 units for manufacturing, 30 units for agriculture and 20 units for services, find the production level x that will satisfy this demand.

OR

Compute the multiplication of partitioned matrices for

$$A = \left(\begin{array}{ccc|cc} 1 & -3 & 2 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hline 0 & 4 & 2 & 7 & -1 \end{array}\right) \text{ and } B = \left(\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hline -1 & 3 \\ 5 & 2 \end{array}\right)$$

18. Let $b_1 = (1, 0, 0)$, $b_2 = (-3, 4, 0)$, $b_3 = (3, -6, 3)$ and $x = (-8, 2, 3)$ then

   (a) Show that $B = \{b_1, b_2, b_3\}$ is a basis of R^3.

   (b) Find the change of co-ordinates matrix from B to the standard basis.

   (c) Find $[x]_B$, for the given x.

19. Diagonalize the matrix, if possible

$$\begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}.$$

20. What is a least – squares solution? Find a least – squares solution of $Ax = b$, where

$$A = \begin{pmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}.$$