Tribhuvan University
Institute of Science and Technology
2067

Bachelor Level/ First Year/ Second Semester/ Science — Full Marks: 80
Computer Science and Information Technology (MTH 155) — Pass Marks: 32
(Linear Algebra) — Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.
Attempt all questions:

Group A (10x2=20)

1. Illustrate by an example that a system of linear equations has either no solution or exactly one solution.
2. Define singular and nonsingular matrices.
3. Using the Invertible matrix Theorem or otherwise, show that
$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$
is invertible.
4. What is numerical drawback of the direct calculation of the determinants?
5. Verify with an example that det (AB) = det (A) det (B) for any n x n matrices A and B.
6. Find a matrix A such that $w = col(A)$.
$$w = \left\{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \in R \right\}.$$
7. Define subspace of a vector with an example.
8. Are the vectors;
$u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ eigen vectors of $\begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$?
9. Find the distance between vectors u = (7, 1) and v = (3, 2). Define the distance between two vectors in R^n.
10. Let w = span $\{x_1, x_2\}$, where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$.
Then construct orthogonal basis for w.

Group B (5x4=20)

11. If a set $s = \{v_1, v_2, \ldots \ldots, v_p\}$ in R^n contains the zero vector, then prove that the set is linearly dependent. Determine if the set
$$\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}$$
is linearly dependent.
12. Given the Leontief input-output model x = Cx + d, where the symbols have their usual meanings, consider any economy whose consumption matrix is given by
$$C \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}$$

Suppose the final demand is 50 units for manufacturing 30 units for agriculture, 20 units for services. Find the production level x that will satisfy this demand.

13. Define rank of a matrix and state Rank Theorem. If A is a 7 x 9 matrix with a two-dimensional null space, find the rank of A.

14. Determine the eigen values and eigen vectors of $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ in complex numbers.

OR

Let $A = \begin{bmatrix} 4 & -9 \\ 4 & 8 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and basis $B = \{b_1, b_2\}$.
Find the B-matrix for the transformation x→ x with $P = [b_1, b_2]$.

15. Let u and v be nonzero vectors in R^2 and the angle between them be θ then prove that
$u.v = \|u\| \, \|v\| \cos\theta$,
where the symbols have their usual meanings.

16. Determine if the following homogeneous system has a nontrivial solution. Then describe the solution set.
$3x_1 + 5x_2 - 4x_3 = 0, \; -3x_1 - 2x_2 + 4x_3 = 0, \; 6x_1 + x_2 - 8x_3 = 0.$

17. An n x n matrix A is invertible if and only if A is row equivalent to I_n , and in this case, any sequence of elementary row operations that reduces A to I_n also transform $I_{n \times m}$ into A^{-1}.
Use this statement to find the inverse of the matrix $= \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if exists.

18. What do you mean by basis change? Consider two bases $B = \{b_1, b_2\}$ and $c = \{c_1, c_2\}$ for a vector space V, such that $b_1 = 4c_1 + c_2$ and $b_2 = 6c_1 + c_2$. Suppose $[x]_B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ i.e., $x = 3b_1 + b_2$. Find $[x]_C$.

OR

Define basis of a subspace of a vector space.
Let $v_1 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 16 \\ -5 \end{bmatrix}$, where $v_3 = 5v_1 + 3v_2$, and let $H = \text{span}\{v_1, v_2, v_3\}$.
Show that $\text{span}\{v_1, v_2, v_3\} = \text{span}\{v_1, v_2\}$ and find a basis for the subspace H.

19. Diagonalize the matrix $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$, if possible.

20. What do you mean by least-squares lines? Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that fits the data points (2, 1), (5, 2), (7, 3), (8, 3).

OR

Find the least-squares solution of Ax = b for

$$= \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}, \; b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}.$$

IOST, TU