Tribhuvan University
Institute of Science and Technology
2072

Bachelor Level / First Year /Second Semester/Science
Computer Science and Information Technology (MTH. 155)
(Linear Algebra)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

Group A (10×2=20)

1. Define linear combination of vectors. When the vectors are linearly dependent and independent?

2. Define linear transformation between two vector spaces.

3. Show that the matrix $\begin{bmatrix} 6 & -9 \\ 4 & 6 \end{bmatrix}$ is not invertible.

4. Define invertible matrix transformation.

5. Let S be the parallelogram determined by the vectors $b_1 = (1, 3)$ and $b_2 = (5, 1)$ and let $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$. Compute the area of the image S under the mapping $x \to Ax$.

6. Define subspace of a vector space.

7. If $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$ and let u = (5, 3, 2), then show that u is in the Nul A.

8. Is u = (6, -5) is an eigen vector of $\begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$?

9. Find the unit vector u of v = (1, -2, 2, 0) along the direction of v.

10. Find the norm of vector v = (1, -2, 3, 0).

Group B (5×4=20)

11. Let $A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ and define $T: R^2 \to R^2$ by $T(x) = Ax$. Find the images under T of $u = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ and $v = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$ and $u + v = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$.

12. Find the determinant of $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 2 & -5 & -1 & -2 \\ 0 & -4 & 5 & 1 \\ -3 & 10 & -6 & 8 \end{bmatrix}$.

13. Show that the vectors (1, 0, 0), (1, 1, 0) and (1, 1, 1) are linearly independent.

14. Find the eigen values of $A = \begin{pmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{pmatrix}$.

15. If $v_1 = (3, 6, 0)$, $v_2 = (0, 0, 2)$ are the orthogonal basis then find the orthonal basis of v_1 and v_2.

OR

Find an orthogonal projection of y onto u, where y = (7, 6), u = (4, 2).

Group C (5×8=40)

16. Determine if the following system is consistent, if consistent solve the system.

$$-2x_1 - 3x_2 + 4x_3 = 5$$
$$x_1 - 2x_3 = 4$$
$$x_1 + 3x_2 - x_3 = 2$$

OR

Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$ and define a transformation T: R^2 by $T(x) = Ax$, so that $T(x) = Ax = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

a) Find T(u)
b) Find x in R^2 whose image under T is b.

17. If the consumption matrix C is

$$C = \begin{pmatrix} 0.5 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{pmatrix}$$

and the final demand is 50 units for manufacturing, 30 units for agriculture and 20 units for services, find the production level x that will satisfy this demand.

18. Let $v_1 = (3, 6, 2)$, $v_2 = (-1, 0, 1)$, $x = (3, 12, 7)$ and $B = \{v_1, v_2\}$. Then B is a basis for H = span $\{v_1, v_2\}$. Determine if x is in H, and if it is, find the co-ordinate vector of x relative to B.

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19. Diagonalize the matrix , if possible

$$\begin{pmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{pmatrix}.$$

20. Find the equation $y = a_0 + a_1x$ for the least squares line that best fits the data points (2, 1), (5, 2), (7, 3), (8, 3).

OR

When two vectors 4 and v are orthogonal? If u and v are vectors, prove that $[\text{dist}(u, -v)]^2 = [\text{dist}(u, v)]^2$ iff $u.v = 0$.