Day-10, Oct-12, 2024 (Dashami-Dashain Tika 2081 BS)

Logistic Regression Mathematics Glimpse

Logistic Regression is used to predict the outcome of input in the range of [0,1] probabilities

→ used to classify either outcome is 0 or 1

→ spam calls or mail, diseased with tuberculosis or not, wine quality good or bad

$p(x) \Rightarrow \frac{p}{1-p}$ where $p(x)$ is sometimes θ called odds.

So,

$$y = \beta_0 + \beta_1 x$$

where, β_0 = y-intercept

$\beta_1 \Rightarrow$ Coefficient of x or slope

$x \Rightarrow$ input.

How to find the coefficient of x and β_0?

1. Linear Combination =

$$z = \beta_0 + \beta_1 x$$

2. Predicted Probabilities : $p = \dfrac{1}{1+e^{-z}}$

③ Gradient Calculations: (Gradient β_0 and Gradient β_1):

for β_0: $\frac{1}{n}\sum_{i=1}^{n}(p_i - y_i)$

for β_1: $\frac{1}{n}\sum_{i=1}^{n}(p_i - y_i)x_i$

where x can be input like hours, speed of rate, wine quality, diabetes level

④ Coefficient updates:

for β_0: $\beta_0 - \alpha \cdot \text{Gradient } \beta_0$

for β_1: $\beta_1 - \alpha \cdot \text{Gradient } \beta_1$

Example:

1. Initialize Parameters

   - Set up β_0 and β_1 $(0, 0)$
   - Define rate of learning α, threshold and iterations (only for programming & coding part)

2. Logistic Regression:

$$p = \frac{1}{1+e^{-z}} \quad \text{where } z = \beta_0 + \beta_1 x$$

and we compute p for each each data points

⑥ Calculate the Gradients:

For: β_0

$\text{gradient}_0 \Rightarrow \frac{1}{n} \sum (p_i - y_i)$ Here p is calculated below

$\Rightarrow \frac{1}{2} \sum ([0.5, 0.5] - [0, 1])$

$\Rightarrow \frac{1}{2} \sum ([0.5, -0.5])$

$\Rightarrow \frac{1}{2} [0.5 + (-0.5)]$

$\Rightarrow \frac{1}{2} \times 0 \Rightarrow 0$

for β_1:

$$\text{Gradient}_1 \Rightarrow \frac{1}{n} \sum (p_i - y_i) \cdot x_i$$

So, before that

$$z = \beta_0 + \beta_1 \cdot x$$

$$\Rightarrow 0 + 0 \cdot [x]$$

$$z \Rightarrow [0, 0]$$

[$\beta_0 = 0$, $\beta_1 = 0$ Assume]

So, $p = \dfrac{1}{1+e^{-z}}$

$$\Rightarrow \frac{1}{1+e^{-0}}$$

$$\left[p \Rightarrow \frac{1}{2} \Rightarrow [0.5, 0.5] \right]$$

$$\text{gradient}_1 \Rightarrow \frac{1}{2}\sum\left(p - \text{pass}(Y_i)\right)\cdot x \qquad \boxed{\begin{matrix} x = 1,2 \\ y = 0,0 \end{matrix}}$$

$$= \frac{1}{2}\sum\left([0.5, 0.5] - [0, 1]\right)\cdot[1, 2]$$

$$\Rightarrow \frac{1}{2}\sum\left([0.5, -0.5]\right)\times[1, 2]$$

$$\Rightarrow \frac{1}{2}\sum\left([0.5, -1]\right)$$

$$\Rightarrow \frac{1}{2}\times(-0.5) \qquad \Rightarrow -0.25$$

Update Gradients for β_0 and β_1 so

Old $\beta_0, \beta_1 \Rightarrow [0, 0]$ But New $\beta_0, \beta_1 \Rightarrow [0, -0.25]$

Continue the Process until Convergence which means β_0 and β_1 have very negligible change example

$$\left[\begin{array}{l} \beta_0 \Rightarrow 0.5678 \\ \beta_0 \Rightarrow 0.5675 \end{array}\right]$$

So, the overall steps is

1. Calcute p
2. Gradient Calculate β_0, β_1.
3. Update Gradient (if Convergences Stop else Continue)