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Image Denoising Base on Non-local Means with Wiener Filtering in Wavelet Domain

Li Lin
Institute of Information science and technology,
Yanshan University
Qinhuangdao, China
lilin@ysu.edu.cn

Kong Lingfu
Institute of Information science and technology,
Yanshan University
Qinhuangdao, China
lfkong@ysu.edu.cn

***Abstract*—Image denoising is a significant inverse problem of image processing and an important image pretreatment. The performance of image denoising is improved by using some statistic characteristics of natural image. In this paper, we combine the extensive self-similarity of images in non-local means algorithm with the minimum mean square error of Wiener filtering in wavelet domain, and then propose an image denoising algorithm based on the non-local means with wiener filtering in wavelet domain. The experimental results demonstrate that one can get denoised image with higher subjective visual quality and peak signal to noise ratio based on the proposed algorithm.**

Keywords-image denoising; non-local means; wiener filtering; wavelet

I. INTRODUCTION

Image denoising has become a very essential exercise of inverse problems in image processing. The imperfection of signal acquisition devices is added with noises, which can be reduced by estimators using prior information on signal properties. A noisy image in a spatial domain is modeled by

$$\boldsymbol{V}=\boldsymbol{U}+\boldsymbol{W} \tag{1}$$

where $\boldsymbol{V}$ is the observed image, $\boldsymbol{U}$ is the unknown original image, and $\boldsymbol{W}$ is assumed to be an i.i.d white Gaussian noise with zero mean and finite variance σ_n^2. The goal is to recover $\boldsymbol{U}$ from the noisy observation $\boldsymbol{V}$.

Signal processing estimation was mostly Bayesian and linear, such that Wiener filter is a minimum mean square error (MMSE) filter. However the denoising result is not satisfying using Wiener filtering in spatial domain. Non-linear smoothing algorithms existed in statistics. Two statisticians, Donoho and Johnstone [1], prove that simple thresholding in sparse representations can yield nearly optimal non-linear estimators. Mihcak, Kozintsev and Ramchandran [2] use an exponential distribution as a prior for the underlying variance field. Based on the LAWMAP model in the wavelet domain, they compute approximate MAP estimates of the variances. Kazubek's [3] denoising method consists of Wiener filtering the wavelet coefficients. However, the above denoising methods just take advantage of the statistic characteristics of the single coefficient or local neighborhood coefficients. Buades, Coll and Morel [4] propose the non-local means (NL-means) filter for image denoising in spatial domain. To estimate a certain pixel, the method uses the similarities between it and all the other pixels in image to act as weight, and the similarities are not computed from pixels themselves but from their neighborhood. For small neighborhood, the algorithm is restricted to suppressing the high-frequency noise and cannot remove the low-frequency noise. For large neighborhood, the algorithm removes the low-frequency noise effectively, but becomes less sensitive to small details [5].

In fact, wavelet coefficients marginal distribution is non-Gaussian and strong correlation between the amplitudes of neighbor coefficients in natural images. NL-means method is acceptable for the coefficients' estimation based on information of all pixels in an image. Because orthogonal wavelet transform obeys the Parseval's Equality, Wiener filter is also a MMSE filter in wavelet domain. To achieve the better denoising result, we propose the image denoising algorithm based on NL-means with Wiener filtering in wavelet domain. In this algorithm, first we remove the high-frequency noise using NL-means method with small radius of the neighborhoods in spatial domain, and then estimate the noise variance σ_n^2. Finally, the denoised image using NL-means method is re-denoised by Wiener filtering in wavelet domain, and the most of low-frequency noises are removed. The experimental results show the proposed algorithm is effective.

II. NL-MEANS DENOSING METHOD

NL-means (see [4]) denoising method is based on the extensive self-similarity of images in spatial domain. The non-local means algorithm assumes that the image contains an extensive amount of redundancy. These redundancies can then be exploited to remove the noise in the image. The NL-means not only compares the gray level in a single point but the geometrical configuration in a whole neighborhood. The NL-means algorithm is introduced below.

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Given a discrete gray noisy image $y = \{ y(i) | \, i \in I \}$, the estimated value $NL[y](i)$, for a pixel i, is computed as a weighted average of all the pixels in the image:

$$NL[y](i) = \sum_{j \in I} w(i,j) \, y(j) \tag{2}$$

where the family of weights $\{ w(i,j) \}_j$ depends on the similarity between the pixels i and j, and satisfies the usual conditions $0 \le w(i,j) \le 1$ and $\sum_j w(i,j) = 1$. Each pixel is a weighted average of all the pixels in the image. The weights are based on the similarity between the neighborhoods of pixels i and j. In order to compute the similarity, a neighborhood must be defined. Let N_i be the square neighborhood centered about a pixel i with a user-defined radius R_{sim}. This similarity is measured as a decreasing function of the weighted Euclidean distance, $\left\| y(N_i) - y(N_j) \right\|_{2,a}^2$, where a>0 is the standard deviation of the Gaussian kernel. These weights are defined as

$$w(i,j) = \frac{1}{Z(i)} \exp\left(- \left\| y(N_i) - y(N_j) \right\|_{2,a}^2 \Big/ h^2 \right) \tag{3}$$

where $Z(i)$ is the normalizing constant

$$Z(i) = \sum_j \exp\left(- \left\| y(N_i) - y(N_j) \right\|_{2,a}^2 \Big/ h^2 \right) \tag{4}$$

and h is the weight-decay control parameter and $h \approx 10\sigma_n$, which controls the decay of the exponential function and therefore the decay of the weights as a function of the Euclidean distances.

The parameter a is the neighborhood filter with radius R_{sim}. The weights of a are computed by the following formula:

$$\frac{1}{R_{sim}} \sum_{i=m}^{R_{sim}} \frac{1}{(2i+1)^2} \tag{5}$$

where m is the distance the weight which is from the center of the filter.

Buades, Coll and Morel [6] prove the NL-means method noise looks like white noise. For visual quality of denoised images, the NL-means performs well on this type of texture, due to its high degree of redundancy. In presence of periodic or stochastic patterns, the NL-means mean square error is significantly more precise than the other algorithms.

The drawback of NL-means is either high-frequency or low-frequency noises are removed well with different parameter R_{sim}, due to it is a spatial algorithm.

III. Wiener Filtering in Wavelet Domain

Assume $\boldsymbol{x}$ represents wavelet coefficients of "clean" image, and $\boldsymbol{x}$ is conditional independent Gaussian random vector. The $\boldsymbol{n}$ represents the coefficient vector of the zero-mean Gaussian with variance σ_n^2. The $\boldsymbol{y}$ represents the coefficient vector of the noisy image and are given by

$$\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{n} \tag{6}$$

For a pixel i, the Wiener filter expression is given by

$$a_i = E\{x_i^2\} \Big/ E\{y_i^2\} \tag{7}$$

The best linear estimate $\hat{x}_i$ of the signal wavelet coefficient x_i is given by

$$\hat{x}_i = a_i y_i \tag{8}$$

On the assumption that the signal and noise are independent, we have

$$E\{x_i^2\} = E\{y_i^2\} - \sigma_n^2 \tag{9}$$

The coefficients of the Wiener filter can be expressed as

$$a_i = \frac{E\{y_i^2\} - \sigma_n^2}{E\{y_i^2\}} \tag{10}$$

Due to the values $E\{x_i^2\}$ is only positive value, so

$$\hat{x}_i = \frac{\max\left(E\{y_i^2\} - \sigma_n^2, 0 \right)}{E\{y_i^2\}} \tag{11}$$

IV. Our Denoising Algorithm

We apply NL-means algorithm to denoising noisy image $\boldsymbol{I}_n$ in spatial domain, and achieve the denoised image $\boldsymbol{I}_{NL}$. Then noise variance $\hat{\sigma}_n$ of $\boldsymbol{I}_{NL}$ is estimated by the robust median estimator in wavelet domain as proposed by David L. Donoho in [7]. The method for estimating the noise is given by

$$\hat{\sigma}_n = Median(|\boldsymbol{c}|) \big/ 0.6745 \tag{12}$$

and $\boldsymbol{c}$ represents the coefficient vector of first level in the diagonal direction of 2D wavelet. The noisy image $\boldsymbol{I}_{NL}$ is decomposed into 3 subbands with J levels using orthogonal wavelet transform. Then we compute an approximate maximum likelihood estimator for the variance $\hat{\boldsymbol{\sigma}}$ of coefficient vector $\boldsymbol{y}$ in each subband [2]:

$$\hat{\sigma}_k^2 = \max\left(0, \frac{1}{M} \sum_{j \in N_k} y_j^2 - \hat{\sigma}_n^2 \right) \tag{13}$$

where N_k is a square window centered at coefficient y_j, and M is the number of in N_k. Next, we use the Wiener filter which is a linear MMSE estimator to estimate the coefficient vector $\hat{\boldsymbol{x}}$ from $\boldsymbol{y}$ in each subband.

$$\hat{x}_k = \frac{\hat{\sigma}_k^2}{\hat{\sigma}_k^2 + \hat{\sigma}_n^2} y_k \tag{14}$$

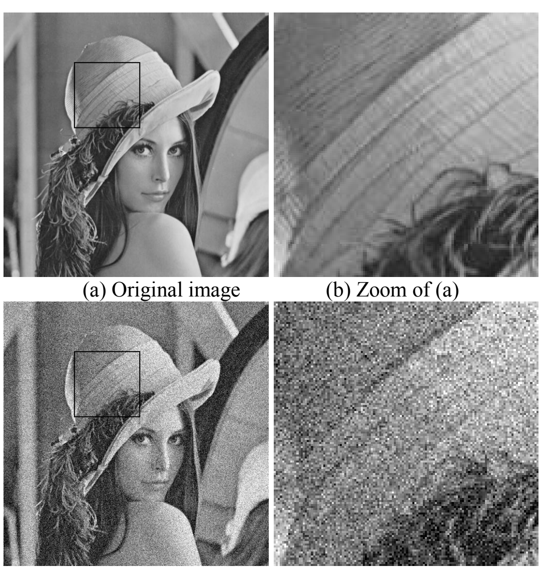
Finally, we reconstruct the image $\hat{\boldsymbol{I}}$ which is an estimation for the "clean" image $\boldsymbol{I}$ from all the coefficient vector $\hat{\boldsymbol{x}}$ in each subband using J levels orthogonal wavelet.

V. Results

A number of test images are used for proving our algorithm, but only the results for *Lena* and *Barbara* is listed. The size of images is 512×512 pixels. The *DWT* represents an orthogonal wavelet transform *db*8 with 5 levels of decomposition in this paper. We compared five different denoising methods. The PSNR results are shown in Table 1. The *DWT-hard* is hard thresholding of wavelet coefficients denoising method. The *Wiener*2 is MATLAB's image denoising algorithm wiener2 in spatial domain. The *DWT-Wiener* is Wiener filter in wavelet domain. The *NL-means* is non-local means denoising in spatial domain. The noise is an i.i.d white Gaussian noise with zero mean and the standard deviation σ_n is any one of 15,20,25,30,35,50,75,100. The size of Wiener filter's local neighborhood N_k is 5×5 in wavelet domain. In the *NL-means* method, the radius of the neighborhoods equal to 3, the radius of a search window equal to 7 , and the weight-decay control parameter *h* is same as [4].

The result is presented in Table 1 which presents the denoised image quality based on our method has more or less improvements than other denoising algorithms. For example, our method improves 1.77dB than DWT-Wiener and 2.95dB for image *Lena* at σ_n=30.

We compare our algorithm with other four different algorithms. Figure 1 presents the comparison results of image *Lena* at σ_n=30. The left column of Figure 1 shows that the PSNR of our algorithm is not only higher than others, but also our algorithm exceeds in the subjective visual quality. The right column of Figure 1 presents the zooms of denoised images compared with original image's zoom. The result shows that our algorithm can keep the details of images very well and reduce the artifacts effectively.



(a) Original image (b) Zoom of (a)

(c) Noisy image (d) Zoom of (c)

(e) DWT-hard (f) Zoom of (e)

(g) Wiener2 (h) Zoom of (g)

(i) DWT-Wiener (j) Zoom of (i)

(k) NL-means (l) Zoom of (k)

(m) Our algorithm (n) Zoom of (m)

Figure 1. Denoised images and zooms compared with original image and its zoom (a) is the original image(Lena). (b) is the zoom of the square of (a). (c) is the noisy image with σ_n=30. (d) is the zoom of (c). (e) is the denoised image based on DWT-hard, and PSNR=26.49dB. (f) is the zoom of (e). (g) is the denoised image based on Wiener2, and PSNR=25.69dB. (h) is the zoom of (g). (i) is the denoised image based on DWT-Wiener, and PSNR=28.43dB. (j) is the zoom of (i). (k) is the denoised image based on NL-means, and PSNR=27.25dB. (l)is the zoom of (k). (m) is the denoised image based on our algorithm, and PSNR=30.20dB. (n) is the zoom of (m).

TABLE I. PSNR RESULTS IN DECIBELS FOR SEVERAL DENOISING METHODS

Image	Method	Noise variance σ_n			
		15	**20**	**25**	**30**
Lena	DWT- hard	30.07	28.51	27.39	26.49
	Wiener2	31.13	28.98	27.19	25.69
	DWT-Wiener	32.45	30.88	29.46	28.43
	NL-means	32.68	31.69	29.78	27.25
	Our method	**32.73**	**32.13**	**31.17**	**30.20**
Barbara	DWT- hard	27.44	25.77	24.56	23.64
	Wiener2	28.29	26.81	25.49	24.28
	DWT-Wiener	30.19	28.59	27.42	26.43
	NL-means	**31.18**	30.21	28.47	26.24
	Our method	31.17	**30.47**	**29.55**	**28.33**
Image	**Method**	**Noise variance σ_n**			
		35	**50**	**75**	**100**
Lena	DWT- hard	25.64	23.81	21.66	20.17
	Wiener2	24.45	21.42	17.92	15.52
	DWT-Wiener	27.59	25.47	22.96	21.04
	NL-means	24.62	18.36	12.99	10.06
	Our method	**28.83**	**25.86**	**23.41**	**21.51**
Barbara	DWT- hard	22.86	21.45	19.77	18.59
	Wiener2	23.26	20.67	17.53	15.24
	DWT-Wiener	25.56	23.66	21.42	19.81
	NL-means	23.87	18.16	12.95	9.99
	Our method	**27.16**	**24.49**	**21.97**	**20.35**

VI. CONCLUSION

In this paper, we describe NL-means in spatial domain and Wiener filter in wavelet domain briefly. Based on the former, we successfully implement an image denoising algorithm base on the non-local means with wiener filtering in the wavelet domain for natural images. The experimental results shows that the proposed denoising algorithm performs better than or is competitive with various denoising algorithm in subjective visual quality and PSNR.

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