Lab 02 - Review of Calculus 1

Overview ¶

In this lab, we will learn how to use SageMath to define functions, to compute limits, derivatives, and integrals, and to plot functions.

Important SageMath Commands Introduced in this Lab

Command	Description	
def **f(x):** return *expression*	Creates the function $f(x) =$ expression	def **f(x):** return
var(‘*letter*’)	Makes the letter or word a variable	$h =$ **var**(‘
limit$(f(x), x = a)$	Evaluates $\lim\limits_{x\to a} f(x)$	**limit**$(f(x$
limit$(f(x), x = a, \text{dir} = \text{'+'})$	Evaluates $\lim\limits_{x\to a^+} f(x)$	**limit**$((f(a$
limit$(f(x), x = a, \text{dir} = \text{'-'})$	Evaluates $\lim\limits_{x\to a^-} f(x)$	**limit**(sqrt
limit$(f(x), x = \text{infinity})$	Evaluates $\lim\limits_{x\to \infty} f(x)$	**limit**$(3x/$
limit$(f(x), x = \text{-infinity})$	Evaluates $\lim\limits_{x\to -\infty} f(x)$	**limit**(e^(3
diff$(f(x), x)$	Finds the derivative of $f(x)$	**diff**(x^2, x)
diff$(f(x), x, n)$	Find the n^{th} derivative of $f(x)$	diff(sin(x)
integrate$(f(x), x)$	Finds an antiderivative of the function	**integrate**
integrate$(f(x), x, a, b)$	Calculates $\int_a^b f(x)\, dx$	**integrate**
plot$(\ldots)$	Plots the function(s) according to the default options or the options specified	**plot**$(x^2$, x yma **plot**$([x^2$, a leger
show$(\ldots)$	Displays the argument in an intelligent way	**show**(3/5

Notes

Note that SageMath's **integrate** command does not include any constants of integration. Whenever you evaluate an indefinite integral, do not forget to include a constant of integration (+C) in your answer.

Basic Function and Expressions: SageMath uses **sqrt(x)** for $\sqrt{x}$, **abs(x)** for $|x|$, **log(x)** for $\ln(x)$, **e** for Euler's constant, **pi** for π, and **infinity** for ∞. Other basic functions can be typed in as they are but you must carefully use $()$ to group together and match up expressions as needed. You must also type $*$ when multiplication is presented. For example, you need to type in **sin(x)** instead of $\sin x$ and 2 * x / (x+y) for $\frac{2x}{x+y}$.

Related Course Material

```
Chapters 1 - 6
```

Example 1

Use SageMath to define the following functions and find their derivative.

1. $f(x) = x^3 \ln(x)$
2. $g(x) = \dfrac{\arctan(x)}{\sqrt{x^2+1}}$
3. $h(x) = \dfrac{1-\ln(x)}{1+\ln(x)}$

In [ ]:
```
def f(x):
    return x^3 * ln(x)
diff(f(x),x)
```

You can make SageMath return the output in an easier to read way by using the $\mathbf{show}(\ldots)$ command. This command works for most outputs in SageMath.

In [ ]:
```
show(diff(f(x),x))
```

$\mathbf{Caution:}$ SageMath uses $\mathbf{log(x)}$ to represent $\mathbf{ln(x)}$.

In [ ]:
```
def g(x):
    return arctan(x)/sqrt(x^2+1)
show(diff(g(x),x))
```

In [ ]:
```
def h(x):
    return (1-log(x))/(1+log(x))
show(diff(h(x),x))
```

Example 2

Use SageMath to compute the first, second, third, and 100th derivative of $f(x) = x\sin(2x)$.

In [ ]:
```
def f(x):
    return x*sin(2*x)
show(diff(f(x),x))
```

In [ ]:
```
def f(x):
    return x*sin(2*x)
show(diff(f(x),x,2))
```

In [ ]:
```
def f(x):
    return x*sin(2*x)
show(diff(f(x),x,3))
```

In [ ]:
```
def f(x):
    return x*sin(2*x)
show(diff(f(x),x,100))
```

Example 3

Use SageMath to evaluate the following indefinite and definite integrals:

1. $\displaystyle\int \frac{x+1}{(x-2)^2}\,dx$
2. $\displaystyle\int_{-1}^{3} xe^{-x^2}\,dx$
3. $\displaystyle\int_{0}^{\pi\,/\,2} \left|\frac{1}{2}-\cos x\right|\,dx$

In [ ]:
```
show(integrate((x+1)/(x-2)^2,x))
```

In [ ]:
```
show(integrate(x*e^(-x^2),x,-1,3))
```

In [ ]:
```
show(integrate(abs(1/2-cos(x)),x,0,pi/2))
```

Example 4

Use SageMath to evaluate the following limits:

1. $\displaystyle\lim_{r\to 0}\frac{r^2\arccos(r)}{(\sin(2r))^2}$
2. $\displaystyle\lim_{n\to\infty}(1+3/n)^{2n}$
3. $\displaystyle\lim_{t\to 1^+}\tan\left(\frac{\pi}{2}t\right)$

Note: In order to use a variable other than x in SageMath, you must first declare it as a variable using the $\mathbf{var()}$ command.

In [ ]:
```
r = var('r')
limit((r^2 * arccos(r))/(sin(2*r)^2), r=0)
```

In [ ]:
```
n = var('n')
limit((1+3/n)^(2*n),n=infinity)
```

In [ ]:
```
t = var('t')
limit(tan((pi/2)*t),t=1,dir='+')
```

Example 5

Define the function $f(x) = \frac{1}{2}x^4 - x^3 - 36x^2 + 108x$ in SageMath.

In [ ]:
```
def f(x):
    return x^4/2-x^3-36*x^2+108*x
```

Find the first and second derivatives of $f(x)$ and assign them to the names $\mathbf{df}$ and $\mathbf{ddf}$.

In [ ]:
```
df = diff(f(x),x)
ddf =diff(f(x),x,2)
```

Plot the graph $y = f'(x)$ on the interval $-10 < x < 10$ and estimate the invervals on which $y = f(x)$ is increasing.

In [ ]:
```
plot(df,-10,10)
```

Plot the graph $y = f''(x)$ on the same interval and estimate the intervals on which $f(x)$ is concave up.

In [ ]:
```
plot(dff,-10,10)
```

Create a single plot containint the graphs $y = f(x)$, $y = f'(x)$, and $y = f''(x)$ on the interval $[-10, 10]$. Make sure to graph each function in a different color and a different linestyle. Also, add a legend identifying each of the three graphs. Were your estimations in the previous two parts correct?

In [ ]:
```
plot([f,df,dff], xmin=-10, xmax=10, color=['green','blue','orange'], linestyle=['solid','dashed','dotted'],legend_label=[f(x),df,dff])
```

In [ ]:
```

```