

CS3081 Assignment 3

Name: Patrick Dillon Ryan

Student Number: 17340382

Question 1 (Problem 4.26)

- (i) (a) = 4, (b) = 7
- (ii) (a) = 2.2, (b) = 7
- (iii) (a) = 4, (b) = 2.2
- (iv) (a) = 7, (b) = 4

Your Answer ((i)-(iv)): (i)

```
threeXthree = [ -2, 1, 0; 1, -2, 1; 0, 1, -1.5];  
fourXfour = [4, -1, 0, 1, 0; -1, 4, -1, 0, 1; 0, -1, 4, -1, 0; 1, 0, -1, 4, -  
1; 0, 1, 0, -1, 4];
```

```
threeXthree = InfinityNorm(threeXthree);  
fourXfour = InfinityNorm(fourXfour);
```

```
disp(threeXthree);  
disp(fourXfour);
```

```
function [max] = InfinityNorm(A)  
    [m, n] = size(A);  
    max = 0;  
    for i = 1:m  
        temp = 0;  
        for j = 1:n  
            temp = temp + abs(A(i,j));  
        end  
        if temp > max  
            max = temp;  
        end  
    end  
end
```

Question 2 (Problem 6.13)

- (i) 420W
- (ii) 420KW
- (iii) 530W
- (iv) 580KW

Your Answer ((i)-(iv)): (iii)

General formula:

$f(x) = \sum_{i=1}^n y_i \cdot L_i(x)$.. where $L_i(x) = \prod_{j=1, j \neq i}^n \frac{(x-x_j)}{(x_i-x_j)}$

fourth:

$f(x) =$

$$\begin{aligned} & ((x-x_2)(x-x_3)(x-x_4)(x-x_5))/((x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)) \cdot y_1 \\ & + ((x-x_1)(x-x_3)(x-x_4)(x-x_5))/((x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)) \cdot y_2 \\ & + ((x-x_1)(x-x_2)(x-x_4)(x-x_5))/((x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)) \cdot y_3 \\ & + ((x-x_1)(x-x_2)(x-x_3)(x-x_5))/((x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)) \cdot y_4 \\ & + ((x-x_1)(x-x_2)(x-x_3)(x-x_4))/((x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)) \cdot y_5 \end{aligned}$$

$x_1=14, x_2=22, x_3=30, x_4=38, x_5=46$ (from the question)

$y_1 = 320, y_2 = 490, y_3 = 540, y_4 = 500, y_5 = 480$ (from the question)

$$\begin{aligned} f(26) = & ((26-22)(26-30)(26-38)(26-46))/((14-22)(14-30)(14-38)(14-46)) \cdot 320 \\ & + ((26-14)(26-30)(26-38)(26-46))/((22-14)(22-30)(22-38)(22-46)) \cdot 490 \\ & + ((26-14)(26-22)(26-38)(26-46))/((30-14)(30-22)(30-38)(30-46)) \cdot 540 \\ & + ((26-14)(26-22)(26-30)(26-46))/((38-14)(38-22)(38-30)(38-46)) \cdot 500 \\ & + ((26-14)(26-22)(26-30)(26-38))/((46-14)(46-22)(46-30)(46-38)) \cdot 480 \end{aligned}$$

$$f(26) = (-12.5) + (229.6875) + (379.6875) + (-78.125) + (11.25)$$

$$f(26) = 530KW$$

Question 3 (Problem 8.7)

The truncation error is:

- (i) $O(h)$
- (ii) $O(h^2)$
- (iii) $O(h^3)$
- (iv) $O(h^4)$

Your Answer ((i)-(iv)): (ii)

$$\begin{aligned}
f(x_{i+1}) &= \\
f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \frac{f'''(x_i)}{3!}h^3 + \frac{f''''(\xi_1)}{4!}h^4 \\
f(x_{i-1}) &= \\
f(x_i) - f'(x_i)(2h) + \frac{f''(x_i)}{2!}(2h)^2 - \frac{f'''(x_i)}{3!}(2h)^3 + \frac{f''''(\xi_2)}{4!}(2h)^4 \\
2f(x_{i+1}) + f(x_{i-1}) &= \\
3f(x_i) + 6\frac{f''(x_i)}{2!}h^2 - 8\frac{f'''(x_i)}{3!}h^3 + \frac{f''''(\xi_1)}{4!}h^4 + 16\frac{f''''(\xi_2)}{4!}h^4 \\
f''(x_i) &= \\
2\frac{f(x_{i-1}) - 3f(x_i) + 2f(x_{i+1}))}{6h^2} + O(h^2) \\
\text{Answer} &= O(h^2)
\end{aligned}$$

Question 4 (Problem 8.9)

- (i) $f'_{\text{male}}(2006)=4965$;
 $f'_{\text{female}}(2006)=10681$;
Predicted_Males(2008)=673601;
Error_Males=0.62%;
Predicted_Females(2008)=277990;
Error_Females=0.58%
- (ii) $f'_{\text{male}}(2006)=4940$;
 $f'_{\text{female}}(2006)=10681$;
Predicted_Males(2008)=673601;
Error_Males=0.62%;
Predicted_Females(2008)=277987;
Error_Females=0.57%
- (iii) $f'_{\text{male}}(2006)=4940$;
 $f'_{\text{female}}(2006)=10681$;
Predicted_Males(2008)=673601;
Error_Males=0.68%;
Predicted_Females(2008)=277987;
Error_Females=0.42%
- (iv) $f'_{\text{male}}(2006)=4965$;
 $f'_{\text{female}}(2006)=10670$;
Predicted_Males(2008)=673601;
Error_Males=0.68%;
Predicted_Females(2008)=277987;
Error_Females=0.52%

Your Answer ((i)-(iv)): (ii)

$$\begin{aligned}f'(x_{i+2}) &= \frac{x_{i+2} - x_{i+1}}{(x_i - x_{i+1})(x_i - x_{i+2})}y_i + \\&\quad \frac{x_{i+2} - x_i}{(x_{i+1} - x_i)(x_{i+1} - x_{i+2})}y_{i+1} + \\&\quad \frac{2x_{i+2} - x_i - x_{i+1}}{(x_{i+2} - x_i)(x_{i+2} - x_{i+1})}y_{i+2} \\x_i &= 2002 \\x_{i+1} &= 2003 \\x_{i+2} &= 2006 \\f'(2006) &= \frac{3}{4}y_i - \frac{4}{3}y_{i+1} + \frac{7}{12}y_{i+2} \\male(2006) &= 4940 \\female(2006) &= 10681\end{aligned}$$

We can then substitute our answers back in from a. This allows us to solve for $y_1 + 2$.

This gives us the values

673601 male doctors

277987 female doctors

Error is the difference between the predicated and the actual values. – Values gotten from previous part and from the question

$$|673601 - 677807| / 677807 = .62\%$$

$$|277987 - 276417| / 276417 = .57\%$$

$$f'_{\text{male}}(2006)=4940;$$

$$f'_{\text{female}}(2006)=10681;$$

$$\text{Predicted_Males}(2008)=673601;$$

$$\text{Error_Males}=0.62\%;$$

$$\text{Predicted_Females}(2008)=277987;$$

$$\text{Error_Females}=0.57\%$$