

**MATHEMATICAL METHODS - II (2024)****INTERPOLATION TUTORIAL**

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- (1) Find a degree 3 polynomial, passing through these points.
- (2) If the function also passes through the point  $(5, -2)$ , find a degree 4 polynomial passing through all 5 points.
- (3) Using the answer to the above part as an polynomial approximation to  $f$ , find an estimate for the integral  $\int_1^5 f(x)dx$ .

Problem 02**Problem 2.**

- (1) Use Lagrange method to find a degree 3 polynomial passing through the points  $(-1, 2)$ ,  $(0, 3)$ ,  $(1, -1)$  and  $(2, 0)$ . Simplify your answer.
- (2) The function  $f$  is passing through the above 4 points. Use the polynomial to estimate the value  $f(1.5)$ .
- (3) It is observed, at a later time, that the function also passes through the point  $(-2, 4)$ . Find a degree 4 polynomial passing through these points and use it to estimate the value  $f(1.5)$ . How does it compare to the previous answer?

Problem 03

Find a degree 2 polynomial passing through the three points (1, 2), (2, 3) and (3, -2).

(1)  $(x - 1)(x - 2)(-3)$

(2)  $2(x-2)(x-3) + 3(x-1)(x-3) - 2(x-1)(x-2)$

(3)  $(x-2)(x-3) - 3(x-1)(x-3) - (x-1)(x-2)$

(4)  $\frac{1}{2} (x-2)(x-3) - (x-1)(x-3) + \frac{1}{2} (x-1)(x-2)$

(5) None of the above.

Problem 04

You are working as a data analyst for an agricultural research institute. The institute has collected data on the growth of a certain crop over a period of time, but due to some missing records, the data is incomplete. The growth of the crop is assumed to follow a polynomial pattern. The available data is as follows:

Day	Height (cm)
1	3.2
2	6.8
3	12.1
4	19.2
5	28.1

Due to a missing record, the height of the crop on Day 3.5 is not available, and the researchers need an estimate of this value to complete their study.

**Task:**

- Use Newton's Forward Difference method to find the interpolating polynomial based on the available data.
- Use this polynomial to estimate the height of the crop on Day 3.5.

Explain how you derived the polynomial and provide the estimated height.

Problem 05

Estimate  $\sqrt{3.9}$  using the Taylor polynomial of degree 2 around the point  $x = 4$ , of the function  $\sqrt{x}$ .

Problem 06

Lakshith is an engineer working on a project involving the design of a spring system alongside his classmates, Aanya and Rohit. The team is responsible for modeling the force exerted by a spring as a function of the displacement  $x$  from its natural length. The force is represented by a polynomial function.

For small displacements around the equilibrium point  $x = 1$ , the team needs to approximate the force using a Taylor series expansion. The force function is given by:

$$f(x) = 2 - 3x + 4x^2 - 5x^3$$

Task:

Lakshith is tasked with finding the Taylor series expansion of the function  $f(x)$  around the point  $c = 1$  up to the third degree.

Aanya needs to use the Taylor series found by Lakshith to estimate the force when the displacement is  $x = 0.9$ .

Rohit will then compare this estimated force with the actual value of the force at  $x = 0.9$  using the original function and discuss the accuracy of the approximation.