

$$f(x) = 19x - 5$$

$$X = \{0.1, 0.3, 0.5, 0.7, 0.9\}$$

$$Y = f(X) = \{-4.9, -4.7, -4.5, -4.2, -3.7\}$$

Полином Ньютона:

$$P_n(x) = f(x_0) + \sum_{k=1}^n \left( f(x_0, \dots, x_k) \cdot \prod_{i=0}^{k-1} (x - x_i) \right), \text{ где } f(x_i, x_{i+1}, \dots, x_{i+k}) = \frac{f(x_{i+1}, x_{i+2}, \dots, x_{i+k}) - f(x_i, x_{i+1}, \dots, x_{i+k-1})}{x_{i+k} - x_i}$$

Честно считаем:

$$P_n(x) = -4.9 + \left( \frac{-4.9}{0.1-0.3} + \frac{-4.7}{0.3-0.1} \right) (x-0.1) + \left( \frac{-4.5}{(0.1-0.3)(0.1-0.5)} + \frac{-4.7}{(0.3-0.1)(0.3-0.5)} + \frac{-4.5}{(0.5-0.1)(0.5-0.3)} \right) \cdot \\ \cdot ((x-0.1)(x-0.3)) + \dots + \left( \frac{-4.9}{(0.1-0.3)(0.1-0.5)(0.1-0.7)(0.1-0.9)} + \frac{-4.7}{(0.3-0.1)(0.3-0.5)(0.3-0.7)(0.3-0.9)} + \dots \right. \\ \left. \dots \frac{-3.7}{(0.9-0.1)(0.9-0.3)(0.9-0.5)(0.9-0.7)} \right) ((x-0.1)(x-0.3)(x-0.5)(x-0.7)) \approx \boxed{2.08x^3 - 1.87x^2 + 1.48x - 5.03}$$

Полином Лагранжа:

$$L(x) = \sum_{j=0}^k y_j l_j(x), \text{ где } l_j(x) = \prod_{\substack{0 \leq m \leq k \\ m \neq j}} \frac{x - x_m}{x_j - x_m}$$

Честно считаем:  $L(x) \approx$

$$-4.9 \frac{(x-0.3)(x-0.5)(x-0.7)(x-0.9)}{(0.1-0.3)(0.1-0.5)(0.1-0.7)(0.1-0.9)} + (-4.7) \frac{(x-0.1)(x-0.5)(x-0.7)(x-0.9)}{(0.3-0.1)(0.3-0.5)(0.3-0.7)(0.3-0.9)} + \dots \\ \dots + (-3.7) \frac{(x-0.1)(x-0.3)(x-0.5)(x-0.7)}{(0.9-0.1)(0.9-0.3)(0.9-0.5)(0.9-0.7)} \approx \boxed{2.08x^3 - 1.87x^2 + 1.48x - 5.03}$$

Полиномы  
совпадают

→ Т.к. степень интер. многочлена на 1 меньше, чем кол-во узлов