University of Moratuwa

Faculty of Engineering

Robotics Mini Project

Kinematic Analysis of a Robot Arm

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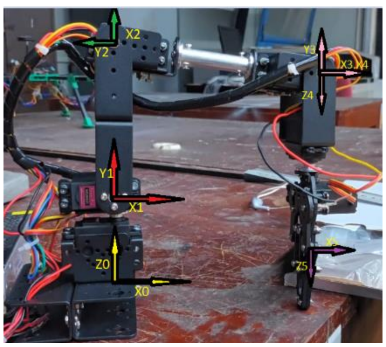
Introduction

The robotic arm chosen for this mini-project is a sophisticated 5-degree-of-freedom (5-DoF) manipulator equipped with a metal mechanical gripper. The design incorporates revolute joints actuated by servo motors, providing rotational movement at each joint.

The project integrates an Arduino Uno for automatic manipulation and control, while MATLAB is employed for the analysis of both forward and inverse kinematics. The system has been developed to excel in two distinct tasks such as joint manipulation using forward kinematics and point-to-point end effector manipulation using inverse kinematics. pick and place task for both implementation and verified.

Frame Assignment & DH Table

Standard DH conventions is used to assign the frames and builds the DH table.



Length(a)	Twist(α)	Offset(d)	Angle(θ)
0	π/2	7	θ_1
10.5	0	0	θ_2
15	0	0	θ_3
0	π/2	0	θ_4
0	0	17.5	θ_5

Forward Kinematic Analysis

Each frame homogeneous transformation matrix obtained from DH tables and the DH convention. Using the composition rule end effector position and orientation determine with respect to base frame for given joint angles.

- Homogeneous transformation matrixes of frames.

$$A1 = \begin{pmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad A2 = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & \frac{21\cos(\theta_2)}{2} \\ \sin(\theta_2) & \cos(\theta_2) & 0 & \frac{21\sin(\theta_2)}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A3 = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 15\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 15\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad A4 = \begin{pmatrix} \cos(\theta_4) & 0 & -\sin(\theta_4) & 0 \\ \sin(\theta_4) & 0 & \cos(\theta_4) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A5 = \begin{pmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & \frac{35}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- End effector transformation matrix with respect to base

$$H^0_5 = A1*A2*A3*A4*A5 \qquad \text{(composition rule)}$$

$$H = \begin{pmatrix} -\sin(\theta_1)\sin(\theta_5) - \cos(\theta_5)\,\sigma_1 & \sin(\theta_5)\,\sigma_1 - \cos(\theta_5)\sin(\theta_1) & \sin(\theta_4)\,\sigma_4 - \cos(\theta_4)\,\sigma_5 & \frac{21\cos(\theta_1)\cos(\theta_2)}{2} - \frac{35\cos(\theta_4)\,\sigma_5}{2} + \frac{35\sin(\theta_4)\,\sigma_4}{2} - 15\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ \cos(\theta_1)\sin(\theta_5) - \cos(\theta_5)\,\sigma_2 & \cos(\theta_1)\cos(\theta_5) + \sin(\theta_5)\,\sigma_2 & \sin(\theta_4)\,\sigma_6 - \cos(\theta_4)\,\sigma_7 & \frac{21\cos(\theta_2)\sin(\theta_1)}{2} - \frac{35\cos(\theta_4)\,\sigma_7}{2} + \frac{35\sin(\theta_4)\,\sigma_6}{2} - 15\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) \\ \cos(\theta_5)\,\sigma_3 & -\sin(\theta_5)\,\sigma_3 & \cos(\theta_4)\,\sigma_8 - \sin(\theta_4)\,\sigma_9 & \frac{21\sin(\theta_2)}{2} + 15\cos(\theta_2)\sin(\theta_3) + 15\cos(\theta_3)\sin(\theta_2) + \frac{35\cos(\theta_4)\,\sigma_8}{2} - \frac{35\sin(\theta_4)\,\sigma_9}{2} + 7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$\sigma_1 = \cos(\theta_4)\,\sigma_4 + \sin(\theta_4)\,\sigma_5$

$\sigma_2 = \cos(\theta_4)\,\sigma_6 + \sin(\theta_4)\,\sigma_7$

$\sigma_3 = \cos(\theta_4)\,\sigma_9 + \sin(\theta_4)\,\sigma_8$

$\sigma_4 = \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$

$\sigma_5 = \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)\sin(\theta_2)$

$\sigma_5 = \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)\sin(\theta_2)$

$\sigma_6 = \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)\sin(\theta_1)$

$\sigma_7 = \cos(\theta_2)\sin(\theta_1)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_1)\sin(\theta_2)$

$\sigma_8 = \cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3)$

$\sigma_9 = \cos(\theta_2)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_2)$

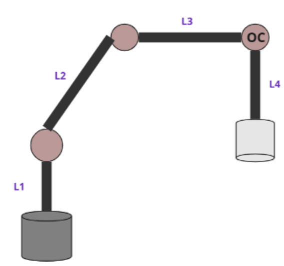
Consider Cartesian position and orientation as x, y, z , Θ, Φ, Ψ

$$R^0_5 = \begin{bmatrix} C\Theta*C\Phi & -S\Phi*C\Psi + S\Theta*C\Phi*S\Psi & S\Psi*S\Phi + S\Theta*C\Phi C\Psi \\ S\Phi*C\Theta & C\Theta*C\Phi + S\Theta*S\Phi*S\Psi & -S\Psi*C\Phi + S\Theta*S\Phi C\Psi \\ -S\Theta & C\Theta*S\Psi & C\Theta*C\Psi \end{bmatrix} \qquad t^0_5 = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

From this convention end effector position and orientation obtain using MATLAB.

Inverse Kinematic Analysis.

Arm links are separated into two parts for the inverse kinematic analysis at the center point of frames 3 and 4(O_c). The fourth link constrained to be vertical in start and end positions (Pick and Place positions) to simplify the inverse kinematics calculation and facilitate manipulation, the universal linkage is consistently maintained at vertical in all pick-and-place tasks.



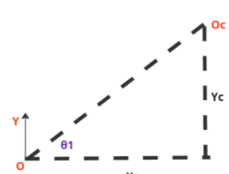
Determine the O_c point coordinate.

$Zc = Ze - 17.5$; $\qquad Xc = Xe$; $\qquad Yc = Ye$

Determine the Θ_1.

Point O lies on the X-Y plane.

Thus $\theta_1 = \tan^{-1}(Yc / Xc)$



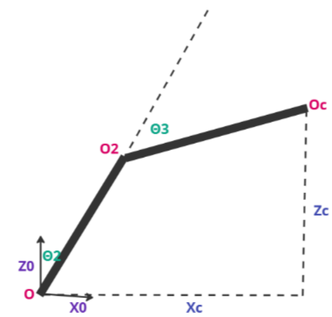
Determine the Θ_3

Apply the cosine rule to triangle $O_0O_2O_c$ to find $\theta 3$

$\Theta_3 = \cos^{-1}((Xc^2 + Yc^2 + Zc^2 - L1^2 - L2^2) / (2 \text{ x } L1 \text{ x} L2$

Determine the Θ_2

$$\Theta_2 = \tan^{-1}\left(\frac{Zc-7}{\sqrt{Xc^2+Yc^2}}\right) - \tan^{-1}\left(\frac{L2*Sin(\Theta 3)}{L1 + L2*Cos(\Theta 3)}\right)$$



Manipulator Jacobian

All the joints in the manipulator are revolute joints. Thus, Joints Jacobian obtain from following equation.

$$J_i = \begin{bmatrix} z_{i-1}^0 \times (t_n^0 - t_{i-1}^0) \\ z_{i-1}^0 \end{bmatrix}$$

Z_i and t_n values obtained from respective joints relative transformation matrix using MATLAB**.**

z_0_0 = 3x1
$$\begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$$

Z_0_1 =
$$\begin{pmatrix} \sin(\theta_1) \\ -\cos(\theta_1) \\ 0 \end{pmatrix}$$

Z_0_2 =
$$\begin{pmatrix} \sin(\theta_1) \\ -\cos(\theta_1) \\ 0 \end{pmatrix}$$

Z_0_3 =
$$\begin{pmatrix} \sin(\theta_1) \\ -\cos(\theta_1) \\ 0 \end{pmatrix}$$

Z_0_4 =
$$\begin{pmatrix} \sin(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)) - \cos(\theta_4)\ (\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)\sin(\theta_2)) \\ \sin(\theta_4)\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)\sin(\theta_1)) - \cos(\theta_4)\ (\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_1)\sin(\theta_2)) \\ \cos(\theta_4)\ (\cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3)) - \sin(\theta_4)\ (\cos(\theta_2)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_2)) \end{pmatrix}$$

t_0_0 = 3x1
$$\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

t_0_1 =
$$\begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix}$$

t_0_2 =
$$\begin{pmatrix} \frac{21\cos(\theta_1)\cos(\theta_2)}{2} \\ \frac{21\cos(\theta_2)\sin(\theta_1)}{2} \\ \frac{21\sin(\theta_2)}{2} + 7 \end{pmatrix}$$

t_0_3 =
$$\begin{pmatrix} \frac{21\cos(\theta_1)\cos(\theta_2)}{2} - 15\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ \frac{21\cos(\theta_2)\sin(\theta_1)}{2} - 15\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) \\ \frac{21\sin(\theta_2)}{2} + 15\cos(\theta_2)\sin(\theta_3) + 15\cos(\theta_3)\sin(\theta_2) + 7 \end{pmatrix}$$

t_0_4 =
$$\begin{pmatrix} \frac{21\cos(\theta_1)\cos(\theta_2)}{2} - 15\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ \frac{21\cos(\theta_2)\sin(\theta_1)}{2} - 15\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) \\ \frac{21\sin(\theta_2)}{2} + 15\cos(\theta_2)\sin(\theta_3) + 15\cos(\theta_3)\sin(\theta_2) + 7 \end{pmatrix}$$

t_0_5 =
$$\begin{pmatrix} \frac{21\cos(\theta_1)\cos(\theta_2)}{2} - \frac{35\cos(\theta_4)\ (\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)\sin(\theta_2))}{2} + \frac{35\sin(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))}{2} - 15\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ \frac{21\cos(\theta_2)\sin(\theta_1)}{2} - \frac{35\cos(\theta_4)\ (\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_1)\sin(\theta_2))}{2} + \frac{35\sin(\theta_4)\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)\sin(\theta_1))}{2} - 15\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 15\cos(\theta_2)\cos(\theta_3)\sin(\theta_1) \\ \frac{21\sin(\theta_2)}{2} + 15\cos(\theta_2)\sin(\theta_3) + 15\cos(\theta_3)\sin(\theta_2) + \frac{35\cos(\theta_4)\ (\cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3))}{2} - \frac{35\sin(\theta_4)\ (\cos(\theta_2)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_2))}{2} + 7 \end{pmatrix}$$

$$J = [J_1\ \ J_2\ J_3\ J_4\ J_5]$$

From the t and Z values manipulator Jacobian Determined using MATLAB.