Л. р. № 10

На сетке (x_i, t_j), где $x_i = ih$,
$t_j = j\tau$

$$Lu(x,t) = \frac{\partial u(x,t)}{\partial t} - a\frac{\partial u(x,t)}{\partial x} = \varphi(x,t)$$

№2 $Ш(x_i, t_j) = \{(x_i, t_{j+1}), (x_i, t_j), (x_{i-1}, t_j)\}$

Предположим, что:

$$L_h u^{(h)}(x_i, t_j) = A(x_i, t_j)u_i^{j+1} + B(x_i, t_j)u_i^j + C(x_i, t_j)u_{i-1}^j$$

Найдем разность:

$$R_h(u(x_i, t_j)) = \Big|\frac{\partial u(x_i, t_j)}{\partial t} - a\frac{\partial u(x_i, t_j)}{\partial x} -$$
$$- [A(x_i, t_j)u(x_i, t_{j+1}) + B(x_i, t_j)u(x_i, t_j) + C(x_i, t_j)u(x_{i-1}, t_j)]\Big|$$

Далее воспользуемся разложениями всех значений функции в „сдвинутых точках" в ряд Тейлора:

$$u(x_i, t_{j+1}) = u(x_i, t_j) + \frac{\tau}{1!}\frac{\partial u(x_i, t_j)}{\partial t} + \frac{\tau^2}{2!}\frac{\partial^2 u(x_i, t_j)}{\partial t^2} + \dots$$

$$u(x_{i-1}, t_j) = u(x_i, t_j) - \frac{h}{1!}\frac{\partial u(x_i, t_j)}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 u(x_i, t_j)}{\partial x^2} - \frac{h^3}{3!}\frac{\partial^3 u(x_i, t_j)}{\partial x^3} + \dots$$

Подставим $u(x_i, t_{j+1})$ и $u(x_{i-1}, t_j)$ в $R_h[u(x_i, t_j)]$ $\Rightarrow$

$$R_h[u(x_i, t_j)] = \Big[(A+B+C)\,u(x_i, t_j) + (-a + Ch)\frac{\partial u(x_i, t_j)}{\partial x} + (1 - A\tau)\frac{\partial u(x_i, t_j)}{\partial t} - A\frac{\tau^2}{2}\frac{\partial^2 u(x_i, t_j)}{\partial t^2} - C\frac{h^2}{2}\frac{\partial^2 (x_i, t_j)}{\partial x^2} + C\frac{h^3}{3!}\frac{\partial^3 u(x_i, t_j)}{\partial x^3}\dots\Big]$$

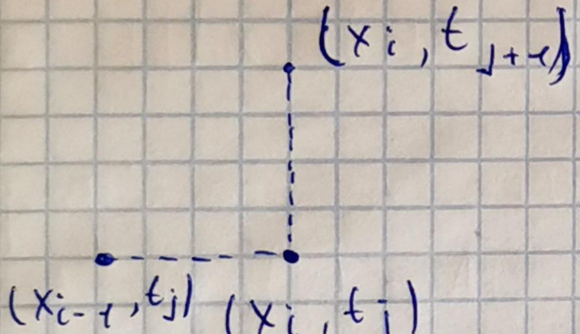
$$\begin{cases} A + B + C = 0 \\ Ch = a \\ A\tau = 1 \end{cases} \Rightarrow \quad \begin{aligned} B &= -\left(\frac{1}{\tau} + \frac{a}{h}\right) \\ C &= \frac{a}{h} \\ A &= \frac{1}{\tau} \end{aligned}$$

$$R_h[u(x_i, t_j)] = \Big[-\frac{\tau}{2}\frac{\partial^2 u(x_i, t_j)}{\partial t^2} - \frac{ah}{2}\frac{\partial^2 (x_i, t_j)}{\partial x^2} + \frac{ah^2}{6}\frac{\partial^3 u(x_i, t_j)}{\partial x^3} + O(h^3) + O(\tau^2)\Big]$$

Разностная схема построена:

$$L_h u^{(h)}(x_i, t_j) = \frac{1}{\tau} u_i^{j+1} - \frac{1}{\tau} u_i^{j} - \frac{a}{h} u_i^{j} +$$

$$+ \frac{a}{h} u_{i-1}^{j} = \varphi(x_i, t_j)$$



$t_j)$ +