№1

$$Lu(x,t) = \frac{\partial u(x,t)}{\partial t} - a\frac{\partial u(x,t)}{\partial x} = \varphi(x,t)$$

$$L_h u^{(h)} = \frac{u_m^{n+1} - u_m^n}{\tau} - a\frac{u_{m+1}^n - u_{m-1}^n}{2h} = \varphi(x_m, t_n)$$

$$\delta f^{(h)} = \frac{u(x_m, t_{n+1}) - u(x_m, t_n)}{\tau} - a\frac{u(x_{m+1}, t_n) - u(x_{m-1}, t_n)}{2h} - \varphi(x,t)$$

$$u(x_m, t_{n+1}) = u(x_m, t_n) + \frac{\tau}{1!}\frac{\partial u(x_m,t_n)}{\partial t} + \frac{\tau^2}{2!}\frac{\partial^2 u(x_m,t_n)}{\partial t^2} + \ldots$$

$$u(x_{m-1}, t_n) = u(x_m, t_n) - \frac{h}{1!}\frac{\partial u(x_m,t_n)}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 u(x_m,t_n)}{\partial x^2} + \ldots$$

$$u(x_{m+1}, t_n) = u(x_m, t_n) + \frac{h}{1!}\frac{\partial u(x_m,t_n)}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 u(x_m,t_n)}{\partial x^2} + \ldots$$

$$\delta f^{(h)} = \frac{\cancel{u(x_m,t_n)} + \frac{\tau}{1!}\frac{\partial u(x_m,t_n)}{\partial t} + \frac{\tau^2}{2!}\frac{\partial^2 u(x_m,t_n)}{\partial t^2} - \cancel{u(x_m,t_n)}}{\tau} -$$

$$- a\,\frac{\cancel{u(x_m,t_n)} + \cancel{\frac{h}{1}}\frac{\partial u(x_m,t_n)}{\partial x} + \cancel{\frac{h^2}{2!}\frac{\partial^2 u(x_m,t_n)}{\partial x^2}} - \cancel{u(x_m,t_n)} + \frac{h}{1}\frac{\partial u(x_m,t_n)}{\partial x} - \cancel{\frac{h^2}{2!}\frac{\partial^2 u(x_m,t_n)}{\partial x^2}}}{\cancel{2h}} - \varphi(x,t)$$

$$\delta f^{(h)} = \frac{\partial u(x_m,t_n)}{\partial t} + \frac{\tau}{2}\frac{\partial^2 u(x_m,t_n)}{\partial t^2} - \cancel{a\frac{\partial u(x_m,t_n)}{\partial x} - \varphi(x,t)}$$

$$\delta f^{(h)} = \frac{\tau}{2}\frac{\partial^2 u(x_m,t_n)}{\partial t^2}$$

$$\|\delta f^{(h)}\|_{F_h} = \max\left|\frac{\tau}{2}\frac{\partial^2 u(x_m,t_n)}{\partial t^2}\right| \leq \frac{\tau}{2} M_\tau$$

Значит, разностная схема имеет первый порядок аппроксимации отн. τ и h

№2

$$\frac{u_m^{n+1}-u_m^n}{\tau} + a\frac{u_{m+1}^n - u_{m-1}^n}{2h} - \frac{a^2\tau}{2}\frac{u_{m+1}^n - 2u_m^n + u_{m-1}^n}{h^2} = 0$$

$$u_m^n = u_m^{n+1} + \frac{a\tau}{2h}\left(u_{m+1}^n - u_{m-1}^n\right) - \frac{a^2\tau^2}{2h^2}\left(u_{m+1}^n - 2u_m^n + u_{m-1}^n\right)$$

$$u_m^n = \lambda^n e^{i\alpha m}, \quad \frac{\tau}{h} = r$$

$$\cancel{\lambda^n} \cancel{e^{i\alpha m}} = \cancel{\lambda^n}\cdot\lambda\cdot\cancel{e^{i\alpha m}} + \frac{ar}{2}\cancel{\lambda^n}\cancel{e^{i\alpha m}}\left(e^{i\alpha} - \frac{1}{e^{i\alpha}}\right) - \frac{a^2 r}{2}\cancel{\lambda^n}\cancel{e^{i\alpha m}}\cdot$$

$$\cdot\left(e^{i\alpha} - 2 + e^{-i\alpha}\right)$$

$$1 = \lambda + \frac{ar}{2}\left(\underbrace{e^{i\alpha} - e^{-i\alpha}}_{= 2i\sin\alpha}\right) - \frac{a^2r^2}{2}\left(\underbrace{e^{i\alpha} - 2 + e^{-i\alpha}}_{= -4\sin^2\frac{\alpha}{2}}\right)$$

$$1 = \lambda + ar\cdot i\sin\alpha + 2\sin^2\frac{\alpha}{2}\cdot a^2r^2$$

$$\lambda = 1 - ari\sin\alpha - 2a^2r^2\cdot\sin^2\frac{\alpha}{2}$$

$$\lambda = 1 + a^2r^2(\cos\alpha - 1) - ari\sin\alpha$$

$$|\lambda|^2 = \left(1 + r^2(\cos\alpha - 1)\right)^2 + r^2\sin\alpha$$

$$|\lambda|^2 = 1 + 2r^2(\cos\alpha - 1) + r^4(\cos^2\alpha - 2\cos\alpha + 1) + r^2(1 - \cos^2\alpha)$$

$$|\lambda|^2 = \cos^2\alpha\,(r^4 - r^2) - 2\cos\alpha\,(r^4 - r^2) + (1 + r^4 - r^2)$$

$$|\lambda|^2 = 1 + r^2(1 - \cos\alpha)^2\cdot(r^2 - 1)$$

$$|\lambda|^2 = 1 + r^2 (1 - \cos\alpha)^2 \cdot (r^2 - 1)$$

$|\lambda|^2 < 1$ при $1 + 4r^2 (r^2 - 1) < 1 \Rightarrow$

$$\Rightarrow r^2 - 1 < 0$$

$$r^2 < 1$$

$$|r| < 1$$

схема устойчива при:

$$\left| \frac{a\tau}{h} \right| < 1$$