Diagonalisation and applications

TOTAL POINTS 7

1. In this quiz you will diagonalise some matrices and apply this to simplify calculations.

1/1 point

Given the matrix $T=\begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$ and change of basis matrix $C=\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ (whose columns are eigenvectors of T), calculate the diagonal matrix $D=C^{-1}TC$.

- $\begin{bmatrix} 9 & 0 \\ 0 & 20 \end{bmatrix}$
- $\bigcirc \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

✓ Correct Well done!

2. Given the matrix $T = \begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix}$ and change of basis matrix $C = \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix}$ (whose columns are eigenvectors of T), calculate the diagonal matrix $D = C^{-1}TC$.

- $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
- $\begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix}$

✓ Correct

3. Given the matrix $T=\begin{bmatrix}1&0\\2&-1\end{bmatrix}$ and change of basis matrix $C=\begin{bmatrix}1&0\\1&1\end{bmatrix}$ (whose columns are eigenvectors of T), calculate the diagonal matrix $D=C^{-1}TC$.

- $\begin{bmatrix}
 1 & 0 \\
 0 & 1
 \end{bmatrix}$
- $\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$
- $\bigcirc \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

- 4. Given a diagonal matrix $D = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$, and a change of basis matrix $C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ with inverse $C = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$, calculate 1/1 point
 - $\begin{bmatrix} -a & 0 \\ 0 & a \end{bmatrix}$
 - $\begin{bmatrix} -a & 0 \\ 0 & -a \end{bmatrix}$

Well donel As it turns out, because D is a special type of diagonal matrix, where all entries on the diagonal are the same, this matrix is just a scalar multiple of the identity matrix. Hence, given any change of co-ordinates, this matrix remains the same.

5. Given that $T=\begin{bmatrix}6&-1\\2&3\end{bmatrix}=\begin{bmatrix}1&1\\1&2\end{bmatrix}\begin{bmatrix}5&0\\0&4\end{bmatrix}\begin{bmatrix}2&-1\\-1&1\end{bmatrix}$, calculate T^3

1/1 point

- \bigcirc $\begin{bmatrix} -61 & 3 \end{bmatrix}$

- $\bigcirc \begin{bmatrix} 3 & 122 \\ 186 & -61 \end{bmatrix}$
- $\bigcirc \begin{bmatrix} 122 & 186 \\ -61 & 3 \end{bmatrix}$

✓ Correct Well done!

6. Given that $T=\begin{bmatrix}2&7\\0&-1\end{bmatrix}=\begin{bmatrix}7&1\\-3&0\end{bmatrix}\begin{bmatrix}-1&0\\0&2\end{bmatrix}\begin{bmatrix}0&-1/3\\1&7/3\end{bmatrix}$, calculate T^3 .

1/1 point

- $\bigcirc \quad \begin{bmatrix} -1 & 21 \\ 8 & 0 \end{bmatrix}$
- $\bigcirc \quad \left[\begin{smallmatrix} 0 & -1 \\ 21 & 8 \end{smallmatrix} \right]$
- $\bigcirc \quad \begin{bmatrix} 21 & 8 \\ 0 & -1 \end{bmatrix}$
- ✓ Correct Well done!
- 7. Given that $T = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, calculate T^6 .

1/1 point

- $\bigcirc \quad \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$
- $\bigcirc \quad \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$
- ✓ Correct Well done!