Solving linear equations using the inverse matrix

TOTAL POINTS 14

1. You go to the shops on Monday and buy 1 apple, 1 banana, and 1 carrot; the whole transaction totals €15. On Tuesday you buy 3 apples, 2 bananas, 1 carrot, all for €28. Then on Wednesday 2 apples, 1 banana, 2 carrots, for €23.

1 / 1 point

Construct a matrix and vector for this linear algebra system. That is, for

$$A\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}s_{\text{Mon}}\\s_{\text{Tue}}\\s_{\text{Wed}}\end{bmatrix}$$

Where a, b, c, are the prices of apples, bananas, and carrots. And each s is the total for that day.

Fill in the components of A and s.

```
# Replace A and s with the correct values below:
A = [[1, 1, 1],
     [3, 2, 1],
     [2, 1, 2]]

s = [15, 28, 23]

```

Run

Reset

✓ Correct

Correct! Well done.

2. Given another system, $B\mathbf{r}=\mathbf{t}$,

1 / 1 point

$$\begin{array}{c}①:\\②:\\③:\end{array}\begin{bmatrix}4&6&2\\3&4&1\\2&8&13\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}9\\7\\2\end{bmatrix}$$

We wish to convert this to echelon form, by using elimination. Starting with the first row, ①, if we divide the whole row by 4, then the top-left element of the matrix becomes 1,

$$\begin{array}{c}①':\\②':\\③':\end{array}\begin{bmatrix}1&3/2&1/2\\3&4&1\\2&8&13\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}9/4\\7\\2\end{bmatrix}$$

Next, we need to fix the second row. This results in the following,

$$\begin{array}{c}①'':\\②'':\\③'':\end{array}\begin{bmatrix}1&3/2&1/2\\0&1&1\\2&8&13\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}9/4\\-1/2\\2\end{bmatrix}$$

What steps did we take?

- (●) The new second row, ②″ is the old second row minus three times the old first row, then all multiplied by -2, i.e., $②''=[②'-3①']\times(-2)$.
- ( ) The new second row, ②″ is the old second row minus three, i.e., $②''=②'-3$.
- ( ) The new second row, ②″ is the old second row divided by four minus the old first row, i.e., $②''=②'/4-①'$.
- ( ) The new second row, ②″ is the old second row minus two times the old first row, i.e., $②''=[②'-2①']$.

✓ Correct

We've made the new second row a linear combination of previous rows.

3. From the previous question, our system is almost in echelon form.

1 / 1 point

$$\begin{array}{c}①''\\②''\\③''\end{array}\begin{bmatrix}1&3/2&1/2\\0&1&1\\2&8&13\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}9/4\\-1/2\\2\end{bmatrix}$$

Fix row 3 to be a linear combination of the other two. What is the echelon form of the system?

- ( ) $\begin{bmatrix}1&3/2&1/2\\0&1&1\\0&5&1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}9/4\\-1/2\\-5/2\end{bmatrix}$
- ( ) $\begin{bmatrix}1&3/2&1/2\\0&1&1\\0&0&1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}9/4\\-1/2\\1/2\end{bmatrix}$
- ( ) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}9/4\\-1/2\\-1/4\end{bmatrix}$
- (●) $\begin{bmatrix}1&3/2&1/2\\0&1&1\\0&0&1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}9/4\\-1/2\\0\end{bmatrix}$

✓ Correct

This system is now in echelon form.

4. Taking your answer from the previous part, use back substitution to solve the system.

1 / 1 point

What is the value of $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$?

- (•) $\mathbf{r} = \begin{bmatrix} 3 \\ -1/2 \\ 0 \end{bmatrix}$
- ( ) $\mathbf{r} = \begin{bmatrix} 9/4 \\ -1/2 \\ 0 \end{bmatrix}$
- ( ) $\mathbf{r} = \begin{bmatrix} 3/2 \\ 1/2 \\ 1 \end{bmatrix}$
- ( ) $\mathbf{r} = \begin{bmatrix} 9 \\ 7 \\ 2 \end{bmatrix}$

✓ **Correct**
Well done!

5. Let's return to the apples and bananas from Question 1. **3 / 3 points**

Take your answer to Question 1 and convert the system to echelon form. I.e.,

$$\begin{bmatrix} 1 & A'_{12} & A'_{13} \\ 0 & 1 & A'_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} s'_1 \\ s'_2 \\ s'_3 \end{bmatrix}.$$

Find values for A' and $\mathbf{s}'$.

```
# Replace A and s with the correct values below:
A = [[ 1 , 1, 1],
     [ 0, 1 , 1],
     [ 0, 0 , 1 ]]
s = [15, 12, 5]

```

Run
Reset

✓ **Correct**
Correct! Well done.

6. Following on from the previous question; now let's solve the system using back substitution. **3 / 3 points**

What is the price of apples, bananas, and carrots?

```
# Replace a, b, and c with the correct values below:
s = [3, 7, 5]

```

Run
Reset

✓ **Correct**
Correct! Well done.

7. If every week, you go to the shops and buy the same amount of apples, bananas, and oranges on Monday, Tuesday, and Wednesday; and every week you get a new list of daily totals - then you should solve the system in general. **3 / 3 points**

That is, find the inverse of the matrix you used in Question 1.

```
# Replace the matrix elements with the correct values below:
Ainv = [[-1.5, 1/2, 1/2],
        [2, 0, -1],
        [1/2, -1/2, 1/2]]

```

Run
Reset

✓ **Correct**
Correct! Well done.

8. In practice, for larger systems, one never solves a linear system by hand as there are software packages that can do this for you - such as *numpy* in Python. **1 / 1 point**

Use this code block to see *numpy* invert a matrix.

You can try to invert any matrix you like. Try it out on your answers to the previous question.

```
import numpy as np

A = [[1, 1, 3],
     [1, 2, 4],
     [1, 1, 2]]
Ainv = np.linalg.inv(A)

```

Run
Reset

In general, one shouldn't calculate the inverse of a matrix unless absolutely necessary. It is more computationally efficient to solve the linear algebra system if that is all you need.

Use this code block to solve the following linear system with *numpy*. $A\mathbf{r} = \mathbf{s}$,

$$\begin{bmatrix} 4 & 6 & 2 \\ 3 & 4 & 1 \\ 2 & 8 & 13 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \\ 2 \end{bmatrix}$$

```
import numpy as np
A = [[4, 6, 2],
     [3, 4, 1],
     [2, 8, 13]]

```

```
s = [9, 7, 2]

r = np.linalg.solve(A, s)

```

Run

Reset

Correct

In cases when you don't need the inverse matrix itself,
linear algebra routines are quicker to solve the system for each case.