



✓ Congratulations! You passed!

TO PASS 80% or higher

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GRADE
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Eigenvalues and eigenvectors

LATEST SUBMISSION GRADE

90%

1. This assessment will test your ability to apply your knowledge of eigenvalues and eigenvectors to some special cases.

1 / 1 point

Use the following code blocks to assist you in this quiz. They calculate eigenvectors and eigenvalues respectively:

```
1 # Eigenvalues
2 M = np.array([[1, 0, 0],
3              [0, 2, 0],
4              [0, 0, 3]])
5 vals, vecs = np.linalg.eig(M)
6 vals
```

```
1 # Eigenvectors - Note, the eigenvectors are the columns of the output.
2 M = np.array([[1, 0, 0],
3              [0, 2, 0],
4              [0, 0, 3]])
5 vals, vecs = np.linalg.eig(M)
6 vecs
7
```

To practice, select all eigenvectors of the matrix, $A = \begin{bmatrix} 4 & -5 & 6 \\ 7 & -8 & 6 \\ 3/2 & -1/2 & -2 \end{bmatrix}$.

☒ $\begin{bmatrix} 1/2 \\ -1/2 \\ -1 \end{bmatrix}$

✓ Correct
This is one of the eigenvectors. Note eigenvectors are only defined upto a scale factor.

☐ $\begin{bmatrix} 1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}$

☐ $\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

☐ $\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$

☒ $\begin{bmatrix} -2/\sqrt{9} \\ -2/\sqrt{9} \\ 1/\sqrt{9} \end{bmatrix}$

✓ Correct
This is one of the eigenvectors. Note eigenvectors are only defined upto a scale factor.

☒ $\begin{bmatrix} -3 \\ -3 \\ -1 \end{bmatrix}$

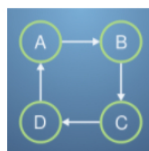
✓ Correct
This is one of the eigenvectors.

☐ None of the other options.

2. Recall from the *PageRank* notebook, that in *PageRank*, we care about the eigenvector of the link matrix, L , that has eigenvalue 1, and that we can find this using *power iteration method* as this will be the largest eigenvalue.

1 / 1 point

PageRank can sometimes get into trouble if closed-loop structures appear. A simplified example might look like this,



With link matrix, $L = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

Use the calculator in Q1 to check the eigenvalues and vectors for this system.

What might be going wrong? Select all that apply.

☐ None of the other options.

☒ Other eigenvalues are not small compared to 1, and so do not decay away with each power iteration.

✓ **Correct**
The other eigenvectors have the same size as 1 (they are $-1, i, -i$)

☐ Some of the eigenvectors are complex.

✓ **Correct**
Because of the loop, *Procrastinating Pats* that are browsing will go around in a cycle rather than settling on a webpage.

✓ **Correct**
If all sites started out populated equally, then the incoming pats would equal the outgoing, but in general the system will not converge to this result by applying power iteration.

☐ The system is too small.

3. The loop in the previous question is a situation that can be remedied by damping.

1 / 1 point

If we replace the link matrix with the damped, $L' = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.7 \\ 0.7 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.7 & 0.1 \end{bmatrix}$, how does this help?

☐ It makes the eigenvalue we want bigger.

☐ The complex number disappear.

✓ **Correct**
There is now a probability to move to any website.

✓ **Correct**
This helps the power iteration settle down as it will spread out the distribution of Pats

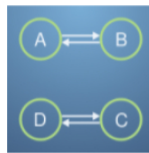
✓ **Correct**
The other eigenvalues get smaller.

✓ **Correct**
So their eigenvectors will decay away on power iteration.

☐ None of the other options.

4. Another issue that may come up, is if there are disconnected parts to the internet. Take this example,

0 / 1 point



with link matrix, $L = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

This form is known as block diagonal, as it can be split into square blocks along the main diagonal, i.e., $L = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$,

with $A = B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ in this case.

What is happening in this system?

☐ The system has zero determinant.

✓ **Correct**
There isn't a unique PageRank.

✓ **Correct**
The power iteration algorithm could settle to multiple values, depending on its starting conditions.

✓ **Correct**
There are two eigenvalues of 1.

✓ **Correct**
The eigensystem is degenerate. Any linear combination of eigenvectors with the same eigenvalue is also an eigenvector.

☐ None of the other options.

☐ There are loops in the system.

You didn't select all the correct answers

5. By similarly applying damping to the link matrix from the previous question. What happens now?

1 / 1 point

☐ Damping does not help this system.

☐ The negative eigenvalues disappear.

✓ **Correct**
None of the other options.

✓ **Correct**
There is now only one eigenvalue of 1, and PageRank will settle to its eigenvector through repeating the power iteration method.

☐ The system settles into a single loop.

☐ There becomes two eigenvalues of 1.

6. Given the matrix $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$, calculate its characteristic polynomial.

1 / 1 point

- ☐ $\lambda^2 + 2\lambda + \frac{1}{4}$
- ☐ $\lambda^2 - 2\lambda - \frac{1}{4}$
- ☐ $\lambda^2 + 2\lambda - \frac{1}{4}$
- ☒ $\lambda^2 - 2\lambda + \frac{1}{4}$

✓ Correct

Well done - this is indeed the characteristic polynomial of A .

7. By solving the characteristic polynomial above or otherwise, calculate the eigenvalues of the matrix $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$.

1 / 1 point

- ☐ $\lambda_1 = -1 - \frac{\sqrt{3}}{2}, \lambda_2 = -1 + \frac{\sqrt{3}}{2}$
- ☒ $\lambda_1 = 1 - \frac{\sqrt{3}}{2}, \lambda_2 = 1 + \frac{\sqrt{3}}{2}$
- ☐ $\lambda_1 = -1 - \frac{\sqrt{5}}{2}, \lambda_2 = -1 + \frac{\sqrt{5}}{2}$
- ☐ $\lambda_1 = 1 - \frac{\sqrt{5}}{2}, \lambda_2 = 1 + \frac{\sqrt{5}}{2}$

✓ Correct

Well done! These are the roots of the above characteristic polynomial, and hence these are the eigenvalues of A .

8. Select the two eigenvectors of the matrix $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$.

1 / 1 point

- ☐ $\mathbf{v}_1 = \begin{bmatrix} 1 - \sqrt{3} \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 + \sqrt{3} \\ 1 \end{bmatrix}$
- ☐ $\mathbf{v}_1 = \begin{bmatrix} -1 - \sqrt{5} \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 + \sqrt{5} \\ 1 \end{bmatrix}$
- ☐ $\mathbf{v}_1 = \begin{bmatrix} 1 - \sqrt{5} \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 + \sqrt{5} \\ 1 \end{bmatrix}$
- ☒ $\mathbf{v}_1 = \begin{bmatrix} -1 - \sqrt{3} \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 + \sqrt{3} \\ 1 \end{bmatrix}$

✓ Correct

These are the eigenvectors for the matrix A . They have the eigenvalues λ_1 and λ_2 respectively.

9. Form the matrix C whose left column is the vector \mathbf{v}_1 and whose right column is \mathbf{v}_2 from immediately above.

1 / 1 point

By calculating $D = C^{-1}AC$ or by using another method, find the diagonal matrix D .

- ☐ $\begin{bmatrix} -1 - \frac{\sqrt{3}}{2} & 0 \\ 0 & -1 + \frac{\sqrt{3}}{2} \end{bmatrix}$
- ☒ $\begin{bmatrix} 1 + \frac{\sqrt{3}}{2} & 0 \\ 0 & 1 - \frac{\sqrt{3}}{2} \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 - \frac{\sqrt{3}}{2} & 0 \\ 0 & 1 + \frac{\sqrt{3}}{2} \end{bmatrix}$
- ☐ $\begin{bmatrix} -1 - \frac{\sqrt{3}}{2} & 0 \\ 0 & -1 + \frac{\sqrt{3}}{2} \end{bmatrix}$

✓ Correct

Well done! Recall that when a matrix is transformed into its diagonal form, the entries along the diagonal are the eigenvalues of the matrix - this can save lots of calculation!

10. By using the diagonalisation above or otherwise, calculate A^2 .

1 / 1 point

- ☐ $\begin{bmatrix} -11/4 & 1 \\ 2 & -3/4 \end{bmatrix}$
- ☐ $\begin{bmatrix} 11/4 & -1 \\ -2 & 3/4 \end{bmatrix}$
- ☐ $\begin{bmatrix} -11/4 & 2 \\ 1 & -3/4 \end{bmatrix}$
- ☒ $\begin{bmatrix} 11/4 & -2 \\ -1 & 3/4 \end{bmatrix}$

✓ Correct

Well done! In this particular case, calculating A^2 directly is probably easier - so always try to look for the method which solves the question with the least amount of pain possible!