EKN812: MICROECONOMICS

PROBLEM SET 1

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1. Find the Marshallian and Hicksian demands for the following preferences. Compute the Marshallian (own-price) elasticity of demand for good x. Use m for the agent's income, p_x for the price of x and p_y for the price of y.

(a) quasilinear utility: let $\epsilon > 0$ be some constant, and let

$$u(x, y) = y + \frac{x^{1-\epsilon^{-1}}}{1-\epsilon^{-1}}$$

Answer:

Set the budget constraint as:

$$P_x X + P_y Y = M$$

Marshallian Demands

Maximise x and y :

$$u(x, y) = y + \frac{x^{1-\varepsilon^{-1}}}{1-\varepsilon^{-1}} \qquad \text{s.t} \qquad P_x X + P_y Y = M$$

$$L = u(x, y) + \lambda(M - P_x X - P_y Y)$$

$$L = y + \frac{x^{1-\varepsilon^{-1}}}{1 - \varepsilon^{-1}} + \lambda(M - P_x X - P_y Y)$$

$$\frac{\partial L}{\partial x} = x^{-\varepsilon^{-1}} - \lambda P_x X = 0$$

$$\frac{\partial L}{\partial y} = 1 - \lambda P_y Y = 0$$

$$\frac{\partial L}{\partial \lambda} = M - P_x X - P_y Y = 0$$

$$\therefore \; x^{-\varepsilon^{-1}} = \lambda P_x X$$

$$1 = \lambda P_y Y$$

$$x^{-\varepsilon^{-1}} = \frac{P_x}{P_y}$$

Equating L_x and L_y :

$$x^{\frac{1}{\varepsilon}} = \frac{P_y}{P_x}$$

$$x^* = \left(\frac{P_y}{P_x}\right)^{\varepsilon}$$

Substitute x^* into M:

$$M = P_x X + P_y Y$$

$$M = \left(\frac{P_y}{P_x}\right)^{\varepsilon} + P_y Y$$

$$P_y Y = M - P_x \left(\frac{P_y}{P_x}\right)^{\varepsilon}$$

$$y^* = \frac{M}{P_y} - \frac{P_x \left(\frac{P_y}{P_x}\right)^{\varepsilon}}{P_y}$$

$$y^* = \frac{M}{P_y} - \frac{P_y}{P_x}\left(\frac{P_y}{P_x}\right)^{\varepsilon}$$

$$\therefore y^* = \frac{M}{P_y} - \left(\frac{P_y}{P_x}\right)^{1-\varepsilon}$$

Computing own price elasticity for x :

$$\frac{dx^*}{P_x} = -\varepsilon P_y^{\varepsilon}\, P_y^{-\varepsilon}$$

$$\frac{dx^*}{P_x} = -\varepsilon P_y^{\varepsilon}\, P_x^{-\varepsilon}\, P_x^{-1}$$

$$\frac{dx^*}{P_x} = -\varepsilon P_y^{\varepsilon}\, P_x^{-\varepsilon}\, P_x^{-1}$$

Computing elasticity for x :

$$\frac{dx^*}{P_x}\, P_x = -\varepsilon \left(\frac{P_y}{P_x}\right)^{\varepsilon} \frac{1}{P_x} \frac{P_x}{x^*}$$

$$\therefore \varepsilon_x = -\varepsilon$$

The elasticity is negative, this represents an inferior good.

Hicksian Demands:

$$Min:$$

$$P_x X + P_y Y = M \qquad \text{s.t} \qquad y + \frac{x^{1-\epsilon^{-1}}}{1-\epsilon^{-1}} = u$$

$$L = P_x X + P_y Y + \lambda\left(u - \frac{x^{1-\epsilon^{-1}}}{1-\epsilon^{-1}}\right)$$

$$\frac{\partial L}{\partial x} = P_x - \lambda x^{-\frac{1}{\varepsilon}} = 0 \quad \therefore P_x = \frac{\lambda}{x^{\frac{1}{\varepsilon}}} \quad \therefore x^{-\frac{1}{\varepsilon}} = \frac{\lambda}{P_x}$$

$$\frac{\partial L}{\partial y} = P_y - \lambda = 0 \quad \therefore P_y = \lambda$$

$$\frac{\partial L}{\partial \lambda} = u - y - \frac{x^{1-\varepsilon^{-1}}}{1-\varepsilon^{-1}} \Rightarrow u = y + \frac{x^{1-\varepsilon^{-1}}}{1-\varepsilon^{-1}}$$

Solving for x :

$$x^{\frac{1}{\varepsilon}} = \frac{P_y}{P_x}$$

$$x^* = \left(\frac{P_y}{P_x}\right)^{\varepsilon}$$

$$u = y + \frac{\left[\left(\frac{P_x}{P_y}\right)^{\varepsilon}\right]^{1-\frac{1}{\varepsilon}}}{1-\frac{1}{\varepsilon}}$$

$$u = y + \frac{\left[\left(\frac{P_x}{P_y}\right)^{\varepsilon}\right]^{\varepsilon-\frac{1}{\varepsilon}}}{1-\frac{1}{\varepsilon}}$$

$$y^* = u - \left(\frac{P_y}{P_x}\right)^{\varepsilon-1} \frac{\varepsilon}{\varepsilon-1}$$

(b) CES utility with two goods: let $-\backslash\infty < \rho \leq 1$ and $\alpha \in (0,1)$ be given constants, and let

$$u(x,y) = (\alpha x^{\rho} + (1-\alpha)y^{\rho})^{1/\rho}$$

s.t

$$M = P_x X + P_y Y$$

The CES utility function represents preferences that are strictly monotonic and strictly concave.

$$L = (\alpha x^{\rho} + (1-\alpha)y^{\rho})^{1/\rho} + \lambda\big(M - P_x X - P_y Y\big)$$

$$\frac{\partial L}{\partial x} = \rho\alpha x^{\rho-1}\left[{}^{1}/_{\rho}\,(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{1/\rho-1}\right] = \lambda P_x$$

$$\frac{\partial L}{\partial y} = \rho(1-\alpha)y^{\rho-1}\left[{}^{1}/_{\rho}\,(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{1/\rho-1}\right] = \lambda P_y$$

$$\frac{\partial L}{\partial \lambda} = M - P_x X - P_y Y = 0 \quad \therefore M = P_x X - P_y y$$

Equating L_x and L_y :

$$\frac{\alpha x^{\rho-1}}{(1-\alpha)y^{\rho-1}} = \frac{P_x}{P_y}$$

$$\frac{(1-\alpha)y^{\rho-1}.\,\alpha x^{\rho-1}}{(1-\alpha)y^{\rho-1}} = \frac{P_x}{P_y.\,(1-\alpha)y^{\rho-1}}$$

$$\alpha x^{\rho-1} = \frac{P_x}{P_y.\,(1-\alpha)y^{\rho-1}}$$

$$y^{\rho-1} = \alpha x^{\rho-1}.\frac{P_y}{(1-\alpha)P_x}$$

$$(y^{\rho-1})^{1/\rho-1} = \left(\frac{\alpha x^{\rho-1}.\,P_y}{(1-\alpha)P_x}\right)^{1/\rho-1}$$

$$y^{*} = x\left[\frac{\alpha P_y}{(1-\alpha)P_x}\right]^{1/\rho-1}$$

Substituting y^{*} into M :

$$M = x\left[P_x + P_y\left[\frac{\alpha P_y}{(1-\alpha)P_x}\right]^{1/\rho-1}\right]$$

$$x^{*} = \left[\frac{M}{P_x + \left(\frac{\alpha P_y^{\rho}}{(1-\alpha)P_x}\right)^{1/1-\rho}}\right].\frac{P_x^{1/1-\rho}}{P_x^{1/1-\rho}}$$

$$x^{*} = \left[\frac{M.\,P_x^{1/1-\rho}}{P_x.\,P_x^{1/1-\rho} + \left(\alpha P_y^{\rho}\big/(1-\alpha)P_x\right)^{1-\rho}}\right].\frac{P_x^{1/1-\rho}}{P_x^{1/1-\rho}}$$

$$x^{*} = \left[\frac{M.\,P_x^{1/1-\rho}}{P_x.\,P_x^{1/1-\rho} + \left(\alpha P_y^{\rho}\big/(1-\alpha)P_x\right)^{1/1-\rho}}\right]$$

$$x^{*} = \frac{M.\,P_x^{1/1-\rho}}{P_x^{1/1-\rho} + \left(\alpha P_y^{\rho}\big/(1-\alpha)P_x\right)^{1/1-\rho}} \cdot \frac{\rho}{\rho-1}$$

Let $r = \frac{\rho}{\rho-1}$

$$x^* = \frac{M.P_x^{r-1}}{P_x^r + P_y^r + \left(\frac{\alpha}{(1-\alpha)}\right)^{r-1}}$$

Computing own elasticity for x^* :

$$\frac{dx^*}{P_x} = \frac{(r-1).M.P_x^{r-2}}{P_x^r + P_y^r + \left[\left(\frac{\alpha}{x1-\alpha}\right)^{r-1} - M.P_x^{r-1} - P_x^{r-1}\right]}$$

$$\therefore = \left(P_y^r + P_y^r\right)^{\frac{(\alpha/(1-\alpha)^{r-1}}{2}}$$

The elasticity for x, η_x^* :

$$\eta_x^* = \frac{d_{x^*}}{P_x} \cdot \frac{P_x}{x^*}$$

Hicksian Demands:

$$Min: M = P_x + P_y y$$

s.t

$$(\alpha x^\rho + (1-\rho)y^\rho)^{1/\rho} - u$$

$$L = P_x X + P_y y - \lambda[(\alpha x^\rho + (1-\alpha)y^\rho]^{1/\rho} - u]$$

$$\frac{\partial L}{\partial x} = P_x - \lambda[(\alpha x^\rho + (1-\rho)y^\rho)]^{1/\rho - 1}\, \alpha x^\rho - 1 = 0$$

$$\frac{\partial L}{\partial y} = Py - \lambda\left[(\alpha x^\rho + (1-\rho)y^\rho)\right]^{1/\rho - 1}(1-\alpha)y^{\rho-1}\Big] = 0$$

$$\frac{\partial L}{\partial \lambda} = (\alpha x^\rho + (1-\rho)y^\rho)^{1/\rho} - u = 0$$

By eliminating λ, then

$$\alpha x = (1-\alpha)y\left(\frac{P_x}{P_y}\right)^{1/\rho - 1}$$

$$u = \left[(1-\alpha)y^\rho\left(\frac{P_x}{P_y}\right)^{\rho/\rho-1} + (1-\alpha)y^\rho\right]^{1/\rho} = (1-\alpha)\left[\left(\frac{P_x}{P_y}\right)^{\rho/\rho-1}.1\right]^{1/\rho}$$

Let $r = \frac{\rho}{\rho-1}$

$$(1-\alpha) = u\left[\left(\frac{P_x}{P_y}\right)^{\frac{\rho}{\rho-1}} + 1\right]^{-\frac{1}{\rho}} = u\left[P_x^{\rho/\rho-1} + P_y^{\rho/\rho-1}\right]^{-\frac{1}{\rho}} P_y^{\rho/\rho-1}$$

$$u = \left(P_x^r + P_y^r\right)^{1/r-1} P_y^{r-1}$$

$$\alpha x = uP_x^{1/\rho-1} P_y^{-1/\rho-1}\left(P_x^r + P_y^r\right)^{1/r-1} P_y^{r-1}$$

$$\therefore = u\left(P_x^r + P_y^r\right)^{1/r-1} P_x^{r-1}$$

$$\alpha x^*(\boldsymbol{p}, u) = \; u\left(P_x^r + P_y^r\right)^{1/r-1} P_x^{r-1}$$

$$(1-\alpha)y^*(\boldsymbol{p}, u) = \left(P_x^r + P_y^r\right)^{1/r-1} P_y^{r-1}$$

The expenditure function,

$$e(\boldsymbol{p}, u) = \; uP_x\left(P_x^r + P_y^r\right)^{1/r-1} P_x^{r-1} + uP_y((P_x^r + P_y^r)^{1/r-1} P_y^{r-1}$$

$$\therefore = u(P_x^r + P_y^r)\left(P_x^r + P_y^r\right)^{1/r-1}$$

$$\therefore = u\left(P_x^r + P_y^r\right)^{\frac{1}{r}}$$

At prices $(P_x, P_{y)}$, utility level u is the maximum that can be attained when the consumer's income is M.

(c) ***quadratic utility: let*** $(\overline{\boldsymbol{x}}, \overline{\boldsymbol{y}}) > \boldsymbol{0}$ ***be given constants: also let*** $\boldsymbol{a}, \boldsymbol{b}$ ***be known with the*** $\boldsymbol{a} > \boldsymbol{0}$. ***Consider***

Answer:

$$Max(u, y) = \; -\frac{1}{2}(a(\overline{x} - x)^2 + 2b(\overline{x} - x)(\overline{y} - y) + (\overline{y} - y)^2)$$
$$\therefore \; = -\frac{1}{2}\left(a(\overline{x} - x)^2 - b(\overline{x} - x)(\overline{y} - y) - \frac{1}{2}(\overline{y} - y)^2\right)$$

s.t

$$P_x X + P_y = M$$
$$L = -\frac{1}{2}\left(a(\overline{x} - x)^2 - b(\overline{x} - x)(\overline{y} - y) + \frac{1}{2}(\overline{y} - y)^2\right) + \lambda\left(M - P_x x - P_y y\right)$$
$$\frac{\partial L}{\partial x} = -a(\overline{x} - x).(1) - b(\overline{y} - y).(1) - \lambda P_x = 0$$

$$\therefore \frac{\partial L}{\partial x} = a(\overline{x} - x) + b(\overline{y} - y) - P_x = 0$$

$$\frac{\partial L}{\partial y} = b(\overline{x} - x) + 1 = 0$$

$$\frac{\partial L}{\partial \lambda} = M - P_x X - P_y Y = 0$$

$$\frac{a(\overline{x} - x) + b(\overline{y} - y)}{b(\overline{x} - x) + b(\overline{y} - y)} = \frac{P_x}{P_y}$$

$$P_y[a(\overline{x} - x) + (\overline{y} - y)] = P_x[b(\overline{x} - x) + (\overline{y} - y)]$$

$$P_y(a\overline{x} + ax + b\overline{y}) - P_y yb = P_x(b\overline{x} - bx + \overline{y})$$

$$P_x y - bP_y y = P_x(b\overline{x} - bx + \overline{y}) - P_y(a\overline{x} + ax + b\overline{y})$$

$$\left(P_x - bP_y\right)y = P_x(\overline{y} + b\overline{x}) - bP_x x - P_y(a\overline{x} = b\overline{y}) + aP_y X$$

$$\left(P_x - bP_y\right)y = P_x(b\overline{x} + \overline{y}) - P_y(a\overline{x} + b\overline{y}) + \left(aP_y - bP_x\right)x$$

$$y = \frac{P_x(b\overline{x} + \overline{y}) - P_y(a\overline{x} + b\overline{y}) + \left(aP_y - bP_x\right)x}{P_x - bP_y}$$

Substitute into M :

$$M = P_x X - P_y\left[\frac{P_x(b\overline{x} + \overline{y}) - P_y(a\overline{x} + b\overline{y})}{P_x - bP_y}\right] + \frac{P_y\left(aP_y - bP_x\right)}{P_x - bP_y}\ x$$

$$M - P_y\left[\frac{P_x(b\overline{x} + \overline{y}) - P_y(a\overline{x} + b\overline{y})}{P_x - bP_y}\right] = P_x x + P_y\frac{\left(aP_y - bP_x\right)}{P_x - bP_y}\ x$$

$$M - P_y\left[\frac{P_x(b\overline{x} + \overline{y}) - P_y(a\overline{x} + b\overline{y})}{P_x - bP_y}\right] = P_x x + P_y\frac{\left(aP_y - bP_x\right)}{P_x - bP_y}\ x$$

$$M(P_x - bP_y) - P_y[P_x(b\overline{x} + \overline{y}) - P_y(a\overline{x} + b\overline{y})] = P_x x\left(P_x - bP_y\right)x + P_y\left(\ aP_y - bP_x\right)x$$

$$M(P_x - bP_y) - P_y[P_x(b\overline{x} + \overline{y}) - P_y(a\overline{x} + b\overline{y})] = [P_x x\left(P_x - bP_y\right)x + P_y\left(\ aP_y - bP_x\right)]x$$

$$x^*M = \frac{M(P_x - bP_y) - P_y[P_x(b\overline{x} + \overline{y}) - P_y(a\overline{x} + b\overline{y})]}{[P_x x\left(P_x - bP_y\right)x + P_y\left(\ aP_y - bP_x\right)]}$$

$$y^*M = \left[\frac{P_x(b\overline{x}+\overline{y}) - P_y(a\overline{x}+b\overline{y})}{P_x - bP_y}\right] + \frac{\left(aP_y - bP_x\right)}{P_x - bP_y}\left\{\frac{M(P_x - bP_y) - P_y[P_x(b\overline{x}+\overline{y}) - P_y(a\overline{x}+b\overline{y})]}{[P_x x\left(P_x - bP_y\right)x + P_y\left(\ aP_y - bP_x\right)]}\right\}$$; Let $a = b = 1$,

$$x^*M = \frac{M(P_x - P_y) - P_y[P_x(b\overline{x} + \overline{y}) - (P_y(\overline{x} + \overline{y})]}{P_x^2 - P_x P_y + P_y^2 - P_x P_y}$$

$$\frac{dx^*M}{dP_x} = \frac{[M - P_y(\overline{x} - \overline{y})][P_x^2 - 2P_x P_y + P_y^2] - \{M\left(P_x - P_y\right) - P_y\left[P_x(b\overline{x}+\overline{y}) - P_y(a\overline{x}+b\overline{y})\right]\}[2P_x - 2P_y]}{[P_x^2 - 2P_x P_y + P_y^2]^2}$$

$$\frac{P_x}{x^*M} = P_x\ .\ \frac{[P_x^2 - 2P_x P_y + P_y^2]^2}{M - P_y(\overline{x} - \overline{y}) - P_y[P_x(\overline{x} + \overline{y}) - P_y((\overline{x} + \overline{y})}$$

Preferences around the “bliss point” violate monoticity.

2. Let $\epsilon > 0$ be given and let

Answer:

$$u(x,y) = \frac{x^{1-\varepsilon^{-1}}}{1 - \varepsilon^{-1}} + \frac{y^{1-(2\varepsilon)^{-1}}}{1 - (2\varepsilon)^{-1}}$$

s.t

$$M = P_x X + P_y Y$$

$$L = u(x, y) + \lambda(M - P_x x + P_y Y)$$

$$L = \frac{x^{1-\varepsilon^{-1}}}{1 - \epsilon^{-1}} + \frac{y^{1-(2\varepsilon)^{-1}}}{1 - (2\varepsilon)^{-1}} + \lambda(M - P_x x + P_y Y)$$

$$\frac{\partial L}{\partial x} = x^{-\varepsilon-1} - \lambda P_x = 0$$

$$\frac{\partial L}{\partial y} = y^{-(2\varepsilon)-1} - \lambda P_y = 0$$

$$\frac{\partial L}{\partial \lambda} = M - P_x X - P_y Y = 0$$

Equating $L_x \: and \: L_y$

$$\frac{x^{-\varepsilon-1}}{y^{-2\varepsilon-1}} = \frac{P_x}{P_y}$$

$$\frac{x}{y^{-\varepsilon-1}} = \frac{P_x}{P_y}$$

$$\frac{x}{y^{\frac{1}{\varepsilon}}} = \frac{P_x}{P_y}$$

Solve for y :

$$\frac{y^{\varepsilon}}{x} = \frac{P_x}{P_y}$$

$$y^{\varepsilon} = \frac{P_x}{P_y} x$$

$$y^{\varepsilon} = \frac{P_x x}{P_y x}$$

$$\therefore y^* = \left(\frac{P_x}{P_y}\right)$$

Substitute into the objective function,

$$u(x, y) = \frac{x^{1-\varepsilon^{-1}}}{1 - \frac{1}{\varepsilon}} + \frac{P_x / P_y}{1 - \frac{1}{2\varepsilon}}$$

$$u(x, y) = x + \left(\frac{P_x}{P_y}\right)^{\varepsilon}$$

Preferences are homothetic. The slope of the indifference curves holds through the origin. The new utils evaluated are twice as much compared to bundle x. Bundle y is a luxury good, and x is a necessity.

3. In class I claimed that in a two-good demand system, the goods have to be substitutes. Prove this claim.

Answer:

Goods represented by bundle (x_1, x_2),

utility function takes the form:

$$u(x_1, x_2) = x_1 + x_2$$

Budget constraint:

$$p_1 + p_2 = y$$

Suppose that a consumer requires two units of x_2 as a form of compensation for giving up one unit of x_1.

Maximise x_1, x_2

$$u(x_1, x_2) = x_1 + 2x_2$$

s.t

$$p_1 + p_2 = y$$

$$L = (x_1 + 2x_2) + \lambda(y - p_1 - p_2)$$

$$\frac{\partial L}{\partial x_1} = 1 - p_1 = 0$$

$$\frac{\partial L}{\partial x_2} = 2 - p_2 = 0$$

$$\frac{\partial L}{\partial \lambda} = y - p_1 - p_2 = 0$$

To get the marginal rate of substitution:

$$\frac{1}{2} = \frac{p_1}{p_2}$$

$$\therefore \ = -\frac{2}{1} = -2$$

Perfect substitutes are the only preferences which are homothetic and quasilinear.

4. Suppose, as in class, that consumers do not maximise utility but passively consume whatever is feasible. In particular, assume that each consumer has a "type" k such that he always spends a fraction k on good x and exhausts his budget.

Answer:

The consume is observed to demand bundle x when the price vector is p_x, and bundle y when price vector is p_y,…, and bundle z_k when the price vector is p_k.

The finite data set is expressed as

$$D = \{(x, p_x,), (y, p_y), \ldots, (n_k, p_k)\}$$

For each $k = 1,2, \ldots, n$

$$u(x, y)(x)) \equiv k$$

Here, preferences are convex. Convexity then implies that $MRS(x, y(x))$ is decreasing in x.

$$y = \frac{k}{x}$$

If the prices rise to p_x'

$$\frac{\partial x(p,m)}{\partial p_x'} = \frac{\partial x^*(p,u^*)}{\partial p_x'} - x'(p,m)\frac{\partial x(p,m)}{\partial m}$$

5. Answer:

(a) Consider a case where there are four groups of consumers with income represented by m_1, m_2, m_3 and m_4. Let:

- p be the price for houses on the private housing market
- c be the price associated with purchasing from this private market, where $c < p$
- hp_1 be the associated with buying lower quality houses, where $p > hp_1$
- hp_2 be the price with buying higher quality houses, where hp_2
- x denote some level of income received by each group prior to any transaction

Therefore,

$$m_1 = x(p - c)$$
$$m_2 = x + (p - c) - h_1$$
$$m_3 = x + (p - c) - h_2$$
$$m_4 = x$$

(b) We can assume groups represented by m_1, m_2 and m_3 have preferred the cash value of the houses. As these groups chose to sell the houses received, they were subject to a price c which is associated with the selling of houses. If recipients, were given the cash value of the houses, they would have an increase in income of c more than their current income.

(c) Beneficiaries of the free housing program would not sell the houses for less than the price they are likely to receive, p, less the cost incurred in selling these houses c. Similarly, the beneficiaries of the free housing program would not be able to sell the houses for more that $p + c$, as buyers as would not pay more than the price p, plus the cost of obtaining these houses, c. Therefore,

$$upperbound = p + c$$

and

$$lowerbaound = p - c$$

However, this assumes that c, representative of all costs associated with selling houses.

(d) The following list ranks the values of the free housing program for the recepients form most valuable to least:

1. Group 4, represented by m_4
2. Group 3, represented by m_3
3. Group 2, represented by m_2
4. Group 1, represented by m_1

Group 4 values the free housing program the most as they do not sell the houses.

7. Does rationing make Marhsallian demands less price elastic? Provide a proof or counterexample.

Answer:

Let $x(p, y)$ be the consumer's Marshallian demand system.

The following relations must hold among income shares and income elasticities of demand:

Proof: we recall that the budget constraints requires

$$y = p.x(p, y)$$

For all p and y

Engel aggregation states that the share-weighted income elasticities must always sum to one

$$1 = \sum_{i=1}^{n} P_i \frac{\partial x_i}{y}$$

Multiply and divide each element in the summation by $x_i y$, rearrange, and get

$$1 = \sum_{i=1}^{n} P_i \frac{P_i x_i}{y} \frac{\partial x_i}{y} \frac{y}{x_i}$$

Substitute from the definitions to get

$$0 = \left[\sum_{i \neq j}^{n} P_i \frac{\partial x_i}{\partial P_j} \right] + x_j + Pi \ \frac{\partial x_i}{\partial_{Pj}}$$

Combine terms and rearrange to get

$$-x_j = \sum_{i=1}^{n} P_i \frac{\partial x_i}{\partial_{Pj}}$$

Multiply both sides of the equation by ${}^{P_j}/_{y}$ and get

$$\frac{-P_j x_j}{y} = \sum_{i=1}^{n} \frac{P_i}{y} \frac{\partial x_i}{\partial_{Pj}} P_j$$

Multiply and divide each element of the summation by x_i and get

$$\frac{-P_j x_j}{y} = \sum_{i=1}^{n} \frac{P_i x_i}{y} \frac{\partial x_i}{\partial_{Pj}} \frac{P_j}{x_i}$$

Substituting from the proof completes the proof:

$$-s_j \sum_{i=1}^{n} s_j \varepsilon_{ij} \ , \qquad j = 1, \dots, n$$

According to homogeneity, demand must respond to change in all prices and income simultaneously, and budget balancedness requires that demand always exhaust the consumer’s income.

The Slutsky equations give us qualitative information on how the system of demand functions must respond to every general kinds of price changes, and gives us the analytical insight into the unobservable components of the demand response to price change.

The aggregation relations provide information on how the quantities demanded – first in response to an income change alone, the response to single price change – must all “hang together” across the system of demand functions.