DIMA OANA-TEODORA 141 13.06.2020

EXAMEN

1. a)
2. b)
3. b)
4. d)

(5) a) $\Gamma: f(x) = x_1^2 - 8x_1x_2 + 7x_2^2 - 12x_1 - 6x_2 - 9$

$$A = \begin{pmatrix} 1 & -4 \\ -4 & 7 \end{pmatrix} \quad \hat{A} = \begin{pmatrix} 1 & -4 & -6 \\ -4 & 7 & -3 \\ -6 & -3 & -9 \end{pmatrix}$$

$\delta = \det A = 7 - 16 = -9 < 0$

$$\det \hat{A} = \Delta = \begin{vmatrix} 1 & -4 & -6 \\ -4 & 7 & -3 \\ -6 & -3 & -9 \end{vmatrix} = -324 \neq 0$$

$\exists!$ centrul P_0, Γ = hiperbola

$\frac{\partial f}{\partial x_1} = 0 \Rightarrow 2x_1 - 8x_2 - 12 = 0$

$\frac{\partial f}{\partial x_2} = 0 \Rightarrow 14x_2 - 8x_1 - 6 = 0$

$$\begin{cases} x_1 - 4x_2 - 6 = 0 \\ 7x_2 - 4x_1 - 3 = 0 \end{cases} \Big| \cdot 4 \Rightarrow \begin{cases} 4x_1 - 16x_2 - 24 = 0 \\ -4x_1 + 7x_2 - 3 = 0 \end{cases} \oplus$$

$-9x_2 - 27 = 0$

$-9x_2 = 27$

$\boxed{x_2 = -3}$

$x_1 = 4x_2 + 6 = -12 + 6 = -6$

$$\boxed{\begin{matrix} x_1 = -6 \\ x_2 = -3 \end{matrix}}$$

θ = translație

$\theta : x = x' + x_0, \quad x_0 = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$

$\theta(\Gamma) = x'^T A x' + \frac{\Delta}{\delta} = 0$

$x_1'^2 - 8x_1'x_2' + 7x_2'^2 + 36 = 0$

$Q: \mathbb{R}^2 \to \mathbb{R}$

$Q(x) = x_1'^2 - 8x_1'x_2' + 7x_2'^2$

$\lambda^2 - Tr(A)\lambda + \det A = 0$

$\lambda^2 - 8\lambda - 9 = 0$

$\lambda^2 - 9\lambda + \lambda - 9 = 0$

$\lambda(\lambda - 9) + \lambda - 9 = 0$

$(\lambda - 9)(\lambda + 1) = 0$

$\lambda_1 = 9$

$\lambda_2 = -1$

$V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid Ax = 9x\}$

$(A - 9I_2)x = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -8 & -4 \\ -4 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{cases} -8x_1 - 4x_2 = 0 \mid (-4) \Rightarrow 2x_1 + x_2 = 0 \\ -4x_1 - 2x_2 = 0 \end{cases}$

$2x_1 + x_2 = 0 \Rightarrow x_2 = -2x_1$

$V_{\lambda_1} = \{(x_1, -2x_1) \mid x_1 \in \mathbb{R}\}$

$V_{\lambda_1} = \langle\{(1, -2)\}\rangle \Rightarrow e_1' = \frac{1}{\sqrt{5}}(1, -2)$

$V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid Ax = -x\}$

$(A + I_2)x = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & -4 \\ -4 & 8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{cases} 2x_1 - 4x_2 = 0 \\ -4x_1 + 8x_2 = 0 \end{cases}$ $\quad x_1 - 2x_2 = 0 \Rightarrow x_1 = 2x_2$

$V_{\lambda_2} = \{ x_2 (2x_2, x_2) \mid x_2 \in \mathbb{R}\}$

$V_{\lambda_2} = \langle\{(2, 1)\}\rangle \Rightarrow e_2' = \frac{1}{\sqrt{5}}(2, 1)$

$\sigma: x' = Rx''$ $\in SO(2)$

$R = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

$\tau \circ \theta: \; X = RX'' + X_0$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x_1'' \\ x_2'' \end{pmatrix} + \begin{pmatrix} -6 \\ -3 \end{pmatrix}$$

$\mathcal{H}: \; \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 1, \; \cancel{a>0, b>0}$

$\tau \in \theta(\Gamma) \quad \lambda_1 \cdot x_1''^2 + \lambda_2 \cdot x_2''^2 + \frac{\det \tilde{A}}{\det A} = 0$

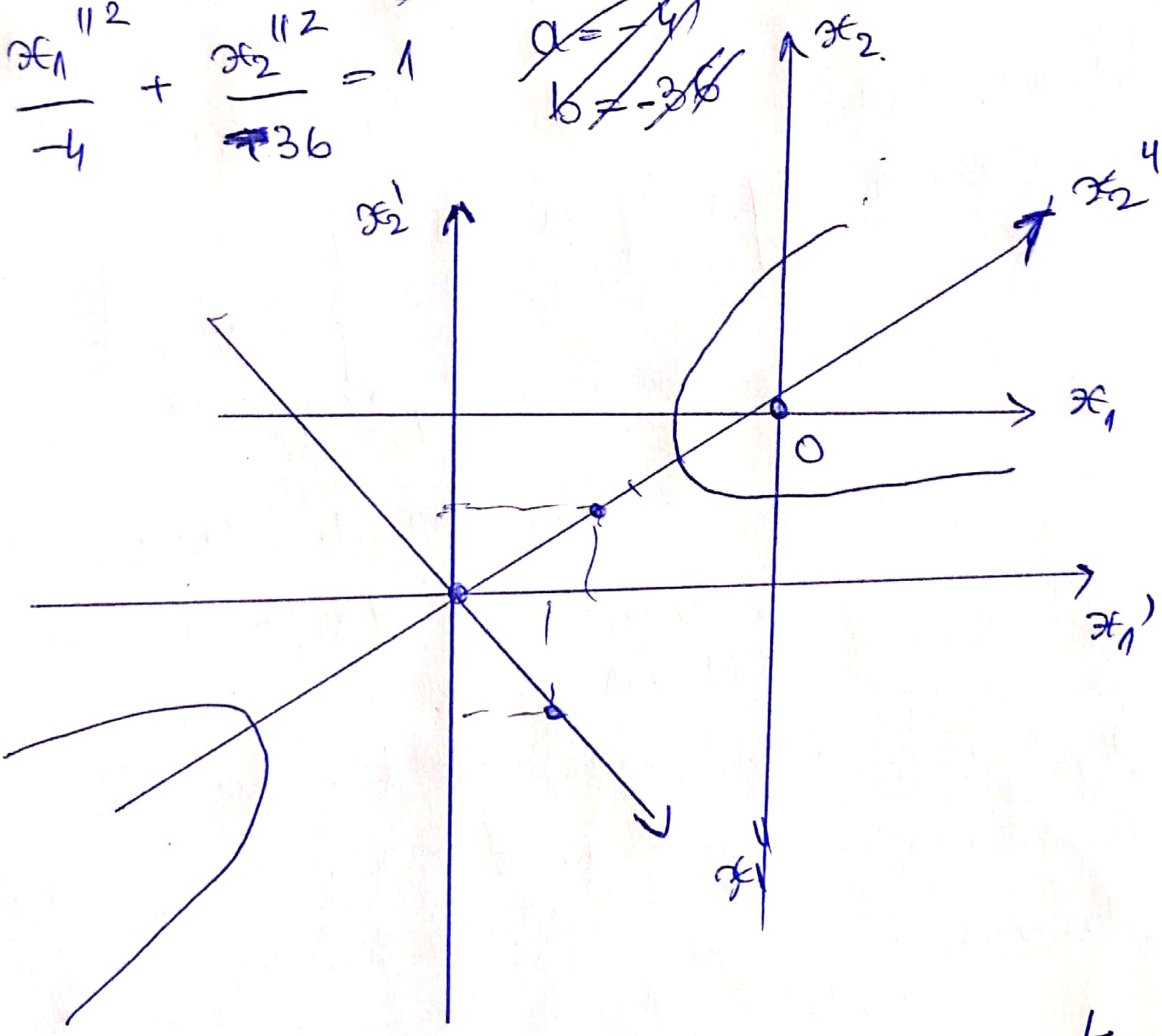
$\Rightarrow \quad 9x_1''^2 - x_2''^2 + 36 = 0 \; | : 36$

$\mathcal{H}: \; \frac{x_1''^2}{4} - \frac{x_2''^2}{36} = -1 \Rightarrow -\frac{x_1''^2}{+4} + \frac{x_2''^2}{+36} = 1$

$\Rightarrow \cancel{\boxed{\begin{matrix} a = +4 \\ b = +36 \end{matrix}}}$

$\frac{x_1''^2}{-4} + \frac{x_2''^2}{36} = 1 \qquad \cancel{\begin{matrix} a = -4 \\ b = -36 \end{matrix}}$

b)



⑥ a) $A = \begin{pmatrix} 3 & -1 & 1 \\ 2 & 0 & 1 \\ 2 & -2 & 3 \end{pmatrix}$

$P(\lambda) = 0$

$P(\lambda) = \det(A - \lambda I_2) = \begin{vmatrix} 3-\lambda & -1 & 1 \\ 2 & -\lambda & 1 \\ 2 & -2 & 3-\lambda \end{vmatrix} =$

$= (\lambda - 1)(\lambda - 2)(\lambda - 3)$

$P(\lambda) = 0 \Rightarrow \lambda_1 = 1 \quad m_1 = 1$

$\lambda_2 = 2 \quad m_2 = 1$

$\lambda_3 = 3 \quad m_3 = 1$

$V_{\lambda_1} = \{ x \in \mathbb{R}^2 \mid f(x) = x \}$

$$\begin{cases} 3x_1 - x_2 + x_3 = x_1 \\ 2x_1 + x_3 = x_2 \\ 2x_1 - 2x_2 + 3x_3 = x_3 \end{cases} \Rightarrow \left.\begin{array}{l} x_3 = x_2 \\ 2x_1 = x_2 - x_3 \end{array}\right\} \Rightarrow$$

$\Rightarrow 2x_1 = 0 \Rightarrow x_1 = 0$

$V_{\lambda_1} = \{ (0, x_2, x_2) \mid x_2 \in \mathbb{R} \}$

$V_{\lambda_1} = \langle \{(0,1,1)\} \rangle \quad \dim V_{\lambda_1} = m_1 = 1$

$V_{\lambda_2} = \{ x \in \mathbb{R}^3 \mid f(x) = 2x \}$

$$\begin{cases} 3x_1 - x_2 + x_3 = 2x_1 \\ 2x_1 + x_3 = 2x_2 \\ 2x_1 - 2x_2 + 3x_3 = 2x_3 \end{cases}$$

$\Rightarrow x_1 = x_2$

$\Rightarrow \underbrace{2x_1 - 2x_1}_{0} + 3x_3 = 2x_3$

$3x_3 - 2x_3 = 0$

$\boxed{x_3 = 0}$

$V_{\lambda_2} = \{ (x_1, x_1, 0) \mid x_1 \in \mathbb{R} \}$

$V_{\lambda_2} = \langle \{(1,1,0)\} \rangle \quad \dim V_{\lambda_2} = m_2 = 1$

$V_{\lambda_3} = \{ x \in \mathbb{R}^2 \mid f(x) = 3x \}$

$$\begin{cases} 3x_1 - x_2 + x_3 = 3x_1 \\ 2x_1 + x_3 = 3x_2 \\ 2x_1 - 2x_2 + 3x_3 = 3x_3 \end{cases}$$

$2x_1 = 2x_2 \Rightarrow$

$\Rightarrow \boxed{x_1 = x_2}$

$x_3 = 3x_2 - 2x_1$

$x_3 = 3x_2 - 2x_2$

$\boxed{x_3 = x_2}$

$x_1 = x_2 = x_3$

$V_{\lambda_3} = \{ (x_1, x_1, x_1) \mid x_1 \in \mathbb{R} \} =$

$V_{\lambda_3} = \langle \{ (1,1,1) \} \rangle$ dim $V_{\lambda_3} = m_3 = 1$

$R_1 = \{ (0,1,1) \}$ reper în V_{λ_1} şi $u_2 = e_2 + e_3 = (0,1,1)$

$R_2 = \{ (1,1,0) \}$ reper în V_{λ_2} şi $u_1 = e_1 + e_2 = (1,1,0)$

$R_3 = \{ (1,1,1) \}$ reper în V_{λ_3} şi $u_3 = e_1 + e_2 + e_3 = (1,1,1)$

$\Rightarrow u_1, u_2, u_3$ ~~val pro.~~ vect proprii ai lui f

b) $R = \{ (1,1,0), (0,1,1), (1,1,1) \}$

$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \Rightarrow \det A \neq 0$ ~~1+0+0-0-0-1~~

R reper dacă $\begin{cases} \operatorname{rg} A \text{ (max)} = |R| \Rightarrow R \text{ e SLI} \\ |R| = \dim \mathbb{R}^3 \Rightarrow R \text{ e bază} \end{cases}$

$\det A \neq 0 \Rightarrow \operatorname{rg} A = 3 = |R| \Rightarrow R$ e SLI

$|R| = \dim \mathbb{R}^3 = 3 \Rightarrow R$ bază

$\Rightarrow R$ reper

$u = e_1 + 2e_2 + e_3$

$$u = a(1,1,0) + b(0,1,1) + c(1,1,1)$$
$$= a(e_1+e_2) + b(e_2+e_3) + c(e_1+e_2+e_3)$$
$$= ae_1 + ae_2 + be_2 + be_3 + ce_1 + ce_2 + ce_3 =$$
$$= e_1(a+c) + e_2(a+b+c) + e_3(b+c)$$

$$\left.\begin{array}{l} a+c=1 \\ a+b+c=2 \\ b+c=1 \end{array}\right\} \Rightarrow a+1=2 \Rightarrow \boxed{a=1}$$
$$a+c=1 \Rightarrow 1+c=1 \Rightarrow \boxed{c=0}$$
$$b+c=1 \Rightarrow b+0=1 \Rightarrow \boxed{b=1}$$

coord lui u în rap cu R sunt $(1,1,0)$

$f(u) = ?$

$$f(x) = (3x_1 - x_2 + x_3,\ 2x_1 + x_3,\ 2x_1 - 2x_2 + 3x_3)$$
$$u = (1,2,1)$$
$$f(u) = (3\cdot 1 - 2 + 1,\ 2\cdot 1 + 1,\ 2\cdot 1 - 2\cdot 2 + 3\cdot 1)$$
$$f(u) = (2,3,1)$$
$$f(u) = 2e_1 + 3e_2 + e_3$$
$$f(u) = a(e_1+e_2) + b(e_2+e_3) + c(e_1+e_2+e_3)$$
$$= e_1(a+c) + e_2(a+b+c) + e_3(b+c)$$

$$\left\{\begin{array}{l} a+c=2 \\ a+b+c=3 \\ b+c=1 \end{array}\right\} \Rightarrow a+1=3 \Rightarrow \boxed{a=2}$$
$$a+c=2 \Rightarrow \cancel{a=}\ 2+c=2 \Rightarrow \boxed{c=0}$$
$$b+c=1 \Rightarrow b+0=1 \Rightarrow \boxed{b=1}$$

coord lui $f(u)$ în rap cu R sunt $(2,1,0)$

⑦ $f: \mathbb{R}^3 \to \mathbb{R}^3$

$f(x) = \frac{1}{3}\left(x_1+x_2+x_3,\ x_1+x_2+x_3,\ x_1+x_2+x_3\right)$

a) $A = [f]_{R_0,R_0} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$f(x) = y \Leftrightarrow Ax = Y$

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3}x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 \\ \frac{1}{3}x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 \\ \frac{1}{3}x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 \end{pmatrix} \Rightarrow$$

$\Rightarrow \left.\begin{matrix} f \text{ liniara} \\ f: \mathbb{R}^3 \to \mathbb{R}^3 \end{matrix}\right\} \Rightarrow f$ este endomorfism

$P(\lambda) = \det(A - \lambda I_3) = 0$

$\underline{e_1' = e_1 + e_2 + e_3}$

$$\det(A - \lambda I_3) = \begin{vmatrix} \frac{1}{3}-\lambda & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3}-\lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}-\lambda \end{vmatrix}$$

$$= (1-\lambda) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{3} & \frac{1}{3}-\lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}-\lambda \end{vmatrix}$$

$(1-\lambda)$

$C_2' = C_2 - C_1$

$C_3' = C_3 - C_1$

$$\begin{vmatrix} \cancel{1} & \cancel{0} & \cancel{0} \\ \frac{1}{3} & -\lambda & 0 \\ \frac{1}{3} & 0 & -\lambda \end{vmatrix}$$

$= (1-\lambda)(\lambda^2) = 0$

$\lambda_1 = 1 \Rightarrow m_1 = 1$

$\lambda_2 = 0 \quad m_2 = 2$

$V_{\lambda_1} = \{ x \in \mathbb{R}^3 \mid f(x) = x \}$

$$\begin{cases} \frac{1}{3}x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 = x_1 \mid \cdot 3 \\ \frac{1}{3}x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 = x_2 \mid \cdot 3 \\ \frac{1}{3}x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 = x_3 \mid \cdot 3 \end{cases}$$

$$\begin{cases} 2x_1 = x_2 + x_3 \quad \Leftrightarrow \quad x_1 = \frac{x_2 + x_3}{2} \\ 2x_2 = x_1 + x_3 \\ 2x_3 = x_1 + x_2 \end{cases}$$

$2x_2 = \frac{x_2 + x_3}{2} + x_3 \Big| \cdot 2 \Rightarrow 4x_2 = x_2 + x_3 + 2x_3 \Rightarrow \boxed{x_2 = x_3}$

$x_1 = \frac{x_2 + x_2}{2} = \frac{2x_2}{2} = x_2$

$\boxed{x_1 = x_2 = x_3}$

$V_{\lambda_1} = \{ (x_1, x_1, x_1) \mid x_1 \in \mathbb{R} \} = \langle \{ (1,1,1) \} \rangle \Rightarrow$

$\Rightarrow R_1 = \{ (1,1,1) \}$ - reper în V_{λ_1}

$\dim V_{\lambda_1} = m_1 = 1$

$V_{\lambda_2} = \{ x \in \mathbb{R}^3 \mid f(x) = 0 \}$

$x_1 + x_2 + x_3 = 0 \Rightarrow x_1 = -x_2 - x_3$

$V_{\lambda_2} = \{ (-x_2 - x_3, x_2, x_3) \mid x_2, x_3 \in \mathbb{R} \}$

$V_{\lambda_2} = \langle \{ (-1,1,0), (-1,0,1) \} \rangle$

$R_2 = \{ (-1,1,0), (-1,0,1) \}$ reper în V_{λ_2}

$\dim V_{\lambda_2} = m_2 = 2$

$R = R_1 \cup R_2 = \{ \underset{f_1}{(1,1,1)}, \underset{f_2}{(-1,1,0)}, \underset{f_3}{(-1,0,1)} \}$

~~Aplic Gram-Schmidt:~~

~~$e_1 = f_1 = (1,1,1)$~~

~~$e_2 = f_2 - \frac{<f_2, e_1>}{<e_1, e_1>} \cdot e_1$~~

$R = \{ (-1,1,0), (-1,0,1), (1,1,1) \}$

$\phantom{R = \{} \overset{\shortparallel}{f_1} \quad \overset{\shortparallel}{f_2} \quad \overset{\shortparallel}{f_3}$

$1+0+0$

$1+1+0$

$-1+1+0$

Aplic Gram - Schmidt:

$e_1 = f_1 = (-1,1,0)$

$e_2 = f_2 - \frac{<f_2, e_1>}{<e_1, e_1>} \cdot e_1 = (-1,0,1) - \frac{1}{2}(-1,1,0) =$

$= (-1,0,1) - (-\frac{1}{2}, \frac{1}{2}, 0) = (-\frac{1}{2}, -\frac{1}{2}, 1) =$

$= \frac{1}{2}(-1,-1,2)$

$e_3 = f_3 - \frac{\overbrace{<f_3, e_1>}^{0}}{<e_1, e_1>} \cdot e_1 - \frac{\overbrace{<f_3, e_2>}^{0}}{<e_2, e_2>} \cdot e_2 =$

$= (1,1,1) - 0 - 0 = (1,1,1)$

$<f_3, e_2> = <(1,1,1)(-\frac{1}{2}, -\frac{1}{2}, 1)> = -\frac{1}{2} - \frac{1}{2} + 1 =$

$= -1 + 1 = 0$

$e_1 = (-1,1,0)$

$e_2 = \frac{1}{2}(-1,-1,2)$

$e_3 = (1,1,1)$

$\{ \frac{1}{\sqrt{2}}(-1,1,0), \frac{1}{\sqrt{6}}(-1,-1,2), \frac{1}{\sqrt{3}}(1,1,1) \}$ reper ortonormat în $\mathbb{R}^3$

10.

⑧ a) $d_1: \begin{cases} x_2 = 0 \\ x_3 = 0 \end{cases}$

$d_1: \frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{0} = t \Rightarrow \begin{cases} x_1 = t \\ x_2 = 0 \\ x_3 = 0 \end{cases}$

$\Rightarrow u = (1,0,0)$

$A_1 (0,0,0)$

$d_2: \begin{cases} x_2 - 1 = 0 \\ x_1 = x_3 \end{cases} \Rightarrow \begin{cases} x_2 - 1 = 0 \\ x_1 - x_3 = 0 \end{cases}$

$x_3 = s \Rightarrow x_1 = s \Rightarrow x_2 = 1$

$d_2: \frac{x_1}{1} = \frac{x_2 - 1}{0} = \frac{x_3}{1} = s \Rightarrow \begin{cases} x_1 = s \\ x_2 = 1 \\ x_3 = s \end{cases}$

$\Rightarrow v = (1,0,1)$

$A_2 (0,1,0)$

$u \neq v \Rightarrow d_1$ și d_2 necoplanare $\Rightarrow \overrightarrow{A_1A_2} (0,1,0)$

$$\Delta c = \begin{vmatrix} \cancel{0} & \cancel{1} & \cancel{0} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 1 \cdot (-1)^2 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0$$

$P_1 (t,0,0) \in d_1$ nd

$P_2 (s,1,s) \in d_2$ nd

$\overrightarrow{P_1P_2} = (s-t, 1, s)$ $\quad u = (1,0,0)$

$\quad v = (1,0,1)$

$\langle \overrightarrow{P_1P_2}, u \rangle = 0 \Rightarrow s - t + 0 + 0 = s - t = 0 \Rightarrow s = t$

$\langle \overrightarrow{P_1P_2}, v \rangle = 0 \Rightarrow s - t + 0 + s = 0 \Rightarrow 2s - t = 0 \Rightarrow$

$\Rightarrow 2t - t = 0 \Rightarrow t = 0 \Rightarrow$

$\Rightarrow s = 0$

M. [signature]

$s = t = 0$

$P_1(0,0,0)$; $P_2(0,1,0)$; $\overrightarrow{P_1P_2}(0,1,0)$

$d: \dfrac{x_1 - 0}{0} = \dfrac{x_2 - 1}{1} = \dfrac{x_3 - 0}{0}$

$d: \dfrac{x_1}{0} = \dfrac{x_2 - 1}{1} = \dfrac{x_3}{0}$

b) $\text{dist}(d_1, d_2) = \text{dist}(P_1, P_2) = \|\overrightarrow{P_1P_2}\| = \sqrt{1^2} = 1$

⑨ a) $\det A = \frac{1}{7^3}\begin{vmatrix} -3 & -2 & 6 \\ 6 & -3 & 2 \\ 2 & 6 & 3 \end{vmatrix} = 1$

$$A\cdot A^T = \frac{1}{49}\begin{pmatrix} -3 & -2 & 6 \\ 6 & -3 & 2 \\ 2 & 6 & 3 \end{pmatrix}\begin{pmatrix} -3 & 6 & 2 \\ 6 & -3 & 6 \\ 2 & 2 & 3 \end{pmatrix} = I_3$$

$\left.\begin{array}{l} \det A = 1 \\ A\cdot A^T = I_3 \end{array}\right\} \Rightarrow f \in O(\mathbb{R}^3)$ de speța 1

$f = R_\varphi$, R_φ este rotația de unghi orientat φ și axa $<\{e_1\}>$

b) $\operatorname{Tr} A = 1 + 2\cos\varphi \Rightarrow \frac{3}{7} = 1 + 2\cos\varphi \Rightarrow 2\cos\varphi = -\frac{4}{7} \Rightarrow$

$\Rightarrow \cos\varphi = -\frac{2}{7} \Rightarrow \varphi = \arccos\left(-\frac{2}{7}\right) = \pi - \arccos\left(\frac{2}{7}\right)$

$$f(x) = x \Leftrightarrow \begin{cases} -3x_1 - 2x_2 + 6x_3 = 7x_1 \\ 6x_1 - 3x_2 + 2x_3 = 7x_2 \\ 2x_1 + 6x_2 + 3x_3 = 7x_3 \end{cases}$$

$$\Rightarrow \begin{cases} -10x_1 - 2x_2 + 6x_3 = 0 \\ 6x_1 - 10x_2 + 2x_3 = 0 \\ 2x_1 + 6x_2 - 4x_3 = 0 \end{cases} \qquad M = \begin{pmatrix} -10 & -2 & 6 \\ 6 & -10 & 2 \\ 2 & 6 & -4 \end{pmatrix}$$

$$\det M = \begin{vmatrix} -10 & -2 & 6 \\ 6 & -10 & 2 \\ 2 & 6 & -4 \end{vmatrix} = 2\cdot 2\cdot 2\begin{vmatrix} -5 & -1 & 3 \\ 3 & -5 & 1 \\ 1 & 3 & -2 \end{vmatrix} =$$

$= 8\cdot 0 = 0 \Rightarrow$

$\Rightarrow \operatorname{rg} M = 2$

înmulțit cu 5 I ec $\Rightarrow$

$$\begin{array}{r} -50x_1 - 10x_2 = -30x_3 \\ 6x_1 - 10x_2 = -2x_3 \quad (-) \\ \hline -56x_1 \ /= -28x_3 \end{array}$$

$\boxed{2x_1 = x_3}$

$3x_3 - 10x_2 = -2x_3$

$-10x_2 = -5x_3 \Rightarrow \boxed{2x_2 = x_3}$

$(x_1, x_2, x_3) = \left(\frac{1}{2}x_3, \frac{1}{2}x_3, x_3\right) = x_3\left(\frac{1}{2}, \frac{1}{2}, 1\right) =$

$= \frac{1}{2}x_3\left(1, 1, 2\right) \Rightarrow$ $<\{(1,1,2)\}>$ axa de rotație