Final formulas (summarized):

doesn't matter here

amplitude at bottom

$$H = \frac{\omega_{tot}^2 \cancel{e^{i\theta}}}{\sqrt{(\omega_{tot}^2 - \omega^2)^2 + \left[\frac{\gamma}{m}\omega \cdot \frac{\omega_{tot}}{\omega_1^2}\left(\frac{\omega^2}{\omega_2^2} - 1\right)\right]^2}} = \frac{B_2}{A}$$

amplitude cryostat

damping coefficient

$$\gamma = 2\pi \delta R \sigma \int_{-h/2}^{h/2} \left[\frac{\mu_0 M_0}{2\pi} \int_{-L/2}^{L/2} \frac{(z - z_1)}{r\sqrt{(b+r)^2 + (z-z_1)^2}} \cdot \left(-E_2(k) + \frac{(b+r)^2 + (z-z_1)^2}{(b-r)^2 + (z-z_1)^2} \cdot E_1(k) \right) dz_1 \right] dz$$

where $E_1(k) = E_1(m, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - m\sin(\theta)}}$

$$E_2(k) = E_2(m, \pi/2) = \int_0^{\phi} \sqrt{1 - m\sin^2(\theta)}\, d\theta$$

$$\omega_{tot}^2 = \frac{\omega_1^2 \omega_2^2}{\omega_1^2 + \omega_2^2}$$

~ = normal pencil

~ = important eq / eq. used later

~ = models / definitions

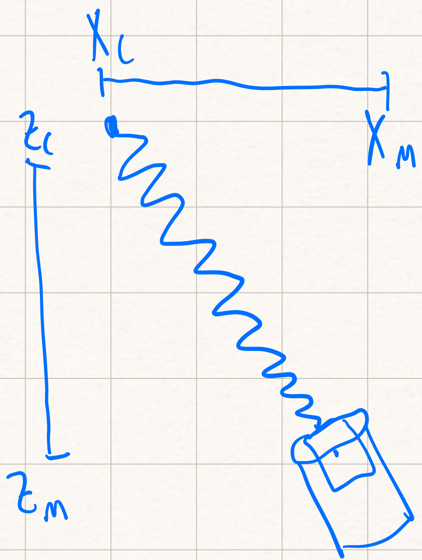
~ = side calculations / more simple calculations that help solve more advanced eq.

~ = divider / title of divider

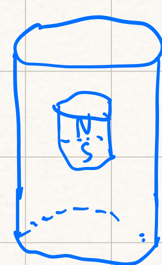
~ = notes for reader/me

- Calculation for the amplitude attenuation that the system creates (pg. 3-15)
- Calculation for the optimal spring lengths that create the biggest amplitude attenuation(pg.15-20)
- Calculation for how much the spring length changes due to change in temperature(pg. 20-23)
- The magnetic field of the system at any given distance from the system (pg. 24-30)
    - Used the textbook by D.J Craik "magnetism: principles and applications"

Calculations for pendulum + spring AND ECD

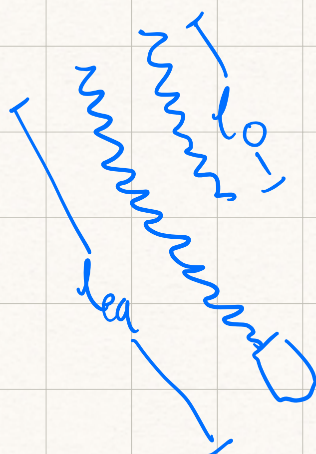


Spring pendulum system



ECD dampens any change in motion

1. Damping for radial motion:



$x_{eq} = 0$

$$\ell^2 = (\ell_0 + z_c - z_m)^2 + (x_m - x_c)^2$$

$$F_{sx} = -k \cdot \frac{(x_m + x_c)}{\ell} \cdot F_s \qquad \sin(\theta) = \frac{x_m - x_c}{\ell}$$

1st order taylor

$F_{sx} = -k\left(\frac{x_m - x_c}{l}\right)\cdot(l - l_0)$ make $l \approx l_{eq}$ because of approximation (1st order)

then, writing these terms in terms of x

$$\boxed{m\ddot{x}_m = -\gamma_z \dot{x}_m - k\frac{x_m - x_c}{l_{eq}}(l_{eq} - l_0)}$$

diff. equation radially

Damping for vertical motion:

$$F_{sz} = \cos(\theta) F_s$$

$$\cos(\theta) = \frac{\sqrt{l^2 - (x_m - x_w)^2}}{l} = \sqrt{1 - \sin^2(\theta)}$$

$$\hookrightarrow = \sqrt{1 - \frac{(x_m - x_c)^2}{l^2}} \approx 1$$

$$F_{sz} = k(l - l_0)\cdot\sqrt{1 - \frac{(x_m - x_c)^2}{l^2}}$$

Because we only need $O(x)$

$$F_{sz} = -k\sqrt{(l_0 + z_c - z_m)^2 + (x_m - x_c)^2}$$

z_0 = Uncompressed unstretched spring

$$-mg - kz_{eq} = 0$$

equation for equilibrium length of spring

$$z_{eq} = \frac{-mg}{k}$$

$$F_{sz} = -k\sqrt{(\ell_0 + z_c - z_m)^2 + (x_m - x_c)^2}$$

$$\Delta z_m = z_m - z_{eq}$$

$$z_m = z_{eq} + \Delta z_m$$

$$= \sqrt{(\ell_0 - z_{eq} + z_c - \Delta z_m)^2 + (x_m - x_c)^2}$$

$$= \sqrt{(\ell_{eq} + z_c - \Delta z_m)^2 + (x_m - x_c)^2}$$

$\ell_0 - z_{eq} = \ell_{eq}$

ℓ_0: Unstretched length of spring

$$\sqrt{l_{eq}^2 + 2l_{eq}(z_c - \Delta z_m} = l_{eq}\sqrt{1 + \frac{z(z_c + \Delta z_m)}{l_{eq}}} = l_{eq}\left[1 + \frac{(z_c - \Delta z_m)}{l_{eq}}\right]$$

binary approximation

$$F_{sz} = k l_{eq} + k(z_c - \Delta z_m)$$

putting in terms of x:

$$m\ddot{z}_m = -\gamma_z \dot{z}_m + k l_{eq} + k(z_c - \Delta z_m)$$

$$m\Delta\ddot{z}_m = -\gamma_z \dot{z}_m + k(z_c - \Delta z_m)$$

$$\boxed{m\Delta\ddot{z}_m = -\gamma_z \dot{z}_m + k(z_c - \Delta z_m)}$$

diff eq. vertically

remembering how to solve the diff. eq.

w_{ren} = renormalized freq.

$$m\ddot{x} + \gamma\dot{x} + m w_0^2 x = f_0 e^{iwt}$$

$$x = Be^{iwt} + B_1 e^{iw_{ren1}t} + B_2 e^{iwt}$$

$$mB(-w^2)e^{iwt} + iw\gamma B e^{iwt} + m w_0^2 B e^{iwt} = f_0 e^{iwt}$$

$$B\left(-w^2+w_0^2+\frac{i\gamma}{m}w\right)=\frac{f_0}{m}$$

$$B=\frac{f_0/m}{-w^2+w_0^2+\frac{i\gamma}{m}w}$$

$$x(t)=Be^{iwt}+B_1e^{iw_{rem1}t}+B_2e^{iw_{rem2}t}$$

$$x(t)=\frac{f_0/m\ e^{iwt}}{-w^2+w_0^2+i\frac{\gamma}{m}w}$$

$$\frac{f_0/m\left(-w^2+w_0^2-i\frac{\gamma}{m}w\right)}{\left(-w^2+w_0^2+i\frac{\gamma}{m}w\right)\left(-w^2+w_0^2-i\frac{\gamma}{m}w\right)}$$

$$=\frac{\frac{f_0}{m}\left(-w^2+w_0^2-i\frac{\gamma}{m}w\right)}{\left(-w^2+w_0^2\right)^2+\left(\frac{\gamma}{m}w\right)^2}$$

$$r=\sqrt{x^2+y^2}$$

$$\frac{x}{r}=\cos(\theta)\qquad\frac{y}{r}=\sin(\theta)$$

$$\tan(\theta)=\frac{y}{x}$$

$$x+iy=r(\cos(\theta)+i\sin(\theta))=re^{i\theta}$$

$$r^2=\left(-w^2+w_0^2\right)^2+\left(\frac{\gamma}{m}w\right)^2$$

$$\tan(\theta)=\frac{-\frac{\gamma}{m}w}{-w^2+w_0^2}$$

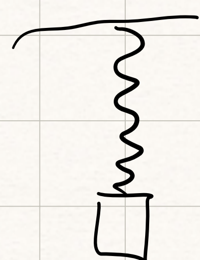
$$= \frac{\frac{f_0}{m}\left(-w^2+w_0^2-i\frac{\gamma}{m}w\right)}{(-w^2+w_0^2)^2+(\frac{\gamma}{m}w)^2}$$

$$= \frac{f_0}{m}\frac{(r\cos(\theta)+ir\sin(\theta))}{r^2} = \frac{f_0}{m}\cdot\frac{1}{r}e^{i\theta}$$

$$\frac{f_0/m}{-w^2+w_0^2+i\frac{\gamma}{m}w} = \frac{f_0/m \cdot e^{i\theta}}{\sqrt{(-w^2+w_0^2)^2+(\frac{\gamma}{m}w)^2}} \qquad \tan(\theta) = \frac{-\frac{\gamma}{m}w}{-w^2+w_0^2}$$

$$x(t) = \frac{f_0/m}{\sqrt{(-w^2+w_0^2)^2+(\frac{\gamma}{m}w)^2}}\,\underbrace{Re(e^{iwt+i\theta})}_{\cos(wt+\theta)}$$

going back to vertical oscillations:



system

$$m\Delta\ddot{z}_m + \gamma_z\Delta\dot{z}_m + k\Delta z_m = kz_c$$

$z_c = A\cos(wt) = Re(Ae^{iwt})$ $\qquad \Delta z_m = Be^{iwt}$

$$m\Delta\ddot{z}_m + \gamma_z\Delta\dot{z}_m + k\Delta z_m = k\underbrace{z_c}_{Ae^{iwt}}$$

z_c = motion cryostat

$f_0 = kA$

$$\Delta z_M(t) = \frac{k \cdot \frac{1}{m} A}{\sqrt{(-w^2+w_0^2)^2 + (\frac{\gamma}{m} w)^2}} e^{i\theta + i w t}$$

$$H = \frac{|B|}{A} = \frac{w_0^2}{\sqrt{(-w^2+w_0^2)^2 + (\frac{\gamma}{m} w)^2}}$$

assuming $w_0 \ll w$

For no friction: $\frac{|B|}{A} = \frac{w_0^2}{-w_0^2+w^2}$ like the montenoibe paper

approximation that $M_{eco} \approx 0$

Calculating how springs in series work:

Spring in series:

$$\Delta X_{tot} = \Delta x_1 + \Delta x_2$$

$$\Delta x_1 = \frac{k_2}{k_1} \Delta x_2$$

$$k_1 \Delta x_1 = k_2 \Delta x_2$$

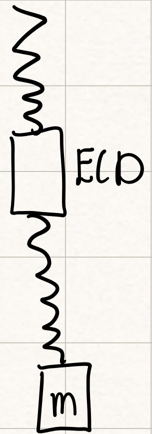
$$\Delta X_{tot} = \Delta x_2 \left(\frac{k_2}{k_1} + 1\right)$$

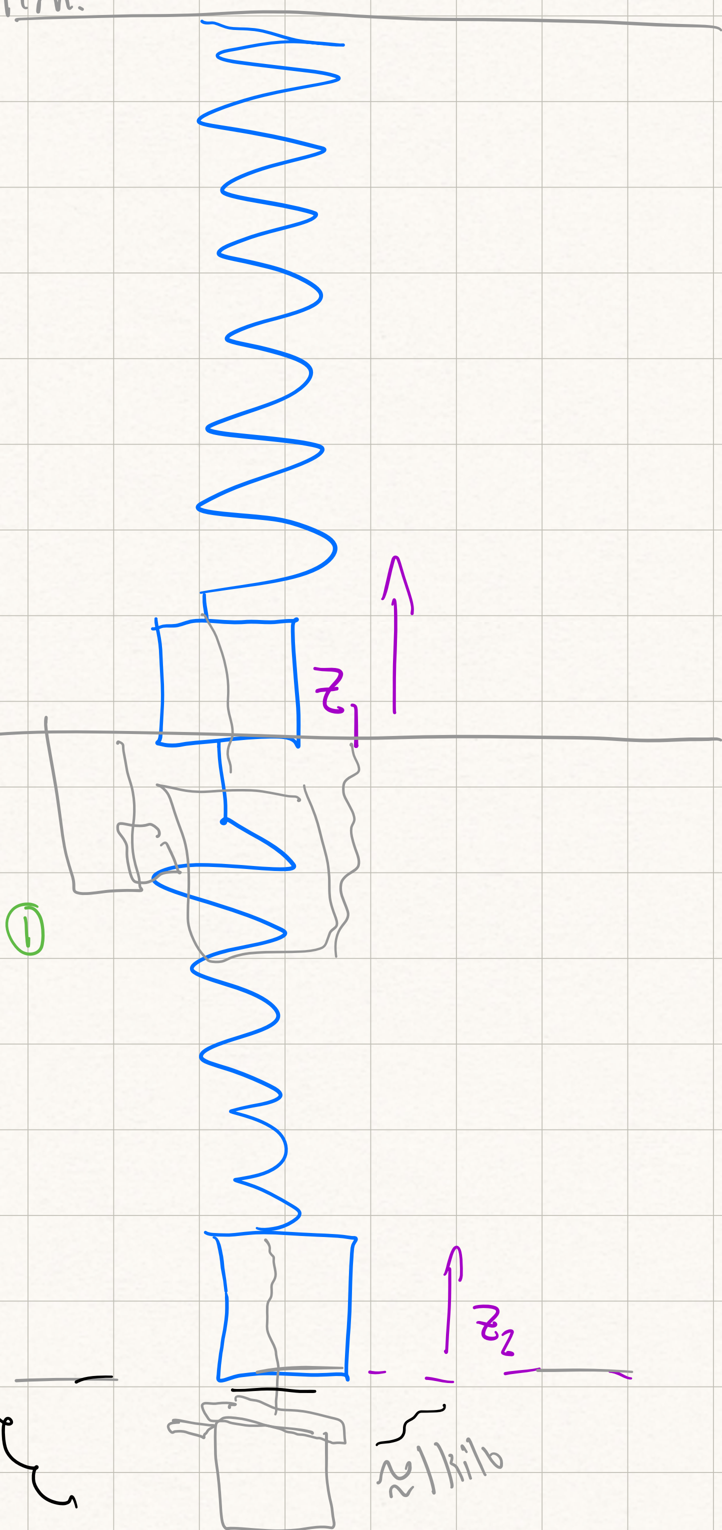
$$\Delta X_2 = \frac{\Delta x_{tot}}{\frac{k_2}{k_1} + 1}$$

$$F_{final} = k_2 \Delta X_2 = k_2 \cdot \frac{X_{tot}}{\frac{1}{k_1} + \frac{1}{k_2}} = k_{tot} + \Delta x_{tot}$$

$$\frac{1}{k_{tot}} = \frac{1}{k_1} + \frac{1}{k_2} \quad \Big| \quad \frac{1}{k_{tot}} > 1/k_1 \text{ or } 1/k_2$$

Now, imagining a more realistic system:





equilibrium conditions:

$$\begin{cases} -k_1 z_1 + k_2 (z_2 - z_1) = 0 & (1) \\ -k_2 (z_2 - z_1) - mg = 0 & (2) \end{cases}$$

$$k_2 z_{2eq} = (k_1 + k_2) z_{1eq}$$

$$z_{1eq} = \frac{k_2}{k_2 + k_1} z_{2eq}$$

$$z_{2eq} - z_{1eq} = z_{2eq}\left(1 - \frac{k_2}{k_2+k_1}\right) = \frac{k_1}{k_2+k_1} z_{2eq}$$

2: $$-k_2 \cdot \frac{k_1}{k_2+k_1} \cdot z_{2eq} = mg$$

$$-\frac{k_1 k_2}{k_1+k_2} z_{2eq} = mg$$

$$z_{2eq} = \frac{-mg}{\left(\frac{k_1 k_2}{k_1+k_2}\right)}$$

I assumed that this is ≈ 0

$m_{edm} \ddot{z}_1$

z cryostat

$$-k_1(z_1 - z_c) + k_2(z_2 - z_1) - \gamma \dot{z}_1 = 0 \quad (3)$$

$$-k_2(z_2 - z_1) - mg = m\ddot{z}_2 \quad (4)$$

$$z_1 = z_{1eq} + \Delta z_1$$

$$z_2 = z_{2eq} + \Delta z_2$$

$$z_{2eq} = -\left(\frac{k_1+k_2}{k_1 k_2}\right) mg$$

$$z_{1eq} = \frac{k_2}{k_1+k_2} \cdot z_{2eq} = -\frac{mg}{k_1}$$

found this earlier, rewriting together for clarity

$$z_{2eq} - z_{1eq} = -mg\left(\frac{k_1+k_2}{k_1k_2} - \frac{1}{k_1}\right) = -\frac{mg}{k_2}$$

④ $$k_2(z_{2eq} - z_{1eq} + \Delta z_2 - \Delta z_1) - mg = m\Delta\ddot{z}_2$$

$$\downarrow$$

$$-k_2(\Delta z_2 - \Delta z_1) = m\Delta\ddot{z}_2 \quad ⑤$$

③ $$-k_1(z_{1eq} + \Delta z_1 - z_c) + k_2(z_{2eq} - z_{1eq} + \Delta z_2 - \Delta z_1) - \gamma\Delta\dot{z}_1 = 0$$

$$\downarrow$$

$$\cancel{-k_1\left(\frac{-mg}{k_1}\right)} + \cancel{k_2\left(-\frac{mg}{k_2}\right)} - k_1(\Delta z_1 - z_c) + k_2(\Delta z_2 - \Delta z_1) - \gamma\Delta\dot{z}_1 = 0$$

$$\downarrow$$

$$-(k_1+k_2)\Delta z_1 + k_2\Delta z_2 - \gamma\Delta\dot{z}_1 = -k_1 z_c \quad ⑥$$

$$z_c = Ae^{i\omega t}$$

$$\Delta z_1 = B_1 e^{i\omega t}$$

$$\Delta z_2 = B_2 e^{i\omega t}$$

⑥ $$\left[-(k_1+k_2) - i\omega\gamma\right]B_1 + k_2 B_2 = -k_1 A$$

$$⑤ \quad k_2 B_1 + (-k_2 + m\omega^2) B_2 = 0$$

$$B_1 = B_2 \cdot \left(\frac{k_2 - m\omega^2}{k_2} \right) \quad ⑦$$

$$k_1 = m\omega_1^2$$
$$k_2 = m\omega_2^2$$

$$\left[k_2 + \frac{(k_2 - m\omega^2)}{k_2} (-)(k_1 + k_2) + \frac{(k_2 - m\omega^2)}{k_2} (-i\omega\gamma) \right] B_2 = -k_1 A$$

$$B_2 \left[\omega_2^2 - (\omega_1^2 + \omega_2^2)\left(1 - \frac{\omega^2}{\omega_2^2}\right) - i \frac{\gamma}{m} \omega \left(1 - \frac{\omega^2}{\omega_2^2}\right) \right] = -\omega_1^2 A$$

$$B_2 = \left[-\omega_1^2 + \frac{(\omega_1^2 + \omega_2^2)}{\omega_2^2} \cdot \omega^2 - i \frac{1}{m} \omega \left(1 - \frac{\omega^2}{\omega_2^2}\right) \right] = -\omega_1^2 A$$

$$B_2 \cdot \frac{(\omega_1^2 + \omega_2^2)}{\omega_2^2} \left[\omega^2 - \frac{\omega_1^2 \omega_2^2}{\omega_1^2 + \omega_2^2} - i \frac{\gamma}{m} \omega \cdot \frac{\omega_2^2}{\omega_1^2 + \omega_2^2} \left(1 - \frac{\omega^2}{\omega_2^2}\right) \right] = -\omega_1^2 A$$

$$\omega_{tot}^2 = \frac{\omega_1^2 \omega_2^2}{\omega_1^2 + \omega_2^2}$$

$$\frac{1}{k_{tot}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_{tot} = m\omega_{tot}^2$$

$$k_1 = m\omega_1^2$$
$$k_2 = m\omega_2^2$$

$$\frac{1}{\omega_{tot}^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}$$

$$B_2\left[\omega^2 - \omega_{tot}^2 - \frac{i\gamma}{m}\omega \cdot \frac{\omega_{tot}^2}{\omega_1^2}\left(1 - \frac{\omega^2}{\omega_2^2}\right)\right] = -\omega_{tot}^2 \cdot A$$

$$\frac{B_2}{A} = \frac{-\omega_{tot}^2}{\omega^2 - \omega_{tot}^2 - i\frac{\gamma}{m}\omega\frac{\omega_{tot}^2}{\omega_1^2}\left(1 - \frac{\omega^2}{\omega_2^2}\right)}$$

$$\gamma_{eff} = \gamma\frac{\omega_{tot}^2}{\omega_1^2}\left(1 - \frac{\omega^2}{\omega_2^2}\right)$$

$$\frac{B_2}{A} = H = \frac{\omega_{tot}^2 \, e^{i\theta}}{\sqrt{\left(\omega_{tot}^2 - \omega^2\right)^2 + \left[\frac{\gamma}{m}\omega \cdot \frac{\omega_{tot}}{\omega_1^2}\left(\frac{\omega^2}{\omega_2^2} - 1\right)\right]^2}}$$

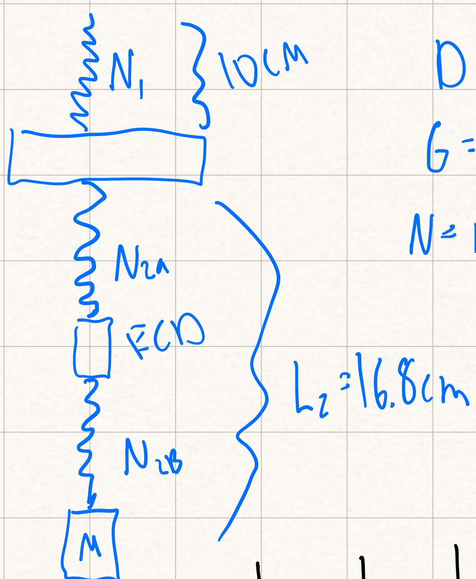
Calculating changing variable spring

finding where Eddy damper should be located

Assumptions: γ constant
$\varkappa$ constant

$$k = \frac{Gd^4}{8D^3N} = \frac{\varkappa}{N} \qquad \varkappa = \frac{Gd^4}{8D^3}$$

d = diamter spring wire
D = diameter spring system
G = modulus of elastivity material
N = number of coils



$$\frac{1}{k_{2tot}} = \frac{1}{k_{2a}} + \frac{1}{k_{2B}} = \frac{N_{2a}}{\varkappa} + \frac{N_{2B}}{\varkappa}$$

$$\frac{1}{k_{2tot}} = \frac{N_{2a} + N_{2B}}{\varkappa}$$

$$\Delta X_{stretched} = \frac{2Mg}{\varkappa} \cdot (N_{2a} + N_{2B})$$

$$N_{2tot} = N_{2a} + N_{2b}$$

$$N_{2tot} \cdot d + \Delta X_{stretched} = L_2 - L_{ECD}$$

$$N_{2tot} \cdot d + \frac{2Mg}{\mathcal{H}} \cdot N_{2tot} = L_2 - L_{ECD}$$

$$N_{2tot}\left(d + \frac{2Mg}{\mathcal{H}}\right) = L_2 - L_{ECD} \qquad ①$$

$N_{2tot} = N_{2a} + N_{2b} \Rightarrow$ fixed by ①

From Matlab: $N_{2tot} = 52.3$

$N_{2A} + N_{2B} = 52.3$

$N_{2B} = 52.3 - N_{2A}$

Need to optimized with respect to N_{2a}

$$(N_{2a} + N_{2b})\left(d + \frac{2Mg}{\mathcal{H}}\right) = L_2 - L_{ECD}$$

Solving for N_1:

$$N_1 d = \Delta X_{1stretched} = L_1$$

$$k_1 \cdot \Delta X_{stretched} = 2Mg$$

$$\Delta X_{1stretched} = \frac{2Mg}{k_1} = \frac{2Mg}{\mathcal{H}} \cdot N_1$$

$$N_1 d + \frac{2Mg}{\mathcal{H}} \cdot N_1 = L_1$$

$$N_1 \left(d + \frac{2Mg}{\mathcal{H}}\right) = L_1$$

From matlab: $N_1 = 48.43$

computing H:

effectively N_1 & N_{2a} act together

$$k_{1eff} = \frac{\mathcal{H}}{N_1 + N_{2a}}$$

$$W_{1eff} = \sqrt{\frac{\mathcal{H}}{M(N_1 + N_{2a})}}$$

$$k_2 = \frac{\mathcal{H}}{N_{2b}} = \frac{\mathcal{H}}{N_{2tot} - N_{2a}}$$

$$W_2 = \sqrt{\frac{\mathcal{H}}{(N_{2tot} - N_{2a})M}}$$

Calculating optimal spring length assuming there is no stage in the middle

- This is to check if the ECD should be on the upper stage

N_1 = number of coils above ECD

N_2 = number of coils below ECD

$$\Delta X_{stretched} = \frac{2Mg}{\mathcal{H}} \cdot (N_1 + N_2)$$

$$N_1 d + N_2 d + \Delta X_{stretched} = L_1 + L_2 - L_{ECD}$$

$$N_{sum}\left(2d + \frac{2mg}{\mathcal{H}}\right) = L_1 + L_2 - L_{ECD}$$

using Matlab: $N_{sum} = 67.87$

$$k_{1eff} = \frac{\mathcal{H}}{N_{sum} - N_2}$$

$$W_{1eff} = \sqrt{\frac{\mathcal{H}}{m(N_{sum} - N_2)}}$$

$$k_{2eff} = \frac{\mathcal{H}}{N_2}$$

H calculation is the same as before

$$W_{2eff} = \sqrt{\frac{\mathcal{H}}{m \cdot N_2}}$$

Website: https://www.sodemann-federn.de/ep4200372250s

Calculating how accurate the k approximation is:

Using a spring company and their measured spring constants:

formula for k: $k = \frac{Gd^4}{8D^3N}$ | $N = \frac{Gd^4}{8D^3k}$

givens: G(302 stainless steel) = 193 GPa

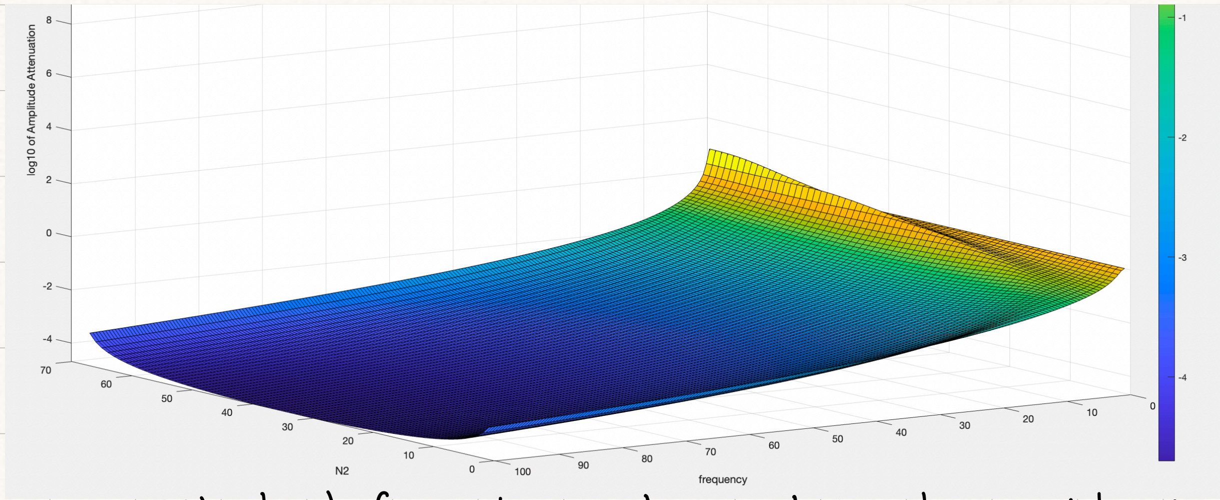
$d = 0.94\,mm$ $k = 0.17\,\frac{N}{mm}$

$D = 10.67\,mm$

$N_{calculated} = 91$ coils

L_o = unloaded length = 57.15 mm

$N_{real} = \frac{L_o}{d} = 63$ coils



results, best for springs on top and bottom be equal lengths

Calculating the ideal length of spring

How steel 302 changes at low temperatures:

coefficient expansion steel[302]: $17.3 \cdot 10^{-6} \frac{1}{C^\circ}$ | $9.6 \cdot 10^{-6} \frac{1}{F^\circ}$

Alloys

T	Steel, SAE 52100 [b]		Steel, AISI 301 [c]		Steel, AISI 302 [d]	
	$10^5 \frac{L_{293}-L_T}{L_{293}}$	$\frac{10^6}{L_{293}} \frac{dL}{dT}$	$10^5 \frac{L_{293}-L_T}{L_{293}}$	$\frac{10^6}{L_{293}} \frac{dL}{dT}$	$10^5 \frac{L_{293}-L_T}{L_{293}}$	$\frac{10^6}{L_{293}} \frac{dL}{dT}$
deg K		*deg⁻¹ K*		*deg⁻¹ K*		*deg⁻¹ K*
0					[a] 316	0
10					316	[a] 0.04
20					316	[a] .09
30					316	.5
40					315	1.4
50					313	2.7
60					309	4.3
70					304	6.5
80	198	6.2			296	8.3
90	192	6.6	267	9.6	287	9.6
100	185	6.9	257	10.1	277	10.4
120	171	7.3	236	11.0	255	11.6
140	156	7.6	213	11.8	231	12.4
160	140	8.2	188	12.5	206	13.2
180	123	8.9	163	13.1	179	13.8
200	105	9.7	136	13.6	151	14.5
220	84.4	10.6	109	14.1	121	15.2
240	62.4	11.3	80.1	14.5	89.6	16.0
260	39.4	11.7	50.5	15.0	57.0	16.7
273	24.0	11.9	30.9	15.3	35.0	17.2
280	15.6	12.0	20.2	15.4	22.9	17.4
293	0.0	12.1	0.0	15.7	0.0	17.9
300	−8.5	12.1	−11.0	15.8	−12.7	18.1
Sources of above data	Werner 1924		Furman 1950		Beenakker and Swenson 1955	
Other refs.			Lucks and Deem 1958			

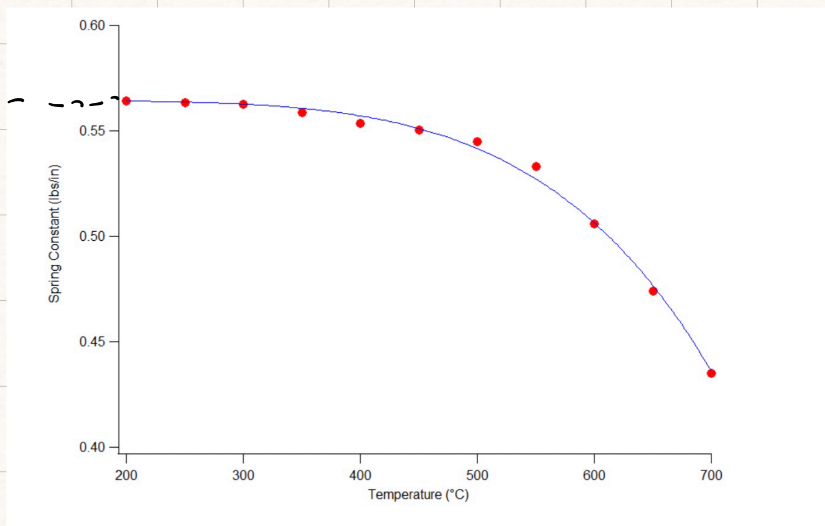
average thermal coefficient form 300-0 K°: $10.76 \cdot 10^{-6}$

app change in length: $\Delta L = \alpha L \Delta T$

$$\Delta L = 10.76 \cdot 10^{-6} \cdot (-293) L = -0.00315 L$$

$$L_{new} = \Delta L + L_{in} = 0.996 L_{orig}$$

But, spring constant supposedly decreases with temperature



I assume the spring constant stays the same at low temperatures

Continuation:

Using Youngs module to find change in spring constant

youngs modulus calculates the stiffness of the material

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L/L}$$

σ = stress
ε = strain
ΔL = change in length
F = force exerted
A = surface area
L = initial length

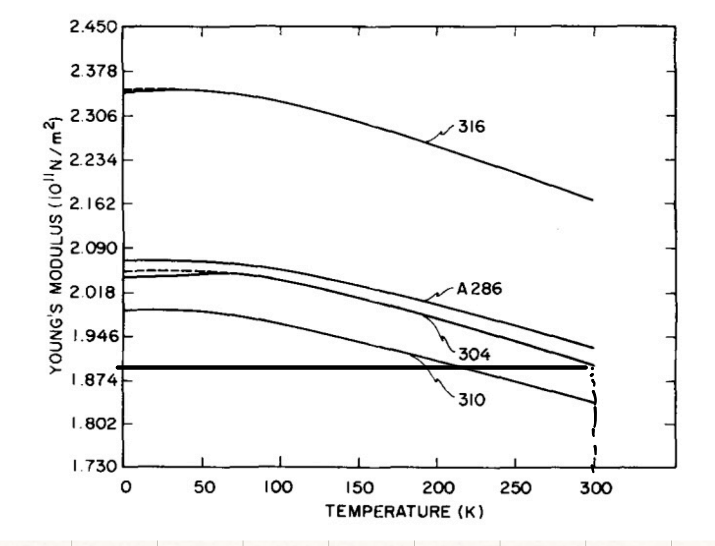
$$F = k\Delta L$$

$$E = \frac{k\Delta L \cdot L}{A\Delta L} = \frac{kL}{A} \quad \Big| \quad k = \frac{EA}{L}$$

This means that $k \propto E$

For Steel 316, 310, A286, 304

- The most similar steel is 304



$$\frac{E_f}{E_i} = \frac{2.08}{1.9} = 1.0947$$

This means that $\frac{k_f}{k_i} = 1.0947$

$\frac{k_f}{k_i} = \frac{\Delta x_i}{\Delta x_f}$, so $\frac{\Delta x_f}{\Delta x_i} = 0.913$

$\Delta x_f = 0.913 \cdot \Delta x_i$

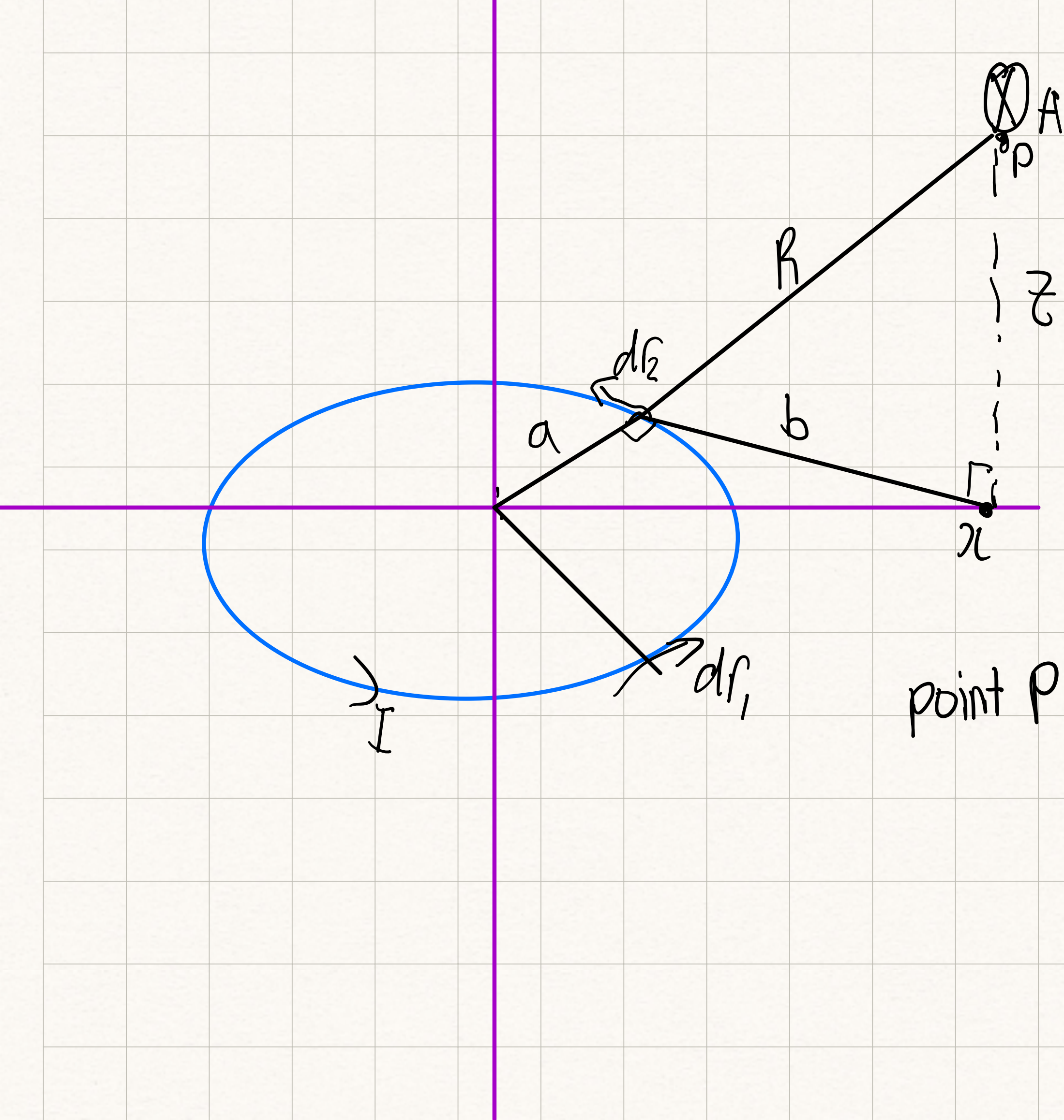
For top spring:

- $\Delta x_i \approx 2cm$ $\Delta x_f = 1.8cm$
- The spring gets 2mm shorter when cooled to $4^\circ K$

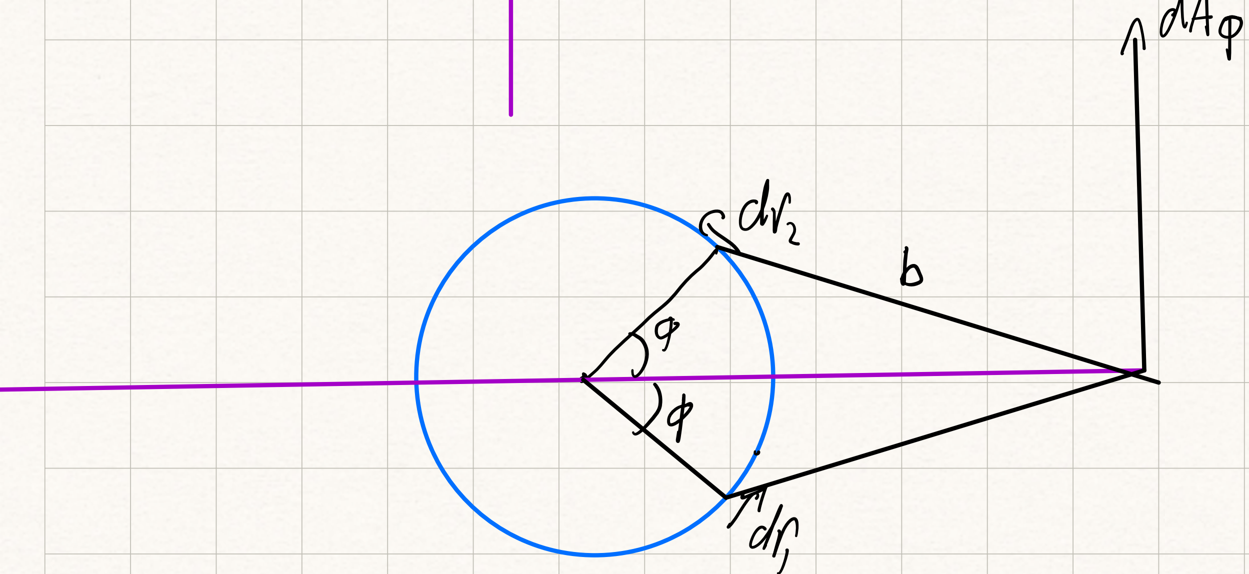
For bottom spring:

- around 12 cm of space
- $\Delta x_i \approx 5cm$, $\Delta x_f = 4.5cm$
- The spring gets 5mm shorter

magnetic field from circular loop



point P at (x, z)



$B = \nabla \times A$

A = vector potential

$$B = \frac{\mu_0}{4\pi}\int_C \frac{i\, dr \times 1_r}{R^2}$$

when $1_r = R$,

$$A = \frac{\mu_0}{4\pi}\int \frac{i\, dr}{R}$$

make $A' = \frac{4\pi}{\mu_0} A = \int_C \frac{i\, dr}{R}$

integrate around semicircle:

$$A'_\phi = 2\int_0^\pi i \cdot \frac{\cos(\phi)\, dr}{R}$$

$dr = a\, d\phi$ $\quad R^2 = z^2 + b^2$ $\quad b^2 = a^2 + x^2 - 2ax\cos(\phi)$

$$A'_\phi = 2i\int_0^\pi \frac{a\cos(\phi)\, d\phi}{(a^2+x^2+z^2-2ax\cos(\phi))^{1/2}}$$

To make it easier using elliptical integrals:

$\phi = \pi + 2\theta$ so that

$$\cos(\phi) = \cos(\pi + 2\theta) = \cos(\pi)\cos(2\theta) - \sin(\pi)\sin(2\theta)$$

$$= -\cos(2\theta) = -2(\cos^2(\theta) - 1) = -(1 - 2\sin^2(\theta))$$

$$d\phi = 2\,d\theta$$

$$A'_\phi = 4ai \int_0^{\pi/2} \frac{(2\sin^2(\theta) - 1)\,d\theta}{[a^2 + x^2 + z^2 - 2ax(2\sin^2(\theta) - 1)]^{1/2}}$$

looking at denominator more closely:

$$k^2 = \frac{4ax}{(a+x)^2 + z^2}$$

$$(a+x)^2 + z^2 - 4ax\sin^2(\theta) = [(a+x)^2 + z^2](1 - k^2\sin^2(\theta))$$

This means: $$A'_\phi = \frac{4ai}{[(a+x)^2 + z^2]^{1/2}} \int_0^{\pi/2} \frac{(2\sin^2(\theta) - 1)\,d\theta}{(1 - k^2\sin^2(\theta))^{1/2}}$$

$$\frac{1}{(1-k^2\sin^2(\theta))^{1/2}} - (1-k^2\sin^2(\theta))^{1/2} = \frac{1-(1-k^2\sin^2(\theta))}{(1-k^2\sin^2(\theta))^{1/2}}$$

$$\frac{\sin^2(\theta)}{(1-k^2\sin^2(\theta))^{1/2}} = \frac{1}{k^2}\cdot\frac{1}{(1-k^2\sin^2(\theta))^{1/2}} - \frac{1}{k^2}(1-k^2\sin^2(\theta))^{1/2}$$

$$A_\phi = \frac{\mu_0 i}{2\pi}\left(\frac{a}{r}\right)^{1/2} k\left[\left(\frac{2}{k^2}-1\right)\int_0^{\pi/2}\frac{d\theta}{(1-k^2\sin^2(\theta))^{1/2}} - \frac{2}{k^2}\int_0^{\pi/2}(1-k^2\sin^2(\theta))^{1/2}d\theta\right]$$

$$= \frac{\mu_0 i}{2\pi}\left(\frac{a}{r}\right)^{1/2}\cdot k\left[\left(\frac{2}{k^2}-1\right)K(k) - \frac{2}{k^2}E(k)\right]$$

A_ϕ is only component, $B = \nabla \times A$

there can be no ϕ component

$$B_r = -\frac{1}{r}\cdot\frac{\partial}{\partial z}(rA_\phi) + \frac{1}{r}\frac{\partial}{\partial\phi}A_z = -\frac{\partial}{\partial z}A_\phi$$

$$B_z = -\frac{1}{r}\frac{\partial}{\partial\phi}A_r + \frac{1}{r}\frac{\partial}{\partial r}rA_\phi = \frac{1}{r}\cdot\frac{\partial}{\partial r}(rA_\phi)$$

property elliptical integral:

$$\frac{dK}{dk} = \frac{E}{k(1-k^2)} - \frac{K}{k} \qquad \frac{dE}{dk} = \frac{E}{k} - \frac{K}{k}$$

$$\frac{\partial K}{\partial x} = \frac{\partial K}{\partial k} \cdot \frac{\partial k}{\partial x}$$

$$\frac{\partial k}{\partial z} = \frac{-z k^3}{4ax} \qquad \frac{\partial k}{\partial x} = (a^2 - x^2 + z^2)\frac{k^3}{8ax^2}$$

then:

$$B_x = \frac{\mu_0 i}{2\pi} \cdot \frac{z}{x} \cdot \frac{1}{[(a+x)^2 + z^2]^{1/2}} \left[E \frac{a^2 + x^2 + z^2}{(a-x)^2 + z^2} - K \right]$$

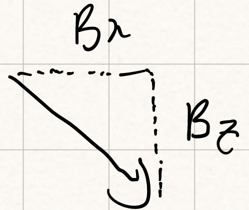
$$B_z = \frac{\mu_0 i}{2\pi} \cdot \frac{1}{[(a+x)^2 + z^2]^{1/2}} \left[E \frac{a^2 - x^2 - z^2}{(a-x)^2 + z^2} + K \right]$$

hen for cylindrical current sheets:

$$B_x = \frac{\mu_0 I}{2\pi} \int_{-l/2}^{l/2} \frac{z - z'}{x\left[(a+x)^2 + (z-z')^2\right]^{1/2}} \left[E(k) \cdot \frac{a^2 + x^2 + (z-z')^2}{(a-x)^2 + (z-z')^2} - K(k) \right] dz'$$

$$B_z = \frac{\mu_0 I}{2\pi} \int_{-l/2}^{l/2} \frac{1}{\left[(a+x)^2 + (z-z')^2\right]^{1/2}} \left[\frac{a^2 - x^2 - (z-z')^2}{(a-x)^2 + (z-z')^2} E(k) + K(k) \right] dz'$$

$$k^2 = \frac{4ax}{(a+x)^2 + (z-z')^2}$$



$$\|B\| = \sqrt{B_z^2 + B_x^2}$$

3.5 +