Optimal multilayer filter design using real coded genetic algorithms

E. Michielssen
S. Ranjithan
R. Mittra

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Abstract: A novel approach for designing optimal multilayer filters based on a real-coded genetic algorithm is presented. Given the total number of layers in the filter, as well as the electrical properties of the materials constituting each layer, the algorithm iteratively constructs multilayers whose frequency response closely matches a desired frequency response. In contrast to existing iterative techniques, this method does not require a preliminary design using classical techniques. Also, the design procedure is independent of the nature of the multilayer as well as the characteristics of the incident and substrate media. The algorithm is applied to the design of various lowpass and highpass optical filters, operating between practical terminal conditions. The performance of the resulting designs matches or improves on that for filters that were synthesised using semiclassical techniques.

1 Introduction

Multilayer filters have been designed using a variety of approaches, which include circuit-analogue methods [1–4], coefficient-matching techniques [5–7], and most recently the turning-point method [8, 9]. The latter technique seems to overcome certain drawbacks of the circuit-analogue and coefficient-matching techniques, since the filters designed using this technique have a smaller passband ripple, larger cut-off and cut-on rates, and a higher rejection than those designed using the classical methods. The turning point method is an iterative technique, as is the method presented in this paper. Starting from an initial design, which can either be a classical stack or a filter obtained using the coefficient-matching method, the turning-point technique iteratively refines the design by altering the thicknesses of the layers, its goal being the design of a filter with equiripple response. The turning-point technique was applied by Chen [9] to design 17 and 18-layer optimal filters which operate between an idealised incident medium and a substrate, as well as towards the synthesis of filters which operate between practical terminal conditions. Filters for operation between practical terminal conditions typically require a two-step design process that involves the synthesis of a prototype operating between idealised terminal conditions, followed by the design of a matching circuit. The design of an elaborate matching system not only complicates the design, but it also interferes with the primary objective of designing filters with a minimum number of layers. The turning-point method allows for the optimisation of the filter characteristic after the matching system has been added. Chen [9] used this direct technique for synthesising highpass filters that operate between practical terminal conditions. However, Chen [9] preferred a technique proposed by Seeley [2] for synthesising multilayers with inherent matching capability in combination with the turning-point method for the design of lowpass filters.

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E. Michielssen and R. Mittra are with the Electromagnetic Communication Laboratory, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, 1406 W. Green Street, Urbana, 12 61801-2991, USA

S. Ranjithan is with the Department of Civil Engineering, University of Illinois at Urbana-Champaign, 1406 W. Green Street, Urbana 12 61801-2991, USA

In this paper, a novel iterative technique is presented for designing multilayer filters, based on a real-coded genetic algorithm. In general, genetic algorithms [10–12] are probabilistic combinatorial optimisation techniques in which genetic operators, which are derived from the natural selection processes occurring in nature, guide a population of potential solutions towards optimal solutions. Genetic algorithms have been used for designing digital filters [13], but their application in this area is new. The design approach presented in this paper offers several advantages over the classical design techniques, as well as over the turning-point method. First, the probabilistic nature of the genetic algorithm allows for random starting configurations, instead of a preliminary design as required in the turning-point method, for the iterative process. The absence of a preliminary design stage greatly simplifies the process.

The second advantage of the technique is its versatility. This technique can be applied directly to design filters which operate between practical terminal conditions. A separate design of a matching circuit becomes superfluous since the algorithm will, given the number of available layers, automatically optimise their thicknesses for optimal operation between arbitrary media. Also, the technique presented herein can be used without modification to design lowpass, highpass, and bandpass filters.

The third advantage of the present technique is its simplicity. The design process only requires the formulation of an objective function to guide the genetic algorithm towards the optimal solution. This objective function is solely determined by the reflection characteristics of the multilayer, and no derivatives have to be calculated as in the Newton–Raphson-based turning-point method. In this paper, only the design of multilayer filters at normal incidence is considered. In addition, we assume all materials to be nondispersive and lossfree. However, these restrictions are not due to any fundamental limitation of the application of the technique, and will be addressed in a future paper.

2 Formulation

Consider the multilayer structure shown in Fig. 1. The multilayer consists of N alternating layers of two avail-

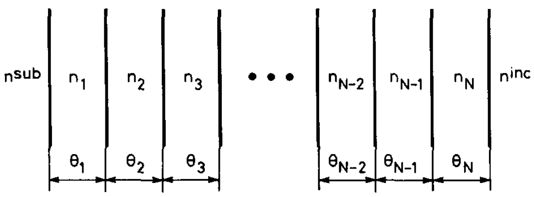

Fig. 1 *Schematic representation of multilayer structure under investigation*

able materials with fixed effective refractive indices n^H and n^L. The multilayer is bounded by the incident medium with refractive index n^{inc} and the substrate medium with refractive index n^{sub}. The goal is to find the thickness of each layer, such that the frequency response of the resulting multilayer closely matches a desired response. The desired response may correspond to either a lowpass, highpass or bandpass characteristic. Given the dispersion-free character of the materials involved, the electrical thicknesses $\tilde{\theta}_1(f)$ through $\tilde{\theta}_N(f)$ of the layers can be expressed in terms of the normalised frequency f as

$$\tilde{\theta}_j(f) = f\theta_j \tag{1}$$

where θ_1 through θ_N are the phase thicknesses at $f = 1$. The reflection coefficient of the multilayer $R = R_N$, where R_N is computed iteratively as follows:

$$\begin{aligned} R_k &= \frac{\tilde{R}_k + R_{k-1}e^{-2j\tilde{\theta}_k(f)}}{1 + \tilde{R}_k R_{k-1}e^{-2j\tilde{\theta}_k(f)}} \qquad 1 \leqslant k \leqslant N \\ &= \frac{n^{sub} - n_1}{n^{sub} + n_1} \qquad k = 0 \end{aligned} \tag{2}$$

where

$$\begin{aligned} \tilde{R}_k &= \frac{n_k - n_{k+1}}{n_k + n_{k+1}} \qquad 1 \leqslant k < N \\ &= \frac{n_N - n^{inc}}{n_N + n^{inc}} \qquad k = N \end{aligned} \tag{3}$$

and n_k is the refractive index of layer k, which is either n^H or n^L.

The algorithm is guided towards the optimal design through the formulation of a proper objective or 'fitness' function. Given a set of frequencies f_m^+ ($m = 1, \ldots, N^+$) at which the filter transmission needs to be maximised, and a set of frequencies f_m^- ($m = 1, \ldots, N^-$) at which the reflection needs to be maximised, a fitness function F suited for the present problem is formulated as

$$\begin{aligned} F = &\sum_{m=1,N^+} w_m^+(1 - |R(f_m^+)|^2)^\alpha \\ &+ \sum_{m=1,N^-} w_m^- |R(f_m^-)|^{2\alpha} \end{aligned} \tag{4}$$

Here, w_m^+ and w_m^- are coefficients that weigh the relative importance of the filter performance for the different frequencies. $\alpha > 1$ is a parameter that expresses the preference of a design with a more or less uniform response over that with a response characterised by a large ripple. Numerical experimentation has shown that α should be chosen between 2 and 4. The relative values of the weighting coefficients can be tailored to yield the desired frequency response and can be used to trade off the filter performance in one part of the frequency spectrum against that in another part. A smaller ripple in the transmission band can be achieved at the cost of a smaller rejection by assigning larger weighting coefficients to the sampling points in the transmission band, and vice versa. The number of sampling points in a specific part of the spectrum also plays an important role. Especially near cut-off or cut-on, it may be preferable to choose several points with moderate weighting coefficients, rather than just a few points with large weighting coefficients. This last-mentioned procedure may result in a design which perfectly matches the desired characteristic at the specified frequency points, but with a large ripple in the neighbourhood of the transition. Specifying the desired response at more points in the transition region leads to designs with a smaller ripple. However, the computational cost associated with the function evaluation increases. Several remarks hold for the passband and rejection band, although numerical experiments have shown that within these bands the requirements on the separation between successive frequency samples are not as crucial as near the transition.

Given the electrical characteristics of the available materials, the current algorithm determines the thicknesses of the layers of the filter required for an optimal response as represented by eqn. 4. The optimisation technique used in this process is based on a genetic algorithm. Genetic algorithms are probabilistic optimisation processes that start from a population of randomly generated potential solutions, and gradually evolve towards better solutions through a repetitive application of genetic operators. These operators are derived from genetic processes occurring in nature, and their application to a population of potential design candidates results in an optimisation procedure which resembles natural selection.

Compared to traditional iterative optimisation techiques, genetic algorithms differ in two respects. First, genetic algorithms are based on probabilistic rather than deterministic transition rules. This greatly improves the capability of the algorithm in searching for a global function maximum, rather than a local function maximum. Other probabilistic optimisation schemes, e.g. simulated annealing, possess similar properties. The relationship between genetic algorithms and simulated annealing was investigated by Davis [14]. The second distinctive property of the typical genetic algorithm is its operation on a discretised representation of the parameters, rather than the parameters themselves. In this paper, a real-coded genetic algorithm is used that directly operates on the parameters, i.e. the thicknesses of the different layers. Real-coded genetic algorithms have been used by other authors [15], most recently to solve the nonlinear transportation problem [16]. The major advantage of the real-coded genetic algorithm, as opposed to the standard genetic algorithm, results from the fact that it does not require a discretisation of the variables. For the present problem, real-coded genetic algorithms provide a simpler and more cost-effective approach.

The genetic algorithm operates on a population of potential design candidates. Once the number of layers and the material choice for each layer are determined, the multilayer can be uniquely represented by a sequence

$$G = \{\theta_1, \theta_2, \ldots, \theta_k, \ldots, \theta_{N-1}, \theta_N\} \tag{5}$$

Using the vernacular of genetic algorithms, the individual parameter θ_k and the sequence G are also referred to as a gene and a chromosome, respectively. The genetic algorithm, a flow chart of which is shown in Fig. 2, starts

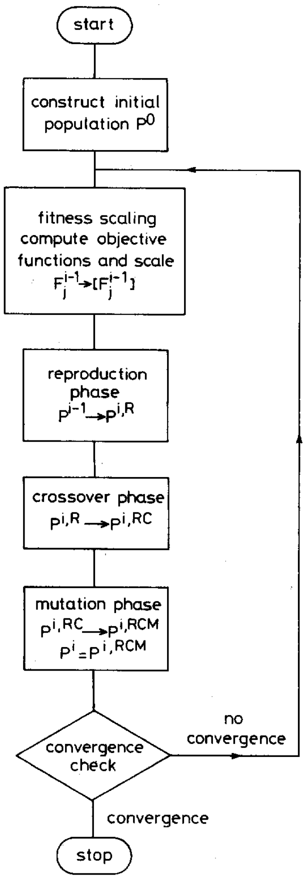

Fig. 2 *Flow chart of genetic algorithm-based design procedure*

with constructing an initial population P^0 consisting of N_{pop} randomly selected sequences G_j^0, i.e.

$$P^0 = \{G_j^0\{\theta_{j,k}^0\}\} = \{G_1^0, G_2^0, \ldots, G_{N_{pop}}^0\} \tag{6}$$

with

$$G_j^0 = \{\theta_{j,1}^0, \theta_{j,2}^0, \ldots, \theta_{j,N}^0\} \tag{7}$$

The sequences G_j^0 are selected randomly in the sense that the electrical lengths of the layers are selected uniformly in the range $0 < \theta_{j,k}^0 < \theta_{max}$ where, typically, $\theta_{max} = \pi$. The imposition of a maximum layer thickness for the sequences in the initial population does not preclude convergence to layer thicknesses which are larger than θ_{max}. The choice for θ_{max} is therefore not crucial. Once the initial population is constructed, the genetic algorithm iteratively generates subsequent populations $P^1, P^2, \ldots, P^{i-1}, P^i$ by using the genetic operators. Each iteration consists of five phases. Different genetic operators are applied during the three key elements of the algorithm, namely, the selection, crossover and mutation operators. The selection operator implements the principle of the survival of the fittest. During the selection phase of iteration cycle i, a population $P^{i,R} = [G_j^{i,R}\{\theta_{j,k}^{i,R}\}]$ is selected from $P^{i-1} = [G_j^{i-1}\{\theta_{j,k}^{i-1}\}]$ by copying sequences with a probability which is proportional to their relative fitness. The probability p_j of sequence G_j^{i-1} being copied from P^{i-1} into $P^{i,R}$ is

$$p_j = \frac{F_j^{i-1}}{\sum_{n=1,N_{pop}} F_n^{i-1}} \tag{8}$$

where F_j^{i-1} represents the fitness function value of sequence j in population P^{i-1}. During the selection phase the size of the population is kept unchanged; i.e. $P^{i,R}$ contains N_{pop} sequences, just like P^{i-1}. This mechanism of selecting sequences is often referred to as weighted roulette-wheel selection. Other selection mechanisms exist [11, 17] but have not yet been applied to the present problem. If the selection process is repeated continuously, the 'good' sequences receive increasing attention while the others vanish. In the end, only the best sequence present in P^{i-1} will survive. Since it is unlikely that this sequence represents the optimal design parameters for the desired filter, two other operators are introduced. The crossover operator produces, starting from $P^{i,R}$, a population $P^{i,RC} = [G_j^{i,RC}\{\theta_{j,k}^{i,RC}\}]$ by first mating two sequences in $P^{i,R}$, and then generating new sequences by crossing over the mated pairs. The random nature in which the sequences in $P^{i,R}$ were selected allows for the generation of $N_{pop}/2$ couples as $(G_1^{i,R}, G_2^{i,R}), \ldots, (G_j^{i,R}, G_{j+1}^{i,R}), \ldots, (G_{N_{pop}-1}^{i,R}, G_{N_{pop}}^{i,R})$ (it is assumed that N_{pop} is even). The crossover process consists of generating two new sequences from each couple $(G_j^{i,P}, G_{j+1}^{i,P})$ as follows. First, a 'crossover site' k_{cross} is selected randomly in the range $1 \leqslant k_{cross} \leqslant N$. Then, two new sequences are created by swapping the genes between positions $k_{cross} + 1$ and N, and by assigning $\theta_{j,k_{cross}}^{i,RC}$ and $\theta_{j+1,k_{cross}}^{i,RC}$ values as

$$\theta_{j,k_{cross}}^{i,RC} = \beta\theta_{j,k_{cross}}^{i,R} + (1-\beta)\theta_{j+1,k_{cross}}^{i,R} \tag{9}$$

$$\theta_{j+1,k_{cross}}^{i,RC} = (1-\beta)\theta_{j,k_{cross}}^{i,R} + \beta\theta_{j+1,k_{cross}}^{i,R} \tag{10}$$

where β is a random real number in the range $0 \leqslant \beta \leqslant 1$. Using these expressions for the crossover genes, the following sequences result after swapping:

$$G_j^{i,RC} = \{\theta_{j,1}^{i,R}, \theta_{j,2}^{i,R}, \ldots, \theta_{j,k_{cross}-1}^{i,R}, \theta_{j,k_{cross}}^{i,RC}, \theta_{j+1,k_{cross}+1}^{i,R}, \ldots, \theta_{j+1,Nl-1}^{i,R}, \theta_{j+1,Nl}^{i,R}\} \tag{11}$$

$$G_{j+1}^{i,RC} = \{\theta_{j+1,1}^{i,R}, \theta_{j+1,2}^{i,R}, \ldots, \theta_{j+1,k_{cross}-1}^{i,R}, \theta_{j+1,k_{cross}}^{i,RC}, \theta_{j,k_{cross}+1}^{i,R}, \ldots, \theta_{j,Nl-1}^{i,R}, \theta_{j,Nl}^{i,R}\} \tag{12}$$

Because each couple produces two offsprings, the crossover operator leaves the size of the population unaffected, just as the selection operator did. Earlier, the selection operator generated new populations from existing populations by probabilistic elimination of the unfit sequences, and the crossover operator improved the quality of the population through a combination of the sequences. The third operator, the mutation operator, constructs a population $P^{i,RCM} = [G_j^{i,RCM}\{\theta_{j,k}^{i,RCM}\}]$ from $P^{i,RC}$ by randomly changing the values of the thickness parameters. Not all thickness parameters are mutated during each iteration cycle. The probability of mutation, p_{mut}, typically remains very low of the order of $0.0001 \leqslant p_{mut} \leqslant 0.1$. The mutation operator changes the thicknesses according to

$$\theta_{j,k}^{i,RCM} = \theta_{j,k}^{i,RC} + \gamma\gamma'\theta_{mut} \tag{13}$$

where γ is a random variable which assumes the value of 0 with a probability of $1 - p_{mut}$, and a value of 1 with a probability of p_{mut}. γ' is a random parameter, in the range $-1 \leqslant \gamma' \leqslant 1$. θ_{mut} is the maximum amount by which the layer thickness is allowed to change, typically $\theta_{mut} =$

$0.1\theta_{max}$. If this operation results in a negative layer thickness, the layer thickness is set to zero. The mutation operator provides important features to the algorithm that are not provided by the other two operators. Most importantly, the mutation operator provides the algorithm with a mechanism for escaping local maxima. This feature becomes especially important near the end of the optimisation process, when all sequences look alike. Also, the mutation operator generates populations with layer thicknesses which are larger or smaller than the existing ones. This allows the layer thickness to grow beyond the original θ_{max}. The population $P^{i,RMC}$ is then used as the reference population for the next iteration cycle, i.e. $P^i = P^{i,RCM}$.

The selection, crossover and mutation operations are typically preceded by a process called fitness scaling, which refers to scaling of the relative values of the objective function for the sequences in a population. This scaling process is invoked to avoid two effects that may hamper proper convergence of the algorithm. First, the early populations typically consist of relatively unfit sequences, along with a few good sequences. Although these good sequences are by no means optimal, the selection operator will provide them with a large number of offsprings which introduces the danger of premature convergence to a weak local maximum. A second convergence problem occurs when the population consists of highly identical sequences, with nearly identical fitness function values. This situation typically occurs near the end of the iteration process. Direct use of the fitness function (eqn. 4) in conjunction with the selection process (eqn. 8) leads to a slow convergence, since the selection operator provides each sequence with a nearly identical number of offsprings. Both convergence problems can be avoided by properly scaling the objective function values for all of the sequences in a population before the selection takes place. This scaling process fixes the relative spread between the highest and average objective function values occurring in a population. In the early stages of the iterative process, the scaling typically reduces the relative spread in order to avoid premature convergence. In the more advanced stages of the process, the scaling operator typically enlarges the spread to speed up convergence. In this paper, the linear scaling rule

$$[F_j^{i-1}]' = AF_j^{i-1} + B \tag{14}$$

was used, where the coefficients A and B are chosen such that the maximum and average fitness function values differ by 20%. The scaled values are now used as a basis for the selection process. If this procedure results in negative $[F_j^{i-1}]'$, A and B are adjusted to maintain $[F_j^{i-1}]' > 0$. The details of this scaling rule, as well as alternative scaling mechanisms, are discussed in Reference 11.

The iterative application of the scaling, selection, crossover and mutation operators leads to new populations with improved average fitness. The convergence of the algorithm is determined based on the ratio of the average population fitness to the best fitness. When this ratio approaches unity, only little improvement can be expected, and the algorithm is stopped. The implementation of a stopping criterion depends highly on the desired filter characteristics and the available computer resources. If the absolute optimal filter is desired, the iterative process should be continued until no improvement in the best solution is encountered over a large number of iterations.

This stopping criterion is often too time consuming because the algorithm only asymptotically reaches the optimal solution. Therefore it is advisable to stop the algorithm at an earlier but mature stage in the design process.

Doing so will lead to the following. First, the computational cost of the algorithm will discuss drastically whereas the performance of the resulting filter will not deviate too much from that of the optimal filter. Secondly, depending on the maturity of the iterative process at the time it is stopped, the final population may contain a diverse set of designs. A specific design may be picked from this diverse population based on criteria, such as ease of manufacturing, which are not explicitly incorporated into the objective function.

3 Numerical results

The design procedure of the previous Section was applied to the synthesis of various filters. Here, the performance of four different filter configurations is discussed. The results are compared to those obtained by Chen, who used the turning-point method.

The first set of filters consists of alternating layers of lead telluride (PbTe, $n^H = 5.1$) and zinc sulphide (ZnS, $n^L = 2.2$). Two different types of filters are investigated, namely a lowpass and a highpass alternative. Both filters are designed for operation between air and a germanium substrate (Ge, $n^{sub} = 4.00$). To compare these designs with those by Chen, who designed 18-layer prototypes matched to air by one matching layer, the number of layers is fixed at 19. Following a notation proposed by Chen [6], our lowpass and highpass filters are denoted as GA19LP and GA19HP ('GA' refers to genetic algorithm, 19 to the total number of layers and 'LP' or 'HP' to the lowpass or highpass character of the filter). The population size, N_{pop}, was fixed at 250. Experimental results have shown that a larger population size does not lead to a significantly improved design. The filter response for the GA19LP and GA19HP filters are shown in Fig. 3 and 4, together with those of Chen [9], (filters NTP19LP and TP19HP). The design parameters for both filters are given in Table 1, and their performance indices (defined in Reference 6) are presented in Table 2.

Comparing the low-pass filter synthesised using the genetic algorithm to Chen's designs, several differences are noted. First, a detailed analysis shows that GA19LP

Table 1: Construction parameters for GA19LP and GA19HP filters

Layer number	Refractive index	GA19LP phase thickness/2π	GA19HP phase thickness/2π
	4.00	massive	massive
1	2.20	0.1282194	0.2920579
2	5.10	0.2394492	0.2857666
3	2.20	0.2627396	0.2831732
4	5.10	0.2194713	0.296617
5	2.20	0.3109459	0.306213
6	5.10	0.2179215	0.2876899
7	2.20	0.3008103	0.2692164
8	5.10	0.2474109	0.26244
9	2.20	0.2676575	0.2595634
10	5.10	0.2796177	0.2540393
11	2.20	0.2453312	0.2535144
12	5.10	0.3032079	0.2569552
13	2.20	0.2231962	0.2531283
14	5.10	0.325131	0.2555472
15	2.20	0.2081572	0.2539169
16	5.10	0.3083373	0.2561098
17	2.20	0.2533512	0.2644407
18	5.10	0.1855231	0.2411517
19	2.20	0.6542158	0.1266311
	1.00	massive	massive

has a slightly larger bandwidth factor, a smaller transition factor and larger maximum rejection than NTP19LP. The transmission for GA19LP is maintained at a level above 0.9 for all $f > 0.3$, whereas the transmission of the NTP19LP design decreases rapidly for $f < 0.5$. The NTP19LP filter, however, has a larger transmission

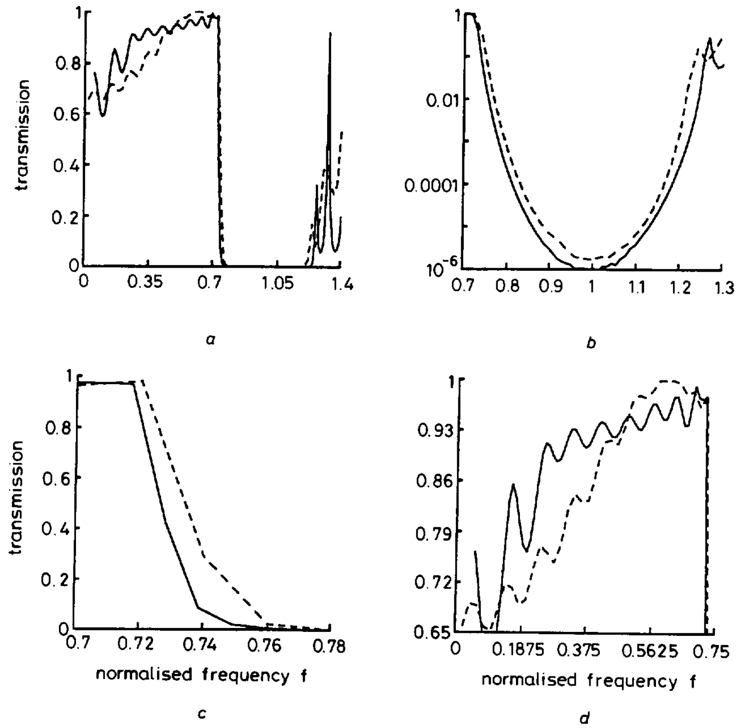

Fig. 3 *Response of 19-layer lowpass filters consisting of alternating layers of PbTe and ZnS for operation between air and germanium substrate compared with that obtained by Chen using turning point method*

—— GA19LP - - - - TP19LP

a Overall filter perforamance *b* Performance in rejection band *c* cut-off performance *d* passband performance

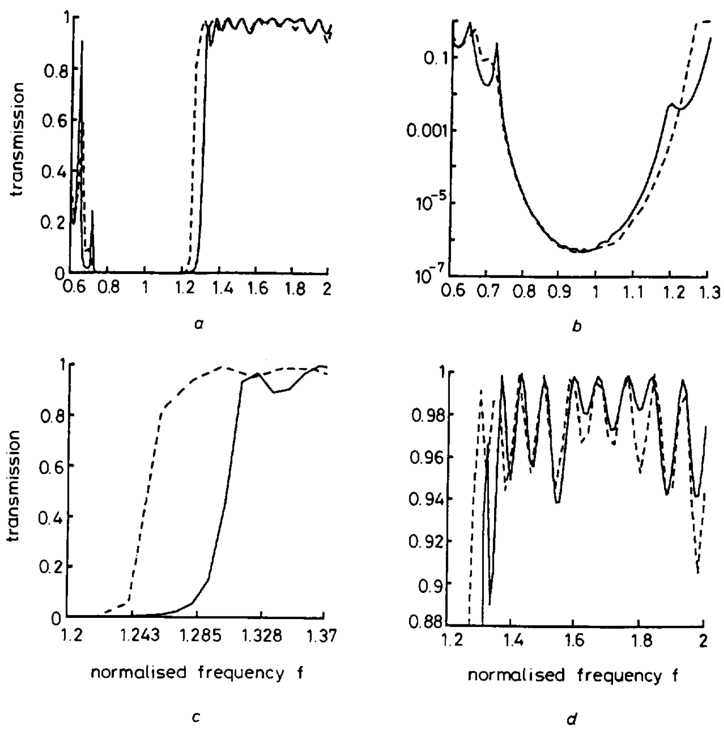

Fig. 4 *Response of 19-layer highpass filters consisting of alternating layers of PbTe and ZnS for operation between air and germanium substrate compared with that obtained by Chen using turning point method*

—— GA19HP - - - - NT19LP

a Overall filter performance *b* Performance in rejection band *c* cut-on performance *d* Passband performance

Table 2: Performance indices of filters designed using genetic algorithm and those by Chen

Filter	Bandwidth factor	Transition factor	Maximum rejection
GA19LP	0.666	0.045	0.102×10^{-5}
NTP19LP	0.572	0.053	0.186×10^{-5}
GA19HP	1.115	0.073	0.429×10^{-6}
TP19HP	1.207	0.046	0.437×10^{-6}
GA18LP	0.898	0.089	0.534×10^{-3}
TP17LP–P	0.780	0.126	0.533×10^{-3}
GA18HP	1.504	0.064	0.521×10^{-3}
TP17HP–P	0.934	0.116	0.535×10^{-3}

in the immediate neighbourhood of the cut-off. The objective function used to obtain GA19LP contained, just as that used for all other filters presented in this Section, 40 sampling points which were uniformly distributed over the pass and rejection bands, and ten sampling points which were located in the immediate vicinity of the cut-off. All sampling points were assigned equal weighting coefficients. Further investigation has shown that objective functions, in which the transmission near cutoff is assigned a larger weight, lead to filters with a transmission characteristic which resembles that of NTP19LP. (The exact same filter is, of course, never obtained.) The foregoing clearly illustrates the flexibility of the design procedure based on genetic algorithms: designs can be fine tuned to meet specific needs by changing the relative weights of the sampling points in the objective function. A program was developed in which this entire process is guided interactively. When comparing the GA19HP and TP19HP designs, both filters have a quasi-identical maximal rejection, but TP19HP has a smaller transition factor and larger bandwidth. A more detailed analysis reveals that the GA19HP design retains a transmission of approximately 0.9 up to $f = 2.12$, whereas that of TP19HP falls below 0.9 starting at $f = 2.05$. GA19HP also has a slightly wider stopband.

The second set of filters consists of 18 alternating layers of zinc sulphide ($n^H = 2.3$) and magnesium fluoride ($n^L = 1.38$). A highpass and a lowpass filter were designed for operation between air and massive glass ($n^{sub} = 1.52$). The frequency responses of these filters, denoted GA18LP and GA18HP, are shown, together with these of Chen's 18-layer designs [9] (filters TP17LP–P and TP17HP–P) in Figs. 4 and 5. The performance indices of both filters are given in Table 2, and their design parameters are presented in Table 3. Comparing GA18LP with TP17LP–P, GA18LP has a smaller transition factor and

Table 3: Construction parameters for GA18 filters

Layer number	Refractive index	GA18LP phase thickness/2π	GA18HP phase thickness/2π
	1.52	massive	massive
1	2.30	0.1622998	0.28630068
2	1.38	0.2393541	0.26840842
3	2.30	0.2296519	0.25135247
4	1.38	0.238123	0.25405198
5	2.30	0.2712908	0.24535034
6	1.38	0.2180728	0.24875983
7	2.30	0.2775777	0.24366563
8	1.38	0.2449599	0.24777628
9	2.30	0.2453928	0.24635053
10	1.38	0.2507482	0.24710184
11	2.30	0.2646756	0.24416205
12	1.38	0.2424523	0.24786457
13	2.30	0.2411647	0.24574105
14	1.38	0.2664087	0.25254172
15	2.30	0.2284045	0.25275895
16	1.38	0.2731469	0.25820929
17	2.30	0.1526431	0.25608819
18	1.38	0.5919	0.12537957
	1.00	massive	massive

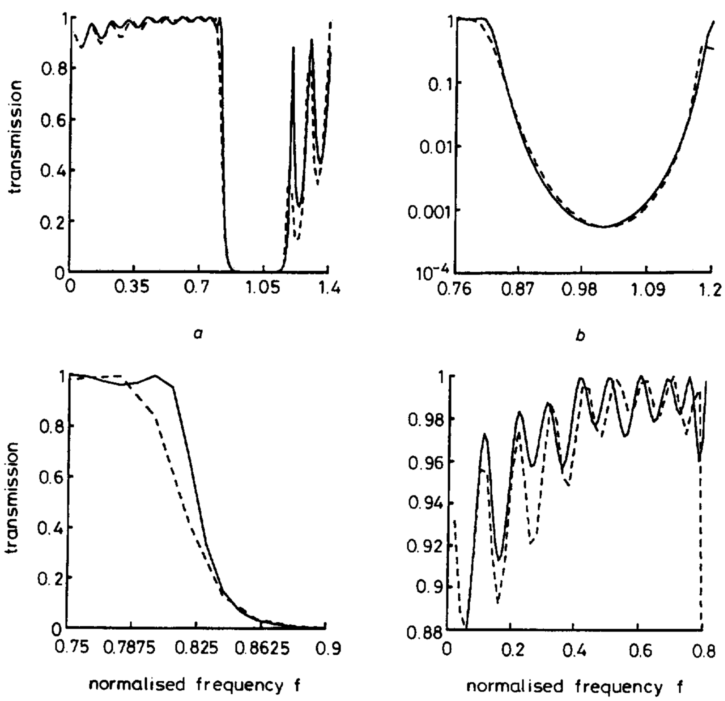

Fig. 5 *Response of 18-layer lowpass filters consisting of alternating layers of ZnS and MgFl for operation between air and glass substrate compared with that obtained by Chen using turning point method*

—— GA18LP ---- TP17LP–P

a Overall filter performance *b* Performance in rejection band *c* Cut-off performance *d* Passband performance larger bandwidth than TP17LP–P, whereas both filters have an equal maximum rejection. GA18LP is therefore the preferred design. A comparison of the GA18HP and TP17HP–P shows that the GA18HP has a smaller transition factor, a larger bandwidth factor and a slightly larger maximum rejection than TP17HP–P. Especially near $f = 2$, the performance of the TP17HP–P degrades sharply compared to that of GA18HP, resulting in a much larger bandwidth for GA18HP, which is therefore the preferred design.

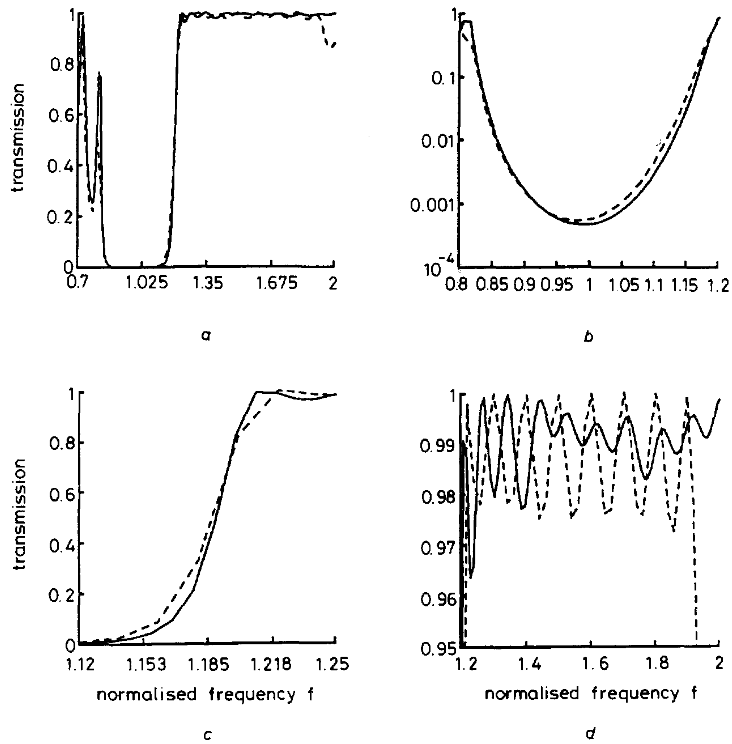

Fig. 6 *Response of 18-layer highpass filters consisting of alternating layers of ZnS and MgFl for operation between air and glass substrate compared with that obtained by Chen using turning point method*
—— GA18HP ---- TP17HP–P
a Overall filter performance *b* Performance in rejection band *c* Cut-on performance *d* Passband performance

We conclude that the filters designed using the genetic algorithm are very similar to those designed using the turning point method. Typically, the multilayers designed using the genetic algorithm marginally outperform those synthesised with the turning-point method. However, many differences between both designs arise from the choice of the weighting coefficients that are used in the formulation of the objective function F, eqn. 4. Note, however, that the turning-point method also requires certain design choices (related to the choice of the points in the passband and at cut-on at which the transmittances are fixed [9]), and the ambiguity of the choice of the objective function should therefore not be regarded as a disadvantage of the present approach when compared with other approaches.

4 Conclusions

A novel approach for designing multilayer optical filters has been developed, based on a real-coded genetic algorithm. Given the total number of layers in the filter and the electrical properties of the materials consisting each layer, the algorithm iteratively constructs multilayers whose frequency response closely matches a desired response. There are three major advantages to the present technique. First, compared to other iterative techniques, including the turning-point method, the technique does not require a crude preliminary design to ensure convergence. This is mainly due to the fact that the genetic algorithm is not a gradient-based search procedure, and therefore is not trapped easily in local maxima. Secondly, the design procedure is independent of the nature of the multilayer as well as the characteristics of the incident and substrate media, which implies that the same program can be used for directly designing lowpass, highpass, and bandpass filters which operate between practical terminal media. A third advantage of the algorithm lies in the simplicity of changing the design objective by manipulating the fitness function. This idea has wider implications than the previously discussed manipulation of the weighting coefficients appearing in the objective function. For instance, given a database of available materials, the algorithm can be used to simultaneously optimise for the material choice for each layer as well as its thickness. This problem is implemented more easily using a binary-coded genetic algorithm, as opposed to the real-coded algorithm used in this paper, but the algorithmic framework remains the same. This particular problem is currently being investigated and the results will be reported in a future paper.

The current algorithm was applied to the design of 19-layer lowpass and highpass filters composed of alternating layers of ZnS with a refractive index $n = 2.20$ and PbTe with a refractive index of 5.10, and operating between air and a germanium substrate. The algorithm was also applied to the design of 18-layer lowpass and highpass filters composed of alternating layers of ZnS with a refractive index $n = 2.30$ and MgFl with a refractive index of 1.38, and operating between air and glass. The performance of all four filters closely matches that of the filters obtained using the turning-point method. The versatility of the technique is evident in its ability to design filters of high performance for varying design conditions. The current implementation of the design technique is certainly more computer intensive than the classical methods and the turning-point technique. However, the full potential of the new technique could be achieved through parallel implementations, since genetic algorithms are implicitly parallel. All designs presented herein were completed in less than ten minutes on a DEC 5000 workstation, making the technique a valuable alternative to the turning-point method.

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