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Nonextensive triplets in cryptocurrency exchanges

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HIGHLIGHTS

- We discover nonextensive triplets in cryptocurrency exchanges.
- Cryptocurrency markets can be properly described by nonextensive statistics.
- This is the first formal link drawn between finance and nonextensive theory.

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ABSTRACT

Cryptocurrencies represent a new type of financial assets that are traded in a decentralized and transparent way. Recently, cryptocurrencies with large market capitalization (mostly Bitcoin) have been studied theoretically, but a deeper understanding of their underlying mechanisms remains elusive. Here we explore the nonextensivity of price changes for 20 cryptocurrency exchanges from 2013 until 2017. We discover nonextensive triplets in the cryptocurrency market, where the three associated values for Bitcoin are remarkably close to those of the logistic map near the edge of chaos. The current findings strongly indicate that the cryptocurrency market represents a system whose physics is properly described by nonextensive statistical mechanics. Our results shed light on the complex and volatile nature of cryptocurrencies, and establish the first formal link with the nonextensive theory.

1. Introduction

The remarkable growth of cryptocurrencies has drawn widespread interest from the media and investors alike in recent years. A cryptocurrency represents a decentralized digital cash system, or a new tradable asset, where there is no single overseeing authority. In contrast to traditional exchange rates, the system operates as an online computer network connecting all those who are involved, to process and check individual transactions [1]. Bitcoin emerges as the oldest and most important one, but many other cryptocurrencies have appeared since its inception, with the same underlying technology. While most cryptocurrency exchanges are clones of Bitcoin with different parameters, some (such as Ethereum and Ripple) consist of more significant innovations. The term ‘cryptocurrency’ refers to the fact that the system relies on cryptography to secure transactions and to create additional currency.

Diverse studies are being employed to address the complex nature of cryptocurrencies, such as descriptive statistics [2–4], fractals and multifractals [5–8], information theory [8,9], and network analysis [10,11]. Yet a complete understanding of the cryptocurrency market remains an open problem. The presence of large price volatilities [5,6], long-range temporal

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dependencies, and heavy-tailed distributions [7], indicate that cryptocurrencies deviate from the normal expectation into out-of-equilibrium states where the traditional statistical mechanics does not work. For a wide class of such systems, it has been shown that they can be described by nonextensive statistics [12] instead. The nonextensive theory thus forms a strong candidate for shedding new light on the cryptocurrency phenomena.

In this work we report evidence of nonextensive triplets [13] (that characterize a system that follows nonextensive statistics) in the cryptocurrency exchange market. This represents the first such discovery in finance. Price changes in cryptocurrencies are characterized by pronounced intermittency, meaning there is an irregular alteration of price dynamics. They also exhibit unusually long temporal dependencies, where the correlations decay slower than exponential. Lastly, tails in the probability distributions follow a power law with an upper limit of two for the exponent. Interestingly, we discover that the nonextensive triplet for Bitcoin is remarkably close to that of the logistic map near the edge of chaos. Our findings of these triplets in a vast number of cryptocurrencies strongly indicates that they fall within the realm of nonextensive statistical mechanics. These results suggest that perhaps an intuitively simple (albeit theoretically intractable) mechanism may be responsible for their widely volatile behavior, promising a deeper insight into the phenomenon.

The remainder of this paper is organized as follows: Section 2 reviews the nonextensive theory; Section 3 describes the cryptocurrency data; Section 4 presents the results and discussions; Section 5 draws the conclusions.

2. Nonextensive theory

The nonextensive theory represents a generalization of Boltzmann–Gibbs (BG) statistical mechanics in order to deal with nonequilibrium systems that were unaccessible before. Tsallis initiated the study of nonextensive statistical mechanics by proposing a nonextensive definition of entropy [14], that can satisfy Clausius' prescription [15] in situations where the standard BG entropy does not work. From a mathematical standpoint, nonextensive statistics can be described by its entropic formulation

$$S_q = -k\frac{1-\sum_i p_i^q}{1-q}, \tag{1}$$

which is obtained by substituting exponentials with q-exponentials

$$e_q(x) = [1+(1-q)x]^{1/(1-q)}, \tag{2}$$

and natural logarithms with q-logarithms

$$\ln_q(x) = \frac{x^{1-q}-1}{1-q}, \tag{3}$$

reducing to the BG entropy, the usual exponential and logarithm as $q \to 1$. The power law exponent q, also known as the *entropic number*, is intimately related to the microscopic dynamics and characterizes the degree of correlations in the system. The implications of this seemingly simple generalization to physical systems are enormous:

i Probability distribution functions (PDFs) acquire heavy tails that are proportional to q-exponentials, where the corresponding states are characterized by a parameter $q \equiv q_{\text{stat}}$.
ii Stationary states exhibit less than exponential sensibility to initial conditions; small initial differences between neighboring states diverge in q-exponential form where $q \equiv q_{\text{sens}}$.
iii Macroscopic variables decay slower than exponential to their equilibrium values, namely q-exponentially with $q \equiv q_{\text{rel}}$.

Therefore, a stationary state that follows nonextensive statistics is characterized by a triplet of q-values $(q_{\text{stat}}, q_{\text{sens}}, q_{\text{rel}}) \neq (1, 1, 1)$, or "$q$-triplet", that satisfies $q_{\text{stat}} > 1$, $q_{\text{sens}} < 1$, and $q_{\text{rel}} > 1$ [13], or even the more rigid set of conditions $q_{\text{sens}} \leq 1 \leq q_{\text{stat}} \leq q_{\text{rel}}$ [12]. A particularly important instance of nonextensive statistics is that of systems that find themselves out of thermal equilibrium but still form stationary states, which can be found in a great variety of complex systems. This triplet has been successfully explored in natural phenomena such as the ozone layer [16], solar plasma [17,18], El Niño/Southern Oscillation [19], geological faults [20], river discharge [21], and in artificial systems including the logistic [22] and standard [23] maps, and scale-free networks [24].

3. Data

The period under study goes from 2013, when there was more than one cryptocurrency in circulation, to late 2017. We analyze time series of the top 50 cryptocurrencies that appear in the website https://coinmarketcap.com/currencies/ by market capitalization as of November 2017. Since cryptocurrencies can last anywhere between a few months and many years, we select a subset of 20 cryptocurrency exchanges with sufficiently long times (>600 days). For each of the cryptocurrencies we calculate over the time interval of one day the logarithmic change in closing price $S(t)$,

$$R_t \equiv \ln\left(\frac{S(t+1)}{S(t)}\right) \tag{4}$$

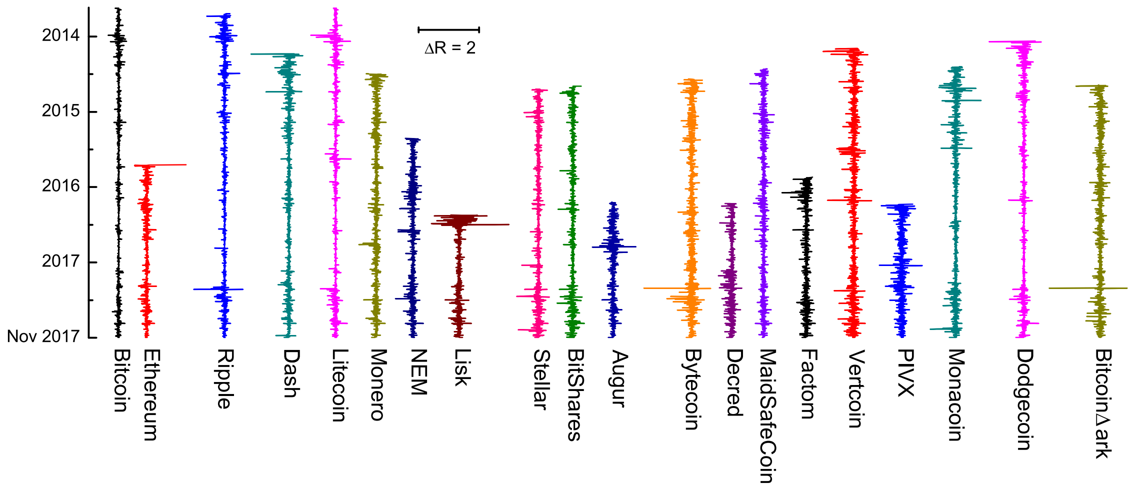

Fig. 1. Time series for the daily logarithmic price changes of 20 out of the top 50 cryptocurrencies ranked by market capitalization. The peaks denote their maximum fluctuations ΔR.

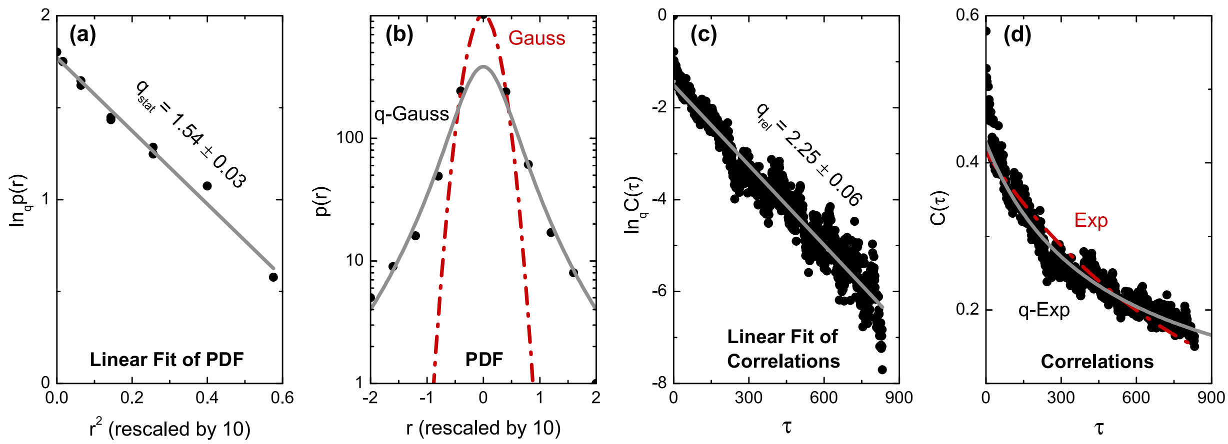

Fig. 2. Fitting the q_{stat} and q_{rel} parameters to Bitcoin data. (a) Linear correlation between $\ln_q[p(r)]$ and r^2. (b) Distribution of price change increments $p(r)$ (circles), the q-Gaussian function that fits $p(r)$ (solid line), and the best fit with a standard Gaussian (dashed line). (c) Linear correlation between $\ln_q[C(\tau)]$ and τ. (d) Autocorrelation coefficient $C(\tau)$ vs. time delay τ for absolute price changes $|R|$ (circles), the q-exponential function that fits $C(\tau)$ (solid line), and the best fit with a standard exponential (dashed line).

and construct a time series from their absolute values $|R_t|$. Fig. 1 shows a clear presence of large price variations in cryptocurrencies, which can be indicative of nonextensive statistics. We also note that cryptocurrencies with smaller market capitalization are more volatile (their prices fluctuate more) than Bitcoin, with the largest market capitalization.

4. Results and discussion

In this section we calculate the nonextensive triplets for Bitcoin and other cryptocurrencies. The values of q_{stat} are obtained from the q-Gaussian, q_{rel} from the q-exponential, and q_{sens} from the multifractality of the time series.

4.1. Stationary $q = q_{stat}$

The suitable value of q for the stationary state is obtained from the PDF associated to price change increments, $r = |R_{t+1}| - |R_t|$. These PDFs are retrieved from fitting q-Gaussians

$$G_q(\beta; r) = \frac{\sqrt{\beta}}{C_q} e_q^{-\beta r^2}, \qquad 1 < q < 3, \quad C_q = \frac{\sqrt{\pi}\,\Gamma\left(\frac{3-q}{2(q-1)}\right)}{\sqrt{q-1}\,\Gamma\left(\frac{1}{q-1}\right)}, \tag{5}$$

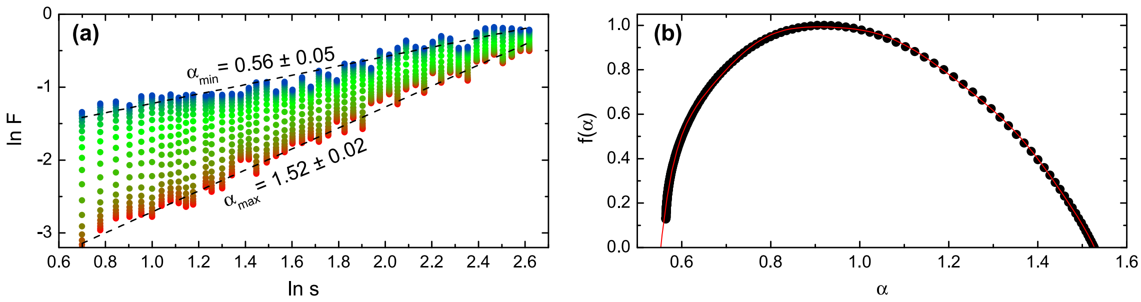

Fig. 3. Fitting the q_{sens} parameter to Bitcoin data. (a) Fluctuation function $\ln F(s)$ versus box size $\ln s$ (circles) and linear regressions for α_{min} and α_{max} (dashed lines). (b) Multifractal spectrum $f(\alpha)$ with a polynomial fit (solid line) where $q_{sens} = 0.14 \pm 0.01$.

to the histogram $\{p(r_i)\}_{i=1}^{N}$. For a proper assessment we consider the graph $\ln_q[p(r)]$ vs. r^2, and vary q from 1 to 5 to find which value makes the best linear adjustment by evaluating the coefficient of determination R^2 [18]. The optimal fit for Bitcoin leads to $q_{stat} = 1.54 \pm 0.03$ with $R^2 = 0.986$ (see Fig. 2(a)). Fig. 2(b) shows the associated q-Gaussian and the best adjustment that can be made with a Gaussian. It is clear that the $p(r)$ values become noticeably non-Gaussian along the tails, and instead can be described by a power law. This is indicative of a Hamiltonian system whose elements do not interact locally but globally [13].

4.2. Relaxation $q = q_{rel}$

The corresponding q-value for the relaxation process is determined from the autocorrelation coefficient

$$C(\tau) = \frac{\sum_t |R_{t+\tau}| \cdot |R_t|}{\sum_t |R_t|^2}. \tag{6}$$

For BG statistics such correlation should decay in exponential fashion. However, the autocorrelation of our series $|R_t|$ clearly decays much slower as shown in Fig. 2(d). The same linear adjustments can be made on the $\ln_q[C(\tau)]$ vs. τ graph to determine which choice of q best linearizes the data. Fig. 2(c) reveals that the autocorrelations of Bitcoin decay as a power law with an exponent $q_{rel} = 2.25 \pm 0.06$ and $R^2 = 0.939$.

4.3. Sensitivity to initial conditions $q = q_{sens}$

It is well known that nonlinear chaotic dynamical systems may have fractal or multifractal attractors [25]. Indeed, deviations of the neighboring trajectories of the attractor set of the dynamics leads to a multifractal structuring of the phase space [18]. Power-law sensitivity to initial conditions provides a natural relation between the scaling properties of the dynamics attractor and its degree of nonextensivity. Particularly, the extreme values, α_{min} and α_{max}, of the multifractal singularity spectrum $f(\alpha)$ are associated with the most concentrated (α_{min}) and sparse (α_{max}) regions in the attractor set. Their scaling properties can then be used to estimate the power-law divergence of nearby orbits, where the smallest separation between two nearby orbits implies the following relation to the q_{sens} parameter [26]:

$$\frac{1}{1 - q_{sens}} = \frac{1}{\alpha_{min}} - \frac{1}{\alpha_{max}}. \tag{7}$$

The multifractal characterization for Bitcoin is evaluated as explained in Ref. [27]. Fig. 3(a) shows that the fluctuation function $F_\eta(s)$ increases with the box size s as a power law $F_\eta(s) \sim s^{h(\eta)}$, where the scaling exponent $h(\eta)$ is calculated as the slope of the linear regression of $\ln F_\eta(s)$ vs. $\ln s$. These exponents are related to the singularity spectrum $f(\alpha)$ through a Legendre transformation [27]. The spectra extrema, α_{min} and α_{max}, can be obtained by extrapolating the curve of a polynomial fit to zero. Fig. 3(b) reveals that the Bitcoin spectra have a wide range of scaling exponents, with $\alpha_{min} = 0.56 \pm 0.05$ and $\alpha_{max} = 1.52 \pm 0.02$, that result in $q_{sens} = 0.14 \pm 0.01$.

4.4. q-triplets for other cryptocurrencies

The values for the Bitcoin q-triplet obey the general relation $q_{sens} \leq 1 \leq q_{stat} \leq q_{rel}$ [12]. We also find a number of other cryptocurrencies that are consistent with the nonextensive scenario, as shown in Table 1. These results reveal that the cryptocurrency market represents a system with a nonequilibrium state, strongly suggesting that long-range correlations exist among the random variables involved in the physical process that controls price activity. Fig. 4(a) further shows that the triplet values vary between cryptocurrencies. Cryptocurrencies with high market capitalization (such as Bitocoin and Etherum) have much weaker sensitivity to initial conditions than those with low market capitalization (for example,

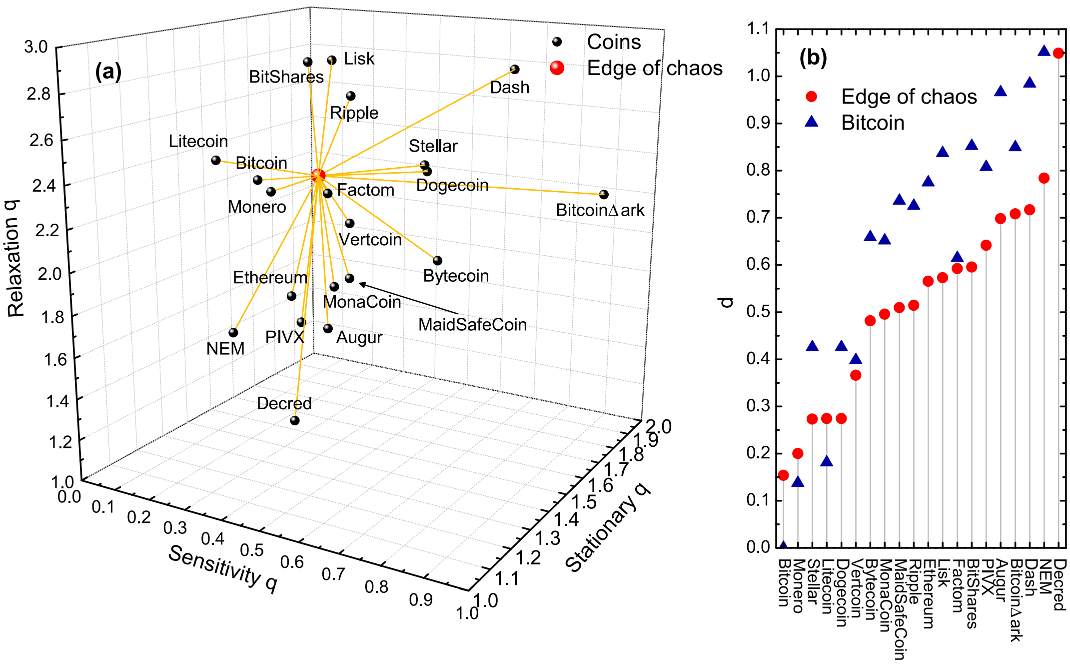

Fig. 4. (a) Scatter plot of the nonextensive triplet values for the 20 considered cryptocurrencies. (b) Their Euclidean distance d in three-dimensional space from Bitcoin (triangles) and from the logistic map near the edge of chaos (circles).

Table 1
Nonextensive triplets for 20 out of the top 50 cryptocurrencies ranked by market capitalization.

Cryptocurrency	q_{sens}	q_{stat}	q_{rel}	Cryptocurrency	q_{sens}	q_{stat}	q_{rel}
#1 Bitcoin	0.14 ± 0.01	1.54 ± 0.03	2.25 ± 0.06	#29 Augur	0.37 ± 0.03	1.49 ± 0.09	1.58 ± 0.05
#2 Ethereum	0.26 ± 0.01	1.50 ± 0.20	1.71 ± 0.05	#30 Bytecoin	0.59 ± 0.07	1.64 ± 0.08	1.92 ± 0.04
#4 Ripple	0.42 ± 0.03	1.52 ± 0.15	2.72 ± 0.16	#32 Decred	0.43 ± 0.03	1.25 ± 0.09	1.30 ± 0.03
#5 Dash	0.64 ± 0.04	1.94 ± 0.53	2.77 ± 0.19	#34 MaidSafeCoin	0.34 ± 0.01	1.64 ± 0.14	1.75 ± 0.03
#6 Litecoin	0.06 ± 0.01	1.48 ± 0.24	2.35 ± 0.09	#39 Factom	0.56 ± 0.05	1.18 ± 0.27	2.43 ± 0.13
#9 Monero	0.24 ± 0.01	1.45 ± 0.04	2.25 ± 0.05	#40 Vertcoin	0.49 ± 0.03	1.39 ± 0.23	2.19 ± 0.06
#10 NEM	0.12 ± 0.01	1.44 ± 0.13	1.51 ± 0.03	#41 PIVX	0.41 ± 0.01	1.31 ± 0.09	1.74 ± 0.05
#12 Lisk	0.26 ± 0.04	1.69 ± 0.14	2.82 ± 0.80	#42 MonaCoin	0.42 ± 0.02	1.44 ± 0.25	1.84 ± 0.06
#20 Stellar	0.48 ± 0.05	1.77 ± 0.22	2.31 ± 0.12	#44 Dogecoin	0.48 ± 0.04	1.79 ± 0.33	2.27 ± 0.11
#24 BitShares	0.26 ± 0.01	1.58 ± 0.01	2.84 ± 0.17	#48 BitcoinDark	0.91 ± 0.23	1.88 ± 0.37	2.22 ± 0.06

BitcoinDark). The relaxation rate to equilibrium solutions varies greatly between cryptocurrencies, while the upper limit of $q_{stat} < 2$ is consistent with results obtained from other studies [28–31]. The distance between triplet pairs a and b can be quantified by taking their Euclidean distance in three dimensions:

$$d = \sqrt{(q^a_{sens} - q^b_{sens})^2 + (q^a_{stat} - q^b_{stat})^2 + (q^a_{rel} - q^b_{rel})^2}. \tag{8}$$

Surprisingly, we discover that the Bitcoin triplet is very close to the q-triplet values of the (well known) logistic map at the edge of chaos [32], $(q^{chaos}_{sens}, q^{chaos}_{stat}, q^{chaos}_{rel}) = (0.24, 1.65, 2.25)$, as shown in Fig. 4(b). This suggests a possible interpretation of the Bitcoin market as an adaptive dynamical system [33] existing on the verge of (but not plunging into) chaos, and could provide an explanation for its skyrocketing prices and large fluctuations. Fig. 4(b) further reveals that roughly five triplets are close to the edge of chaos, while two of them (those of Litecoin and Monero) fall very near to that of Bitcoin, implying that the cryptocurrencies possess similar nonextensive behavior.

5. Conclusions

In this paper we study the nonextensive behavior of daily price changes for a number of top cryptocurrencies by market capitalization, including Bitcoin, from 2013 until 2017. From these data we are able to estimate the values of the nonextensive triplet. The three associated indices for Bitcoin adopt the values

$$(q_{stat}, q_{sens}, q_{rel}) = (1.54 \pm 0.03, 0.14 \pm 0.01, 2.25 \pm 0.06), \tag{9}$$

and similar relations are found for other cryptocurrencies. The current results strongly indicate that the cryptocurrency market represents a system in an out-of-equilibrium stationary state whose physics is properly described by nonextensive statistical mechanics, the first such finding in finance. This opens up new questions towards a deeper understanding of the price dynamics in cryptocurrencies, that may prove relevant for future research.

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References

[1] S. Nakamoto, Bitcoin: A peer-to-peer electronic cash system, 2008. http://bitcoin.org/bitcoin.pdf. (Accessed 1 January 2018).
[2] J. Chu, S. Nadarajah, S. Chan, Statistical analysis of the exchange rate of bitcoin, PLoS One 10 (7) (2015) 1–27.
[3] S. Chan, J. Chu, S. Nadarajah, J. Osterrieder, A statistical analysis of cryptocurrencies, J. Risk Financ. Manage. 10 (2) (2017).
[4] S. Wang, J.-P. Vergne, Buzz factor or innovation potential: What explains cryptocurrencies' returns? PLoS One 12 (1) (2017) 1–17.
[5] A.F. Bariviera, M.J. Basgall, W. Hasperué, M. Naiouf, Some stylized facts of the Bitcoin market, Physica A 484 (2017) 82–90.
[6] J. Alvarez-Ramirez, E. Rodriguez, C. Ibarra-Valdez, Long-range correlations and asymmetry in the Bitcoin market, Physica A 492 (2018) 948–955.
[7] T. Takaishi, Statistical properties and multifractality of bitcoin, arXiv:1707.07618.
[8] S. Lahmiri, S. Bekiros, Chaos, randomness and multi-fractality in Bitcoin market, Chaos Solitons Fractals 106 (2018) 28–34.
[9] L. Kristoufek, M. Vosvrda, Gold, currencies and market efficiency, Physica A 449 (2016) 27–34.
[10] D. Kondor, I. Csabai, J. Szüle, M. Pósfai, G. Vattay, Inferring the interplay between network structure and market effects in Bitcoin, New J. Phys. 16 (12) (2014) 125003.
[11] D. Kondor, M. Pósfai, I. Csabai, G. Vattay, Do the rich get richer? An empirical analysis of the bitcoin transaction network, PLoS One 9 (2) (2014) 1–10.
[12] C. Tsallis, Introduction to Nonextensive Statistical Mechanics - Approaching a Complex World, Springer, New York, 2009.
[13] C. Tsallis, Dynamical scenario for nonextensive statistical mechanics, Physica A 340 (1) (2004) 1–10.
[14] C. Tsallis, Possible generalization of Boltzmann-Gibbs statistics, J. Stat. Phys. 52 (1) (1988) 479–487.
[15] R. Clausius, On several convenient forms of the fundamental equations of the mechanical theory of heat, Philos. Mag. J. Sci. (1865) 327–365.
[16] G. Ferri, M.R. Savio, A. Plastino, Tsallis' q-triplet and the ozone layer, Physica A 389 (9) (2010) 1829–1833.
[17] L. Burlaga, A. F.-V. nas, Triangle for the entropic index q of non-extensive statistical mechanics observed by voyager 1 in the distant heliosphere, Physica A 361 (1) (2006) 173–179.
[18] G. Pavlos, A. Iliopoulos, G. Zastenker, L. Zelenyi, L. Karakatsanis, M. Riazantseva, M. Xenakis, E. Pavlos, Tsallis non-extensive statistics and solar wind plasma complexity, Physica A 422 (2015) 113–135.
[19] G.L. Ferri, A. Figliola, O.A. Rosso, Tsallis' statistics in the variability of el nio/southern oscillation during the holocene epoch, Physica A 391 (5) (2012) 2154–2162.
[20] D.B. de Freitas, G.S. Frana, T.M. Scherrer, C.S. Vilar, R. Silva, Nonextensive triplet in a geological faults system, Europhys. Lett. 102 (3) (2013) 39001.
[21] T. Stosic, B. Stosic, V.P. Singh, q-triplet for Brazos river discharge: The edge of chaos? Physica A (2017).
[22] U. Tirnakli, C. Tsallis, Chaos edges of z-logistic maps: Connection between the relaxation and sensitivity entropic indices, Phys. Rev. E 73 (2006) 037201.
[23] G. Ruiz, U. Tirnakli, E.P. Borges, C. Tsallis, Statistical characterization of the standard map, J. Stat. Mech. Theory Exp. 2017 (6) (2017) 063403.
[24] C. Tsallis, M. Gell-Mann, Y. Sato, Asymptotically scale-invariant occupancy of phase space makes the entropy sq extensive, Proc. Natl. Acad. Sci. 102 (43) (2005) 15377–15382.
[25] F.C. Moon, Chaotic and Fractal Dynamics: An Introduction for Applied Scientists and Engineers, Wiley, 2007.
[26] M.L. Lyra, C. Tsallis, Nonextensivity and multifractality in low-dimensional dissipative systems, Phys. Rev. Lett. 80 (1998) 53–56.
[27] J.W. Kantelhardt, S.A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde, H. Stanley, Multifractal detrended fluctuation analysis of nonstationary time series, Physica A 316 (1) (2002) 87–114.
[28] C. Tsallis, C. Anteneodo, L. Borland, R. Osorio, Nonextensive statistical mechanics and economics, Physica A 324 (1) (2003) 89–100.
[29] R. Osorio, L. Borland, Tsallis, Distributions of high-frequency stock-market observables, in: M. Gell-Mann, C. Tsallis (Eds.), Nonextensive Entropy: Interdisciplinary Applications, Oxford University Press, New York, 2004.
[30] A. Cortines, R. Riera, Non-extensive behavior of a stock market index at microscopic time scales, Physica A 377 (1) (2007) 181–192.
[31] S. Devi, Financial market dynamics: Superdiffusive or not? J. Stat. Mech. Theory Exp. 2017 (8) (2017) 083207.
[32] C. Tsallis, The nonadditive entropy sq and its applications in physics and elsewhere: Some remarks, Entropy 13 (10) (2011) 1765–1804.
[33] C. Gros, Complex and Adaptive Dynamical Systems, Springer-Verlag, Berlin Heidelberg, 2013.