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Stock market trading rule based on pattern recognition and technical analysis: Forecasting the DJIA index with intraday data

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ABSTRACT

This work presents empirical evidence which confronts the classical Efficient Market Hypothesis, which states that it is not possible to beat the market by developing a strategy based on a historical price series.

We propose a risk-adjusted profitable trading rule based on technical analysis and the use of a new definition of the flag pattern. This rule defines when to buy or sell, the profit pursued in each operation, and the maximum bearable loss. In order to untie the results from randomness, we used a database comprised of 91,307 intraday observations from the US Dow Jones index. We parameterized the trading rule by generating 96 different configurations and reported the results of the whole sample over 3 subperiods. In order to widen its validity we also replicated the analysis on two leading European indexes: the German DAX and the British FTSE. The returns provided by the proposed trading rule are higher for the European than for the US index, which highlights the greater inefficiency of the European markets.

1. Introduction

The possibility of predicting the future price of financial assets (stocks, ETFs, options, futures, etc.) from historical price series is one of the most important challenges both for individual investors and for companies linked to the financial environment.

Price prediction confronts the Efficient Market Hypothesis introduced by Fama (1970). According to this hypothesis, the efficient nature of the market makes it impossible to predict prices by means of historical series, which implies that it is not possible to develop an investment strategy that can beat the market under the classical criteria of risk and return. However, there is abundant evidence in the literature against this hypothesis, as we shall see later, much of which is based on the use of technical analysis.

As stated by Bagheri, Peyhani, and Akbari (2014), professional traders use two major types of analysis to make accurate decisions in financial markets: *fundamental* and *technical*. Fundamental analysis uses global economic, industrial and business indicators. The technical analysis makes its decisions on the basis of historical prices, under the assumption that past behaviors have an effect on the future evolution of prices. In technical analysis it is common to use indicators (Hu, Feng, Zhang, Ngai, & Liu, 2015; Patel, Shah, Thakkar, & Kotecha, 2015; Żbikowski, 2015), which are created by applying more or less complex formulas to historical prices. Together with these indicators, it is also common to use chart pattern analysis (Bagheri et al., 2014; Zapranis & Tsinaslanidis, 2012), which tries to predict the future behavior of prices from chart patterns which are constantly repeated in financial markets, regardless of the financial assets considered or the temporary window analyzed.

Those who have developed trading rules based on technical analysis use information based on indicators, chart patterns, or both of these. From a methodological point of view, these studies incorporate models from econometrics, statistics and artificial intelligence. In all cases trading rules are generated which allow investors to beat the market, confronting the Efficient Market Hypothesis.

Examples include the work of Hu et al. (2015), who propose a hybrid long- and short-term evolutionary trend-following algorithm that combines trend-following investment strategies with extended classifier systems (XCS). Through this methodology they introduce a trading rule which selects stocks by different indicators. Silva, Neves, and Horta (2015) apply a Multi-Objective Evolutionary Algorithms (MOEA) with two objectives, return and

Abbreviations: DAX, Deutscher Aktien index; DJIA, Dow Jones industrial index; FTSE, the FTSE 100 index; VAR, value at risk.

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risk, to optimize portfolio management. They conclude that to obtain stocks with high valuation potential, it is necessary to choose companies with a lower or average market capitalization, low PER, high rates of revenue growth and high operating leverage.

Bagheri et al. (2014) combine an Adaptive Network-based Fuzzy Inference System with a Quantum-behaved Particle Swarm Optimization to forecast a financial time series from the foreign exchange market (Forex), developing a prediction system by means of chart patterns. De Oliveira, Nobre, and Zárate (2013) use economic and financial theory, combining technical analysis, fundamental analysis and analysis of time series, to predict price behavior in the Brazilian stock market by an Artificial Neural Network.

Kao, Chiu, Lu, and Chang (2013) propose a new stock price forecasting model which integrates wavelet transform, multivariate adaptive regression splines (MARS), and support vector regression (SVR) to improve price forecasting accuracy. Patel et al. (2015) compare four prediction models to forecast the trend direction in financial markets: Artificial Neural Network (ANN), Support Vector Machine (SVM), random forest and Naive-Bayes. The results suggest random forest outperforms the three other prediction models on overall performance. Yu, Chen, and Zhang (2014) also use SVM to construct a stock selection model, which can classify stocks nonlinearly. Guresen, Kayakutlu, and Daim (2011) present an excellent compilation of studies which use neural networks in order to predict stock market indexes.

A hybrid stock trading system based on Genetic Network Programming (GNP) and Mean Conditional Value-at-Risk Model (GNP–CVaR) is proposed by Chen and Wang (2015). Dymova, Sevastianov, and Kaczmarek (2012) use fuzzy logic to build trading rules and develop a stock trading expert system based on the rule-base evidential reasoning. Ng, Liang, Li, Yeung, and Chan (2014) point that a major problem in machine learning based stock trading researches is the imbalance between *buy*, *hold* and *sell* decisions: the hold decision is in the majority in comparison to both buy and sell decisions. In order to solve this problem they propose the use of a genetic algorithm that minimizes the weighted localized generalization error (wL-GEM). Liao and Chou (2013) analyze co-movement in the Taiwan and China stock markets using association rules and cluster analysis.

Gottschlich and Hinz (2014) propose a decision support system (DSS) that enables investors to include the crowd's recommendations in their investment decisions and use it to manage a portfolio.

Booth, Gerding, and McGroarty (2014) propose an expert system based on machine learning techniques to predict the price return on seasonal events, and develop a profitable trading strategy. The technique applied is known as *random forest*, and the market in consideration is the German DAX. The authors consider not only return but also risk. As in our paper, Booth et al. (2014) measure risk by drawdown.

All the above studies analyze stocks, indexes or currencies. One of their limitations is the size of the sample from which the model is built. When working with daily data, it is difficult to build a database wide enough to rule out possible random results. In order to overcome this limitation, our work introduces intraday data, which widens the size of the sample in a very significant way.

In this context, the present study develops a chart pattern based trading rule using the flag pattern, which has received a lot of attention in academic circles. Leigh, Paz, and Purvis (2002), Leigh, Purvis, and Ragusa (2002), Leigh, Modani, and Hightower (2004) and Wang and Chan (2007), Wang and Chan (2009) have reported the positive performance of trading rules based on the flag pattern by employing different stock market indexes. The profits obtained from using this trading rule were greater than the index selected as a benchmark, even after including the transaction costs. However, the size of the sample used in these works is limited.

In order to test the statistical significance of these results, and to mitigate the effects of data snooping, attention should be paid to Brock, Lakonishok, and LeBaron (1992) advice: (1) to report results from all trading rules, (2) to use very long data series, and (3) to emphasize the robustness of results across various non-overlapping sub-periods.

The aim of the present work is to follow the line of research started with chart patterns, by proposing a new version of the flag pattern which includes similarities with the IF-THEN rule. To our knowledge, the last study which analyzed this pattern was the one carried out by Wang and Chan (2009).

The present work introduces significant contributions to the existing literature. Firstly, we introduce a new definition of the flag pattern. When empirically validating its return, we strengthen the statistical robustness of the pattern and its use in the design of the trading rule.

Secondly, the validation of the trading rule based on the flag pattern also presents important novelties with respect to previous works: (1) two new parameters are included, *stop loss* and *take profit*, which allow the dynamic modeling of the closing of operations; (2) intraday data are employed, which allows considerable width in the number of observations in the sample; (3) not only closing prices are considered, but also opening prices; thus, the information considered when deciding whether or not to start an operation is widened.

Thirdly, besides evaluating the performance of the trading rules through the profits they obtain, we also consider risk in the form of the maximum drawdown of the return curves, since the non normality of the returns prevents us from using the *t*-student.

The results confirm the positive performance of the flag pattern over the intraday data of the DJIA for a time horizon of more than 13 years.

Fourthly, the results provide empirical evidence which confronts the Efficient Market Hypothesis and show how it is possible to develop an investment strategy capable of beating the market in the mean–variance sense. This is obtained by applying the trading rule to 91,307 observations of the US DJIA index. Furthermore, we complete the analysis by including the results we obtained from the application of the trading rule to the two main European indexes: the German DAX and the British FTSE.

This paper is structured as follows: the next section presents the weight matrix that identifies the flag pattern, linking the rule IF-THEN. In the third section the trading rule is developed and the *stop loss* and *take profit* values are defined according to price range. In the fourth section we present the results obtained when applying the trading rule to the intraday data of the Dow Jones Industrial Average (DJIA). The fifth section presents the results of the 3 non-overlapped sub-periods, the sixth section presents two additional case studies on the DAX and FTSE indexes. Finally, the last section summarizes the findings and our conclusions are given.

2. A new version of the flag pattern

Charting analysis is based on the recognition of chart patterns in price changes and, eventually, in the volume of operations. This work focuses on price chart analysis, specifically on one of the most analyzed patterns in the literature: the flag. Downes and Goodman (1998) defined it thus: "Technical chart pattern resembling a flag shaped like a parallelogram with masts on either side, showing a consolidation within a trend. It results from price fluctuations within a narrow range, both preceded and followed by sharp rises or declines".

To the best of our knowledge, the works by Leigh, Modani, Purvis, and Roberts (2002), Leigh et al. (2002) and Leigh, Purvis, and Ragusa (2002) are the first to deal with the graphical recognition of this kind of pattern and to introduce a trading rule linked to it. In order to identify the pattern, we used a 10x10 grid of weights, like the one in Fig. 1, which allows the bull flag pattern to be recognized (Leigh et al., 2002).

The first 7 columns of the weights matrix represent the Downes and Goodman *consolidation* process, whereas the last 3 columns represent the *breakout* (sharp increase in the price). The bear flag variation would be obtained as a mirror reflection of the horizontal axis.

In order to detect possible flag patterns, we fit the template over the price window we want to check; thus, the highest price in the window is made to correspond with the top of the grid, and the lowest price in the window is made to correspond with the bottom of the grid.

The difference between the maximum and the minimum constitutes the price range (R) or height of the window. The goal of this process is to obtain the fit value that will show the level of matching between the matrix and the price window: when the price in the cell labeled 1 falls at a certain time, the fit value will increase by a unit; if the price falls in a cell labeled 0.5, the fit value will increase by 0.5 units; this is done for each of the different values that appear in the template in Fig. 1 and for each of the times t, $t = 1\ldots10$.

Since each element of the price window can only coincide with one cell of the grid (in the cited literature the authors work with closing prices), a window with 10 prices will restrict the maximum value of the fitting function to 10, and this total fitting value will provide the maximum fitting of the price window to the flag pattern. In the most frequent case, in which prices will not only fall in cells labeled 1, the total fitting value will be below 10, the greater the difference between the price window and the bull flag template (Fig. 1), the lower the total fitting value. The fitting value threshold (minimum total fitting value) should be established by the researcher or the trader in advance. A very demanding threshold will reduce the number of identified flags, which will restrict the statistical significance of the results; while a more permissive one might take some window prices as the flag pattern which would hardly be considered in other cases.

The determination of the weights can result in a black box for the users (Wang & Chan, 2007). As Zapranis and Tsinaslanidis (2012) have stated, "in template matching techniques the user has to set specific weights in the template's grid before the identification of a pattern. These weights in some cases are set somehow arbitrarily and the process embeds a level of subjectivity".

The choice of the threshold value is closely related to the weight grid, and the choice of these weights is critical when identifying the pattern. Fig. 2 presents two window prices which, considering the weight grid in Fig. 1, provide the same total fitting value of 6.5. While in the first case (Fig. 2(a)) it can be assumed that the price window corresponds to a bull flag pattern, in the second case (Fig. 2(b)) the price goes down without confirming a following breakout and the shape deviates considerably from a bull flag pattern. One solution might be to increase the threshold of the total fitting value, but in this case we would discard price windows that would fit the pattern (Fig. 2(a)).

This work tries to mitigate this disadvantage by making an alternative choice of weights. Fig. 3 contains the proposed weight grid that will allow a bull flag pattern to be identified and which makes a new contribution to the literature on the subject.

The first difference with respect to the matrix in Fig. 1 lies in the shape of the bull flag pattern. While Leigh, Modani, et al. (2002), Leigh, Paz, et al. (2002), Leigh, Purvis, et al. (2002) and Wang and Chan (2007), Wang and Chan (2009) use the consolidation and breakout version, we explore the breakout and consolidation approach.

.5	0	-1	-1	-1	-1	-1	-1	-1	0
1	.5	0	-.5	-1	-1	-1	-1	-.5	0
1	1	.5	0	-.5	-.5	-.5	-.5	0	.5
.5	1	1	.5	0	-.5	-.5	-.5	0	1
0	.5	1	1	.5	0	0	0	.5	1
0	0	.5	1	1	.5	0	0	1	1
-.5	0	0	.5	1	1	.5	.5	1	1
-.5	-1	0	0	.5	1	1	1	1	0
-1	-1	-1	-.5	0	.5	1	1	0	-2
-1	-1	-1	-1	-1	0	.5	.5	-2	-2.5

Consolidation ⟷ Breakout

Fig. 1. Bull flag template from Leigh, Purvis, et al. (2002).

.5	0	-1	-1	-1	-1	-1	-1	-1	0
1	.5	0	-.5	-1	-1	-1	-1	-.5	0
1	1	.5	0	-.5	-.5	-.5	-.5	0	.5
.5	1	1	.5	0	-.5	-.5	-.5	0	1
0	.5	1	1	.5	0	0	0	.5	1
0	0	.5	1	1	.5	0	0	1	1
-.5	0	0	.5	1	1	.5	.5	1	1
-.5	-1	0	0	.5	1	1	1	1	0
-1	-1	-1	-.5	0	.5	1	1	0	-2
-1	-1	-1	-1	-1	0	.5	.5	-2	-2.5

(a)

.5	0	-1	-1	-1	-1	-1	-1	-1	0
1	.5	0	-.5	-1	-1	-1	-1	-.5	0
1	1	.5	0	-.5	-.5	-.5	-.5	0	.5
.5	1	1	.5	0	-.5	-.5	-.5	0	1
0	.5	1	1	.5	0	0	0	.5	1
0	0	.5	1	1	.5	0	0	1	1
-.5	0	0	.5	1	1	.5	.5	1	1
-.5	-1	0	0	.5	1	1	1	1	0
-1	-1	-1	-.5	0	.5	1	1	0	-2
-1	-1	-1	-1	-1	0	.5	.5	-2	-2.5

(b)

Fig. 2. Examples of bull flag template. The grey cells indicate price falls. In (a) there is a price window that can fit a bull flag pattern. If we add the values of the dark cells, we will get a total fitting value of 6.5. In (b) there is a price window whose total fitting value is also 6.5, although it cannot be considered a bull flag pattern.

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	-1	-1	-1	-1	-1	-1
0	0	0	-1	-2	-2	-2	-2	-2	-2
0	0	-1	-3	-3	-3	-3	-3	-3	-3
0	-1	-3	-5	-5	-5	-5	-5	-5	-5
0	-1	-5	-5	-5	-5	-5	-5	-5	-5
0	-1	-5	-5	-5	-5	-5	-5	-5	-5
5	-1	-5	-5	-5	-5	-5	-5	-5	-5

Fig. 3. Grid of weights proposed to identify a bull flag pattern. The weights configuration is related to the IF-THEN rule.

The second difference lies in the range of weights considered and in the distribution of the weights around the matrix: the weights configuration is a key point in our proposal.

The matrix in Fig. 3 contains only one cell with a positive value, which indicates the area from which the price should start: the bottom left-hand corner. It should be noted that in order to get a strictly positive value for the fitting function, the price must pass through this cell. The cells with negative values indicate areas in which the price must not fall if we want the price window to be considered a bull flag pattern. The cells with value 0 mark the areas where the price could move without affecting the total value of the fit function.

This configuration of weights shows the pattern more clearly in comparison to other kinds of grid, since other grids might consider that different price windows could adopt the shape of a flag (Fig. 2). Our proposal is more closely related to an IF-THEN rule. For example, if we decide that the price windows whose fitting value is equal to 4 are the only ones to be considered flags, then the following conditions must be satisfied:

- The price must fall in the cell labeled 5.
- The price must visit only one cell with a negative weight.

These two conditions strictly limit the cells in which the price can fall, thus 8 of the 10 price columns must fall in the cells labeled 0. If both conditions are fulfilled (IF) then the price window is considered a bull flag pattern (THEN).

Another important difference with respect to other studies is the matching of the historical prices and the weight grid; we do not take the closing prices, but the body of the candlesticks. This implies that a candlestick can fall in more than one cell per column. Therefore, the fitting function will not be found by only adding the value of the 10 cells, since it could include a greater number.

In order to include more information on price evolution, we adopted the candlestick body variation instead of the simpler closing price. The use of the Japanese candlesticks in charting analysis has become widespread; in fact, most of the trading platforms now work with this kind of representation by default. We have not used the total range of the candlestick's high and low difference, since these extremes represent price levels which have already been reached after the opening and rejected before the closing.

3. Trading rule specification

In the previous section we defined the way in which the flag patterns are identified. The following step will consist of defining the trading system. According to Park and Irwin (2007), a technical trading system consists of a set of trading rules that generate trading signals according to various parameter values. In order to implement a trading system it is necessary to determine the starting moment of a buying (or selling) operation of an asset, and the conditions required to close the operation.

Once the proper matching is obtained from the comparison between the price window and the weight grid, the buying or selling operation starts, according to whether a bull or bear flag appears. Therefore, if the matching takes place for the price window between t and $t+9$, the operation will start with the opening price in $t+10$.

In order to evaluate the return of the operation, the exit point should be defined. Most studies consider a holding period of d candlesticks. The value of d will change according to different authors: $d = 6$ in Lee and Jo (1999), $d = 20$ in Leigh et al. (2002) and $d = 100$ in Leigh et al. (2002). In order to increase the validity of the results and mitigate the data snooping effect, some authors propose considering a group of values for d instead of only one: $d \in \{10, 20, 40, 80\}$ in Leigh et al. (2002), $d \in \{20, 40, 60, 80, 100\}$ in Leigh, Modani, and Hightower (2004) and in Wang and Chan (2007), and $d \in \{20, 40, 60, 80, 100, 120, 160, 200, 240\}$ in Wang and Chan (2009).

An interesting alternative will not statically fix a determined value or group of values, but instead adopt a dynamic process in which operations will close according to the price evolution and not the time. Teixeira and De Oliveira (2010) propose the use of a variation often used by traders: to put a *stop loss* and a *take profit* in each operation, which will limit both the loss and the profit of each of the operations. Once a position is initiated, the *stop loss* will mark the price level at which the maximum supported loss will be reached, thus if prices reach the *stop loss* then the operation will close and assume the loss. In the same way, the *take profit* will mark the price level at which the expected profit or target of the operation should be taken, thus when the price reaches this level the operation will close and earn the corresponding profit.

As a general rule, the gain at the *take profit* level is usually greater than the loss at the *stop loss* level. This makes the resulting average profit on the operation greater than the average loss experienced, so that the total profit will depend on the success ratio of the operations.

In our case, we defined the *stop loss* and the *take profit* in relation to the price range R of the pattern. That is to say, if the flag has been developed over a narrow range of prices, then the *stop loss* and the *take profit* will also be small. If the flag had been developed over a wide range of prices, the *stop loss* and *take profit* would have been larger.

Fig. 4 contains an illustrative example for 15-min candlesticks of DJIA futures, from 5:00 pm to 10:00 pm on June 16th, 2003. We can see (a) a bull flag identification and (b) the *stop loss* and *take profit* that depend on the price range R.

In this case, the matching among the first 10 candlesticks and the weight grid provides a fitting value of 5, or the maximum possible. We have set a *stop loss* of 0.5 and a *take profit* of 1 over the price range $R = 48$.

Since the opening price of the 11th candlestick is 9249, the *stop loss* will be at the level $9249 - 0.5 * 48 = 9225$ and the *take profit* will be at the level $9249 + 1 {}^{*} 48 = 9297$. The 9:45 pm candlestick reaches the *take profit*, so that the full operation will provide a positive result of 48 points.

4. Results

Following the trading rule introduced in the previous section, we chose one of the world's best known indexes: the DJIA. In order to get a historical series wide enough to infer significant results and to mitigate the data snooping effect, we selected a 15-min timeframe during the period from May 22nd 2000 to November 29th 2013, during which there were bullish, bearish and lateral market stages. The total number of candlesticks amounts to 91,307.

Most authors use daily data, which (1) limits the number of observations and (2) requires a delayed historical price series in order to reach a representative number of observations. The use of previous historical data with respect to the present time can be a problem when trying to validate a trading rule: "Suppose that some technical trading rules can be found that unambiguously outperform the benchmark over the sample period, but that these are based on technology (e.g. neural networks) that only became available after the end of the sample. Since the technique used was not available to investors during the sample period, we do not believe that such evidence would contradict weak-form market efficiency" (Sullivan, Timmermann, & White, 1999). In this line of argument

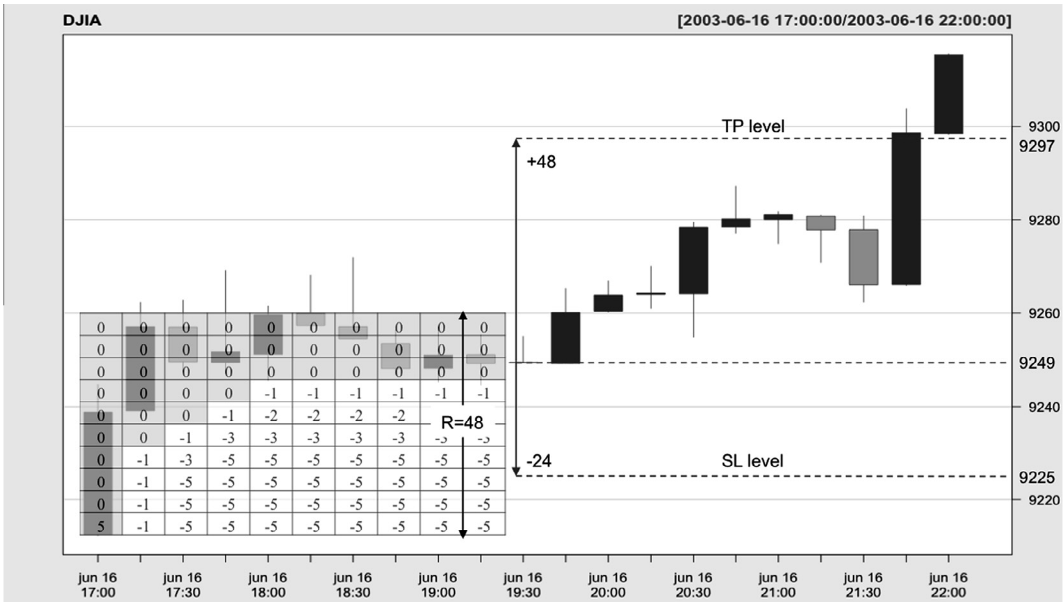

Fig. 4. Price series of futures over the DJIA index from 5:00 pm to 10:00 pm on June 16th 2003, with a 15-min timeframe. A bull flag can be identified, which corresponds to the 10 first candlesticks of the period, and a fitting value of 5. The price range during the flag is $R = 48$. The *stop loss* (*SL*) is calculated as the difference between the opening price of candlestick number 11 minus 0.5 times *R*, and the *take profit* (*TP*) is the same as opening price plus 1 *R*. The operation closes at the 9:45 pm candlestick when the price reaches *take profit*.

Timmermann and Granger (2004) question the use of strategies designed from the historical data in the short term, since when the most recent information is included, the price strategy stops being successful.

The use of intraday data does not affect the pattern identification, due to the fractality which underlies stock market data. Elder (2002), one of the most reputed traders, states "if you remove price and time markings from a chart, you won't be able to tell whether it is weekly, daily, or intraday. Markets are fractal". Also Bollinger (2002), who introduced the Bollinger bands, defends the existence of fractality in financial markets: "It turns out that fractal patterns are very common. For example, take a long-term W bottom. When examined closely, the W may turn out to have intermediate-term W bottoms embedded in its footings".

Given a price range *R*, the multiples considered in order to fix the *stop loss* (*SL*) and the *take profit* (*TP*) were:

$$SL \in \{0.2, 0.4, 0.6, 0.8\}$$

$$TP \in \{1.0, 1.2, 1.4, 1.6, 1.8, 2.0\}$$

The combination of these values will allow working with 24 different *stop loss* and *take profit* configurations.

The threshold fitting value is also a configurable parameter:

$$Threshold\ fitting\ value \in \{2, 3, 4, 5\}$$

Therefore, the total number of configurations amounts to 96. For each one we calculated the total number of operations, the number of successful operations, the number of failed operations, the hit ratio, the total return, the average of the returns and the maximum drawdown. Table 1 summarizes the results.[2]

As expected, the number of operations is inversely proportional to the threshold fitting value (*Thr*). In the most selective case $Thr = 5$ only 483 flag patterns were identified, which is 0.53% of the total sample. In the case $Thr = 2$ the number of identified patterns is 1402; i.e. 1.54% of the sample.

Although they may seem significantly low percentages, we should point out that the flag pattern is only one of a multitude of technical figures that can be found on the charts. If we considered the whole range of technical figures the percentage would increase considerably. However, it is reasonable to think that the market does not always offer investment opportunities and that these will only occur from time to time and will usually be linked to specific information.

By analyzing the trading rule's performance we can check how the total return *TR*, total sum of the returns on the operations, is positive in the 96 cases; and the same thing happens with the average return *AR*. The maximum return 180.2% is obtained by the configuration $Thr = 2$, $SL = 0.2$ and $TP = 2$. In the worst case scenario, a total return of 28.8% is obtained by combining $Thr = 3$, $SL = 0.8$ and $TP = 1$. For this configuration we also obtained the worst average returns (0.18%) by the combination $Thr = 5$, $SL = 0.6$ and $TP = 1.8$.

In comparison with the DJIA, we obtained average returns of 0.0000042. Therefore, all the configurations provided a higher average of returns than the DJIA index.

Viewing the results, we can conclude that the trading rule achieves more positive results than the benchmark. The only requirement that an investor should consider is that the transaction costs should not exceed the average return obtained for the different configurations given in Table 1.

The percentage of successful operations is always lower than the failed operations ($HR < 50$), which is in accordance with the imposed restriction that *take profit* should be greater than *stop loss*. However, as has already been commented, the total return and the average return are positive in all cases.

Fig. 5 shows the total return curves for different configurations of the trading rule, in which all the considered threshold levels have been included.

Besides the return, we also estimated the risk for each of the configurations. The non normality of the trading rule returns impedes the application of the statistical *t* (Leigh et al., 2004) and the estimation of the mean return intervals. In the present study, the non normality of the returns is even higher than in other authors' work, due to the levels of *stop loss* and *take profit* chosen.

[2] The trading rule was programmed by means of the statistical software R and some of its bundles were specially developed in order to handle stock market data.

Table 1
Trading rule results of the DJIA index for the whole period.

Thr. (#Ops)	SL	TP	#PosOps	#NegOps	HR	TR	AR	MD	TR > MD
5 (483)	0.8	2.0	137	346	0.284	0.810	0.0017	0.301	T
		1.8	150	333	0.311	0.806	0.0017	0.243	T
		1.6	161	322	0.333	0.636	0.0013	0.237	T
		1.4	177	306	0.366	0.718	0.0015	0.241	T
		1.2	194	289	0.402	0.644	0.0013	0.171	T
		1.0	208	275	0.431	0.401	0.0008	0.232	T
	0.6	2.0	116	367	0.240	0.776	0.0016	0.233	T
		1.8	129	354	0.267	0.883	0.0018	0.191	T
		1.6	138	345	0.286	0.701	0.0015	0.174	T
		1.4	153	330	0.317	0.777	0.0016	0.134	T
		1.2	168	315	0.348	0.693	0.0014	0.126	T
		1.0	183	300	0.379	0.523	0.0011	0.126	T
	0.4	2.0	87	396	0.180	0.481	0.0010	0.203	T
		1.8	98	385	0.203	0.603	0.0012	0.149	T
		1.6	103	380	0.213	0.431	0.0009	0.147	T
		1.4	114	369	0.236	0.544	0.0011	0.112	T
		1.2	126	357	0.261	0.436	0.0009	0.116	T
		1.0	138	345	0.286	0.341	0.0007	0.086	T
	0.2	2.0	58	425	0.120	0.614	0.0013	0.118	T
		1.8	65	418	0.135	0.686	0.0014	0.076	T
		1.6	71	412	0.147	0.605	0.0013	0.086	T
		1.4	80	403	0.166	0.722	0.0015	0.070	T
		1.2	84	399	0.174	0.558	0.0012	0.074	T
		1.0	93	390	0.193	0.496	0.0010	0.069	T
4 (739)	0.8	2.0	211	528	0.286	0.812	0.0011	0.376	T
		1.8	231	508	0.313	0.847	0.0011	0.296	T
		1.6	250	489	0.338	0.821	0.0011	0.281	T
		1.4	269	470	0.364	0.790	0.0011	0.267	T
		1.2	295	444	0.399	0.722	0.0010	0.275	T
		1.0	314	425	0.425	0.379	0.0005	0.306	T
	0.6	2.0	183	556	0.248	0.981	0.0013	0.244	T
		1.8	201	538	0.272	1.055	0.0014	0.191	T
		1.6	215	524	0.291	0.966	0.0013	0.186	T
		1.4	233	506	0.315	0.944	0.0013	0.171	T
		1.2	257	482	0.348	0.868	0.0012	0.166	T
		1.0	274	465	0.371	0.553	0.0007	0.172	T
	0.4	2.0	143	596	0.194	0.882	0.0012	0.201	T
		1.8	158	581	0.214	0.963	0.0013	0.152	T
		1.6	167	572	0.226	0.839	0.0011	0.145	T
		1.4	182	557	0.246	0.873	0.0012	0.121	T
		1.2	200	539	0.271	0.728	0.0010	0.137	T
		1.0	215	524	0.291	0.525	0.0007	0.124	T
	0.2	2.0	98	641	0.133	1.077	0.0015	0.104	T
		1.8	107	632	0.145	1.093	0.0015	0.094	T
		1.6	118	621	0.160	1.084	0.0015	0.096	T
		1.4	129	610	0.175	1.126	0.0015	0.071	T
		1.2	138	601	0.187	0.931	0.0013	0.085	T
		1.0	151	588	0.204	0.808	0.0011	0.084	T
3 (1.077)	0.8	2.0	301	776	0.279	0.698	0.0006	0.417	T
		1.8	322	755	0.299	0.590	0.0005	0.375	T
		1.6	350	727	0.325	0.750	0.0007	0.356	T
		1.4	379	698	0.352	0.795	0.0007	0.345	T
		1.2	415	662	0.385	0.719	0.0007	0.340	T
		1.0	445	632	0.413	0.288	0.0003	0.384	F
	0.6	2.0	263	814	0.244	1.113	0.0010	0.274	T
		1.8	281	796	0.261	1.029	0.0010	0.230	T
		1.6	301	776	0.279	1.054	0.0010	0.233	T
		1.4	329	748	0.305	1.108	0.0010	0.229	T
		1.2	362	715	0.336	1.027	0.0010	0.222	T
		1.0	391	686	0.363	0.642	0.0006	0.192	T
	0.4	2.0	210	867	0.195	1.161	0.0011	0.185	T
		1.8	227	850	0.211	1.134	0.0011	0.162	T
		1.6	241	836	0.224	1.051	0.0010	0.162	T
		1.4	266	811	0.247	1.118	0.0010	0.166	T
		1.2	289	788	0.268	0.937	0.0009	0.161	T
		1.0	314	763	0.292	0.666	0.0006	0.149	T
	0.2	2.0	143	934	0.133	1.430	0.0013	0.093	T
		1.8	154	923	0.143	1.373	0.0013	0.093	T
		1.6	169	908	0.157	1.409	0.0013	0.077	T
		1.4	188	889	0.175	1.480	0.0014	0.067	T
		1.2	202	875	0.188	1.256	0.0012	0.075	T
		1.0	222	855	0.206	1.071	0.0010	0.089	T

Table 1 (*continued*)

Thr. (#Ops)	SL	TP	#PosOps	#NegOps	HR	TR	AR	MD	TR > MD
2 (1.402)	0.8	2.0	400	1002	0.285	0.952	0.0007	0.479	T
		1.8	428	974	0.305	0.842	0.0006	0.476	T
		1.6	464	938	0.331	0.944	0.0007	0.449	T
		1.4	501	901	0.357	1.056	0.0008	0.493	T
		1.2	548	854	0.391	1.079	0.0008	0.405	T
		1.0	594	808	0.424	0.613	0.0004	0.416	T
	0.6	2.0	348	1054	0.248	1.357	0.0010	0.316	T
		1.8	369	1033	0.263	1.165	0.0008	0.323	T
		1.6	397	1005	0.283	1.162	0.0008	0.311	T
		1.4	432	970	0.308	1.281	0.0009	0.306	T
		1.2	476	926	0.340	1.342	0.0010	0.260	T
		1.0	519	883	0.370	0.934	0.0007	0.213	T
	0.4	2.0	279	1123	0.199	1.498	0.0011	0.215	T
		1.8	299	1103	0.213	1.388	0.0010	0.219	T
		1.6	322	1080	0.230	1.303	0.0009	0.206	T
		1.4	354	1048	0.252	1.440	0.0010	0.192	T
		1.2	390	1012	0.278	1.424	0.0010	0.153	T
		1.0	428	974	0.305	1.078	0.0008	0.135	T
	0.2	2.0	186	1216	0.133	1.802	0.0013	0.115	T
		1.8	200	1202	0.143	1.703	0.0012	0.126	T
		1.6	221	1181	0.158	1.723	0.0012	0.112	T
		1.4	245	1157	0.175	1.752	0.0012	0.087	T
		1.2	265	1137	0.189	1.523	0.0011	0.078	T
		1.0	292	1110	0.208	1.267	0.0009	0.081	T

Thr. = Threshold fitting value; #Ops = Number of operations; SL = (*R* times) stop loss; TP = (*R* times) take profit; #PosOps = Number of positive operations; #NegOps = Number of negative operations; HR = Hit ratio (#PosOps/#Ops); TR = Total return; AR = Average of the returns; MD = Maximum drawdown; TR > MD = is TR greater than MD?

The restriction that the former must be lower than the latter means that the returns follow a clearly asymmetric distribution, as can be inferred from the hit ratio in Table 1.

The non-normality of the returns made us look for an alternative which could help measure the risk involved in the strategy: the maximum drawdown (*MD*).

The drawdown at time *t*, D_t, is defined in Eqs. (1) and (2) as the drop of the return curve from the previous maximum at *s*, $s < t$. For instance, if at time *s* the return curve reaches a new maximum of 60%, and after several operations the total return at *t* drops to 45%, then the drawdown at time *t* would be 15%.

$$D_t = \max_{s<t} (TR_s - TR_t) \tag{1}$$

$$\mathrm{MD} = \max_{\mathrm{t}} \mathrm{D_t} \tag{2}$$

The maximum drawdown corresponds with the maximum loss experienced during the entire period, and has a certain similarity with the value at risk (VAR). For instance, considering the configuration with the greatest total return 180.2% we estimated a maximum drawdown of 11.5%. This value indicates that if we had started the trading rule at the worst possible time, the maximum loss we could support would be 11.5%.

In the last column of Table 1 it can be seen that the total return is greater than the maximum drawdown in 95 of the total of 96 configurations. Therefore, the use of this trading rule would have guaranteed a greater profit than the risk supported in 98.96% of the configurations.

In comparison with the DJIA, the drawdown was 78.42% and the total return was 37.53% during the whole period, so that the risk-adjusted return of the trading rule is better than the risk-adjusted return of the DJIA.

The relationship between the trading rule performance and the parameters used in its configuration can be seen in Table 2.

It is worth noting that the correlation between the threshold and both the variable total return and the average return. The total return is negatively related to the threshold level (−0.70), while the average return per operation is positively related (0.50). This means there is a positive relationship between the return on operations and the similarity between the price window and the flag pattern. Therefore, the more recognizable and clearer the flag pattern, the greater the return on the trading rule. The negative relationship between the threshold and the total return is explained by the greater number of operations for low threshold values. In fact, the correlation between the threshold and the total return after removing the effect of the number of operations is not statistically significant, with a coefficient value of −0.08 and a *p*-value of 0.42.

It is also interesting to see how both the total return and the average return are negatively related to *stop loss* and positively related to *take profit*. This confirms the correct choice of the *take profit*, which should be higher than *stop loss* when defining an investment strategy.

With respect to risk, the maximum drawdown is negatively related to the average return (−0.52). This would contradict the fundamentals of financial market: return and risks are positively related. In order to confirm this hypothesis and after removing the effect of the number of operations, the correlation coefficient and the *p*-value were found to be 0.01 and 0.90, respectively. Therefore, the number of operations explains the relationship between the average return and the maximum drawdown.

To summarize, a higher threshold will involve a greater average return, without assuming a greater risk, i.e. the flag pattern provides a positive and significant risk-adjusted return.

5. Analysis of non-overlapped periods

Brock et al. (1992) state that the data snooping problem can be mitigated by using (1) a very long data series, (2) reporting results from all trading rules, and (3) providing results across various non-overlapping sub-periods. In the present work a long historical data series was used in the form of intraday data. We also configured the trading rule in order to analyze its behavior with different plausible configurations. Therefore, it only remains to use several non-overlapping sub-periods.

This section presents the results of applying the trading rule over 3 non-overlapping sub-periods, all of which are of the same size with nearly 31,500 15-min candlesticks .The first sub-period lasts from May 22nd 2000 to November 26th 2004, the second

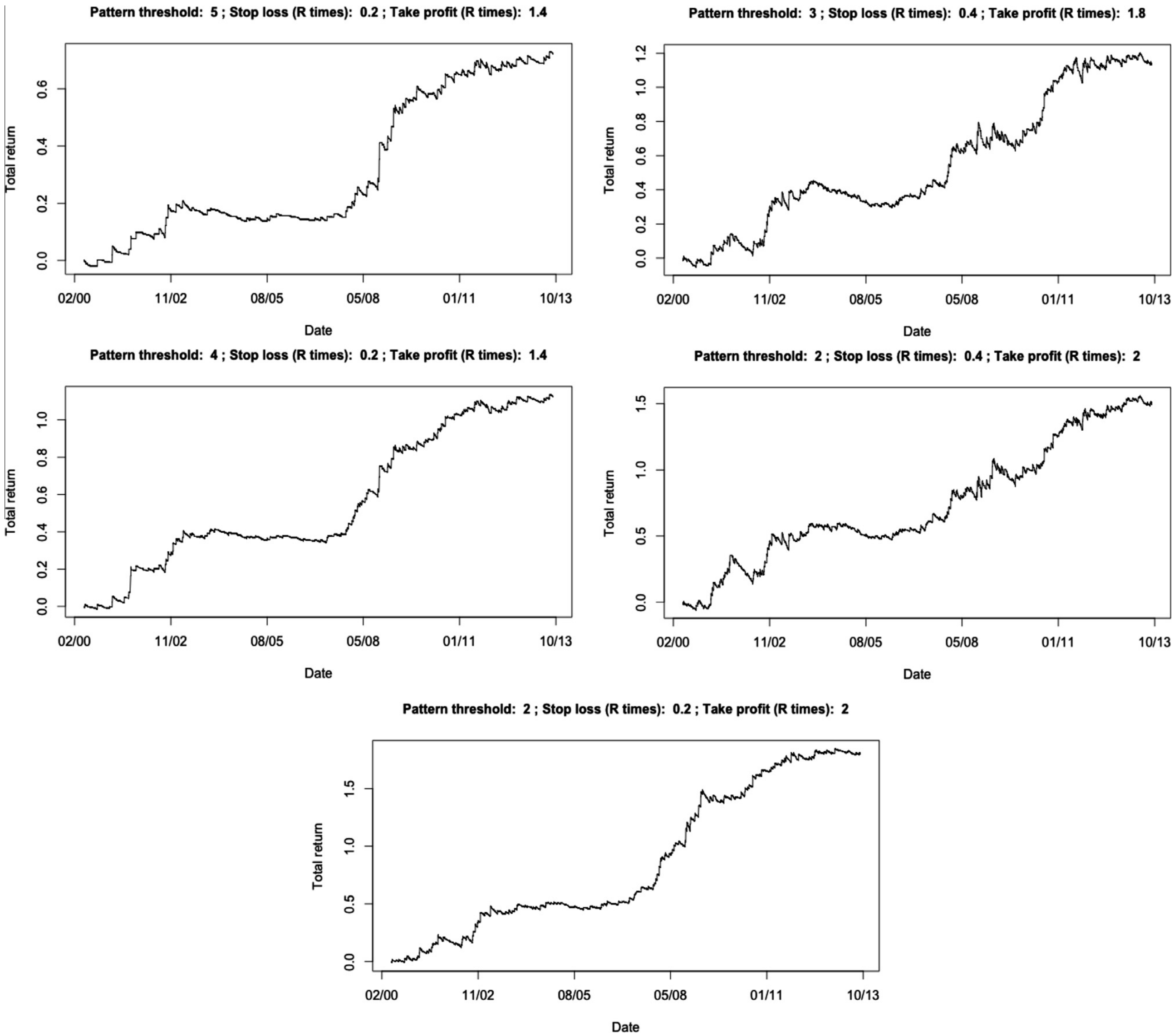

Fig. 5. Total return curves for 5 different configurations of the trading rule.

Table 2
Correlation matrix.

	Thr.	SL	TP	#Ops	#PosOps	#NegOps	TR	AR	MD
Thr.	1.00								
SL	0.00	1.00							
TP	0.00	0.00	1.00						
#Ops	−1.00**	0.00	0.00	1.00					
#PosOps	−0.77**	0.52**	−0.29**	0.77**	1.00				
#NegOps	−0.95**	−0.24*	0.13	0.96**	0.55**	1.00			
TR	−0.70**	−0.39**	0.29**	0.69**	0.18	0.82**	1.00		
MR	0.50**	−0.35**	0.39**	−0.50**	−0.72**	−0.32**	0.23*	1.00	
MD	−0.35**	0.85**	0.20	0.35**	0.69**	0.14	−0.16	−0.52**	1.00

Thr. = Threshold fitting value; SL = Stop loss (as R times); TP = Take profit (as R times); #Ops = Number of operations; #PosOps = Number of positive operations; #NegOps = Number of negative operations; TR = Total return; AR = Average of the returns; MD = Maxim Drawdown.
* 5% level of significance.
** 1% level of significance.

from November 26th 2004 to February 27th 2007 and the third from February 27th 2007 to November 29th 2013. The aim was to check if the behavior of the trading rule for each of the sub-periods is similar to the behavior of the whole sample.

For this purpose we again applied the 96 configurations of the trading rule over the 3 sub-periods (the results are given in Table 3). For the sake of brevity, only information on the total return, average return and maximum drawdown is given.

Table 3
Results of the trading rule over the DJIA index for 3 non-overlapped sub-periods. Firstsub-period: From May 22nd, 2000 to November 26th, 2004.Second sub-period: November 26th, 2004 to February 27th, 2007. Third sub-period: February 27th, 2007 to November 29th, 2013.

Thr.	SL	TP	Sub-period 1			Sub-period 2			Sub-period 3		
			TR	AR	MD	TR	AR	MD	TR	AR	MD
5	0.8	2.0	0.463	0.0041	0.135	0.203	0.0015	0.184	0.089	0.0004	**0.264**
		1.8	0.421	0.0037	0.119	0.215	0.0016	0.158	0.201	0.0009	0.159
		1.6	0.398	0.0035	0.081	0.101	0.0008	**0.209**	0.171	0.0008	0.158
		1.4	0.404	0.0036	0.062	0.240	0.0018	0.135	0.113	0.0005	**0.216**
		1.2	0.338	0.0030	0.059	0.233	0.0018	0.131	0.117	0.0005	**0.171**
		1.0	0.309	0.0027	0.052	0.153	0.0012	0.111	−**0.014**	−0.0001	**0.230**
	0.6	2.0	0.450	0.0040	0.154	0.191	0.0014	0.130	0.057	0.0003	**0.143**
		1.8	0.416	0.0037	0.144	0.314	0.0023	0.095	0.158	0.0007	0.089
		1.6	0.370	0.0033	0.121	0.209	0.0016	0.103	0.131	0.0006	0.105
		1.4	0.381	0.0034	0.093	0.313	0.0023	0.095	0.096	0.0004	**0.113**
		1.2	0.325	0.0029	0.089	0.296	0.0022	0.080	0.089	0.0004	**0.099**
		1.0	0.300	0.0027	0.056	0.219	0.0016	0.092	0.023	0.0001	**0.126**
	0.4	2.0	0.256	0.0023	0.130	0.107	0.0008	**0.108**	0.145	0.0006	0.115
		1.8	0.228	0.0020	0.108	0.172	0.0013	0.065	0.231	0.0010	0.091
		1.6	0.178	0.0016	0.093	0.092	0.0007	0.065	0.188	0.0008	0.091
		1.4	0.204	0.0018	0.064	0.165	0.0012	0.073	0.202	0.0009	0.059
		1.2	0.178	0.0016	0.060	0.102	0.0008	0.065	0.184	0.0008	0.059
		1.0	0.179	0.0016	0.037	0.109	0.0008	0.069	0.082	0.0004	**0.082**
	0.2	2.0	0.140	0.0012	0.091	0.348	0.0026	0.054	0.136	0.0006	0.085
		1.8	0.138	0.0012	0.066	0.394	0.0029	0.032	0.165	0.0007	0.075
		1.6	0.119	0.0011	0.070	0.320	0.0024	0.032	0.178	0.0008	0.078
		1.4	0.149	0.0013	0.060	0.380	0.0028	0.032	0.206	0.0009	0.053
		1.2	0.122	0.0011	0.056	0.308	0.0023	0.032	0.141	0.0006	0.056
		1.0	0.092	0.0008	0.054	0.291	0.0022	0.032	0.127	0.0006	0.066
4	0.8	2.0	0.460	0.0023	0.134	0.180	0.0009	**0.248**	0.145	0.0005	**0.197**
		1.8	0.463	0.0023	0.101	0.204	0.0010	**0.206**	0.228	0.0008	0.153
		1.6	0.582	0.0029	0.075	0.096	0.0005	**0.221**	0.150	0.0005	**0.153**
		1.4	0.531	0.0027	0.068	0.216	0.0011	0.208	0.044	0.0001	**0.213**
		1.2	0.425	0.0021	0.081	0.275	0.0014	0.216	0.042	0.0001	**0.135**
		1.0	0.362	0.0018	0.083	0.166	0.0008	**0.206**	−**0.120**	−0.0004	**0.232**
	0.6	2.0	0.546	0.0028	0.105	0.199	0.0010	0.166	0.193	0.0006	0.124
		1.8	0.484	0.0024	0.107	0.336	0.0017	0.130	0.265	0.0009	0.115
		1.6	0.536	0.0027	0.097	0.237	0.0012	0.135	0.192	0.0006	0.120
		1.4	0.500	0.0025	0.092	0.325	0.0016	0.120	0.111	0.0004	0.108
		1.2	0.412	0.0021	0.081	0.368	0.0018	0.135	0.095	0.0003	0.091
		1.0	0.333	0.0017	0.059	0.264	0.0013	0.128	−**0.021**	−0.0001	**0.138**
	0.4	2.0	0.474	0.0024	0.085	0.186	0.0009	0.140	0.257	0.0008	0.120
		1.8	0.416	0.0021	0.072	0.263	0.0013	0.070	0.323	0.0011	0.096
		1.6	0.441	0.0022	0.070	0.162	0.0008	0.079	0.244	0.0008	0.096
		1.4	0.421	0.0021	0.070	0.227	0.0011	0.070	0.223	0.0007	0.078
		1.2	0.346	0.0017	0.060	0.198	0.0010	0.081	0.193	0.0006	0.080
		1.0	0.295	0.0015	0.044	0.198	0.0010	0.086	0.052	0.0002	**0.100**
	0.2	2.0	0.306	0.0015	0.043	0.471	0.0023	0.063	0.290	0.0010	0.100
		1.8	0.291	0.0015	0.039	0.494	0.0024	0.046	0.304	0.0010	0.091
		1.6	0.359	0.0018	0.038	0.405	0.0020	0.043	0.288	0.0010	0.093
		1.4	0.341	0.0017	0.038	0.458	0.0023	0.035	0.287	0.0009	0.066
		1.2	0.281	0.0014	0.035	0.422	0.0021	0.045	0.199	0.0007	0.070
		1.0	0.233	0.0012	0.035	0.403	0.0020	0.057	0.154	0.0005	0.081
3	0.8	2.0	0.181	0.0006	**0.207**	0.351	0.0012	0.252	0.124	0.0003	**0.277**
		1.8	0.184	0.0006	**0.200**	0.305	0.0010	0.237	0.145	0.0004	**0.238**
		1.6	0.373	0.0013	0.114	0.340	0.0011	0.248	0.051	0.0001	**0.235**
		1.4	0.428	0.0015	0.089	0.489	0.0016	0.216	−**0.082**	−0.0002	**0.284**
		1.2	0.295	0.0010	0.108	0.472	0.0016	0.235	−**0.021**	−0.0001	**0.167**
		1.0	0.189	0.0007	0.116	0.342	0.0011	0.193	−**0.224**	−0.0006	**0.305**
	0.6	2.0	0.387	0.0014	0.166	0.357	0.0012	0.175	0.266	0.0007	0.173
		1.8	0.297	0.0010	0.181	0.435	0.0015	0.159	0.279	0.0007	0.164
		1.6	0.375	0.0013	0.159	0.472	0.0016	0.167	0.186	0.0005	0.168
		1.4	0.440	0.0015	0.129	0.589	0.0020	0.138	0.079	0.0002	**0.172**
		1.2	0.338	0.0012	0.097	0.559	0.0019	0.162	0.118	0.0003	0.109
		1.0	0.209	0.0007	0.104	0.459	0.0015	0.129	−**0.021**	−0.0001	**0.150**
	0.4	2.0	0.413	0.0014	0.117	0.308	0.0010	0.185	0.370	0.0010	0.156
		1.8	0.331	0.0012	0.114	0.341	0.0011	0.151	0.398	0.0010	0.145
		1.6	0.382	0.0013	0.106	0.299	0.0010	0.142	0.304	0.0008	0.145
		1.4	0.448	0.0016	0.094	0.344	0.0011	0.149	0.276	0.0007	0.108
		1.2	0.341	0.0012	0.063	0.269	0.0009	0.101	0.293	0.0008	0.093
		1.0	0.247	0.0009	0.069	0.291	0.0010	0.098	0.107	0.0003	**0.115**
	0.2	2.0	0.318	0.0011	0.070	0.711	0.0024	0.093	0.293	0.0008	0.073
		1.8	0.286	0.0010	0.074	0.693	0.0023	0.073	0.288	0.0007	0.073
		1.6	0.382	0.0013	0.066	0.656	0.0022	0.065	0.258	0.0007	0.073
		1.4	0.429	0.0015	0.047	0.679	0.0023	0.065	0.271	0.0007	0.060
		1.2	0.342	0.0012	0.042	0.599	0.0020	0.042	0.224	0.0006	0.067

(continued on next page)

Table 3 (*continued*)

Thr.	SL	TP	Sub-period 1			Sub-period 2			Sub-period 3		
			TR	AR	MD	TR	AR	MD	TR	AR	MD
		1.0	0.243	0.0008	0.042	0.577	0.0019	0.045	0.153	0.0004	0.088
2	0.8	2.0	0.218	0.0006	**0.332**	0.431	0.0012	0.274	0.116	0.0003	**0.270**
		1.8	0.274	0.0007	**0.321**	0.375	0.0010	0.249	0.084	0.0002	**0.267**
		1.6	0.417	0.0011	0.243	0.311	0.0009	0.256	−**0.013**	0.0000	**0.303**
		1.4	0.502	0.0014	0.194	0.528	0.0015	0.260	−**0.174**	−0.0004	**0.365**
		1.2	0.301	0.0008	0.158	0.665	0.0019	0.275	−**0.137**	−0.0003	**0.294**
		1.0	0.147	0.0004	**0.153**	0.512	0.0014	0.222	−**0.284**	−0.0006	**0.347**
	0.6	2.0	0.435	0.0012	0.279	0.387	0.0011	0.168	0.311	0.0007	0.138
		1.8	0.337	0.0009	0.293	0.447	0.0012	0.159	0.275	0.0006	0.139
		1.6	0.383	0.0010	0.266	0.401	0.0011	0.161	0.179	0.0004	0.167
		1.4	0.482	0.0013	0.213	0.589	0.0016	0.136	0.045	0.0001	**0.197**
		1.2	0.325	0.0009	0.171	0.735	0.0020	0.174	0.062	0.0001	**0.134**
		1.0	0.160	0.0004	**0.170**	0.599	0.0017	0.138	−**0.028**	−0.0001	**0.148**
	0.4	2.0	0.426	0.0011	0.185	0.486	0.0013	0.106	0.463	0.0010	0.102
		1.8	0.347	0.0009	0.189	0.495	0.0014	0.106	0.447	0.0010	0.119
		1.6	0.396	0.0011	0.176	0.462	0.0013	0.090	0.335	0.0007	0.136
		1.4	0.501	0.0014	0.145	0.554	0.0015	0.089	0.285	0.0006	0.132
		1.2	0.385	0.0010	0.110	0.617	0.0017	0.089	0.305	0.0007	0.105
		1.0	0.241	0.0006	0.122	0.587	0.0016	0.089	0.178	0.0004	0.120
	0.2	2.0	0.345	0.0009	0.108	0.949	0.0026	0.054	0.334	0.0007	0.063
		1.8	0.315	0.0008	0.112	0.899	0.0025	0.053	0.324	0.0007	0.060
		1.6	0.365	0.0010	0.087	0.870	0.0024	0.045	0.289	0.0006	0.067
		1.4	0.436	0.0012	0.065	0.829	0.0023	0.045	0.289	0.0006	0.075
		1.2	0.332	0.0009	0.068	0.745	0.0021	0.041	0.252	0.0005	0.076
		1.0	0.209	0.0006	0.067	0.695	0.0019	0.045	0.180	0.0004	0.085

Thr. = Threshold fit value; SL = (*R* times) Stop loss; TP = (*R* times) Take profit; TR = Total return; AR = Average of returns; MD = Maxim drawdown.
Negative TR in black; Black MD indicates a maximum drawdown which is greater than the total return.

It can be noticed that the total return is positive in all cases for sub-period 1 and in most configurations the maximum drawdown is lower than the total return for the whole sub-period.

Sub-period 2 shows very similar behavior to sub-period 1. Total return is positive in all instances, and only in a few cases did the maximum drawdown exceed the total return.

The sub-period 3 results are more complex. Total return is negative for 12 configurations, and 34 configurations provide a maximum drawdown greater than the total return.

Therefore, it can be concluded that 2 of the 3 sub-periods perform in a similar way to the entire sample, whereas the third sub-period presents some cases of negative returns and risks greater than the return.

Furthermore, the lower the values of the *stop loss* the better the performance of the trading rule. When this parameter takes a value of 0.2, it can be seen that the total return is positive for all sub-periods and the rest of the parameter values, and also that total return is greater than the maximum drawdown. A similar conclusion is reached when taking a value of 0.4: all the total returns are positive and only in some cases is the risk greater than the return. These results are consistent with the correlations shown in Table 2, in which low levels of *stoploss* favored a greater return and a lower level of drawdown. In fact, in Table 2 it can be checked that the greater correlation coefficient in absolute value is related to *stop loss* and the maximum drawdown.

6. Additional empirical evidence: the case of European financial markets

In the preceding sections we strengthened the robustness and significance of the results by analyzing a high number of observations, reporting the results of the trading rule for a wide number of configurations (threshold, stop loss, take profit), and reporting the results over a whole sample period and 3 subperiods.

It can therefore be ruled out that our results are due to random factors and it would appear that financial markets are not efficient.

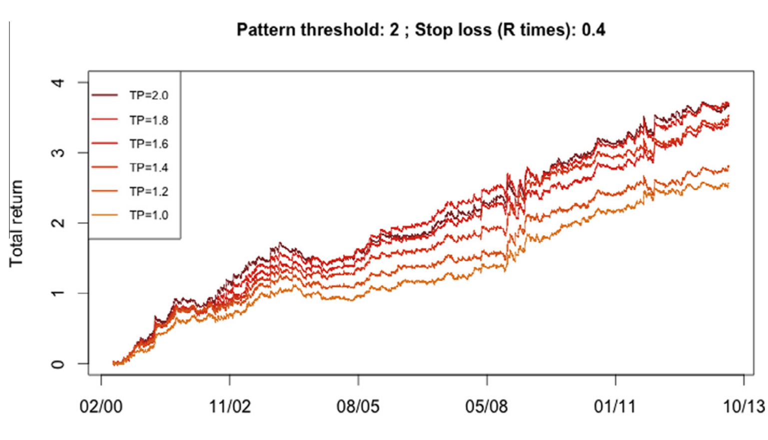

Fig. 6. Total return curves for DAX index, with pattern threshold = 2 and stop loss = 0.4.

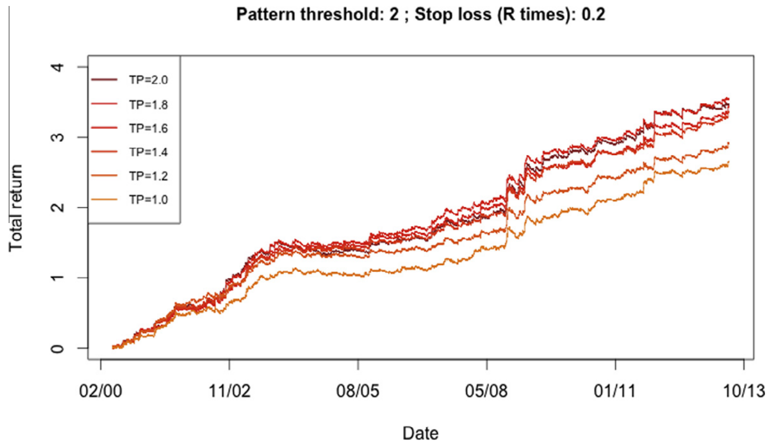

Fig. 7. Total return curves for DAX index, with pattern threshold = 2 and stop loss = 0.2.

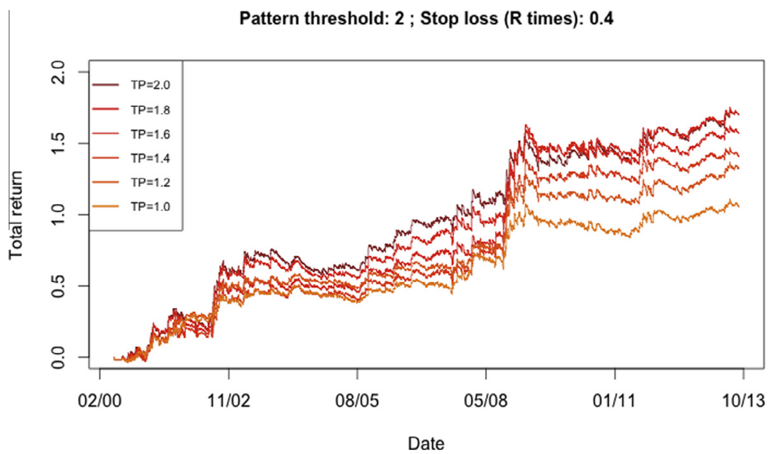

Fig. 8. Total return curves for FTSE index, with pattern threshold = 2 and stop loss = 0.4.

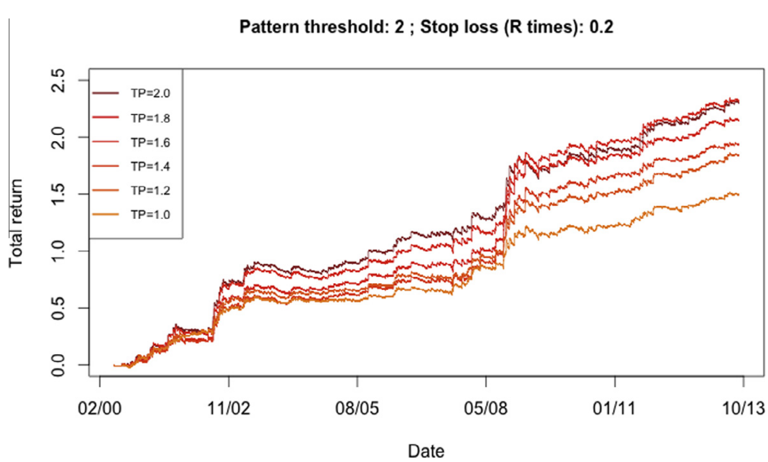

Fig. 9. Total return curves for FTSE index, with pattern threshold = 2 and stop loss = 0.2.

However, and in order to widen the number of case studies, it would be appropriate to validate the trading rule over a great number of stock market indexes, from different financial markets. In this section, we present the results obtained from applying the trading rule to two of the main European indexes: the German DAX and the British FTSE. In Figs. 6 and 7 we can see the evolution of the total return obtained from the trading rule applied to the DAX index for a threshold value of 2 and two different configurations of the stop loss: 0.4 and 0.2. These levels were selected from the conclusions drawn from the analysis of correlations

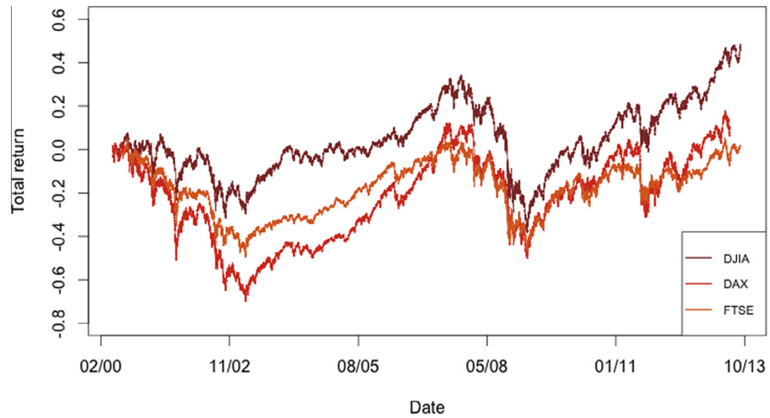

Fig. 10. Comparison of the return evolution for Dow Jones, DAX and FTSE.

shown in Table 2. At the same time inside each figure we can see the return corresponding to each one of the 6 possible *take profit* levels considered, from 1.0 to 2.0.

Figs. 8 and 9 show the results of the same analysis applied to the FTSE index.

As these results are in line with those obtained from the Dow Jones index we can also conclude that the Efficient Market Hypothesis can also be discarded in these markets. Although we do not include the detailed results table for reasons of space, we observed a higher return adjusted to the risk of the trading rule applied to the European indexes than to the US one.

This would imply that the European markets present more inefficiencies than the US markets and therefore give greater opportunities to both individual and institutional traders.

The robustness of the trading rule is confirmed more significantly when we observe the evolution of each of the three analyzed markets and compare the return on the trading rule. Fig. 10 shows the evolution of the returns on the Dow Jones, DAX and FTSE indexes in which bull and bear periods are combined. The trading rule shows a higher return than each of the analyzed indexes and moreover works with a much lower risk (drawdown) than the risks involved in the indexes.

Whereas the indexes experienced large falls during the last financial crisis, the behavior of our proposed trading rule remained steady and lineal and thus ensured returns adjusted to the risk independently of the stock market cycle.

7. Conclusions

This work provides empirical evidence which confronts the classical Efficient Market Hypothesis. A trading rule based on the flag patterns is presented and important contributions are introduced with respect to previous publications in the literature.

Firstly, we introduced a new version of the flag pattern: the breakout and consolidation flag pattern version. A new definition of the weight grid allows us to relate this kind of patterns to the IF-THEN rule, which we understand to be closer to the decisions made by stock market investors. This new approach reinforces the positive results reported for the flag pattern by previous studies.

Secondly, in order to address the data snooping problem, different variations were considered: the first one was linked to the use of intraday data, which allowed us to use historical data series of more than 90,000 observations for the US Dow Jones index. This large sample avoids any relationship of the results to random factors, and supports their statistical robustness. The second one introduces a dynamic approach when specifying the trading strategy, by closing the operations when the price level reached *stop loss* or *take profit*.

The definition of these two parameters together with the threshold, which limits the fitting of the price series to the flag pattern, allowed us to consider 96 different configurations. The third variation was to validate the trading rule over 3 non-overlapping sub-periods. We thus confirmed that the trading rule provided positive results for all the configurations, with a positive performance in the three sub-periods.

Thirdly, our results confirm that the trading rule provides a positive return, even after considering risk and beats the benchmark in the mean variance sense, which is also a new contribution to this field. The only consideration that an investor would need to consider would be to avoid transaction costs exceeding the average return obtained with the strategy.

The best results were obtained when considering adjusted levels of *stop loss* of between 0.2 and 0.4 times the price rank and also for high values of *take profit* with respect to the given *stop loss*. The explanation of these *stop loss* levels can be found when defining the weight grid which identifies the flag pattern. The proposed weights clearly limit one consolidation area which the price should not pass through, this area is located over the 30% limit (first three rows of the figure). In fact, this level corresponds to a *stop loss* of 0.2/0.4, where the strategy gets its best results. This result indicates that this level is acting as "support" to price, so that when the price passes through this level it will be quite likely to finish the operation with losses, whereas when it stays below this level it will be very likely to obtain a successful operation.

Last but not least, the strategy was validated not only for the US Dow Jones index, but also for two leading European indexes: the DAX and FTSE. In both cases the results are in line with the ones obtained for the Dow Jones, strengthening the robustness and significance of the trading rule.

The higher returns of the trading rule for the European indexes highlights the greater inefficiency of the European markets, in comparison to the US market.

Although the results show the inefficiency of the analyzed markets, a further analysis is needed to limit the number of failed operations and to increase the return of the trading rule. As a future line of research we consider that the results might be improved by using other indicators. By including information on indicators like the moving averages, stochastic and/or MACD with techniques like decision trees, neural networks or Support Vector Machines, we believe it will be possible to filter the operations and to increase the return adjusted to the risk of the trading rule.

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