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Bayesian change point analysis of Bitcoin returns

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ABSTRACT

This paper studies existence of structural breaks in the average return and volatility of the Bitcoin price. We utilize a Bayesian change point model to detect structural breaks and to partition the time series into segments. We find that structural breaks in average returns and volatility of Bitcoin are very frequent. By merging segments with similar properties into regimes we identify several regimes with positive average returns and one regime with negative average returns. Across regimes, higher volatility is associated with higher average returns, with exception of the most volatile regime, which is the only regime with negative average returns.

1. Introduction

Bitcoin is the first and most popular decentralized, open-source cryptocurrency. Its popularity has been increasing tremendously, both in terms of media coverage and in terms of its market value. As of December 11, 2017, Bitcoins market capitalization exceeds 280 billion USD. An overview of the economics of Bitcoin and private digital currencies in general can be found in e.g. Dwyer, 2015.

Recently, Bitcoin has been extensively studied from the perspective of finance. Ciaian et al. (2016) study the economics of Bitcoin price formation and find that market forces and Bitcoin attractiveness have a significant impact on the Bitcoin price. Bouri et al. (2017a) conclude that the Bitcoin price reacts positively to uncertainty. Urquhart (2016), Bariviera (2017), Nadarajah and Chu (2017) and Tiwari et al. (2018) find that the Bitcoin market is informational efficient, particularly in recent period. However, Jiang et al. (2017) arrives to opposite conclusion. Feng et al. (2017) find evidence of informed trading in the Bitcoin market. Research whether Bitcoin should be considered primarily a currency, or a speculative asset, mainly agree that the Bitcoin market is highly speculative (Baek and Elbeck, 2015; Cheah and Fry, 2015; Kristoufek, 2015; Dyhrberg, 2016; Blau, 2017; Corbet et al., 2017)

For investors considering whether to include Bitcoin in their portfolios, it is important to understand Bitcoins risk-return characteristics, as well as its correlation with other assets. It has been documented in the literature that Bitcoin is largely uncorrelated with other financial assets (Bouri et al., 2017c; 2017b; Baur et al., 2018).

Average return and volatility are usually estimated from historical data. Since the Bitcoin market experiences rapid changes, it is important to understand whether average return and volatility of Bitcoin remain constant over time, and if they keep changing, how often it happens. Average returns and volatility are required inputs for uncountable types of analysis, with portfolio optimization being probably the most important one. Knowledge about stability of these parameters is therefore paramount of any analysis of Bitcoin as a financial asset. We utilize Bayesian change point (BCP) analysis to investigate the presence of various segments in the distribution of Bitcoin returns, which are allowed to be independent of one another and are not restricted to a prespecified number. This allows for an unrestricted view of the dynamics of the expected return and volatility of the Bitcoin price. We find that there are many segments, i.e. distribution of Bitcoin returns is subject to frequent changes.

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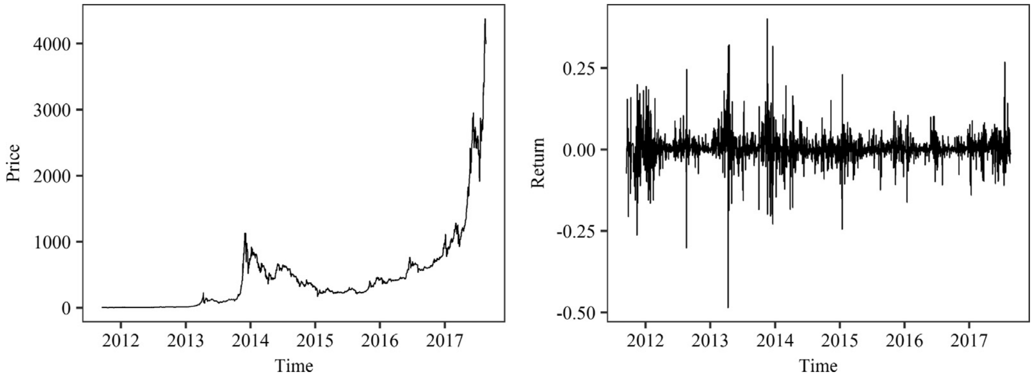

Fig. 1. Price index and returns of Bitcoin.

In financial markets, higher average return is usually a reward for bearing higher risk. We investigate whether this relationship holds also for Bitcoin. In order to easily see whether higher volatility is associated higher expected returns, we merge segments with similar statistical properties into regimes. We find seven regimes. On average, there is a positive association between volatility and the average returns of Bitcoin across these regimes. Low volatility regimes are associated with lower (but still positive) average returns and regimes with high volatility are associated with high average returns. However, we also document the presence of the most volatile regime, which is the only regime associated with negative average returns.

The rest of this paper is organized as follows: Section 2 introduces the data, Section 3 explains the methodology, Section 4 presents the results and Section 5 concludes.

2. Data

We obtain the Bitcoin price from the Bitstamp marketplace which is one of the larges Bitcoin exchanges (Brandvold et al., 2015). The time series spans a period from September 2011 to August 2017, resulting in 2170 daily observations. We calculate the return of the Bitcoin price as the logarithm of the ratio of two consecutive prices. Fig. 1 shows the price index as well as the log-returns.

Table 1 presents descriptive statistics for daily Bitcoin returns. The mean and the median are both positive and measure 0.41% and 0.22% respectively. The standard deviation of daily Bitcoin returns is 4.90%, which is higher than for most financial assets. The largest price decrease is $-$ 48.52% and the largest price increase is 40.14%. Skewness and kurtosis further characterize the return distribution as nearly symmetric but highly leptokurtic.

The return plot in Fig. 1 and the summary statistics suggest that the Bitcoin returns series can be modeled by a symmetric distribution with periods of extreme events. In the next section we describe a model which is able to partition a time series into segments in which the returns are independent of any other segment and follow a normal distribution.

3. Methodology – Bayesian change point model

Suppose a univariate time series of T Bitcoin returns denoted $\boldsymbol{r}_{1:T} = (r_1, ..., r_T)$. We can partition this time series into $m + 1$ segments divided by change points at locations $\tau_{1:m} = (\tau_1, ..., \tau_m)$, and set $\tau_0 = 0$ and $\tau_{m+1} = T$. Within each segment i the return-data $\boldsymbol{r}_i = \boldsymbol{r}_{\tau_{i-1}+1:\tau_i}$ is assumed to follow a normal distribution with segment-specific parameters (μ_i, σ_i^2). Conditional on the change points and segment-specific parameters, the data are independent and identically distributed (iid). To make an inference about the unknown number and the location of the change points and about the segment-specific parameters we proceed as follows (Fearnhead, 2006).

First, we introduce a prior on the number and location of change points indirectly through a prior on the length of each segment. In particular, we model the segment length using a geometric distribution:

$$(\tau_i - \tau_{i-1}) \sim Geo(p). \tag{1}$$

Table 1
Descriptive statistics for daily Bitcoin returns.

Mean	0.41%
Median	0.22%
STD	4.90%
Min	−48.52%
Max	40.14%
Skewness	−0.04
Kurtosis	17.16
n	2170

The probability mass function and distribution function of the geometric distribution are defined as

$$g(t|\boldsymbol{\psi}) = p(1-p)^{t-1}, \tag{2}$$

$$G(t|\boldsymbol{\psi}) = \sum_{s=1}^{t} g(s|\boldsymbol{\psi}), \tag{3}$$

where $\boldsymbol{\psi} = \{p\}$ is a vector of hyperparameters which is used to summarize all parameters of the prior distributions.

Second, we introduce priors for the segment-specific parameters of the normal distribution:

$$r_t \sim N(\mu_i, \sigma_i^2), \quad t \in [\tau_{i-1}+1, \tau_i]. \tag{4}$$

In order to enable analytic calculus, a natural prior, when the data follows a normal distribution, is the normal-inverse-gamma (NIG) distribution. This prior implies an inverse gamma (IG) distribution on the segment-specific variance and a conditional normal distribution on the segment-specific mean:

$$\sigma_i^2 \sim IG(\alpha_0/2, \beta_0/2), \tag{5}$$

$$\mu_i|\sigma_i^2 \sim N(\mu_0, \sigma_i^2\kappa_0). \tag{6}$$

Third, we calculate the marginal likelihood of the data, which gives us a quantity that measures the likelihood of the data independent of any model specifications. The marginal likelihood is derived by multiplying the likelihood of the data by the respective priors for the parameters and then integrating out these parameters. Hence, we have

$$Q(\boldsymbol{r}_i|\boldsymbol{\psi}) = \int_{\mu_i}\int_{\sigma_i^2} N(\boldsymbol{r}_i|\mu_i, \sigma_i^2) \times N(\mu_i|\mu_0, \sigma_i^2\kappa_0) \times IG(\sigma_i^2|\alpha_0, \beta_0)d\mu_i d\sigma_i^2. \tag{7}$$

We assume that (7) can be calculated for every possible segment.

Finally, we can use standard filtering recursions to calculate the posterior distributions of the quantities of interest (Fearnhead and Liu, 2007). The posterior probabilities for the number and location of change points are calculated for $t = 2, \ldots, T$ and $j = 1, \ldots, t-2$ by

$$p(C_t = j|\boldsymbol{r}_{1:t}, \boldsymbol{\psi}) \propto \left(\frac{Q(r_{j+1:t}|\boldsymbol{\psi})}{Q(r_{j+1:t-1}|\boldsymbol{\psi})}\right) \times \left(\frac{(1-G(t-j|\boldsymbol{\psi}))}{(1-G(t-j-1|\boldsymbol{\psi}))}\right) \times p(C_{t-1} = j|\boldsymbol{r}_{1:t-1}, \boldsymbol{\psi}) \tag{8}$$

and

$$p(C_t = t-1|\boldsymbol{r}_{1:t}, \boldsymbol{\psi}) \propto Q(r_{t:t}|\boldsymbol{\psi}) \times \sum_{k=0}^{t-2} p(C_{t-1} = k|\boldsymbol{r}_{1:t-1}, \boldsymbol{\psi}) \times \left(\frac{g(t-k-1|\boldsymbol{\psi})}{(1-G(t-k-1|\boldsymbol{\psi}))}\right). \tag{9}$$

Simulating the number and location of change points is straightforward. Based on the simulation output, we can further infer the posterior mean and the posterior variance, which is calculated for every segment and weighted by the probability of the segment.

The BCP model has some advantages over the models that are commonly used for nonlinear analysis of financial time series. First, the number of segments is not limited. Second, new segments are independent of previous segments. If new and unseen situations occur, e.g. due to the changing environment of the financial market, this may be advantageous over models which only assume prespecified number of recurring regimes. Third, the posterior parameters are weighted by the likelihood of the segment and thus may also change continuously over time. This increases flexibility as the transition periods between segments can be modeled. Therefore, we find the BCP model a flexible tool for modeling the nonlinear behavior of a time series e.g. in contrast to a markov regime switching model. In the next section we employ the BCP model to analyze the Bitcoin return series.

4. Results

The upper part of Fig. 2 presents the Bitcoin returns, the posterior mean and a 95%-interval where the boundaries are calculated as 1.96 times the posterior volatility.

In the Bitcoin return series we find 48 structural breaks over the total period, which on average, is roughly one change point every 1.5 months. The identified segments differ in length and show strong variation in posterior mean and volatility. The transitions between the segments show both patterns abrupt from one point in time to another (e.g. in February 2012) and continuous lasting several months (e.g. from January 2015 to May 2015).

The first segment is rather long and has a posterior volatility of 8%, which is higher than most of the other segments. However, there are about four periods with segments whose posterior volatility is even higher than this. These include one week in the middle of August 2012 with a posterior volatility of 14%, the period from January 2013 to May 2013 where the posterior volatility is between 11% and 18%, the period from October 2013 to December 2013 with a volatility of around 13% and two weeks in January 2015 where the volatility ranges between 8% and 11%. In contrast, all other segments exhibit lower posterior volatility, which is mostly either 1.5% or 3.5%.

As we partition the Bitcoin time series into independent segments, we avoid the model to be overly restrictive. This provides a lot

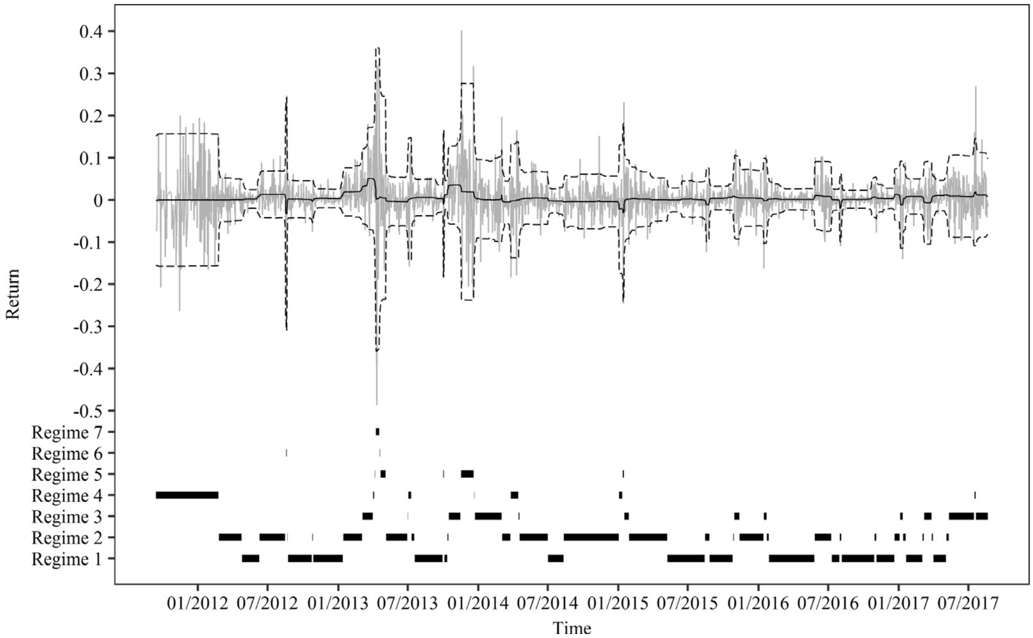

Fig. 2. Return, posterior mean, posterior 95%-interval and volatility cluster of Bitcoin.

of flexibility so that we can freely explore the dynamics of the time series. However, in order to go one step further, we combine segments that exhibit the same statistical properties. For example, most of the segments identified between January 2015 and August 2017 have posterior volatility of either 1.5% or 3.5%. This indicates a regime-switching process particular to these segments. We combine the independent segments by clustering them based on their posterior volatility.[1]

The clustering results are displayed in the lower part of Fig. 2. Regime 1 is associated with low volatility and regime 7 with high volatility. Table 2 shows summary statistics for each regime. While regimes 1 to 4 can be interpreted as normal situations with a volatility of up to 8.42% and a high frequency, regimes 5, 6 and 7 mark short periods with high volatility of up to 27.76%.

In financial time series the leverage effect describes an inverse relationship between stock prices and volatility. We do not observe such a relationship for the Bitcoin price. By investigating Fig. 2 in more detail and taking a closer look at the posterior mean in the high-volatility regimes (at the beginning and end of 2013), we first find a simultaneous rise in the mean return and volatility. However, as time goes on, the average return drops sharply while the volatility increases even more. In contrast, in January 2015 for example, we only find a rise in volatility while average return decreases. Hence, based solely on the posterior mean and volatility, we do not find evidence of a leverage effect in Bitcoin prices.

However, the comparison of various regimes created by the volatility clustering method (Table 2) reveals an interesting observation. Across the first six regimes, there is a positive association between mean return and volatility. The least volatile regime exhibits the lowest (but still positive) mean return and vice versa. However, this relationship does not hold for the most volatile regime, regime 7, which is the only regime with negative average returns.

This may be interpreted as follows. In most situations on the Bitcoin market (regime 1 to regime 6), investors exposed to higher volatility are rewarded with higher average returns. However, average return was negative during the most volatile regime 7, which occurred in 2013 and lasted ten days, marking a situation that is to extreme and uncertain for Bitcoin investors.

5. Conclusion

We have used the Bayesian change point (BCP) model to analyze how do the mean and volatility of Bitcoin returns change over time. This approach has several advantages over other nonlinear time series models used in the financial literature. The dynamics in financial time series can be studied without making restrictive assumptions about the data-generating process, for example about the number of regimes. The BCP model enables us to partition the time series into independent segments and calculate the posterior mean and volatility in each segment.

We find that Bitcoin average returns and volatility are indeed subject to a high number of change points. This observation has important implications. In various kinds of financial analysis, expected return and volatility are required as an input. However, in case of Bitcoin, these parameters should be considered as highly uncertain because they have been subject to very frequent changes in

[1] We cluster segments into regimes based on the posterior volatility because volatility clusters are well known in finance and this is a simple first step. It would be straightforward to apply clustering based on the posterior mean instead, or on both criteria. We use hierarchical clustering and find the optimal number of regimes using different criteria, namely the within-cluster sum of squares, the average silhouette and the gap statistic method.

Table 2
Mean and volatility of Bitcoin returns within regimes.

Regime	Mean	Volatility	n
1	0.31%	1.63%	819
2	0.33%	3.27%	765
3	0.89%	5.57%	298
4	0.31%	8.42%	180
5	0.85%	13.40%	57
6	2.43%	19.05%	6
7	−1.30%	27.76%	10

the past. Any research that utilizes expected return and volatility of Bitcoin as inputs should be aware that this issue.

Next we merge the segments with similar statistical properties into regimes. We find six regimes with positive average returns and one regime with negative average returns. Across the six regimes with positive average returns, higher volatility is associated with higher average returns. In other words, Bitcoin investors are rewarded by higher expected returns during more volatile periods. However, the only regime with negative average returns is the most volatile regime. A possible interpretation is that this short period represents an extreme event. It is interesting to observe that established risk-reward mechanism of traditional financial markets seems to hold even for a highly dynamic and evolving Bitcoin market.

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