

Short Communication

Minute-ahead stock price forecasting based on singular spectrum analysis and support vector regression



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ABSTRACT

Time series modeling and forecasting is an essential and hard task in financial engineering and optimization. Various models have been proposed in the literature and tested on daily data. However, a limited attention has been given to intraday data. In this regard, the current work presents a model for intraday stock price prediction that uses singular spectrum analysis (SSA) and support vector regression (SVR) coupled with particle swarm optimization (PSO). In particular, the SSA decomposes stock price time series into a small number of independent components used as predictors. The SVR is applied to the task of forecasting and PSO is employed to optimize SVR parameters. The performance of our proposed model is compared to the performance of four models widely used in financial prediction: the wavelet transform (WT) coupled with feedforward neural network (FFNN), autoregressive moving average (ARMA) process, polynomial regression (PolyReg), and naïve model. Finally, the mean absolute error (MAE), mean absolute percentage error (MAPE), and the root mean of squared errors (RMSE) are used as main performance metrics. By applying all models to six intraday stock price time series, the forecasting results from simulations show that the presented SSA-PSO-SVR largely outperforms the conventional WT-FFNN, ARMA, polynomial regression, and naïve model in terms of MAE, MAPE and RMSE. Therefore, our proposed predictive system SSA-PSO-SVR shows evident potential for noisy financial time series analysis and forecasting.

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1. Introduction

Time series modeling and prediction is an important task in different applications [1–6]. In particular, several multiresolution decomposition methods have been adopted to analyze time series; including the traditional Fourier transform which is a basic tool in signal processing used to approximate the signal in terms of sinusoids and the wavelet transform [7] that simultaneously decomposes a signal into subsequences at different resolution time scales. In recent years, the singular spectrum analysis (SSA) of time series [8] has received a growing attention as a non-parametric time series modeling technique where an observed time series is unfolded into the column vectors of a Hankel structured matrix, known as a trajectory matrix [9]. In particular, the purpose is to unfold a time series into a trajectory matrix whose singular values are then determined to reconstruct a smoother time series which can be used for explaining structure and for forecasting [9]. Indeed, it decomposes the original time series into a sum of a small number of components: slowly varying trend, oscillatory components, and noise [10].

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Since forecasting stock market time series is receiving a large attention in the literature [11–15] because it is a major issue in economics and business applications and decision making, the purpose of this paper is to apply the SSA technique in forecasting intraday stock market prices to enrich the literature related to SSA application in this topic. Indeed, in this paper, we present a prediction system that uses SSA for time series decomposition, support vector regression (SVR) [16] for learning and prediction, and particle swarm optimization (PSO) which is a global optimization method [17] for SVR initial parameters optimization. As a version of the support vector machine (SVM) [18], the main advantage of the SVR is applying the structural risk minimization principle to minimize an upper bound on the generalization error rather than implementing the empirical risk minimization principle to minimize the training error [18]. Therefore, it theoretically guarantees to achieve the global optimum. In this regard, recently, the SVM and SVR have been successfully employed to solve non-linear regression and classification of time series problems in finance and engineering [19,20].

The performance of the presented forecasting system will be compared to that of conventional system used in the literature which is based on wavelet transform (WT) [7] for financial data decomposition and feedforward neural network (FFNN) [21] for forecasting [22]. In addition, the autoregressive moving average (ARMA), the polynomial regression (PolyReg) and naïve models are also considered in this study for comparison as well.

In sum, the contributions and highlights of this paper can be summarized as follows. First, we develop a hybrid approach based on the SSA financial time series processing technique and support vector regression for time series forecasting. Second, particle swarm optimization which is a global optimization heuristic technique is adopted to optimize SVR initial parameters. Third, the presented forecasting model is applied to a set of individual stocks rather than stock indices to gain further insight on the applicability of the proposed predictive system. Fourth, results are compared against three benchmark models: FFNN trained with WT coefficients (WT-FFNN), polynomial regression (PolyReg), naïve model, and the classical ARMA process.

The remainder of this paper is organized as follows. In Section 2, the technical methods are presented. The forecasting results are presented in Section 3. Finally, Section 4 concludes the paper.

2. Methods

2.1. Singular spectrum analysis

In this section we provide a brief review of the SSA as adapted from [23–26]. The SSA is based on the singular value decomposition (SVD) of the trajectory matrix, derived from the original time series [25]. In particular, it decomposes the time series into an additive set of independent principal components [27]. The basic SSA method consists of two complementary steps [24]; namely the decomposition and the reconstruction step; where each step includes two separate steps. The original signal is decomposed and reconstructed respectively in the first and second step. For instance, the decomposition step includes an embedding operation followed by singular value decomposition (SVD). Finally, the reconstruction step includes grouping and diagonal averaging operation. Each step is described following [24] and [25] notations.

In the embedding procedure, the purpose is to map a one-dimensional time series f of length N into an $l \times k$ matrix with rows of length l as follows:

$$\mathbf{X} = [\mathbf{X}_1, \dots, \mathbf{X}_k] = \begin{bmatrix} f_0 & f_1 & \dots & f_{k-1} \\ f_1 & f_2 & \dots & f_k \\ \vdots & \vdots & \ddots & \vdots \\ f_{l-1} & f_l & \dots & f_{r-1} \end{bmatrix} \quad (1)$$

where the trajectory matrix \mathbf{X} is a Hankel matrix, vectors \mathbf{x}_i are called l -lagged vectors, $k = r - l + 1$ is the number of windows ($1 \leq l \leq r$). The embedding operation is followed by applying SVD to the trajectory matrix \mathbf{X} to represent it as a sum of rank-one bi-orthogonal elementary matrices. The SVD of the trajectory matrix is given by:

$$\mathbf{X} = \sum_{i=1}^d \mathbf{X}_i = \sum_{i=1}^d \sqrt{\lambda_i} \mathbf{u}_i \mathbf{v}_i^t \quad (2)$$

where λ_i ($i = 1, \dots, l$) are the eigenvalues, in decreasing order of magnitude, of the covariance matrix $\mathbf{C}_x = \mathbf{X}^t \mathbf{X}$, d_i ($i = 1, \dots, l$) are the corresponding orthogonal eigenvectors, subscript t denotes the transpose of a vector, and \mathbf{v}_i is given by:

$$\mathbf{v}_i = \mathbf{X}^t \mathbf{u}_i / \sqrt{\lambda_i} \quad (3)$$

In the grouping step, the elementary matrices \mathbf{X}_i are split into several groups and a summation of matrices is performed within each group. For instance, the indices $j = 1, \dots, d$ are grouped into M disjoint subsets I_1, \dots, I_M corresponding to split the elementary matrices $\mathbf{X}_i = 1, \dots, d$ into M groups; where Each group contains set of indices as $I = \{i_1, \dots, i_p\}$. As a result, the matrices \mathbf{X}_I and \mathbf{X} are respectively given by:

$$\mathbf{X}_I = \mathbf{X}_{i_1} + \dots + \mathbf{X}_{i_p} \quad (4)$$

$$\mathbf{X} = \mathbf{X}_{I_1} + \dots + \mathbf{X}_{I_M} \quad (5)$$

Finally, the diagonal averaging step seeks to transform each matrix I into a time series. In particular, an approximation of the original time series is reconstructed by diagonal averaging the M subsets of the grouped elementary matrices in Eq. (5). Assume X is of size $L \times K$ with elements x_{ij} and let $L^* = \min(L, K)$, $K^* = \max(L, K)$, $x_{ij}^* = x_{ij}$, if $L < K$; and $x_{ij}^* = x_{ji}$, otherwise. Then, the diagonal averaging transforms the matrix X into a series (principal component) g_0, \dots, g_N as follows [25]:

$$\begin{aligned} g_{k+1} &= (k+1)^{-1} \sum_{m=1}^{k+1} x_{m, k-m+2}^* \text{ if } 0 \leq k < L^* - 1 \\ g_{k+1} &= L^{*-1} \sum_{m=1}^{L^*} x_{m, k-m+2}^* \text{ if } L^* - 1 \leq k < K^* \\ g_{k+1} &= (N-k)^{-1} \sum_{m=k-k^*+2}^{N-k+1} x_{m, k-m+2}^* \text{ if } K^* - 1 \leq k < N \end{aligned} \quad (6)$$

The diagonal averaging of each resultant matrix produces a sub-series with the length N [25]: $g^k = (g_1^k, \dots, g_N^k)$. Then, the original time series f of length N can be reconstructed by summation over the produced sub-series as follows:

$$\tilde{f}_n = \sum_{k=1}^m g_n^k \quad (7)$$

where $1 \leq n \leq N$. The series g^k are also known as reconstructed components [28]. A compact description of the SSA can be found in [26]. They consist of a trend representing the time series mean at each instant, a set of periodic series, and an aperiodic noise [29]. As a result, they will be used to predict next-day stock price.

2.2. Support vector regression

Let $\{(\mathbf{x}_k, \mathbf{y}_k)\}_{k=1}^N$ denote the k th input vector \mathbf{x} of the k th training pattern and \mathbf{y}_k represent its corresponding desired output. Then, regression function f which is performed by a linear SVM [18] is expressed as follows:

$$f(\mathbf{x}) = \omega \mathbf{x}^T + b \quad (8)$$

where \mathbf{x} , $\omega = (\omega_1, \omega_2, \dots, \omega_n) \in \mathbb{R}^n$, $b \in \mathbb{R}$ and T are respectively the input vector, the weight vector, the intercept, and transpose operator. The optimization problem for training the linear SVR is given by:

$$\text{Minimize } \frac{1}{2} \|\omega\|^2 + C \sum_{k=1}^N (\xi_k + \xi_k^*) \quad (9)$$

Subject to,

$$y_k - \omega \mathbf{x}_k^T - b \leq \varepsilon + \xi_k \text{ and } \omega \mathbf{x}_k^T + b - y_k \leq \varepsilon + \xi_k^* \quad (10)$$

where C is the penalty for incorrectly estimating the output associated with input vectors, $\varepsilon > 0$ is the regularization factor that weights the trade-off between the \mathbf{y} estimated value and the target value, and ξ and ξ^* are slack variables, and $k = 1, \dots, N$. Briefly speaking, the nonlinear support vector regression (SVR) [16] seeks to solve the following nonlinear regression problems:

$$f(\mathbf{x}) = \omega \varphi(\mathbf{x})^T + b = \sum_{k=1}^N (\alpha_k - \alpha_k^*) \varphi(\mathbf{x}_k) \varphi(\mathbf{x}_k)^T + b \quad (11)$$

where $\varphi(\mathbf{x})$ denotes a mapping function that maps the input vector \mathbf{x} into a higher dimensional feature space, and where α and α^* are the Lag range multipliers. The inner product of functions $\varphi(\mathbf{x})$ and $\varphi(\mathbf{x})^T$ can be replaced by a kernel function $K(\cdot)$. Thus, the general form of the SVR is given by:

$$f(\mathbf{x}) = \sum_{k=1}^N (\alpha_k - \alpha_k^*) K(\mathbf{x}, \mathbf{x}_k) + b \quad (12)$$

In this study, third order polynomial kernel function $K(\cdot)$ is chosen.

In this study, PSO [17] is used to optimize the SVR main parameters; namely C and ε by finding the optimal values that minimize the SVR root mean square error (RMSE) during its training phase. The PSO algorithm is an evolutionary optimization technique based on population of particles used to represent a potential problem solution. These particles move through a multidimensional search space, where each particle changes its position in the direction of its previously best position and best position of all other particles to find the optimal solution. In our work, the matrix encoding strategy is adopted as it is suitable for training [30]. The initial velocities of particles are randomly generated in the interval $[0, 1]$. Finally, the population size is set to 50.

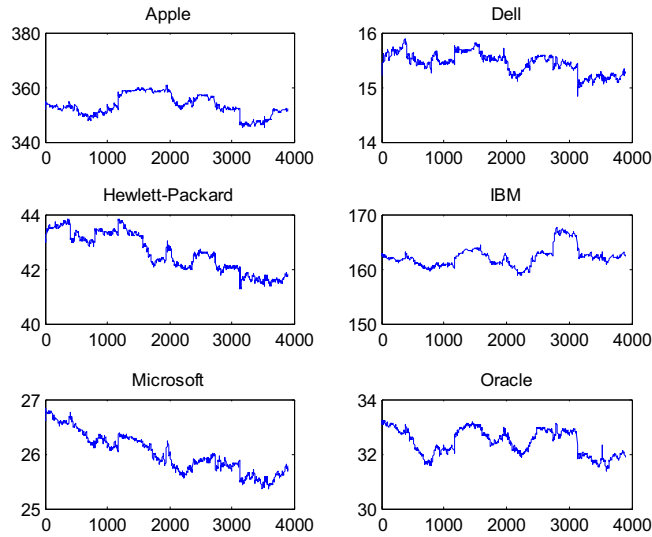


Fig. 1. Time series of stock prices.

2.3. Benchmarks

For comparison purpose several models are considered. For instance, we use FFNN [21] trained with stock price approximation coefficients obtained by wavelet transform (WT) [7]. In addition, we employ polynomial regression [31], naïve model, and the well-known traditional ARMA process. The FFNN is a nonlinear system with neurons used to process data. A standard architecture has one input layer with x predictive variables, one hidden layer for input–output mapping, and an output layer with the predicted variable y . The sigmoid function is adopted in this work as a transfer function for data processing. The polynomial regression (PolyReg) [31] is a linear regression in which the relationship between the independent variable x (for instance, actual stock price) and the dependent variable y (the future stock price) is expressed in the form of a polynomial equation. In this study, a third degree (cubic) polynomial is considered. Besides, the naïve model is based on a no-change criterion. For instance, this naïve process establishes that stock price estimated for one day ahead is simply the actual price of the previous day. Finally, the forecasting performance is evaluated using the following performance metrics: the mean absolute error (MAE), mean absolute percentage error (MAPE), and the root mean of squared errors (RMSE).

3. Experimental results

In our work, the SSA-FFNN-PSO, conventional WT-FFNN model, polynomial regression (PolyReg), naïve model, and ARMA process are tested on intraday stock prices of Apple, Dell, Hewlett-Packard (HPQ), IBM, Microsoft (MSF) and Oracle. The Daubechies-4 (DB4) wavelet is adopted in this study at third decomposition level. Fig. 1 exhibits the times series of each individual stock price. All of the data are minute-in-day closing prices for the period from February 28th 2011 to March 11th 2011. There are 3910 data points for each dataset. The first 80% of the total sample points are selected as the training sample, while the remaining 20% are selected for testing purpose. As mentioned previously, the goal is to predict next minute stock price level. The obtained prediction performances are provided in Table 1.

According to Table 1, it is evident that our model (M1) outperforms all other ones used in our study in terms of MAE, MAPE, and RMSE. In addition, both polynomial regression (M3) and naïve model (M4) perform better than the WT-FFNN system (M2) and ARMA process (M5) based on all three performance measures. Based on these findings, our predictive model SSA-PSO-SVR, polynomial regression and naïve model achieve quite small forecasting errors in comparison with conventional WT-FFNN system and classical ARMA process when predicting intraday prices of individual stocks under study. Besides, Fig. 2a and b exhibits the predicted stock prices by each model in comparison with true stock price values. For all stocks, it is clearly shown that our proposed model (SSA-PSO-SVR) fits well the curve of true price time series. In this regard, it is also observed that naïve model and polynomial regression (PolyReg) closely fitted the time series of true prices. However, both ARMA process and WT-FFNN system yielded to poor fitting results; particularly, the ARMA process. Indeed, all conclusions from Fig. 2a and b are in accordance with performance metrics provided in Table 1.

In summary, the above comparisons (see Table 1) between SSA-PSO-SVR (M1), WT-FFNN system (M2), polynomial regression (M3), naïve model (M4), and ARMA process (M5) show that SSA-PSO-SVR system is the superior one in its performance to forecast one-minute-ahead stock price of each individual stock used in our experiments. This result can be explained by the ability of SSA to analyze the time series under study to provide its hidden patterns, the role of PSO in finding optimal initial parameters of the SVR, and the ability of the latter to apply the structural risk minimization principle to minimize

Table 1
Results for individual stocks.

	MAE	MAPE Apple	RMSE	MAE	MAPE IBM	RMSE
M1	0.00108	0.00031	0.00185	0.00002	0.00001	0.00003
M2	4.29550	1.23425	5.30180	3.37560	2.07890	3.41450
M3	0.06010	0.01732	0.07400	0.00112	0.00069	0.00150
M4	0.14655	0.04211	0.24890	0.06404	0.03944	0.10345
M5	7.85680	2.25753	41.72570	4.02780	2.48057	19.36990
		Dell			MSF	
M1	0.00004	0.00025	0.00020	0.00002	0.00010	0.00004
M2	0.12930	0.85505	0.14250	0.69870	2.73402	0.71200
M3	0.00331	0.02188	0.00347	0.00409	0.01605	0.00457
M4	0.01059	0.07002	0.02508	0.01123	0.04396	0.01845
M5	0.44200	2.92291	1.84140	0.50330	1.96942	3.03900
		HPQ			Oracle	
M1	0.00064	0.00155	0.00086	0.00003	0.00010	0.00007
M2	2.60990	6.27088	2.63290	1.04130	3.26911	3.87510
M3	0.01684	0.04051	0.01785	0.00178	0.00560	0.00227
M4	0.01956	0.04700	0.03041	0.02002	0.06287	0.03254
M5	0.79000	1.89815	4.94380	1.04130	3.26911	3.87510

M1: SSA-PSO-SVR; M2: WT-FFNN; M3: PolyFit; M4: Naïve; M5: ARMA.

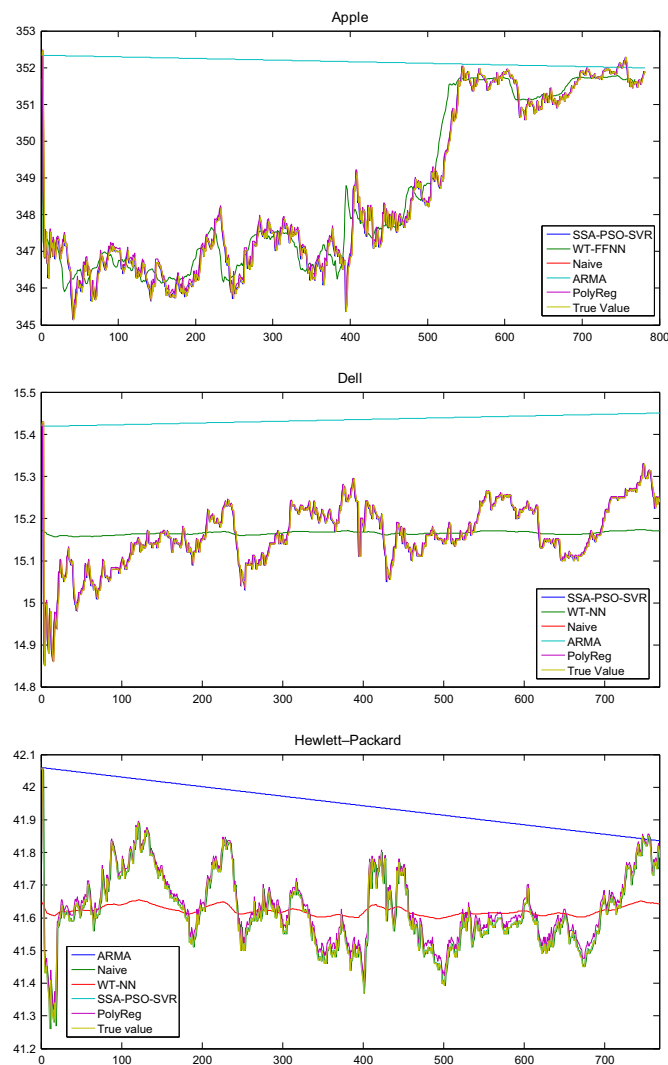


Fig. 2. (a) Comparison of models in terms of predicted values: Apple, Dell, Hewlett-Packard. (b) Comparison of models in terms of predicted values: IBM, Microsoft, Oracle.

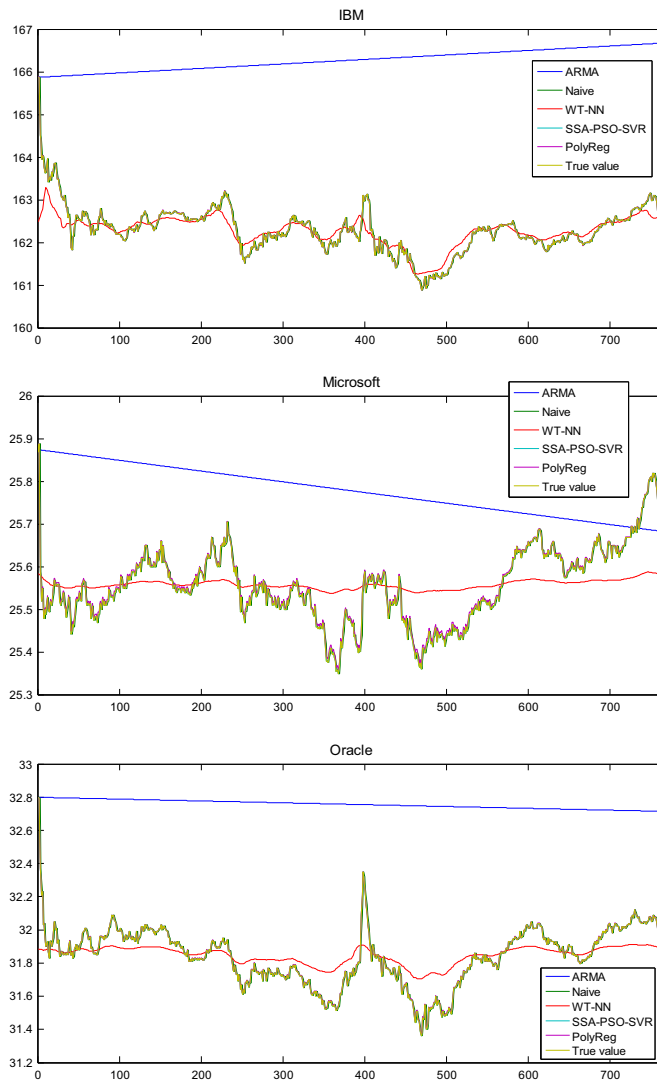


Fig. 2. Continued

upper bound on the generalization error to achieve the global optimum. Indeed, the SSA-PSO-SVR is effective; but, it requires relatively a long processing time to model data under study. This is mainly due to fine tuning of parameters by PSO and learning process of the SVR. However, PSO is still a fast and effective heuristic optimization technique in comparison with alternative heuristic optimization techniques [32,33]. In contrary, the conventional approach which is based on the WT-FFNN and widely used in the literature has some drawbacks. First, the WT requires a predetermined wavelet function. In this study, the popular Daubechies-4 is chosen although other functions could be used and tested. This issue is left to future work. Second, the FFNN adopts the empirical risk minimization principle to minimize the training error; as a result, it could be trapped in local optimum. Besides, it is interesting to notice that the performance of the WT-FFNN model can be improved by incorporating PSO for FFNN parameters tuning and also by employing both approximation and details coefficients—obtained by WT after applying it to original time series—as predictive patterns [34]. It is also interesting to indicate that the simple naïve model was found to be valuable in predicting intraday stock prices considered in our work. This predictive model is straightforward to implement and is based on simple statistical assumption. Further, although the polynomial regression is basically a linear estimation, it is also fast and effective to fit the nonlinear relationship between actual and future price level. Finally, as ARMA process is basically a linear model based on strong statistical assumptions, it yielded to poor results when used to model and predict noisy intraday stock prices used in our study.

In our study, we enriched previous works on stock price forecasting [11–15,35,36] by using singular spectrum analysis with conjunction with artificial neural networks and particle swarm optimization. In particular, we proposed a new intelligent predictive system with application to intraday stock prices known to be highly noisy. The results were found to be encouraging in comparison with various existing models. Indeed, time series analysis, modeling and forecasting is a hot

topic not only in financial applications [4,11–15,35,36], but also in environmental engineering [1–3], mechanical engineering [5,6], and computational biology [37–42] to name a few. In this regard, our proposed model could be applied to some science and engineering problems to check its effectiveness.

4. Conclusion

This paper compared a new stock price forecasting model based on singular spectrum analysis and support vector regression coupled with particle swarm optimization with other four techniques widely used for stock market price prediction. We performed an empirical study with six intraday stock price time series. The forecasting performance results showed that the presented predictive model achieved the lowest MAE, MAPE, and RMSE for all time series used in the study. As a result, the presented approach; based on singular spectrum analysis and support vector regression for which particle swarm optimization is adopted to optimize parameters; is a promising tool for intraday stock price prediction due to its excellent forecasting capability associated with its ability to capture hidden information in intraday financial time series; such as trend and harmonic components.

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