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Time-varying long-term memory in Bitcoin market

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ABSTRACT

This study attempts to investigate the time-varying long-term memory in the Bitcoin market through a rolling window approach and by employing a new efficiency index (Sensoy and Hacihasanoglu, 2014). The daily dataset for the period from 2010 to 2017 is utilized, and some interesting findings emerge that: (i) all of the generalized Hurst exponents in the Bitcoin market are above 0.5; (ii) long-term memory exists in the Bitcoin market; (iii) high degree of inefficiency ratio; (iv) the Bitcoin market does not become more efficient over time; and (v) rolling window approach can help to obtain more reliable results. Some implications for investors and policy-makers are concluded.

1. Introduction

Since the first time it was introduced by Nakamoto (2008), Bitcoin has received extensive attention among policymakers, investors, customers, and economists due to both the opportunities and challenges it presents (Nakamoto, 2008). As a new kind of cryptocurrency, Bitcoin can be traded online at any time and exchanged into several kinds of major currencies at a low cost of foreign exchange (Fry and Cheah, 2016; Kim, 2017). Compared to other traditional financial assets, Bitcoin provides investors a new instrument in portfolio management. Over the last few years, there have been many studies about the Bitcoin market (see e.g. Brandvold et al., 2015; Dwyer, 2015; Urquhart, 2016; Nadarajah and Chu, 2017; Bouri et al., 2016; Baur et al., 2017). Balcilar et al. (2017) empirically analyze the relationship between Bitcoin trading volume and return but find no evidence that trading volume helps to predict the volatility of Bitcoin returns.

The efficient market hypothesis (EMH) is one of the fundamental theories for analyzing financial assets (Fama, 1970). There are three forms of market efficiency. The one most commonly discussed is the weak form, where investors cannot earn consistent abnormal profits by analyzing past information, and the returns series follows a random walk. According to the research of Mandelbort (1971) and many others (Fama and French, 1988; Lo and Mackinley, 1988; Brock et al., 1992), long-term memory exists in financial asset returns and investors can earn abnormal profits with this existence, meaning that the weak form market hypothesis can be rejected. There have been some studies on the inefficiency of the Bitcoin market recently, for instance, Urquhart (2016) uses many tests to analyze the efficiency of Bitcoin market and conclude that it becomes more efficient in the latter time sample. Nadarajah and Chu (2017) utilize eight different tests on an odd integer power transformation of Bitcoin returns and show that returns are weakly efficient. However, to the best of our knowledge, the studies, which focus on the time-varying long-term memory of the Bitcoin market, have never been discussed before. Sensoy and Hacihasanoglu (2014) estimate time-varying generalized Hurst exponents to investigate the presence of long-term memory among several energy futures contracts, providing a new perspective to study the market efficiency of financial assets.

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We aim to investigate the existence of long-term dependence in the Bitcoin market and gap the literature about the time-varying inefficiency of Bitcoin. Empirical results show that long-term memory exists in the Bitcoin market and no evidence has been found to prove the improvement of the efficiency in it. Through the results of this study, we expect to increase the information efficiency for investors and policymakers.

This paper contributes to the literature in three ways. First, we introduce generalized Hurst exponents to analyze the long-term memory of the Bitcoin market and apply estimation techniques of the index to yield results that are more robust compared to other traditional Hurst exponents (Barunik and Kristoufek, 2010; Bariviera et al., 2017). Second, compared to some static analysis of the financial markets (Jiang et al., 2017; Urquhart, 2016), we utilize a rolling window approach to examine the time-varying inefficiency of the Bitcoin market and consider different time windows and day shifts to do a robustness check to obtain decent results. Finally, we compare the tests of Urquhart (2016), indicating that rolling window approach can help to obtain more reliable results.

2. Methodology

2.1. Generalized Hurst exponents

In past literature, the long memory in time series is usually analyzed by using the two popular methods namely modified *R/S* analysis and the variance ratio tests. In this paper, we will utilize a different methodology. We emphasized on the degree of long memory with a stochastic log process of Bitcoin price series $S(t)$. Variable t is defined as $(1, 2, \ldots, \Delta t)$ with Δt as the length of the time window with unitary time steps. To measure the extent of long memory, we use function $H(q)$. $H(q)$, which is a simplification of the approach proposed by Hurst (1951) and can be evaluated using the qth order moments of the increase distribution. This is a good characterization of function $S(t)$ statistical progression. The scaling behavior of the absolute increments is described by equation $K_q(\tau)$ for $q = 1$. $K_q(\tau)$, which is proportional to the autocorrelation function $C(t,\ \tau) = \langle S(t + \tau)S(t)\rangle$ (Di Mateo et al., 2005).

$$K_q(\tau) = \frac{\langle |S(t+\tau) - S(t)|^q\rangle}{\langle |S(t)|^q\rangle}. \tag{1}$$

For the equation above, τ can vary between 1 and τ_{max} and $\langle \ldots \rangle$ refers the sample average over the time period. $H(q)$ is then denoted for each time scale τ and each parameter q.

$$K_q(\tau) \propto \tau^{qH(q)}. \tag{2}$$

$H(q)$ is obtained simply through a linear least squares fitting by using a set of values corresponding to different values of τ_{max} in Eq. (2). Value of $H(q) = 0.5$ indicates that $S(t)$ does not display a sign of long memory. On the other hand $H(q) > 0.5$ implies that $S(t)$ is persistent and $H(q) < 0.5$ implies reversion to the mean[1,2]

2.2. Calculation of the standard errors

By using a post-blackening bootstrap approach and pre-whitening, we can find the standard errors of the $H(1)$ estimates. This method is proposed by Grau-Carles (2005) and is also previously used by Souza et al. (2008). The methodology structure can be summarized into the following steps:

1. Obtain the log returns series $r(t)$ from prices data.
2. Conduct the pre-whitening process by estimating an $AR(p)$ model for $r(t)$ with sufficient p (1 to 30). Akaike information criteria are used to estimate the order of the AR.
3. Obtain the residuals $\varepsilon(t)$ from the AR model historical sequence.
4. Obtain the simulated innovations by bootstrapping the residuals using circular block bootstrap (Politis and Romano, 1992).[3]
5. Obtain the synthetic log return series by making post blackening and adding the new series generated by bootstrap to the model whose parameter was generated in the pre-whitening.
6. Recover the synthetic prices recursively through its bootstrap samples and estimate the $H_b(1)$ for each synthetic log price by running 100 bootstrap samples.

Standard deviation of $S(H_b(1))$ is taken as the standard error of generalized Hurst exponents. Finally, we run Wald statistics[4] given by $W = \left(\frac{H(1) - 0.5}{S(H_b(1))}\right)^2$ and test the null hypotheses that *long-term memory does not exist.*

[1] Processes with a scaling behavior of (2) can be divided into two kinds: (i) unifractal processes where $H(q)$ is independent from q; (ii) multifractal processes where $H(q)$ is not constant and each moment scales with a different exponent. Previous studies show that financial time series show multifractal scaling behavior (Sensoy, 2013).

[2] The source code of generalized Hurst exponents can be found at: http://www.mathworks.com/matlabcentral/fileexchange/30076.

[3] The rule of choice of block length comes from Politis and White (2004).

[4] The W has the χ_1^2 distribution (Cajueiro and Tabak, 2008, 2009).

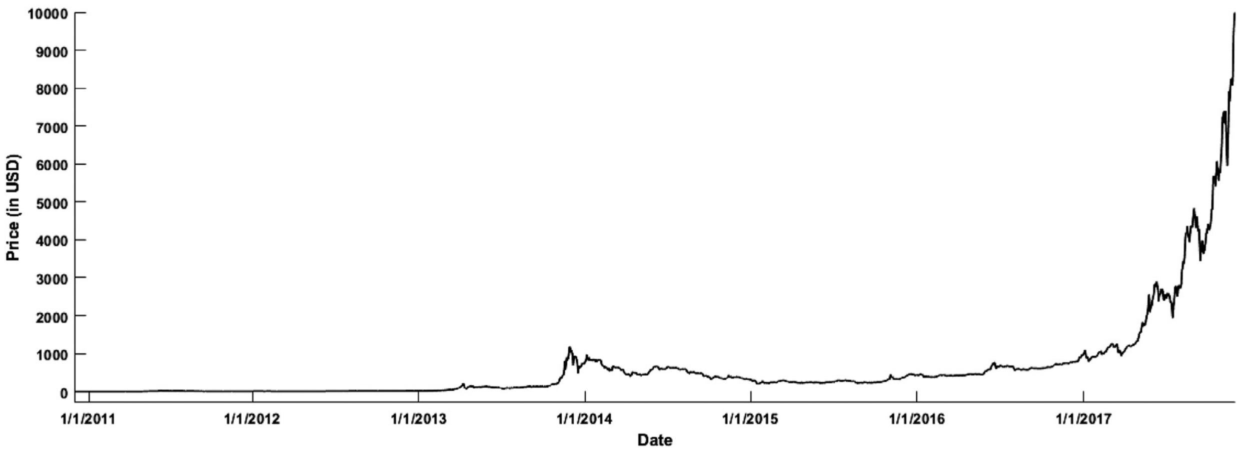

Fig. 1. Bitcoin daily price.

3. Data and results

The Bitcoin price dataset used in this paper includes 2551 daily observations throughout the time period from 1st December 2010 to 30th November 2017.[5] Fig. 1 shows that Bitcoin price is stable until a sharp rise that occurs at the end of 2013, after which the price of Bitcoin experiences intense fluctuation, especially in 2017. Regardless of this severe fluctuation, Bitcoin price has increased by over 50,000 times up to November 2017, showing a strong persistence of returns series. We define Bitcoin returns as

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right)\cdot 100 \tag{3}$$

where r_t is the return of Bitcoin price and p_t and p_{t-1} denote the prices of Bitcoin at time t and $t-1$ respectively. Table 1 presents the descriptive statistics of log price and r_t. The mean logs of price are positive with excess kurtosis and negative skewness, but the returns series are not normally distributed and display fat-tailed behavior. According to Barunik and Kristoufek (2010), compared to other traditional Hurst exponents, generalized Hurst exponent provides the lowest variance and is less sensitive to outliers regardless of the sample size. Furthermore, generalized Hurst exponent suits the returns series, which are not normally distributed and are heavy-tailed (Sensoy and Tabak, 2016). Therefore, considering the features associated with returns time series, we opt to use generalized Hurst exponent to test the existence of long-term memory, and expect more accurate results.

Recent studies reveal that some structural breaks should be considered when analyzing financial time series, but just using a few sub-samples or non-overlapping intervals could not capture this dynamic (Sensoy and Tabak, 2016; Nie et al., 2017; Hendrickson and Luther, 2017). Hence we apply a rolling window approach to examine the inefficiency of the Bitcoin market. We choose a 2-year (725 observations) time window and a 14-day shift (14 observations) since it is long enough to reflect its full effect and provides satisfactory statistical significance (Sensoy and Tabak, 2016). We use an approach similar to the one used by Sensoy and Hacihasanoglu (2014): Firstly, we calculate $H(1)$ and the standard errors for each window to obtain the Wald statistic W. Then mark a window inefficient if the null hypothesis of weak form efficiency is rejected. The inefficiency ratio is defined as the percentage of inefficient windows among the total number of windows.

In Fig. 2, black curves display the time-varying $H(1)$ for the Bitcoin market and blue and red markers denote the rejection of weak form efficiency at 5% and 1% significance levels respectively. The date in x-axis stands for the end of the sample used in estimation of the generalized Hurst exponents. For example, for the date Jan. 2013, the Hurst exponents were evaluated against the sample beginning with 725 observations earlier and ending in Jan. 2013 and so forth. It can be seen that although the $H(1)$ declined in late 2015, it quickly rose and returned to its initial level in a short time. Moreover, all of the $H(1)$ are consistently above 0.5, implying that long-term memory exists in Bitcoin returns series, the market does not become more efficient over time. With a high $H(1)$, we can also easily conclude that a strong persistence exists in the Bitcoin market.

Table 2 presents the number of significant window and the inefficiency ratio at 5% and 1% significance levels respectively. There are 127 windows are inefficient at 1% significance levels, while 129 windows are inefficient at 5% significance levels, hence the Bitcoin market has a very high degree of inefficiency ratio.

Considering that the choice of the length of time window and the day shift with a rolling window approach is controversial (Balcilar and Ozdemir, 2013; Sensoy and Tabak, 2016), we repeat our work with different time windows and day shifts to do a robustness check. And all the results shown in Table 3 are similar with our previous findings, indicating that our results are sufficiently robust.

Urquhart (2016) divides the full sample period into two subsamples, and Ljung–Box test as well as AVR test exhibit that the

[5] Available at: www.bitcoinaverage.com.

Table 1
Descriptive statistics.

Variables	N	Mean	Std	Min	Max	Kurt	Skew
$\ln p_t$	2551	4.713885	2.406602	−1.660731	9.208758	2.480045	−0.642595
r_t	2550	0.422285	4.915910	−44.46070	37.22395	17.88438	−0.498384

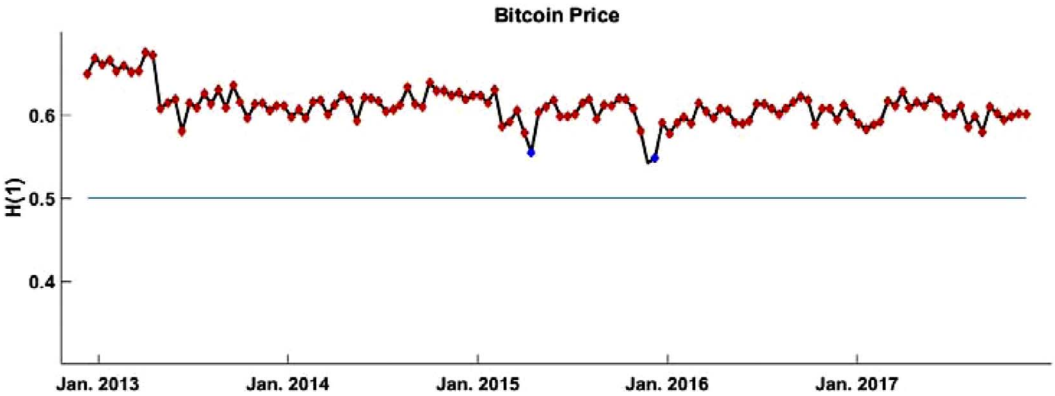

Fig. 2. Time-varying $H(1)$ for Bitcoin market. Blue and red markers denote the rejection of weak form efficiency at 5% and 1% significance levels respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Results of Bitcoin market on analysis with a rolling window of 2-year length.

Time window	Day shift	Sig. windows (1%)	Sig. windows (5%)	Total windows	Ineff. ratio (1%)	Ineff. ratio (5%)
725	14	127	129	130	97.69%	99.23%

Notes: A significant window (α%) is a window where market efficiency is rejected at α% significance level. Inefficiency ratio is calculated by dividing the number of significant windows by the total number of windows.

Table 3
Results of robustness check based on different time windows and day shifts.

Time window	Day shift	Sig. windows (1%)	Sig. windows (5%)	Total windows	Ineff. ratio (1%)	Ineff. ratio (5%)
725	21	85	86	86	98.84%	100%
725	28	64	65	65	98.46%	100%
1090	14	104	104	104	100%	100%
1090	21	69	69	69	100%	100%
1090	28	52	52	52	100%	100%
1455	14	78	78	78	100%	100%
1455	21	52	52	52	100%	100%
1455	28	39	39	39	100%	100%

Notes: The 725, 1090 and 1455 time window means 2, 3 and 4 years' time window respectively. A significant window (α%) is a window where market efficiency is rejected at α% significance level. Inefficiency ratio is calculated by dividing the number of significant windows by the total number of windows.

Bitcoin market is efficient in the latter period. We apply a rolling window approach to conduct Ljung–Box test and AVR test with a 725-day time window and a 14-day shift. The results show that null hypotheses of randomness (efficiency of the Bitcoin market) can be rejected in most of the periods but accepted in very few time windows, and we obtain similar results with different time windows and day shifts.[6] Hence, the results gained from a few sub-samples may be affected by some random factors and rolling window approach can help to reduce the random errors and obtain more reliable conclusions.

4. Conclusions

In this study, a new efficiency index is applied to test the existence of long-term memory in the Bitcoin market with a rolling window approach. Such a time-varying approach is useful in tracking the dynamic efficiency of the market. The results indicate that

[6] Due to limited space, the rolling window results of Ljung–Box test and AVR test are not shown here. However, we are glad to provide these if requested.

the Bitcoin market is inefficient and that the returns series presents a strong persistence in the full sample period. As an emerging market, the existence of long-term memory found in the Bitcoin market is not surprising. The rationales behind such results are related to the irrational behaviors of investors and the lack of a reasonable pricing mechanism. Hence, the results reveal some implications for investors and policymakers. On the one hand, before analyzing the actual value and pricing it prudently, the investors should not enter such a risky market arbitrarily and invest with only speculative purposes. On the other hand, the policymakers ought to strengthen the supervision on this kind of emerging market as Bitcoin. We hope that our results will provide guidance for investors, policymakers, and others. Further studies can focus on the development of efficiency in the Bitcoin market and find theories to price it correctly.

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