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A Forex trading expert system based on a new approach to the rule-base evidential reasoning

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ABSTRACT

Currently *FOREX* (foreign exchange market) is the largest financial market over the world. Usually the Forex market analysis is based on the Forex time series prediction. Nevertheless, trading expert systems based on such predictions do not usually provide satisfactory results. On the other hand, stock trading expert systems called also "mechanical trading systems", which are based on the technical analysis, are very popular and may provide good profits. Therefore, in this paper we propose a Forex trading expert system based on some new technical analysis indicators and a new approach to the rule-base evidential reasoning (*RBER*) (the synthesis of fuzzy logic and the Dempster–Shafer theory of evidence). We have found that the traditional fuzzy logic rules lose an important information, when dealing with the intersecting fuzzy classes, e.g., such as *Low* and *Medium* and we have shown that this property may lead to the controversial results in practice. In the framework of the proposed in the current paper new approach, an information of the values of all membership functions representing the intersecting (competing) fuzzy classes is preserved and used in the fuzzy logic rules. The advantages of the proposed approach are demonstrated using the developed expert system optimized and tested on the real data from the Forex market for the four currency pairs and the time frames 15 m, 30 m, 1 h and 4 h.

1. Introduction

The most of papers devoted to the Forex market analysis are based on the Forex time series prediction. Pang, Song, and Kasabov (2011) proposed a correlation-aided support vector regression (*cSVR*) for the Forex time series prediction. The proposed *cSVR* is experimented for time series prediction with 5 future contracts (*NZD/AUD, NZD/EUD, NZD/GBP, NZD/JPY*, and *NZD/USD*). Nevertheless, the *cSVR* prediction is found sometime surfing unexpectedly far away from the truth value. In Shmilovici, Kahiri, and Ben-Gal (2009), a universal Variable Order Markov model is proposed and used for testing the weak form of the Efficient Market Hypothesis (*EMH*). The *EMH* is tested for 12 pairs of international intra-day currency exchange rates. However, the authors noted that predictability of the model is not sufficient to generate a profitable trading strategy. In contrast to the earlier techniques, Bahrepour et al. (2011) proposed a high-order fuzzy time series identification scheme which utilises an adaptive order selection scheme and partitions the universe of discourse using self organising maps. This partitioning scheme allows different granularity at different parts of decision spaced. The proposed technique is then applied to the prediction of FOREX daily dataset. However, the possibility of the proposed method to generate profitable trading strategies is not studied. Amiri, Zandieh, Vahdani, Soltani, and Roshanaei (2010) proposed a new integrated eigenvector-DEA-TOPSIS methodology to evaluate the risks of portfolio in the Forex market. The proposed eigenvector-DEA-TOPSIS methodology uses eigenvector method to determine the weights of criteria, linguistic terms such as high, medium and low to assess Portfolio risks under each criterion. Bagheri, Peyhani, and Akbari (2014) proposed an approach to financial forecasting using ANFIS networks with Quantum-behaved Particle Swarm Optimization. They stated that, by implementing and testing the proposed method on real Forex data, they could forecast the market direction and make correct trading decisions with approximately 69% accuracy. Undoubtedly, this a very good result for prediction, but there are no real trading system or trading strategy in this paper.

Opposite to the above papers, in Mendes, Godinho, and Dias (2012) a Forex trading system based on a genetic algorithm and a set of ten technical trading rules is presented. Summarising, the authors wrote "The strategies that were obtained showed very good performance in the training series but, if we take transaction costs into account, they were often unable to achieve positive

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results in the out-of-sample test series." Obviously, such strategies cannot be used in practice.

Since ideas and methods used for the development of Forex trading system are similar to those used for building stock market trading systems (*SMTS*), here we present a short review of the works devoted to problems of *SMTS*.

The most of papers in this field were devoted to stock prise prediction. Kao, Chiu, Lu, and Chang (2013) proposed a hybrid approach by integrating wavelet-based feature extraction with MARS and SVR for stock index forecasting. A system for predicting stock and stock price index movement using Trend Deterministic Data Preparation and machine learning techniques was developed by Patel, Shah, Thakkar, and Kotecha (2015). Nevertheless, it was shown by Kuo, Chen, and Hwang (2001) that the most of works devoted to the stock price prediction generally used the time series analysis techniques (Kendall & Ord, 1990) and multiple regression models. It was noted by Haefke and Helmenstein (2000) that the models based on the stock prices prediction produced less than successful results when were used for building stock trading expert systems.

Artificial neural networks (*ANNs*) and genetic algorithms (*GAs*) have been applied for the stock prices prediction. However, these researches cannot produce at least satisfactory stock trading expert systems (Baba & Kozaki, 1992; Kim & Han, 2000; Kuo et al., 2001; Mahfoud & Mani, 1996; Mehta & Bhattacharyya, 2004).

The methods of Rough Sets Theory (Pawlak, 1982) for trading rules extraction was used in (Shen & Loh, 2004; Wang, 2003) with rather negative results.

The use of expert's wisdom makes it possible to develop more reliable stock trading systems. Gottschlich and Hinz (2014) proposed a decision support system design that enables investors to include the crowd's recommendations in their investment decisions and use it to manage a portfolio. The stock trading rule discovery with an evolutionary trend following model proposed by Hu, Feng, Zhang, Ngai, and Liu (2015) is implicitly based on the expert's experience and provides good results. The limitation of this study is that, the trading strategy rules are sub-optimal, which may lead to the fact that the rules discovered by eTrend are not global optimal. The use of expert's experience and intuition for the formulation of trading rules may be considered as a basic idea of a new approach presented in the current paper.

The first step in the development of trading system based on the expert's wisdom was made by Dourra and Siy (2002). They presented the expert system based on the fuzzy logic representation of trading rules used by traders. They used the fuzzy logic representation of technical analysis to create the rules which provide sell, buy and hold signals. They directly used the classical Mamdani's general form of fuzzy rules (Mamdani & Assilian, 1975), which provide real-valued outcomes. The Mamdani's approach was developed for fuzzy logic controllers and may lead to creation of somewhat artificial system of fuzzy rules when dealing decision making problems. Therefore, Santiprabhob, Nguyen, Pedrycz, and Kreinovich (2001) developed a new system of the so-called "Logic-Motivated Fuzzy Logic Operators" (*LMFL*) which better reflects the specificity of human reasoning in decision making processes. This system is based on a new approach to the mathematical representation of *t*-norm and Yager's implication rule (Turksen, Kreinovich, & Yager, 1998). Sewastianow and Rozenberg (2004) used the *LMFL* approach to develop stock trading decision support systems. It was shown that these systems provide significantly better results than Mamdani's approach proposed by Dourra and Siy (2002). In Sevastianov and Dymova (2009), two stock trading decision support systems based on *LMFL* and on the synthesis of fuzzy logic and *DST* were developed and compared using the real-world *NYSE* data. Based on the obtained results it was shown that *RBER* approach provides better and more reliable outcomes than those obtained using *LMFL* approach. Dymova, Sevastianov, and Bartosiewicz (2010) developed a new approach to the *RBER* which can be considered as an extension of the known *RIMER* method (Yang, Liu, Wang, Sii, & Wang, 2006; Yang, Liu, Xu, Wang, & Wang, 2007). The advantages of this approach were demonstrated using the developed stock trading decision support system. In Dymova, Sevastianov, and Kaczmarek (2012) this approach was successfully used for the development of stock trading expert system based on the rule-base evidential reasoning using Level 2 Quotes. Rule-base evidential reasoning is based on the synthesis of methods of Fuzzy Sets theory (*FST*) and the Dempster–Shafer theory (*DST*). This approach combine these theories in a synergic way, preserving their strengths while avoiding disadvantages they present when used solely: a capacity for the representation of fuzzy classifiers is enhanced by introduction the measure of ambiguity; limitations of *DST* in providing effective procedures to draw inferences from belief function are softened by integrating the rule of propagation of evidence within the fuzzy deduction paradigm. The synthesis of *FST* and *DST* primarily was used for the solution of control and classification problems (Binaghi, Gallo, & Madella, 2000; Binaghi & Madella, 1999; Ishizuka, Fu, & Yao, 1982; Yager, 1982; Yen, 1990). With the use of the known *COA* method (Yager & Filev, 1995), these models generated the system's output in the form of real values. Nevertheless, when we deal with decision support systems, the outputs can be only the names or labels of corresponding actions or decisions.

Therefore, a more appropriate for the development of decision support systems seems to be the so-ca' *RIMER* method proposed by Yang et al. (2006, 2007). With the use of this method, the final results obtained as the aggregation of belief rules are as $O = \{(D_j, \beta_j)\}$, where β_j, $j = 1$ to N, is the aggregated degree of belief in the decision (hypothesis, action, diagnosis) D_j. Then the decision with a maximal aggregated degree of belief is the best choice.

In Dymova et al. (2010, 2012), a new approach which can be treated as an extension of *RIMER* method was proposed and used to develop the effective stock trading expert systems. The method used in these works is based on the classical Dempster's rule of combination of evidence from different sources. Generally, the method provides good results, but in some cases the unreasonable results were obtained. In the part of such cases, undesirable results were coursed by a large conflict between evidence,which often cannot be eliminated when dealing with the real-world problem (it is well known that the Dempster's rule provide controversial results in such situations). On the other hand, in some cases we also have obtained unreasonable results, when the conflict was very small or even equal to zero.

Inspired by these results, in the current paper we have carried out a study aimed to find a more appropriate combination rules.

Therefore, in this paper, using critical examples, we analyze the known combination rules (including their aggregations) and show their restriction and drawbacks.

At the end, we show that in the cases of small and large conflict, the use of simple averaging rule for combination of basic probability assignments (*bpas*) seems to be a best choice.

We have found that the traditional fuzzy logic rules lose an important in- formation, when dealing with the intersecting fuzzy classes, e.g., such as *Low* and *Medium*, and this property may lead to the controversial results. In the framework of the proposed in the current paper approach, an information of the values of all membership functions representing the intersecting (competing) fuzzy classes is preserved and used in the fuzzy logic rules.

Therefore, the rest of the paper is organised as follows. In Section 2, we present the basic definitions of *DST* needed for the subsequent analysis and with the use of critical examples consider the problems concerned with the known combination rules in *DST*.

Section 3 presents our new approach to the rule-base evidential reasoning illustrated by a simple example of decision making in Forex trading. In Section 4, four new and adapted known technical analysis indicators with their fuzzy representation used in the developed Forex trading system are presented. The developed Forex trading system based on our new approach is presented and illustrated by real-world examples in Section 5. Section 6 concludes with some remarks.

2. The problems of combination rules in *DST*

The basics of *DST* were formulated by Dempster (1967, 1968) who developed a system of upper and lower probabilities. Later Shafer (1976) proposed a more thorough explanation of belief functions.

Assume A are subsets of X. Generally a subset A may be treated also as a question or proposition and X as a set of propositions or mutually exclusive hypotheses or answers. A *DS* belief structure has associated with it a mapping m, called basic probability assignment (*bpa*), from subsets of X into a unit interval, m: $2^X \to [0, 1]$ such that $m(\emptyset) = 0$, $\sum_{A\subseteq X} m(A) = 1$. The subsets of X for which the mapping does not assume a zero value are called focal elements.

An important (and often used in different applications) part of the evidence theory is the rule of combination of evidence from different sources. Nowadays many different rules for evidence combination are proposed in the literature. Ferson (2002) presented a review of them, where the rules are analyzed according to their algebraic properties and using different examples. A recent review of most popular rules is presented by Smarandache (2005). The most popular in the different applications of *DST* are the conjunction Dempster's rule of combination (Shafer, 1976) and the Dubois and Prade's disjunctive combination rule (Dubois & Prade, 1986, 1988).

Let m_1 and m_2 be (*bpas*). The Dempster's rule is defined us follows.

$$m_{12}^D(A) = \frac{\sum_{B\cap C=A} m_1(B)m_2(C)}{1-K}, A \neq \emptyset, m_{12}(\emptyset) = 0, \tag{1}$$

where $K = \sum_{B\cap C=\emptyset} m_1(B)m_2(C)$ is called the degree of conflict which measures the conflict between pieces of evidence and the process of dividing by $1-K$ is called normalization.

The main problem with this rule is that it provides counterintuitive results when the conflict K is close to its maximal value equal to 1 (Zadeh, 1986).

Let us consider the following example.

Example 1. Suppose two experts presented their recommendation for the decisions *Buy, Hold* and *Sell* of some financial instrument in the form of *bpas* as follows.

The firs expert proposed $m_1(Buy) = 0.95$, $m_1(Hold) = 0.05$, $m_1(Sell) = 0$,

The second expert proposed $m_2(Buy) = 0$, $m_2(Hold) = 0.05$, $m_2(Sell) = 0.95$.

Then according to the Dempster's rule we get:

$$K = m_1(Buy)m_2(Hold) + m_1(Buy)m_2(Sell) + m_1(Hold)m_2(Sell) = 0.9975,$$
$$m_{12}^D(Buy) = \frac{m_1(Buy)m_2(Buy)}{1-K} = \frac{0.95\cdot 0}{1-0.9975} = 0,$$
$$m_{12}^D(Hold) = \frac{m_1(Hold)m_2(Hold)}{1-K} = \frac{0.05\cdot 0.05}{1-0.9975} = 1,$$
$$m_{12}^D(Sell) = \frac{m_1(Sell)m_2(Sell)}{1-K} = \frac{0\cdot 0.95}{1-0.9975} = 0.$$

We can see that Dempster's rule yields the counterintuitive result since the basic probability assignment for the decision *Hold* is equal to 1 whereas both experts assigned to this decision the small value of *bpa* equal to 0.05. This points to the inconsistency when Dempster's rule is used in the circumstances of significant relevant conflict. Dubois and Prade disjunctive combination rule is defined as follows (Dubois & Prade, 1986, 1988):

$$m_{12}^{DP}(X) = \sum_{X_1\cup X_2=X} m_1(X_1)m_2(X_2). \tag{2}$$

For Example 1, from (2) we obtain $m_{12}^{DP}(Buy) = 0.99$, $m_{12}^{DP}(Hold) = 0.0199$, $m_{12}^{DP}(Sell) = 0.99$. Of course, this result seems to be more reasonable than that obtained from the Dempster's rule. Therefore, to reduce the disadvantage of Dempster's rule in the cases of considerable conflict, the different combinations of Dempster's and Dubois and Prade's rules were proposed in the literature (Delmotte, Dubois, Desodt, & Borne, 1995; Dubois & Prade, 1998; Inagaki, 1991; Martin, Osswald, Dezert, & Smarandache, 2008; Smets, 1997).

These combinations are usually based on the of conjunction

$$m_\wedge(X) = \sum_{X_1\cap X_2=X} m_1(X_1)m_2(X_2) \tag{3}$$

and disjunction

$$m_\vee(X) = \sum_{X_1\cup X_2=X} m_1(X_1)m_2(X_2) \tag{4}$$

rules.

The hybrid rules are usually based on weighted sums of conjunction and disjunction rules with weights dependent on the conflict K.

For example, Florea, Dezert, Valin, Smarandache, and Jousselme (2006) proposed the following rule:

$$m(X) = \alpha(K)m_\vee(X) + \beta(K)m_\wedge(X), \tag{5}$$

where $\alpha(K) = \frac{K}{1-K+K^2}$, $\beta(K) = \frac{1-K}{1-K+K^2}$.

In Florea et al. (2006), a set of functions $\alpha(K)$ and $\beta(K)$ is proposed. These functions have the following properties:

(1) $\alpha(K)$ is an increasing function with $\alpha(0) = 0$ and $\alpha(1) = 1$,
(2) $\beta(K)$ is a decreasing function with $\beta(0) = 1$ and $\beta(1) = 0$,
(3) $\alpha(K) = 1 - (1-K)\beta(K)$.

The links of approach based on (5) and proposed set of functions $\alpha(K)$ and $\beta(K)$ with other existing rules is analyzed. On the other hand, the use of disjunction rule $m_\vee(X)$ to reduce the undesirable properties of conjunction rule $m_\wedge(X)$ in the case of considerable conflict is not exclusive and the best way to develop hybrid rules with acceptable properties. Let us turn to the Example 1.

We can see that in this example the results obtained using the simple averaging

$$m_{12}(A) = \tfrac{1}{2}(m_1(A) + m_2(A)) = \tfrac{1}{2}(0.99 + 0) = 0.445,$$
$$m_{12}(B) = \tfrac{1}{2}(m_1(B) + m_2(B)) = \tfrac{1}{2}(0.01 + 0.01) = 0.01,$$
$$m_{12}(C) = \tfrac{1}{2}(m_1(C) + m_2(C)) = \tfrac{1}{2}(0.99 + 0) = 0.445$$

are reasonable enough.

Let us consider the following critical example:

Example 2. Suppose two experts presented identical recommendation for the decisions *Buy, Hold* and *Sell* of some financial instrument in the form of *bpas* as follows.

$m_1(Buy) = 0.1$, $m_1(Hold) = 0.2$, $m_1(Sell) = 0.7$,
$m_2(Buy) = 0.1$, $m_2(Hold) = 0.2$, $m_2(Sell) = 0.7$.

Here the decision *Hold*, in the spirit of *DST*, will be treated as the compound decision (*Buy, Sell*) when a trader hesitates in his/her choice between *Buy* and *Sell* decisions. Then using the Dempster's rule (1) we get:

$$K = m_1(Buy)\cdot m_2(Sell) + m_1(Sell)\cdot m_2(Buy) = 0.1\cdot 0.7 + 0.7\cdot 0.1 = 0.14,$$

$m_{12}^D(Buy) = \frac{m_1(Buy)\cdot m_2(Buy)+m_1(Buy)\cdot m_2(Hold)+m_1(Hold)\cdot m_2(Buy)}{1-K} = 0.058,$
$m_{12}^D(Sell) = \frac{m_1(Sell)\cdot m_2(Sell)+m_1(Sell)\cdot m_2(Hold)+m_1(Hold)\cdot m_2(Sell)}{1-K} = 0.895,$
$m_{12}^D(Hold) = \frac{m_1(Hold)\cdot m_2(Hold)}{1-K} = 0.047.$

With the use of rule (2) proposed by Dubios and Prade we obtain:

$$m_{12}^{DP}(Buy) = m_1(Buy)\cdot m_2(Buy) + m_1(Buy)\cdot m_2(Sell) + m_1(Sell)\cdot m_2(Buy) + m_1(Buy)\cdot m_2(Hold) + m_1(Hold)\cdot m_2(Buy) = 0.19.$$
$$m_{12}^{DP}(Sell) = m_1(Sell)\cdot m_2(Sell) + m_1(Sell)\cdot m_2(Buy) + m_1(Buy)\cdot m_2(Sell) + m_1(Sell)\cdot m_2(Hold) + m_1(Hold)\cdot m_2(Sell) = 0.19.$$
$$m_{12}^{DP}(Hold) = m_1(Hold)\cdot m_2(Hold) + m_1(Hold)\cdot m_2(Buy) + m_1(Hold)\cdot m_2(Sell) + m_1(Sell)\cdot m_2(Hold) + m_1(Buy)\cdot m_2(Hold) = 0.36.$$

It is seen that there are no any conflict between experts (sources of evidence) in the considered example. Therefore, according to common sense the only one correct result of combination should be:

$$m_{12}(Buy) = 0.1,\ \ m_{12}(Sell) = 0.7,\ \ m_{12}(Hold) = 0.2.$$

Nevertheless, using the combination rule (1) we get $m_{12}^D(Buy) = 0.058$, $m_{12}^D(Sell) = 0.895$, $m_{12}^D(Hold) = 0.047$ and with the use of combination rule (2) we obtain $m_{12}^{DP}(Buy) = 0.19$, $m_{12}^{DP}(Sell) = 0.19$, $m_{12}^{DP}(Hold) = 0.36$.

It is well known that the combination rules (1) and (2) are not idempotent, but in considered example, the results obtained from combination rules (1) and (2) should be considered even as wrong ones. Taking into account the above consideration, it is easy to see that the hybrid rule (5) may provide illogical results in the case of absence or low conflict as well.

On the other hand, using the simple averaging rule we obtain the correct result:

$$m_{12}(Buy) = \tfrac{1}{2}(0.1 + 0.1) = 0.1,$$
$$m_{12}(Sell) = \tfrac{1}{2}(0.7 + 0.7) = 0.7,$$
$$m_{12}(Hold) = \tfrac{1}{2}(0.2 + 0.2) = 0.2.$$

Obviously, the averaging combination rule (as well as any other method) has some disadvantages (see Murphy, 2000; Smarandache, 2005). Nevertheless, in this paper we will show that the averaging rule is justified enough and provides reasonable results in the case of large conflict and a true result in the case when there is no conflict between sources of evidence.

For this purpose, let us consider some critical examples presented in Table 1.

We can see that in the Example 3 we have $K = 0$ and therefore formally we should treat the sources of evidence as not conflicting, whereas they are not identical, and therefore some non-zero real conflict exists. Since the differences between corresponding focal elements of the sources in Example 3 are greater than in the Example 4, we can conclude that the conflict in Example 3 is greater than in the Example 4, and therefore the value of K in Example 3 should be greater than in Example 4. But in Table 1, we see the opposite situation. In the Examples 6 and 7 we deal with the identical sources of evidence and a true measure of conflict in these example should be equal to 0, whereas in these cases we have $K > 0$.

We can see that only in the cases of large conflict (see Examples 8 and 9), the value of K reflects well the sense of conflict.

Therefore, Liu (2006) states that “K only represents the mass of uncommitted belief (or falsely committed belief) as a result of combination” and that the “value K cannot be used as a quantitative measure of conflict between two beliefs, contrary to what has long been taken as a fact in the Dempster–Shafer theory community”. Martin et al. (2008) showed that the value of K is not appropriate to characterise the conflict between mass functions. Therefore, other approaches to the evaluation of the measure of conflict between sources of evidence were proposed.

Table 1
Examples 3–10.

Ex.	Expert rating			K	The results obtained using Dempster's rule			The result obtained using the averaging rule		
	$m(B)$	$m(S)$	$m(H)$		$m_{12}^D(B)$	$m_{12}^D(S)$	$m_{12}^D(H)$	$m_{12}(B)$	$m_{12}(S)$	$m_{12}(H)$
3	0 0	0 0.8	1 0.2	0	0	0.8	0.2	0	0.4	0.6
4	0.02 0.02	0.02 0.58	0.96 0.4	0.01	0.03	0.58	0.39	0.02	0.3	0.68
5	0.8 0.15	0.15 0.8	0.05 0.05	0.66	0.50	0.50	0.01	0.48	0.48	0.05
6	0.2 0.2	0.5 0.5	0.3 0.3	0.2	0.2	0.69	0.11	0.2	0.5	0.3
7	0.2 0.2	0.6 0.6	0.2 0.2	0.24	0.16	0.79	0.05	0.2	0.6	0.2
8	0.96 0.02	0.02 0.96	0.02 0.02	0.99	0.50	0.50	0.01	0.49	0.49	0.02
9	0 1	1 0	0 0	1	0/0	0/0	0/0	0.5	0.5	0
10	0.35 0.35	0 0	0.65 0.65	0	0.58	0	0.42	0.35	0	0.65

(In Table 1: B – *Buy*, H – *Hold*, S – *Sell*.)

However, this problem is out of scope of the current work and will be analyzed in the separate paper.

Here we can only say that the value of K should be used only in the calculation of normalization factor in the Dempster's rule of combination, not for the evaluation of conflict between *bpa*s. Let us compare the results obtained using Dempster's rule (1), and the averaging rule $m(A) = \frac{1}{N}\sum_{i=1}^{N} m_i(A)$ on the base of Examples 3–10.

The results are presented in Table 1.

It is seen that in the Examples 3 and 4, the sums (from both *bpa*s) of values of arguments in favor of (*Hold*) are greater than in favor of *Sell*. Therefore, it is intuitively obvious that in these examples after combination we should expect $m_{12}(Hold) > m_{12}(Sell)$. We can see that such results are obtained for averaging rule, but Dempster's rule provides counterintuitive results (see Table 1).

All the analyzed rules provide intuitively obvious results for the Examples 5 and 8. In the Examples 6 and 7, we deal with the identical *bpa*s. Therefore, only idempotent averaging rule provides true results. In the Example 9, the result of Dempster's rule is not defined as in this case we deal with the dividing by 0 since $K = 1$ and 1-K = 0 (see Table 1). In this example, the averaging combination rule provides good results which can be naturally treated as fifty-fifty chances for *Buy* and *Sell*.

Therefore, we can say that the averaging rule performs better than the Dempster's rule, as it provides true results in both asymptotic cases: in the case of full conflict and in the case of lack of conflict.

It is worth noting that in practice we often deal with *bpa*s characterised by relatively low conflict. It is clear that in such cases the use of not idempotent Dempster's rule may provide inappropriate numerical results (see Examples 6 and 7).

3. A new approach to the rule-base evidential reasoning

In many cases, real trading systems consist of hundreds rules and the use of them to represent the basics of new approach in the transparent form seems to be difficult. Therefore, in this section we present our approach using a simplified, but real-world example of decision-making in the Forex trading. Let us consider the currency pair EUR/USD (Euro/U.S. Dollar). The currency pair tells the reader how many U.S. dollars (the quote currency) are needed to purchase one euro (the base currency).

A two-way price quotation that indicates the best price at which a security can be sold and bought at a given point in time.The *Bid* price represents the maximum price that a buyer or buyers are willing to pay for a security. The *Ask* price represents the minimum price that a seller or sellers are willing to receive for the security. A trade or transaction occurs when the buyer and seller agree on a price for the security.

Suppose for the pair EUR/USD we have $Bid = 1.17556$ and $Ask = 1.17559$. Then if we want to buy 1000 EUR we must pay 1175.59 USD and if we sell 1000 EUR we get 1175.56.

To simplify our subsequent analysis hereinafter we will use the averaged price $p = (Bid + Ask)/2$.

Suppose a trader makes transactions on the currency pair EUR/USD based on the opinions of two independent experts ($E1$, $E2$). The experts on the base of analysis of changes of price propose the possible transactions: *Buy, Sell* and *Hold* (the lack of transactions). Here we will treat the decision *HOLD* as an intermediate one when an expert hesitates in his/her choice between *Buy* and *Sell*. Therefore, the decision *Hold* in the spirit of *DST* will be treated as the compound decision (*Buy, Sell*) In practice, experts intuitively and based on their experience transform the numerical information of prices into linguistic terms such as *Low, Medium, High* and use them as preconditions for possible decisions. In such a case, in our example (the pair EUR/USD) the expert's opinions concerned with the choice of transaction and based on the actual or predicted prises may be presented in the form of membership functions $\mu_{E1}^{Buy}(p)$, $\mu_{E1}^{Hold}(p)$, $\mu_{E1}^{SEll}(p)$ (based on the opinion of expert $E1$) and $\mu_{E2}^{Buy}(p)$, $\mu_{E2}^{Hold}(p)$, $\mu_{E2}^{SEll}(p)$ (based on the opinion of expert $E2$). These membership functions are presented in Fig. 1, The values of them represent degrees of expert's belief in the reasonableness of transactions.

Based on the classical fuzzy logic, in this case we can conclude that according to the expert $E1$ the right decision is *Sell* and according to the expert $E2$ the right decision is *Buy*. It is easy to see that in our example (symmetrical membership functions) we get the decision *Sell* if $\mu_{E1}^{Sell} \geq 0.5$, decision *Hold* if $\mu_{E1}^{Hold} \geq 0.5$, decision *Buy* if $\mu_{E1}^{Buy} \geq 0.5$ and so on. Then according to the known approach based of the synthesis of classical fuzzy logic and *DST* (Dymova et al., 2010) and treating the experts as two different sources of evidence, we get the following set of rules:

$$
\begin{aligned}
&R_1: \text{ If } mu_{E1}^{Buy} \geq 0.5 \text{ then } m_{E1}^*(Buy) = \mu_{E1}^{Buy},\\
&R_2: \text{ If } mu_{E1}^{Hold} \geq 0.5 \text{ then } m_{E1}^*(Hold) = \mu_{E1}^{Hold},\\
&R_3: \text{ If } mu_{E1}^{Sell} \geq 0.5 \text{ then } m_{E1}^*(Sell) = \mu_{E1}^{Sell},\\
&R_4: \text{ If } mu_{E2}^{Buy} \geq 0.5 \text{ then } m_{E2}^*(Buy) = \mu_{E2}^{Buy},\\
&R_5: \text{ If } mu_{E2}^{Hold} \geq 0.5 \text{ then } m_{E2}^*(Hold) = \mu_{E2}^{Hold},\\
&R_6: \text{ If } mu_{E2}^{Sell} \geq 0.5 \text{ then } m_{E2}^*(Sell) = \mu_{E2}^{Sell},
\end{aligned}
\tag{6}
$$

where $m_{E1}^*(Buy)$, $m_{E1}^*(Hold)$, $m_{E1}^*(Sell)$, $m_{E2}^*(Buy)$, $m_{E2}^*(Hold)$, $m_{E2}^*(Sell)$ are *bpas* which in our case can be treated as the values of arguments in favour of corresponding transactions.

These *bpas* should be normalised as follows:

$$
\begin{aligned}
&m_{E1}(Buy) = m_{E1}^*(Buy)/S_{E1},\\
&m_{E1}(Hold) = m_{E1}^*(Hold)/S_{E1},\\
&m_{E1}(Sell) = m_{E1}^*(Sell)/S_{E1},\\
&S_{E1} = m_{E1}^*(Buy) + m_{E1}^*(Hold) + m_{E1}^*(Sell),\\
&m_{E2}(Buy) = m_{E2}^*(Buy)/S_{E2},\\
&m_{E2}(Hold) = m_{E2}^*(Hold)/S_{E2},\\
&m_{E2}(Sell) = m_{E2}^*(Sell)/S_{E2},\\
&S_{E2} = m_{E2}^*(Buy) + m_{E2}^*(Hold) + m_{E2}^*(Sell).
\end{aligned}
$$

For $p = p_1^*$ and $p = p_2^*$ (see Fig. 1) using the above rules (6) and the conventional fuzzy logic, from the first expert $E1$ we get the decision *Sell* (with $m_{E1}^*(Sell) = \mu_{E1}^{Sell}(p_1^*)$) and from the second one - *Buy* (with $m_{E2}^*(Buy) = \mu_{E1}^{Buy}(p_2^*)$).

Since these two experts are different sources of evidence, to obtain the final decision the obtained results should be combined using an appropriate combination rule.

Nevertheless, the use of classical fuzzy logic (as in (6)) may lead to the counterintuitive results. That may be explained as follows. Following to rules of classical fuzzy logic in the first source of evidence we take into account only the decision *Sell*, whereas the decision *Hold* is possible as well with a non-zero value of membership function μ_{E1}^{Hold} (see Fig. 1). Similarly, in the second source of evidence we ignore the possible decision *Hold*, too.

Obviously, this may lead to significant loss of information especially when, e.g., $\mu_{E1}^{Hold} \approx \mu_{E1}^{Sell}$.

Another problem of classical approach is that it is possible in the considered case to use two different rules:

$$\text{if } \mu_{E1}^{Sell} = 0.5 \text{ and } \mu_{E2}^{Hold} = 0.5 \text{ then Sell}$$

and

$$\text{if } \mu_{E1}^{Sell} = 0.5 \text{ and } \mu_{E2}^{Hold} = 0.5 \text{ then Hold.}$$

Since we cannot use these rules simultaneously, we need an additional information to choose a correct rule and this information usually may be found only out of the framework of classical fuzzy logic.

Summarising, we can say that the known methods of rule-base evidential seasoning lead to the loss of important information which may affect the final results.

To avoid these problems, we propose a new approach which is free of above mentioned shortcomings. To represent transparently the basic ideas of our approach we will use the example presented in Fig. 1. To preserve all the information available in our example, we represent the decision-making rules as follows:

$$
\begin{aligned}
&\text{if}(p = p_1^*) \text{ then } m_{E1}^*(Buy) = \mu_{E1}^{Buy}(p_1^*),\ m_{E1}^*(Hold) = \mu_{E1}^{Hold}(p_1^*),\\
&\quad m_{E1}^*(Sell) = \mu_{E1}^{Sell}(p_1^*),\\
&\text{if}(p = p_2^*) \text{ then } m_{E2}^*(Buy) = \mu_{E2}^{Buy}(p_2^*),\ m_{E2}^*(Hold) = \mu_{E2}^{Hold}(p_2^*),\\
&\quad m_{E2}^*(Sell) = \mu_{E2}^{Sell}(p_2^*).
\end{aligned}
\tag{7}
$$

To represent the advantages of proposed approach, consider the critical example presented in Fig. 1.

Using conventional fuzzy logic, from (6) we get *bpas* $m_{E1}^*(Sell) = 0.6$ and $m_{E2}^*(Buy) = 0.625$. In this case, we can say only that we deal with a high conflict between the pieces of evidence and cannot obtain a reasonable decision in the considered example. It is important that in this case we do not use information concerned the values of $m_{E1}^*(Hold)$, $m_{E2}^*(Hold)$ and as the result of $m_{12}^*(Hold)$. On the other hand, according to the trader's (decision maker) opinion, in our case the decision *Hold* seems to be more justified than *Sell* and *Sell* is somewhat more preferable than *Buy*. The decision *Hold* is intuitively obvious for a trader in the considered example. Moreover, in his informal, but based on common sense analysis, a trader considered the values $m_{E1}^*(Buy) = 0$, $m_{E1}^*(Hold) = 0.4$, $m_{E1}^*(Sell) = 0.6$ and $m_{E2}^*(Buy) = 0.625$, $m_{E2}^*(Hold) = 0.375$, $m_{E2}^*(Sell) = 0$ as arguments in favour of corresponding decisions. It is easy to see that the sum of arguments in favour of *Hold* ($m_{E1}^{Hold} + m_{E2}^{Hold}$) is grater than the sums of arguments in favour of *Buy* ($m_{E1}^{Buy} + m_{E2}^{Buy}$) and *Sell* ($m_{E1}^{Sell} + m_{E2}^{Sell}$).

For our example we get:

$$
\begin{aligned}
&\text{if}(p = p_1^*) \text{ then } m_{E1}^*(Buy) = 0),\ m_{E1}^*(Hold) = 0.4,\\
&\quad m_{E1}^*(Sell) = 0.6,\\
&\text{if}(p = p_2^*) \text{ then } m_{E2}^*(Buy) = 0.625,\ m_{E2}^*(Hold) = 0.375,\\
&\quad m_{E2}^*(Sell) = 0.
\end{aligned}
\tag{8}
$$

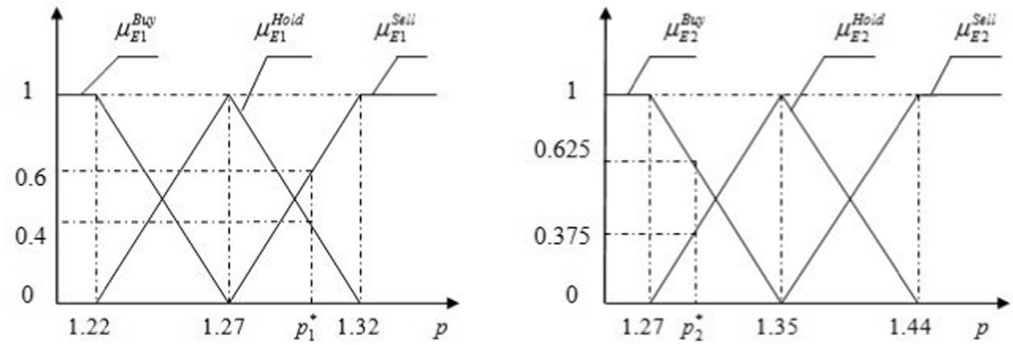

Fig. 1. The membership function of transaction decision based on the opinion of two experts $E1$ and $E2$.

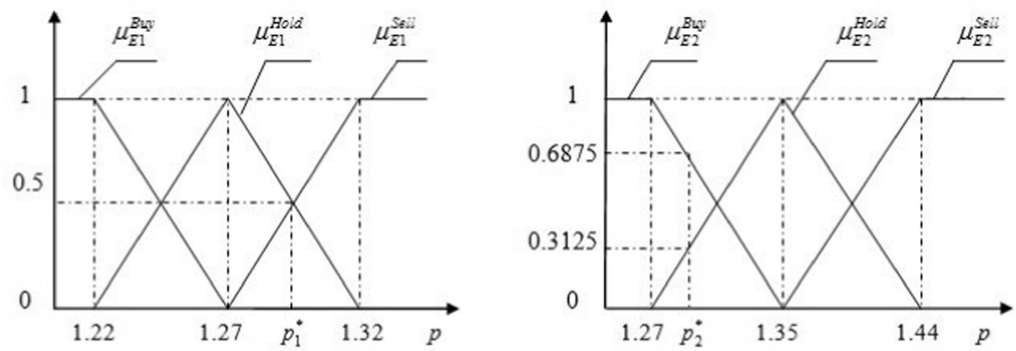

Fig. 2. The membership function of transaction decision based on the opinion of two experts $E1$ and $E2$.

It is seen that obtained *bpa*s for both sources of evidence are normalised. Then using averaging combination rule we finally obtain:

$m_{12}(Buy) = 0.3125,\ m_{12}(Hold) = 0.3875,\ m_{12}(Sell) = 0.3.$

We can see that this result qualitatively coincides with the results of informal, but based on common sense analysis presented above.

On the other hand, using the Dempster's combination rule (1), from (8) we obtain

$m_{12}^D(Buy) = 0.4,\ m_{12}^D(Hold) = 0.36,\ m_{12}^D(Sell) = 0.24.$

It is seen that this result can be considered as controversial one, as it doesn't qualitatively coincide with the results of above analysis. Let us consider the critical example presented in Fig. 2.

In this example, we have $p_1^* = p_2^* = 1.295\,USD$ and $m_{E1}^*(Sell) = m_{E1}^*(Hold) = 0.5$.

In this case, the use of classical fuzzy logic leads to obtaining from (6) two possible results: $m_{E1}^*(Sell) = 0.5,\ m_{E2}^*(Buy) = 0.6875$ and $m_{E1}^*(Hold) = 0.5,\ m_{E2}^*(Buy) = 0.6875$.

It is seen that in this case we deal with an ambiguity and have no reasons to choose the right result.

On the other hand, using a new approach, from (7) for this example we get

$$\begin{aligned}
&\text{if}(p = p_1^*)\ \text{then}\ m_{E1}^*(Buy) = 0),\ m_{E1}^*(Hold) = 0.5,\\
&\quad m_{E1}^*(Sell) = 0.5,\\
&\text{if}(p = p_2^*)\ \text{then}\ m_{E2}^*(Buy) = 0.6875,\ m_{E2}^*(Hold) = 0.375,\\
&\quad m_{E2}^*(Sell) = 0.
\end{aligned} \tag{9}$$

Then using the averaging combination rule, we get:

$m_{12}(Buy) = 0.34375,\quad m_{12}(Hold) = 0.40625,\quad m_{12}(Sell) = 0.25.$ Obviously, in this case we have no ambiguity and the decision *Hold* is the best choice. This result seems to be intuitively obvious, whereas the use of the Dempster's rule (1) (as in the Section 3, *Hold* is treated here as the compound decision (*Buy, Sell*)) provides: $m_{12}^D(Buy) = 0.5238,\ \ m_{12}^D(Hold) = 0.2381, m_{12}^D(Hold) = 0.2381.$ The obtained result (decision *Buy* seems to be not intuitively clear since we have $m_{12}^D(Hold) = m_{12}^D(Sell)$, whereas the sum $m_{E1}^*(Buy) + m_{E2}^*(Buy)$ is lesser than the sum $m_{E1}^*(Hold) + m_{E2}^*(Hold)$. So far we have considered the examples characterised by the significant conflict between sources of evidence . Therefore, let us consider the example presented in Fig. 3, where the conflict is absent.

Since in this case we have $m_{E1}^*(Buy) = m_{E2}^*(Buy) = 0.4$, $m_{E1}^*(Hold) = m_{E2}^*(Hold) = 0.6$ and $m_{E1}^*(Sell) = m_{E2}^*(Sell) = 0$, it is clear that in this case the only right result of combination of evidence should be $m_{12}(Buy) = 0.4$, $m_{12}(Hold) = 0.6$ and $m_{12}(Sell) = 0$.

The use of classical fuzzy logic and rules (6) provides only $m_{12}^*(Hold) = 0.6$ and this result cannot be treated as a satisfactory one.

Using the averaging combination rule, in this case from (7) we get the right result $m_{12}(Buy) = 0.4$, $m_{12}(Hold) = 0.6$ and $m_{12}(Sell) = 0$, whereas using the Dempster's rule (1) from (7) we obtain the result which seems to be undoubtedly wrong: $m_{12}(Sell) = 0.48$, $m_{12}(Hold) = 0.36$ and $m_{12}(Buy) = 0$. Summarising, we can say that based on the considered critical examples we have shown that the proposed new approach to the rule-base evidential reasoning performs better than known ones in the cases of large and small conflict. We have shown that the classical Dempster's combination rule may provide controversial and undoubtedly wrong result not only in the case of large conflict, but when the conflict is absent, whereas the use of averaging combination rule makes it possible to avoid these problems.

Therefore in the following sections we will use our approach for the development of Forex trading system.

4. Technical analysis indicators and their fuzzy representation

Generally, trading expert systems (*TES*) are based on the analysis of charts such as shown in Fig. 4.

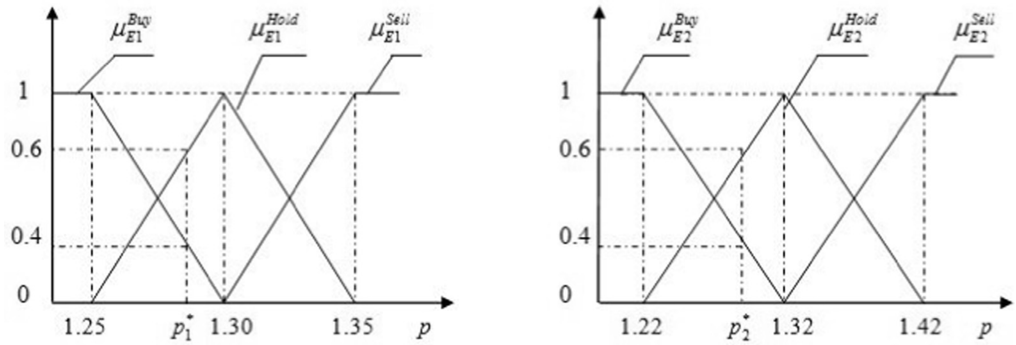

Fig. 3. The membership function of transaction decision based on the opinion of two experts *E*1 and *E*2.

Fig. 4. The example of *EUR/USD* quotation for the 1-hour time frame.

The objects presented by *High, Low, Open* and *Close* prices on a chosen time frame are called "Bars". In Fig. 4, the 1-hour time frame (1h-Bars) for the pair *EUR/USD* is presented.

In the most of *TESs*, different trend following strategies are used which are based on the technical analysis of past prises and volumes (number of transaction during a unit of a time frame) with a help of different technical analysis indicators.

Therefore, to develop the Forex trading system we will use some new and adapted known indicators with their fuzzy representation.

4.1. The adapted moving average indicator dEMA

As the base here we use the known moving average indicator which for n past Bars may be presented as follows:

$$EMA_t(n) = \frac{C_t + (1-\alpha)C_{t-1} + (1-\alpha)^2 C_{t-2} + \ldots + (1-\alpha)^n C_{t-n}}{1 + (1+\alpha) + (1-\alpha)^2 + \ldots + (1-\alpha)^n}, \quad (10)$$

where $\alpha = \frac{2}{n+1}$, C_t is the *Close* price on tth Bar.

Taking into account the specificity of Forex trading, we propose the adaptation of this indicator as follows:

$$dEMA_t(n) = \frac{Bid + Ask}{2} - EMA_{t-1}(n). \quad (11)$$

In the case, when there are no pronounced trends, the value of *dEMA* is near 0 and the decision *Hold* is the best choice. On the hand, if a significant Uptrend takes place, the value of *dEMA* is considerably greater than 0 and the decision *Buy* is preferable. In the case of Downtrend, the value of *dEMA* is considerably less than 0

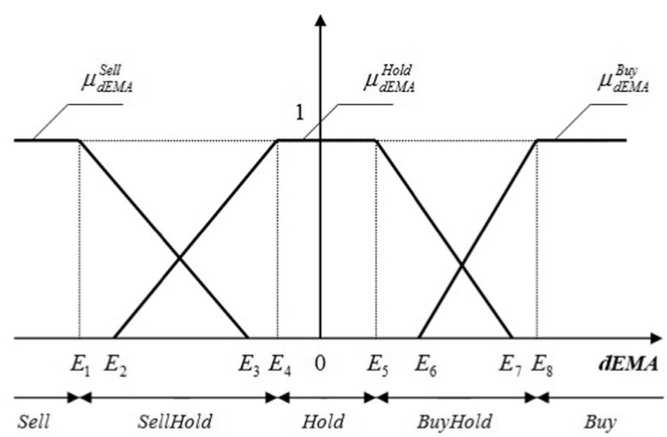

Fig. 5. The memberships of buying, holding and selling signals based on the *dEMA*.

and the decision *Sell* is reasonable. Since used above estimations "near", "significant","considerably" and so on are rather fuzzy ones, we present the signals for buying, holding and selling based on the *dEMA* in the form of corresponding membership functions μ_{dEMA}^{Buy}, μ_{dEMA}^{Hold} and μ_{dEMA}^{Sell}. These membership functions presented in Fig. 5 were designed on the base of expert's opinions, but the actual values of parameters $E_1 - E_8$ and n are determined on the stage of optimization (teaching) of expert system as a whole (with all rules and indicators) with the natural restrictions $E_1 \le E_2 < E_3 \le E_4 \le 0$ and $E_8 \ge E_7 > E_6 \ge E_5 \ge 0$.

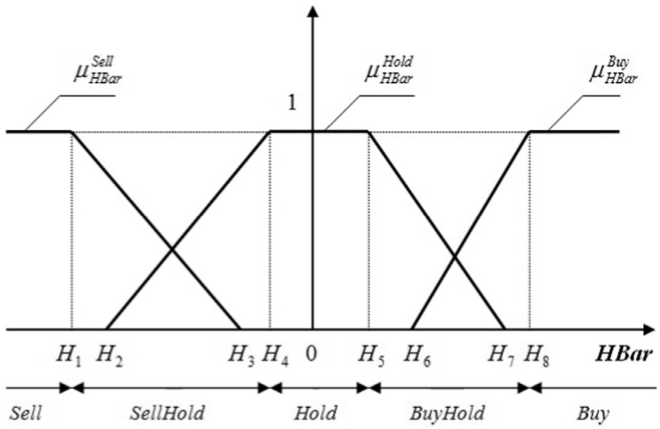

Fig. 6. The memberships of buying, holding and selling signals based on the *HBar*.

4.2. *A new indicator* HBar

The most of known technical analysis indicators are based on the *Close* prices. Nevertheless, according to our experience in the Forex trading, the analysis of *High, Low* and *Open* prices makes it possible to enhance the quality of trading decisions.

Therefore, we proposed a new indicator *HBar* based these prices. For time t and m past Bars the value of this indicator is calculated as follows:

$$HBar(m)_t = HBar'(m)_t fH(m)_t, \tag{12}$$

where

$$HBar'(m)_t = \frac{|Close_{t-1} - Open_{t-1}|}{\frac{1}{m}\sum_{j=t-1-m}^{m} \left|Close_j - Open_j\right|}, \tag{13}$$

$$fH'(m)_t = \frac{\frac{Close_{t-1}-Open_{t-1}}{High_{t-1}-Low_{t-1}}}{\frac{1}{m}\sum_{i=t-1-m}^{t-2}\frac{|Close_i-Open_i|}{High_i-Low_i}}, \tag{14}$$

$$\begin{aligned} &\text{if } fH'(m)_t > 1 \text{ then } fH(m)_t = 1,\\ &\text{if } fH'(m)_t < -1 \text{ then } fH(m)_t = -1,\\ &\text{if } fH'(m)_t = 0 \text{ then } fH(m)_t = 0. \end{aligned} \tag{15}$$

This indicator constructed in such a way that if the value of *HBar* lies in interval $[-1, 1]$ then the decision *Hold* is most preferable, if $HBar > 1$ then the best decision is *Buy* and if $HBar < -1$ then *Sell.*

We avoid here from the detailed and cumbersome description and justification of this indicator since this may be interested only for those who develop new technical analyses indicators. Nevertheless, we can say that indicator *HBar* works well and in many cases provides good results when is used in the trading system solely.

As in the case of previous indicator *dEMA*, we present the signals for buying, holding and selling based on the *HBar* in the form of corresponding membership functions $\mu^{\text{Sell}}_{\text{HBar}}$, $\mu^{\text{Hold}}_{\text{HBar}}$ and $\mu^{\text{Buy}}_{\text{HBar}}$ presented in Fig. 6.

The values of parameters $H_1 - H_8$ and m are determined on the stage of optimization (teaching) of expert system as a whole (with all rules and indicators) with the natural restrictions $H_1 \le H_2 < H_3 \le H_4 \le 0$ and $H_8 \ge H_7 > H_6 \ge H_5 \ge 0$.

4.3. *A new indicator* iV *based on volumes*

There are many indicators based on the volumes proposed in the literature. Nevertheless the use of them in our trading system provided less than satisfactory results. Probably this was caused by the specificity of Forex trading.

Therefore, a new indicator has been proposed:

$$iV_t(k, d) = \frac{\frac{1}{k}\sum_{j=t-k}^{t-1} Volume_j}{\frac{1}{d}\sum_{j=t-d}^{t-1} Volume_j}, \tag{16}$$

where $k < d$ are the numbers of past Bars. In the case when $iV \approx 1$ there are no activity in the market and the *Hold* decision may be recommended. When *iV* is considerably greater than 1 this indicates the growing activity in the market and *iV* serves as an additional argument in favour of *Buy* or *Sell* decisions. Obviously, *iV* cannot be solely used in trading. Therefore the membership function $\mu^{BuySell}_{iV}$ was introduced for the fuzzy representation of *iV* shown in Fig. 7. The values of parameters k, d, $V_1 - V_4$ are determined on the stage of optimization (teaching) of expert system as a whole with the restrictions $V_1 \le V_2 < V_3 \le V_4$ (in practice $V_1 \approx 1$) and $k < d$.

4.4. *A modified indicator* BB *based on Bollinger Bands*

Bollinger Bands is one of the most popular technical analysis indicators. Bollinger Bands consist of: an b-period moving average

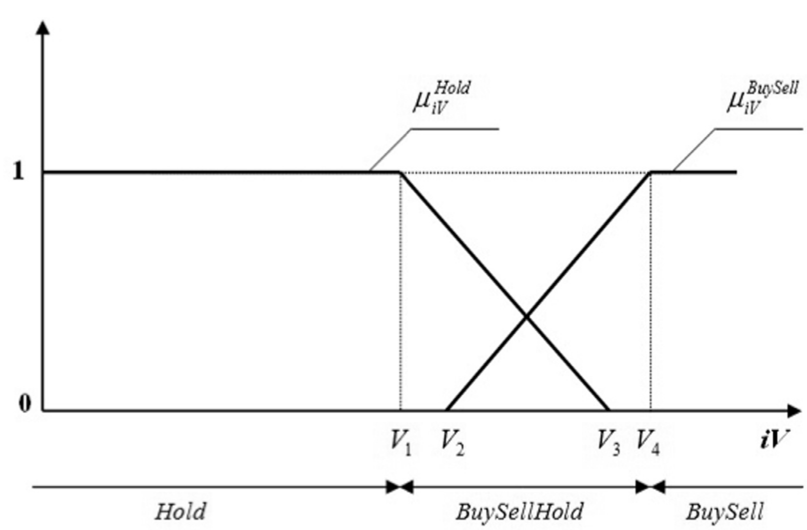

Fig. 7. The memberships of signals based on the *iV*.

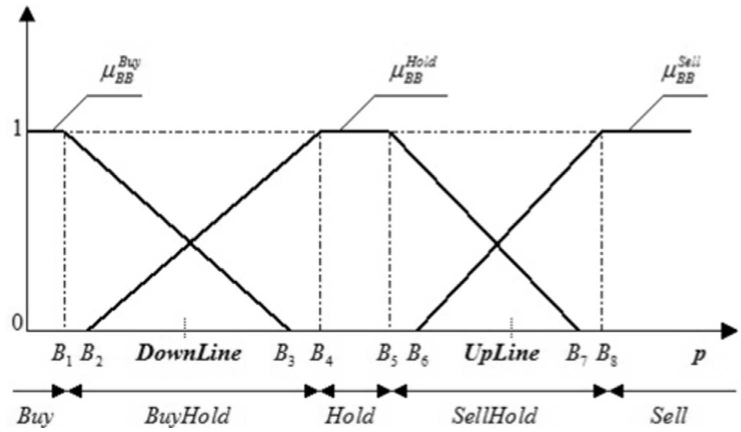

Fig. 8. The membership functions of signals based on the Bollinger Bands.

(SMA)

$$SMA_t(b) = \frac{1}{b}\sum_{j=1}^{b} Close_j, \tag{17}$$

an upper band at f_{sd} times an b-period standard deviation σ above the moving average ($SMA + f_{sd}\sigma$):

$$UpLine_t(b) = SMA_t(b) + f_{sd}\sqrt{\frac{1}{b}\sum_{j=t-b}^{t-1}\left(Close_j - SMA_t(b)\right)^2}, \tag{18}$$

an lower band at f_{sd} times an b-period standard deviation σ below the moving average ($SMA - f_{sd}\sigma$):

$$DownLine_t(b) = SMA_t(b) - f_{sd}\sqrt{\frac{1}{b}\sum_{j=t-b}^{t-1}\left(Close_j - SMA_t(b)\right)^2}. \tag{19}$$

Typical values for b and f_{sd} are 20 and 2, respectively. There are many methods proposed by traders for the decision making based on BB. The simplest approach is based on the fact that for $f_{sd} = 2$ the width of BB ($UpLine_t - Downline_t$) is equal 2σ and approximately 95% of prices $p = \frac{Bid+Ask}{2}$ fall in this band.

Therefore, when $p > UpLine_t$ we can treat this as a signal to *Sell* and if $p < DownLine_t$ we have a signal to *Buy*. On the other hand such an approach seems to be too restrictive since, e.g., in the case when p is close to, but below $UpLine_t$ the *Sell* decision may be enough justified if some other indicators generate the *Sell* decision.

Therefore, here we propose a more flexible approach based on the three membership functions $\mu_{BB}^{Buy}(p)$, $\mu_{BB}^{Hold}(p)$ and $\mu_{BB}^{Sell}(p)$ presented in Fig. 8.

The values of parameters b, f_{sd} needed for calculation of $UpLine_t$ and $DownLine_t$ in (17)–(19) and parameters B_1–B_8 (see Fig. 8) are obtained on the stage of optimization (teaching) of expert system as a whole with the restrictions $B_1 \le B_2 < B_3 \le B_4 \le B_5 \le B_6 < B_7 \le B_8$, $B_1 < DownLine < B_4$, $B_5 < UpLine < B_8$, $b > 0$ and $1 \le f_{sd} \le 2$.

Presented above technical analysis indicators were used to develop an expert system for the Forex trading.

5. Trading system

In this section we present the developed Forex trading system based on the four described in the previous section indicators treating as independent sources of information (sources of evidence) and the set of rules based on common sense and the opinions of experienced traders. The procedure of optimization (teaching) and result of testing of the system will be presented as well.

Table 2
The distribution of teaching and testing time periods.

Step	Teaching period		Testing period	
	Start	End	Start	End
1	01.11.2013	31.12.2013	01.01.2014	31.01.2014
2	01.12.2013	31.01.2014	01.02.2014	28.02.2014
3	01.01.2014	28.02.2014	01.03.2014	31.03.2014
4	01.02.2014	31.03.2014	01.04.2014	30.04.2014
5	01.03.2014	30.04.2014	01.05.2014	31.05.2014
6	01.04.2014	31.05.2014	01.06.2014	30.06.2014
7	01.05.2014	30.06.2014	01.07.2014	31.07.2014
8	01.06.2014	31.07.2014	01.08.2014	31.08.2014
9	01.07.2014	31.08.2014	01.09.2014	30.09.2014
10	01.08.2014	30.09.2014	01.10.2014	31.10.2014
11	01.09.2014	31.10.2014	01.11.2014	30.11.2014
12	01.10.2014	30.11.2014	01.12.2014	31.12.2014

Table 3
The results of testing of the developed trading system.

Symbol	Interval	Profit trades (% of total)	Total trades		ROI [%]	Price change |B&H| [%]
			Long pos.	Short pos.		
EUR/USD	1H	240 (52.06)	274	187	98.44	16.08
	4H	107 (55.73)	101	91	83.68	
GBP/USD	30M	316 (48.92)	431	216	161.76	10.75
	4H	124 (60.78)	115	89	160.15	
EUR/CHF	1H	143 (61.11)	124	110	57.06	2.63
	4H	66 (51.97)	83	44	7.37	
USD/CHF	15M	455 (45.77)	645	349	70.70	11.17
	1H	239 (50.32)	353	122	52.98	

ROI - Return on investment, pos. - positions, Testing period: 01.01–31.12.2014.

5.1. A set of trading rules

The proposed set of rules is based on our approach to the rule-base evidential reasoning presented in Section 3. For the convenience we will use the following simplified notation. For example, if $dEMA$ is greater than $E8$ (see Fig. 5), we will write $dEMA$ is *Buy*; if $dEMA$ lies in the interval [$E5$, $E8$] denoted in Fig. 5 as *BuyHold*, we will write $dEMA$ is *BuyHold* and so on.

Then the set of rules may be presented as follows:

if $dEMA$ **is** Buy **then** $m_1(Buy) = \mu_{dEMA}^{Buy}$,
$m_1(Hold) = 0, m_1(Sell) = 0,$

$$
\begin{aligned}
&\textbf{if } dEMA \textbf{ is } BuyHold \textbf{ then } m_1(Buy) = \mu^{Buy}_{dEMA},\\
&\quad m_1(Hold) = \mu^{Hold}_{dEMA},\ m_1(Sell) = 0,\\
&\textbf{if } dEMA \textbf{ is } Hold \textbf{ then } m_1(Hold) = \mu^{Hold}_{dEMA},\\
&\quad m_1(Buy) = 0, m_1(Sell) = 0,\\
&\textbf{if } dEMA \textbf{ is } SellHold \textbf{ then } m_1(Sell) = \mu^{Sell}_{dEMA},\\
&\quad m_1(Hold) = \mu^{Hold}_{dEMA},\ m_1(Buy) = 0,\\
&\textbf{if } dEMA \textbf{ is } Sell \textbf{ then } m_1(Sell) = \mu^{Sell}_{dEMA},\\
&\quad m_1(Buy) = 0, m_1(Hold) = 0,\\
&\textbf{if } HBar \textbf{ is } Buy \textbf{ then } m_2(Buy) = \mu^{Buy}_{HBar},\\
&\quad m_2(Hold) = 0, m_2(Sell) = 0,\\
&\textbf{if } HBar \textbf{ is } BuyHold \textbf{ then } m_2(Buy) = \mu^{Buy}_{HBar},\\
&\quad m_2(Hold) = \mu^{Hold}_{HBar},\ m_2(Sell) = 0,\\
&\textbf{if } HBar \textbf{ is } Hold \textbf{ then } m_2(Hold) = \mu^{Hold}_{HBar},\\
&\quad m_2(Buy) = 0, m_2(Sell) = 0,\\
&\textbf{if } HBar \textbf{ is } SellHold \textbf{ then } m_2(Sell) = \mu^{Sell}_{HBar},\\
&\quad m_2(Hold) = \mu^{Hold}_{HBar},\ m_2(Buy) = 0,\\
&\textbf{if } HBar \textbf{ is } Sell \textbf{ then } m_2(Sell) = \mu^{Sell}_{HBar},\\
&\quad m_2(\mathrm{Buy}) = 0, m_2(Hold) = 0,\\
&\textbf{if } iV \textbf{ is } BuySell \textbf{ then } m_3(Buy) = \mu^{BuySell}_{iV},\\
&\quad m_3(Sell) = \mu^{BuySell}_{iV},\ m_3(Hold) = 0,\\
&\textbf{if } iV \textbf{ is } BuySellHold \textbf{ then } m_3(Buy) = \mu^{BuySell}_{iV},\\
&\quad m_3(Hold) = \mu^{Hold}_{iV},\ m_3(Sell) = \mu^{BuySell}_{iV},\\
&\textbf{if } iV \textbf{ is } Hold \textbf{ then } m_3(Hold) = \mu^{Hold}_{iV},\\
&\quad m_3(Buy) = 0, m_3(Sell) = 0,\\
&\textbf{if } p \textbf{ is } Buy \textbf{ then } m_4(Buy) = \mu^{Buy}_{BB},\\
&\quad m_4(Hold) = 0, m_4(Sell) = 0,\\
&\textbf{if } p \textbf{ is } BuyHold \textbf{ then } m_4(Buy) = \mu^{Buy}_{BB},\\
&\quad m_4(Hold) = \mu^{Hold}_{BB},\ m_4(Sell) = 0,\\
&\textbf{if } p \textbf{ is } Hold \textbf{ then } m_4(Hold) = \mu^{Hold}_{BB},\\
&\quad m_4(Buy) = 0, m_4(Sell) = 0,\\
&\textbf{if } p \textbf{ is } SellHold \textbf{ then } m_4(Sell) = \mu^{Sell}_{BB},\\
&\quad m_4(Hold) = \mu^{Hold}_{BB},\ m_4(Buy) = 0,\\
&\textbf{if } p \textbf{ is } Sell \textbf{ then } m_4(Sell) = \mu^{Sell}_{BB},\\
&\quad m_4(Buy) = 0, m_4(Hold) = 0. \qquad (20)
\end{aligned}
$$

The consequences of the rules (20) are *bpas* based on the four independent technical analysis indicators. Since we have no reasons to prefer some of them, here we assume that used indicators are of equal importance.

Then according to our approach (see Section 3) to obtain final results we will use the averaging rule of combination of *bpas* from different sources of evidence:

$$
m(Buy)_t = \frac{1}{4}\sum_{i=1}^{4} m_i(Buy),\ m(Sell)_t = \frac{1}{4}\sum_{i=1}^{4} m_i(Sell),
$$
$$
m(Hold)_t = \frac{1}{4}\sum_{i=1}^{4} m_i(Hold). \qquad (21)
$$

Finally, on each Bar the transaction decision TD_t is generated as follows:

$$
TD_t = \max\{m(Buy)_t, m(Sell)_t, m(Hold)_t\}. \qquad (22)
$$

5.2. Optimization(teaching) and testing of trading system

The proposed Forex trading system was implemented with the use of a Forex trading platform MetaTrader 4, on the base of data from Investor online FX. Generally, the developed trading system include such parameters as Stop Loss, Take profit and special parameters that enhance the quality of decision when, e.g., $m(Sell)_t \approx m(Hold)_t$ and so on. The best values of these parameters are obtained at the stage optimization(teaching) of trading systems on the base of historical data. Nevertheless,the detailed description of trading system seems to be too cumbersome.

On the other hand, the aim of this paper is to illustrate the efficiency of proposed new approach to the rule-base evidential reasoning on the real-world example of Forex trading. Therefore, here we illustrate our approach using the simplified version of trading system based only on the rules (20)–(22).

The trading strategy based on (20)–(22) generates the *Buy, Sell* and *Hold* signals. Based on this strategy, the model simulating the decision making process has been developed and implemented using trading platform MetaTrader 4. The main output of this model is the total return *R*, i.e., a profit gained by the model during a chosen time period. In the developed trading system, the total return *R* is the function of parameters defining membership functions used in the rule-base system and the auxiliary parameters used in the calculations of technical analysis indicators. The developed trading system should be optimized. Optimization (teaching) pertains to the ability to determine the values of trading system parameters which result in the most favourable performance for the developed trading system.

Table 4
Monthly results of testing of the developed trading system.

Step	EUR/USD			GBP/USD			EUR/CHF			USD/CHF		
	1H [%]	4H [%]	B&H [%]	30M [%]	4H [%]	B&H [%]	1H [%]	4H [%]	B&H [%]	15M [%]	1H [%]	B&H [%]
1	13.95	14.15	−1.99	30.64	6.19	−0.87	16.4	5.95	−0.61	18.79	1.86	1.7
2	24.43	18.45	2.03	32.61	33.98	1.78	8.58	−3.73	−0.85	0.56	−0.84	−2.83
3	4.79	9.34	0.09	14.99	16.44	−0.28	9.45	1.69	0.57	12.9	6.58	0.49
4	15.82	6.34	0.7	19.37	22.41	1.25	6.65	−0.4	0.21	3.94	9.86	−0.47
5	12.32	10.66	−1.66	14.17	11.12	−0.64	1.05	0.33	−0.05	4.39	6.66	1.59
6	7.81	9.03	0.41	20.94	32.55	2.04	3.65	−3.59	−0.48	12.09	−3.9	−0.86
7	−6.9	−2.16	−2.21	−6.15	−1.34	−1.29	−2.09	0.69	0.21	1.31	−1.29	2.48
8	−3.84	−1.06	−1.96	4.01	21.43	−1.73	5.53	5.01	−0.83	−1.63	7.7	1.12
9	8.59	0.08	−3.78	0.34	4.26	−2.29	−1.27	−1.9	−0.05	10.45	18.3	3.93
10	6.44	1.41	−0.98	10.66	1.45	−7.72	3.39	2.08	−0.03	7.39	−0.83	0.97
11	5.8	5.66	−0.35	20.11	1.34	4.35	3.88	0.84	−0.19	−6.49	−6.72	0.16
12	9.23	11.78	−2.93	0.07	10.32	−0.2	1.85	0.38	−0.05	6.99	15.61	2.88

Step: 1 (January), 2 (February), ..., 12 (December).

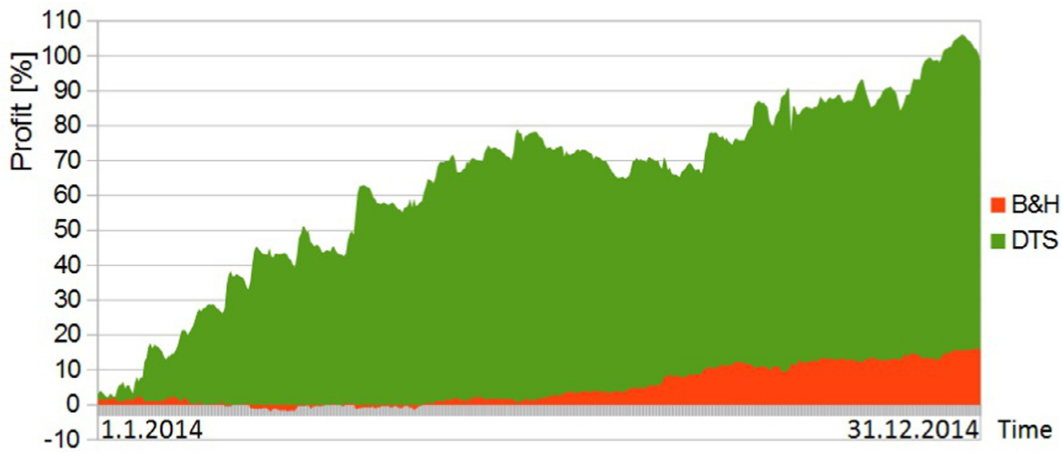

Fig. 9. The profit curves obtained using developed trading system (*DST*) and Buy and Hold strategy (*B&H*) for the pair EUR/USD and time frame 1H.

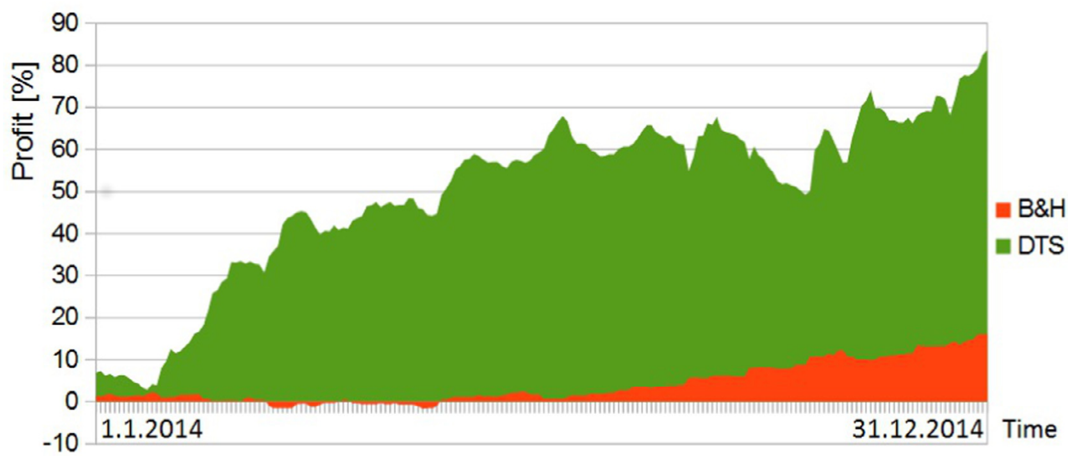

Fig. 10. The profit curves obtained using developed trading system (*DST*) and Buy and Hold strategy (*B&H*) for the pair EUR/USD and time frame 4H.

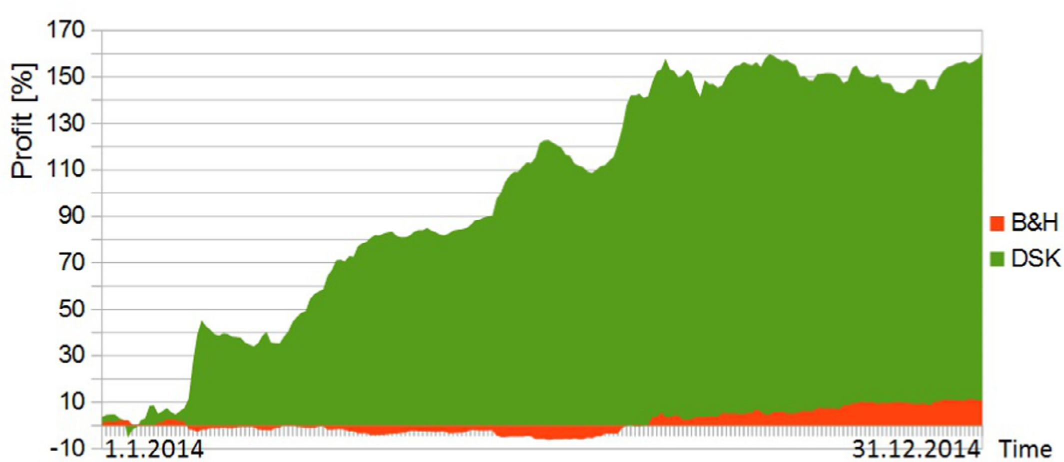

Fig. 11. The profit curves obtained using developed trading system (*DST*) and Buy and Hold strategy (*B&H*) for the pair GBP/USD and time frame 4H.

In out case, the optimization problem has been formulated as follows:

$$\max R(n, E_1 - E_8, m, H_1 - H_8, k, d, V_1 - V_4, b, f_{sd}, B_1 - B_8) \quad (23)$$

with the restriction described in Section 4.

To solve this problem, the trading platform MetaTrader 4 was used. In our studies, the quotations of currency pairs EUR/USD (time frames 1H and 4H), GBP/USD (30M and 4H), EUR/CHF (1H and 4H) oraz USD/CHF (15M and 1H) from 01.11.2013 to 31.12.2014 were used.

For the simplicity, in out trading system, we use only one lot, i.e., we buy one lot if we have a buying signal and sell one lot when a selling signal appears.

It is well known that even very successful trading systems providing excellent results at the stage of optimization, usually produce good or satisfactory results only during some limited (testing) time after optimization period. Therefore a regular reoptimization of the trading systems on the base of new data is needed. The choice of an appropriate durations of optimization (teaching) and testing stages depends on the time frames and, generally, may be considered as the optimization problem.

Nevertheless, based on our experience we have used two months for the teaching and one month for the testing periods. Therefore, the whole time duration of our study from 01.11.2013 to 31.12.2014 has been divided by twelve steps as it is shown in Table 2.

Obviously, the estimation of trading systems efficacy may be provided taking into account only the returns at the testing periods. Then in our case, we have the summarised testing time from 01.01.2014 to 31.12.2014 (see Table 2.

The summarised results obtained at the testing periods from 01.01.2014 to 31.12.2014 using the procedure of reoptimization are presented in the Table 3 (overall results) and in the Table 4 (monthly results). We can see that for all used time frames from 15 min to 4 h and all studied currency pairs we get undoubtedly good results (high Return on investment (ROI) and number of profit trades) that are considerably better than those obtained using the passive strategy Buy and Hold.

In Figs. 9–11, the profit curves obtained using developed trading system in comparison with the Buy and Hold strategy are presented. We can see that our trading system provides considerably better results that the Buy and Hold strategy. In Figs. 9 and 10, we can see the substantial drawdowns. This is a consequence of the lack of StopLoss signals in our trading system. We deliberately excluded such signals at the stages of optimization and testing of our system to represent its efficiency in arduous conditions.

Summarising we can say that the developed Forex trading system based on the proposed new approach to the rule-base evidential reasoning may be successfully used in practice for different currency pairs and time frames.

6. Conclusions

A new approach to the rule-base evidential reasoning (*RBER*) is proposed. Since the combination rules play a key role in *RBER*, a comparative analysis of them and their combinations is provided.

In is shown that the classical Dempster's rule may provide unacceptable results not only in the case of high conflict between evidence, but in the case of absence of it. In practice this negative property may lead to the wrong result (we have shown that the same drawback has the combination rule proposed by Dubios and Prade). It is shown that the simple averaging combination rule is free of the drawbacks of Dempster's rule and may be successfully used in *RBER*.

We have found that the traditional fuzzy logic rules lose an important information, when dealing with the intersecting fuzzy classes, e.g., such as *Low* and *Medium* and we have shown that this property may lead to the controversial results in practice. In the framework of the proposed in the this paper new approach, an information of the values of all membership functions representing the intersecting (competing) fuzzy classes is preserved and used in the fuzzy logic rules.

The basics ideas of a new approach to *RBER* are illustrated on the base of simplified example of Forex trading strategy. Based on the new approach to *RBER* and four technical analysis indicators presented in the fuzzy form (two of them are proposed by the authors and others are adaptations of known ones) a real Forex trading system was developed.

One of the limitation of presented trading system is lack of StopLoss and TakeProfit signals, whereas the use of them may provide a more reliable trading system. In this work, we deliberately excluded such signals at the stages of optimization and testing of our system to represent its efficiency in arduous conditions. Another limitations is that we use in the trading only one lot, whereas a more profitable system should regulate the numbers of open positions (lots) according the power of buying and selling signals.

Nevertheless is shown that even using this relatively simple trading system (based on the new approach to *RBER* optimized and tested on the real data from the forex market for the four currency pairs and the time frames 15m, 30m, 1h and 4h it is possible to obtain more than satisfactory results. We can say that the proposed system may be successfully used in practice.

Our future researches will be focused mainly on the introducing of StopLoss and TakeProfit signals and introducing the rules managing by the numbers of opening and closing position. Another potentially fruitful direction of studies is the representation of *RBER* in the framework intuitionistic fuzzy sets, since in our recent papers (not cited in the current one) a strong link between intuitionistic fuzzy sets and *DST* was revealed and used for the solution of multiple criteria decision making problems.

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