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On long memory effects in the volatility measure of Cryptocurrencies

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ABSTRACT

Cryptocurrencies as of late have commanded global attention on a number of fronts. Most notably, their variance properties are known for being notoriously wild, unlike their fiat counterparts. We highlight some stylized facts about the variance measures of Cryptocurrencies using the logarithm of daily return range and relate these results to their respective cryptographic designs such as intended transaction speed. The results favor oscillatory long run autocorrelations over standard long run autocorrelation filters to model the log daily return range. The overarching implication of this result is the volatility of Cryptocurrencies can be better understood and measured via the use of fast moving autocorrelation functions, as opposed to smoothly decaying functions for fiat currencies.

1. Introduction

Financial controllers globally are now at a cross road of accepting Cryptocurrencies as a medium of exchange, or purely as a speculative alternative asset class. In this note, a time series model is used to further address such issues, by providing a novel approach to better understand their unique volatility properties.

As of late, there has been an emergence of methods attempting to explain the long run autocorrelation properties of Cryptocurrencies, particularly Bitcoin. For instance, Jiang et al. (2017) finds evidence of a standard long run autocorrelation in Bitcoin returns only, but does not consider coupling this finding with the possibility of time-varying daily volatility. Time-varying daily volatility models are appealing as they are intuitive and capture more empirical properties compared to non-time varying volatility models, but are often avoided because they are difficult to estimate. Lahmiri et al. (2018) models long run autocorrelations in the daily time varying volatility component itself, but they do not consider the unique long run trend behaviors of Cryptocurrencies such as jumps.

In this work a model is proposed extending the work of Phillip et al. (2018) by the inclusion of daily time varying volatility measures which have been shown to greatly improve model performance (Koopman et al., 2005). This is especially true when market nuances such as time-of-day effects are present. Incorporating volatility measures into financial time series models, such as the CBOE Volatility Index (VIX), have gained traction as of late since they are efficient measures of the true volatility and reduce model error. Given the extreme volatility of Cryptocurrencies, such measures are extremely valid and a worthwhile pursuit. Hence, including such measures in the time series model is an important step in the volatility estimation process and by not doing so, a weaker signal about the current level of volatility is obtained. This is the case for the Stochastic Volatility (SV) model of Taylor (1986), one of the most commonly used models to measure daily time-varying volatility, as the model assumes the volatility process is latent based on the

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information of returns. Imposing the limitations of a weaker volatility signal within the context of Cryptocurrencies is extremely debilitating since they are notorious for wild volatility characteristics. Takahashi et al. (2009) first suggested incorporating additionally realized volatility into the SV model to supplement such a limitation in estimating time varying volatility and the model is referred to as the Realized Stochastic Volatility (RSV) model.

Traditionally, realized volatility is defined as the sum of squared intraday returns over a specific time interval (Andersen and Bollerslev, 1998; Barndorff-Nielsen and Shephard, 2001). The purpose of including such a volatility measure is to provide a robust estimator to filter the volatility component of the time series. Although it is a popular choice, the use of realized volatility as an additional measure suffers a significant drawback of being dependent on the intraday sampling interval which can bias the results. Alizadeh et al. (2002) find that daily range based volatility measures are highly efficient extracts of structural volatility components and are robust to market microstructure noise.

The presence of long run autocorrelation in range based volatility measures has been known to exist within some financial assets such as stocks and currencies. A routine solution to modeling this is to include a standard long run autocorrelation filter, which assumes exponential decay over time (Raggi and Bordignon, 2012; Corsi, 2009; Koopman et al., 2005). This is due to the fact that long run autocorrelation persistence in fiat assets is slowly decaying, with no oscillatory behavior. We however find a completely different case for the range based volatility measure of Cryptocurrencies which show oscillatory long run autocorrelation behavior in general. These sporadic long run autocorrelations can be measured using suitable Gegenbauer long run autocorrelation filters, which are able to capture oscillatory behaviors.

However, one of the main criticisms of long run autocorrelation estimation in general is that such effects may indeed be confused for regime changes in the long run trend component; see Guégan (2005) for a detailed review. We respond to such criticisms by incorporating for the first time, the so-called Buffered Autoregressive (BAR) model of Zhu et al. (2014) in conjunction with the time varying SV model of Taylor (1986). By doing so, we justify the use of the long-run autocorrelations together with structural changes. In addition to including BAR effects, we also simultaneously allow for occasional jumps, as are often reported about Cryptocurrencies. These jumps are assumed to occur in the long-run trend component.

The aim of this note is to advance Cryptocurrency models by addressing the above mentioned issues with respect to their wild volatilities. Our proposed model is novel in the Crypotocurrency literature in three aspects: an additional daily range volatility model within the SV model structure, Gegenbauer long run autocorrelation filters for the volatility measure and the BAR model with jumps in the trend model. We assume persistence in the volatility measures rather than returns as the persistence in returns can be distorted by jumps-especially Bitcoin. Our model can capture any jump features in the returns so that the persistence in volatility can be more easily detected. This advanced model not only provides substantial improvement in model performance but also offers new implications about the volatility features of Cryptocurrencies.

The remainder of this note is organised as follows: in Section 2, we discuss the data source and the model; Section 3 discusses empirical findings and concludes with Section 4.

2. Data and methodology

The Cryptocurrency data is sourced from the Brave New Coin (BNC) Digital Currency indices database. Currently, there are more than 2800 Cryptocurrency index series available on the BNC database. However, some of these have market capitalizations which are small (< \$1, 000, 000 USD) and traded very little. After filtering out for a meaningful investable basket, this leaves a total of 149 Cryptocurrencies and the inception date of these time series vary but all end on the 31st of December, 2017.

We assume in this note that Cryptocurrency behaviour can be decomposed into two components. The first being a long run trend (or mean) of the time series y_t defined as the daily index price percentage change $y_t = (P_t - P_{t-1})/P_{t-1}$ (conceptually similar to return) where P_t is the daily index value at time (day) t. The second component is the volatility measure, the log daily return range, which is defined as

$$v_t = \log(R_{h,t} - R_{l,t}), \tag{1}$$

where the high and low daily return on day t are $R_{k,t} = (P_{k,t} - P_{c,t-1})/P_{c,t-1}$, $k = h, l$ respectively and $P_{k,t}$, $k = h, l, c$ represents the high, low and closing price of day t. We use this particular definition because it is guaranteed to have support that agrees with the normal distribution. Our model attempts to explain the behavior of this daily super imposed volatility component of Cryptocurrencies.

Cryptocurrencies are also plagued with a host of other competing issues due to their infrastructure set-up. To address the issues, the long run trend component in y_t is assigned to a buffered threshold model with different jump features in each structural component. Moreover, the autocorrelation functions (ACFs) of the log daily return range volatility for the top six Cryptocurrencies displayed in Fig. 1 confirm the presence of oscillating long run autocorrelations. This oscillatory behavior strongly suggests the use of a Gegenbauer long run autocorrelation filter to properly estimate such effects. This extended model makes it a natural contender by correctly measuring these oscillatory effects in the presence of a large investable universe of Cryptocurrencies.

We describe our proposed Jump BAR SV Gegenbauer Log Range (JBAR-SV-GLR) model below. Let the return y_t, $t = 1, 2, \cdots, T$ and its volatility measure v_t, $t = 1, 2, \cdots, T$ satisfy the equations

$$\text{JBAR:} \quad y_t = \begin{cases} \phi_1 y_{t-1} + k_t q_t + e_t, & \text{if } R_t = 1, \\ \phi_2 y_{t-1} + k_t q_t + e_t, & \text{if } R_t = 0, \end{cases} \tag{2}$$

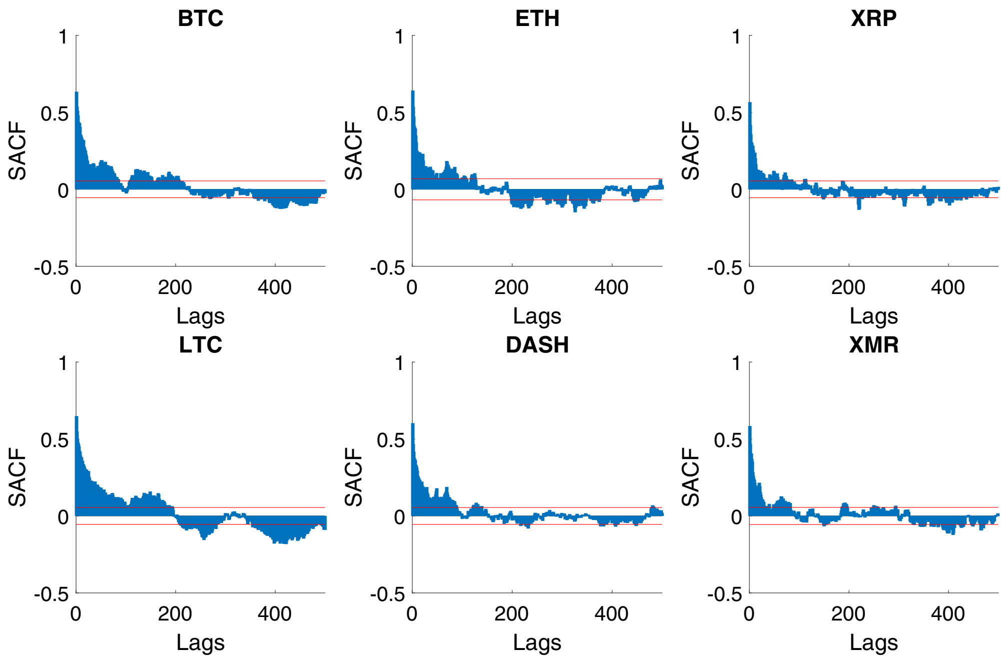

Fig. 1. Sample ACF plots of the log daily return range of the 6 largest Cryptocurrencies measured by market capitalization on 31/12/2017. BTC: Bitcoin. ETH: Ethereum. XRP: Ripple. LTC: Litecoin. DASH: Dash. XMR: Monero.

$$\text{SV:} \quad h_t = \alpha + \beta(h_{t-1} - \alpha) + \eta_t, \quad \eta_t \sim N(0, \sigma^2), \tag{3}$$

$$\text{GLR:} \quad (1 - 2uB + B^2)^d v_t = \gamma + h_t + \xi_t, \quad \xi_t \sim N(0, \sigma_v^2), \tag{4}$$

where $e_t \sim N(0, e^{h_t})$, and the buffer regime indicator and initial model at $t = 1$ are respectively

$$R_t = \begin{cases} 1, & \text{if } y_{t-d} \le r_L, \\ R_{t-1}, & \text{if } r_L < y_{t-d} \le r_U, \\ 0, & \text{if } y_{t-d} > r_L, \end{cases} \quad y_1 \sim \begin{cases} \mathcal{N}\left(\dfrac{k_1 q_1}{1-\phi_1}, \dfrac{e^{h_1}}{1-\phi_1^2}\right) & \text{if } R_1 = 0, \\ \mathcal{N}\left(\dfrac{k_1 q_1}{1-\phi_2}, \dfrac{e^{h_1}}{1-\phi_2^2}\right) & \text{if } R_1 = 1. \end{cases}$$

The jump indicator $q_t \in \{0, 1\}$ has probability of jumping equal to $\mathbb{P}(q_t = 1) = \varkappa$ and the jump size $k_t \sim \mathcal{N}(\mu_k, \sigma_k^2)$. It is known the volatility measure v_t has long run autocorrelation effects when $(\{|u| < 1, 0 < d < 0.5\} \cup \{|u| = 1, 0 < d < 0.25\})$. Additionally, γ is the level of the volatility measure, and σ_v^2 is the volatility of the volatility measure. When $u = 1$, v_t has standard long run autocorrelation effects such that Eq. (4) becomes $(1 - B)^{2d} v_t = \gamma + h_t + \xi_t$. The volatility component h_t in the SV model evolves according to the state Eq. (3) for $t = 1, \ldots, T$, α is the constant level of the volatility, β is the persistence of the volatility process and σ^2 is the volatility of the volatility process. We assume $|\beta| < 1$ so h_{t+1} is not explosive.

This model allows the return y_t to experience buffered regime changes (Eq. (2)) and the logarithm of the volatility component to have an autoregressive structure (Eq. (3)). It also relates the volatility measure v_t via another linear model with Gegenbauer long run autocorrelation filter (Eq. (4)).

3. Empirical results

In order to contest the current literature of utilizing a standard long run autocorrelation specification to model volatility, we estimate model Eqs. (2)–(4) and also its special case, which is in fact the standard case by setting $u = 1$ in Eq. (4). Results are reported in Table 1 for the six largest Cryptocurrencies measured by market capitalization.

The main points of interest from Table 1 are the long run autocorrelation parameters, u and d. Both models share a d parameter, which measures the strength of long run autocorrelations present in the volatility measures. As shown, the standard model does not show any significance of long run autocorrelations, as most estimates of d are close to 0. This is in stark contrast, however, with the Gegenbauer model, as most estimates of d are around 0.25. Since d is limited to the range [0, 0.5], these estimates of d are economically significant. The parameter u is limited between the range $[-1, 1]$ and measures the level of oscillation in the long run ACF of the volatility measure. The closer u is to -1, the more oscillatory the ACF is, and $u = 1$ means there is no long run autocorrelation oscillation. All values of u are statistically and economically significant for the Gegenbauer filter, with most values being estimated around -0.7. After allowing for the long run autocorrelation structure, most trend components for y_t do not possess jump behavior,

Table 1
Parameter estimates for each dataset for JBAR-SV-GLR model and its special case with standard long memory (u = 1).

Data	Model	r^U	r^L	ϕ^U	ϕ^L	u	d	α	β	σ^2	z	σ_v^2	κ	μ_k	σ_k^2	DIC
BTC	Standard															
	$\hat{\theta}$	0.0535	−0.2036	0.0222	−0.6498		0.0005	−7.5249	0.6539	0.4106	4.0328	0.0049	0.0010	0.0241	2.0379	−8810
	Std.	0.0829	0.0059	0.0227	0.1706		0.0003	0.0640	0.0208	0.0163	0.0372	0.0019	0.0011	0.0917	1.5542	
	Gegenbauer															
	$\hat{\theta}$	0.0032	−0.1900	0.0209	−0.2077	−0.7717	0.2106	−7.5493	0.7933	0.4003	2.9470	0.0048	0.0009	0.0025	2.2259	−8874
	Std.	0.0853	0.0767	0.0850	0.2319	0.0157	0.0080	0.0906	0.0171	0.0159	0.0630	0.0016	0.0010	0.0475	1.5020	
ETH	Standard															
	$\hat{\theta}$	0.0121	−0.1252	0.0384	−0.3803		0.0004	−6.0038	0.6765	0.3707	3.2992	0.0070	0.0016	0.0232	2.1043	−4304
	Std.	0.0447	0.0189	0.0310	0.1116		0.0004	0.0824	0.0259	0.0191	0.0484	0.0031	0.0019	0.1219	1.5275	
	Gegenbauer															
	$\hat{\theta}$	−0.0012	−0.1198	0.0338	−0.3570	−0.7920	0.2397	−6.0833	0.8157	0.3707	2.3768	0.0057	0.0017	0.0204	2.0976	−4365
	Std.	0.0468	0.0285	0.0347	0.1393	0.0453	0.0068	0.1265	0.0205	0.0188	0.0658	0.0021	0.0022	0.0752	1.5533	
XRP	Standard															
	$\hat{\theta}$	0.0194	−0.1325	−0.0301	−0.1791		0.0008	−6.4377	0.5953	0.4701	3.6175	0.0066	0.0039	0.8756	0.9691	−6762
	Std.	0.0571	0.0409	0.0264	0.0599		0.0005	0.0603	0.0223	0.0188	0.0329	0.0027	0.0032	0.5179	1.0500	
	Gegenbauer															
	$\hat{\theta}$	−0.0408	−0.1028	0.0057	−0.1833	−0.7630	0.2267	−6.5116	0.7547	0.4773	2.6783	0.0051	0.0023	1.1437	1.2943	−7349
	Std.	0.0482	0.0344	0.0280	0.0718	0.0096	0.0106	0.0834	0.0192	0.0189	0.0716	0.0016	0.0017	0.4868	1.1905	
LTC	Standard															
	$\hat{\theta}$	0.0720	−0.2166	−0.0208	−0.0911		0.0011	−7.1119	0.6720	0.4882	3.8409	0.0055	0.0122	0.2981	0.0137	−7394
	Std.	0.0957	0.1058	0.0327	0.1135		0.0006	0.0823	0.0204	0.0194	0.0609	0.0021	0.0047	0.0475	0.0230	
	Gegenbauer															
	$\hat{\theta}$	0.0008	−0.0767	−0.0102	−0.0886	−0.8512	0.2819	−7.2094	0.8395	0.5014	2.3199	0.0045	0.0102	0.2594	0.0216	−7961
	Std.	0.0994	0.1077	0.0481	0.1338	0.0043	0.0067	0.1309	0.0154	0.0198	0.0423	0.0015	0.0041	0.0537	0.0163	
DASH	Standard															
	$\hat{\theta}$	0.0164	−0.1961	0.0143	−0.3646		0.0008	−6.1169	0.6278	0.3195	3.4832	0.0056	0.0038	0.5961	0.4318	−6528
	Std.	0.0833	0.0649	0.0236	0.2271		0.0005	0.0577	0.0216	0.0127	0.0425	0.0021	0.0029	0.2433	0.8365	
	Gegenbauer															
	$\hat{\theta}$	−0.0045	−0.1960	0.0031	−0.2642	−0.7501	0.3284	−6.1748	0.8180	0.3397	2.0001	0.0042	0.0011	0.0839	2.0251	−6954
	Std.	0.0880	0.0862	0.0310	0.2624	0.0049	0.0097	0.0936	0.0163	0.0137	0.0696	0.0012	0.0013	0.2088	1.5468	
XMR	Standard															
	$\hat{\theta}$	−0.0229	−0.1885	−0.0188	−0.4546		0.0012	−5.7350	0.6119	0.2543	3.3578	0.0052	0.0022	0.4239	0.9184	−6033
	Std.	0.0816	0.0480	0.0250	0.1888		0.0008	0.0570	0.0229	0.0104	0.0443	0.0019	0.0018	0.2858	1.3065	
	Gegenbauer															
	$\hat{\theta}$	0.0033	−0.1873	−0.0171	−0.3320	−0.8601	0.1727	−5.7598	0.7453	0.2546	2.7331	0.0045	0.0012	0.0974	1.9898	−6192
	Std.	0.0798	0.0680	0.0294	0.2009	0.0240	0.0168	0.0672	0.0204	0.0106	0.0750	0.0016	0.0014	0.2333	1.5438	

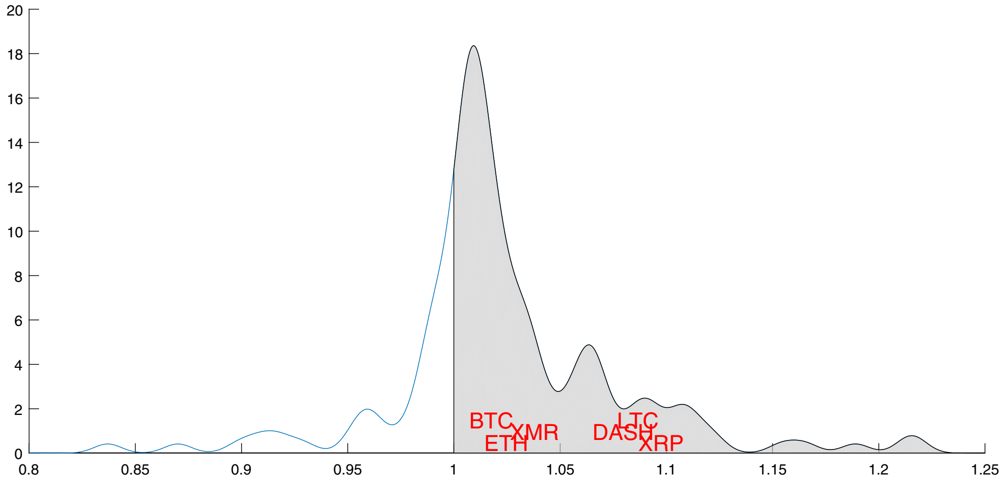

Fig. 2. Density plot of VOMRs of Gegenbauer to standard long run autocorrelation filter for log daily return range. The top six Cryptocurrencies by market capitalization are overlaid.

even for BTC which displays sporadic y_t. The only exceptions are LTC and XRP. This interesting finding also confirms the necessity of modeling persistence in the volatility measures rather than returns as in most Cryptocurrency models.

We next expand the analysis to a large and practically investable universe of 149 Cryptocurrencies, and provide an intuitive handle on the results. We note the DICs reported in Table 1 measure the model misfit and hence a lower DIC (more negative) indicates better model fit. In order to gauge a broad overview of the data, the DIC ratios of the standard model ($u = 1$) to the Gegenbauer model for all 149 Cryptcurrencies are measured. These DIC ratios which we call 'Volatility Oscillation Memory Ratios' (VOMRs) are powerful metrics that provide a deeper understanding on the properties of Cryptocurrencies: a VOMR greater than one indicates that higher model misfit for the standard model relative to the Gegenbauer model and hence a preference for the Gegenbauer model over the standard model. Fig. 2 depicts the density plot of the VOMRs for all 149 Cryptocurrencies. In total, 118 (79%) of the Cryptocurrencies have a VOMR which is considered high (greater than one), compared to only 31 (21%) which are low.

It is clear from Fig. 2 that all of the top six Cryptocurrencies have a VOMR greater than one. Upon closer inspection, it is evident the VOMR is closely related to completion time (transaction speed). This completion time issue presents new challenges which are not present in fiat currencies. One of the pioneering aspects of Cryptocurrencies is the use of Blockchain technology; which can be intuitively interpreted as a clearing house for transactions. Arguably, the most appealing aspect of such technology is that transactions are intended to be almost instantaneous and have a negligible bid-ask spread. This feature is very different from fiat currencies, which do have these market frictions. The most commonly discussed example where this would benefit the most is within the international money transfer services community (such as Western Union) in which there is a clear need to send cash overseas very cheaply and instantly.

The intuitive relationship of VOMR with completion time seems to explain how the day-to-day volatility correlation is dependent on completion times, and therefore liquidity. To illustrate the transaction speed (and hence liquidity issue), we look at the top six Cryptocurrencies. It is commonly known that BTC, ETH and XMR have long completion times, compared to LTC, DASH and XRP which have shorter completion times. For example, BTC, the largest and most widely traded Cryptocurrency today, can take up to two days to transact, whereas XRP takes only seconds. This intuitive understanding helps to settle important speculative debates over Cryptocurrencies as it reveals their transaction speeds, an important factor to the role of currency, are related to the oscillatory long run autocorrelation structure of their volatility measures.

Regarding transaction speed, BTC receives the biggest criticism as its infrastructure set-up was not designed to handle such a large volume of trades that it currently experiences. As such, critics argue that it is not a sustainable Cryptocurrency, since it now has extremely slow transaction speeds and is therefore not a long-term viable solution. This can shed light from its close-to-one VOMR showing weak preference for Gegenbauer specification. Another interesting finding is the VOMR of ETH being again close to one. ETH claims to have embedded "Smart Contracts" to circumvent the slow transaction fallacy of BTC. However in reality, the transaction time of ETH has also increased considerably due to a lack of infrastructure upgrades to deal with growing pains. XMR is a coin which mainly focuses itself on security and privacy, but not on speed. This finding too is evidenced on the chart, since it has a VOMR close to one. These three Cryptocurrencies as a group are in sharp contrast to LTC, DASH and XRP who pride themselves on having faster transactions (almost instant), and this is indeed the case depicted in Fig. 2 as they are clustered on the right of the chart.

An extremely important example which further illustrates the relationship between transaction speed and preference for Gegenbauer specification is XRP which is one of the most popular Cryptocurrencies and has one of the highest VOMRs (1.09). XRP is the most commonly used Cryptocurrency by financial institutions since there is virtually no overnight risk. By design of the cryptographic integrity of XRP, there is no positive dampening correlation across time for overnight risk, and is therefore very liquid. XRP is now the preferred Cryptocurrency used by large banks and is the main Cryptocurrency used by banks to connect with other banks, with an emphasis on almost instantaneous transaction speeds of up to apparently 17 s, compared to traditional transaction times. As such, financial institutions now routinely convert fiat to XRP for liquidity.

4. Conclusions and future research

This note addresses several important issues. Digital assets present challenges which are unlike their fiat counterparts, and require specific treatment. Previous debates on the role of Cryptocurrencies mainly focus on measuring their volatility, but do not provide a practical handle on the broader financial implications of this.

We label a trend that stronger oscillating long run volatility autocorrelations are associated with shorter transaction times. Upon closer observation of the top six Cryptocurrencies, it is found that slower transacted Cryptocurrencies, such as Bitcoin, have less oscillatory features (VOMR ≈ 1) whereas faster transacted coins, such as Ripple (VOMR > 1), display oscillatory features. As faster transacted Cryptocurrencies have lower liquidity risk during transactions, these are more preferable purely as a medium of exchange. This trend of oscillatory long run autocorrelations and transaction time is important and has broader practical implications to investors and policy regulators as it provides an alternative tool to explain the speculative nature of Cryptocurrencies based on their volatility measures. Finally, it is confirmed the long run autocorrelation patterns found in Cryptocurrency time series are not regime changes; and their investigation should be orientated through their realized volatility measures instead of returns, contrary to current methodologies. Future avenues of research include conditioning on further stylized facts, such as the leverage effect, or fat-tails.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.frl.2018.04.003.

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