Function

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Outline

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Function

Let A and B be sets, a function f from A into B, denoted by $f : A \to B$, is a *rule* that assigns to each element of A exactly one element of B.

- A is called *domain*
- B is called *codomain*

Notation & terminology

Let f be a function. For each x belonging to the domain of f, the corresponding element (in the codomain of f) assigned by f is denoted by $f(x)$, and called the *image* of x under f.

Example

In this course, we mostly consider both domain and codomain that are subsets of real numbers, that is $f : \mathbb{R} \to \mathbb{R}$, for example,

$$f(x) = x^2 - 5. \tag{1}$$

Exercise

Given function f defined in (1), find

1. $f(5)$
2. $f(c + 1)$
3. $f(k^3)$
4. $f(x + h)$

Function: domains & ranges

Domain of a function f can be determined from the rule defining the function. For example, given

$$f(x) = \sqrt{x}, \tag{2}$$

is defined for all real numbers $x \geq 0$ but not for $x < 0$. The rule determines the domain of f, that is, $[0, \infty]$. The domain obtained in such a way is called *natural domain*.

Exercise

Find the natural domains of the functions

1. $f(x) = x^2 + 3$
2. $g(x) = 1/(x - 2)$
3. $h(x) = \sqrt{1 + 5x}$

Function: domains & ranges

Given a function $f : A \to B$ and $S \subseteq A$, the image of S under f denoted by $f[S]$ is the subset of B, that is

$$f[S] = \{y \in B : y = f(x) \text{ for some } x \in S\}. \tag{3}$$

For example, let $f : \mathbb{R} \to \mathbb{R}$ be a function given by $f(x) = x + 3$. Then for $S = \{1, 3, 5\}$, we have $f[S] = \{4, 6, 8\}$.

Given a function $A \to B$, the *range* of f, denoted by $\text{ran}(f)$, is the image of A under f, i.e., $\text{ran}(f) = f[A]$.

Write a function in R

The following R scripts implement the functions f from (1) and (2).

```
1
2  #example 1
3  f1 <- function(x) {
4        result <- x^2-5
5        return(result)
6  }
7
8  #example 2
9  f2 <- function(x) {
10       result <- sqrt(x)
11       return(result)
12 }
```

`f1` and `f2` are the function names, and `x` is the "input variable" or argument. `return()` is used to return output of the function.

Exercise

1. Write the following functions in R.
   1.1 $f(x) = x^3 + x^2 - 6$
   1.2 $g(a, b) = ab(b - a)$
   1.3 $h(m, n) = (\sqrt{m}/n) + m - 2n$
2. The following functions accepts matrices as "input variables". Assume the size of input matrices follow the desired operations.
   2.1 $f(a, b) = (a + b)ab$
   2.2 $h(m, n) = |m|n - mn$
   2.3 $g(x) = x'x - 2x$

Graph of functions

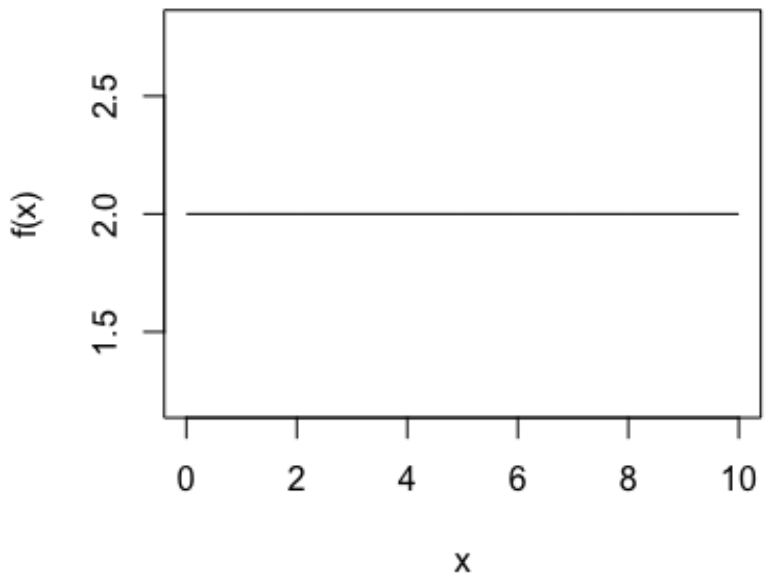
Given a function $f : A \to B$, with $A \subseteq \mathbb{R}$, the *graph* of f is

$$\{(x, y) \in \mathbb{R}^2 : x \in A \text{ and } y = f(x)\} \tag{4}$$

Constant functions

$$f(x) = c \tag{5}$$

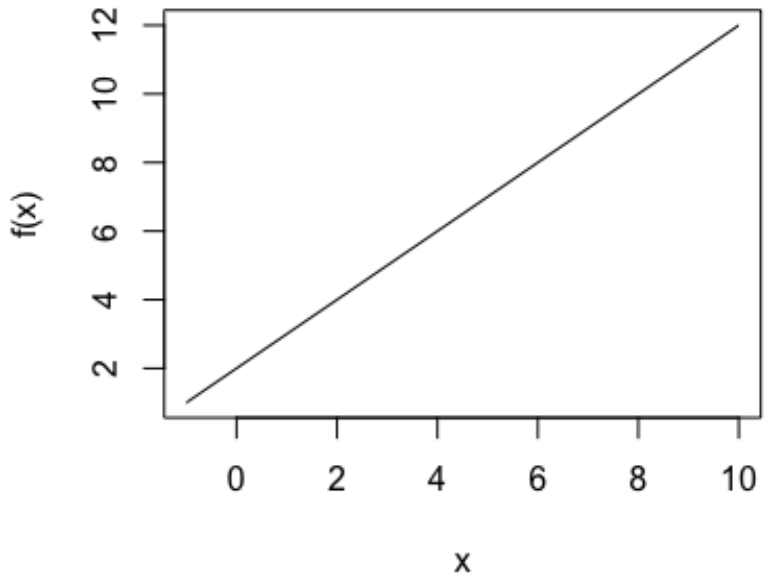
```
1  f <- function(x) {
2    #suppose c=2
3    fx <- 2
4    return(fx)
5  }
6
7  input <- 0:10
8  plot(input,
9  sapply(input, f),
10 type="l", xlab = "x",
11 ylab = "f(x)")
```



Linear functions

$$f(x) = ax + b \tag{6}$$

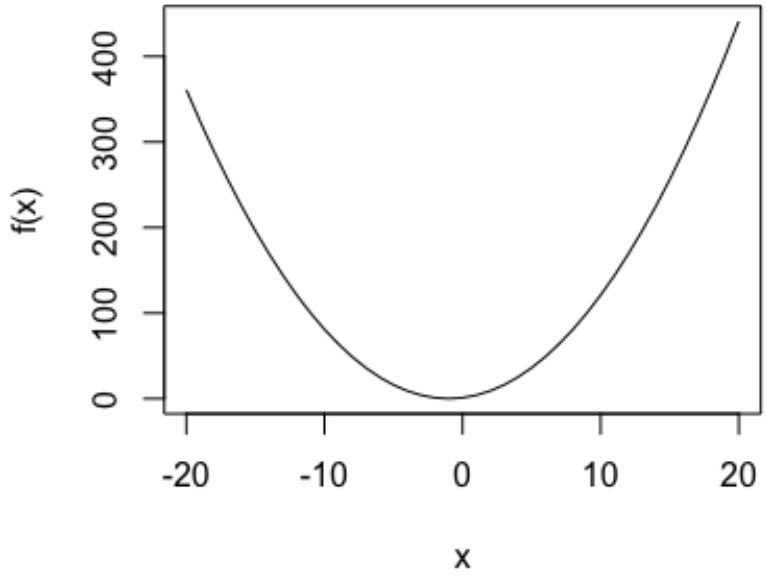
```
1  f <- function(x) {
2    #a=1 and b=2
3    fx <- 1 * x + 2
4    return(fx)
5  }
6
7  input <- -1:10
8  plot(input,
9  sapply(input, f),
10 type="l",
11 xlab = "x",
12 ylab = "f(x)")
```



Quadratic functions

$$f(x) = ax^2 + bx + c \tag{7}$$

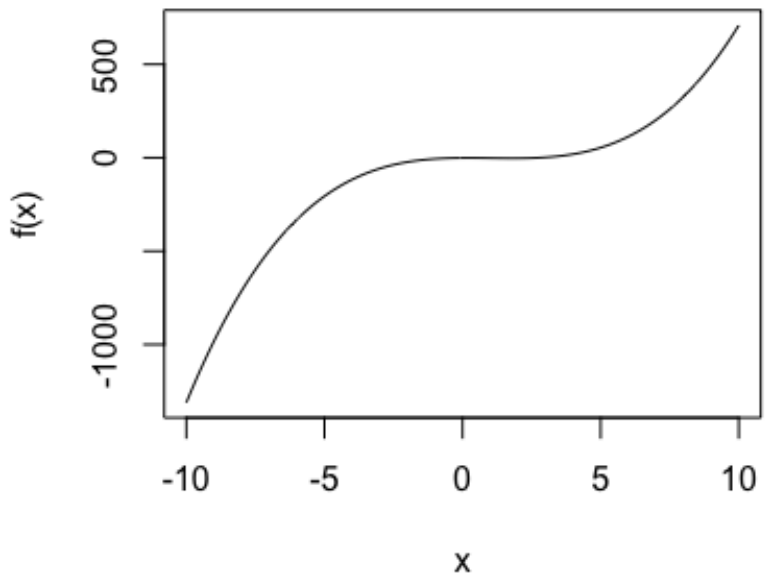
```
1  f <- function(x) {
2    #a=1 also try a=-1, b=2,
         c=1
3    fx <- 1*x^2 + 2*x + 1
4    return(fx)
5  }
6
7  input <- -20:20
8  plot(input,
9  sapply(input, f),
10 type="l",
11 xlab = "x",
12 ylab = "f(x)")
```



Polynomial functions

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \tag{8}$$

```
1  f <- function(x) {
2    fx <- x^3 - 3*x^2 + x - 1
3    return(fx)
4  }
5  input <- seq(-10, 11, 0.1)
6  plot(input,
7  sapply(input, f),
8  type="l",
9  xlab = "x",
10 ylab = "f(x)")
```

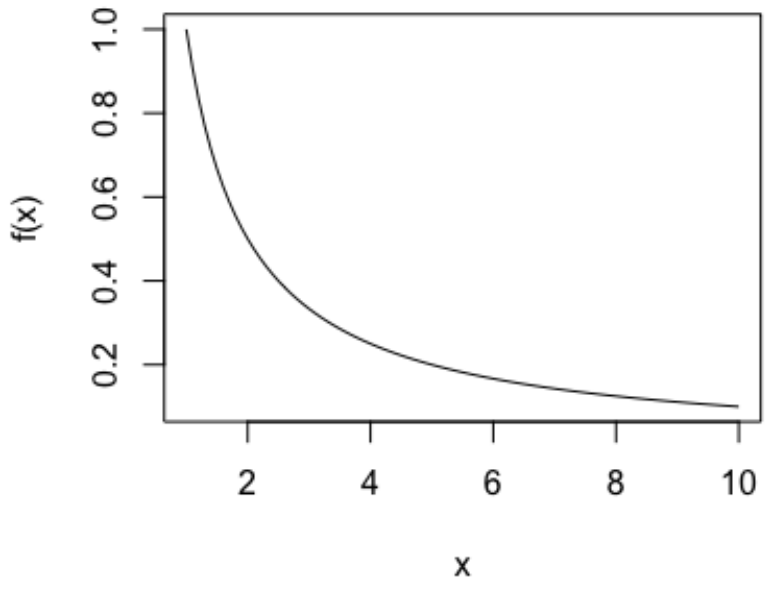


Rational functions

$$f(x) = \frac{p(x)}{q(x)}, \tag{9}$$

with p and q are polynomial functions, e.g., $f(x) = \frac{1}{x}$.

```
1  f <- function(x) {
2     fx <- 1/x
3     return(fx)
4  }
5
6  input <- seq(1, 10, 0.1)
7  plot(input,
8  sapply(input, f),
9  type="l",
10 xlab = "x",
11 ylab = "f(x)")
```



Exercise

Implement the following functions with R, and experiment with the domain and the plot.

- $f(x) = sin(x)$
- $f(x) = log(x)$
- $\sqrt{x} - 2$
- $\sqrt{x - 2}$

Reference

S.K. Chung (2014), "Understanding Basic Calculus".

Available at `http://www.math.nagoya-u.ac.jp/~richard/teaching/f2015/BasicCalculus.pdf`