Date: ___/___/___

Activity

Find quadratic splines for the following dataset

x_i	y_i
0	1.0
1	3.0
2	2.0
3	4.0

$a_1 x^2 + b_1 x + C_1$

(3,4)

(1,3)   (2,2) $a_2 x^2 + b_2 x + C_2$

(0,1)
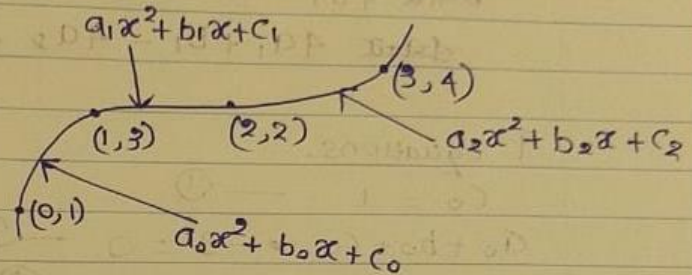
$a_0 x^2 + b_0 x + C_0$

Substituting points

$a_0 x^2 + b_0 x + C_0$

$(x=0 \;,\; x=1)$ $a_0 (0)^2 + b_0 (0) + C_0 = 1$
$a_0 (1)^2 + b_0 (1) + C_0 = 3$

$a_1 x^2 + b_1 x + C_1$

$(x=1 \;,\; x=2)$ $a_1 (1)^2 + b_1 (1) + C_1 = 3$
$a_1 (2)^2 + b_1 (2) + C_1 = 2$

$a_2 x^2 + b_2 x + C_2$

$(x=2 \;,\; x=3)$ $a_2 (2)^2 + a_2 (2) + C_2 = 2$
$a_2 (3)^2 + a_2 (3) + C_2 = 4$

Equating the derivatives at $(1,3)$

$$\frac{d(a_0 x^2 + b_0 x + C_0)}{dt}\Big|_{x=1} = \frac{d(a_1 x^2 + b_1 x + C_1)}{dt}\Big|_{x=1}$$

$2a_0 x + b_0 = 2a_1 x + b_1$
$2a_0 (1) + b_0 = 2a_1 (1) + b_1$
$2a_0 - 2a_1 + b_0 - b_1 = 0$

Equating the derivatives at $(2,2)$

$$\frac{d(a_1 x^2 + b_1 x + c_1)}{dt}\Big|_{x=2} = \frac{d(a_2 x^2 + b_2 x + c_2)}{dt}\Big|_{x=2}$$

$$2a_1 x + b_1 = 2a_2 x + b_2$$

$$\cancel{4a_1 x} \quad 4a_1 + b_1 - 4a_2 - b_2 = 0$$

9 equations.

$$C_0 = 1 \qquad\qquad \text{①}$$
$$a_0 + b_0 + C_0 - 3 = 0 \qquad \text{②}$$
$$a_1 + b_1 + C_1 - 3 = 0 \qquad \text{③}$$
$$4a_1 + 2b_1 + C_1 - 2 = 0 \qquad \text{④}$$
$$4a_2 + 2b_2 + C_2 - 2 = 0 \qquad \text{⑤}$$
$$9a_2 + 3b_2 + C_2 - 4 = 0 \qquad \text{⑥}$$
$$2a_0 + b_0 - 2a_1 - b_1 = 0 \qquad \text{⑦}$$
$$4a_1 + b_1 - 4a_2 - b_2 = 0 \qquad \text{⑧}$$
$$a_0 = 0 \qquad\qquad \text{⑨}$$

	a_i	b_i	C_i
0	0	2	1
1	3	2	-3
2	2	-4	6