Suggestions for an end of term project in "Physics with Mathematica"

Let ϕ be a real scalar field in $1+1$ dimensions with a potential $V(\phi) = \lambda(\phi^2 - 1)^2$ for a coupling constant $\lambda \geq 0$. Its equation of motion is

$$\partial_t^2 \phi = \partial_x^2 \phi - V'(\phi).$$

It has stationary solutions $\phi_\pm(t, x) = \pm 1$, but there are also kink solultions that are time independent but approach -1 for $x \to -\infty$ and $+1$ for $x \to +\infty$ (hint: set $\partial_t^2 \phi$ to zero and then interprete x as the new time and use energy conservation to reduce it to a first order equation).

You can boost the kink-solution to obtain a moving kink. Study perturbations of these kink solutions as well as the scattering of a kink and an anti-kink (a solution interpolating in the opposite direction).

You can also break the degeneracy between the two minima by adding a term to the potential that is linear in ϕ. What happens to the solutions?

Instead of a potential with two minima, you can also study the sine-Gordon-model with $V(\phi) = \lambda \cos(\phi)$ that has a richer structure of kink solutions.

When you study numerical solutions, you need to take care of boundary conditions. You can investigate several choices like periodic boundary conditions, Dirichlet or Neumann boundary conditions, you can try to find absorbing boundary conditions or do a coordinate transformation that maps the infinite x-axis to a finite interval.

There are many possibilities for visualizatons.

You don't have to follow all these suggestions. Be creative, do something interesting!

In the end, you should submit a well documented mathematica notebook via moodle. Please submit both an executable .nb file as well as a PDF.

You have until March 31st to submit your solution. Until then, I expect you to spend no more than two working days on this project.

You are expected to work on this alone without discussing your project or your solution with other people before your (and their) final submission. You can use the internet for inspiration but simply copying other people's work does not constitute a solution. You can contact me for questions any time on `helling@lmu.de`.