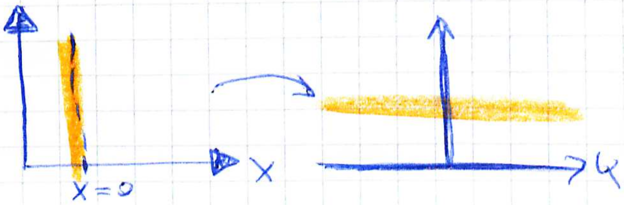
$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}\,dk$

$\tilde{\delta}(x) = \text{const}$



$e^{-x^2} \overset{FT}{\rightleftarrows} e^{-k^2}$

Zettel 14

a) $\omega_p^2 = \frac{Ne^2}{\epsilon_0 \cdot m_e}$ $\quad N = \frac{\text{Teilchenzahl}}{\text{Volumen}} = \frac{n}{V}$

$= \frac{n \cdot e^2}{\frac{4}{3}\pi r_{Ag}^3 \cdot \epsilon_0 \cdot m_e} = 1{,}362 \cdot 10^{16}\, s^{-1}$

$\omega = 2\pi f = \frac{2\pi c}{\lambda}$

$\omega(\lambda = 450\,nm) \approx 4{,}189 \cdot 10^{15}\,Hz$

$\omega(\lambda = 650\,nm) \approx 2{,}9 \cdot 10^{15}\,Hz$

$\omega(\lambda = 700\,nm) \approx 2{,}693 \cdot 10^{15}\,Hz$

$\left.\right\}$ $\omega < \omega_p$ $\Rightarrow k = i\kappa$ $\Rightarrow n'' = \mathrm{Im}(n)$

$n = \sqrt{\epsilon_r} = \sqrt{1 - \underbrace{\frac{\omega_p^2}{\omega^2}}_{>1}}$ da hier $\omega < \omega_p$, n imaginär

$n_{blau} \approx 1{,}5i$

$n_{rot} \approx 2i$

Senkrechter Lichteinfall $\Rightarrow$ Polarisation des Lichtes spielt keine Rolle

$r_p(\alpha) = \frac{n_2 - n_1}{n_2 + n_1}$

$n_1 = 1$ (Luft) $\quad n_2 = in'' \Rightarrow r_p = \frac{in'' - 1}{in'' + 1}$ $\leftarrow$ Amplitudenreflexion Betrag & Phase

Intensitätsreflexion Betrag:

$R = |r_p|^2 = \left|\frac{in''-1}{in''+1}\right|^2 = \left|\frac{1-in''}{1+in''}\right|^2 = 1 = \left|\frac{z}{z^*}\right|^2$

b) $\delta = \frac{1}{K}$ $\quad \omega^2 = \omega_p^2 - c^2k^2 \iff k = \sqrt{\frac{\omega^2 - \omega_p^2}{-c^2}}$ $\quad \delta_{blau} = 23\,mm$

$\Rightarrow$ Leichter Blaustich, da $\delta_{blau} > \delta_{rot}$ $\quad \delta_{rot} = 22{,}5\,nm$

c) $I = \frac{1}{100} I_0 \quad I \sim (E)^2 \Rightarrow |E| = |\frac{1}{10} E_0|$

$E(z,t) = E_0 \cdot e^{-kz - i\omega t}$

$E(z, t=0) = E_0 \cdot e^{-kz}$

$\frac{1}{10} \cancel{E_0} = \cancel{E_0}\, e^{-kz} \iff z = \ln(10) \cdot \delta \approx 50\,nm$

d) Betrachtung Grenzfall $\omega \to \omega_p$

$\omega^2 = \omega_p^2 + c^2k^2 \Rightarrow k = \sqrt{\frac{\omega^2 - \omega_p^2}{c^2}}$

$\Rightarrow E(z,t) = E_0 \cdot e^{i(kz - \omega t)}$

$\overset{t=0}{\Rightarrow} E_0 \cdot e^{i\sqrt{\frac{\omega^2 - \omega_p^2}{c^2}}}$

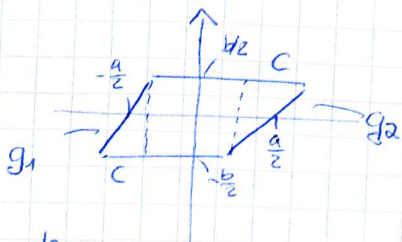
$\underline{\omega < \omega_p}$: $e^{\cdots}$ reell, schwächt sich also um $e^{\cdots}$ ab.

$\underline{\omega > \omega_p}$: $e^{\cdots}$ komplex, durchlässig

$\omega = \frac{2\pi c}{\lambda} \iff \lambda = \frac{2\pi c}{\omega}$; $\lambda < \frac{2\pi c}{\omega} = \cancel{138}\ 138\,nm$

$\omega^2 = \omega_p^2 \pm c^2k^2$

Aufgabe 2



Steigung $m = \frac{b}{c}$

y-Achsenabschnitt $n_1 = \frac{ab}{2c}$, $n_2 = -\frac{ab}{2c}$

$\Rightarrow g_1(x) = \frac{b}{c}x + \frac{ab}{2c}$, $g_2(x) = \frac{b}{c}x - \frac{ab}{2c}$

$\psi(\vec{r}, \omega) = C_1 \cdot \int_{M_1} 1 \cdot e^{-i(ux + vy)}\, dA$

mit $M_1 = \{(x,y) \in \mathbb{R}^2 \mid \frac{c}{b}y - \frac{a}{2} \le x \le \frac{c}{b}y + \frac{a}{2},\ -\frac{b}{2} \le y \le \frac{b}{2}$

$= C_1 \int_{-b/2}^{b/2} dy\, e^{-ikvy} \int_{\frac{c}{b}y - \frac{a}{2}}^{\frac{c}{b}y + \frac{a}{2}} dy\, e^{-ikux}$

$= C_1 \int_{-\frac{b}{2}}^{\frac{b}{2}} dy\, e^{ikvy} \cdot \frac{1}{-iku}\left(e^{-iku(\frac{c}{b}y + \frac{a}{2})} - e^{-iku(\frac{c}{b}y - \frac{a}{2})}\right)$

$= \frac{C_1}{-iku}\left[\frac{1}{-ik(v + \frac{uc}{b})} e^{-ik(vy + u(\frac{c}{b}y + \frac{a}{2}))} - \frac{1}{-ik(v + \frac{uc}{b})} e^{-ik(vy + u(\frac{c}{b}y - \frac{a}{2}))}\right|_{-b/2}^{b/2}$

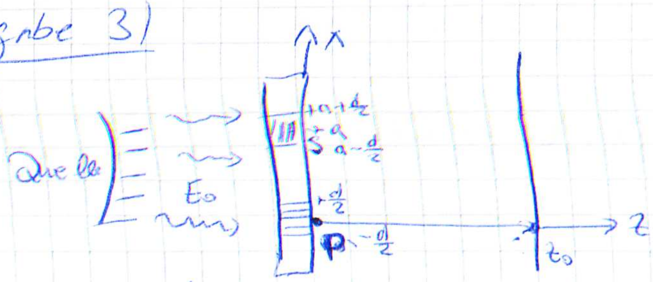
$\ldots = \frac{C}{k^2 u(v + \frac{uc}{b})} \sin\left(\frac{kua}{2}\right) \sin\left(\frac{k(vb + uc)}{2}\right) 4i^2$

$$= \dots = C_1 \frac{2}{k}\frac{2}{k} \frac{ab}{auvb + u^2 ca} \sin\left(\frac{kua}{2}\right) \sin\left(\frac{k(vb+uc)}{2}\right) \quad \Big| \quad \frac{\sin x}{x} = \operatorname{sinc} x$$

$$= -C_1\, ab\, \operatorname{sinc}\left(\frac{kua}{2}\right) \operatorname{sinc}\left(\frac{k(vb+uc)}{2}\right)$$

$$I_1 = |\Psi^2| = |C|^2 a^2 b^2 \operatorname{sinc}^2\left(\frac{kua}{2}\right) \operatorname{sinc}^2\left(\frac{k(ub+uc)}{2}\right)$$

Aufgabe 3)



(y-Achse zeigt aus der Ebene hinaus)

a) unpolarisiertes ~~unpolasiertes~~ Licht

$$\rightsquigarrow \binom{x}{y} = \vec{E}_0 = \binom{1}{1} A_0 \qquad |\vec{E}_0| = \sqrt{2}\, A_0 = E_0$$

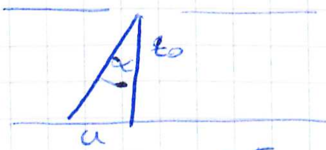
p-Polarisation

$$\vec{E}_p = \binom{0}{1} A_0 \Rightarrow \vec{E}_p = \frac{1}{\sqrt{2}} E_0$$

S-Polarisation

$$\Rightarrow E_s = \frac{1}{\sqrt{2}} E_0$$

Spalt 1)



$$\Psi_1(r',\omega) = \frac{C E_0}{\sqrt{2}} \int_{-d/2}^{d/2} dx\, e^{-ikux} = \frac{d\, C E_0}{\sqrt{2}} \operatorname{sinc}\left(\frac{kdz_0 \sin(\alpha)}{2}\right)$$

Spalt 2)

$$\Psi_2(r',\omega) = \frac{C E_0}{\sqrt{2}} \int_{-\frac{d}{2}+a}^{\frac{d}{2}+a} dx\, e^{-ikux} = \frac{dcE}{\sqrt{2}}\, e^{-aikz_0 \sin(\alpha)} \operatorname{sinc}\left(\frac{kdz_0 \sin\alpha}{2}\right)$$

keine Interferenz zwischen Ψ_1, Ψ_2

$$I_a = |\Psi_1|^2 + |\Psi_2|^2 = d^2 C^2 E_0^2 \operatorname{sinc}^2\left(\frac{kdz_0 \sin(\alpha)}{2}\right)^2$$

b) beide Filter Parallel

$$\Psi_{int} = \Psi_1 + \Psi_2 \quad \Big| \quad I_b = |\Psi_{int}|^2$$

$$= 2 d^2 c^2 E_0^2 \operatorname{sinc}^2\left(\frac{kdz_0 \sin(\alpha)}{2}\right) \cos^2\left(\frac{kdz_0 \sin(\alpha)}{2}\right)$$

c) linear polarisierten Licht (senkrecht) $\uparrow$ in Spaltrichtung

$$\binom{x}{y} = \vec{E} = \binom{0}{1} A_0 \quad \Big| \quad |\vec{E}_s| = \frac{1}{\sqrt{2}} E_0$$

$\longrightarrow$ a) Einzelspalt $I_{c,1} = |\Psi_1|^2 = \frac{d^2 C^2 E_0^2}{2} \operatorname{sinc}^2\left(\frac{kdz_0 \sin(\alpha)}{2}\right)$

$\longrightarrow$ b) Doppelspalt $I_{c,2} = |\Psi_1 + \Psi_2|^2 = I_b$

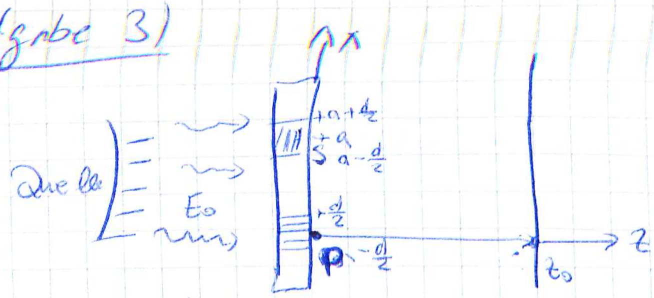
$$= \dots = C_1 \frac{2}{k}\frac{2}{k} \frac{ab}{auvb+u^2 ca} \sin\left(\frac{kua}{2}\right) \sin\left(\frac{k(vb+uc)}{2}\right) \quad \Big| \quad \frac{\sin x}{x} = \mathrm{sinc}\, x$$

$$= -C_1\, ab\, \mathrm{sinc}\left(\frac{kua}{2}\right) \mathrm{sinc}\left(\frac{k(vb+uc)}{2}\right)$$

$$I_1 = |\psi|^2 = |C|^2 a^2 b^2\, \mathrm{sinc}^2\left(\frac{kua}{2}\right) \mathrm{sinc}^2\left(\frac{k(ub+uc)}{2}\right)$$

Aufgabe 3)

(y-Achse zeigt aus der Ebene hinaus)



a) ~~unpolasierten~~ unpolarisiertes Licht

$$\rightsquigarrow \begin{pmatrix} x \\ y \end{pmatrix} = \vec{E}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} A_0 \quad |\vec{E}_0| = \sqrt{2}\, A_0 = E_0$$

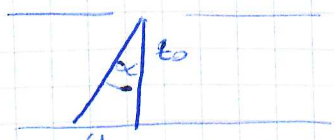
p-Polarisation

$$\vec{E}_p = \begin{pmatrix} 0 \\ 1 \end{pmatrix} A_0 \Rightarrow \vec{E}_p = \frac{1}{\sqrt{2}} E_0$$

S-Polarisation

$$\Rightarrow E_s = \frac{1}{\sqrt{2}} E_0$$

Spalt 1)



$$\psi_1(r',\omega) = \frac{C E_0}{\sqrt{2}} \int_{-d/2}^{d/2} dx\, e^{-ikux} = \frac{d\, C E_0}{\sqrt{2}} \mathrm{sinc}\left(\frac{k d z_0 \sin(\alpha)}{2}\right)$$

Spalt 2)

$$\psi_2(r',\omega) = \frac{C E_0}{\sqrt{2}} \int_{-\frac{d}{2}+a}^{\frac{d}{2}+a} dx\, e^{-ikux} = \frac{d C E}{\sqrt{2}} e^{-a i k z_0 \sin(\alpha)}\, \mathrm{sinc}\left(\frac{k d z_0 \sin\alpha}{2}\right)$$

keine Interferenz zwischen ψ_1, ψ_2

$$I_a = |\psi_1|^2 + |\psi_2|^2 = d^2 C^2 E_0^2\, \mathrm{sinc}^2\left(\frac{k d z_0 \sin(\alpha)}{2}\right)^2$$

b) beide Filter parallel

$$\psi_{int} = \psi_1 + \psi_2 \quad \Big| \quad I_b = |\psi_{int}|^2$$

$$= 2 d^2 c^2 E_0^2\, \mathrm{sinc}^2\left(\frac{k d z_0 \sin(\alpha)}{2}\right) \cos^2\left(\frac{k d z_0 \sin(\alpha)}{2}\right)$$

c) linear polarisiertes Licht (senkrecht) $\uparrow$ m Spaltrichtung

$$\begin{pmatrix} x \\ y \end{pmatrix} = \vec{E} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} A_0 \quad \Big| \quad |\vec{E}_s| = \frac{1}{\sqrt{2}} E_0$$

$\longrightarrow$ a) Einzelspalt $I_{c,1} = |\psi_1|^2 = \frac{d^2 C^2 E_0^2}{2} \mathrm{sinc}^2\left(\frac{k d z_0 \sin(\alpha)}{2}\right)$

$\longrightarrow$ b) Doppelspalt $I_{c,2} = |\psi_1 + \psi_2|^2 = I_b$

$\psi_2 = \psi_1 e^{-i\gamma}$

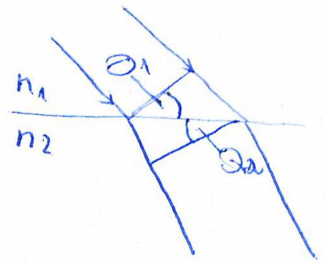
a) $\vec{\psi} = \begin{pmatrix}\psi_p\\ \psi_s\end{pmatrix} = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \psi_1\begin{pmatrix}1\\ e^{-i\gamma}\end{pmatrix}$ $\quad |\vec{\psi}| = |\psi_1|^2 \underbrace{\sqrt{1+|e^{-2i\gamma^2}|}^{\,2}}_{=2}$

b) $\vec{\psi} = \begin{pmatrix}\psi_p\\ \psi_s\end{pmatrix} = \begin{pmatrix}\psi_1+\psi_2\\ 0\end{pmatrix} = \psi_1\begin{pmatrix}1+e^{-i\gamma}\\ 0\end{pmatrix}$; $\quad |\vec{\psi}|^2 = \underbrace{|\psi_1|^2}_{\text{Einzelspalt}}\, \underbrace{4\cos^2\left(\frac{\gamma}{2}\right)}_{\text{Interferenz}}$

Optik: $\omega = k v_{ph} = k \frac{c}{n}$

$v_{ph} \neq v_{gr} = \frac{\partial\omega}{\partial k}$ aber für Licht $v_{ph} = v_{gr}$.

Brechungsgesetz von Snellius



$n_1 \sin(\Theta_1) = n_2 \sin(\Theta_2)$ (13,3)

optisch dünne → dichtere
Strahl zur Senkrechten hin
optisch dicht → dünn
Strahl von Senkrechter weg

Totalreflexion

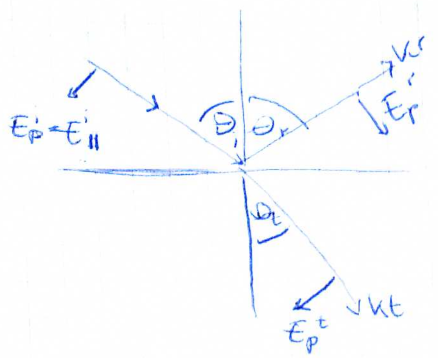
$\Theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$ → Übergang von dicht → dünn $n_1 > n_2$

Zusatz: evaneszente Welle im dünnen Medium

Kontinuitätsbedingungen aus Maxwell an Grenzflächen

$E_\parallel$ stetig $D_\perp$ stetig

$H_\parallel$ stetig $B_\perp$ stetig



Stichwort Fresnel-Gleichungen

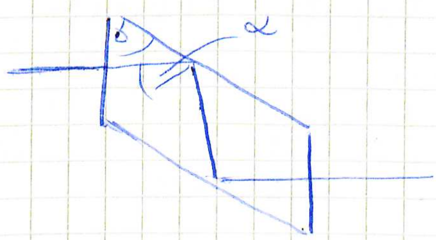
$r_p = r_\parallel = \frac{\cos\vartheta_i n_2 - \cos\vartheta_t n_1}{\cos\vartheta_i n_2 + \cos\vartheta_t n_1}$ $\qquad t_p = t_\parallel = \frac{2 n_1 \cos\vartheta_r}{\cos\vartheta_i n_2 + \cos\vartheta_t n}$

$r_s = r_\perp = \frac{\cos\vartheta_i n_1 - \cos\vartheta_t n_2}{\cos\vartheta_i n_1 + \cos\vartheta_t n_2}$ $\qquad t_s = t_\perp = \frac{2 n_1 \cos\vartheta_i}{\cos\vartheta_i n_1 + \cos\vartheta_t n}$

r, t → Amplituden

R, T → Intensitäten

Aufgabe 4 $\quad \alpha = \gamma$



a) $r_s = \dfrac{n_1 \cos(\alpha) - \sqrt{n_2^2 - n_1^2 \sin^2\alpha}}{*}$

$r_p = \dfrac{n_2 \cos(\alpha) - \sqrt{\frac{n_1}{n_2}\, n_2^2 - n_1^2 \sin\alpha}}{n_2 \cos(\alpha) + \frac{n_1}{n_2}\sqrt{\ldots}} \quad *$

Snellius

$n_1 \sin(\alpha) = n_2 \sin(\beta)$

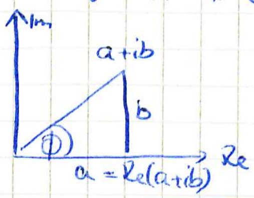
$n_1 \sin(\alpha) > n_2 \cdot 1 \quad \leftarrow$ Totalreflexion

$\Rightarrow n_2^2 - n_1^2 \sin^2(\alpha) < 0$

$\Rightarrow \sqrt{\ldots} = + i \sqrt{- \ldots}$

$$r_s = \frac{n_1 \cos(\alpha) - i\sqrt{n_1^2 \sin^2\alpha - n_2^2}}{n_1 \cos(\alpha) + i\sqrt{n_1^2 \sin\alpha - n_2^2}} = \frac{a - ib}{a + ib} = \frac{re^{-i\Phi}}{re^{i\Phi}}$$

$= e^{-i2\Phi}$



$$\tan(\Phi) = \frac{b}{a} = \frac{\sqrt{n_1^2 \sin\alpha - n_2^2}}{n_1 \cos(\alpha)} \quad (2)$$

analog für r_p

$$r_p = \frac{n_2 \cos(\alpha) - i\,\frac{n_1}{n_2}\sqrt{n_1^2 \sin^2(\alpha) - n_2^2}}{n_2 \cos(\alpha) + i\,\frac{n_1}{n_2}\sqrt{\ldots}} = \frac{c - id}{c + id} = \frac{\tilde{r} e^{-i\Psi}}{\tilde{r} e^{i\Psi}} = e^{-i2\Psi}$$

$$\tan(\Psi) = \frac{\frac{n_1}{n_2}\sqrt{n_1^2 \sin^2(\alpha) - n_2^2}}{n_2 \cos(\alpha)} \quad (2')$$

Tipp: $\tan(a \pm b) = \dfrac{\tan(a) \pm \tan(b)}{1 \pm \tan(a)\tan(b)}$

Phasenverschiebung nach Prisma (1 mal reflektiert)

$\left.\begin{array}{l} \text{s-Pol.: } 2\Phi = \Delta\varphi_s \\ \text{p-Pol.: } 2\Psi = \Delta\varphi_p \end{array}\right\}$ zwischen s- & p-Pol.: Phase $\delta = 2\Psi - 2\Phi$

$\tan\left(\frac{\delta}{2}\right) = \tan(\Psi - \Phi) \overset{\text{Tipp}}{=} \dfrac{\tan(\Psi) - \tan(\Phi)}{1 - \tan(\Psi)\tan(\Phi)} \ldots$ setze (2) und (2') ein

$$= \frac{\cos(\alpha)\sqrt{\sin^2(\alpha) - \frac{n_2}{n_1}}}{\sin^2(\alpha)} \quad (3)$$

$\cos\alpha = \sqrt{1-\sin^2(\alpha)}$

b) $(3) \iff 0 = \sin^4(\alpha) - \sin^2(\alpha)\,\dfrac{1+\left(\frac{n_2}{n_1}\right)^2}{1+\tan^2\left(\frac{\delta}{2}\right)} + \dfrac{\left(\frac{n_2}{n_1}\right)^2}{1+\tan^2\left(\frac{\delta}{2}\right)}$

$z := \sin^2(\alpha)$ & p-q-Formel

$$z = \frac{1+\left(\frac{n_2}{n_1}\right)^2}{2+2\tan^2\left(\frac{\delta}{2}\right)} \pm \sqrt{\left[\frac{1+\left(\frac{n_2}{n_1}\right)^2}{2+2\tan^2\left(\frac{\delta}{2}\right)}\right]^2 - \frac{\left(\frac{n_2}{n_1}\right)^2}{1+\tan^2(\delta/2)}}$$

mit $\alpha = \tan(\sqrt{2})$

1-mal Reflexion: Phasenverschub δ
2-mal " : möchte $\frac{\pi}{2}$ haben
(ganzer Prisma)

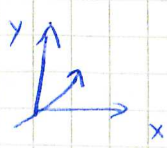
$\Big\}\ 2\delta = \frac{\pi}{2} \iff \delta = \frac{\pi}{4}$

$z_+ \approx 0{,}51$ $\Longrightarrow$ $\alpha_1 = 57{,}2^\circ$

$z_- \approx 0{,}65$ $\quad\ \alpha_2 = 53{,}3^\circ$

linear polarisiert

c) linear $\vec{E} = E_0 \begin{pmatrix} \sin(\varphi) \\ \sin(\varphi) \end{pmatrix} = E_0 \cdot \sin\varphi \begin{pmatrix} 1 \\ 1 \end{pmatrix}$



nach Prisma:

$\vec{E} = E_0 \begin{pmatrix} \sin(\varphi) \\ \sin(\varphi+\frac{\pi}{2}) \end{pmatrix} = E_0 \begin{pmatrix} \sin(\varphi) \\ \cos(\varphi) \end{pmatrix}$

$\boxed{\text{Eigene Aufgabe}}$

$l(k)$, $m(k)$, $n(k)$

$Q = q^k p^l$, $P = q^m p^n$

$\Rightarrow$ kanonische Trafo

$\{f,g\} = \dfrac{\partial f}{\partial q}\dfrac{\partial g}{\partial p} - \dfrac{\partial g}{\partial q}\dfrac{\partial f}{\partial p}$

$\{q,p\} = 1\cdot 1 - 0\cdot 0 = 1$

$\{Q,P\} = 1\,?$

$\dfrac{\partial Q}{\partial q} = k q^{k-1} p^l \qquad \dfrac{\partial Q}{\partial p} = l q^k p^{l-1}$

$\dfrac{\partial P}{\partial p} = n q^n p^{n-1} \qquad \dfrac{\partial P}{\partial q} = m q^{m-1} p^n$

$\{Q,P\} = (kn - lm)\, q^{k+m-1}\, p^{l+n-1} \doteq 1$

$\Longrightarrow m = 1-k$, $l = k-1$; $n = 2-k$

$Q = p^{-1}(q,p)^k$, $P = p\,(qp)^{1-k}$

$F_1(q,Q) = (1-k)\cdot Q^{\frac{1}{k-1}}\ q^{\frac{1}{(1-k)}}$

② $Q = p \,;\; P = -q \,,\; F_1(q,Q)$

$$F_1(q,Q) = \int p(q,Q)\,dq + g(Q)$$
$$= \int Q\,dq + g(Q)$$
$$= qQ + g(Q)$$

$$\frac{\partial F_1(q,Q)}{\partial Q} = q + g'(Q) \stackrel{!}{=} -P(q,Q) = q$$

$$g'(Q) = 0 \Rightarrow g = \text{const}$$

$$\Rightarrow F_1(q,Q) = qQ$$

(*) $Q = p,\; P = p$

$$F_1(q,Q) = qQ + g(Q)$$

$$\frac{\partial F_1}{\partial Q} = q + g'(Q) = -P(q,Q) = -q$$

$$g'(Q) = -2q$$

③ a) $F(p,Q) = p^2 + Q^2$

b) $F(p,Q) = -(e^Q - 1)^2 \tan p$

$$P = -\frac{\partial F}{\partial Q} \qquad q = -\frac{\partial F}{\partial p}$$

a) $P = -\frac{\partial F}{\partial Q} = -2Q$ \
$q = -\frac{\partial F}{\partial p} = 2p$ \
$\Big\}$ keine Trafo $(q,p) \to (Q,P)$?

b) $P = 2e^Q(e^Q - 1)\tan p$

$$q = \frac{(e^Q - 1)^2}{\cos^2 p}$$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = -\sin x \cos^{-2}x\,(-\sin x) + \frac{\cos x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos^2 x} + 1 = \tan^2 x + 1 = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$Q = \ln(1 + \sqrt{q\cos^2 p})$$

$$P = \pm 2(1 \pm \sqrt{q\cos^2 p})\sqrt{q\cos^2 p}\,\tan p$$