$$M_{ges} = M_3 M_2 M_1$$

$$\underbrace{\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}}_{M_2} \underbrace{\begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix}}_{M_1} = \underbrace{\begin{pmatrix} \alpha & \alpha d_1 + \beta \\ \gamma & \gamma d_1 + \delta \end{pmatrix}}_{M_{2/1}}$$

$$\underbrace{\begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix}}_{M_3} \underbrace{\begin{pmatrix} \alpha & \alpha d_1 + \beta \\ \gamma & \gamma d_1 + \delta \end{pmatrix}}_{M_{2/1}} = \underbrace{\begin{pmatrix} \alpha + d_2\gamma & (\alpha d_1 + \beta) + d_2(\gamma d_1 + \delta) \\ \gamma & \gamma d_1 + \delta \end{pmatrix}}_{M_{ges}}$$ ✓

b)

Der Einfallwinkel darf keine Rolle für die Darstellung des Bildes spielen

$$\Rightarrow (\alpha d_1 + \beta) + d_2(\gamma d_1 + \delta) \overset{!}{=} 0$$ ✓

$$\Leftrightarrow \alpha d_1 + \beta + d_2 \gamma d_1 + d_2 \delta = 0 \quad \text{mit } d_1 = g,\ d_2 = \ell - g$$ ✓

$$\Leftrightarrow \alpha g + \beta + \gamma(g(L-g)) + \delta(L-g) = 0$$

$$\Leftrightarrow -\gamma g^2 + g(\alpha + \gamma \ell - \delta) + \beta + \delta \ell = 0 \quad | \cdot \frac{-1}{\gamma}$$

$$g^2 + -\frac{g}{\gamma}(\alpha + \gamma\ell - \delta) + \frac{\beta}{-\gamma} + \frac{\delta \ell}{-\gamma} = 0$$

$$g_{1/2}\ \frac{\alpha - \delta}{2\gamma} + \frac{L}{2} \pm \sqrt{\left(\frac{\alpha - \delta}{2\gamma} + \frac{L}{2}\right)^2 + \frac{\beta + \delta L}{\gamma}}$$ ✓

c)

$$\frac{1}{f} = \frac{1}{g} + \frac{1}{b} \quad \left(\Leftrightarrow g = \frac{1}{\frac{1}{f} - \frac{1}{b}}\right)$$

$$= \frac{1}{g} + \frac{1}{\ell - g}$$

$b = \ell - g$

$$= \frac{\ell}{g\ell - g^2}$$

$$\Leftrightarrow g\ell - g^2 - \ell f = 0$$

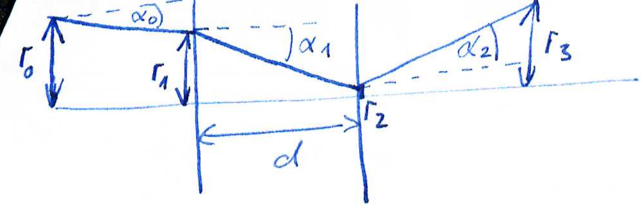
$$g^2 - g\ell + \ell f = 0$$

$$g_{1/2} = \frac{\ell}{2} \pm \sqrt{\frac{\ell^2}{4} - \ell f}$$ ✓

Scharfe Abb nur, wenn

$$\frac{\ell^2}{4} \geq \ell f \Leftrightarrow f \leq \frac{\ell}{4}$$ ✓

5/5



$$M\begin{pmatrix} r_1 \\ \alpha_0 \end{pmatrix} = \begin{pmatrix} r_2 \\ \alpha_2 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 0 \\ -D_2 & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -D_1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & d \\ -D_2 & -D_2 d + 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -D_1 & 1 \end{pmatrix} = \begin{pmatrix} 1 - D_1 d & d \\ \underbrace{-D_2 + D_1 D_2 d - D_1}_{\simeq -\frac{1}{\tilde{f}}} & -D_2 d + 1 \end{pmatrix}$$

$$\tilde{f} = \frac{1}{D_1 + D_2 - D_1 D_2 d}$$

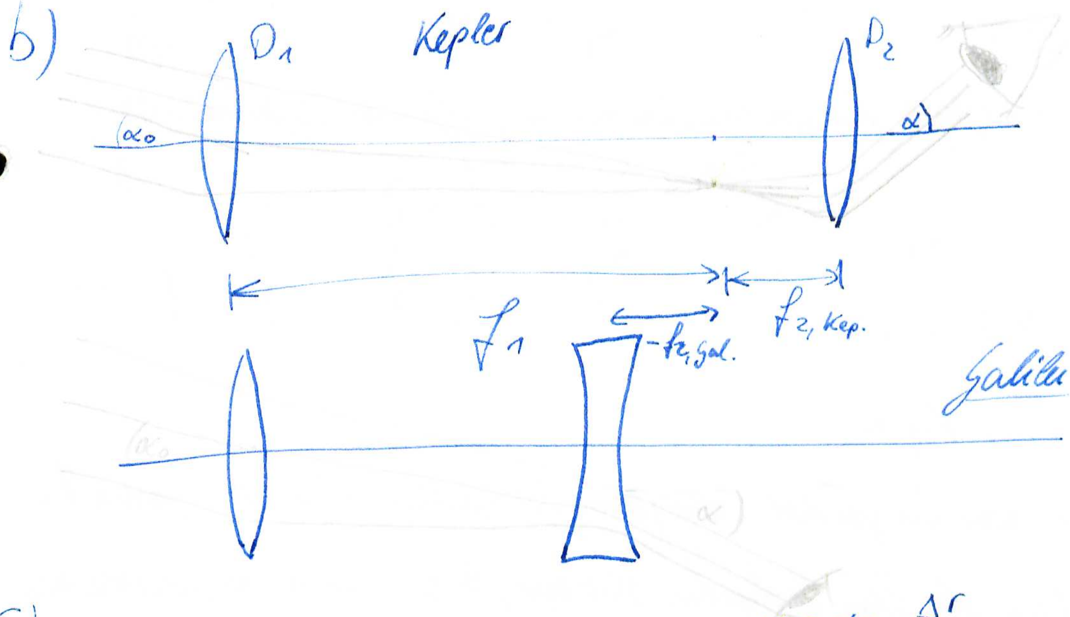
Forderung: $-\frac{1}{\tilde{f}} = 0$ bzw. $\tilde{f} \rightarrow$

Parallel eintretende Strahlen sollen auch parallel austreten.

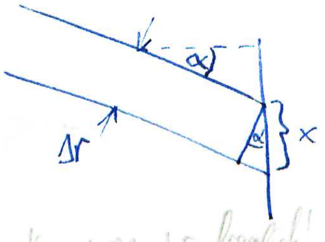
$\Rightarrow 0 = D_1 + D_2 - D_1 D_2 d$

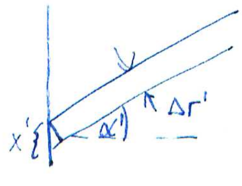
$\Rightarrow \alpha_2 = (1 - D_2 d) \cdot \alpha_0$

b) Kepler



c)





$\cos\alpha = \frac{\Delta r}{x}$; $\cos\alpha' = \frac{\Delta r'}{x'}$

$x = r_0'' - r_0'$; $x' = r_2'' - r_2'$

$x' = (r_0''(1 - D_1 d) + d\alpha) - (r_0'(1 - D_1 d) + d\alpha)$

$= (1 - D_1 d)x$

Definition von Δr fraglich!
Für vlt. sinnvollere Alternative siehe d)

A1	A2	A3	A4	Σ
5	5	4,5	4,5	19

A1) 13. Übungsblatt   Herr M. Sc. Jaeger, Herr Hölting

a) $M_1 = \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix}$; $M_2 = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$; $M_3 = \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix}$

$M_{ges} = M_3 M_2 M_1$

$$\underbrace{\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}}_{M_2} \underbrace{\begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix}}_{M_1} = \underbrace{\begin{pmatrix} \alpha & \alpha d_1 + \beta \\ \gamma & \gamma d_1 + \delta \end{pmatrix}}_{M_{2/1}}$$

$$\underbrace{\begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix}}_{M_3} \underbrace{\begin{pmatrix} \alpha & \alpha d_1 + \beta \\ \gamma & \gamma d_1 + \delta \end{pmatrix}}_{M_{2/1}} = \underbrace{\begin{pmatrix} \alpha + d_2\gamma & (\alpha d_1 + \beta) + d_2(\gamma d_1 + \delta) \\ \gamma & \gamma d_1 + \delta \end{pmatrix}}_{M_{ges}}$$ ✓

b)

Der Einfallwinkel darf keine Rolle für die Darstellung des Bildes spielen

$\Longrightarrow (\alpha d_1 + \beta) + d_2(\gamma d_1 + \delta) \stackrel{!}{=} 0$ ✓

$\Longleftrightarrow \alpha d_1 + \beta + d_2 \gamma d_1 + d_2 \delta = 0$ mit $d_1 = g$, $d_2 = \ell - g$ ✓

$\Longleftrightarrow \alpha g + \beta + \gamma(g(L-g)) + \delta(L-g) = 0$

$\Longleftrightarrow -\gamma g^2 + g(\alpha + \gamma\ell - \delta) + \beta + \delta\ell = 0 \quad | \cdot \frac{-1}{\gamma}$

$g^2 + -\frac{g}{\gamma}(\alpha + \gamma\ell - \delta) + \frac{\beta}{-\gamma} + \frac{\delta\ell}{-\gamma} = 0$

$g_{1/2} \quad \frac{\alpha - \delta}{2\gamma} + \frac{L}{2} \pm \sqrt{\left(\frac{\alpha-\delta}{2\gamma} + \frac{L}{2}\right)^2 + \frac{\beta + \delta L}{\gamma}}$ ✓

c)

$\frac{1}{f} = \frac{1}{g} + \frac{1}{b} \quad \left(\Longleftrightarrow g = \frac{1}{\frac{1}{f} - \frac{1}{b}} \right)$

$= \frac{1}{g} + \frac{1}{\ell - g}$

$b = \ell - g$

$= \frac{\ell}{g\ell - g^2}$

$\Longleftrightarrow g\ell - g^2 - \ell f = 0$

$g^2 - g\ell + \ell f = 0$

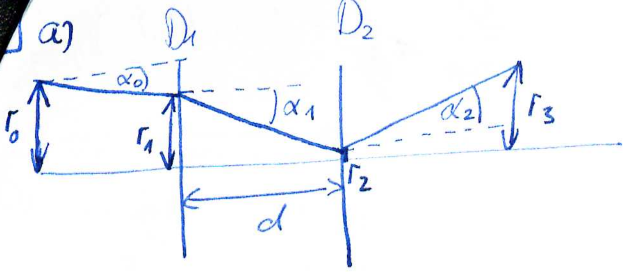
$g_{1/2} = -\frac{\ell}{2} \pm \sqrt{\frac{\ell^2}{4} - \ell f}$ ✓

Scharfe Abb nur, wenn

$\frac{\ell^2}{4} \geq \ell f \Longleftrightarrow f \leq \frac{\ell}{4}$ ✓

5/5

a)



$$M\begin{pmatrix} r_1 \\ \alpha_0 \end{pmatrix} = \begin{pmatrix} r_2 \\ \alpha_2 \end{pmatrix}$$

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$$\tilde{f} = \frac{1}{D_1+D_2-D_1 D_2 d}$$

Forderung: $-\frac{1}{\tilde{f}} = 0$ bzw. $\tilde{f} \to \infty$

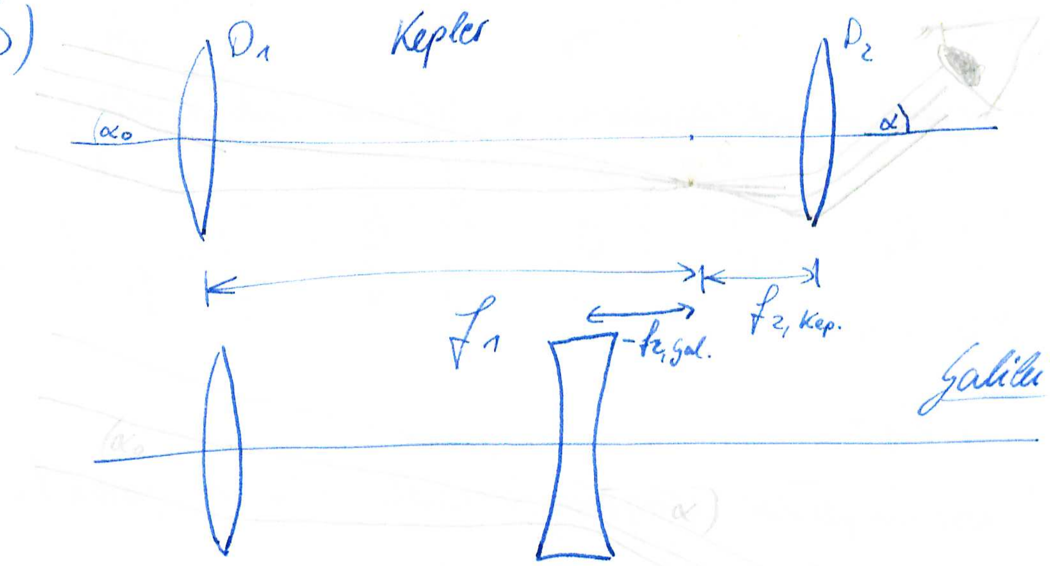
Parallel eintretende Strahlen sollen auch parallel austreten.

$$\Rightarrow 0 = D_1 + D_2 - D_1 D_2 d \quad (*)$$

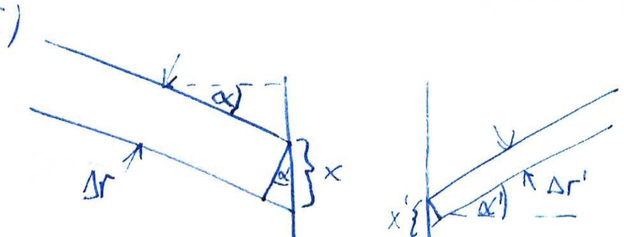
$\Rightarrow d = f_1 + f_2$ (siehe d))

$$\Rightarrow \alpha_2 = (1 - D_2 d) \cdot \alpha_0$$

b)



c)



$$\cos\alpha = \frac{\Delta r}{x} \quad ; \quad \cos\alpha' = \frac{\Delta r'}{x'}$$

$$x = r_0'' - r_0' \quad ; \quad x' = \cancel{x_2}\, r_2'' - r_2'$$

$$x' = \left(r_0''(1-D_1 d) + d\alpha\right) - \left(r_0'(1-D_1 d) + d\alpha\right)$$

$$= (1 - D_1 d)\, x$$

Definition von Δr fraglich!
Für obj. sinnvollere Alternative siehe d)

→ $\Delta r' = \cos\alpha' (1-D_1 d)\, x = \cos\alpha' \cdot (1-D_1 d) \cdot \frac{\Delta r}{\cos\alpha} = \Delta r \frac{\cos((1-D_2 d)\alpha)}{\cos\alpha} (1-D_1 d)$

$\alpha' = (1-D_2 d)\alpha$

für weitere Vereinfachung siehe d)

→ $V = \left|\frac{\alpha'}{\alpha}\right| = |1-D_2 d| \overset{(*)}{=} \left|1 - D_2 \cdot \frac{D_1+D_2}{D_1 D_2}\right| = \left|\frac{D_2}{D_1}\right| = \left|\frac{f_1}{f_2}\right|$

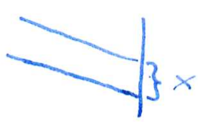
d) Mit (*) ist

$\cancel{V = \left|1 - D_2 \cdot \frac{D_1+D_2}{D_1 D_2}\right| = \left|\frac{+D_2}{D_1}\right| =}$

Beide Vergrößerungen weisen bei Linsen mit gleichen Brennweiten

$f_{1,\text{Galilei}} = f_{1,\text{Kepler}}$; $f_{2,\text{Galilei}} = -f_{2,\text{Kepler}}$

dieselbe Vergrößerung auf. Wenn wir die Terme $\frac{\cos((1-D_2 d)\alpha)}{\cos\alpha} = 1$ setzen, d.h. uns eigentlich nur die Projektion der Strahlen auf die senkrechte Linsenfläche anschauen:



dann ergibt sich im Vergleich der Strahlaufweitungen folgender Unterschied:

$$\frac{\Delta r'}{\Delta r} = 1 - D_1 d \overset{(*)}{=} 1 - D_1 \cdot \frac{D_1+D_2}{D_1 D_2} = 1 - \frac{1}{f_1} \cdot \frac{\frac{1}{f_1}+\frac{1}{f_2}}{\frac{1}{f_1}\cdot\frac{1}{f_2}} = \frac{f_1}{f_1} - \frac{1}{f_1}\cdot\underbrace{(f_1+f_2)}_{=d}$$

$$\frac{\Delta r'}{\Delta r} = -\frac{f_2}{f_1} \begin{cases} >0 & \text{Galilei} \\ <0 & \text{Kepler} \end{cases}$$

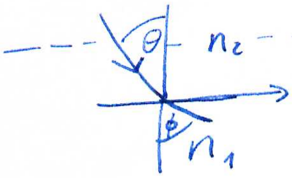
Das Galilei-Fernrohr ist zwar bei gleicher Vergrößerung kompakter, aber das Gesichtsfeld ist deutlich kleiner als beim Keplerschen Fernrohr, da die Strahlen aufgeweitet werden.

5/5

3 a) $n = \frac{c}{v} = \frac{s}{c \cdot t} \Rightarrow \Delta t = \frac{\Delta n}{c \cdot s} = 1\,\mu s$ (✓) $n_1 = 1{,}52$ $n_2 = 1{,}66$

↳? $\frac{c}{v} = \frac{c \cdot t}{s}$ $\frac{\Delta n s}{c}$

b) $\frac{n_1}{n_2} = \frac{\sin\alpha_2}{\sin\alpha_1}$ ✓ Totalreflex.: $\sin\alpha_2 = 1 = \sin\phi$

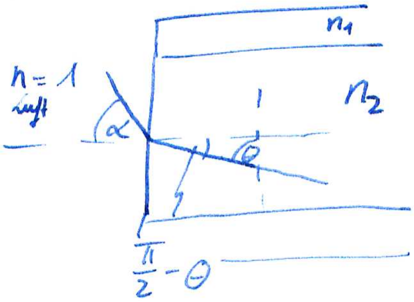


bzw. hier

$$n_2 \cdot \sin\Theta_c = n_1 \cdot \underbrace{\sin\phi}_{1}$$

$$\sin\Theta_c = \frac{n_1}{n_2} \rightarrow \Theta_c = \arcsin\left(\frac{n_1}{n_2}\right) = 66{,}3°$$ ✓

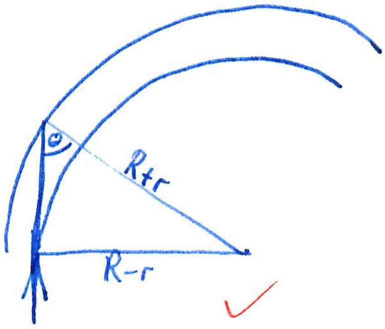
c)



$$1 \cdot \sin\alpha = n_2 \cdot \sin\left(\frac{\pi}{2} - \Theta\right) = n_2 \cdot \cos(\Theta)$$ ✓

$$\alpha = \arcsin(n_2 \cdot \cos 66{,}3°) = 41{,}8° = \varphi_{max}$$ ✓

d) und e) WTF?



✓

$$\sin\Theta = \frac{R-r}{R+r}$$ ✓ $$\Leftrightarrow (R+r)\sin\Theta = R-r$$

$$\Leftrightarrow R_{min} = r \cdot \frac{1+\sin\Theta}{1-\sin\Theta}$$ ✓ $= 2{,}2\,cm$ ✓

↯

Setze $\Theta = \Theta_c$ ein.

4,5/5

Hausaufgabe 4

$F(y, y')$ erfüllt E-L $\quad$ mit $\frac{dy}{dx} = y'$

$$\frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y} = 0$$

$$\Leftrightarrow \frac{\partial F}{\partial y'} = \int \frac{\partial F}{\partial y}\, dx \quad \text{mit } dx = \frac{dy}{y'}$$

$$= \int \frac{1}{y'}\frac{\partial F}{\partial y}\, dy \quad \Big| \cdot y'$$

$$\Leftrightarrow \frac{\partial F}{\partial y'} \cdot y' = F + C \quad (*)$$

b) $T = \int dt = \int ds \frac{1}{v}$ $\qquad ds = \sqrt{dx^2 + dy^2}$

$$= dx\,\Big(1 + \underbrace{\frac{dy^2}{dx^2}}_{y'}\Big)^{\frac{1}{2}}$$

$$= \int dx \underbrace{\frac{n(y)}{c}(1+y'^2)^{\frac{1}{2}}}_{F}$$

$v = \frac{c_0}{n(y)}$

$$\Rightarrow F = \frac{n(y)}{c}(1+y'^2)^{\frac{1}{2}}$$

c) F in $(*)$ $\quad \frac{n}{c_0}(1+y'^2)^{\frac{1}{2}} - y'\frac{n}{c_0}\frac{\partial (1+y'^2)^{\frac{1}{2}}}{\partial y'} = C$

$$\Leftrightarrow \frac{n}{c_0}(1+y'^2)^{1/2} - y'^2\frac{n}{c_0}(1+y'^2)^{-\frac{1}{2}} = C$$

$$\Leftrightarrow \frac{n}{c_0}(1+y'^2)^{-1/2} = C \quad \Big|\cdot\frac{c}{n}\ \Big|\,kw\,\Big|^2\Big|-1\Big|\sqrt{\ }$$

$$\Leftrightarrow dx\sqrt{\left(C\frac{c_0}{n}\right)^{-2} - 1} = dy \Rightarrow \int dx = \int dy \frac{1}{\sqrt{\left(C\frac{c_0}{n}\right)^{-2} - 1}}$$

$$\Leftrightarrow \int dx = \int dy \frac{c_0 C}{\sqrt{n^2 - c_0^2 C^2}}$$

d) $\int dx = c_0 C \int dy \frac{1}{\sqrt{n_0^2 + 2\alpha y n_0^2 + \alpha^2 y^2 n_0^2 - c_0^2 C^2}}$ $\qquad$ mit $n(y) = n_0(1+\alpha y)$

Subst. $u = \text{const}\cdot\left(y + \frac{1}{\alpha}\right)$

$$\frac{du}{dy} = \underbrace{\text{const}}_{:= d} \rightarrow dy = \frac{1}{d}\,du \quad , \quad y = \frac{u}{d} - \frac{1}{\alpha}$$

$$= \frac{1}{d}\int du \frac{c_0 C}{\sqrt{n_0^2\left(1 + \frac{\alpha u}{d} - 1\right)^2 - c_0^2 C^2}}$$

$$= \frac{1}{d}\int du \frac{1}{\sqrt{\frac{n_0^2\alpha^2}{d^2 c_0^2 C^2}u^2 - 1}} \qquad \text{Wähle } d = \frac{n_0\alpha}{c_0 C}$$

$$\Leftrightarrow x-a = \frac{c_0 C}{h_0 \alpha} \operatorname{arcosh}(u) = \frac{c_0 C}{h_0 \alpha} \operatorname{arcosh}\left(\frac{h_0 \alpha}{c_0 C}\left(y+\frac{1}{\alpha}\right)\right)$$

$$\Leftrightarrow y(x) = -\frac{1}{\alpha} + \frac{c_0 C}{\alpha h_0} \cosh\left(\frac{\alpha h_0}{c_0 C}(x-a)\right)$$

✓

Begründung / Erklärung

4,5 / 5