9. Übungsblatt Matthias Jaeger, Marius Hölting

Frohes Fest

A1	A2	A3	A4	Σ
4,5	5	4	4	17,5 /20

Aufgabe 1:

a) $f_1(x) = \begin{cases} x + 2\pi, & -\pi \leq x < 0 \\ x, & 0 \leq x < \pi \end{cases}$

$A_k = \frac{1}{2\pi} \left(\int_{-\pi}^{0} (x+2\pi) e^{-ikx} dx + \int_{0}^{\pi} x e^{-ikx} dx \right)$ ✓

$= \int_{-\pi}^{0} e^{-ikx} dx + \frac{1}{2\pi} \left(\int_{-\pi}^{0} x e^{-ikx} dx + \int_{0}^{\pi} x e^{-ikx} dx \right)$

$= \frac{-1}{ik} e^{-ikx} \Big|_{-\pi}^{0} + \frac{1}{2\pi} \int_{-\pi}^{\pi} x e^{-ikx} dx$ ✓

$= -\frac{1}{ik} + \frac{1}{ik} e^{+ik\pi} + \frac{1}{2\pi} \left[\frac{x e^{-ikx}}{-ik} \right]_{-\pi}^{\pi} - \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{-ik} e^{-ikx} dx$

$= -\frac{1}{ik} + \frac{1}{ik} e^{+ik\pi} + \frac{1\pi}{2\pi(-ik)} e^{-ik\pi} - \frac{1\pi}{2\pi ik} e^{ik\pi} - \frac{1}{2\pi (ik)^2} e^{-ikx} \Big|_{-\pi}^{\pi}$ (✓)

$= -\frac{1}{ik} + \frac{1}{k} \sin(k\pi) - \frac{i}{\pi k^2} \sin(k\pi)$ (✓)

für $k \in \mathbb{Z} \Rightarrow A_k = -\frac{1}{ik}$

$A_0 = \frac{1}{2\pi} \int_{-\pi}^{0} (x+2\pi) e^0 dx + \frac{1}{2\pi} \int_{0}^{\pi} e^0 x \, dx$

$= \frac{1}{2\pi} \left(\left[\frac{1}{2}x^2\right]_{-\pi}^{0} + \left[2\pi x\right]_{-\pi}^{0} + \left[\frac{1}{2}x^2\right]_{0}^{\pi} \right)$

$= \frac{1}{2\pi} \left(-\frac{\pi^2}{2} + 2\pi^2 + \frac{\pi^2}{2} \right) = \pi$ ✓

$\Longrightarrow S^n(x) = \left(\pi + \sum_{k=-n}^{-1} -\frac{1}{ik} \right) \cdot \underbrace{\sum_{k=-n}^{0\,?}}_{f} e^{ikx}$

b) $f_2(x) = \begin{cases} -1, & -\pi \leq x < 0 \\ 1, & 0 \leq x < \pi \end{cases}$

$A_k = \frac{1}{2\pi} \left(\int_{-\pi}^{0} -1 e^{-ikx} dx + \int_{0}^{\pi} 1 e^{-ikx} dx \right.$ ✓

$= \frac{1}{2\pi} \left(\frac{1}{ik} e^{-ikx} \Big|_{-\pi}^{0} + \frac{1}{-ik} e^{-ikx} \Big|_{0}^{\pi} \right)$

$= \frac{1}{2\pi} \left(\frac{1}{ik} + \frac{1}{ik} - \frac{1}{ik} e^{ik\pi} - \frac{1}{ik} e^{-ik\pi} \right)$

$= \frac{1}{ik\pi} - \frac{1}{2ik\pi} \left(e^{ik\pi} + e^{-ik\pi} \right)$

$= \frac{1}{ik\pi} - \frac{1}{ik\pi} \cos(k\pi) = -i/k\pi + \frac{i}{k\pi} \cos(k\pi)$ ✓

für $k \in \mathbb{Z} \Rightarrow -\frac{i}{k\pi} + \frac{i}{k\underset{\pi}{x}} (-1)^k$ (✓)

$A_0 = 0$ ✓

$\Rightarrow S^n(x) = \sum_{k=-n}^{n} \left(-\frac{i}{k\pi} + \frac{1}{k\pi} (-1)^k \right) e^{+ikx}$

c) $f_3(x) = x^2$

$$A_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} x^2 e^{-ikx} \, dx$$

$$= \frac{1}{2\pi} \left(\left[x^2 \frac{1}{-ik} e^{-ikx} \right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{2x}{-ik} e^{-ikx} \, dx \right) ✓$$

$$= \frac{1}{2\pi} \left(-\frac{\pi^2}{ik} e^{-ik\pi} - \frac{\pi^2}{-ik} e^{ikx} - \left[-\frac{2}{ik} \left(\left[\frac{x}{+ik} e^{-ikx} \right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{1}{-ik} e^{-ikx} \, dx \right] \right.$$

$$= \frac{1}{2\pi} \left(-\frac{\pi^2}{ik} e^{-ik\pi} + \frac{\pi^2}{ik} e^{ik\pi} + \frac{2}{ik} \left(\frac{-\pi}{ik} e^{-ik\pi} - \frac{\pi}{ik} e^{ik\pi} - \frac{1}{k^2} e^{-ikx} \Big|_{-\pi}^{\pi} \right) \right.$$

$$= \frac{\pi}{k} \sin(k\pi) + \frac{1}{ik\pi} \left(-\frac{\pi}{ik} 2\cos(k\pi) + \frac{1}{ik\pi} \left(-\frac{\pi}{ik} \cos(k\pi) - \frac{1}{k^2} i \sin(k\pi) \right) \right.$$

$$= \frac{\pi}{k} \sin(k\pi) + \frac{2}{k^2} \cos(k\pi) - \frac{1}{ik^3\pi} \sin(k\pi)$$

$k \in \mathbb{Z} \implies \frac{2}{k^2} (-1)^k$ ✓

$$A_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} x^2 e^0 \, dx = \frac{1}{2\pi} \left[\frac{1}{3} x^3 \right]_{-\pi}^{\pi} = \frac{1}{6}\pi^2 + \frac{1}{6}\pi^2 = \frac{1}{3}\pi^2$$

$$\Longrightarrow S^n(x) = \frac{1}{3}\pi^2 + \sum_{k=-n}^{-1} \frac{2}{k^2} (-1)^k e^{+ikx} + \sum_{k=1}^{n} \frac{2}{k^2} (-1)^k e^{+ikx}$$

(✓)

4,5/5

2)

a) $\sqrt{2\pi}\, F(\omega) = \cancel{\frac{1}{\sqrt{2\pi}}} \int_{-\infty}^{\infty} dt\, e^{-t^2/a^2} \underbrace{e^{-i\omega t}}_{\cos\omega t + i\sin\omega t} \;\cancel{= \frac{1}{\sqrt{2\pi}} \int dt\, e^{-t^2/a^2} \cos\omega t\, dt + i \int dt\, e^{-t^2/a^2} \sin\omega t\, dt}$

$\cancel{= -e^{-t^2/a^2} \sin\omega t \Big|_{-\infty}^{\infty} - \int dt\, \frac{2t}{a^2} e^{-t^2/a^2} \sin\omega t}$

$\cancel{v' := \frac{2t}{a^2} \sin\omega t}$

$$\int_{-\infty}^{\infty} dt \exp\left(-\frac{t^2}{a^2} - i\omega t + \frac{1}{4}\omega^2 a^2 - \frac{\omega^2 a^2}{4}\right)$$

$$= \int_{-\infty}^{\infty} dt \exp\left(-\left(\frac{t}{a} + \frac{i\omega a}{2}\right)^2\right) \cdot e^{-\frac{\omega^2 a^2}{4}}$$

$$\beta := \frac{t}{a} + \frac{i\omega a}{2} \Rightarrow \frac{d\beta}{dt} = \frac{1}{a} \Leftrightarrow dt = a\, d\beta$$

$$= a \int_{-\infty}^{\infty} d\beta \exp(-\beta^2) \cdot e^{-\frac{\omega^2 a^2}{4}} = e^{-\frac{\omega^2 a^2}{4}} \cdot \sqrt{\pi} \cdot a$$

$$\Rightarrow F(\omega) = \frac{1}{\sqrt{2}} e^{-\frac{\omega^2 a^2}{4}} \cdot a$$

b) Max.: $f'(t) = 0 = -\frac{2t}{a^2} \exp\left(-\frac{t^2}{a^2}\right) \Rightarrow t = 0$

$f(t=0) = 1$ Maximum, da

$f''(t) = -\frac{2}{a^2} \exp\left(-\frac{t^2}{a^2}\right) + \frac{4t^2}{a^4} \exp\left(-\frac{t^2}{a^2}\right) \Rightarrow f''(t=0) = -\frac{2}{a^2} \underline{< 0}$

$F(\omega)$ äquivalent. $F(0) = \frac{1}{\sqrt{2}} \cdot a$ f

Wann ist $f(t) = \frac{1}{e} \Leftrightarrow e^{-1} = \exp\left(-\frac{t^2}{a^2}\right) \Leftrightarrow t = \pm a \Rightarrow \Delta t = 2a$

$F(\omega) \overset{!}{=} \frac{1}{\sqrt{2}\, e}$ f $\Leftrightarrow \frac{1}{\sqrt{2}} e^{-1} = \frac{1}{\sqrt{2}} e^{-\frac{\omega^2 a^2}{4}} \Leftrightarrow \omega = \pm \frac{2}{a} \Rightarrow \underline{\Delta\omega = \frac{4}{a}}$

c) $\Delta t \cdot \Delta \omega = \frac{4}{a} \cdot 2a = 8$ ✓

Δt ist so etwas wie die Standardabweichung der Gauß-Verteilung
und $\Delta\omega$ ist so etwas wie die " " Fouriertransformierten
Gauß-Verteilung.

5/5

$\boxed{5}$

a) $f_A(t) = f(t) = \begin{cases} t\cdot\frac{20}{\pi}+10 & -\pi \le t \le 0 \\ -t\cdot\frac{20}{\pi}+10 & 0 \le t \le \pi \end{cases}$ ✓ Dreieckspannung

$$f(t) = \sum_{n=1}^{\infty} \left(a_n \cos(nt) + b_n \sin(nt)\right) + a_0$$

$$a_0 = \frac{1}{L}\int_{-\pi}^{\pi} f(t)\,dt = \frac{1}{2\pi}\int_{-\pi}^{0}\left(t\cdot\frac{20}{\pi}+10\right)dt + \frac{1}{2\pi}\int_{0}^{\pi}\left(-t\cdot\frac{20}{\pi}+10\right)dt \checkmark$$

$$= \frac{1}{2\pi}\cdot\left(\frac{10t^2}{\pi}+10t\right)\Big|_{-\pi}^{0} + \frac{1}{2\pi}\left(-\frac{10t^2}{\pi}+10t\right)\Big|_{0}^{\pi}$$

$$= \frac{1}{2\pi}\cdot\left(-(10\pi - 10\pi) + (-10\pi + 10\pi)\right) = 0 \checkmark$$

$$a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)\,dt = \frac{1}{\pi}\int_{-\pi}^{0}\left(t\cdot\frac{20}{\pi}+10\right)\cdot\cos(nt)\,dt + \frac{1}{\pi}\int_{0}^{\pi}\left(-t\cdot\frac{20}{\pi}+10\right)\cos(nt)\,dt$$

$$= \frac{10}{\pi}\left(\int_{-\pi}^{0}\underbrace{\frac{2t}{\pi}}_{u}\underbrace{\cos(nt)}_{v'}dt + \int_{0}^{\pi}\frac{-2t}{\pi}\cos(nt)\,dt + \cancel{\int_{-\pi}^{\pi}\cos(nt)\,dt}\right)$$

$$= \frac{10}{\pi}\left(\frac{2t}{n\pi}\sin nt\Big|_{-\pi}^{0} - \frac{2t}{n\pi}\sin nt\Big|_{0}^{\pi} \pm \int_{-\pi}^{0}\frac{2}{\pi}\cdot\frac{1}{n}\sin(nt)\,dt + \int_{0}^{\pi}\frac{2}{\pi n}\sin(nt)\,dt\right)$$

$$= \frac{10}{\pi}\left[\left(-\frac{2}{n}\underbrace{\sin n\pi}_{=0\ \forall n} - \frac{2}{n}\underbrace{\sin n\pi}_{=0}\right) + \frac{2}{\pi n^2}\cdot 2\cdot(+\cos(nt))\Big|_{-\pi}^{0}\right]$$

$$= \frac{10}{\pi}\left(0 + \frac{+4}{\pi n^2}\left(+1 \mp \cos(\pi\cdot n)\right)\right) = \begin{cases} 0 & n \text{ gerade} \\ +\frac{80}{\pi^2 n^2} & n \text{ ungerade}\end{cases} \quad (\checkmark)$$

$$b_n = \frac{1}{\pi}\left(\int_{-\pi}^{0}\left(t\cdot\frac{20}{\pi}+10\right)\sin(nt)\,dt + \int_{0}^{\pi}\left(-t\cdot\frac{20}{\pi}+10\right)\sin(nt)\,dt\right)$$

$\boxed{b_n = 0\text{, da } f(t) \text{ gerade}}$ ✓

$$= \frac{10}{\pi}\left(\int_{-\pi}^{0}\underbrace{t}_{u}\cdot\frac{2}{\pi}\underbrace{\sin(nt)}_{v'}dt - \int_{0}^{\pi}t\cdot\frac{2}{\pi}\sin(nt)\,dt\right)$$

da cos gerade

$$= \frac{10}{\pi}\left(-\frac{2t}{n\pi}\cos(nt)\Big|_{-\pi}^{0} + \frac{2t}{n\pi}\cos(nt)\Big|_{0}^{\pi} + \cancel{\int_{-\pi}^{0}\frac{2}{n\pi}\cos(nt)\,dt - \int_{0}^{\pi}\frac{2}{n\pi}\cos(nt)\,dt}\right)$$

) $f(t) = \sum_{n=1}^{\infty} a_n \cos(nt)$

$$U_i = \sqrt{\frac{1}{T}\int_0^{2\pi} a_i^2 \cos^2(i t)} = \sqrt{\frac{1}{2\pi} a_i^2 \int_0^{2\pi} \cos^2 i t} = \sqrt{\frac{a_i^2}{2\pi i}\left(\frac{2it + \sin(2it)}{4i}\right)\Big|_0^{2\pi}}$$

$$= \sqrt{\frac{a_i^2}{2\pi \cancel{i}} \cdot \frac{2\pi}{2}} = a_i \cdot \frac{1}{\sqrt{2}}$$ ✓

c) $$k^2 = \frac{\sum_{i=2}^{\infty} a_i^2}{\sum_{i=1}^{\infty} a_i^2}$$ $a_i = \frac{+80}{\pi^2 n^2}$ für ungerades n

$$= \frac{\sum_{n=2}^{\infty} \frac{1}{n^4}}{\sum_{n=1}^{\infty} \frac{1}{n^4}} = \frac{\sum_{n=1}^{\infty} \frac{1}{n^4}}{\sum_{n=1}^{\infty} \frac{1}{n^4}} - \frac{1}{\sum_{n=1}^{\infty} \frac{1}{n^4}} = 1 - \frac{1}{\sum_{n=0}^{\infty} \frac{1}{(2n+1)^4}}$$

$$= 1 - \frac{1}{\sum_0^{\infty} \frac{1}{(2n+1)^4} + \frac{1}{(2n)^4}}$$?

NR: $$\sum_1^{\infty} \frac{1}{n^4} = \sum_1^{\infty} \frac{1}{(2n+1)^4} + \frac{1}{(2n)^4} + 1 = \sum_0^{\infty}\left(\frac{1}{(2n+1)^4} - 1\right) + \sum_1^{\infty} \frac{1}{(2n)^4} + 1)$$

$$= \sum_0^{\infty} \frac{1}{(2n+1)^4} + \sum_1^{\infty} \frac{1}{(2n)^4} = \frac{\pi^4}{96} + \frac{1}{2^4}\sum_1^{\infty} \frac{1}{n^4} \qquad \Big| - \frac{1}{2^4}\sum_1^{\infty}\frac{1}{n^4}$$

$$\Leftrightarrow \frac{15}{16}\sum_{1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{96} \Leftrightarrow \sum_1^{\infty} \frac{1}{n^4} = \frac{16}{15}\frac{\pi^4}{96} = \frac{\pi^4}{90}$$ (✓)

$$\Rightarrow k^2 = 1 - \frac{90}{\pi^4} \Rightarrow k^2 \approx 0{,}076$$

$k \approx 0{,}276$ (✓)

d) Klirrdämpfungsmaß (Wiki):

$$L_k = 20 \cdot \log \frac{1}{k}\ \mathrm{dB} = 11{,}2\ \mathrm{dB}$$ ✓ (✓)

4/5

a) A4 ② $u(x=a, y, t) = 0$, ③ $u(x, y=0, t) = 0 = u(x=0, y, t)$ ①

2D-Wellengl.:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2}\right)\vec{u}(x_1, x_2, t) = 0$$

Keine Dilatation, Scherung oder Rotation, weil ~~Membran~~ 3 Seiten der Membran eingespannt sind.

Ansatz: $u(x_1, x_2, t) = g(t) u_1(x_1) u_2(x_2)$

$$\Rightarrow \frac{1}{c^2}(\partial_t^2 g(t)) u_1(x_1) u_2(x_2) = (\partial_{x_1}^2 u_1(x_1)) g(t) u_2(x) + (\partial_{x_2}^2 u_2(x_2)) g(t) u_1(x)$$

$$\Leftrightarrow \frac{1}{c^2} \frac{\partial_t^2 g(t)}{g(t)} = \frac{\partial_{x_1}^2 u_1(x_1)}{u_1(x_1)} + \frac{\partial_{x_2}^2 u_2(x_2)}{u_2(x_2)} \quad (*)$$

Ansatz:

$g(t) = A\sin(\omega t) + B\cos(\omega t)$

$u_1(x_1) = A_1 \sin(k_1 x_1) + B_1 \cos(k_1 x_1)$

$u_2(x_2) = A_2 \sin(k_2 x_2) + B_2 \cos(k_2 x_2)$

RB:

① $u(x=0, y, t) = 0 \Leftrightarrow u_1(0) = 0 = B_1$

② $u(x=a, y, t) = 0 \Leftrightarrow u_1(a) = 0 = A_1 \sin(k_1 a)$

$\Leftrightarrow k_1 = n_1 \cdot \pi \cdot \frac{1}{a}$, $n_1 \in \mathbb{N}$

③ $u(x, y=0, t) = 0 \Leftrightarrow u_2(0) = 0 = B_2$

④ $\partial_y u(x, y=b, t) = 0$

$= \frac{\partial}{\partial x_2} u = g(t) \cdot u_1(x_1) \cdot \frac{\partial}{\partial x_2} u_2(x_2)$ oBdA

$= g(t)\, u_1(x_1) \cdot A_2 k_2 \cos(k_2 x_2)$

$\Leftrightarrow A_2 k_2 \cos(k_2 b) = 0$

$\Leftrightarrow k_2 = \left(n_2 \cdot \pi + \frac{\pi}{2}\right) \cdot \frac{1}{b}$, $n_2 \in \mathbb{N}$

$\left[\sin(n_2 \cdot \pi + \frac{\pi}{2}) = \cos(n_2 \pi) \right]$

allg. Lsg.:

$$u(x_1,x_2,t) = g(t)\cdot u_1(x_1)\cdot u_2(x_2)$$

$$= \sum_{n_1,n_2=0}^{\infty} \sin\left(\frac{n_1\pi}{a}x_1\right)\cdot \overset{\sin}{\cos}\left(\frac{n_2\pi}{b}x_2\right)\cdot\left[A_{n_1,n_2}\sin(\omega_{n_1,n_2}t) + B_{n_1,n_2}\cos(\omega_{n_1,n_2}t)\right]$$

$B_1 = B_2 = 0$

b) ① $\cancel{u(x,y,t=0)=0}$ $\cancel{\Rightarrow}$ oBdA

$$\frac{\partial_t^2 g(t)}{c^2\cdot g(t)} = \frac{-\omega^2}{c^2} = -k^2$$ ↑ Dispersionsrelation

$$\frac{\partial_{x_1}^2 u_1(x_1)}{u_1(x_1)} = -k_1^2 \quad ; \quad \frac{\partial_{x_2}^2 u_2(x_2)}{u_2(x_2)} = -k_2^2$$

$$\overset{(*)}{\Longrightarrow} k^2 = k_1^2 + k_2^2 = \frac{\omega^2}{c^2} \Rightarrow \omega = c\sqrt{k_1^2+k_2^2} = \omega_{n_1,n_2}$$

- $u(x,y,t=0) = 0 = \sum_{n_1,n_2=0}^{\infty} \sin\left(\frac{n_1\pi}{a}x_1\right)\cdot\overset{\sin}{\cos}\left(\frac{n_2\pi}{b}y\right) B_{n_1,n_2}$

  oBdA $\Rightarrow B_{n_1,n_2} = 0 \quad \forall n_1,n_2$

- $\dot u(x,y,t=0) = v_0\sin\left(\frac{\pi}{a}x\right)\frac{y}{b} = \sum_{n_1,n_2} \sin\left(\frac{n_1\pi}{a}x\right)\overset{\sin}{\cos}\left(\frac{n_2\pi}{b}y\right) A_{n_1,n_2}\,\omega_{n_1,n_2}\underbrace{\cos(\omega_{n_1,n_2}t)}_{=1}$

  $\left|\cdot\sin\left(\frac{n_2\pi}{a}x\right)\right. \quad \left|\cdot\cos\left(\frac{n_1\pi}{b}y\right)\right. \quad \left|\int_0^b dy\right.$

$$\Leftrightarrow v_0\sin\left(\frac{\pi}{a}x\right)\cdot\sin\left(\frac{n_2\pi}{a}x\right)\cdot\int_0^b \frac{y}{b}\cdot\cos\left(\frac{n_1\pi}{b}y\right)dy = \sum_{n_1}\sin\left(\frac{n_1\pi}{a}x\right)\sin\left(\frac{n_2\pi}{a}x\right) A_{n_1n_2}\,\omega_{n_1n_2}\cdot\frac{b}{2} \quad \left|\int_0^a dx\right.$$

$$v_0\underbrace{\int_0^a \sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{n_2\pi}{a}x\right)dx}_{=\frac{a}{2}\cdot\delta_{1n_2} = \begin{cases}\frac{a}{2} & n_2=1\\ 0 & \text{sonst}\end{cases}}\cdot\int_0^b \frac{y}{b}\cos\left(\frac{n_1\pi}{b}y\right)dy = A_{n_1n_2}\,\omega_{n_1n_2}\cdot\frac{a\cdot b}{4}$$

NR: $\int_0^b \frac{y}{b}\cos\left(\frac{n_1\pi}{b}y\right)dy = \underbrace{\left[\frac{y}{b}\cdot\frac{b}{n_1\pi}\sin\frac{n_1\pi}{b}y\right]_0^b}_{=0,\ \text{da}\ \sin(\pi n_1)=0} - \int_0^b \frac{1}{b}\cdot\frac{b}{n_1\pi}\sin\left(\frac{n_1\pi}{b}y\right)dy$

u' v'

$= \frac{b}{n_1^2\pi^2}\cdot\cos\left(\frac{n_1\pi}{b}y\right)\Big|_0^b = \frac{b}{n_1^2\pi^2}\cdot(\cos n_1\pi - 1) = \begin{cases} \frac{-2b}{n_1^2\pi^2} & n_1 \text{ ungerade} \\ 0 & n_1 \text{ gerade}\end{cases}$

$\Rightarrow A_{n_1 n_2} = \begin{cases} 0 & n_2 \neq 1 \\ a_{n_1} & n_2 = 1\end{cases}$

mit $a_{n_1} = \begin{cases} 0 & n_1 \text{ gerade} \\ \frac{4}{\cancel{ab}}\frac{1}{\omega_{n_1 1}}v_0\cdot\frac{\cancel{a}}{\cancel{2}}\cdot\frac{-\cancel{2}b}{n_1^2\pi^2} = -\frac{4v_0}{\omega_{n_1 1}n_1^2\pi^2}\end{cases}$

allg. Lsg. $\omega_{n_1 1} =: \omega_{n_1}$

$n_2 = 1$!

$u(x,y,t) = \sum_{n_1=0}^{\infty}\sin\left(\frac{(2n_1+1)\pi}{a}x\right)\cos\left(\frac{\pi}{b}y\right)\cdot\left(\frac{-4v_0}{\omega_{n_1}(n_1\cdot 2+1)^2\pi^2}\right)\sin(\omega_{n_1}t)$ (✓)

c) $\omega_{n_x n_y} = c\pi\sqrt{\frac{n_x^2}{a^2}+\frac{n_y^2}{b^2}} = c\pi\cdot\frac{1}{2a}\cdot\sqrt{4n_x^2+n_y^2}$ ✓

Vorlesung

$b = 2a$

Es muss für Entartung gelten:

$\omega_{n_{x1}n_{y1}} = \omega_{n_{x2}n_{y2}}$ ✓ mit $n_{x1}, n_{y1} \neq n_{x2}, n_{y2}$

$\Leftrightarrow \sqrt{4n_{x1}^2+n_{y1}^2} = \sqrt{4n_{x2}^2+n_{y1}^2}$

$\Leftrightarrow 4\cdot(n_{x1}^2 - n_{x2}^2) = n_{y1}^2 - n_{y2}^2 \quad \Leftrightarrow 4 = \left(\frac{n_{x1}^2 - x_{x2}^2}{n_{y1}^2 - n_{y2}^2}\right)^{-1}$

$\omega_{12} = \omega_{24} = \omega_{48} = \omega_{5\,10} = \omega_{36} = \dots$ ∞ oft entartet mit $n_y = 2\cdot n_x$

nicht entartet: ω_{11} ✓ → ~~ω_{05} ist gleich ⇒ 2-fach entartet~~ 1-fach: $\omega_{15} = \omega_{5\,11}$

wie geht das analytisch? Raten funktioniert nicht so gut. Doch, für die kleinen Zahlen

gar nicht, vermutlich

~~$\omega_{17} = \omega_{39}$~~

$\omega_{21} = \omega_{47}$

(ich nehm jetzt Python und checke bis 30 für n_{x2} und n_{y2} ...)

2-fach: $\omega_{17} = \omega_{39} = \omega_{11\,23}$ f

~~ω_{17}~~

3-fach: finde ich nicht.

4/5