

$$U = \frac{1}{2} k (x_1 - x_2)^2 \quad \checkmark$$

$$F = m\ddot{x} = -\nabla U$$

$$m_1 \ddot{x}_1 = -k(x_1 - x_2) \quad \text{I} \quad \checkmark$$

$$m_2 \ddot{x}_2 = k(x_1 - x_2) \quad \text{II} \quad \checkmark$$

schön

b)

$$\text{I} + \text{II}$$

$$m_1 \ddot{x}_1 + m_2 \ddot{x}_2 = 0 \quad \checkmark$$

$$\ddot{x}_1 = 0 \quad \checkmark$$

$$x_1 = m_1 x_1 + m_2 x_2 \quad \checkmark$$

$$x_2 = a_{21} x_1 + a_{22} x_2$$

$$- \text{I} \cdot m_2 + \text{II} \cdot m_1$$

$$m_1 m_2 (\ddot{x}_2 - \ddot{x}_1) = k(x_1 - x_2) \cdot \frac{(m_2 + m_1)}{m_1 m_2}$$

$$\ddot{x}_2 = k \cdot \frac{(m_2 + m_1)}{m_1 m_2} x_2 \quad \checkmark$$

$$x_2 = x_2 - x_1 \quad \checkmark$$

schön

$$c) \frac{\partial^2 \vec{x}}{\partial t^2} = M \vec{\ddot{x}} \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\ddot{x}_1 = -\frac{k}{m_1} (x_1 - x_2)$$

$$\ddot{x}_2 = \frac{k}{m_2} (x_1 - x_2)$$

$$\Rightarrow \frac{\partial^2 \vec{x}}{\partial t^2} = \underbrace{\begin{pmatrix} -\frac{k}{m_1} & \frac{k}{m_1} \\ \frac{k}{m_2} & -\frac{k}{m_2} \end{pmatrix}}_{M} \vec{x} \quad \checkmark$$

$$\det(M - \lambda E) \stackrel{!}{=} 0 \quad \checkmark$$

$$= \left| \begin{pmatrix} -\frac{k}{m_1} - \lambda & \frac{k}{m_1} \\ \frac{k}{m_2} & -\frac{k}{m_2} - \lambda \end{pmatrix} \right| = \frac{k^2}{m_1 m_2} + \lambda \frac{k}{m_1} + \lambda \frac{k}{m_2} + \lambda^2 - \frac{k^2}{m_1 m_2}$$

$$= \lambda \left(\lambda + \left(\frac{k}{m_1} + \frac{k}{m_2} \right) \right) \stackrel{!}{=} 0$$

$$\Rightarrow \lambda = 0$$

$$\Rightarrow \lambda = -k \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \quad \checkmark$$