Integration by Parts

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ILATE

$\int u\, dv = u\, v - \int v\, du$

But what is u & what is dv?

A rule of thumb for choosing which of two functions is to be u and which is to be dv is to

Choose <u>u</u> as whichever function comes <u>first</u> in this list:

I: Inverse trigonometric functions: arctan x, arcsin x, etc.

L: Logarithmic functions: ln x, $\log_2(x)$, etc.

A: Algebraic functions: x, x², $\sqrt{x}$ etc.

T: Trigonometric functions: sin x, tan x, etc.

E: Exponential functions: e^x, 13^x, etc.

Then make dv the other function. You can remember the list by the mnemonic ILATE. The reason for this choice is that functions farther down the list have easier or more helpful anti-derivatives than the functions above them.

$\int e^x \cos x\, dx$ by Parts

Although $\int e^x \cos x\, dx$ requires repeated integration by parts, a <u>different</u> trick is needed. The trick for $\int e^x \cos x\, dx$ is that after double integration by parts we get:

$\int e^x \cos x\, dx = e^x \cos x + e^x \sin x - \int e^x \cos x\, dx$.

Note that we still have an integral on the right side, but the <u>same</u> integral!

This gives: $\int e^x \cos x\, dx = \frac{1}{2}(e^x \cos x + e^x \sin x)$

$\int \ln(x)dx, \int \arcsin(x)dx, \int \arctan(x)dx$, etc. are done with parts

Tabular Repeated Integration by Parts

Integration by parts uses the formula: $\int u\, dv = uv - \int v\, du$

It is the way to integrate, for example: $\int$arctan x dx, $\int$x $\sin^{-1}$x dx, $\int$ln x dx, $\int$x tan x dx, $\int$x e^x dx.

After integration by parts some expressions, for example, $\int x^2 e^{-x}\, dx, \int x^2 \cos x\, dx$ and $\int e^{-x} \cos x\, dx$ require a second application of integration by parts. This repeated integration by parts is on the IB syllabus.

Repeated integration by parts requires a lot of tedious bookkeeping. But instead of explicitly listing out each of the steps, you can create a table of two columns, for example for $\int x^2 e^{-x}\, dx$, listing x² and each of its successive derivatives in the first column and e^{-x} and each of its successive integrals in the second column. Then in the first column multiply the first term by +1, the second term by -1, the third term by +1, etc.

You then multiply the first entry in the first column by the second entry in the second column and then multiply the second entry in the first column by the third entry in the second column, and continue this process. The last row will have a derivative of 0 in the first column, which does not need to be multiplied by anything. Then add each of these terms and the constant C.

Example $\int x^2 e^{-x}\, dx$

+ x²	e^{-x}
− 2x	$-e^{-x}$
+ 2	e^{-x}
0	$-e^{-x}$

Adding the diagonal products: $\int x^2 e^{-x}\, dx = -x^2 e^{-x} - 2x\, e^{-x} - 2e^{-x} + C$