Formulas Not in the IB Formula Booklets

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Complex Numbers (AA HL & AI HL only)

$z = a + bi = r\,cis\,\theta = r\left(\cos\theta + i\sin\theta\right)$

$r =$ the modulus $= |\,z\,| = \text{mod}\,z = \sqrt{a^2 + b^2}$

$\theta =$ the argument $= \arg(z) = \arctan\left(\dfrac{b}{a}\right), \pm\pi$ if in 2nd or 3rd Quadrant

$(a+bi)(a-bi) = a^2 + b^2$

$\left(r\,cis\,\theta\right)^n = r^n \text{cis}\,n\theta$

$z\,z^* = \left|z\right|^2$

$\text{cis}\theta \times \text{cis}\phi = cis(\theta+\phi), \dfrac{\text{cis}\theta}{\text{cis}\phi} = cis(\theta-\phi),$

$\cos\left(-\theta\right) = \cos\theta, \sin\left(-\theta\right) = -\sin\theta$

$z^n - 1 = \left(z-1\right)\left(z^{n\text{-}1} + z^{n\text{-}2} + \ldots + z^2 + z + 1\right)$

Exponents (all syllabi)

$b^m \times b^n = b^{m+n}$

$\left(b^m\right)^n = b^{m\times n}$

$\dfrac{a^m}{a^n} = a^{(m-n)}$

$(a\ b)^m = a^m \times b^{\text{m}}$

$a^{-n} = \dfrac{1}{a^n}$

$\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

$a^0 = 1$

$\sqrt[n]{a} = a^{\frac{1}{n}}$

Geometry

Perpendicular lines have gradients and $m_2 = \dfrac{-1}{m_1}$ or $m_1 \times m_2 = -1$

Circle with radius r & centre at (h, k): $(x - h)^2 + (y - k)^2 = r^2$

Trigonometry

$y = a\sin(b(x-c)) + d : |a| = \text{amplitude},\ b = \frac{2\pi}{\text{period}}\ or\ \frac{360}{\text{period}},\ c = \text{horizontal shift (}c\text{ is not in AI SL)},\ d = \text{midline}$

sin θ = *cos* $(90° - \theta)$, *cos* θ = *sin* $(90° - \theta)$

The values of sine, cosine, and tangent for the special angles (AA only)

Trig identities on unit circle; examples: $\cos x = \cos(-x), \sin(x+\pi) = -\sin(x)$ (AA only)

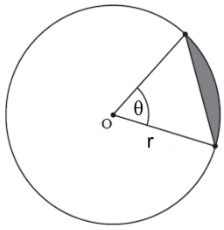
$\sin(180° - \theta) = \sin\theta,\ \cos\ x = \cos(360° - x)$ (AA only)

$\cos(180° - \theta) = -\cos\theta, \tan(180° - \theta) = -\tan\theta$ (AA HL only)

If sin(A)=sin(B), then either $A = B + 2\pi k$ or $A = \pi - B + 2\pi k$ (AA HL only)

If cos(A)=cos(B), then either $A = B + 2\pi k$ or $A = -B + 2\pi k$ (AA HL only)

To convert radians to degrees multiply by $\frac{180°}{\pi}$. To convert degrees to radians multiply by $\frac{\pi}{180°}$.

The area of the shaded region  is $\frac{1}{2}r^2(\theta - \sin\theta)$ with θ in radians (Not AI SL)

For a line through the origin, $y = mx$, $m = \tan\theta$,

where θ is the angle between the line & the positive x-axis. (AA only)

Inverse Trig function	Domain	Range (AA HL only)
y = arcsin x	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
y = arccos x	$-1 \le x \le 1$	$0 \le y \le \pi$
y = arctan x	$\mathbb{R}$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Statistics

x is an outlier if $x < Q_1 - 1.5 \times IQR$ or $x > Q_3 + 1.5 \times IQR$

χ^2 test for independence: $f_{exp} = \frac{(\text{row sum}) \times (\text{column sum})}{\text{table sum}}$ (AI only)

χ^2 test for independence: df = (# rows - 1) × (# columns - 1) (AI only)

χ^2 GOF-Test: df = # categories - 1 (AI SL)

χ^2 GOF-Test: df = # categories - 1 - # of parameters estimated (AI HL only)

Calculus

$v = \frac{ds}{dt},\ a = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times v$ (HL only)

$\int (ax+b)^n dx = \frac{1}{a} \times \frac{1}{n+1}(ax+b)^{n+1} + C,\quad \int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C,\quad \int \sin(ax+b)dx = \frac{-1}{a}\cos(ax+b) + C$

$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b| + C,\quad \int e^{(ax+b)}dx = \frac{1}{a}e^{(ax+b)} + C,$ (Not AI SL)