Κατανομές δειγματοληψίας
(Sampling distributions)

Κατανομή δειγματοληψίας (sampling distribution)

- Το **στατιστικό στοιχείο** είναι μια **τυχαία μεταβλητή** που **περιγράφει** συνολικά το **δείγμα**

- Η κατανομή που ακολουθεί το στατιστικό στοιχείο ονομάζεται **κατανομή δειγματοληψίας** και αποτελεί μία **γνωστή κατανομή**

- Μελετώντας το στατιστικό στοιχείο, μέσω της κατανομής δειγματοληψίας, μπορούμε να κάνουμε:
  - ✓ **εκτίμηση** της τιμής της παραμέτρου
  - ✓ **έλεγχο** κάποιας **υπόθεσης** πάνω στην παράμετρο

Σημαντικά στατιστικά στοιχεία

- Ο **δειγματικός μέσος** (***sample mean***) $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

- Η **δειγματική διακύμανση** (***sample variance***)

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$$

Ισχύει η σχέση: $S^2 = \frac{1}{n-1}\sum_{i=1}^{n} X_i^2 - \frac{n}{n-1}\bar{X}^2$

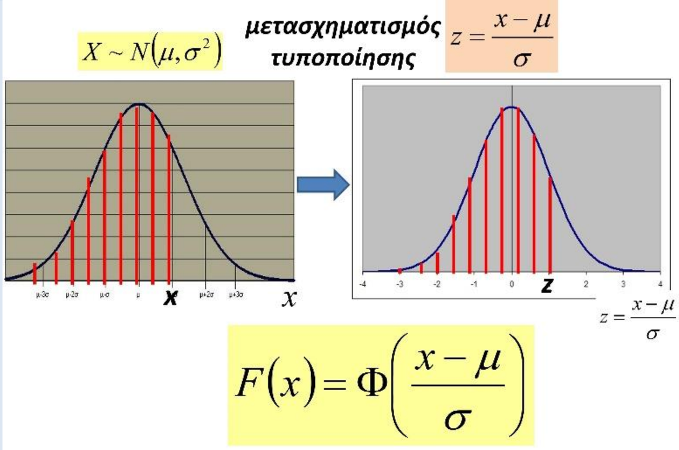
S (>0) είναι η **δειγματική τυπική απόκλιση**

- Άλλα στατιστικά στοιχεία είναι:
  - το **μέγιστο (*max*)**, το **ελάχιστο(*min*)**,
  - το **εύρος τιμών (*|max-min|*)**,
  - κλπ.

Σπουδαιότερες κατανομές δειγματοληψίας

- Η **Κανονική** κατανομή - $\boxed{N(\mu, \sigma^2)}$
- Η κατανομή ***Χι-τετράγωνο*** - $\boxed{X^2_n}$
- Η κατανομή ***t-Student*** - $\boxed{t_n}$
- Η ***F-κατανομή*** - $\boxed{F_{n,m}}$

(Α). Η Κανονική κατανομή N(μ, σ²)



- Έστω τ.μ. ***X ~ N(μ, σ²)***, τότε:

$$X \sim N\left(\mu,\sigma^2\right) \Rightarrow Z = \frac{X-\mu}{\sigma} \sim N(0,1)$$

(***τυπική κανονική***)

Πρόταση 1α

- Αν X_1, X_2, …, X_n ***n*** ανεξάρτητες και ισόνομες κανονικές μεταβλητές, δηλ. $X_i \sim N(\mu, \sigma^2)$, τότε για το **άθροισμα** ισχύει ότι:

$$S_n = \sum_{i=1}^{n} X_i \sim N\left(n\mu, n\sigma^2\right)$$

$$Z = \frac{S_n - n\mu}{\sqrt{n}\sigma} \sim N(0,1)$$

- ***<u>Παρατήρηση</u>:*** *Από το* ***Κ.Ο.Θ.*** *το παραπάνω ισχύει για κάθε κατανομή όταν το* ***n*** *είναι μεγάλο.*

Πρόταση 1β

- Αν $X_1, X_2, \ldots, X_n$ n ανεξάρτητες και ισόνομες κανονικές μεταβλητές, δηλ. $X_i \sim N(\mu, \sigma^2)$, τότε για τον **δειγματικό μέσο** ισχύει ότι:

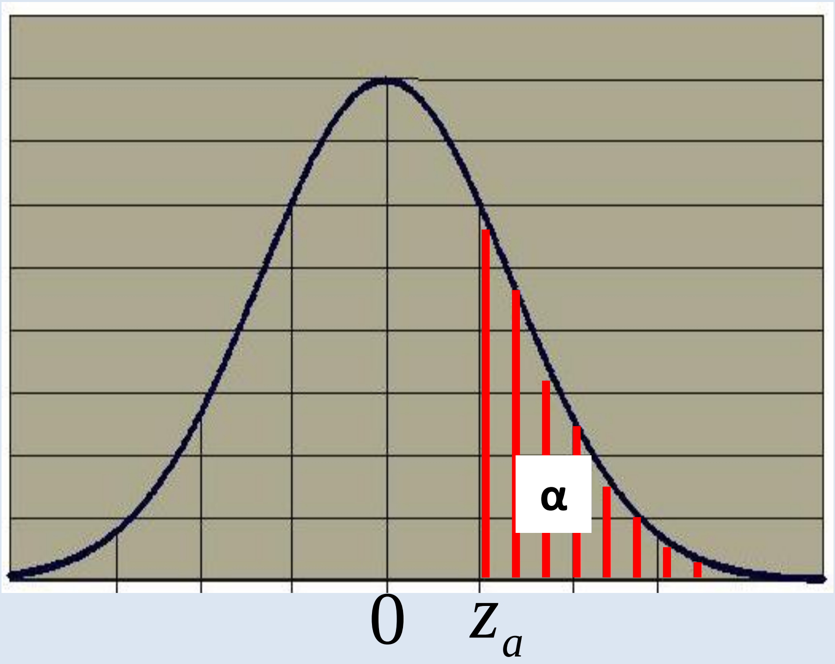
$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \Rightarrow \boxed{Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)}$$

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

- Το **στατιστικό στοιχείο** του δειγματικού μέσου (ή μέσου όρου) σχετίζεται με τις **παραμέτρους** του μέσου ***μ*** και της διασποράς ***σ²***

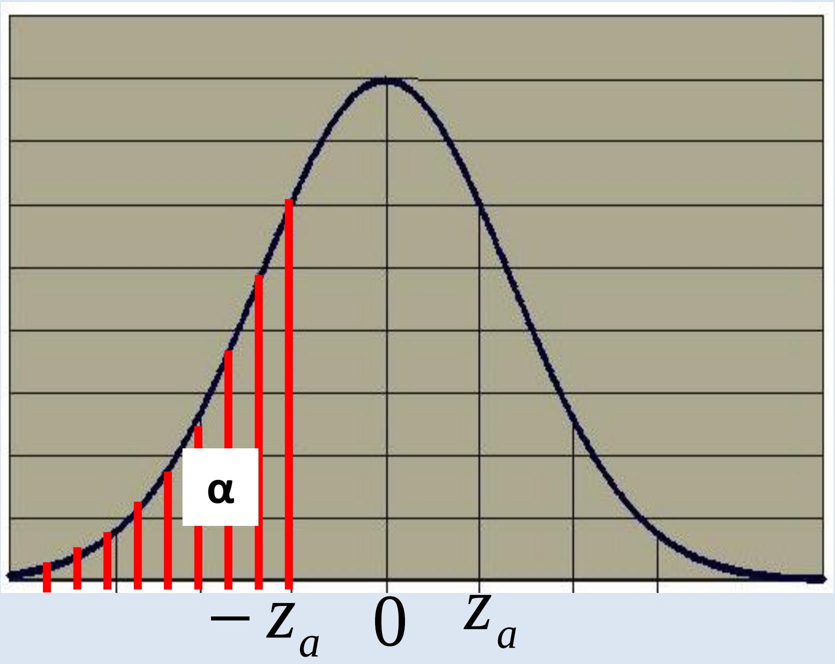
- Η κατανομή αυτού του στοιχείου είναι η **τυπική κανονική** (*με μέσο 0 και διακύμανση 1*)

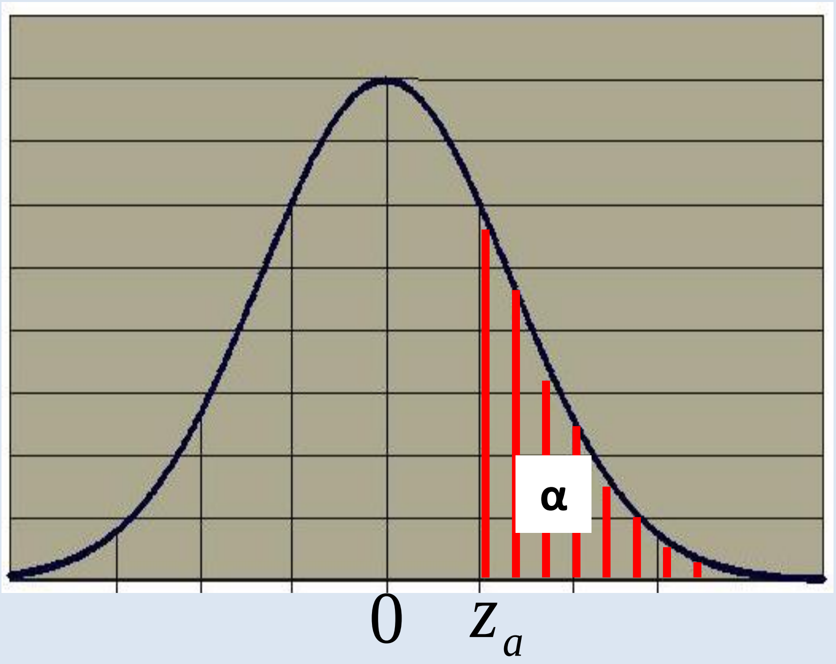
Πρόταση 2: Το σημείο z_a



- Ορίζουμε ως z_a εκείνο το σημείο εκείνο για το οποίο ισχύει :

$$z_a : P(X > z_a) = a$$



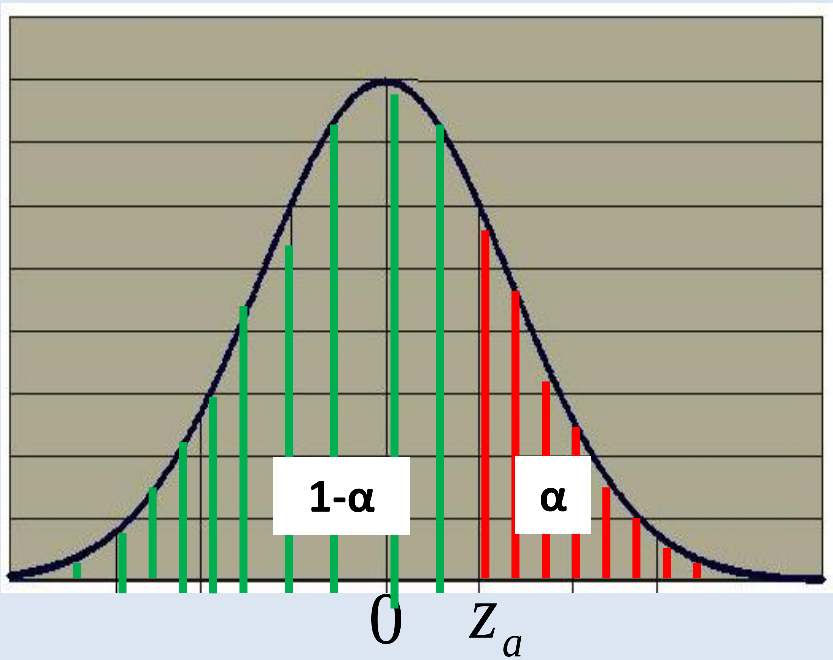


- Ορίζουμε ως $\boldsymbol{z_a}$ εκείνο το σημείο εκείνο για το οποίο ισχύει (**λόγω συμμετρίας**) :

$$z_a : P(X > z_a) = P(X \leq -z_a) = a$$

Διαδικασία εύρεσης του σημείου $z_α$

- Εύρεση από τον **πίνακα** του σημείου που αντιστοιχεί στην **πιθανότητα** ***1-α*** (στοιχείο πίνακα)



$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt:$$

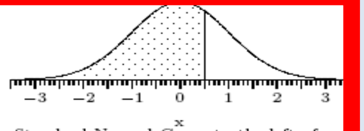


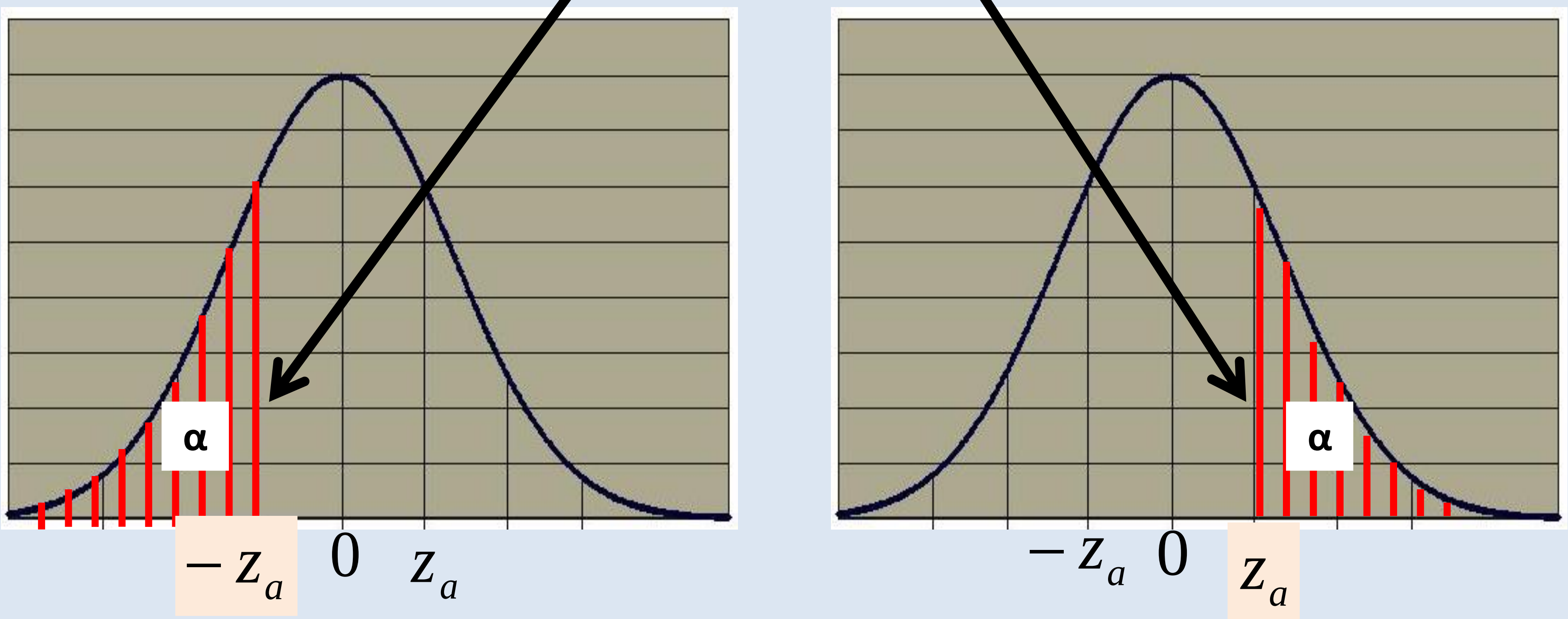
Table 5.1 Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

- Έτσι

$$z_a: \qquad \Phi(z_a) = 1 - a$$

$$a = \Phi(-z_a) = 1 - \Phi(z_a)$$



Παράδειγμα: Να βρεθεί το σημείο $z_{0.02}$

$$z_a: \quad \Phi(z_a) = 1 - a = 0.98$$

$\frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt$:

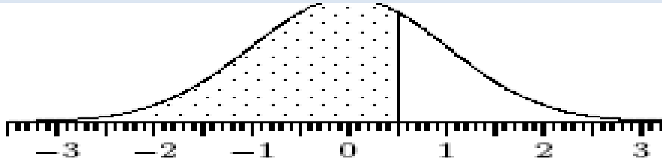


Table 5.1 Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
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1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
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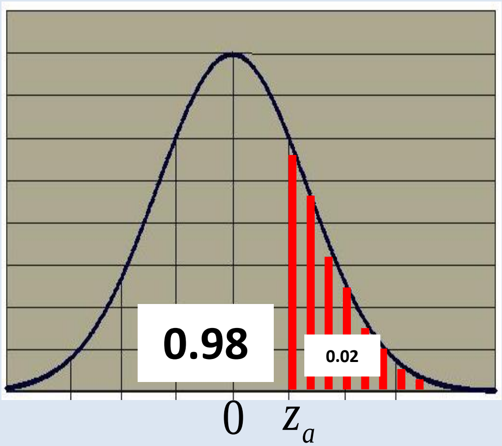
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$z_a = 2.05$



Η κατανομή **Χι-τετράγωνο** X^2_n με n βαθμούς ελευθερίας

- Η κατανομή **Γάμμα** *G(α, λ)* έχει σ.π.π.

$$f(x) = \frac{\lambda^a}{\Gamma(a)} x^{a-1} e^{-\lambda x} \qquad x \geq 0$$

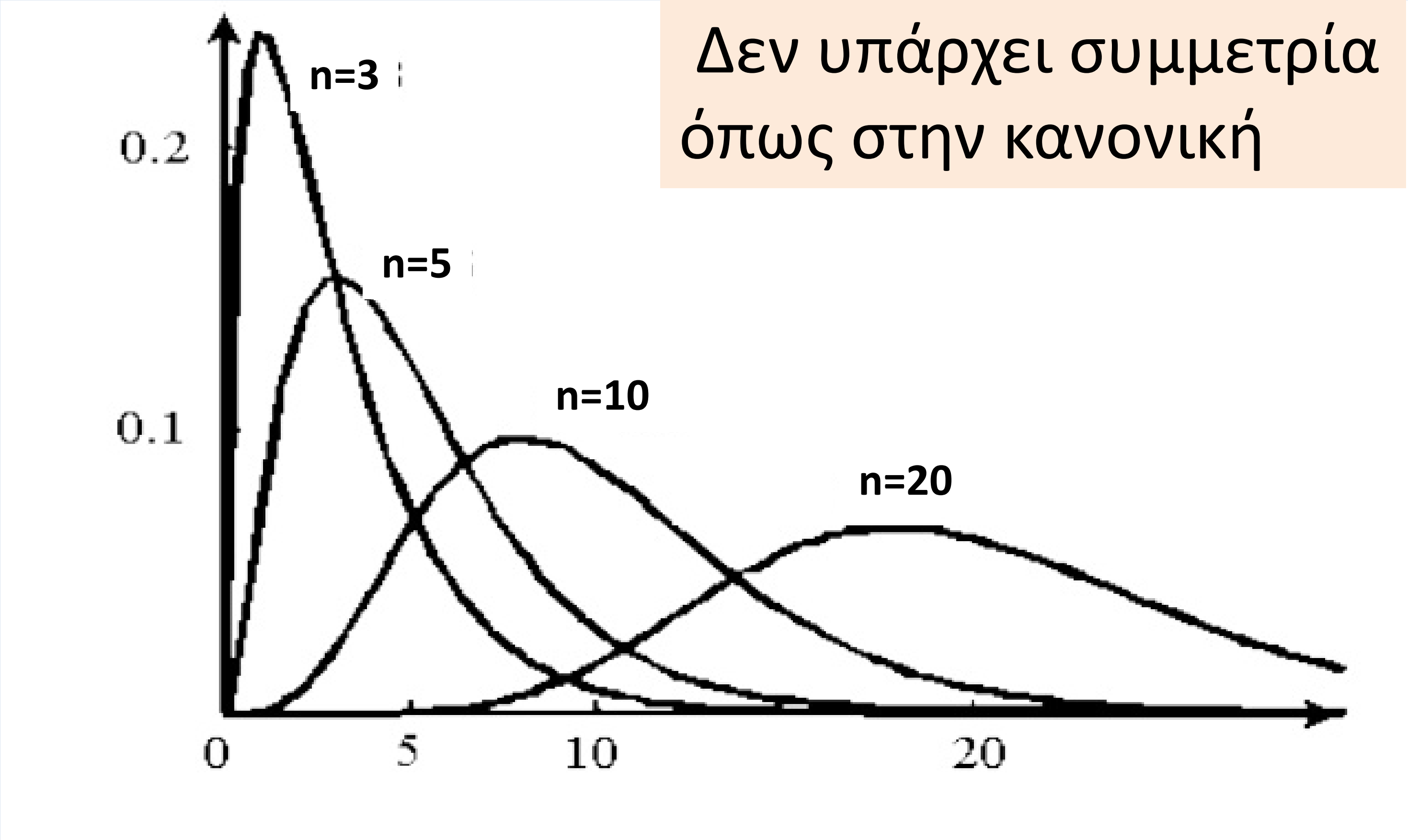
- Αν ***α=n/2*** και ***λ=1/2*** τότε:

$$f(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2}$$

προκύπτει η συνάρτηση της κατανομής **Χι-τετράγωνο** με ***n*** βαθμούς ελευθερίας, X^2_n

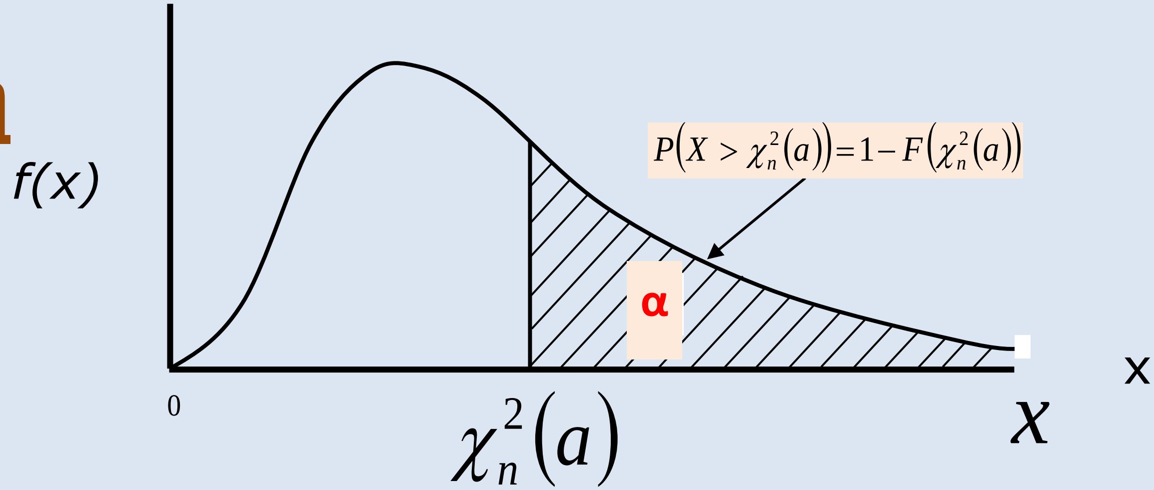
Η μορφή της συνάρτησης πυκνότητας

:



Δεν υπάρχει συμμετρία όπως στην κανονική

Πρόταση



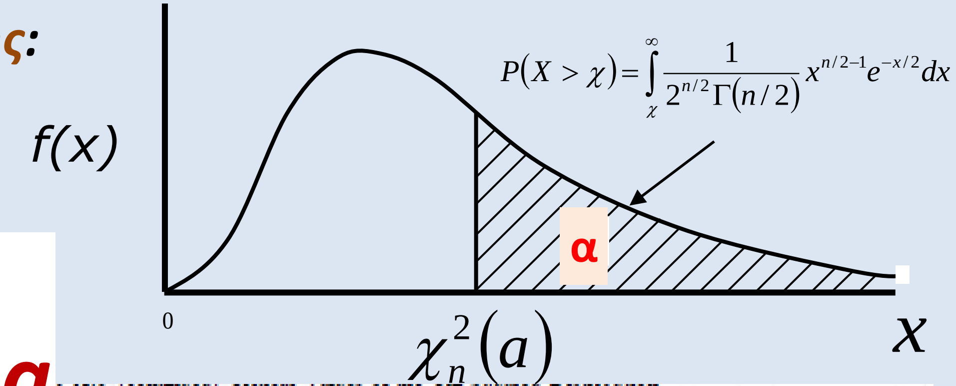
- Ορίζουμε ως ***χ²ₙ (α)*** εκείνο το σημείο εκείνο για το οποίο ισχύει:

$$P\left(X > \chi_n^2(a)\right) = a$$

$$P(X > \chi) = \int_{\chi}^{\infty} f(x)dx = \int_{\chi}^{\infty} \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2} dx$$

όχι αναλυτικός τύπος (αριθμητικές μέθοδοι υπολογισμού του ολοκληρώματος)

Πίνακας τιμών της χι-τετράγωνο κατανομής:
για διάφορες τιμές της πιθανότητας ***α*** και
του βαθμού ελευθερίας ***n***



$$\chi_n^2(a): P\left(X>\chi_n^2(a)\right)=a$$

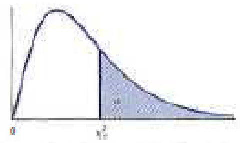


Table A.5 Critical Values of the Chi-Squared Distribution

a

n

v	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50	0.3	0.25	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001
1	0.0^4393	0.0^3157	0.0^3628	0.0^3982	0.00393	0.0158	0.0642	0.102	0.148	0.455	1.074	[illegible]	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.466
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.515
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	0.989	1.239	1.561	1.690	2.167	2.833	3.822	4.255	4.671	6.346	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.321
8	1.344	1.647	2.032	2.1.80	2.733	3.490	4.594	5.071	5.527	7.344	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	2.1.56	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.031	11.340	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	3.565	4.107	4.765	5.009	5.892	7.041	8.634	9.299	9.926	12.340	15.119	15.984	16.985	19.812	22.362	24.736	25.471	27.688	29.819	34.527
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.124
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339	17.322	18.245	19.311	[illegible]	[illegible]	[illegible]	28.259	30.578	32.801	37.698
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338	18.418	19.369	20.465	[illegible]	[illegible]	[illegible]	29.633	32.000	34.267	39.252
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338	19.511	20.489	21.615	[illegible]	[illegible]	[illegible]	30.995	33.409	35.718	40.791
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338	[illegible]0.601	21.605	22.760	[illegible]	[illegible]	[illegible]	32.346	34.805	37.156	42.312
[illegible]	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338	[illegible]1.689	[illegible]	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.819
20	[illegible].434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337	[illegible]2.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.314
[illegible]	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337	[illegible]3.858	[illegible]	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.796
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337	[illegible]4.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337	[illegible]6.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337	[illegible]7.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337	[illegible]8.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.619
20	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336	[illegible]9.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.051
27	11.808	12.878	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336	[illegible]0.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.475
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336	[illegible]1.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.994	56.892
29	13.121	11.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336	[illegible]2.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.335	58.301
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336	[illegible]3.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.702
40	20.707	22.164	23.838	24.433	26.509	29.051	32.345	33.66	34.872	39.335	[illegible]4.165	45.616	47.269	51.805	55.758	59.342	60.436	63.691	66.766	73.403
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335	[illegible]4.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.660
60	35.534	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335	[illegible]5.226	66.981	68.972	74.397	79.082	83.298	84.58	88.379	91.952	99.608

$$\chi_{20}^2(0.25)$$

$$P(X > 7.261) - P(X > 24.996) =$$
$$= 0.95 - 0.05$$

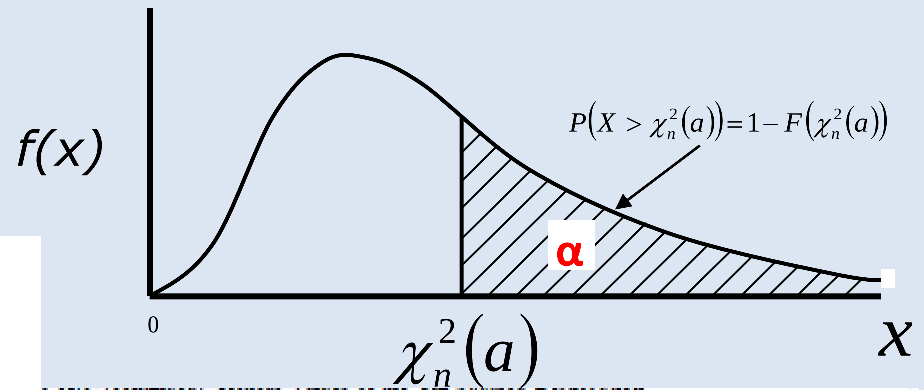
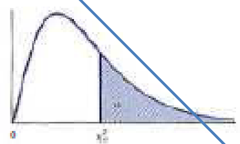


Table A.5 Critical Values of the Chi-Squared Distribution



n

v	0.995	0.99	0.98	0.97	0.95	0.90	0.80	0.75	0.70	0.50	0.30	0.25	0.20	0.1	0.05	.025	0.02	0.01	0.005	0.001
1	0.0^4393	0.0^3157	0.0^3628	0.0^3982	[illegible]	0.0158	0.0642	0.102	0.148	0.455	1.074	1.323	1.642	2.706	[illegible]	5.024	5.412	6.635	7.879	10.827
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.466
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.515
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	0.989	1.239	1.561	1.690	2.167	2.833	3.822	4.255	4.671	6.346	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.321
8	1.344	1.647	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.031	11.340	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	3.565	4.107	4.765	5.009	5.892	7.041	8.634	9.299	9.926	12.340	15.119	15.984	16.985	19.812	22.362	24.736	25.471	27.688	29.819	34.527
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.124
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.698
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.791
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.819
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.314
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.796
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.619
20	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.051
27	11.808	12.878	14.125	14.573	16.151	"18.114	20.703	21.749	22.719	26.336	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.475
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.994	56.892
29	13.121	11.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.335	58.301
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.702
40	20.707	22.164	23.838	24.433	26.509	29.051	32.345	33.66	34.872	39.335	44.165	45.616	47.269	51.805	55.758	59.342	60.436	63.691	66.766	73.403
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335	54.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.660
60	35.534	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335	65.226	66.981	68.972	74.397	79.082	83.298	84.58	88.379	91.952	99.608

Πρόταση

- Έστω ***n*** τ.μ. *{X$_1$, X$_2$, …, X$_n$}* **ανεξάρτητες** και **ισόνομες** που ακολουθούν την **τυπική κανονική**, δηλ. ***$X_i \sim N(0, 1)$***.

- Τότε η $Y = \sum_{i=1}^{n} X_i^2$ ακολουθεί την κατανομή **Χι-τετράγωνο** **με *n* βαθμούς ελευθερίας X^2_n**.

$$\left\{X_i \sim N(0,1)\right\}_{\mathrm{i}=1}^{\mathrm{n}} \Rightarrow Y = \sum_{i=1}^{n} X_i^2 \sim \mathrm{X}_n^2$$

με συνάρτηση πυκνότητας πιθανότητας:

$$f(y) = \frac{1}{2^{n/2}\Gamma(n/2)} y^{n/2-1} e^{-y/2}$$

Εφαρμογές της κατανομής Χι-τετράγωνο στη Στατιστική

Πρόταση 1

- Το **στατιστικό στοιχείο** $\dfrac{\sum_{i=1}^{n}\left(X_i-\mu\right)^2}{\sigma^2}$ ακολουθεί την κατανομή **Χι-τετράγωνο** με ***n*** **βαθμούς ελευθερίας**

$$\frac{\sum_{i=1}^{n}\left(X_i-\mu\right)^2}{\sigma^2} \sim \mathrm{X}_n^2$$

$$\frac{\sum_{i=1}^{n}\left(X_i - \mu\right)^2}{\sigma^2} \sim \mathrm{X}_n^2$$

- Το **στατιστικό στοιχείο** συνδέει το δείγμα με τις **παραμέτρους** του μέσου ***μ*** και της διασποράς ***σ²***

- Η **κατανομή** που ακολουθεί είναι η **Χι-τετράγωνο** με ***n*** **βαθμούς ελευθερίας.**

Πρόταση 2

- Το **στατιστικό στοιχείο** $\dfrac{(n-1)S^2}{\sigma^2} = \dfrac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{\sigma^2}$

  ακολουθεί την κατανομή **Χι-τετράγωνο**

  με ***n-1*** βαθμούς ελευθερίας

$$\frac{(n-1)S^2}{\sigma^2} = \frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{\sigma^2} \sim \mathrm{X}^2_{n-1}$$

- όπου $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ η **δειγματική διασπορά**

$$\frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{\sigma^2} \sim \mathrm{X}_{n-1}^2 \Rightarrow \frac{(n-1)S^2}{\sigma^2} \sim \mathrm{X}_{n-1}^2$$

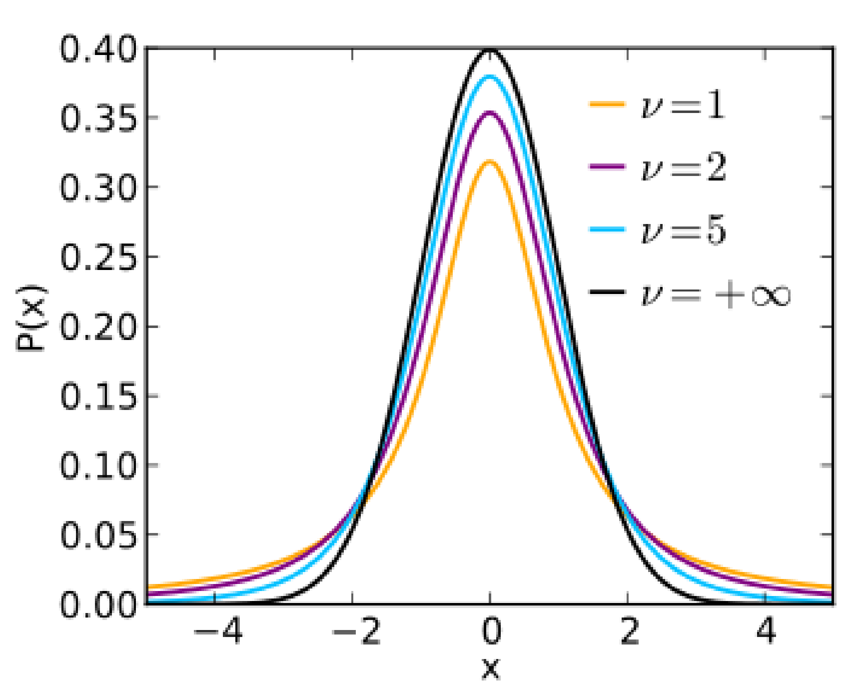
- Αυτό το **στατιστικό στοιχείο** συνδέει το δείγμα με την **παράμετρο** της διασποράς ***σ²***

- Η **κατανομή** που ακολουθεί είναι η **Χι-τετράγωνο** με ***n-1*** **βαθμούς ελευθερίας**

(Γ). Η κατανομή t-Student *(Gosset 1908)*

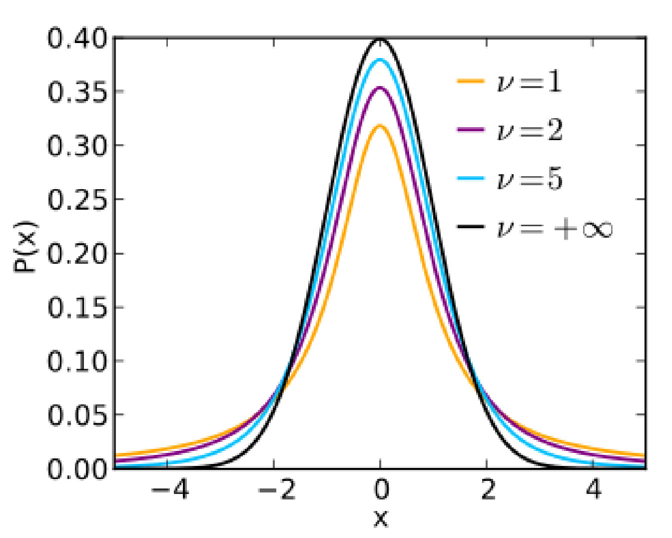
Συνάρτηση πυκνότητας πιθανότητας (**σ.π.π.**)

$$f_n(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{\pi}\sqrt{n}\,\Gamma\left(\frac{n}{2}\right)}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}} = \frac{1}{\sqrt{\pi}\,\mathrm{B}\left(\frac{1}{2},\frac{n}{2}\right)}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$$



$$B(a+b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Ιδιότητες της t-Student κατανομής



- **Μέση τιμή και διακύμανση**:

  E(X)=0 και *V(X)=n/(n-2)*

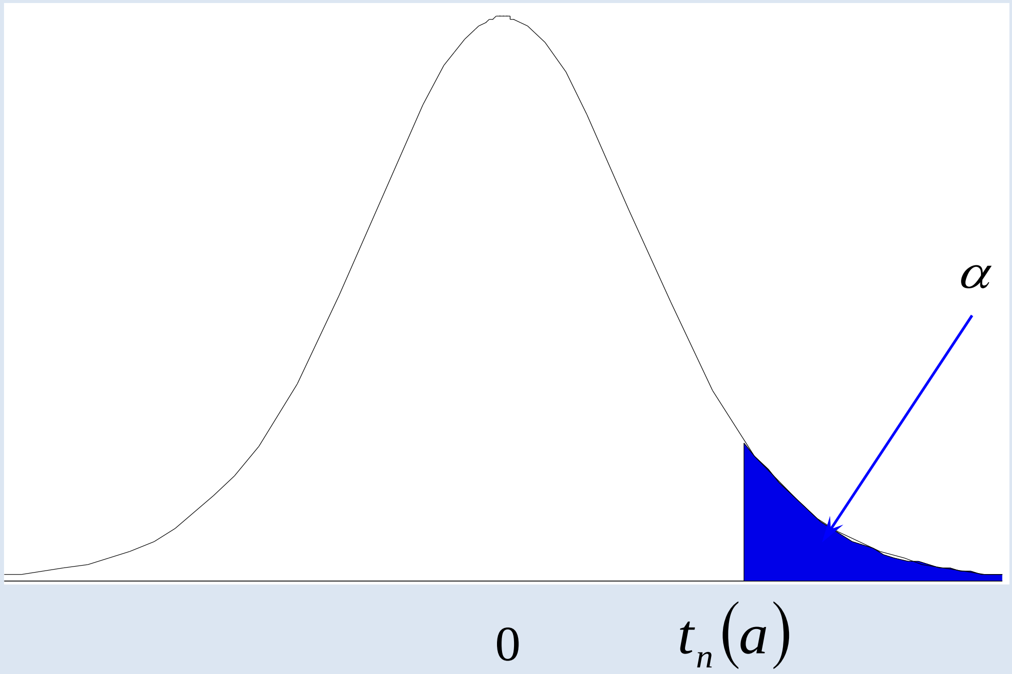
- **Είναι συμμετρική**

  μοιάζει με τη Κανονική κατανομή, αλλά έχει πιο πολύ πιθανότητα στις άκρες.

- Για **μεγάλη τιμή του '*n*'** συγκλίνει στην τυπική Κανονική

$$\lim_{n\to\infty} f_n(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Πρόταση



Ορίζουμε ως $t_n(a)$ *το σημείο για το οποίο ισχύει:*

$$P(X > t_n(a)) = a$$

π.χ. εύρεση του $t_{12}(0.005)$

$$t_n(a): P\big(X > t_n(a)\big) = a$$

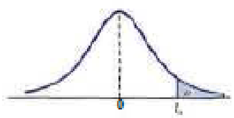


Table A.4 Critical Values of the t-Distribution

a

n

v	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

v	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290

$$t_{12}(0.005) = 3.055$$

π.χ. εύρεση του $t_{11}(0.05)$

$$t_n(a): P\big(X > t_n(a)\big) = a$$

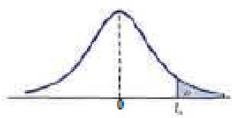


Table A.4 Critical Values of the *t*-Distribution

a

n

v	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	[illegible]	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	[illegible]	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	[illegible]	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
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v	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
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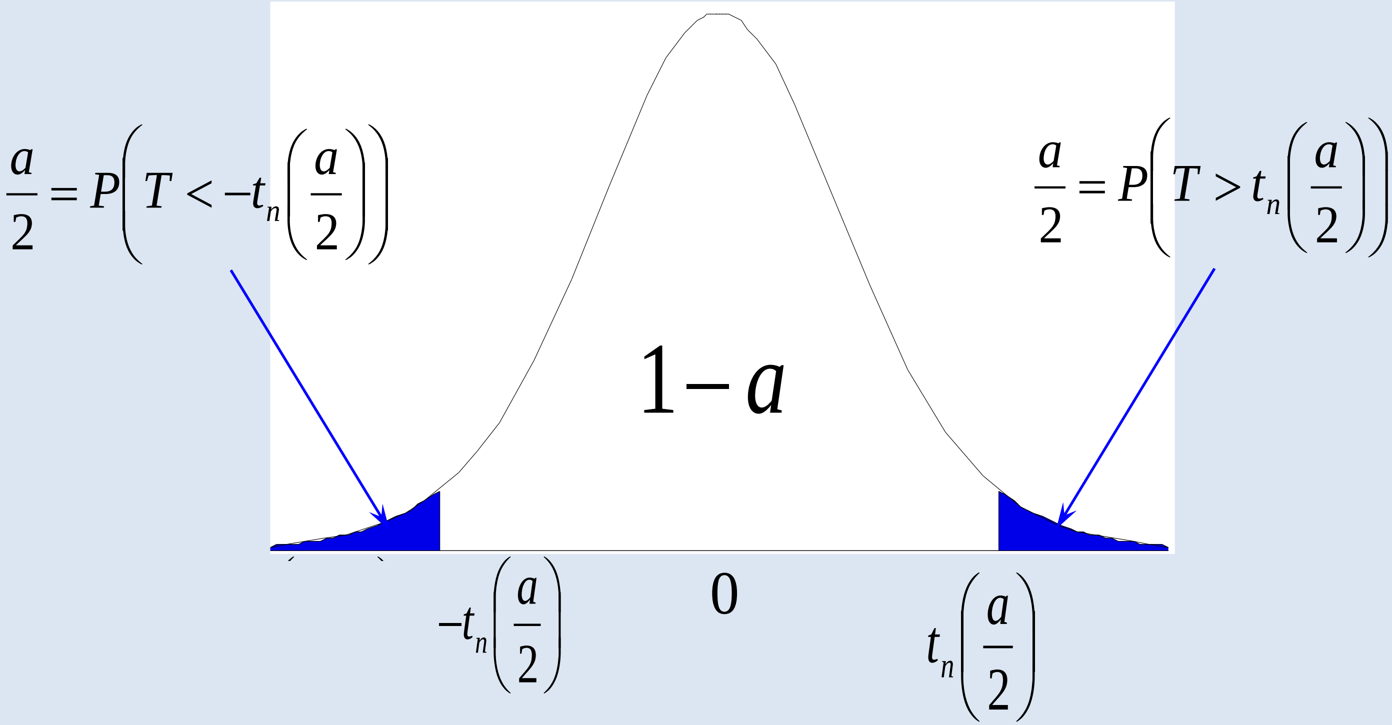
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290

$t_{11}(0.05) = 1.796$



$$P\left(-t_n\left(\frac{a}{2}\right) < T < t_n\left(\frac{a}{2}\right)\right) = P\left(T < t_n\left(\frac{a}{2}\right)\right) - P\left(T < -t_n\left(\frac{a}{2}\right)\right) =$$

$$P\left(T > -t_n\left(\frac{a}{2}\right)\right) - P\left(T > t_n\left(\frac{a}{2}\right)\right) = 1 - 2P\left(T > t_n\left(\frac{a}{2}\right)\right)$$

Πρόταση 3

- Έστω 2 **ανεξάρτητες** τυχαίες ***Z, Y*** μεταβλητές με κατανομές:

  Z~N(0,1) και ***Y~ X^2_n***, τότε η τυχαία μεταβλητή $T = \dfrac{Z}{\sqrt{Y/n}}$

  ακολουθεί την κατανομή ***t-Student*** με ***n*** βαθμούς ελευθερίας.

$$\begin{array}{c} Z \sim N(0,1) \\ Y \sim X_n^2 \end{array} \Rightarrow T = \frac{Z}{\sqrt{Y/n}} \sim t_n$$

Εφαρμογή της Πρότασης 3

Για $Z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$ και $Y = \dfrac{(n-1)S^2}{\sigma^2} \sim \mathrm{X}^2_{n-1}$

έχουμε:

$$T = \frac{Z}{\sqrt{\dfrac{Y}{n-1}}} = \frac{\dfrac{\bar{X} - \mu}{\cancel{\sigma} / \sqrt{n}}}{\sqrt{\dfrac{\cancel{(n-1)}S^2}{\cancel{\sigma^2}\cancel{(n-1)}}}} = \frac{\bar{X} - \mu}{\dfrac{S}{\sqrt{n}}} \sim t_{n-1}$$

Κατανομή t-Student με *n-1* βαθμούς ελευθερίας

- Αυτό το **στατιστικό στοιχείο** συνδέει το δείγμα με τη **παράμετρο** του μέσου ***μ*** του πληθυσμού

$$\frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1}$$

- Η κατανομή που ακολουθεί είναι η ***t-Student*** με ***n-1*** βαθμούς ελευθερίας.

(Δ). Η κατανομή **F** *(Fisher – Snedecor 1934)*

Ronald Fisher (1880–1962), a pioneer in statistics

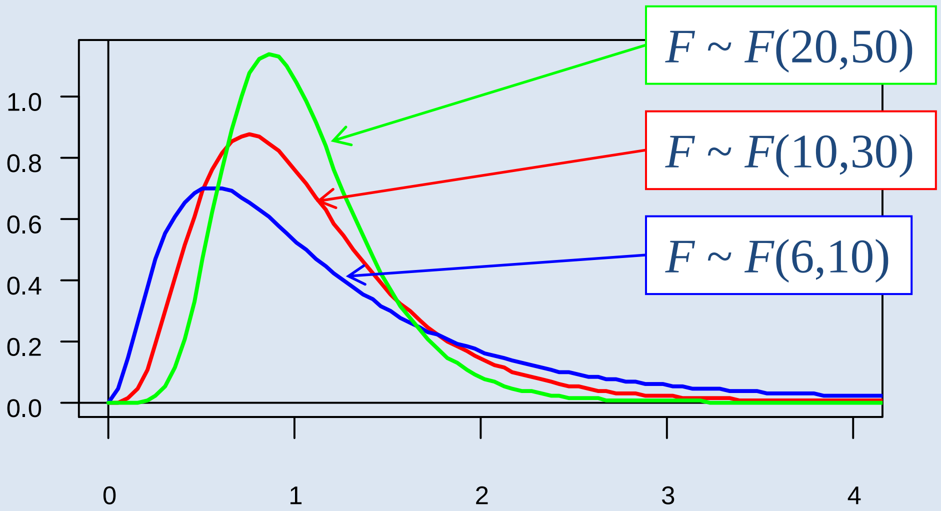
George Snedecor (1881–1974)

Αν $Y_1 \sim X_n^2$ και $Y_2 \sim X_m^2$ τότε το πηλίκο

$$F = \frac{Y_1 / n}{Y_2 / m} \sim F_{n,m}$$

ακολουθεί την **κατανομή F** με συνάρτηση πυκνότητας:

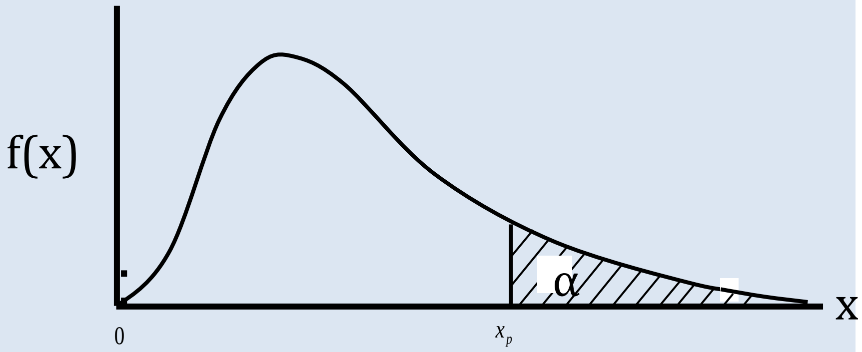
$$f_{n,m}(x) = \frac{\Gamma\left(\frac{n+m}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(\frac{m}{2}\right)} \frac{x^{n/2-1}}{(1+x)^{(n+m)/2}}$$



Ιδιότητες της κατανομής F

- Η κατανομή εξαρτάται από τις τιμές των *n, m* αλλά και από την σειρά με την οποία εμφανίζονται, δηλ. $F(n,m) \neq F(m,n)$
- Παρουσιάζει **θετική ασυμμετρία** η οποία μειώνεται όσο αυξάνονται οι βαθμοί ελευθερίας *n*, *m*.
- **Μέση τιμή & διακύμανση**:

$$E(X)=\frac{m}{m-2} \qquad V(X)=\frac{2m(n+m-2)}{n(m-2)^2(m-4)}$$



Ορίζουμε ως $f_a(v_1, v_2)$

το σημείο για το οποίο ισχύει:

$$P\left(F > x_a(v_1, v_2)\right) = a$$

Ισχύει ότι:

$$f_{1-\alpha}(v_1, v_2) = \frac{1}{f_\alpha(v_2, v_1)}$$

για συγκεκριμένο α=0.05

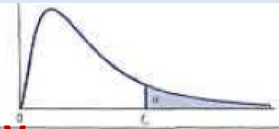


Table **A.6*** Critical Values of the *F*-Distribution

$f_{0.05}(v_1, v_2)$

v_2

v_1

v_2	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5,11	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2,59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2,59	2,54
17	4.45	3.59	3.20	2.96	2.81	2.70	2,61	2,55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2,51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.5-1	2.48	2,12
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2,57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2,55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2,53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2,51	2.42	2.36	2.30
25	4.21	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2,59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2,57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2,53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

Πρόταση 5

- Έστω *2* **ανεξάρτητα δείγματα** μεγέθους n_1, n_2 με δειγματικές διασπορές S^2_1 και S^2_2 αντίστοιχα Τότε:

$$A_{1,2} = \frac{\dfrac{S_1^2}{\sigma_1^2}}{\dfrac{S_2^2}{\sigma_2^2}} = \frac{S_1^2 \sigma_2^2}{S_2^2 \sigma_1^2} \sim F_{n_1 - 1, n_2 - 1}$$

όπου σ_1^2 , σ_2^2 οι πραγματικές διασπορές των δύο πληθυσμών.

Απόδειξη

Γνωρίζουμε ότι η μεταβλητή $Y = \frac{(n-1)S^2}{\sigma^2} \sim X^2_{n-1}$

Έτσι έχουμε ότι $\frac{(n_1-1)S_1^2}{\sigma_1^2} \sim X^2_{n_1-1}$ και $\frac{(n_2-1)S_2^2}{\sigma_2^2} \sim X^2_{n_2-1}$

Επομένως σύμφωνα με τον ορισμό της κατανομής F έχουμε:

$$\frac{\frac{(n_1-1)S_1^2}{\sigma_1^2} \Big/ n_1-1}{\frac{(n_2-1)S_2^2}{\sigma_2^2} \Big/ n_2-1} \sim F_{n_1-1,n_2-1} \Rightarrow \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1,n_2-1}$$

$$\frac{S_1^2 \sigma_2^2}{S_2^2 \sigma_1^2} \sim F_{n_1-1,n_2-1}$$

- Αυτό το στατιστικό στοιχείο συνδέει τα δύο διαθέσιμα δείγματα με τον λόγο των **παραμέτρων** της διασποράς των δύο πληθυσμών $\boldsymbol{\sigma_1^2}$ προς $\boldsymbol{\sigma_2^2}$

- Η κατανομή που ακολουθεί είναι η $\boldsymbol{F_{n1-1,\ n2-1}}$

Σημαντικά στατιστικά στοιχεία και κατανομές τους

- ✓ $$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$
- ✓ $$\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\sigma^2} \sim X_n^2 \qquad \frac{(n-1)S^2}{\sigma^2} \sim X_{n-1}^2$$
- ✓ $$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$$
- ✓ $$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1,n_2-1}$$

Ως προς το είδος της **κατανομής δειγματοληψίας**

Σημαντικά στατιστικά στοιχεία και κατανομές τους

- ✓ $$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$ $$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$$

- ✓ $$\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\sigma^2} \sim X_n^2$$ $$\frac{(n-1)S^2}{\sigma^2} \sim X_{n-1}^2$$

- ✓ $$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1,n_2-1}$$

Ως προς το είδος των **παραμέτρων** που ελέγχουν