Design Document

Ground Station Squad
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1 Introduction

1.1 Background

Every year more and more CubeSats are deployed into orbit, and, therefore, the demand of accessible and inexpensive ground stations which to capture the signals increases. What is more, the New Space approach - one where space data and the space sector as a whole is more accessible to the common public - is growing in popularity as time passes. As a result, the number of amateur radio operators who would like to build their own ground station progressively expands. In this document, we describe the design and process of building such station. The project is part of the Space Challenges Bootcamp 2021, Gela village, Bulgaria.

1.2 The Space Challenges 2021 Ground Station

The Space Challenges ground station (GS) follows the open-source design of the SatNOGS project[1]. It is capable of receiving signals from orbiting satellites. Antenna rotation is achieved by a two degrees-of-freedom (DoF) rotator. A Yagi-Uda antenna was designed for 436.5 MHz in order to capture transmitted data over UHF. Signal reception is achieved by a wide-band low-noise amplifier (LNA) and a Software-defined Radio (SDR). The signal is then processed by applying digital-to-analog conversion (DAC), Doppler compensation, demodulation, and de-packetization. Finally, the decoded satellite data is uploaded to a web platform where ground station users can download it or book time for the ground station in order to track a specific satellite on a certain date and time.

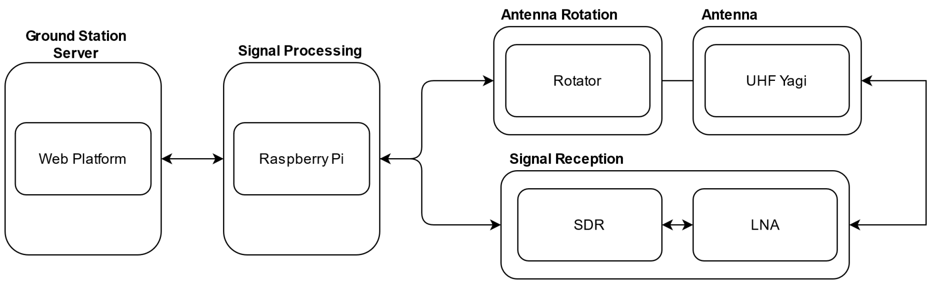

Figure 1: Block diagram of the Space Challenges ground station

[1] https://wiki.satnogs.org/SatNOGS_Rotator_v3

2 Analysis

2.1 Mission Requirements

The mission aims to capture data from EnduroSat's QMR-KWT satellite via a ground station over UHF, capturing 2 passes a day and processing the received signal.

2.2 System Requirements

S1: The system shall track satellites with altitude in the range of 250 to 2,000 km (LEO)

S2: The system shall capture radio signals at 436.5 MHz

S3: The system shall have BER $<=10^{-3}$

S4: The system shall store satellite data and allow users to download it via a web platform

S5: The system shall be connected to the SatNOGS network

S6: The system shall interpret signal from the QMR-KWT satellite

2.3 Subsystem Requirements

SB1: The rotator subsystem shall have angular velocity of azimuth axis in the range of >=5 deg/s (related to S1.)

SB2: The antenna subsystem shall have gain of >=12 dBm (related to S3.)

SB3: The RF subsystem shall demodulate 2GFSK signal at 9600 bps (related to S6.)

SB4: The RF subsystem shall decode unnumbered information (UI) frames in AX.25 format (related to S6.)

SB5: The internet connection shall be with speed of at least 64 Mbps for uploading and 8 Mbps for downloading (related to S4.)

3 Design

3.1 Communication

3.1.1 Air to Ground Segment

Communication between ground station and satellite is illustrated in Fig.2. The QMR-KWT satellite transmits data at 436.5 Hz, 9600 bits per second with 2GFSK modulation and encodes the data over the AX.25 protocol. The bandwidth of the signal is approximately 10 KHz.
The structure of a UI frame of the AX.25 protocol is depicted in Figure 3. The data field is surrounded by a Preamble and Postamble. The protocol uses bit stuffing and scrambling to enable transmission channels to be synchronised and maximize the available bandwidth.

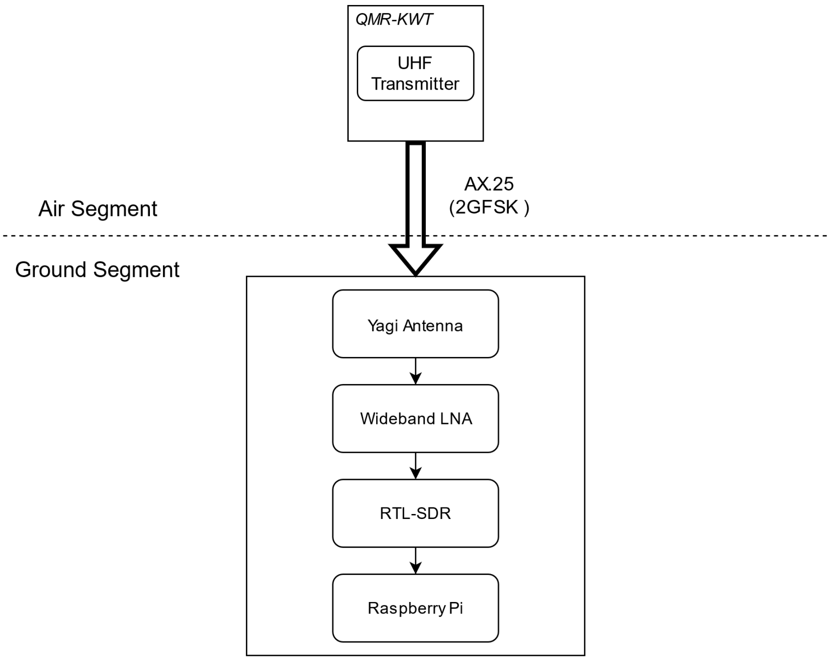

Figure 2: Communication between ground and air segment

Figure 3: Structure of the AX.25 frame

3.1.2 Rotator controller

Once the tracking procedures start, active communication between the PC (Raspberry Pi) and the rotator controller (Arduino Pro Micro) automatically begins. EasyComm 2 has been used as a communication protocol between the two which also allows full station control and manual intervention. The host PC issues commands to the controller by sending a 2-character command identifier followed by the command value. Commands are separated by either a space or carriage return. Not all commands have been implemented - the current firmware version interprets only rotator control commands:

Command	Meaning	Argument
AZ	Azimuth	1 decimal place number [deg]
EL	Elevation	1 decimal place number [deg]
SA	Stop azimuth moving	-
SE	Stop elevation moving	-
RESET	Return to HOME position	-

3.2 Antenna

An Yagi-Uda antenna was chosen to be build due to its simple structure and high performance. The Yagi-Uda is a type of directional antenna that consists of a long rod called “boom” to which the other elements are attached. The boom can be made of conductive or non-conductive material (the material does not heavily impact the performance of the antenna, but keeping it light in mass would be beneficial for its operation). The main element that catches the signal is the “driven element” and it represents the two dipoles of the antenna. Those dipoles are two rods that connect to a coaxial cable through a BALUN (BALanced to UNbalanced convertor) - the reason is that antennas have a balanced impedance, but coaxial cables work better with unbalanced impedance. The induced magnetic field in the coaxial cable will amount for losses in the signal when the cable is routed through walls or the ground and other materials. The unbalanced impedance reduces those losses. Baluns can be configured to have different output impedance from the input impedance of the antenna - a 1:1 balun has equal input and output impedance and a common 1:4 balun has 4 times bigger impedance than its input. Since we need to have an impedance of 50 ohms we are using a 1:1 balun as shown in the Figure 4. The length of the driven element should be roughly equal to half the wavelength of the signal. The driven element has a gap in the middle where it connects to the balun. Both the dipoles should be electrically isolated

from the boom if it is made of conductive material. The reflector sits behind the driven element and its length is 5% longer than half the wavelength. On the other side of the dipoles sit the directors, typically 5% shorter than half the wavelength. The elements have a specific spacing between them. The director and the driven element are positioned a quarter of the wavelength and the spacing between the driven element and the directors is between a quarter and third of the wavelength (we chose a factor of 0.28 times the wavelength). Every reflector is spaced apart from the others at a distance of one third the wavelength. The directors and the reflector can be connected to conductors such as a cable that goes through the middle of each element (excluding the two dipoles). This can be omitted if the boom is conductive and is not isolated from the elements. The boom should be only as long as needed to hold the other elements together. [1]
The simplest yagi antenna is composed of only one of each of those parts (one reflector, one driven element and one director). More reflectors can be added to form a reflector plane, perpendicular to the boom which will reduce interference and signal losses coming from behind the antenna. The role of the directors is to “guide” the wave-front towards the driven element and amplify it based on wave interference physics. More directors are added to increase the gain of the antenna. This gain depends directly on the number of elements. With a minimum of 3 elements as stated above, the gain of the antenna is calculated by multiplying the amount of elements by a factor of 1.66dB. Each director contributes an additional 1.66dB and increases the directivity of the antenna and reduces the HPBW.

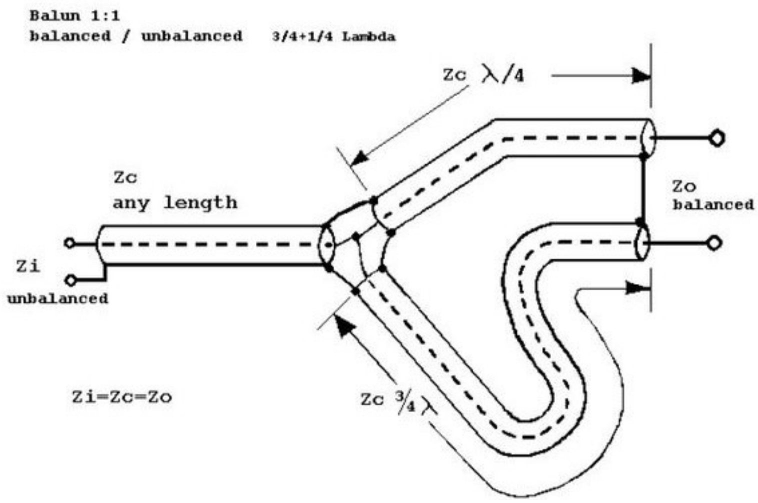

Figure 4: Structure of 1:1 Balun

A simple 1:1 balun can be constructed from two pieces of coaxial cable if we know the wavelength of the signal and its speed of propagation (a ratio or fraction of the speed of light) through the cable (coaxial cables have different velocity of propagation depending on their specifications, usually given as RG/U. The first piece of coax cable for the balun should be as long as ¼ the wavelength and multiplied by the velocity of propagation and the other should be 3 times longer.

3.2.1 Antenna Design

- Driven element length = $\frac{\lambda}{2}$ (The two dipoles should be a bit shorter than half of that and the gap distance between them should fill the remainder)
- Reflector length = $\frac{\lambda}{2}(1+5\%)$
- Director length = $\frac{\lambda}{2}(1-5\%)$
- Spacing between reflector and driven element = $\frac{\lambda}{4}$
- Spacing between driven element and first director = $\frac{\lambda}{3}$
- Spacing between directors (less than $\frac{\lambda}{3}$, eg. 15% shorter) = $\frac{\lambda}{3}(1-15\%)$
- Boom length = Spacing between reflector and driven element + Spacing between driven element and first director + Spacing between directors * (number of directors - 1) + diameter of elements * number of elements (+ a few centimeters excess on the two ends)
- Gain = 1.66 * Number of elements

3.2.2 Balun Configuration

- Balun cable length 1 = $\frac{\lambda}{4}V_c$
- Balun cable length 2 = $\frac{3\lambda}{4}V_c$
- Velocity of propagation = speed of light / dielectric constant of the cable

3.2.3 Antenna Specifications

- Minimum frequency (MHz): 435
- Maximum frequency (MHz): 438
- Number of directors: 8
- Optimized for frequency of 436.5 MHz
- Length of Reflector: 36.1 cm
- Driven element length: 34.36 cm (a folded dipole antenna has 4 times higher impedance compared to a non-folded dipole; for space applications lower impedance of 50ohms is preferred using a non-folded/linear dipole)
- Director length: 30.9 cm
- Spacing between reflector and driven element: 17.2 cm
- Spacing between driven element and director: 19.2 cm
- Spacing between directors: 22.9 cm
- Approximate gain: 16.6 dBi
- Boom length: 2.1 m
- Balun (for RG174 cable) L1 = 11.4 cm, L2 = 34.2 cm
- Minimum elevation[2]: 15°

3.2.4 Link Budget

In antennas, it is good to know how to take account of all of the power gains and losses that a communication signal experiences in its path from source to receiver. This is the referred as link budget. In digital communication or data transmission, Eb/N0 (energy per bit to noise power spectral density ratio) is a normalized signal-to-noise ratio (SNR) measure. The features that contribute to a good ratio are the power of the transmitter, the gain of the transmitter and the gain of the receiver. The losses come from the distance (free space losses), atmospheric attenuation of the signal (dependent

[2] Typically, the latter varies between 7 - 8°, but our numerical evaluation is due to the fact that the antenna will be placed in a forested area.

Parameter	Meaning	Value	Unit
G_t	Gain of Tx antenna	0	dBi
P_t	Tx Power	-3.5	dBW
G_r	Gain of Rx antenna	16.6	dBi
T_s	System temperature	320	K
K_b	Boltzmann constant	-228.5	dBW/(K · Hz))
L_a	Amospheric losses	1	dB
L_s	Free space losses	153	dB
L_p	Polarization mismatch	3	dB
R	Data rate	9600	bps

on the weather), polarisation mismatch (if the transmitter and the receiver use different polarisation), system temperature (a measure of the inherent noise of the receiver system) and also the increase of transmission rate. It is more convenient to do these calculations in decibel units to avoid operations with large numbers. If the estimated received power is sufficiently large (typically relative to the receiver sensitivity), which is also dependent on the modulation scheme (in our case 2GFSK), the link will be sufficient for signal reception. The amount by which the received power exceeds receiver sensitivity is called the link margin.
Our 10-element yagi antenna has an estimated gain of 16.6 dBi. The $\frac{E_b}{N_o}$ can be calculated as follows:

$$\frac{E_b}{N_o} = G_t + P_t + \frac{G_r}{T_s} - K_b - L_a - L_s - L_p - 10 \log R \qquad (1)$$

where G_t is gain of satellite antenna in dB, P_t is transmission power in dBW, $\frac{G_r}{T_s}$ is dB ratio between gain of ground station and system temperature, K_b is Boltzmann constant in dB, L_a represents atmospheric losses, L_s free space losses, L_p losses due to match in polarization mismatch, all three measured in dB, and R is the transmission rate.
The system $\frac{E_b}{N_o}$ was calculated to be 19.5 dB which results in a bit error rate (BER) of 10^{-5}. The packet error rate (PER) can be calculated as follows:

$$PER = 1 - (1 - BER)^{N_{bytes}.bits} = 1 - (1 - BER)^{77 \cdot 8} = 0.05 \qquad (2)$$

3.2.5 Half-Power Beam Width

Half-power beam width (HPBW) is the angle in which relative power is more than 50% of the peak power, in the effective radiated field of the antenna. This is considered to be the part of the antenna output that has maximum consistency and utility and is closely related to the gain or the directivity of the antenna. The maximum directivity of an antenna is equal to its gain in decibels and is approximately expressed as $D = \frac{4\pi}{\theta_{HP}\Phi_{HP}}$, where Φ_{HP} is the half-power beam width in the Φ direction and θ_{HP} is the half-power beam width in the θ direction, measured in radians. The directivity for an yagi-uda antenna is almost the same in both directions. The radiation pattern can be visualized as seen in Figure 5.

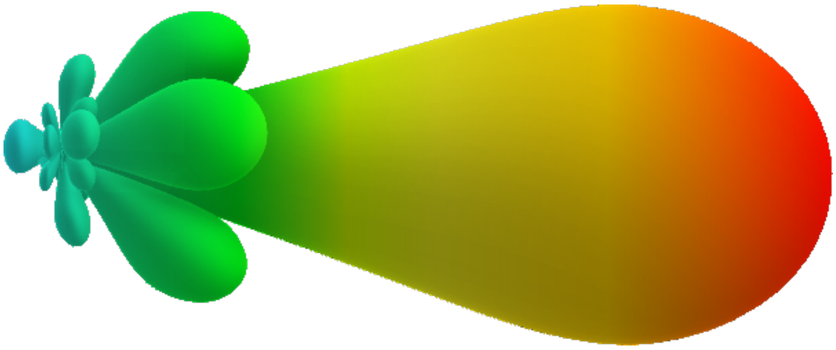

Figure 5: Radiation pattern of an yagi-uda antenna

Therefore, we can calculate the half-power beam width of the whole antenna as follows:

$$HPBW = 2\left(\sqrt{\frac{4\pi}{D-3}} - \sqrt{\frac{4\pi}{D}}\right) = 2\,(55.1 - 49.9) = 10.4^\circ \tag{3}$$

First, we take half the directivity, hence -3db because it's measured in decibels and then subtract all of it and multiply by two to get the whole angle. In other words, if the antenna is pointed 5° off the satellite position, our system will lose half of the signal.

3.2.6 Doppler Compensation

Doppler effect is the observed change in frequency when the emitter of a wave (such as RF signals and sound waves) is moving relative to the observer. It has a noticeable impact on communication systems when working with satellites in LEO which have relatively high speeds compared to the Earth surface. For this reason, the SDR of the ground station should be tuned to compensate for this change in order to get more reliable results. The Doppler shift can be estimated using the observer and satellite position and their relative velocity. The GNU Radio toolkit has a dedicated Doppler compensation flowgraph that can be used to correct the received signal frequency.
Further details can be found in Appendix A.

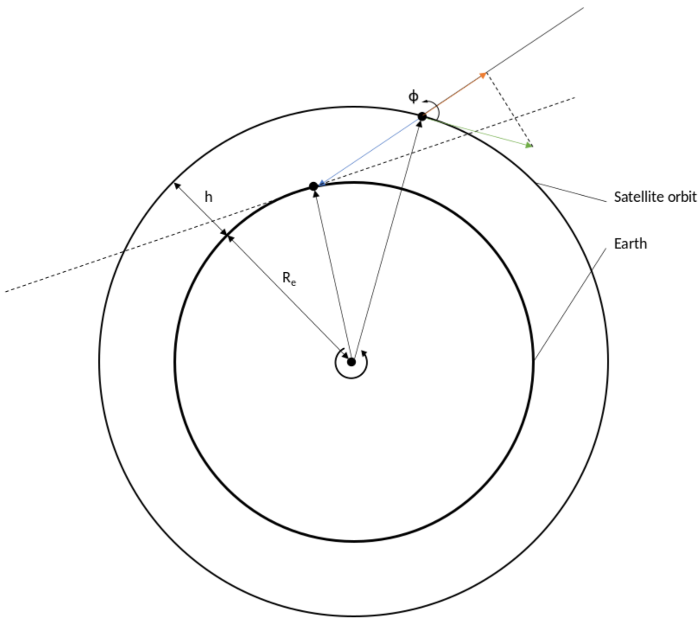

Figure 6: Estimating Doppler's effect on the transmitted signal from the QMR-KWT satellite

3.3 Mechanical Analysis Rotator

3.3.1 Break Torque

The greatest pressure the antenna needs to withstand is the force applied to it when it is at 90 degrees (vertical) and is exposed to wind blowing perpendicularly to it. The torque that the rotator needs to withstand is calculated according to the following formula [2]:

$$F = A \cdot P \cdot C_d \cdot K_z \cdot G_h \tag{4}$$

where A is the effective area of the antenna, P is wind pressure, C_d constant related to the cylindrical shape of the antenna boom, K_z is a constant related to the length of the antenna in horizontal position, G_h is a constant related to the height of the antenna in vertial position. The average wind speed measured in Gela, Bulgaria, in the years 2015-2021, is 7 km/h. The resultant brake torque is equal to 0.72 N.

3.3.2 Moment of Inertia

Moment of inertia is calculated as:

$$F = \frac{1}{12} m L^2 \tag{5}$$

where m is the mass of the antenna and L is its length. This results in $1.3kg.m^2$

3.3.3 Angular Velocity

To track a satellite, the antenna has to rotate towards it throughout the whole observation time. If it cannot support the speed at which the satellite flies above the ground station, it will go out of range and there will be no reliable downlink. To ensure that our antenna can track any satellite reliably, the rotator must be capable of delivering certain angular velocity. To calculate it, a worst-case scenario is considered with a theoretical satellite in LEO, which passes right above us, which will have the antenna rotate the fastest. Assuming our observer antenna is a point on a circle with the radius of Earth, tracking a point satellite on a circle in lowest possible position (160km altitude and speed of 7.8km/s), we can get the angular speed at an elevation approaching zenith, where the distance of the satellite to the antenna approaches its altitude above the Earth radius and its trajectory is an arc. Knowing the velocity of the satellite, using the length of the arc we can get the time needed for rotating through its angle measurement.

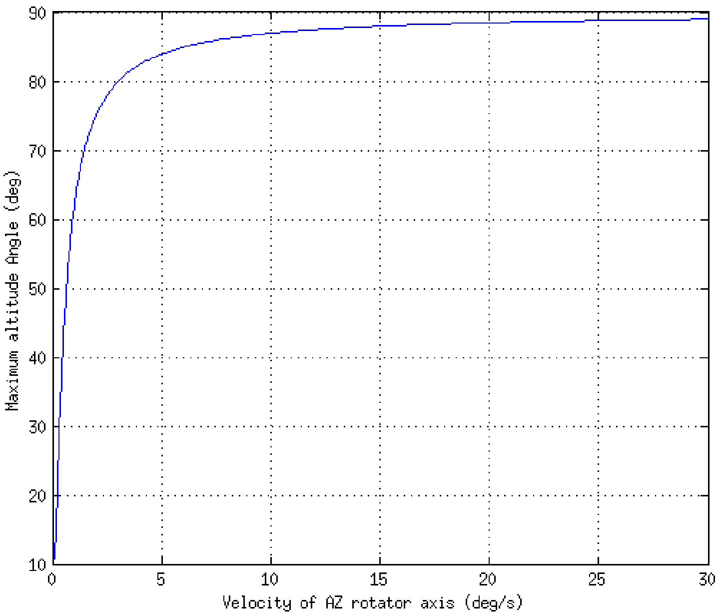

Figure 7: Maximum altitude angle vs Velocity of azimuth rotator axis (deg/s)

We can calculate the length of the arc similarly to how we did the 2D plane calculations for the doppler effect. We calculate the distance from the satellite

to the observer and the angle connecting them to the center of Earth. With a cosine theorem we can find the distance from the satellite to its highest point above the observer and then again with a cosine theorem we get the angle connecting the satellite and its highest point with the observer antenna. This is the approximated angle with highest angular velocity. If we divide that angle by the time the satellite needs to travel through the arc we get the maximum angular velocity.
The lowest height limit is 160 km and the speed unit to orbit earth in this altitude is 7,8 km/s. As a result, required angular velocity in elevation is 2,8 deg/s. This is not enough, however, due to known limitations near zenith. According to the diagram in Figure 7, at least angular velocity of at least 5 deg/s is needed to achieve decent range of rotation.

3.4 Main computer

The system uses a Raspberry Pi 4 which realizes the bridge between the rotator and the Web Platform. There is a constant listener on the Web Platform, and when a query is received from there, a scheduler for the beginning of the rotation procedures is set. Since the python library which is responsible for the orbital calculations - pyorbital - needs updated TLE file locally to operate with, such downloading operation is also scheduled. A back-up for critical situations like lack of internet is also foreseen, and therefore, the downloading course of action is done regularly every 2 days, and as a result, relatively updated version of the file is always available locally.
During the tracking procedures, the calculations are performed in the Raspberry Pi and the results are then sent to the rotator controller. This is repeated constantly until the end of the procedure.

3.5 Web Platform

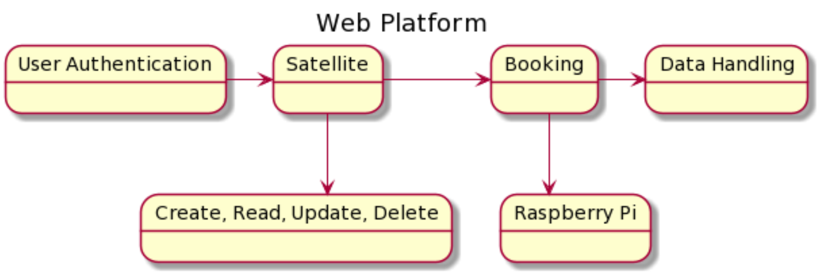

Figure 8: Web platform's concept of operation

With the construction of the ground station, a web platform[3] (WP) where users can book time for our ground station for a certain satellite and time was also built. Even though the whole system was designed specifically for the QMR-KWT, the web platform supports other satellites which transmit on the same frequency and pass over the receiving zone. From a design perspective, the WP is a REST API based on Django, jQuery and Bootstrap. As illustrated on Figure 8 and Figure 10, 9, after having the user authenticated, they can choose a satellite which they want to track for the reserved time and book it via CRUD operations. Once this is done, the server sends a JSON file to the host PC which contains the NORAD ID and the reservation time so the PC can later process the scheduling itself. Finally, after the tracking procedures have finished, the data is sent back to the server and uploaded to the platform, where the users can download it.

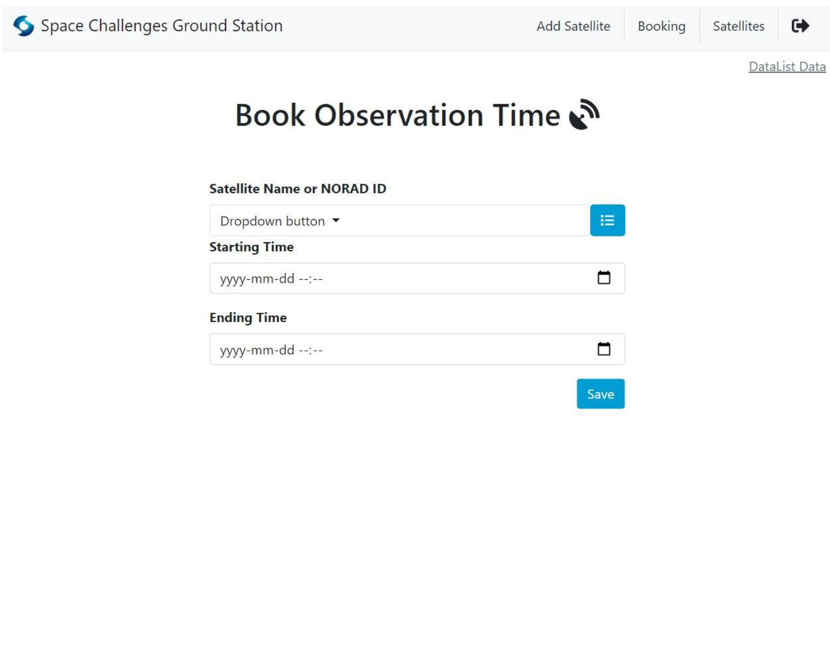

Figure 9: Web platform's booking form

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[3] https://github.com/PetarDamyanov/SpaceApp

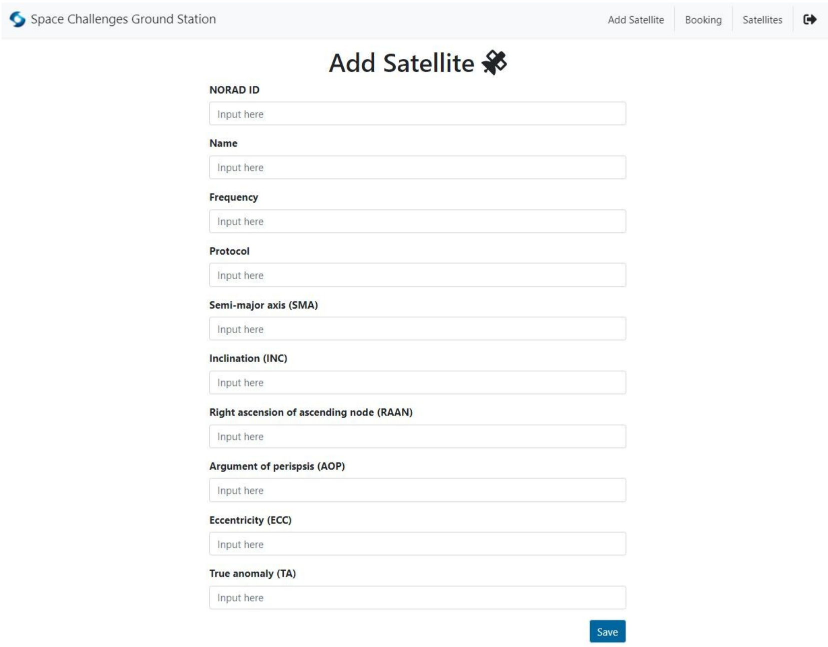

Figure 10: Web platform's satellite registration

3.5.1 Network

By observations, we discovered that a record of a waterfall for 20 seconds is 160 Mb of data. Therefore, via simple mathematical calculations we can guarantee that the system will work sufficiently with uploading speed of:

$$Internet\ speed = \frac{160\ MB * 8\ bits}{20\ seconds} = 64\ Mbps \tag{6}$$

4 Implementation

4.1 Construction of Tripod

The tripod construction was started by gathering wood planks from a saw mill. A wooden equilateral triangle (10cm side) was made and put between

Part	Quantity
Wooden Lath (64 cm)	3
Wooden Lath (11x3x2 cm)	2
Wooden Lath (40x5x2 cm)	3
Wooden Lath (4x3x2 cm)	2
Wooden Equilateral triangle (10cm)	1
Wooden Plank (17x23 cm)	1
Torx Screw 5cm	22
Wing Nut M6	4
Washers M6	4
Threaded Rod M6 (15cm)	2

the 3 laths to make the tripod more stable. After that a wooden rectangular (17x23cm) was cut and put over the tips of the 3 wooden laths.

4.2 Construction of Antenna

A 2.1m telescopic rod was used as a boom for the antenna. At the back of this antenna base we made a 36.1 cm reflector and in front of it cut holes to make a 34.4 cm long driven element. The antenna was designed to have 8 30.9 cm long directors. The spacing between the driven element and the first director is 19.2cm and the spacing between every director is 19.2 cm.

Part	Quantity	Used for
Extendable Rod (2m)	1	Boom
Steel Threaded Rod (36.1 cm)	1	Reflector
Steel Threaded Rod (34.4 cm)	1	Driven Element
Steel Threaded Rod (30.9 cm)	9	Directors
RF Coaxial Cable, SMA, 50 ohm, 15cm	1	Balun
RF Coaxial Cable, SMA, 50 ohm, 60cm	1	Balun
RF Coaxial Cable, SMA, 50 ohm, 1m	1	LNA->SDR
Nut M6	20	

4.3 Construction of Rotator

The construction of the rotator consisted of several different parts: enclosure frame, worm gear mount, worm gear, shaft collar, encoder gear, motor mount, and bearing side. Most of the parts were 3D printed based on the files provided from SatNOGS [3].
The rotator frame was supposed to be assembled with T-slot aluminium profiles, however, due to lack of availability it was constructed out of wood like the tripod. The new design altered some of the inner dimensions of the enclosure and required change in the driving mechanism. The solution with extra pulleys is depicted in Appendix B.
A 625 ball bearing was press-fit in to the deep groove of the mount. The worm gear mount was then attached to the wooden frame with 2 screws.
A M5 threaded rod waws cut into 10 cm length and used as a worn shaft for the worm gear. The worm gear and a GT2 36T pulley were then put through the shaft and secured by two M5 washers plus nuts on both sides.

4.3.1 Error compensation

To compensate for the accumulated error we add an extra micro switch that acts as a sensor in order to allow the Arduino to recalculate the rotators' gears' position.

4.4 Signal Processing

A GNU Radio flowgraph was used to process the received signal. The methodology consisted of several steps: sourcing, demodulation, and deframing.
To be able to do the following steps you will need to download Miniconda and then download gnu radio companion with gr-satellites [4]. If you did those successfully you will have GNU Radio version >=3.9.2 with gr-satellites as an additional library. gr-osmosdr is also necessary to be able to receive RF signals live.

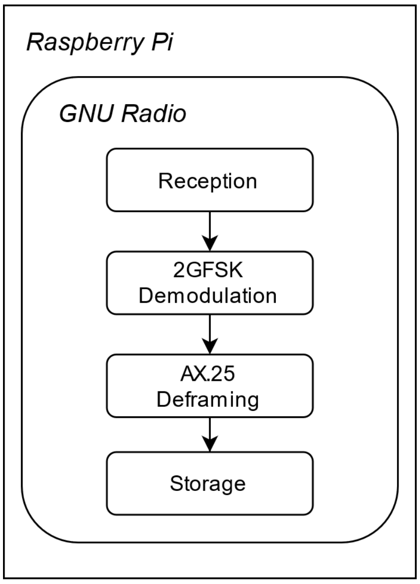

Figure 11: Signal processing block diagram

5 Evaluation

Here we describe the test cases we performed in order to verify the proper execution of our system.

Correct antenna rotation

Problem description: In this test we want to confirm that during the rotation sequence the antenna turns in the prescribed degrees and the belts do not skip steps. We are looking for a match in the assigned data (for instance 30°) and the actual angle deviation of the antenna.

Method: Through Arduino Pro Micro we command the antenna to rotate at 15°, 30°, 60° and 90°. With a stationary camera we take multiple photos of the antenna deviation. Knowing the length of the metal rod, we can calculate the angle between the antenna and the ground.

Results: The antenna rotates in the given magnitude.

Accuracy of pyorbital calculations

Problem description: The orbital calculations on our ground station are performed via the pyorbital python library. In this library the angle at which we see a specific satellite is given. However, we want to ensure that the system calculations are accurate, so that the tracking is precise and error-free.

Method: First, we look at the source from which pyorbital starts: the library generates a Two-line element (TLE) file containing the orbital parameters of the satellites and then internally computes their orbit. Then, we compare the TLE file with the database of a reliable source, such as CelesTrack. From there, we extract the position of the satellite in an Earth-centered coordinate system in x, y and z axes, then calculate our own (x, y, z) coordinates and subtract the two vectors to get the distance vector from the satellite to the observer antenna. If we then use the dot product (using dot product and inverse cosine you can get the angle between two vectors) of the distance vector and the local zenith vector of the observer (pointing directly upwards), using some trigonometry, we can find the angle between and this is our custom algorithm that finds the azimuth and elevation angles and refer to the calculated values using the pyorbital library.

Results: We noticed only negligible deviation below half of degree from the given angle values.

Working state of the antenna

Problem description: We want to confirm that the received signal is thanks to the antenna and not due to the external devices only.

Method: Using CubicSDR, we can examine the spectrum of the signal before and after the integration of the antenna. If the signal is clearer and has higher amplitude, then this is a result of the antenna.

Results: With the antenna integrated the signal is magnified.

6 Index of Abbreviations

DAC - Digital-to-Analog Conversion

DoF - Degrees of Freedom

GS - Ground Station

LEO - Low Earth Orbit

LNA - Low Noise Amplifier

SDR - Software-Defined Radio

Appendix A - Doppler compensation

To get the change of frequency we derive a formula from the equation for Doppler effect Equation 7. First we substitute the speed of the wave with the speed of light (since we're working with electromagnetic waves) and the observer speed with 0m/s to get formula Equation 8 which will give us the new observed frequency. To find the difference we subtract the frequency of the source from the observed frequency Equation 9

$$f_0 = \frac{V + V_o}{V + V_s} f_s \tag{7}$$

$$f_0 = \frac{C}{C + V_s} f_s \tag{8}$$

$$\Delta f = f_s \left(\frac{C}{C + V_{s_x}} - 1 \right) \tag{9}$$

The unknown parameter is the relative velocity of the satellite with respect to the observer. As shown in Figure 2, we use trigonometry to find it. We know our position on Earth, the pyorbital python library gives us the satellite's position, we know its azimuth and elevation with respect to our observer. Their positions and the center of the Earth form a triangle of which we know two of the sides and the angle between them. Using the cosine theorem Equation 10 we can get the third side of that triangle marked with x.

$$c^2 = a^2 + b^2 - 2ab \cos \gamma \tag{10}$$

By substituting our parameters in the formula we get Equation 11 and reformulate it as a quadratic equation respecting x Equation 12.

$$(R_e + a_s)^2 = x^2 + (R_e + a_0)^2 - 2x(R_e + a_0) \cos el + 90 \tag{11}$$

$$x^2 - 2x(R_e + a_0) \cos (el + 90) + (R_e + a_0)^2 - (R_e + a_s)^2 = 0 \tag{12}$$

Solving it as a standard quadratic equation we find x in the form of Equation 13 which is actually the distance between the sat and the observer.

$$x = (R_e + a_0) \cos (el + 90) + \sqrt{(R_e + a_0)^2 [\cos^2 (el + 90)] + (R_e + a_s)^2} \tag{13}$$

For the relative velocity we need the vector component of the absolute velocity of the satellite which is the orthogonal projection on the x line. Using similar triangles we can derive that the angle between the vector and x is equal to the elevation plus the angle equal to the angle measured to the satellite and observer from the center of Earth. We can get the measure of that angle using another cosine theorem Equation 14 and then using the *arccos* function. Finally, the magnitude of the vector component of the velocity towards us can be calculated and substituted in Equation 9.

$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab} \tag{14}$$

Since the earth is a squashed sphere, its radius may vary depending on latitude, which is calculated using formula Equation 15 and then put into the formula above.

$$R_e = \sqrt{\frac{[r_1^2 \cos lat]^2 + [r_2^2 \sin lat]^2}{[r_1 \cos lat]^2 + [r_2 \sin lat]^2}} \tag{15}$$

Alternatively, in 3D we can calculate the Doppler effect by taking the vector distance from the center of earth to the observer by knowing its geological position in longitude, latitude and elevation and the radius of Earth at that position. Since the earth is slightly squashed at the poles, the radius varies depending on latitude as described in Equation 15 . Then we calculate the distance vector from the center of earth to the satellite either by calculating its orbit via Keplerian components or in our case, using a software that gives us the exact distance vector. The two vectors are subtracted to get the distance vector from the satellite to our observer, which contains both the direction and the real distance in the magnitude of the vector. The satellite's velocity vector is then calculated. For the Doppler effect we only need the component of that vector that lies on the line connecting the observer with the source of the electromagnetic wave. Therefore, we need to project the vector on that line for which we need the cosine of the angle between the two. By taking the dot product of the distance vector and the velocity vector we get the angle and do the projection. Then, to compensate for the rotation of earth, which might be moving us towards or away from the satellite, we also project the observer's velocity on the same line from before and plug them into the Doppler effect formula. To get the observer's velocity we use the following formulas:

$$\overrightarrow{v} = \overrightarrow{\omega} r \tag{16}$$

$$r = (R_e + al)\cos(lat) \tag{17}$$

$$|\overrightarrow{v}| = \omega(R_e + al)\cos(lat) \tag{18}$$

$$\overrightarrow{v_x} = |\overrightarrow{v}| \cos{(lon)} \tag{19}$$

$$\overrightarrow{v_y} = |\overrightarrow{v}| \sin{(lon)} \tag{20}$$

$$\overrightarrow{v} = \overrightarrow{v_x} + \overrightarrow{v_y} \tag{21}$$

$$\overrightarrow{v_x} = \omega(R_e + al) \cos{(lat)} \cos{(lon)} \tag{22}$$

$$\overrightarrow{v_y} = \omega(R_e + al) \cos{(lat)} \sin{(lon)} \tag{23}$$

Appendix B - Short Timing Belt Fix

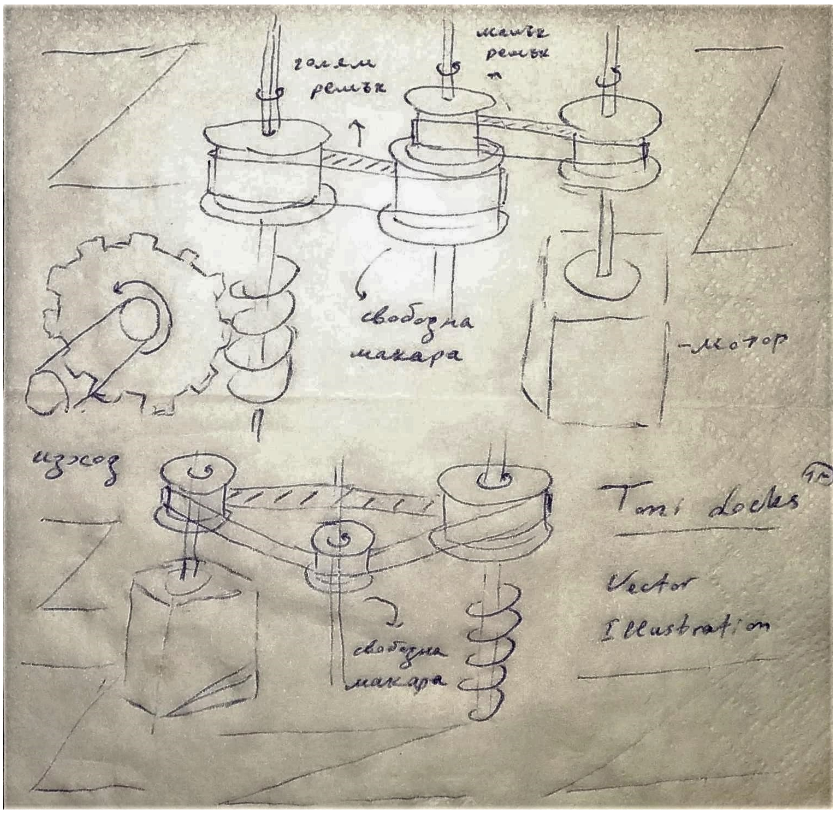

Figure 12: Timing belt fix with extra pulleys

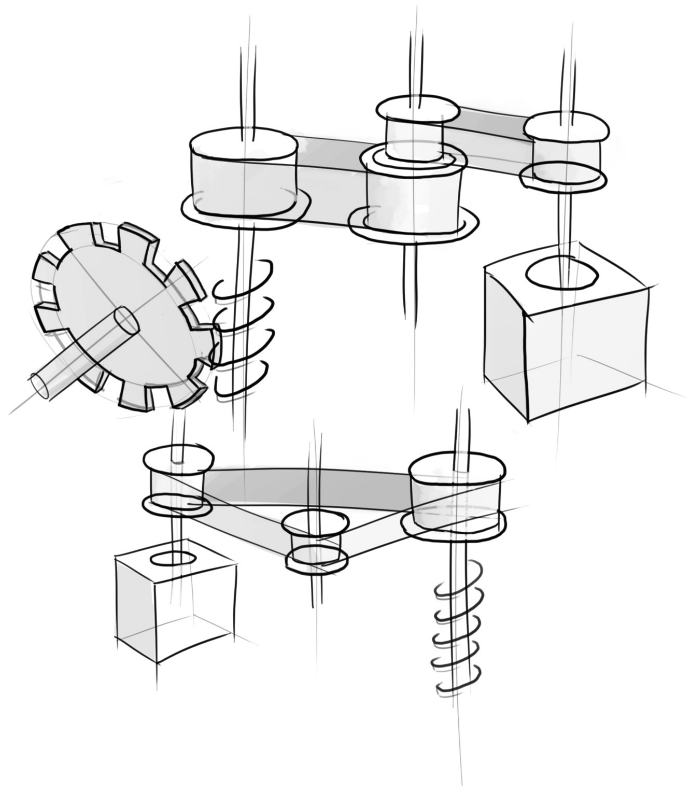

Figure 13: Timing belt fix with extra pulleys, Credits - Mariya

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