# Preadmitere Poli 2025 - Mate

## Varianta A

1) $\sqrt{6x-8} = x$

C.E $\left.\begin{array}{l} 6x-8 \geq 0 \Rightarrow 6x \geq 8 \Rightarrow x \geq \frac{4}{3} \\ x \geq 0 \end{array}\right| \Rightarrow$

$\Rightarrow x \in \left[\frac{4}{3}, \infty\right)$

$\Rightarrow 6x-8 = x^2 \Rightarrow x^2-6x+8=0 \Rightarrow x^2-4x-2x+8=0$

$\Rightarrow x(x-4)-2(x-4)=0 \Rightarrow (x-2)(x-4)=0 \Rightarrow$

$\Rightarrow x \in \{2, 4\}$ (Respectă C.E)

$\Rightarrow R$: d) $\{2, 4\}$

2) $3x+1 > 2x+2 \Leftrightarrow 3x-2x > 2-1 \Leftrightarrow$

$\Leftrightarrow x > 1 \Rightarrow R$: b) $x \in (1, \infty)$

3) $\log_a b = \frac{4}{3} \Rightarrow b = a^{\frac{4}{3}} = \sqrt[3]{a^4}$

$\log_c d = \frac{5}{6} \Rightarrow d = c^{\frac{5}{6}} = \sqrt[6]{c^5}$

$a, b, c, d \in \mathbb{N}^* \Rightarrow \sqrt[3]{a^4}, \sqrt[6]{c^5} \in \mathbb{N}^* \Rightarrow$

$\Rightarrow$ a și c trebuie să se poată scrie de forma $k^3$, respectiv $p^6$, unde $k, p \in \mathbb{N}^*$.

Observăm că $c-a=37$, iar c și a fiind numere naturale $\Rightarrow c > a$.

Folosind aceste observații deducem că pentru $k=3$ și $p=2 \Rightarrow a = 3^3 = 27$ și $c = 2^6 = 64$, numere ce verifică relația $c-a=37 \Rightarrow$

$$\left.\begin{array}{l} \Rightarrow b = \sqrt[3]{(3^3)^4} = 3^4 = 81 \\ c = \sqrt[6]{(2^6)^5} = 2^5 = 32 \end{array}\right| \Rightarrow b-a = 81-32 = 49$$

$\Rightarrow R$: a) 49

4) $\det(A) = \begin{vmatrix} m & -1 \\ 2 & m+2 \end{vmatrix} = m(m+2)+2 =$

$$\left.\begin{array}{l} = m^2+2m+2 \\ \det(A) = 1 \end{array}\right| \Rightarrow m^2+2m+2 = 1 \Rightarrow$$

$\Rightarrow m^2+2m+1=0 \Rightarrow (m+1)^2 = 0 \Rightarrow$

$\Rightarrow m = -1$

$\Rightarrow m^2+2 = (-1)^2+2 = 1+2 = 3 \Rightarrow R$: c) 3

5) $3^{x^2+4x+6} = 27 \Rightarrow 3^{x^2+4x+6} = 3^3 \Rightarrow$

$\Rightarrow x^2+4x+6 = 3 \Rightarrow x^2+4x+3 = 0 \Rightarrow$

$\Rightarrow x^2+3x+x+3=0 \Rightarrow x(x+3)+x+3=0 \Rightarrow$

$$\Rightarrow (x+3)(x+1) = 0 \Rightarrow \left.\begin{array}{l} x_1 = -3 \\ x_2 = -1 \end{array}\right| \Rightarrow$$

$\Rightarrow |x_1| + |x_2| = |-3| + |-1| = 3+1 = 4 \Rightarrow$

$\Rightarrow R: 8) 4$

6) Fie $M(x_M, y_M)$ punctul comun celor două grafice $\Rightarrow$ $\left.\begin{array}{l} M \in G_f \\ M \in G_g \end{array}\right| \Rightarrow \begin{array}{l} y_M = \operatorname{arctg}\sqrt{x_M} \\ y_M = \frac{1}{4}(\pi + \ln x_M) \end{array}$

$\Rightarrow \operatorname{arctg}\sqrt{x_M} = \frac{1}{4}(\pi + \ln x_M)$ (*)

$f'(x) = \frac{1}{1+(\sqrt{x})^2} \cdot (\sqrt{x})' = \frac{1}{x+1} \cdot \frac{1}{2\sqrt{x}} =$
$= \frac{1}{2\sqrt{x}(x+1)}$

$g'(x) = \frac{1}{4} \cdot \frac{1}{x} = \frac{1}{4x}$

Ecuația tangentei la graficul funcției:

$y - y_0 = \underline{f'(x_0)}(x - x_0)$

tangenta comună

$\left.\begin{array}{l} M \in G_f \\ M \in G_g \end{array}\right| \Rightarrow f'(x_M) = g'(x_M) \Rightarrow$

$\Rightarrow \frac{1}{2\sqrt{x_M}(x_M+1)} = \frac{1}{4x_M} \Rightarrow$

$\Rightarrow 2\sqrt{x_M}(x_M+1) = 4x_M \Rightarrow$

$\Rightarrow 2(x_M\sqrt{x_M} + \sqrt{x_M} - 2x_M) = 0$

$\Rightarrow \sqrt{x_M}\left(x_M + 1 - 2\frac{x_M}{\sqrt{x_M}}\right) = 0 \Rightarrow$

$\Rightarrow \sqrt{x_M}\left(x_M + 1 - 2 \cdot \frac{x_M \cancel{\sqrt{x_M}}}{\cancel{\sqrt{x_M}}}\right) = 0 \Rightarrow$

$$\Rightarrow \sqrt{x_M}\,(x_M + 1 - 2x_M) = 0 \Rightarrow$$

$$\Rightarrow \sqrt{x_M}\,(1 - x_M) = 0 \Rightarrow$$

I $\sqrt{x_M} = 0 \Rightarrow x_M = 0 \overset{(*)}{\Longrightarrow}$

$$\Rightarrow \operatorname{arctg}\sqrt{0} = \frac{1}{4}(\pi + \underbrace{\ln 0}_{\nexists})$$

II $1 - x_M = 0 \Rightarrow x_M = 1 \overset{(*)}{\Longrightarrow}$

$$\Rightarrow \operatorname{arctg}\sqrt{1} = \frac{1}{4}(\pi + \underbrace{\ln 1}_{=0}) \Rightarrow$$

$$\Rightarrow \frac{\pi}{4} = \frac{\pi}{4} \quad ✓$$

Deci $x_M = 1$ și $y_M = \frac{\pi}{4} \Rightarrow$

$$\Rightarrow f'(x_M) = f'(1) = \frac{1}{2\cdot\sqrt{1}\cdot(1+1)} = \frac{1}{4}$$

Scriem ecuația tangentei pentru punctul M:

$$d: y - y_M = f'(x_M)(x - x_M) \Rightarrow$$

$$\Rightarrow d: y - \frac{\pi}{4} = \frac{1}{4}(x - 1) \Rightarrow$$

$$\Rightarrow d: y = \frac{x - 1 + \pi}{4}$$

$P(\alpha, 0) \in d$

$$\Rightarrow \frac{\alpha - 1 + \pi}{4} = 0$$

$$\Rightarrow \alpha = 1 - \pi \Rightarrow$$

$$\Rightarrow R: c)\ 1 - \pi$$

7)

$$\det(A)=\begin{vmatrix} 2m & 1 & m+1 \\ m+2 & m+1 & m+2 \\ 3m & 1 & 2m+1 \end{vmatrix} \overset{L_3=L_3-L_1}{=\!=} \begin{vmatrix} 2m & 1 & m+1 \\ m+2 & m+1 & m+2 \\ m & 0 & m \end{vmatrix} =$$

$$\overset{C_1=C_1-C_3}{=\!=} \begin{vmatrix} 2m & 1 & m+1 \\ m+2 & m+1 & m+2 \\ m & 0 & m \end{vmatrix} = \begin{vmatrix} m-1 & 1 & m+1 \\ 0 & m+1 & m+2 \\ 0 & 0 & m \end{vmatrix} =$$

$$= (m-1)\left[m(m+1) - 0\cdot(m+2)\right] = m(m-1)(m+1)$$

Sistem incompatibil $\Leftrightarrow \begin{cases} \det(A)=0 & (1) \\ \Delta_{car} \neq 0 & (2) \end{cases}$

$(1) \Rightarrow m \in \{-1, 0, 1\}$

Verificăm fiecare caz pentru care se verifică (2):

I $m=-1 \Rightarrow A = \begin{pmatrix} -2 & 1 & 0 \\ 1 & 0 & 1 \\ -3 & 1 & -1 \end{pmatrix}$

$\Delta_p = \begin{vmatrix} -2 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0 \Rightarrow \Delta_{car} = \begin{vmatrix} -2 & 1 & -1 \\ 1 & 0 & 2 \\ -3 & 1 & 1 \end{vmatrix} =$

$= 0 - 6 - 1 - 0 + 4 - 1 = -2 \neq 0 \overset{(2)}{\Rightarrow}$ Sistem incomp. ✓

II $m=0 \Rightarrow A = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix} \Rightarrow \Delta_p = \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -2 \Rightarrow$

$\Rightarrow \Delta_p \neq 0$

$$\Delta_{car} = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 0+0+2-0-0-2 = 0 \Rightarrow$$

$\xRightarrow{(2)}$ Sistem Comp. Nedeterminat

III $m = 1 \Rightarrow A = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & 3 \\ 3 & 3 & 3 \end{pmatrix} \Rightarrow \Delta_p = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} =$

$= 4 - 3 = 1 \neq 0$

$$\Delta_{car} = \begin{vmatrix} 2 & 1 & 3 \\ 3 & 2 & 2 \\ 3 & 3 & 1 \end{vmatrix} = 4+6+27-18-12-3 =$$

$= 5 \neq 0 \xRightarrow{(2)}$ Sistem incompatibil

$\Rightarrow R$: d) $A = \{-1, 1\}$

8) Din Relațiile lui Viète $\Rightarrow$:

$S = x_1 + x_2 = -\frac{b}{a} = -\frac{3}{1} = -3$

$P = x_1 \cdot x_2 = \frac{c}{a} = \frac{1}{1} = 1$

$$\left(\frac{x_1}{x_2+1}\right)^2 + \left(\frac{x_2}{x_1+1}\right)^2 = \frac{x_1^2(x_1+1)^2 + x_2^2(x_2+1)^2}{(x_1+1)(x_2+1)} =$$

$$= \frac{x_1^2(x_1^2+2x_1+1) + x_2^2(x_2^2+2x_2+1)}{x_1x_2+x_1+x_2+1} =$$

! $x_k^2 + 3x_k + 1 = 0 \Rightarrow x_k^2 + 2x_k + 1 = -x_k$, $k \in \{1, 2\}$

$$= \frac{x_1^2 \cdot (-x_1) + x_2^2(-x_2)}{x_1x_2 + x_1 + x_2 + 1} = -\frac{x_1^3 + x_2^3}{x_1x_2+x_1+x_2+1} =$$

! $x_k^2 + 3x_k + 1 = 0 \Rightarrow x_k^3 + 3x_k^2 + x_k = 0 \Rightarrow$

$\Rightarrow x_k^3 = -3x_k^2 - x_k$, $\forall k \in \{1, 2\} \Rightarrow$

$\Rightarrow x_1^3 + x_2^3 = -3x_1^2 - x_1 - 3x_2^2 - x_2 =$

$= -3(x_1^2 + x_2^2) - (x_1 + x_2)$ !

$$= \frac{-3(x_1^2 + x_2^2) - (x_1 + x_2)}{x_1 x_2 + x_1 + x_2 + 1} =$$

Din Viéte $\frac{-3(S^2 - 2P) - S}{P + S + 1} = \frac{-3(9-2)+3}{1-3+1} =$

$$= \frac{-21+3}{-1} = 18 \Rightarrow R: e)\ 18$$

g) $x^2 + 2\int_0^x t \cdot f(t)\,dt + 2 = (x^2+1) \cdot f(x) + \ln 2 \Rightarrow$

f derivabilă (Derivăm) $2x + 2(x f(x) - 0 \cdot f(x)) = 2x f(x) + (x^2+1) \cdot f'(x) \Rightarrow$

$\Rightarrow 2x + \cancel{2x f(x)} = \cancel{2x f(x)} + (x^2+1) f'(x) \Rightarrow$

$\Rightarrow (x^2+1) f'(x) - 2x = 0 \Rightarrow f'(x) = \frac{2x}{x^2+1}$ (*)

$$\int_0^1 t \cdot f(t)\,dt = \int_0^1 \left(\frac{t^2}{2}\right)' \cdot f(t)\,dt = \frac{t^2}{2} f(t) \Big|_0^1$$

$$- \frac{1}{2}\int_0^1 t^2 f'(t)\,dt \overset{(*)}{=} \frac{1}{2} f(1) - \frac{1}{2}\int_0^1 t^2 \cdot \frac{2t}{t^2+1}\,dt =$$

$u = t^2 + 1 \Rightarrow du = 2t\,dt$

$$= \frac{1}{2} f(1) - \frac{1}{2}\int_1^2 \frac{u-1}{u}\,du = \frac{1}{2} f(1)$$

$$- \frac{1}{2}\int_1^2 \left(1 - \frac{1}{u}\right) du = \frac{1}{2} f(1) - \frac{1}{2}(u - \ln u)\Big|_1^2 =$$

$$= \frac{1}{2} f(1) - \frac{1}{2}(2 - \ln 2 - 1 - \ln 1) =$$

$$= \frac{1}{2} f(1) - \frac{1}{2}(1 - \ln 2) = \frac{1}{2}(f(1) - 1 + \ln 2) \quad (1)$$

Revenim în ecuația originală și înlocuim cu $x=1$

$$\Rightarrow 1+2\int_0^1 t f(t)\,dt + 2 = 2f(1) + \ln 2 \overset{(1)}{\Rightarrow}$$

$$\Rightarrow 1+\cancel{2}\cdot\frac{1}{\cancel{2}}\left(f(1)-1+\ln 2\right)+2 = 2f(1)+\ln 2 \Rightarrow$$

$$\Rightarrow \cancel{1}+f(1)\cancel{-1}+\cancel{\ln 2}+2 = 2f(1)+\cancel{\ln 2} \Rightarrow$$

$$\Rightarrow f(1)=2$$

$$\int_0^1 f(x)\,dx = \int_0^1 x' f(x)\,dx = x f(x)\Big|_0^1 - \int_0^1 x f'(x)\,dx =$$

$$\overset{(*)}{=} f(1) - 0\cdot f(0) - \int_0^1 x\cdot\frac{2x}{x^2+1}\,dx =$$

$$= 2 - 2\int_0^1 \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\,dx = 2-2\int_0^1\left(1-\frac{1}{x^2+1}\right)dx =$$

$$= 2-2\left(x-\operatorname{arctg} x\right)\Big|_0^1 = 2-2(1-\operatorname{arctg}1 - 0 + \underbrace{\operatorname{arctg}0}_{=0}) =$$

$$= 2-2\left(1-\frac{\pi}{4}\right) = 2-2+2\cdot\frac{\pi}{4} = \frac{\pi}{2} \Rightarrow$$

$$\Rightarrow R:\ a)\ \frac{\pi}{2}$$

10) $S_5 = 5a_1 + r\cdot\frac{4\cdot 5}{2} = 5a_1 + 10r$

$S_{10} = 10a_1 + r\cdot\frac{9\cdot 10}{2} = 10a_1 + 45r$

$$\Rightarrow \begin{cases} 5a_1+10r=40 \\ 10a_1+45r=155 \end{cases} \Leftrightarrow \begin{cases} a_1+2r=8 \\ 2a_1+9r=31 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 2a_1+4r=16 \\ 2a_1+9r=31 \end{cases}$$

$$/\quad 5r=15$$

$$r=3 \Rightarrow a_1+2\cdot 3=8 \Rightarrow a_1=2$$

$$S_{15} = 15a_1 + \frac{14 \cdot 15}{2} r = 15a_1 + 105r =$$

$$= 15 \cdot 2 + 105 \cdot 3 = 30 + 315 = 345 \Rightarrow R: \ c)\ 345$$