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Longitudinal Dynamic Stability

Airplane E (case 1- power approach)

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MS3 - IPSA

Longitudinal Dynamic Stability

of

Airplane E (case 1- power approach)

The objective of this homework is to study the stability modes of an aircraft which we knew his geometric, inertial and aerodynamic data.

For attempting this objective, you will find by using Matlab:

1. Equations of longitudinal motion.
2. The matrix A of aircraft
3. The characteristic equation
4. The eigenvalues (roots of equation) of the system
5. Different modes of longitudinal stability
   a. Short period mode
   b. Phegoid mode or
6. Different modes of longitudinal stability
   a. Short period mode (Natural Frequency, Damping Factor)
   b. Phegoid mode (Natural Frequency, Damping Factor)
7. Curves of longitudinal motion:
   a. Axial velocity in function of time
   b. Angle of attack
   c. Pitch rate
   d. Pitch angle
8. Transfer Functions of Each variable

Data for airplane E

Figure 5 presents a three-view for Airplane E. This airplane is representative of supersonic fighter bomber airplane. Stability and control-derivatives for this airplane are provide in Table C5

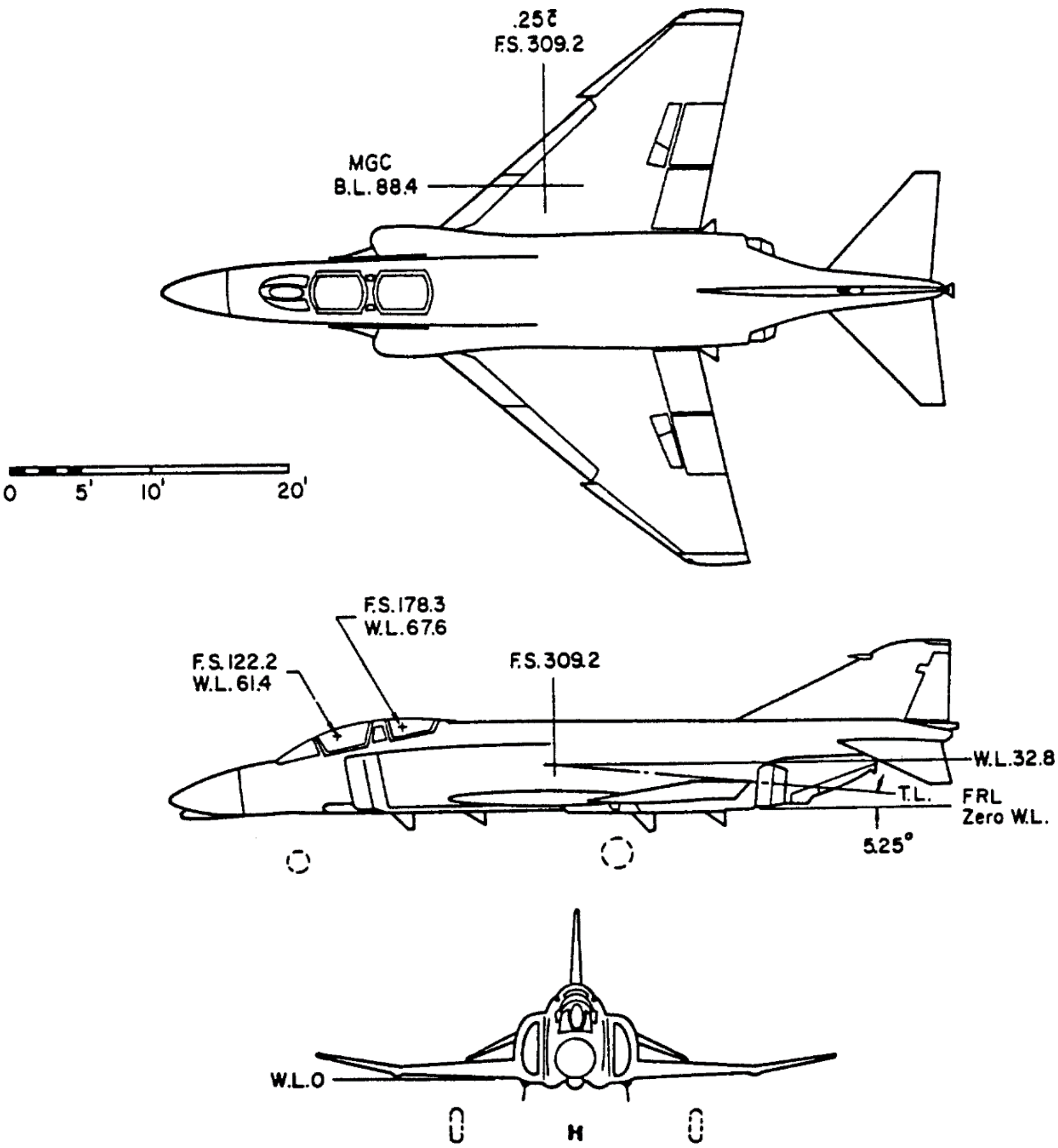


Figure C5 Three-View of Airplane E

Table C5 Stability and Control Derivatives for Airplane E

Flight Condition	1
	Power Approach
Altitude (ft)	Sealevel
Air Density (slugs/ft^3)	.002378
Speed (fps)	230
Center of Gravity ($\bar{x}_{cg}$)	.29
Initial Attitude (deg)	11.7
Geometry and Inertias	
Wing Area (ft^2)	530
Wing Span (ft)	38.7
Wing Mean Geometric Chord (ft)	16.0
Weight (lbs)	33,200
I_{xx_B} (slug ft^2)	23,700
I_{yy_B} (slug ft^2)	117,500
I_{zz_B} (slug ft^2)	133,700
I_{xz_B} (slug ft^2)	1,600
Steady State Coefficients	
C_{L_1}	1.0
C_{D_1}	.2
$C_{T_{X_1}}$	.2
C_{m_1}	0
$C_{m_{T_1}}$	0

Table C5 Stability and Control Derivatives for Airplane E (Cont.)

Longitudinal Derivatives	1
C_{m_u}	0
C_{m_α}	-.098
$C_{m_{\dot\alpha}}$	-.95
C_{m_q}	-2.0
$C_{m_{T_u}}$	0
$C_{m_{T_\alpha}}$	0
C_{L_u}	0
C_{L_α}	2.8
$C_{L_{\dot\alpha}}$	0
C_{L_q}	0
C_{D_α}	.555
C_{D_u}	0
$C_{T_{X_u}}$	0
$C_{L_{i_H}}$	.24
$C_{D_{i_H}}$	-.14
$C_{m_{i_H}}$	-.322

Parameter	*Symbol*	*Imperial unit*	*Equivalent SI unit*
Mass	m	1 slug	14.594 kg
Length	l	1 ft	0.3048 m
Velocity	V	1 ft/s	0.3048 m/s
Acceleration	a	1 ft/s^2	0.3048 m/s^2
Force	F	1 lb	4.448 N
Moment	M	1 lb ft	1.356 N m
Density	ρ	1 slug/ft^3	515.383 kg/m^3
Inertia	I	1 slug ft^2	1.3558 kg m^2

1) Equations of longitudinal motion.

The longitudinal equation is:

$$\begin{cases} \left[\frac{\mathrm{d}}{\mathrm{d}t} - X_u\right] u + g_0 \cos\Theta_0 \theta - X_w w = X_{\delta_e}\delta_e + X_{\delta_T}\delta_T \\ -Z_u u + \left[(1 - Z_{\dot{w}})\frac{\mathrm{d}}{\mathrm{d}t} - Z_w\right] w - [u_0 + Z_q] q + g_0 \sin\Theta_0 \theta = Z_{\delta_e}\delta_e + Z_{\delta_T}\delta_T \\ -M_u u - \left[M_{\dot{w}}\frac{\mathrm{d}}{\mathrm{d}t} + M_w\right] w + \left[\frac{\mathrm{d}}{\mathrm{d}t} - M_q\right] q = M_{\delta_e}\delta_e + M_{\delta_T}\delta_T \end{cases}$$

Represented using A and B by $\mathrm{X} = \mathrm{Ax} + \mathrm{B}$

2) The matrix A of aircraft

Using MATLAB, we have:

$$\mathrm{A} = \begin{bmatrix} -0.0562 & 0.0350 & 0 & -6.3529 \\ -0.2873 & -0.4215 & 70.1040 & 7.4751 \\ 0.0006 & -0.0054 & -0.4658 & -0.0160 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

We also have B that is needed for the longitudinal motion equation:

$$\mathrm{B} = \begin{bmatrix} -0.0197 & 0 \\ -0.0034 & 0 \\ -0.0208 & 0 \\ 0 & 0 \end{bmatrix}$$

3) The characteristic equation

We know that:

$$\det(\mathrm{A} - \lambda \mathrm{I}) = 0$$

So, thanks to MATLAB command we have the characteristic equation of the matrix A:

$$|(\mathrm{A} - \lambda\mathrm{I})| = \lambda^4 + 0.9435 \times \lambda^3 + 0.6539 \times \lambda^2 + 0.0879 \times \lambda + 0.01425 = 0$$

4) The eigenvalues (roots of equation) of the system

Thanks to the command eig() on MATLAB we get the eigenvalues of the matrix A:

$$\lambda_1 = -0.40913 + \imath 0.59735$$

$$\lambda_2 = -0.40913 - \imath 0.59735$$

$$\lambda_3 = -0.06263 + \imath 0.15249$$

$$\lambda_4 = -0.06263 - \imath 0.15249$$

We have eigen values for both modes that are phugoid and short-period.

5) Different modes of longitudinal stability

We find these modes for each eigen value given the size:

$$\lambda_1 = -0.40913 + i0.59735 \text{ (Phugoid mode)}$$

$$\lambda_2 = -0.40913 - i0.59735 \text{ (Phugoid mode)}$$

$$\lambda_3 = -0.06263 + i0.15249 \text{(Short} - \text{period mode)}$$

$$\lambda_4 = -0.06263 - i0.15249 \text{ (Short} - \text{period mode)}$$

6) Different modes of longitudinal stability (Natural Frequency, Damping Factor)

a. Short period mode

The damping ratio of the short period mode is thus given by:

$$\zeta = \sqrt{\frac{1}{1 + \left(\frac{\mathrm{Im}(\lambda_{3,4})}{\mathrm{Re}(\lambda_{3,4})}\right)}}$$

$$\zeta = 0.56507$$

The undamped natural frequency of this mode is:

$$\omega_{\mathrm{n}} = \frac{|\mathrm{Re}(\lambda_{3,4})|}{\zeta}$$

$$\omega_{\mathrm{n}} = 0.72403 \text{ rad/s}$$

b. Phugoid mode

The damping ratio of the Phugoid mode is thus given by:

$$\zeta = \sqrt{\frac{1}{1 + \left(\frac{\mathrm{Im}(\lambda_{1,2})}{\mathrm{Re}(\lambda_{1,2})}\right)}}$$

$$\zeta = 0.3799$$

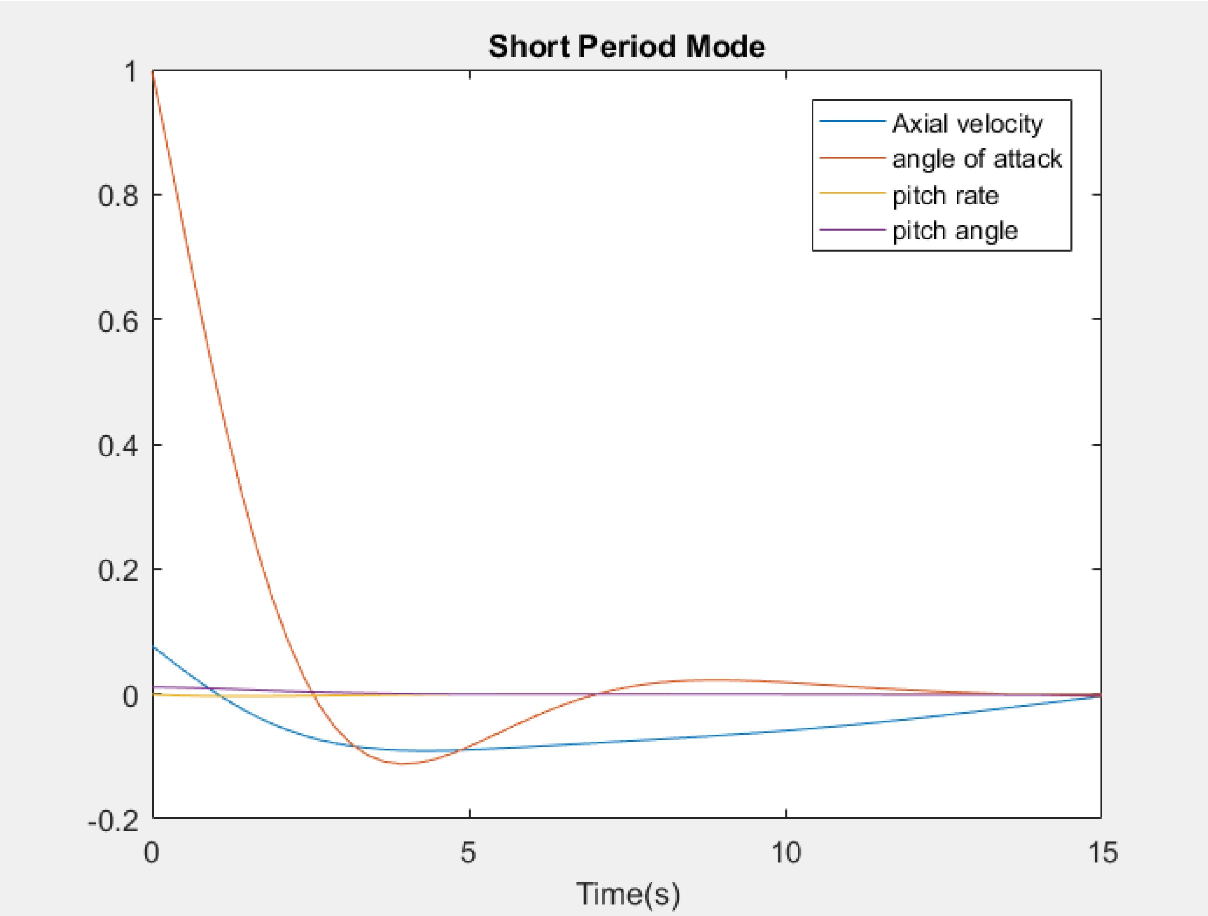
The undamped natural frequency of this mode is:

$$\omega_{\mathrm{n}} = \frac{|\mathrm{Re}(\lambda_{1,2})|}{\zeta}$$

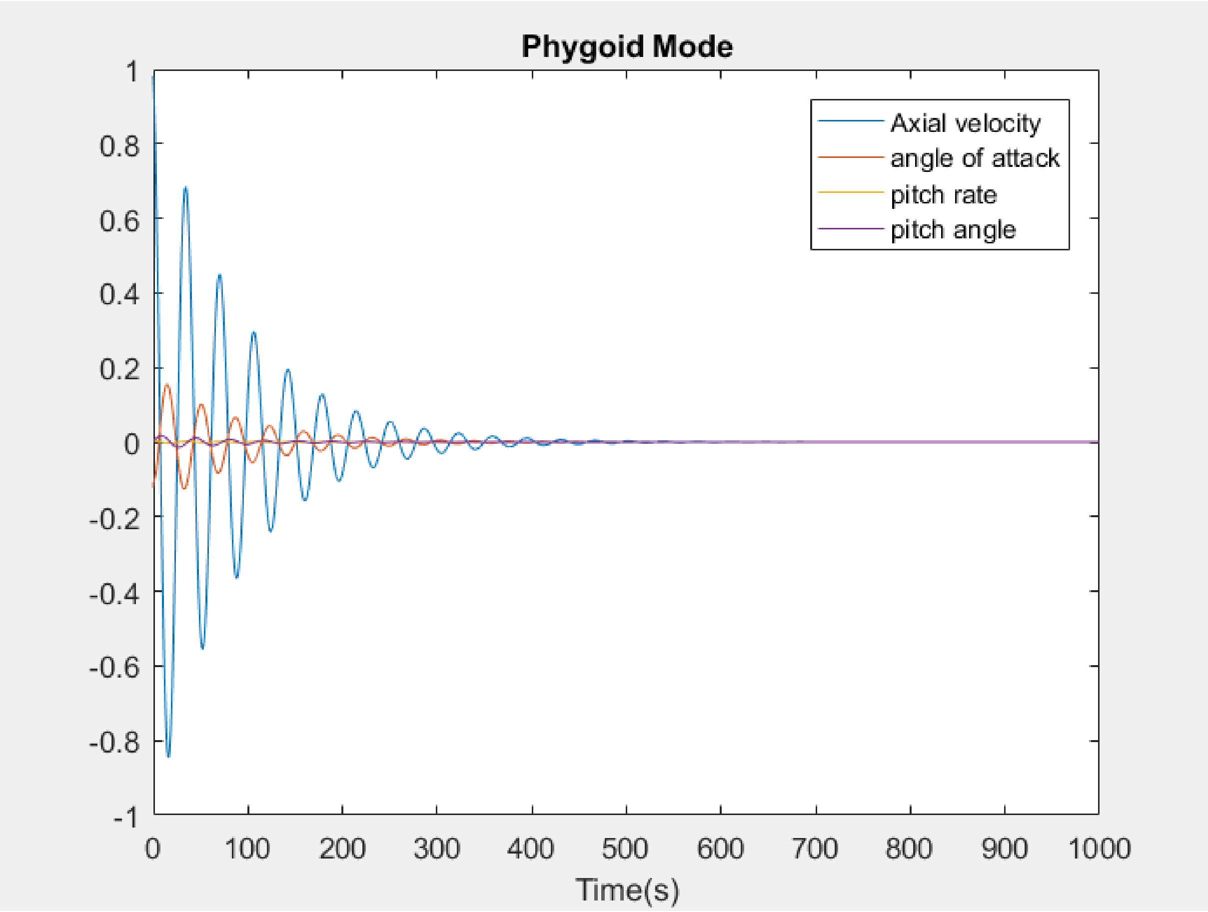
$$\omega_{\mathrm{n}} = \text{-}0.16485 \text{ rad/s}$$

7) Curves of longitudinal motion
   a. Axial velocity in function of time
   b. Angle of attack
   c. Pitch rate
   d. Pitch angle

For the Short-period mode we have:



For the Phugoid mode we have:



8) Transfer Functions of Each variable and inverse Laplace transform

a. Axial velocity

Using MATLAB, we find the continuous time transfer Function of the Axial velocity:

$$\mathrm{TF_{av}} = \frac{-0.01967 \times \mathrm{S} + 0.0004719}{\mathrm{S}^2 + 0.0562 \times \mathrm{S} - 0.0402}$$

The inverse Laplace transform is found using ilaplace function:

$$-0.0096 \times e^{0.17t} - 0.01 \times e^{-0.23t}$$

b. Pitch angle

Using MATLAB, we find the continuous time transfer Function of the pitch angle:

$$\mathrm{TF_{pa}} = \frac{-0.000126}{\mathrm{S}^2 + 0.0562 \times \mathrm{S} - 0.0402}$$

The inverse Laplace transform is found using ilaplace function:

$$0.00032i \times -e^{(-0.028-0.2i)t} + 0.00032i \times e^{(-0.028+0.2i)t}$$

c. Angle of attack

Using MATLAB, we find the continuous time transfer Function of the Angle of attack:

$$\mathrm{TF_{aoa}} = \frac{-0.0000481 \times \mathrm{S} - 0.02087}{\mathrm{S}^2 + 0.3239 \times \mathrm{S} + 0.008244}$$

The inverse Laplace transform is found using ilaplace function:

$$0.078 \times e^{-0.3t} - 0.078 \times e^{-0.028t}$$

d. Pitch rate

Using MATLAB, we find the continuous time transfer Function of the Pitch rate:

$$\mathrm{TF_{pr}} = \frac{-0.02085 \times \mathrm{S} - 0.0001251}{\mathrm{S}^2 + 0.3239 \times \mathrm{S} + 0.008244}$$

The inverse Laplace transform is found using ilaplace function:

$$0.0017 \times e^{-0.028t} - 0.023 \times e^{-0.3t}$$

ANNEXE

MATLAB in 2 parts

Part 1:

```
%longitudinal Dynamic Stability  of Airplane E (case 1-
power approach)
clear clc;
close all;
clear all;
%Data

    %Flight Condition
alt = 0;%feet
a_dens = 0.002378; %slug/feet3
speed = 230;%fps
gravity_center = 0.29; % x_cg
attitude_ini = 11.7;% in rad
g_0 = 9.81;% m/s2

    %Geometry and Inertias
wing_area = 530; %feet2
wing_span = 38.7;%feet
wing_mean_chord = 16;% feet
weight = 33200;%lbs
i_xx = 23700;%slug feet2
i_yy = 117500;%slug feet2
i_zz = 133700;%slug feet2
i_xz = 1600; %slug feet2
    %Steady State Coefficients
c_L = 1;
c_D = 0.2;
c_T_X = 0.2;
c_m_ = 0;
c_m_T = 0;
    %longitudinal-Directional Derivatives
%Longitudinal Derivatives
Cmu = 0;
Cmalpha = -0.098;
Cmalphapoint = -0.95;
Cmq = -2;
CmTu = 0;
CmTalpha = 0;
```

```
CLu = 0;
CLalpha = 2.8;
Clalphapoint = 0;
CLq = 0;
CDalpha = 0.555;
CDu = 0;
CTXu = 0;
CLiH = -.024;
CDiH = -0.14;
CmiH = -0.322;

CLdeltaT = 0 ;
CDdeltaT = 0 ;
CMdeltaT = 0 ;

ft = 0.3048;
lb = 4.448; %kg*gravity
slug = 14.62;
ft2 = 0.0929;
slugft2 = 1.3558;
slugft3 = 515.383;

%Data conversion
    %Flight Condition
alt = alt*ft;%m
a_dens = a_dens*slugft3; %kg/m3
speed = speed*ft;%m/s
M = speed/340.3;

    %Geometry and Inertias
wing_area = wing_area*(ft2); %m2
wing_span = wing_span*ft; %m
wing_mean_chord = wing_mean_chord*ft; %m
weight = weight*lb/g_0; %kg
i_xx = i_xx*slugft2; %kg m2
i_yy = i_yy*slugft2; %kg m2
i_zz = i_zz*slugft2; %kg m2
i_xz = i_xz*slugft2; %kg m2
A=wing_span^2/wing_area;

%1. Equation of longitudinal motion.
        %X=A*x+B*eta;
disp('1.    Equations of longitudinal motion.');
disp('X=A*x+B*eta');
% Calculation of A Matrix composant

 %flight condition
```

```
q=(1/2)*a_dens*speed^2;
coeff=(q*wing_area)/(weight*speed);
coeff2=(q*wing_area*wing_mean_chord)/(speed);

X_u=-coeff*(2*c_D+M*CDu);
X_w=coeff*(c_L-((2*c_L*CLalpha)/(pi*0.84*A)));
Z_u=-coeff*(2*c_L+(M^2*c_L)/(1-M^2));
Z_w=-coeff*(c_D+CLalpha);
Z_w_point =
((q*wing_area*wing_mean_chord)/(2*weight*(speed^2)))*(c_
D+Clalphapoint);
Z_q=coeff2*(CLq)/(2*weight);
M_u=coeff2*M*Cmu/i_yy;
M_w=coeff2*Cmalpha/i_yy;
M_w_point=coeff2*(wing_mean_chord*Cmalphapoint)/(2*i_yy*
speed);
M_q=coeff2*(wing_mean_chord*Cmq)/(2*i_yy);


%2. The matrix A of aircraft

A=[X_u,X_w,0,-g_0*cos(attitude_ini);
   Z_u,Z_w,speed,-g_0*sin(attitude_ini);

M_u+(M_w_point*Z_u),M_w+(M_w_point*Z_w),M_q+(speed*M_w_p
oint),-M_w_point*g_0*sin(attitude_ini);
   0,0,1,0];
disp('A = ')
disp(A)

% Calculation of B Matrix composant

Xdeltae=coeff*CDiH;
Zdeltae=coeff*CLiH;
Mdeltae=coeff2*CmiH/i_yy;
XdeltaT=coeff*CDdeltaT;
ZdeltaT=coeff*CLdeltaT;
MdeltaT=coeff2*CMdeltaT/i_yy;

%2. The matrix B of aircraft

B=[Xdeltae,XdeltaT;
  Zdeltae,ZdeltaT;
  Mdeltae+Zdeltae*M_w_point,MdeltaT+ZdeltaT*M_w_point;
  0,0 ];
disp('B = ')
disp(B)
```

```
%eta
eta(1,:)=B(:,1).';
eta(2,:)=B(:,2).';
disp('eta = ')
disp(eta)

%characteristic polynom
coeffPoly = poly(A);
syms x ;
polynome = vpa(simplify(det(A-x*eye(4))),4);

%eigen values
[k,l]=eig(A);

eigenvect1=l(1,1);
eigenvect2=l(2,2);
eigenvect3=l(3,3);
eigenvect4=l(4,4);

%----------Short-period mode-------------
disp('short-period mode');
zeta_sp=sqrt(1/(1+((imag(eigenvect1)/real(eigenvect1))^2
)));
disp(zeta_sp);
omega_sp = -(real(eigenvect1))/zeta_sp;
disp(omega_sp);

%----------Phugoid mode------------
zeta_ph =
sqrt(1/(1+((imag(eigenvect3)/real(eigenvect3))^2)));
omega_ph = (real(eigenvect3))/zeta_ph;
disp('rad/s');

%---------Curves of longitudinal motion------
T1=(0:1:500);
T2=(0:0.1:10);

figure (1)

u=k(:,2);
[t,x] = ode45('DATA', [0 15], u);
plot(t,x(:,1),t,x(:,2),t,x(:,3),t,x(:,4))

legend('Axial velocity ','angle of attack','pitch rate
','pitch angle ')
xlabel('Time(s)')
title('Short Period Mode')
```

```
figure (2)

u=k(:,3);
[t,x] = ode45('DATA', [0 1000], u);
plot(t,x(:,1),t,x(:,2),t,x(:,3),t,x(:,4))


legend('Axial velocity ','angle of attack ','pitch rate
','pitch angle ')
xlabel('Time(s)')
title('Phygoid Mode')

%------------------Transfer function--------------------
syms s ;
format short e

%-----------------Axial velocity-----------------------
num1=vpa([Xdeltae g_0; -Zdeltae/speed s],3);
deno1=vpa([s-X_u g_0; (-Z_u/speed) s],3);
detnum1=det(num1);
detdeno1=det(deno1);

TF_U=vpa(detnum1/detdeno1,3);

tfnum1=[- 0.019671637229293992277234792709351
0.00047189993940525493349228708743358];
tfdeno1=[1 0.056204677798632474150508642196655 -
0.040196427499300170986772926174551];

tf_U=tf(tfnum1,tfdeno1)

vpa(ilaplace(TF_U),2)

[z,gain]=zero(tf_U)


%-----------------Pitch angle---------------------------
num4=vpa((Zdeltae/speed)+((X_u*Zdeltae)/speed)-
((Z_u*Xdeltae)/speed),3);
deno4=vpa((s^2)-(X_u*s)-((Z_u*g_0)/speed),3);

TF_Theta=num4/deno4;

tfnum4=[-1.26e-4];
tfdeno4=[1 0.0562 0.0402];

tf_Theta=tf(tfnum4,tfdeno4)
```

```
vpa(ilaplace(TF_Theta),2)
%[z,gain]=zero(tf_Theta)

% -----------------Angle of attack----------------------
num2=vpa([Zdeltae/speed -1 ;
Mdeltae+((M_w_point*Zdeltae)/speed) s-
(M_q+M_w_point)],3) ;
deno2=vpa([s-(Z_w/speed) -1 ; (-
(M_w+((M_w_point*Z_w)/speed)) (s-(M_q+M_w_point))],3) ;

detnum2=det(num2);
detdeno2=det(deno2);

tfnum2=[-0.000481039693580598282096616458 14776 -
0.020865074388508721822862459897003];
tfdeno2=[1 0.32393121940413038828410208225 25
0.0082443925494719386795529395837908];

TF_alpha=detnum2/detdeno2;

tf_alpha=tf(tfnum2,tfdeno2)

vpa(ilaplace(TF_alpha),2)

%[z,gain]=zero(tf_alpha)

% -----------------Pitch rate---------------------------

num3=vpa([s-(Z_w/speed) Zdeltae/speed ; (-
(M_w+((M_w_point*Z_w)/speed))
Mdeltae+((M_w_point*Zdeltae)/speed)],3) ;
deno3=vpa([s-(Z_w/speed) -1 ; (-
(M_w+((M_w_point*Z_w)/speed)) (s-(M_q+M_w_point))],3) ;

detnum3=det(num3) ;
detdeno3=det(deno3) ;

tfnum3=[-0.020849781260039890184998512268066 -
0.00012506502437473327746899455895901];
tfdeno3=[1 0.32393121940413038828410208225 25
0.0082443925494719386795529395837908];

TF_q =detnum3/detdeno3;

tf_q=tf(tfnum3,tfdeno3)

vpa(ilaplace(TF_q),2)
```

Part 2:

```
function sys=DATA(t,u)

%,w,q,Theta
Xu= -5.6205e-02;
Xw= 3.4997e-02;


Zu=-2.8725e-01;
Zw=-4.2154e-01;
Zq=0 ;
Zw_dot =9.7747e-04;

Mu=0;
Mw=-6.3456e-03;
Mq= -3.1578e-01;
Mw_dot= -2.1396e-03;

U0=7.0104e+01;

g=9.81;
theta0=0;


V=u(1);
W=u(2);
Q=u(3);
Theta=u(4);

du =Xu*V+Xw*W +0*Q -g*cos(theta0)*Theta;
dw =Zu/(1-Zw_dot)*V + Zw/(1-Zw_dot)*W+ (Zq+U0)/(1-
Zw_dot)*Q+(-g*sin(theta0)/(1-Zw_dot))*Theta;
dq =(Mu+(Mw_dot*Zu)/(1-Zw_dot))*V+ (Mw+(Mw_dot*Zw)/(1-
Zw_dot))*W+(Mq+(Mw_dot*(Zq+U0 ))/(1-Zw_dot))*Q-
(g*Mw_dot*sin(theta0)/(1-Zw_dot))*Theta;
dtheta = Q ;

sys=[du;dw;dq;dtheta];
end
```