In[28]:=
```
Clear["Global`*"]
```

- Probabilities of patterns with 1, 2, 3 and 4 individuals

In[29]:=
$$\text{PatProb1ind} = \left\{\left\{\frac{1-x}{n}\right\}, \left\{x + (1-x)(1-1/n)\right\}\right\};$$

$$\text{PatProb2ind} = \left\{\{x^2\}, \left\{2\,x\,(1-x) * \frac{1}{n} + (1-x)^2 * \frac{(n-1)(n-2)}{n^3}\right\}, \left\{2\,x\,(1-x) * \frac{2}{n} * \left(1-\frac{2}{n}\right) + (1-x)^2 * \frac{4\,(n-1)(n-2)}{n^3}\right\}, \left\{2\,x\,(1-x)\left(1-\frac{2}{n}\right)\left(1-\frac{3}{n}\right) + (1-x)^2 * \frac{(n-1)(n-2)(n-3)}{n^3}\right\}\right\};$$

$$\text{PatProb3ind} = \left\{\{x^3\}, \left\{x^2\,(1-x) * \frac{\left(1-\frac{2}{n}\right)}{n}\right\}, \left\{2\,x^2\,(1-x) * \frac{\left(1-\frac{2}{n}\right)}{n}\right\}, \left\{x^2\,(1-x) * \left(1-\frac{2}{n}\right)\left(1-\frac{3}{n}\right)\right\},\right.$$
$$\left\{2\,x * (1-x)^2 * \frac{(n-2)(n-3)(n-4)}{n\,\text{^}\,4} + \left((1-x)^3\right) * \frac{(n-1)(n-2)(n-3)(n-4)}{n^5}\right\},$$
$$\left\{\left(4\,x * (1-x)^2 * \frac{(n-2)(n-3)(n-4)}{n^4} + 4\,(1-x)^3 * \frac{(n-1)(n-2)(n-3)(n-4)}{n^5}\right)\right\},$$
$$\left.\left\{3\,x\,(1-x)^2 * \frac{(n-2)(n-3)(n-4)(n-5)}{n\,\text{^}\,4} + (1-x)^3 * \frac{(n-1)(n-2)(n-3)(n-4)(n-5)}{n^5}\right\}\right\};$$

$$\text{PatProb4ind} = \left\{\{x^4\}, \left\{\frac{2\,(-2+n)(1-x)\,x^3}{n^2}\right\}, \left\{\frac{(-2+n)(1-x)\,x^3}{n^2}\right\},\right.$$
$$\left\{\frac{4\,(-3+n)(-2+n)(1-x)\,x^3}{n^2}\right\}, \left\{\frac{(-4+n)(-3+n)(-2+n)(1-x)^2\,x^2}{n^4}\right\},$$
$$\left\{\frac{2\,(-4+n)(-3+n)(-2+n)(1-x)^2\,x^2}{n^4}\right\}, \left\{\frac{(-5+n)(-4+n)(-3+n)(-2+n)(1-x)^2\,x^2}{n^4}\right\},$$
$$\left\{\frac{(-6+n)(-5+n)(-4+n)(-3+n)(-2+n)(-1+n)(1-x)^4}{n^7} + \frac{(-6+n)(-5+n)(-4+n)(-3+n)(-2+n)(1-x)^3\,x}{n^6}\right\},$$
$$\left\{\frac{4\,(-6+n)(-5+n)(-4+n)(-3+n)(-2+n)(-1+n)(1-x)^4}{n^7} + \frac{(-6+n)(-5+n)(-4+n)(-3+n)(-2+n)(1-x)^3\,x}{n^6}\right\},$$
$$\left.\left\{\frac{(-7+n)(-6+n)(-5+n)(-4+n)(-3+n)(-2+n)(-1+n)(1-x)^4}{n^7} + \frac{4\,(-7+n)(-6+n)(-5+n)(-4+n)(-3+n)(-2+n)(1-x)^3\,x}{n^6}\right\}\right\};$$

- Matrices containing the conditional probabilities (i.e. Mendelian randomness)

In[33]:=
```
Axi111HR = {{1/4, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 1}, {0, 0}, {0, 0}};
Axi112HR = {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 1/2, 0, 0},
  {0, 0, 0, 0}, {0, 1/2, 0, 0}, {1, 0, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
Axi102HR = {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 1/2, 0, 0}, {0, 0, 0, 0},
   {1/4, 0, 1/4, 0}, {0, 0, 0, 0}, {1/2, 0, 1/4, 0}, {0, 1, 1/2, 1}, {0, 0, 0, 0}};
Axi002HR = {{1/32, 0, 1/64, 0}, {1/32, 0, 0, 0}, {1/8, 0, 1/16, 0}, {0, 0, 1/32, 0},
   {1/8, 0, 1/16, 0}, {0, 0, 1/32, 0}, {1/4, 1, 3/16, 1/4}, {0, 1, 3/8, 1/2}, {0, 0, 0, 1/4}};
Axi103HR = {{0, 0, 0, 0, 0, 0, 0}, {0, 1/4, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
   {0, 1/4, 0, 0, 1/2, 0, 0}, {1/4, 0, 1/8, 0, 0, 0, 0}, {0, 0, 3/8, 1/4, 0, 1/8, 0},
   {1/2, 0, 1/2, 0, 0, 0, 0}, {0, 2, 3/2, 2, 0, 1, 0}, {0, 0, 0, 1/2, 2, 3/4 + 1, 1}};
Axi003HR = {{1/32, 0, 3/64, 0, 0, 0, 0},
   {1/32, 0, 1/32, 0, 0, 0, 0}, {1/8, 1/8, 1/4 + 1/16, 1/8, 0, 1/8, 0},
   {0, 1/8, 1/32 + 1/16, 1/8, 0, 1/8, 0}, {1/8, 1/8, 1/4, 1/8, 0, 1/8, 0},
   {0, 1/8, 1/32 + 1/16, 1/8, 0, 1/8, 0}, {1/4, 1/2, 5/8, 1/2, 0, 0, 0},
   {0, 3/2, 3/8 + 1/2, 3/2, 2, 3/2, 1/2}, {0, 0, 0, 0, 1, 3/8 + 1/2, 1/2}};
Axi004HR = {{1/32, 1/32, 0, 0, 0, 0, 0, 0, 0, 0},
   {1/32, 1/32, 0, 0, 1/8, 0, 0, 0, 0, 0}, {1/8, 1/8, 1/8, 1/16, 0, 0, 0, 0, 0, 0},
   {0, 1/16, 1/8, 1/16, 1/4, 1/4, 1/4, 0, 1/8, 0}, {1/8, 1/8, 1/8, 1/16, 0, 0, 0, 0, 0, 0},
   {0, 1/16, 1/8, 1/16, 1/4, 1/4, 1/4, 0, 1/8, 0}, {1/4, 1/8, 0, 0, 1/8, 0, 0, 0, 0, 0},
   {0, 1/8, 2, 1/2, 1/2, 1, 2, 0, 1, 0}, {0, 0, 0, 0, 9/4, 3, 3, 4, 9/2, 1}};
```

- Derive now the rows of the matrix by multiplying the Mendelian tables with the vectors containing the probabilities of the patterns

In[39]:=
```
vxi111 = Expand[Axi111HR.PatProb1ind];
vxi111 = Flatten[Reverse[Expand[Insert[vxi111, Simplify[1 - Total[vxi111]], 1]]]];
vxi111order1 = Limit[vxi111, n → Infinity];
vxi111orderN = Limit[n * (vxi111 - vxi111order1), n → Infinity];
vxi111 = vxi111order1 + vxi111orderN / n;
vxi112 = Expand[Axi112HR.PatProb2ind];
vxi112 = Flatten[Reverse[Expand[Insert[vxi112, Simplify[1 - Total[vxi112]], 1]]]];
vxi112order1 = Limit[vxi112, n → Infinity];
vxi112orderN = Limit[n * (vxi112 - vxi112order1), n → Infinity];
vxi112 = vxi112order1 + vxi112orderN / n;
vxi102 = Expand[Axi102HR.PatProb2ind];
vxi102order1 = Limit[vxi102, n → Infinity];
```

```
vxi102orderN = Limit[n*(vxi102 - vxi102order1), n → Infinity];
vxi102 = vxi102order1 + vxi102orderN / n;
vxi102 = Flatten[Reverse[Expand[Insert[vxi102, Simplify[1 - Total[vxi102]], 1]]]];
vxi012 = {Part[vxi102, 1], Part[vxi102, 2],
    Part[vxi102, 3], Part[vxi102, 4], Part[vxi102, 7], Part[vxi102, 6],
    Part[vxi102, 5], Part[vxi102, 8], Part[vxi102, 9], Part[vxi102, 10]};
vxi002 = Expand[Axi002HR.PatProb2ind];
vxi002order1 = Limit[vxi002, n → Infinity];
vxi002orderN = Limit[n*(vxi002 - vxi002order1), n → Infinity];
vxi002 = vxi002order1 + vxi002orderN / n;
vxi002 = Flatten[Reverse[Expand[Insert[vxi002, Simplify[1 - Total[vxi002]], 1]]]];
vxi103 = Expand[Axi103HR.PatProb3ind];
vxi103order1 = Limit[vxi103, n → Infinity];
vxi103orderN = Limit[n*(vxi103 - vxi103order1), n → Infinity];
vxi103 = vxi103order1 + vxi103orderN / n;
vxi103 = Flatten[Reverse[Expand[Insert[vxi103, Simplify[1 - Total[vxi103]], 1]]]];
vxi013 = {Part[vxi103, 1], Part[vxi103, 2],
    Part[vxi103, 3], Part[vxi103, 4], Part[vxi103, 7], Part[vxi103, 6],
    Part[vxi103, 5], Part[vxi103, 8], Part[vxi103, 9], Part[vxi103, 10]};
vxi003 = Expand[Axi003HR.PatProb3ind];
vxi003order1 = Limit[vxi003, n → Infinity];
vxi003orderN = Limit[n*(vxi003 - vxi003order1), n → Infinity];
vxi003 = vxi003order1 + vxi003orderN / n;
vxi003 = Flatten[Reverse[Expand[Insert[vxi003, Simplify[1 - Total[vxi003]], 1]]]];
vxi004 = Expand[Axi004HR.PatProb4ind];
vxi004 = Flatten[Reverse[Expand[Insert[vxi004, Simplify[1 - Total[vxi004]], 1]]]];
vxi004order1 = Limit[vxi004, n → Infinity];
vxi004orderN = Limit[n*(vxi004 - vxi004order1), n → Infinity];
vxi004 = vxi004order1 + vxi004orderN / n;
vxicoal = {0, 0, 0, 0, 0, 0, 0, 0, 0, 1};
PHR =
  {vxi004, vxi003, vxi002, vxi013, vxi012, vxi103, vxi102, vxi112, vxi111, vxicoal};
```

- Recovering the WF matrix

```
(*The Wright Fisher matrix is given as PWF*)
```

In[ ]:=
$$\text{PWF} = \left\{\left\{1-\frac{6}{n}, \frac{4}{n}, 0, \frac{1}{2\,n}, 0, \frac{1}{2\,n}, 0, 0, 0, \frac{1}{n}\right\},\right.$$
$$\left\{\frac{1}{2}-\frac{3}{n}, \frac{1}{2}+\frac{1}{2\,n}, 0, \frac{1}{2\,n}, \frac{1}{2\,n}, \frac{1}{2\,n}, \frac{1}{2\,n}, 0, 0, \frac{1}{2\,n}\right\}, \left\{\frac{1}{4}-\frac{3}{2\,n}, \frac{1}{2}-\frac{1}{2\,n}, \frac{1}{4}+\frac{1}{4\,n},\right.$$
$$\left.\frac{1}{8\,n}, \frac{1}{4\,n}, \frac{1}{8\,n}, \frac{1}{4\,n}, 0, \frac{1}{16\,n}, \frac{15}{16\,n}\right\}, \left\{1-\frac{6}{n}, \frac{4}{n}, 0, \frac{1}{2\,n}, 0, \frac{1}{2\,n}, 0, 0, 0, \frac{1}{n}\right\},$$
$$\left\{0, 1-\frac{3}{n}, \frac{1}{n}, 0, \frac{1}{2\,n}, 0, \frac{1}{n}, 0, 0, \frac{1}{2\,n}\right\}, \left\{1-\frac{6}{n}, \frac{4}{n}, 0, \frac{1}{2\,n}, 0, \frac{1}{2\,n}, 0, 0, 0, \frac{1}{n}\right\},$$
$$\left\{0, 1-\frac{3}{n}, \frac{1}{n}, 0, \frac{1}{n}, 0, \frac{1}{2\,n}, 0, 0, \frac{1}{2\,n}\right\}, \left\{1-\frac{6}{n}, \frac{4}{n}, 0, \frac{1}{2\,n}, 0, \frac{1}{2\,n}, 0, 0, 0, \frac{1}{n}\right\},$$
$$\left.\left\{0, 0, 1-\frac{1}{n}, 0, 0, 0, 0, 0, \frac{1}{4\,n}, \frac{3}{4\,n}\right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 1\}\right\};$$

In[ ]:=
```
x = 0;
```

In[ ]:=
```
Simplify[PHR - PWF]
```

Out[ ]=
```
{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

- The HR replaces the entire population, i.e. x=1

In[ ]:=
```
x = 1;
```

In[ ]:=
```
TableForm[PHR]
```

Out[ ]//TableForm=

0	0	$\frac{1}{4}$	0	$\frac{1}{8}$	0	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{7}{16}$
0	0	$\frac{1}{4}$	0	$\frac{1}{8}$	0	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{7}{16}$
0	0	$\frac{1}{4}$	0	$\frac{1}{8}$	0	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{7}{16}$
0	0	$\frac{1}{2}$	0	0	0	$\frac{1}{4}$	0	0	$\frac{1}{4}$
0	0	$\frac{1}{2}$	0	0	0	$\frac{1}{4}$	0	0	$\frac{1}{4}$
0	0	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0	0	0	$\frac{1}{4}$
0	0	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0	0	0	$\frac{1}{4}$
0	0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1