Optimum Constellation for Symbol-Error-Rate to PAPR Ratio Minimization in SWIPT

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2022 IEEE 95th Vehicular Technology Conference (VTC2022-Spring) | 978-1-6654-8243-1/22/$31.00 ©2022 IEEE | DOI: 10.1109/VTC2022-Spring54318.2022.9860473

***Abstract*—In this work, a spike-QAM, a regular QAM constellation where one to four of the elements with the largest amplitudes are placed further from the center, is proposed as the optimum constellation that minimizes the ratio of the symbol-error-rate (SER) to peak-to-average-power-ratio (PAPR) in simultaneous-wireless-information-and-power-transfer (SWIPT) applications. This constellation is capable of maximizing the SER performance for a given design PAPR, or maximize the PAPR given a SER performance. The relationship between these parameters is defined. An approximation to the SER performance is given for the spike-QAM in AWGN channels and it is demonstrated to be valid via numerical simulations.**

***Index Terms*—Optimization, spike-QAM, SWIPT, PAPR, SER**

I. Introduction

Simultaneous wireless information and power transfer (SWIPT) is a new wireless technology that allows to transmit information and energy wirelessly in the same resource [1]. This technology is a promising solution for massive machine type communications (mMTC), characterized by a vast number of low-rate low power devices.

There is a fundamental property that the signals with high peak-to-average power ratio (PAPR) are appropriate for transferring energy [2], [3]. In [4], conventional communication modulations are compared in terms of SWIPT. It is worth taking into account that due to the nonlinearity of the rectification process, SWIPT efficiency highly depends on the shape of the transmitted signal and requires a re-design. Thus, some works investigate innovative modulation schemes by considering this nonlinearity. Modulation constellations implemented using deep neural network autoencoders are presented in [5], [6] proposes an asymmetric PSK, where symbols are distributed in a limited phase range and achieve higher rate-energy tradeoff regimes and [7] proposes a SWIPT-based probabilistic shaping modulation scheme. Moreover, [8] and [9] propose constellations that can approach a rate-energy region. In [10], the authors propose circular quadrature amplitude modulation (CQAM) to find a trade-off between symbol-error-rate (SER) and PAPR for SWIPT applications.

In this work, we propose a classical QAM constellation in which one to four of the elements with the largest amplitude are placed further from the center so that the PAPR is increased, at the expense of decreasing the information transfer performance, since the mean transfer power is maintained. This constellation is proposed as a solution to an optimization problem that aims at maximizing the minimum euclidean distance while maximizing the PAPR, which are two conflicting problems. Later, we analyze the performance of the proposed constellation in additive white Gaussian noise (AWGN) channels and provide a mean to define a specific PAPR and find the constellation that maximizes the minimum euclidean distance. We show that these constellations outperform the ones presented in [10] in terms of SER to PAPR ratio.

II. System Model

A. Signal Model

A single-input single-output (SISO) point-to-point SWIPT system, where the transmitter transmits information with a specific constellation shape and the receiver can jointly harvest energy and decode the information is considered. We do not assume any specific constellation shape, so the modulated symbols are distributed all over the complex plane. As an example, a QPSK constellation is composed of the symbols $s_n = \pm 1 \pm j$, where j denotes $\sqrt{-1}$. To study the fundamental characteristics of our proposed constellation for SWIPT, we consider the transmission over an additive white Gaussian noise (AWGN). The baseband channel model of the received signal is given by

$$y = x + \nu, \tag{1}$$

where x is a $M = 2^m$ complex baseband modulated symbol with m bits per symbol, that belongs to an M-ary irregular quadrature amplitude modulation (iQAM) of the form $x \in \mathfrak{M}, \mathfrak{M} = \{x_m, m = 0, 1, \cdots, M-1\}$, y is the baseband received signal, and ν is an AWGN realization with zero mean and variance $2N_0$.

In what follows, we focus on the tradeoff between PAPR and the minimum Euclidean distance of our constellation. The PAPR of the constellation can be written as

$$\text{PAPR} = \frac{\max_m |\{x_m\}|^2}{E_m\{|x_m|^2\}}, \tag{2}$$

where $E_m\{|x_m|^2\}$ is the mean over the elements $|x_m|^2$. Now, as the energy harvesting performance is a function of the PAPR, the next section will focus on the proposal and analysis of the constellation that minimizes the symbol-error-rate performance while maximizing the PAPR.

III. Constellation Proposal, Analysis and Usage

In this section, we first propose an optimization problem whose solution gives the constellation that minimizes the PAPR to error rate ratio, second we analyze the proposed constellation and last we indicate under which conditions should each variant of the proposed constellation be used.

A. Constellation Proposal

A constellation that minimizes the symbol-error-rate while maximizing the PAPR is proposed in this paper. To minimize the symbol-error-rate, the minimum distance among the elements in the constellation should be maximized, this is, the following optimization problem should be solved

$$\max d^{\text{min}} = \max \min_{n\neq m} |x_n - x_m| \quad \forall m,n \in 0,\cdots,M-1, \tag{3}$$

where $d_{mn} = |x_m - x_n|$ is the distance between the elements x_m and x_n. The solution to this problem would be d_{mn}^{min} and the constellation would resemble the well-known lattice-like constellation that was proposed in the 70's, and more concretely the hexagonal lattice [11].

To maximize the PAPR in a given constellation, the following optimization problem should be solved

$$\max \text{PAPR} = \max \frac{\max_m\{|x_m|^2\}}{E_m\{|x_m|^2\}}. \tag{4}$$

The solution to this problem is a constellation that places one element at an amplitude of $\sqrt{M}$ at any phase and the rest of the elements at the center of the plane, all with amplitude 0. This constellation, suffers from a symbol-error-rate very close to 1, since the detection of the elements with amplitude 0 always fails. This solution should therefore be avoided.

To combine the two previous optimization problems, a constellation that leaves one element far from the center and the others centered following a lattice-like distribution would be an intermediate solution to both problems, thus, it would be a solution to the following optimization problem that combines the two previous optimization problems

$$\max \left(\beta\left(\min_{n\neq m}|x_n - x_m|\right) + \frac{|\max_m\{x_m\}|^2}{E_m\{|x_m|^2\}}\right), \tag{5}$$

where $\forall m,n \in 0,\cdots,M-1 \quad$ and $\quad m \neq n$, and $\beta \in [0, \inf)$ is a parameter to find intermediate solutions between the solutions to the two independent optimization problems.

B. Constellation Analysis

Basically, and to simplify the problem, in this paper we propose a rectangular QAM constellation, which is a particular case of lattice-like constellations, and take one of the elements that is furthest from the center, and take it even further from the center. This constellation will be an optimal solution to the optimization problem defined in (4). We will call this constellation M-*ary spike*-QAM (M-sQAM). A set of examples of a 16-sQAM is shown in Fig. 1.

By utilizing the factor β, we can create different constellations with different PAPR and d^{min}, which in any case gets the best possible d^{min} given a certain PAPR and viceversa. For instance, for $\beta = 0$ the result constellation by solving (4) is obtained, while for $\beta = \infty$ the result constellation is that obtained by solving (3). For intermediate values, we can design the optimal constellation in terms of SER for a certain PAPR. The symbol error rate of rectangular QAM is

$$P_{\text{QAM}}^{\gamma=1} = 2\left(1 - \frac{1}{\sqrt{M}}\right)\text{erfc}\left(k\sqrt{\frac{E_s}{N_0}\gamma}\right) - \left(1 - \frac{2}{\sqrt{M}} + \frac{1}{M}\right)\text{erfc}^2\left(k\sqrt{\frac{E_s}{N_0}\gamma}\right) \tag{6}$$

with $k = (2(M-1)/3)^{-(1/2)}$ and $\gamma = 1$.

The symbol error rate (SER) of the M-sQAM is going to be practically the same as that of M-QAM for large M, but there will be larger differences for smaller M. An alternative to placing only one element further apart from the center, is to place up to 4 elements. To calculate the SER, we have to apply the following approximation formula

$$P_{\text{sQAM}} = \frac{M-i}{M}P_{\text{QAM}}^{\gamma} + \frac{i}{M}P_{\text{QAM}}^{\text{max}}, \tag{7}$$

where i is the number of elements further apart from the center, P_{QAM}^{γ} is the symbol error probability of a rectangular QAM but with a mean power smaller than 1 due to the extra power taken by the i elements placed further apart and $P_{\text{QAM}}^{\text{max}}$ is the symbol error probability of the element with the largest power against the rest of the scaled elements. This formula approximates the total error probability as the sum of the P_s of the $M-i$ elements that are centered plus the P_s of the i elements that are further apart.

Taking into account that the constellation should be normalized in transmission, there will be a maximum achievable minimum distance in the constellation, which equals $d_{ij}^{\text{max}} = 2k$. The minimum distance between the elements in the proposed constellation given a certain PAPR will be $d_{ij}^{\text{min}} = 2k\sqrt{\gamma}$, where γ is a scaling factor caused by the increment in PAPR.

The mean power of the centered elements can be computed by not considering the i ones with the largest power, for $E\{|x_m|^2\} = 1$ as

$$\gamma = \frac{M - i\cdot\text{PAPR}}{M-i} = \frac{M - i\cdot|x_M|^2}{M-i}, \tag{8}$$

where $|x_M|^2$ is the largest power. The derivation of γ is found in Appendix A.

The term P_{QAM}^{γ} is already found in (6), and the term $P_{\text{QAM}}^{\text{max}}$ can be approximated as the error probability of the spike element versus the rest of the elements. In this case, we can approximate the error by approximating the decision regions for the spike elements, as shown in Fig. 2 for $i = 1$.

It can be approximated by utilizing an approach similar to that of the calculation of the pairwise symbol error probability [12], defining a decision region as the one showed in Fig. 2. Taking this into account, $P_{\text{QAM}}^{\text{max}}$ can be defined as

$$P_{\text{QAM}}^{\text{max}} = Q\left(\frac{D_{\text{dec}}}{\sqrt{2N_0}}\right) = Q\left(\frac{|x_M| - D_{\text{mid}}}{\sqrt{2N_0}}\right), \tag{9}$$

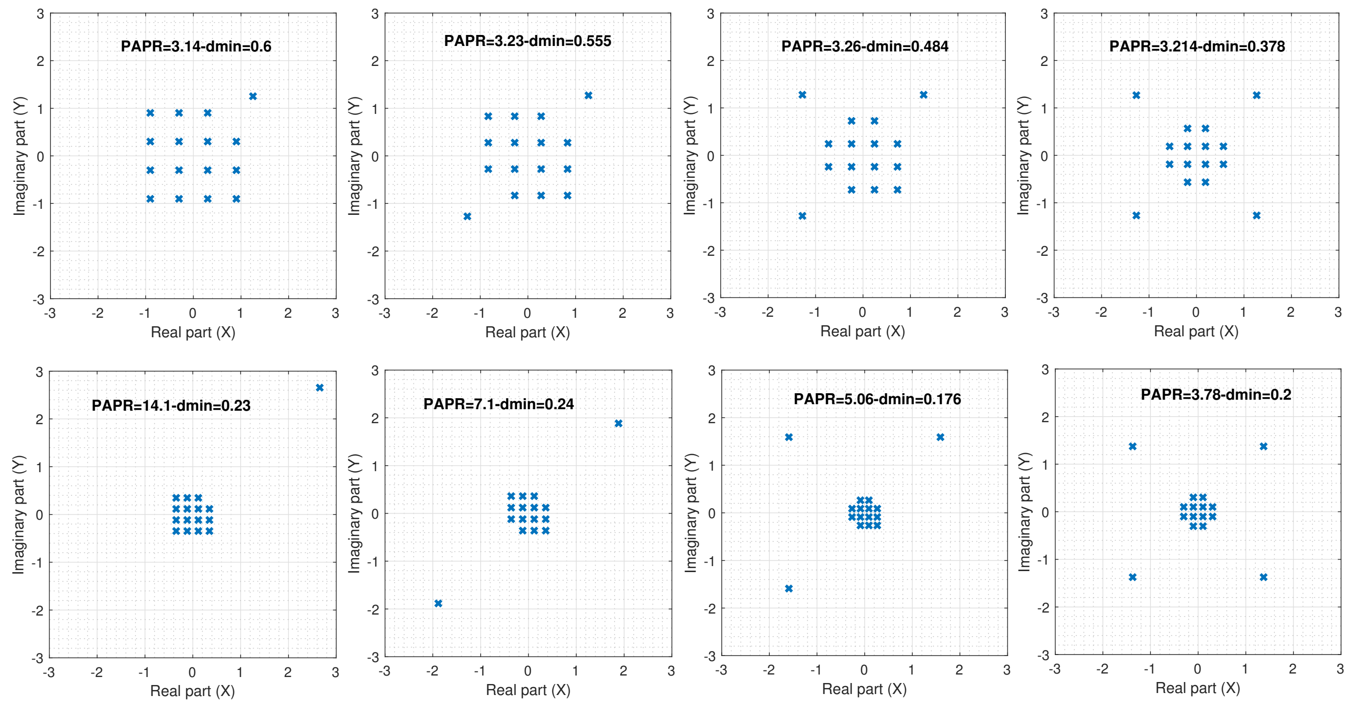

Fig. 1. 16 spike-QAM for 2 different PAPR-dmin ratios (smaller PAPR on top and larger at the bottom) and $i = 1, 2, 3, 4$ from left to right, respectively.

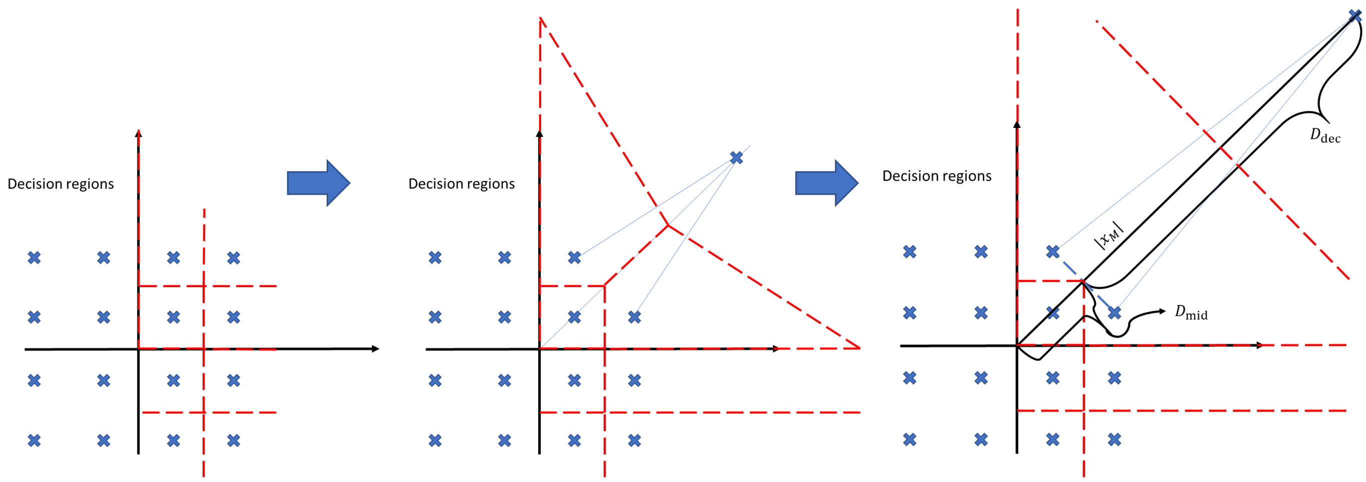

Fig. 2. Decision region approximation for the spike element in the M-ary spike-QAM.

where $|x_M|$ is known and equals $|x_M| = \sqrt{\text{PAPR}}$, and D_{mid} is the amplitude of the point found in the middle of the two symbols that are closer to the spike element. It is defined as

$$D_{\text{mid}} = k\sqrt{2M - 8\sqrt{M} + 8}, \tag{10}$$

which is obtained via straightforward trigonometric derivations, shown in Appendix B.

It is worth noting that the minimum definable PAPR value (which is the same as that of a regular QAM) equals $\text{PAPR}_{\text{min}} = k\sqrt{2M - 4\sqrt{M} + 2}$ and the maximum PAPR value is $\text{PAPR}_{\text{max}} = M/i$.

In a single carrier configuration, the frequency of spike elements will be i/M, which is an important factor in the SWIPT application, since only the strongest elements may excite the EH circuit.

C. Constellations Usage

To use the proposed constellations, we must define under which conditions should each configuration be used. The transmitted power (P_T) minus the path-loss (PL) (P_L) equals the received power (P_R) in the energy harvesting circuit ($P_T - P_L = P_R$). To excite this circuit, a certain threshold power P_{th} should reach it, thus $P_T - P_L \geq P_{th}$. By defining the $P_T = G_T \cdot P_p$, where P_p is the peak power of the normalized M-sQAM constellation (normalize the mean power to 1mW for simplicity), and G_T represents the gain obtained by the RF circuitry. By defining a maximum transmission power $P_T^{\max}$, a corresponding $G_T^{\max} = P_T^{\max}/P_p$ can be also defined. With this, the following equation arises

$$P_p \geq \frac{P_{th} + P_L}{G_T^{\max}} = \alpha + \delta P_L. \tag{11}$$

In the previous equation, P_{th} can be regarded as constant and dependent on the energy harvesting (EH) circuit, $G_T^{\max}$ depends on the transmitter, so it can also be regarded as constant, and P_L typically varies with the distance between transmitter and receiver. Three scenarios are foreseen:

1) Small P_L: the P_p does not need to be large and a regular QAM can do the job of charging the EH circuit.
2) Medium P_L: the P_p needs to be large but not extremely, in which case more than 1 spike elements are chosen ($i = 2, 3, 4$) and the PAPR may not be too large.
3) Large P_L: the P_p needs to be very large, so only one spike element is selected ($i = 1$) and the PAPR is thus, very large, but less frequent.

Specific values for each previous P_L depend on the scenario.

IV. Performance Results

In this section, we are going to show via numerical simulations that the error probability analysis performed in Section III is valid and can be used to measure the performance of the proposed constellation. We also get some curves that related the PAPR versus the minimum distance in the constellation.

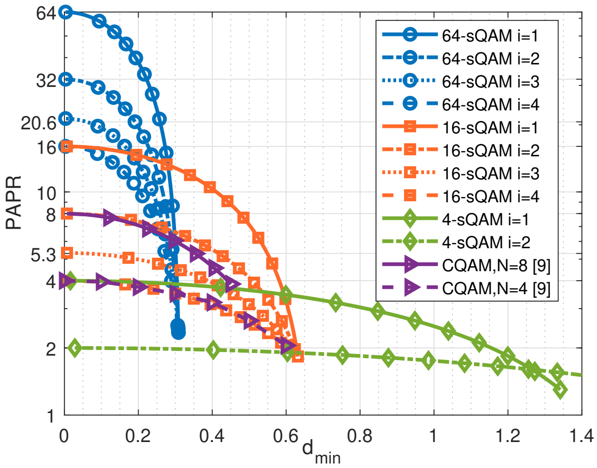

Fig. 3. PAPR versus $d_{\min}$ for 4, 16, 64-sQAM. Comparison with CQAM of [10] for N levels.

Fig. 3 shows the PAPR to $d_{\min}$ ratio of the 4,16,64-sQAM. This comes to show how sQAM constellations with more elements can achieve larger PAPRs but lower $d_{\min}$. On the contrary, constellations with less elements can achieve larger $d_{\min}$ but smaller PAPR. We compare our proposed constellations with the ones in [10], and find that our constellations are more versatile, since they allow larger ranges of PAPR and $d_{\min}$, and can achieve larger PAPRs while maintaining larger $d_{\min}$, thus smaller SER. The CQAM with $N = 4$ of [10] is virtually the same as our proposed 16-sQAM with $i = 4$.

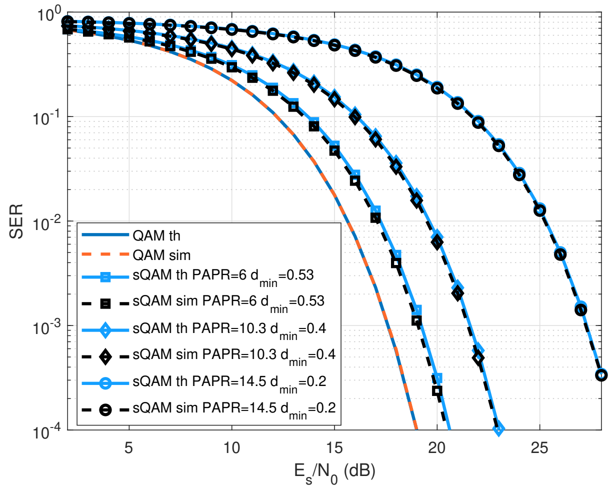

Fig. 4. SER of 16-sQAM for PAPR versus $d_{\min}$ ratios ($i = 1$).

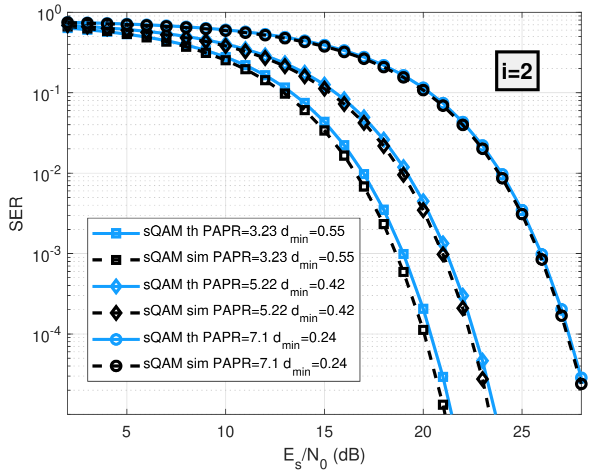

Fig. 5. SER of 16-sQAM for PAPR versus $d_{\min}$ ratios ($i = 2$).

In Figs. (4,5,6) and 7, we show the theoretical (th), obtained with (7) versus the simulated (sim) SER of the 16-spike-QAM ($i = 1$, $i = 2$ and, $i = 3$ and $i = 4$) and the 64-spike-QAM ($i = 1$). It can be seen how the theoretical analysis and the simulated performance have a great match. Thus, this analysis can be regarded as useful in case spike-QAMs are used as constellations that maximize the PAPR to $d_{\min}$ ratio.

V. Conclusions

We propose a spike-QAM constellation that maximizes the PAPR to $d_{\min}$ ratio for SWIPT applications. The spike-QAM consists on a regular QAM that takes some elements and place them further from the center. We performed an analysis of the SER performance of the proposed constellation for any PAPR value and provided an analysis on how to calculate the different parameters of the proposed constellation. The SER analysis shows a very good agreement with the simulations, thus confirming the validity of our analysis.

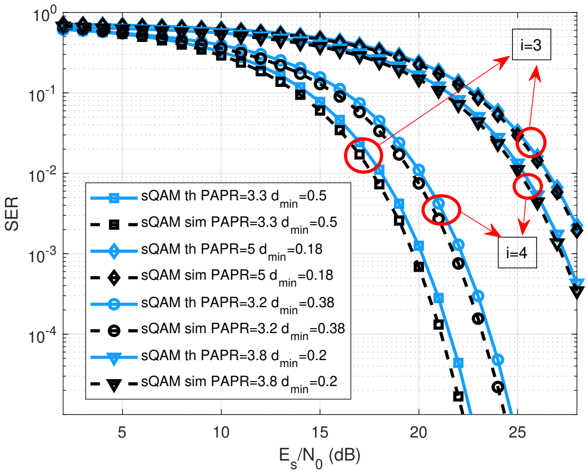

Fig. 6. SER of 16-sQAM for PAPR versus d_{min} ratios ($i = 3$ and $i = 4$).

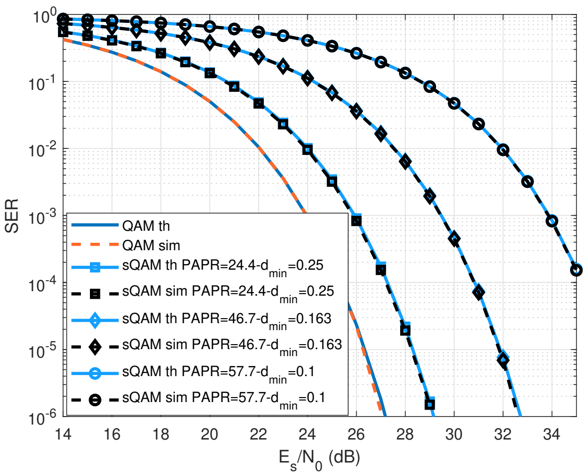

Fig. 7. SER of 64-sQAM for PAPR versus d_{min} ratios ($i = 1$).

ACKNOWLEDGEMENTS

This work has received funding from the European Union Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie ETN TeamUp5G, grant agreement No. 813391, and the Spanish National Project IRENE-EARTH (PID2020-115323RB-C33) (MINECO/AEI/FEDER, UE).

APPENDIX A
DERIVATION OF THE POWER SCALING FACTOR γ

We consider the elements $M - i + 1$ to M to be the ones with the largest amplitude, without loss of generality, and with $\frac{1}{M}\sum_{m=1}^{M}|x_m|^2 = E\{|x_m|^2\}$:

$$\frac{M-i}{M}\left(\frac{1}{M-i}\sum_{m=1}^{M-i}|x_m|^2\right) + \frac{i}{M}|x_M|^2 = E\{|x_m|^2\} \quad (12)$$

$$\left(\frac{1}{M-i}\sum_{m=1}^{M-i}|x_m|^2\right) = \frac{ME\{|x_m|^2\} - i \cdot |x_M|^2}{M-i} \quad (13)$$

Using $E\{|x_m|^2\} = |x_M|^2/PAPR$, and with $E\{|x_m|^2\} = 1$, then $|x_M|^2 = PAPR$, so

$$\left(\frac{1}{M-i}\sum_{m=1}^{M-i}|x_m|^2\right) = \frac{M - i \cdot \text{PAPR}}{M-i}. \quad (14)$$

APPENDIX B
DERIVATION OF THE DISTANCE D_{MID}

We derive the distance between the spike element and the center of its two closest elements, without normalization, by first calculating the distance of one of these elements as

$$D = \sqrt{\left(\sqrt{M}-1\right)^2 + \left(\sqrt{M}-3\right)^2} = \sqrt{2M - 8\sqrt{M} + 10}. \quad (15)$$

The distance between the two elements is calculated as $d_2 = 2\sqrt{2}$, so the distance from the center to the middle point between the two elements closest to the spike element can be calculated via trigonometrics as

$$d_{\text{obj}} = \sqrt{D^2 - \left(\frac{d_2}{2}\right)^2} = \sqrt{2M - 8\sqrt{M} + 8}, \quad (16)$$

and by multiplying d_{obj} with k we convert it to D_{mid}.

REFERENCES

[1] T. D. Ponnimbaduge Perera, D. N. K. Jayakody, S. K. Sharma, S. Chatzinotas, and J. Li, "Simultaneous wireless information and power transfer (swipt): Recent advances and future challenges," *IEEE Communications Surveys Tutorials*, vol. 20, no. 1, pp. 264–302, 2018.

[2] B. Clerckx and E. Bayguzina, "Waveform Design for Wireless Power Transfer," *IEEE Transactions on Signal Processing*, vol. 64, no. 23, pp. 6313–6328, 2016.

[3] D. I. Kim, J. H. Moon, and J. J. Park, "New SWIPT Using PAPR: How It Works," *IEEE Wireless Communications Letters*, vol. 5, no. 6, pp. 672–675, 2016.

[4] W. Liu, X. Zhou, S. Durrani, and P. Popovski, "SWIPT with practical modulation and RF energy harvesting sensitivity," in *2016 IEEE International Conference on Communications (ICC)*, 2016, pp. 1–7.

[5] M. Varasteh, E. Piovano, and B. Clerckx, "A Learning Approach to Wireless Information and Power Transfer Signal and System Design," in *ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2019, pp. 4534–4538.

[6] E. Bayguzina and B. Clerckx, "Asymmetric Modulation Design for Wireless Information and Power Transfer With Nonlinear Energy Harvesting," *IEEE Transactions on Wireless Communications*, vol. 18, no. 12, pp. 5529–5541, 2019.

[7] A. Rajaram, D. N. K. Jayakody, B. Chen, R. Dinis, and S. Affes, "Modulation-Based Simultaneous Wireless Information and Power Transfer," *IEEE Communications Letters*, vol. 24, no. 1, pp. 136–140, 2020.

[8] C. Thomas, M. Weidner, and S. Durrani, "Digital Amplitude-Phase Keying with M-Ary Alphabets," *IEEE Transactions on Communications*, vol. 22, no. 2, pp. 168–180, 1974.

[9] Y.-B. Kim, D. K. Shin, and W. Choi, "Rate-Energy Region in Wireless Information and Power Transfer: New Receiver Architecture and Practical Modulation," *IEEE Transactions on Communications*, vol. 66, no. 6, pp. 2751–2761, 2018.

[10] G. M. Kraidy, C. Psomas, and I. Krikidis, "Fundamentals of Circular QAM for Wireless Information and Power Transfer," in *2021 IEEE 22nd International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2021, pp. 616–620.

[11] G. Foschini, R. Gitlin, and S. Weinstein, "Optimization of Two-Dimensional Signal Constellations in the Presence of Gaussian Noise," *IEEE Transactions on Communications*, vol. 22, no. 1, pp. 28–38, 1974.

[12] Proakis, *Digital Communications 5th Edition*. McGraw Hill, 2007.