Aristotle University of Thessaloniki
Department of Electrical and Computer Engineering
Telecommunications Division

Ioannis Dimoulios 10641
Dimitrios Diakoloukas 10642

Communication Systems II

Constellation Design for Simultaneous Wireless Information and Power Transfer (SWIPT)

0 Introduction

The task requires the analysis and design of various constellations considering the balance in simultaneous wireless information and power transfer (SWIPT) systems. Specifically, the focus is on the relationship between peak-to-average power ratio (PAPR) and minimum Euclidean distance (d_{min}) in designing optimal constellations for SWIPT and reviewing their performance in terms of symbol error rate (SER).
We examine known modulation schemes such as 16-PAM, 16-PSK, and 16-QAM, as well as the state of the art 16-Circular QAM (CQAM) and 16-Spike QAM (sQAM) as presented in [1] and [2] respectively. In the final sections of the assignment we propose a new modulation scheme, namely 16-Bee QAM (BQAM), which shows promising results.
Our approach includes both a theoretical and a simulation analysis. Constellations are represented with simple arrays of complex numbers in Julia with the convention that the first two symbols of the array always have distance d_{min}. A more thorough explanation of our software setup is included in the Appendix section.

1 PAPR versus Minimum Euclidean Distance

In what follows the average symbol energy for all the constellations is normalized on $E_s = 1$. This ensures that the comparisons are fair energy-wise. Moreover, $M = 16$ is the number of symbols for each constellation.

1.1 Theoretical Analysis

16-PAM

Assume that the energy of the main pulse of the PAM is E_g. It is well known that

$$E_g = \frac{3E_s}{M^2 - 1} = 0.0118. \tag{1}$$

Then, given that the symbol with the highest energy, i.e. the symbol furthest from the origin has coordinates

$$x_M = \left\{ (M-1)\sqrt{E_g} \right\}, \tag{2}$$

we can calculate PAPR as follows

$$\text{PAPR}_{\text{PAM}} = \frac{|x_M|^2}{E_s} = 2.647. \tag{3}$$

As far as the minimum euclidean distance is concerned, we have

$$d_{\text{min}} = 2\sqrt{E_g} = 0.217. \tag{4}$$

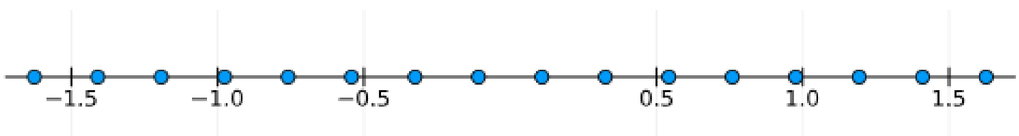

Figure 1: 16-PAM constellation diagram

16-PSK

The symbols of PSK have coordinates

$$x_i = \left\{ \sqrt{E_s} \cos\theta_i, \sqrt{E_s} \sin\theta_i \right\} \tag{5}$$

where $\theta_i = \dfrac{2\pi(i-1)}{M}$ for all $i = 1, \ldots, M$. Hence for all i, we have

$$E_i = |x_i|^2 = \sqrt{\cos^2\theta_i + \sin^2\theta_i} = 1, \tag{6}$$

which in turn means $E_{\max} = 1$ and

$$\mathrm{PAPR}_{\mathrm{PSK}} = \frac{E_{\max}}{E_s} = 1. \tag{7}$$

It also follows from simple trigonometric calculations that

$$d_{\min} = 2\sqrt{E_s} \sin\frac{\pi}{M} = 0.390. \tag{8}$$

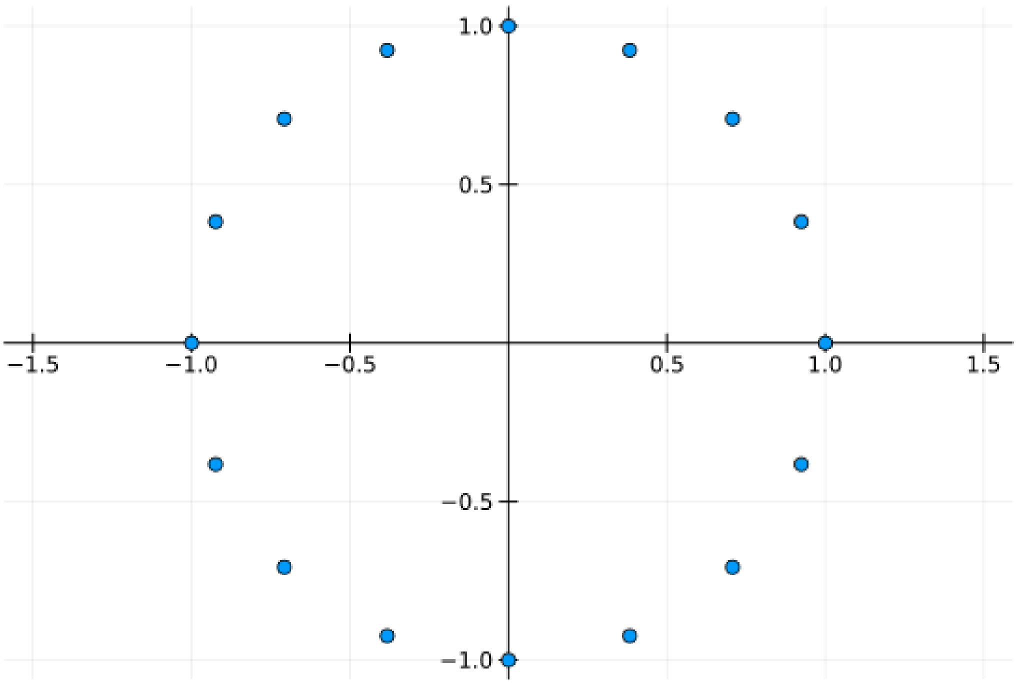

Figure 2: 16-PSK constellation diagram

16-QAM

M-QAM is essentially the composition of two orthogonal $\sqrt{M}$-PAM constellations representing the I/Q components of the constellation. The energy is equally shared between each component; hence

$$E_s = 2E_{s\left(\sqrt{M}-\mathrm{PAM}\right)} \tag{9}$$

and if E_g is the energy of the main pulse of each $\sqrt{M}$-PAM component we have

$$E_g = \frac{3E_{s\left(\sqrt{M}-\mathrm{PAM}\right)}}{\sqrt{M}^2 - 1} = \frac{1.5E_s}{M-1} = 0.1. \tag{10}$$

Then the coordinates of the symbol with the highest energy are

$$x_M = \left\{ \left(\sqrt{M} - 1\right) \sqrt{E_g}, \left(\sqrt{M} - 1\right) \sqrt{E_g} \right\}. \tag{11}$$

Now, we can calculate

$$\mathrm{PAPR}_{\mathrm{QAM}} = \frac{|x_M|^2}{E_s} = 0.1 \cdot 2 \cdot 3^2 = 1.8. \tag{12}$$

On the other hand

$$d_{\min} = 2\sqrt{E_g} = 0.632. \tag{13}$$

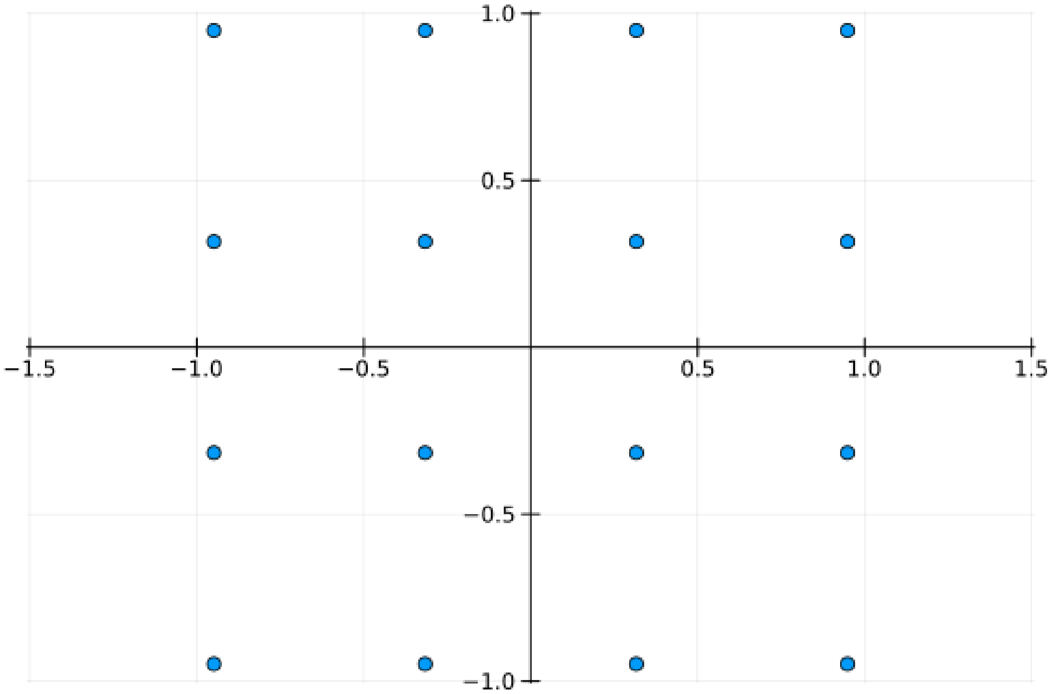

Figure 3: 16-QAM constellation diagram

16-CQAM

According to the algorithm provided [1] we initialize CQAM as a function of two parameters:

- N, the number of circles of the constellation and
- $d_{\min}$, the minimum euclidean distance, from which we can directly calculate the radius of the first circle.

Each circle must contain $n = M \div N$ symbols. Indeed, the first radius is

$$R_1 = \frac{d_{\min}}{2 \sin \frac{\pi}{n}}. \tag{14}$$

Next, we create the first circle with n symbols such that adjacent symbols are distanced at $d_{\min}$. The next $N - 2$ circles are formed with n with the minimum possible radius such that the $d_{\min}$ condition is not violated.

The remaining energy is then used to form the last circle with radius $R_N > R_j$ for all $j \in 1, \ldots, N-1$.
Hence, considering that the symbols carrying the maximum energy are those of the last circle we can calculate,

$$\mathrm{PAPR}_{\mathrm{CQAM}} = \frac{R_N^2}{E_s} = R_N^2. \tag{15}$$

In order to create these constellations on software, we used rotating vectors of modulus $d_{\min}$ to form equilateral triangles on the first $N-1$ circles. The results are presented in the diagram below.

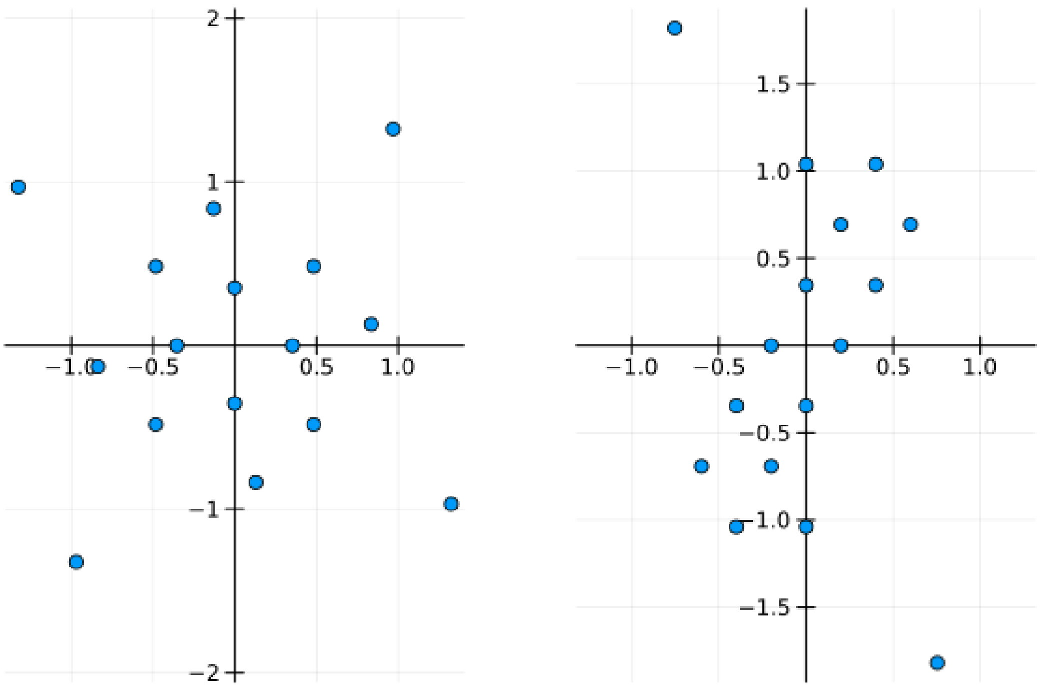

Figure 4: 16-CQAM constellations, a) with $N = 4$, $d_{\min} = 0.5$ and b) with $N = 8$, $d_{\min} = 0.4$

1.2 Simulation Results

For PAPR calculations, we used the normalized average $E_s = 1$ for each constellation and found the symbol with maximum energy by means of linear search.
For $d_{\min}$ calculations, we made sure we adhered to the convention stated in the introduction, namely that the first two symbols of the constellation are distanced at $d_{\min}$.
The simulations are therefore summarized in the following figure.

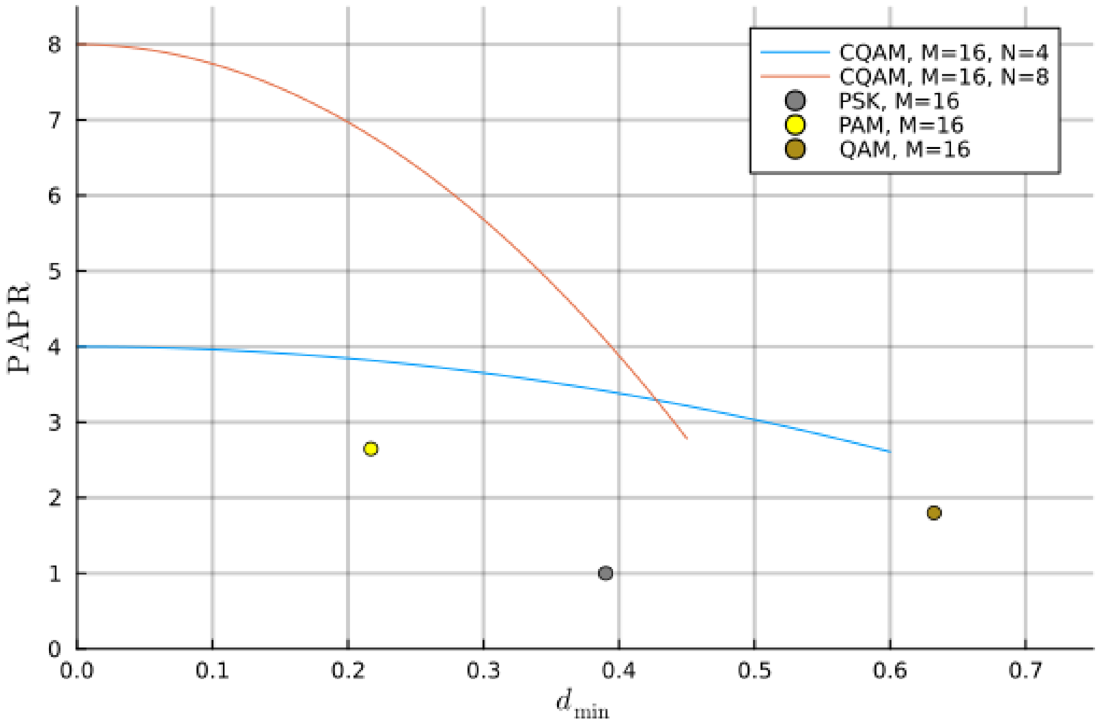

Figure 5: PAPR versus $d_{\min}$ plot for various modulation schemes

Notice that the PSK point differs from the one presented in [1]. Additionally, there is a slight variation for both the CQAM curves compared to [1].

2 SER versus Harvested Energy and SNR

Our goal is to study the SER as a function of both the normalized harvested energy and the Signal to Noise Ratio (SNR). Note that we define SNR as

$$\mathrm{SNR} = \frac{E_b}{N_0} = \frac{E_s}{N_0 \log_2 M}. \tag{16}$$

To that end, we have made a few assumptions regarding the receiver. Firstly, the energy is harvested and then the noise is added. Moreover, the demodulator is aware of the energy transfer the adjusts the constellation accordingly. Lastly, we work in a very high SNR environment.

2.1 Theoretical Analysis

Harvesting a portion of the energy of a transmitted symbol corresponds to the following transform of the symbol in the constellation space; if x is the transmitted symbol represented by a complex number, $\varepsilon(x)$ is the amount of energy harvested, and $\delta^+ = \max(0, \delta)$, then

$$x = re^{i\theta} \mapsto \sqrt{(r^2 - \varepsilon(x))^+} e^{i\theta}. \tag{17}$$

Of course, this transform applied to symbols with $d_{\min}$ distance generates a new and reduced minimum distance $d'_{\min}$ for the constellation. In that spirit and taking into consideration the high SNR environment we approximate the SER of a symbol by assuming that it can only be mistaken for a “neighbor”, i.e. another symbol which is $d'_{\min}$ away. In order to account for the complex constellation geometries generated by the energy harvest transform, we extend the definition of a “neighbor” by adding a small tolerance of $\dfrac{d'_{\min}}{c \cdot \mathrm{SNR}}$, where $c > 1$ is a positive constant. For the sake of this assignment we pick $c = 5$.

Formally, we define the set of neighbors of a symbol in the transformed constellation as

$$G(i) = \left\{ x_j \mid |x_i - x_j| \leq d'_{\min} \left(1 + \frac{1}{c \cdot \mathrm{SNR}}\right) \right\}, \tag{18}$$

where $\{x_k\}_{1 \leq k \leq M}$ is the set of the symbols of the transformed constellation.
However, we are mostly interested in the cardinality of G; hence, define

$$v(i) = |G(i)| \tag{19}$$

for all $1 \leq i \leq M$ to be the number of neighbors of the symbol x_i. Note that $v(i)$ changes as $\varepsilon(x)$ increases.
For example, consider a 16-PAM constellation and let $\varepsilon(x_i) = \varepsilon$ for all $1 \leq i \leq 16$. Then, if

- $\varepsilon = 0$, $d_{\min} = 0.217$ and $v(16) = 1$,
- $\varepsilon = 0.5$, $d'_{\min} = 0$ and $v(16) = 0$.

This volatility on the values of $v(i)$ makes the construction of a formula for SER as a function of ε extremely hard. Luckily, we can bypass this by calculating for each ε the new $d'_{\min}$ and the array of $v(i)$s.
We proceed by providing constellation specific insights for SER calculation.

16-PAM

It is straightforward that

$$\mathrm{SER}_j = v(j) Q\left(\sqrt{\frac{6E_s}{(M^2-1)N_0}}\right) = v(j) Q\left(\frac{d'_{\min}}{\sqrt{2N_0}}\right) \tag{20}$$

$$\mathrm{SER}_{\mathrm{PAM}} = \frac{1}{M}\sum_{j=1}^{M} \mathrm{SER}_j = \frac{1}{M}\sum_{j=1}^{M} v(j) Q\left(\frac{d'_{\min}}{\sqrt{2N_0}}\right) \tag{21}$$

For $\varepsilon = 0$, this becomes

$$\mathrm{SER} = Q\left(\sqrt{\frac{6E_s}{(M^2-1)N_0}}\right) = Q\left(\sqrt{\frac{6\log_2 M \cdot \mathrm{SNR}}{(M^2-1)}}\right). \tag{22}$$

16-QAM

When $\varepsilon > 0$ the constellation loses its straight line grid structure, but we can still approximate it as such. Hence, by considering the I/Q decomposition in two orthogonal 4-PAM's we get the estimation

$$\mathrm{SER}_j \approx 1 - (1 - \mathrm{SER}_{Ij})(1 - \mathrm{SER}_{Qj}) \tag{23}$$

$$= 1 - \left[1 - v_I(j) Q\left(\frac{d'_{\min}}{\sqrt{2N_0}}\right)\right]\left[1 - v_Q(j) Q\left(\frac{d'_{\min}}{\sqrt{2N_0}}\right)\right], \tag{24}$$

where $v_I(\cdot)$ and $v_Q(\cdot)$ represent the number of neighbors on the I and Q component respectively.
By taking the mean over all SER_j, we can find a good approximation for $\mathrm{SER}_{\mathrm{QAM}}$.
For $\varepsilon = 0$, this becomes

$$\mathrm{SER} = 1 - \left[1 - Q\left(\sqrt{\frac{1.5\log_2 M \cdot \mathrm{SNR}}{M-1}}\right)\right]^2 \tag{25}$$

as the energy is equally distributed among the I and Q components and we have $\sqrt{M}$ instead of M.

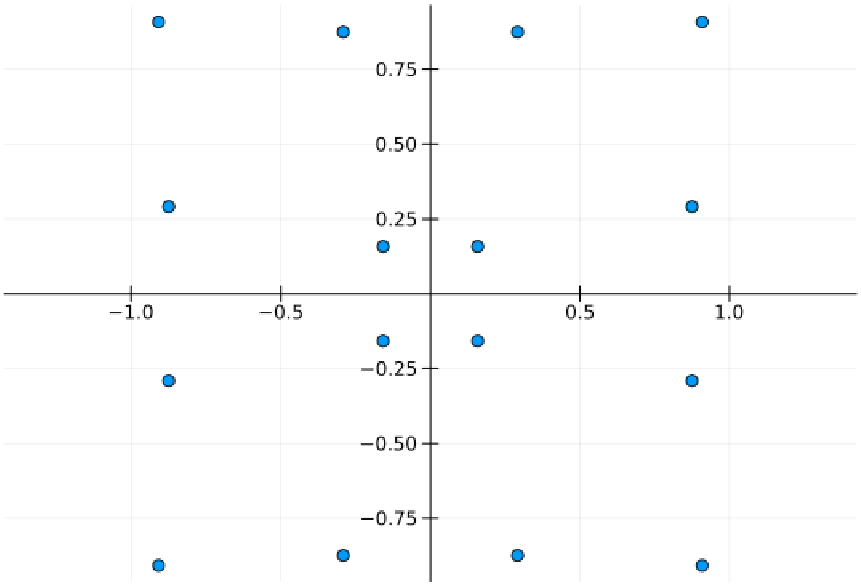

Figure 6: 16-QAM constellation diagram with $\varepsilon = 0.15$

16-CQAM

Assume the variables introduced in the former analysis in section 1 are the same. According to [1], we can approximate the error rate as follows

$$\mathrm{SER}_{\mathrm{CQAM}} = \frac{1}{N}\sum_{j=1}^{N} v(i) Q\left(\sqrt{\mathrm{SNR}\sin\frac{\pi}{n}\left(\sqrt{R_j^2-\varepsilon}\right)^{+}}\right). \tag{26}$$

16-sQAM

We will use the configuration from [2] with 4 spikes. On this constellation it only makes sense to harvest energy from the symbols corresponding to the spikes. Otherwise, the reason of existence of the spikes would be neutralized. Of course, it will have to be normalized to match the total harvested energy of the other constellations with constant ε across all their symbols. For clarity, this means if we have ℓ spikes and ε was the constant harvested energy of every single symbol on the rest of the constellations, then,

$$\varepsilon' = \varepsilon(\text{spike}) = \frac{\varepsilon M}{\ell}. \tag{27}$$

This is shown on the constellation diagram below. Note that the symbols not corresponding to spikes are identical, while energy has only be harvested from the spiked symbols.
For the theoretical calculation of the symbol error probability, we emply the approximation outlined in [2]. However we first need to calculate the PAPR. This is straight-forward as given the $d_{\min}$, we can find the PAPR of the unnormalized constellation corresponding to that minimum distance by reverse engineering the process in the theoretical analysis of the first section. This is essentially the energy of 4 corner symbols, which will also be spiked.
To achieve this we add the remaining energy to those corner elements to reach average energy $E_s = 1$ and then the PAPR $= |x_M|^2$, where x_M is one of the corner symbols. Then, we define

the following constants:

$$k = \left(\frac{(2(M-1)}{3} \right)^{-(1/2)} \tag{28}$$

$$\gamma = \frac{M - \ell \cdot \text{PAPR}}{M - \ell} \tag{29}$$

$$D_{\text{mid}} = k\sqrt{2M - 8\sqrt{M} + 8}. \tag{30}$$

Now we are ready to calculate the SER. Indeed,

$$\text{SER}_{\text{QAM}}^{\gamma} = 1 - \left[1 - Q\left(\sqrt{\frac{1.5 \log_2 M \cdot \text{SNR}}{M-1}\gamma} \right) \right]^2, \tag{31}$$

$$\text{SER}_{\text{QAM}}^{\max} = Q\left(\frac{|x_M| - D_{\text{mid}} - \varepsilon'}{\sqrt{2N_0}} \right), \tag{32}$$

Note the normalizing constant γ, as well as the extraction of energy from the spike symbols. Then,

$$\text{SER}_{\text{sQAM}} = \frac{M-\ell}{M}\text{SER}_{\text{QAM}}^{\gamma} + \frac{\ell}{M}\text{SER}_{\text{QAM}}^{\max} \tag{33}$$

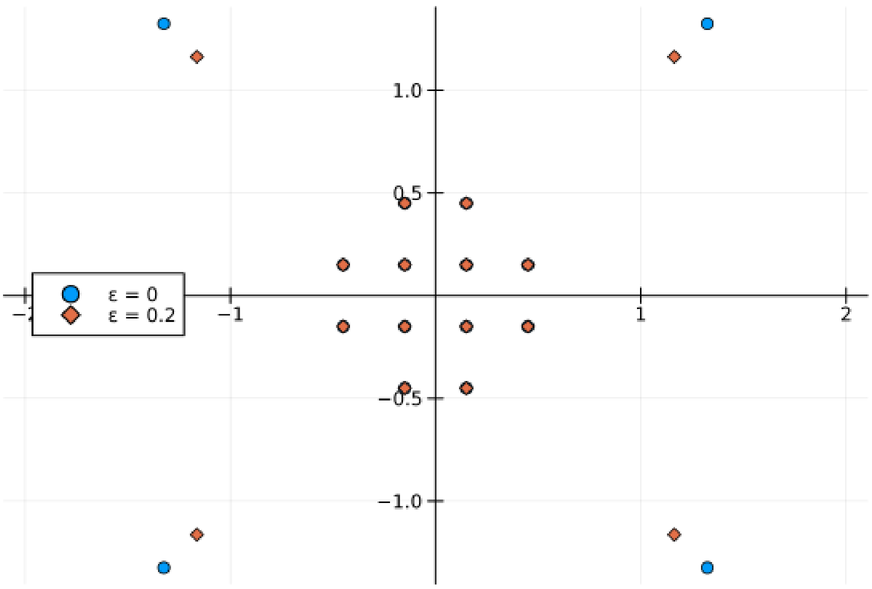

Figure 7: 16-sQAM constellation diagram of $d_{\min} = 0.3$ with $\varepsilon = 0$ and $\varepsilon = 0.2$

2.2 Simulation Results

We have devised a Monte-Carlo simulation environment where we repeat the same experiment for a specific number of attempts and log the ratio of the errors to the number of the attempts in each of the two cases.

SER over $\varepsilon(x)$

The results of SER over $\varepsilon(x)$ simulation are summarized in the following graph.
Excluding CQAM, the results are identical to those presented in [1].
Regarding sQAM, it outperforms all other modulation schemes up to the point that the spikes are completely degraded to the origin.

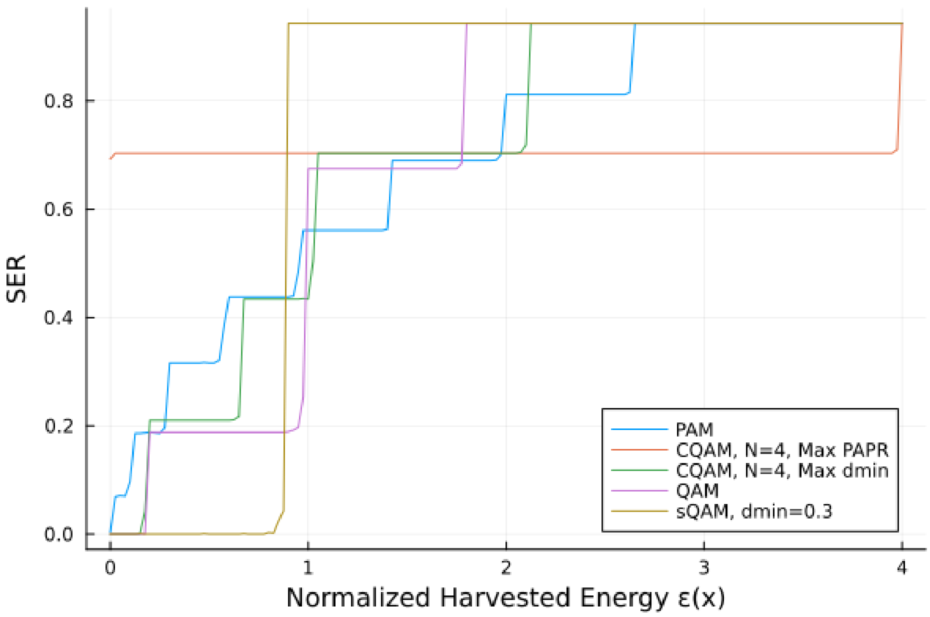

Figure 8: SER as a function of $\varepsilon(x)$ simulation graph

SER over SNR

The results of SER over SNR simulation are summarized in the following graph.

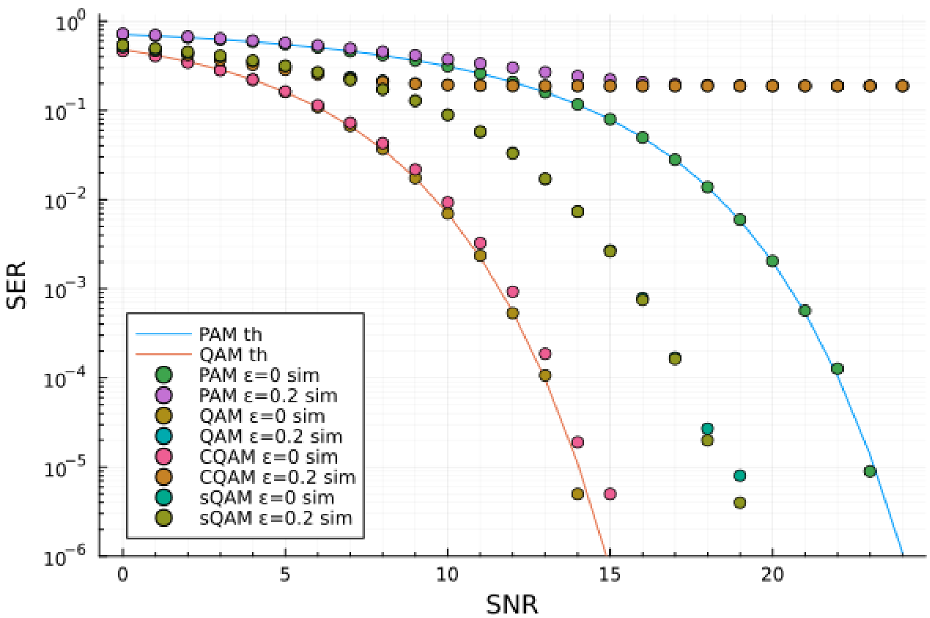

Figure 9: SER as a function of SNR simulation graph

Note that CQAM with harvesting performance is radically worse to the one in [1]. The other curves are very similar though. Moreover, notice that the perfomance of sQAM is not degraded at all despite the harvesting. This is to be expected when harvesting only from the spikes in a very high SNR environment.

3 16-Bee QAM (16-BQAM)

BQAM is a new modulation scheme optimized for SWIPT drawing inspiration from both CQAM and sQAM.

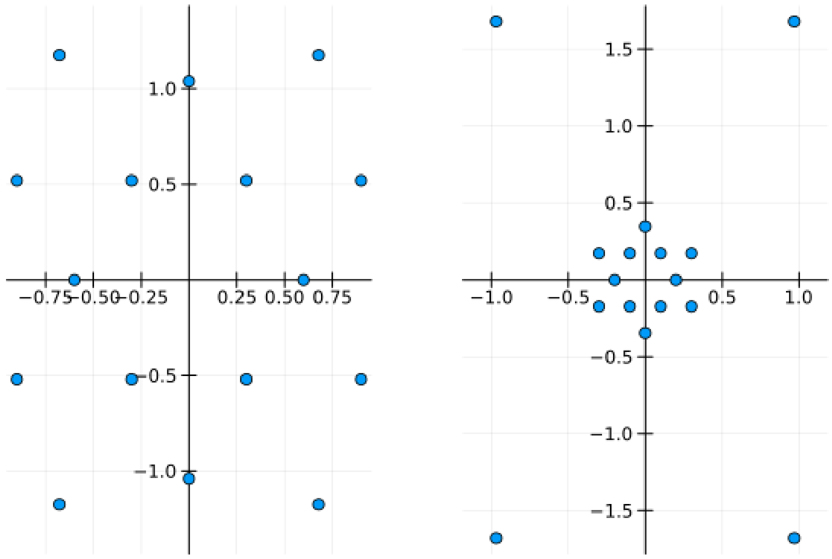

Figure 10: 16-BQAM constellations, a) with $d_{\min} = 0.6$ and b) with $d_{\min} = 0.2$

It consists of two 6-PSKs such that the distance between any two of their 12 symbols in total is $d_{\min}$. The remaining energy is used for the other 4 symbols, which form a 4-PSK with the maximal energy. The idea can be generalized for higher order modulations with $M = 4^k$ symbols, such that $M \equiv 4 \pmod 6$ with appropriate restrictions for $d_{\min}$.
We now repeat our former simulation analysis, but this time we include BQAM.

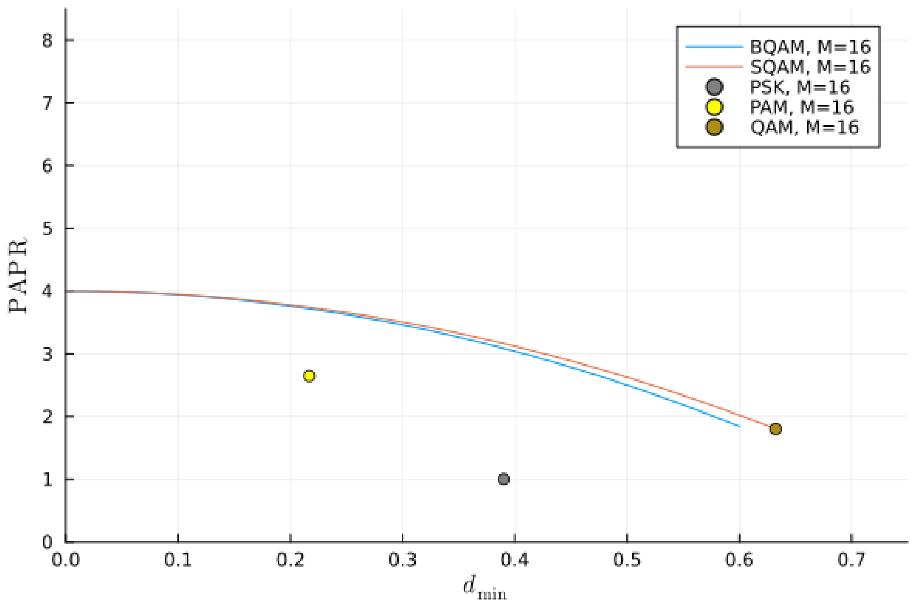

Figure 11: PAPR over $d_{\min}$ plot for BQAM

For all possible $d_{\min}$ the PAPRs of BQAM and sQAM are almost identical.

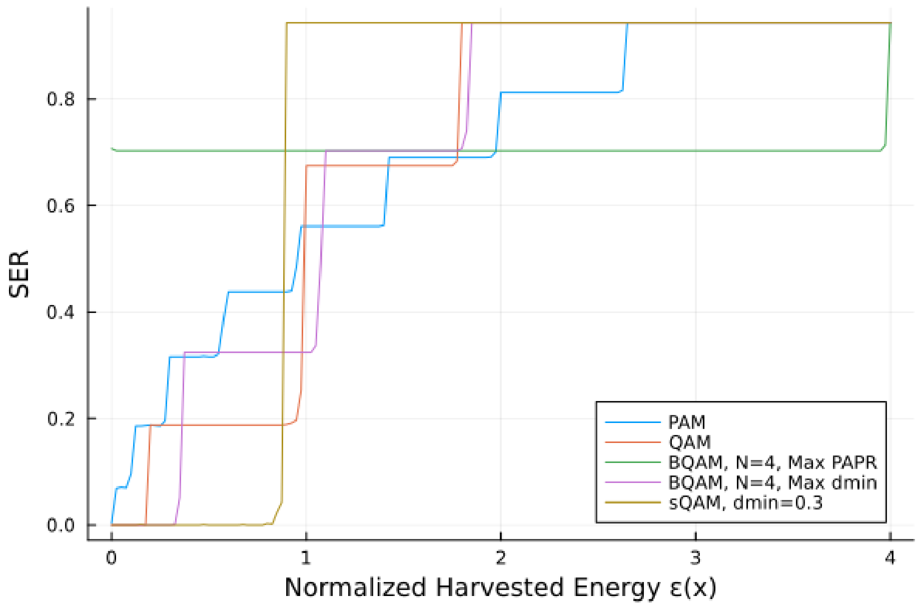

Figure 12: SER as function of $\varepsilon(x)$ graph with BQAM included

It is apparent that a tradeoff between $d_{\min}$ and PAPR can be made such that the performance of QAM is matched. Note that we are not harvesting energy exclusively on the spikes of BQAM; hence, the difference with sQAM which does exactly that.

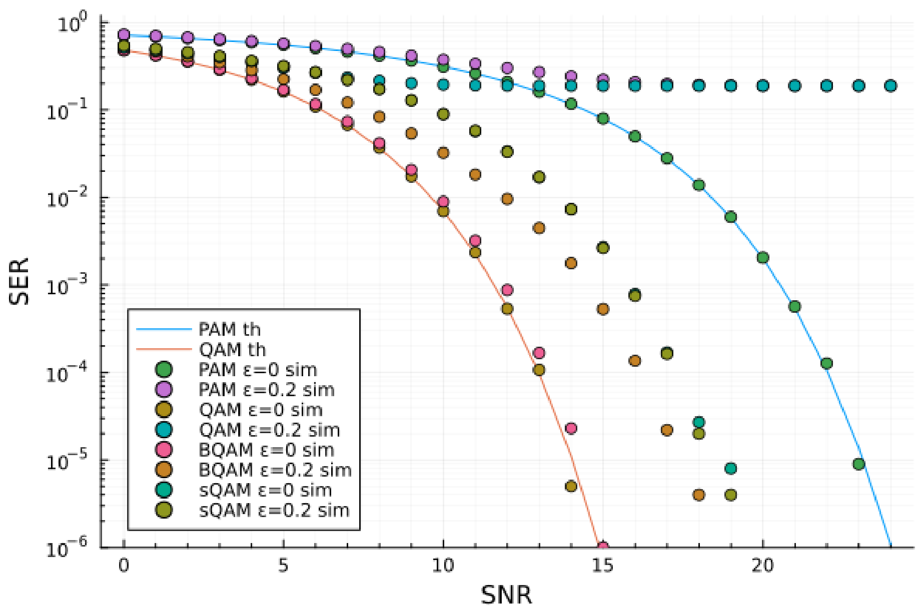

Figure 13: SER as function of $\varepsilon(x)$ graph with BQAM included

Note that BQAM has almost identical SER with QAM without energy harvesting. On the other hand it is cleary superior to all other modulation schemes including sQAM when energy harvesting is taken into account.
In conclusion, we have shown that in simulations BQAM is indeed better that other known modulation schemes. In future iterations we plan to perform a theoretical analysis for this constellation and further optimize it.

References

[1] G. M. Kraidy, C. Psomas, and I. Krikidis, "Fundamentals of circular qam for wireless information and power transfer," in *2021 IEEE 22nd International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2021, pp. 616–620.

[2] M. J. L. Morales, K. Chen-Hu, and A. G. Armada, "Optimum constellation for symbol-error-rate to papr ratio minimization in swipt," in *2022 IEEE 95th Vehicular Technology Conference: (VTC2022-Spring)*, 2022, pp. 1–5.