



## Homework 02

### Numpy Introduction

**1a) Create two numpy arrays (a and b). a should be all integers between 10-19 (inclusive), and b should be ten evenly spaced numbers between 1-7. Print all the results below:**

- i) Square all the elements in both arrays (element-wise)
- ii) Add both the squared arrays (e.g.,  $[1,2] + [3,4] = [4,6]$ )
- iii) Sum the elements with even indices of the added array.
- iv) Take the square root of the added array (element-wise square root)\_\_\_

```
In [13]: import numpy as np

a = np.arange(10,20)
print("i:")
print(a)
print()
b = np.linspace(1,7,10)
print(b)

print("\nnii:")

a = np.square(a)
print(a)
print()
b = np.square(b)
print(b)
print()
c = (a+b)
print(c)

print("\nniii:")

print(c[::2])
sum_c = c[::2]
sum_c = sum_c.sum()
print(sum_c)

print("\niv:")
sqrt_c = np.sqrt(c)
print(sqrt_c)

i:
[10 11 12 13 14 15 16 17 18 19]

[ 1.          1.66666667  2.33333333  3.          3.66666667  4.33333333
  5.          5.66666667  6.33333333  7.          ]

ii:
[100 121 144 169 196 225 256 289 324 361]

[ 1.          2.77777778  5.44444444  9.          13.44444444
 18.77777778 25.          32.11111111 40.11111111 49.          ]

[ 101.          123.77777778 149.44444444 178.          209.44444444
 243.77777778 281.          321.11111111 364.11111111 410.          ]

iii:
[ 101.          149.44444444 209.44444444 281.          364.11111111]
1105.0

iv:
[ 10.04987562  11.12554618  12.22474721  13.34166406  14.47219556
 15.61338457  16.76305461  17.91957341  19.08169571  20.24845673]
```

**1b) Append b to a, reshape the appended array so that it is a 5x4, 2d array and store the results in a variable called m. Print m.**

```
In [14]: m = np.append(a,b)
print(m)
m = np.reshape(np.ravel(m), (5,4))
print(m)
```

```
[ 100.      121.      144.      169.      196.      225.
  256.      289.      324.      361.      1.
   2.77777778  5.44444444  9.      13.44444444  18.77777778
  25.      32.11111111  40.11111111  49.      ]
[[ 100.      121.      144.      169.      ]
 [ 196.      225.      256.      289.      ]
 [ 324.      361.      1.      2.77777778]
 [ 5.44444444  9.      13.44444444  18.77777778]
 [ 25.      32.11111111  40.11111111  49.      ]]
```

1c) Extract the second and the third column of the m matrix. Store the resulting 5x2 matrix in a new variable called m2. Print m2.

```
In [15]: m2 = m[:, [1, 2]]
print(m2)
```

```
[ [ 121.      144.      ]
  [ 225.      256.      ]
  [ 361.      1.      ]
  [ 9.      13.44444444]
  [ 32.11111111  40.11111111]]
```

1d) Take the dot product of m2 and m store the results in a matrix called m3. Print m3. Note that Dot product of two matrices  $A \cdot B = A^T B$

```
In [26]: m3 = np.dot(m.T,m2)
m3
```

```
Out[26]: array([[ 174015.77777778,  65975.97530864],
 [ 196699.12345679,  76794.01234568],
 [ 76794.01234568,  88062.65432099],
 [ 88219.22222222, 100540.67901235]])
```

1e) Round the m3 matrix to two decimal points. Store the result in place and print the new m3.

```
In [27]: m3 = np.around(m3,2)
print(m3)
```

```
[ [ 174015.78  65975.98]
  [ 196699.12  76794.01]
  [ 76794.01  88062.65]
  [ 88219.22 100540.68]]
```

1f) Sort the m3 array so that the highest value is at the top left, the next highest value to the right of the highest, and the lowest value is at the bottom right. Print the sorted m3 array.

```
In [28]: m4 = np.sort(m3, axis=None)
m4 = np.reshape(np.ravel(m4[::-1]),(4,2))
m4
```

```
Out[28]: array([[ 196699.12, 174015.78],
 [ 100540.68,  88219.22],
 [ 88062.65,  76794.01],
 [ 76794.01,  65975.98]])
```

## NumPy and Masks

2a) create an array called 'f' where the values are sin(x) for x from 0 to pi with 100 values in f

- print f
- use a 'mask' and print an array that is True when  $f \geq 1/2$  and False when  $f < 1/2$
- create and print an array sequence that has only those values where  $f \geq 1/2$

```
In [8]: f = np.array(np.sin(np.linspace(0,np.pi,100)))
print(f)

print()

mask = f>=(1/2)
print(mask)
```

```
print()

true_array = f[mask]
print(true_array)
```

```
[ 0.00000000e+00  3.17279335e-02  6.34239197e-02  9.50560433e-02
 1.26592454e-01  1.58001396e-01  1.89251244e-01  2.20310533e-01
 2.51147987e-01  2.81732557e-01  3.12033446e-01  3.42020143e-01
 3.71662456e-01  4.00930535e-01  4.29794912e-01  4.58226522e-01
 4.86196736e-01  5.13677392e-01  5.40640817e-01  5.67059864e-01
 5.92907929e-01  6.18158986e-01  6.42787610e-01  6.66769001e-01
 6.90079011e-01  7.12694171e-01  7.34591709e-01  7.55749574e-01
 7.76146464e-01  7.95761841e-01  8.14575952e-01  8.32569855e-01
 8.49725430e-01  8.66025404e-01  8.81453363e-01  8.95993774e-01
 9.09631995e-01  9.22354294e-01  9.34147860e-01  9.45000819e-01
 9.54902241e-01  9.63842159e-01  9.71811568e-01  9.78802446e-01
 9.84807753e-01  9.89821442e-01  9.93838464e-01  9.96854776e-01
 9.98867339e-01  9.99874128e-01  9.99874128e-01  9.98867339e-01
 9.96854776e-01  9.93838464e-01  9.89821442e-01  9.84807753e-01
 9.78802446e-01  9.71811568e-01  9.63842159e-01  9.54902241e-01
 9.45000819e-01  9.34147860e-01  9.22354294e-01  9.09631995e-01
 8.95993774e-01  8.81453363e-01  8.66025404e-01  8.49725430e-01
 8.32569855e-01  8.14575952e-01  7.95761841e-01  7.76146464e-01
 7.55749574e-01  7.34591709e-01  7.12694171e-01  6.90079011e-01
 6.66769001e-01  6.42787610e-01  6.18158986e-01  5.92907929e-01
 5.67059864e-01  5.40640817e-01  5.13677392e-01  4.86196736e-01
 4.58226522e-01  4.29794912e-01  4.00930535e-01  3.71662456e-01
 3.42020143e-01  3.12033446e-01  2.81732557e-01  2.51147987e-01
 2.20310533e-01  1.89251244e-01  1.58001396e-01  1.26592454e-01
 9.50560433e-02  6.34239197e-02  3.17279335e-02  1.22464680e-16]
```

```
[False False False False False False False False False False False False
 False False False False False True True True True True True True
 True True True True True True True True True True True True True
 True True True True True True True True True True True True True
 True True True True True True True True True True True True True
 True True True True True True True True True True True True False
 False False False False False False False False False False False False
 False False False False]
```

```
[ 0.51367739  0.54064082  0.56705986  0.59290793  0.61815899  0.64278761
 0.666769    0.69007901  0.71269417  0.73459171  0.75574957  0.77614646
 0.79576184  0.81457595  0.83256985  0.84972543  0.8660254    0.88145336
 0.89599377  0.909632    0.92235429  0.93414786  0.94500082  0.95490224
 0.96384216  0.97181157  0.97880245  0.98480775  0.98982144  0.99383846
 0.99685478  0.99886734  0.99987413  0.99987413  0.99886734  0.99685478
 0.99383846  0.98982144  0.98480775  0.97880245  0.97181157  0.96384216
 0.95490224  0.94500082  0.93414786  0.92235429  0.909632    0.89599377
 0.88145336  0.8660254    0.84972543  0.83256985  0.81457595  0.79576184
 0.77614646  0.75574957  0.73459171  0.71269417  0.69007901  0.666769
 0.64278761  0.61815899  0.59290793  0.56705986  0.54064082  0.51367739]
```

## NumPy and 2 Variable Prediction

Let 'x' be the number of miles a person drives per day and 'y' be the dollars spent on buying car fuel (per day).

We have created 2 numpy arrays each of size 100 that represent x and y.

x ( number of miles) ranges from 1 to 10 with a uniform noise of (0,1/2)

y (money spent in dollars) will be from 1 to 20 with a uniform noise (0,1)

```
In [32]: # seed the random number generator with a fixed value
import numpy as np
np.random.seed(500)

x=np.linspace(1,10,100)+ np.random.uniform(low=0,high=.5,size=100)
y=np.linspace(1,20,100)+ np.random.uniform(low=0,high=1,size=100)
print ('x = ',x)
print ('y= ',y)
```

```
x = [ 1.34683976  1.12176759  1.51512398  1.55233174  1.40619168
 1.65075498  1.79399331  1.80243817  1.89844195  2.00100023
 2.3344038  2.22424872  2.24914511  2.36268477  2.49808849
 2.8212704  2.68452475  2.68229427  3.09511169  2.95703884
 3.09047742  3.2544361  3.41541904  3.40886375  3.50672677
 3.74960644  3.64861355  3.7721462  3.56368566  4.01092701
 4.15630694  4.06088549  4.02517179  4.25169402  4.15897504
```

```

4.26835333 4.32520644 4.48563164 4.78490721 4.84614839
4.96698768 5.18754259 5.29582013 5.32097781 5.0674106
5.47601124 5.46852704 5.64537452 5.49642807 5.89755027
5.68548923 5.76276141 5.94613234 6.18135713 5.96522091
6.0275473 6.54290191 6.4991329 6.74003765 6.81809807
6.50611821 6.91538752 7.01250925 6.89905417 7.31314433
7.20472297 7.1043621 7.48199528 7.58957227 7.61744354
7.6991707 7.85436822 8.03510784 7.80787781 8.22410224
7.99366248 8.40581097 8.28913792 8.45971515 8.54227144
8.6906456 8.61856507 8.83489887 8.66309658 8.94837987
9.20890222 8.9614749 8.92608294 9.13231416 9.55889896
9.61488451 9.54252979 9.42015491 9.90952569 10.00659591
10.02504265 10.07330937 9.93489915 10.0892334 10.36509991]
y= [ 1.6635012 2.0214592 2.10816052 2.26016496 1.96287558
2.9554635 3.02881887 3.33565296 2.75465779 3.4250107
3.39670148 3.39377767 3.78503343 4.38293049 4.32963586
4.03925039 4.73691868 4.30098399 4.8416329 4.78175957
4.99765787 5.31746817 5.76844671 5.93723749 5.72811642
6.70973615 6.68143367 6.57482731 7.17737603 7.54863252
7.30221419 7.3202573 7.78023884 7.91133365 8.2765417
8.69203281 8.78219865 8.45897546 8.89094715 8.81719921
8.87106971 9.66192562 9.4020625 9.85990783 9.60359778
10.07386266 10.6957995 10.66721916 11.18256285 10.57431836
11.46744716 10.94398916 11.26445259 12.09754828 12.11988037
12.121557 12.17613693 12.43750193 13.00912372 12.86407194
13.24640866 12.76120085 13.11723062 14.07841099 14.19821707
14.27289001 14.30624942 14.63060835 14.2770918 15.0744923
14.45261619 15.11897313 15.2378667 15.27203124 15.32491892
16.01095271 15.71250558 16.29488506 16.70618934 16.56555394
16.42379457 17.18144744 17.13813976 17.69613625 17.37763019
17.90942839 17.90343733 18.01951169 18.35727914 18.16841269
18.61813748 18.66062754 18.81217983 19.44995194 19.7213867
19.71966726 19.78961904 19.64385088 20.69719809 20.07974319]

```

### 3a) Find Expected value of x and the expected value of y

```

In [33]: E_x = np.mean(x)
         print(E_x)

         print()

         E_y = np.mean(y)
         print(E_y)

5.78253254159

11.0129816833

```

### 3b) Find variance of distributions of x and y

```

In [34]: E_x = np.var(x)
         print(E_x)

7.03332752948

In [35]: E_y = np.var(y)
         print(E_y)

30.1139035755

```

### 3c) Find co-variance of x and y.

```

In [36]: Cov_xy = np.cov(x,y)
         print(Cov_xy)

[[ 7.10437124 14.65774383]
 [14.65774383 30.41808442]]

```

### 3d) Assuming that number of dollars spent in car fuel is only dependant on the miles driven, by a linear relationship. Write code that uses a linear predictor to calculate a predicted value of y for each x ie $y_{\text{predicted}} = f(x) = y_0 + mx$ .

```

In [37]: x_trp = np.vstack([x, np.ones(len(x))]).T
         print(x_trp)
         m, c = np.linalg.lstsq(x_trp, y)[0]
         print()
         print(m,c)

```

```

[[ 1.34683976  1.      ]
 [ 1.12176759  1.      ]
 [ 1.51512398  1.      ]
 [ 1.55233174  1.      ]
 [ 1.40619168  1.      ]
 [ 1.65075498  1.      ]
 [ 1.79399331  1.      ]
 [ 1.80243817  1.      ]
 [ 1.89844195  1.      ]
 [ 2.00100023  1.      ]
 [ 2.3344038   1.      ]
 [ 2.22424872  1.      ]
 [ 2.24914511  1.      ]
 [ 2.36268477  1.      ]
 [ 2.49808849  1.      ]
 [ 2.8212704   1.      ]
 [ 2.68452475  1.      ]
 [ 2.68229427  1.      ]
 [ 3.09511169  1.      ]
 [ 2.95703884  1.      ]
 [ 3.09047742  1.      ]
 [ 3.2544361   1.      ]
 [ 3.41541904  1.      ]
 [ 3.40886375  1.      ]
 [ 3.50672677  1.      ]
 [ 3.74960644  1.      ]
 [ 3.64861355  1.      ]
 [ 3.7721462   1.      ]
 [ 3.56368566  1.      ]
 [ 4.01092701  1.      ]
 [ 4.15630694  1.      ]
 [ 4.06088549  1.      ]
 [ 4.02517179  1.      ]
 [ 4.25169402  1.      ]
 [ 4.15897504  1.      ]
 [ 4.26835333  1.      ]
 [ 4.32520644  1.      ]
 [ 4.48563164  1.      ]
 [ 4.78490721  1.      ]
 [ 4.84614839  1.      ]
 [ 4.96698768  1.      ]
 [ 5.18754259  1.      ]
 [ 5.29582013  1.      ]
 [ 5.32097781  1.      ]
 [ 5.0674106   1.      ]
 [ 5.47601124  1.      ]
 [ 5.46852704  1.      ]
 [ 5.64537452  1.      ]
 [ 5.49642807  1.      ]
 [ 5.89755027  1.      ]
 [ 5.68548923  1.      ]
 [ 5.76276141  1.      ]
 [ 5.94613234  1.      ]
 [ 6.18135713  1.      ]
 [ 5.96522091  1.      ]
 [ 6.0275473   1.      ]
 [ 6.54290191  1.      ]
 [ 6.4991329   1.      ]
 [ 6.74003765  1.      ]
 [ 6.81809807  1.      ]
 [ 6.50611821  1.      ]
 [ 6.91538752  1.      ]
 [ 7.01250925  1.      ]
 [ 6.89905417  1.      ]
 [ 7.31314433  1.      ]
 [ 7.20472297  1.      ]
 [ 7.1043621   1.      ]
 [ 7.48199528  1.      ]
 [ 7.58957227  1.      ]
 [ 7.61744354  1.      ]
 [ 7.6991707   1.      ]
 [ 7.85436822  1.      ]
 [ 8.03510784  1.      ]
 [ 7.80787781  1.      ]
 [ 8.22410224  1.      ]
 [ 7.99366248  1.      ]
 [ 8.40581097  1.      ]
 [ 8.28913792  1.      ]
 [ 8.45971515  1.      ]
 [ 8.45971515  1.      ]

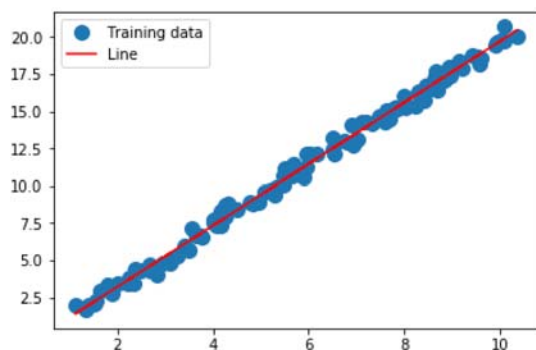
```

```
[ 8.54227144  1. ]
[ 8.6906456  1. ]
[ 8.61856507  1. ]
[ 8.83489887  1. ]
[ 8.66309658  1. ]
[ 8.94837987  1. ]
[ 9.20890222  1. ]
[ 8.9614749  1. ]
[ 8.92608294  1. ]
[ 9.13231416  1. ]
[ 9.55889896  1. ]
[ 9.61488451  1. ]
[ 9.54252979  1. ]
[ 9.42015491  1. ]
[ 9.90952569  1. ]
[ 10.00659591  1. ]
[ 10.02504265  1. ]
[ 10.07330937  1. ]
[ 9.93489915  1. ]
[ 10.0892334  1. ]
[ 10.36509991  1. ]]
```

```
2.06320071597 -0.917543596587
```

```
In [38]: import matplotlib.pyplot as plt

plt.plot(x, y, 'o', label='Training data', markersize=10)
plt.plot(x, m*x + c, 'r', label='Line')
plt.legend()
plt.show()
```



3e) Predict y for each value in x, put the error into an array called y\_error

```
In [39]: y_predict = c + m*x
print(y_predict)
```

```
[ 1.86125717  1.39688809  2.20846128  2.28522836  1.98371207
  2.48829527  2.78382468  2.80124813  2.9993232  3.21092152  3.8988
  3.67152796  3.7228942  3.9571493  4.23651436  4.9033035
  4.62116978  4.61656787  5.46829307  5.18342105  5.45873164
  5.79701128  6.12915141  6.11562653  6.31753758  6.81864709
  6.61027849  6.86515115  6.43505522  7.35780389  7.65775187
  7.46087825  7.38719373  7.85455455  7.66325667  7.88892606
  8.00622544  8.33721481  8.95468038  9.08103323  9.33034895
  9.78539799  10.00879629  10.06070164  9.53754157  10.38056671
  10.36512531  10.72999716  10.42269073  11.25028634  10.81276185
  10.97218988  11.35052091  11.83583685  11.38990445  11.51849632
  12.58177632  12.49147206  12.98850691  13.14956122  12.50588416
  13.35028889  13.5506705  13.31658991  14.17094102  13.947246
  13.74018137  14.51931443  14.74126735  14.79877137  14.96739089
  15.28759454  15.66049665  15.1916755  16.05043004  15.57498655
  16.42533161  16.18461169  16.53654675  16.70687695  17.01300263
  16.86428603  17.31062607  16.95616347  17.54476017  18.08227006
  17.57177784  17.49875711  17.92425351  18.80438359  18.91989301
  18.77061069  18.51812677  19.5277969  19.72807224  19.76613158
  19.8657155  19.58014745  19.89856998  20.46773797]
```

```
In [40]: y_error = y - y_predict
print(y_error)
```

```
[-0.19775597  0.62457111 -0.10030076 -0.02506341 -0.02083649  0.46716823
  0.24499418  0.53440482 -0.24466541  0.21408918 -0.50209852 -0.27775029
  0.06213923  0.42578118  0.0931215  -0.86405311  0.1157489  -0.31558388
  -0.67666017 -0.10166110 -0.16107277 -0.17054211 -0.36070017 -0.17828001]
```

```
-0.02000017 -0.40100143 -0.40107377 -0.47534311 -0.5007047 -0.17030304
-0.58942116 -0.10891094 0.07115518 -0.29032384 0.74232081 0.19082863
-0.35553767 -0.14062095 0.39304511 0.0567791 0.61328502 0.80310676
0.77597321 0.12176065 -0.06373323 -0.26383402 -0.45927925 -0.12347238
-0.60673379 -0.20079382 0.0660562 -0.30670405 0.33067419 -0.062778
0.75987212 -0.67596798 0.65468531 -0.02820071 -0.08606832 0.26171143
0.72997592 0.60306068 -0.40563939 -0.05397013 0.02061681 -0.28548928
0.7405245 -0.58908804 -0.43343988 0.76182107 0.02727604 0.32564401
0.56606805 0.11129392 -0.46417555 0.27572093 -0.5147747 -0.16862142
-0.42262995 0.08035574 -0.72551112 0.43596616 -0.71282602 0.11027337
0.16964259 -0.14132301 -0.58920807 0.31716141 -0.17248631 0.73997278
-0.16712997 -0.17284167 0.33165948 0.52075457 0.43302563 -0.6359709
-0.30175553 -0.10998314 0.29405306 -0.07784496 -0.00668554 -0.04646431
-0.07609646 0.06370343 0.79862812 -0.38799477]
```

3f) Write code that calculates the root mean square error(RMSE), that is root of average of y-error squared

```
In [43]: RMSE=np.sqrt(((y_error**2).sum())/np.size(y_error))
print(RMSE)
```

```
0.417677723669
```

```
In [ ]:
```

```
In [ ]:
```