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Communication Systems II

Constellation Design for Simultaneous Wireless Information and Power Transfer (SWIPT)

1 Task 1

We will first provide the theoretical analysis about PAPR (Peak to Average Power Ratio) behavior versus $d_{\min}$ of 16-Circular QAM compared with known modulation schemes, namely 16-PAM, 16-PSK, and 16-QAM. Then we will provide the correspoding simulation results. PAPR equation can be seen in equation (1).

$$PAPR = \frac{P_{\max}}{P_{avrg}} = \frac{E_{\max}}{E_{avrg}} \tag{1}$$

1.1 16-PAM

The average energy of an M-PAM constellation can be found using equation (2):

$$E_s = \frac{E_g \cdot (M^2 - 1)}{3} \tag{2}$$

And the maximum energy of an M-PAM constellation is the energy of the symbol with the greater euclidean distance from the origin. The energy of this symbol can be found as shown in equation (3):

$$E_{\max} = ((2M - M - 1) \cdot \sqrt{E_g})^2 = (M - 1)^2 \cdot E_g \tag{3}$$

So PAPR will be:

$$PAPR = \frac{E_{\max}}{E_s} = \frac{3 \cdot (M - 1)^2}{(M^2 - 1)} \tag{4}$$

We can notice that $PAPR$ is independent of $d_{\min}$

We also know that $d_{\min} = 2 \cdot \sqrt{E_g}$.

We considered multiple values for $d_{\min}$ with $d_{\min,begin} = 0$ to $d_{\min,end} = 0.7$. And we calculated E_g for each value of d_{min}. However, since $PAPR$ is independent of $d_{\min}$ it's value will be constant for a 16-PAM constellation. Thus, we will have $PAPR = 2.6471$.

The above theoretical analysis can be confirmed through simulation, whose results can be seen in Figure 1.

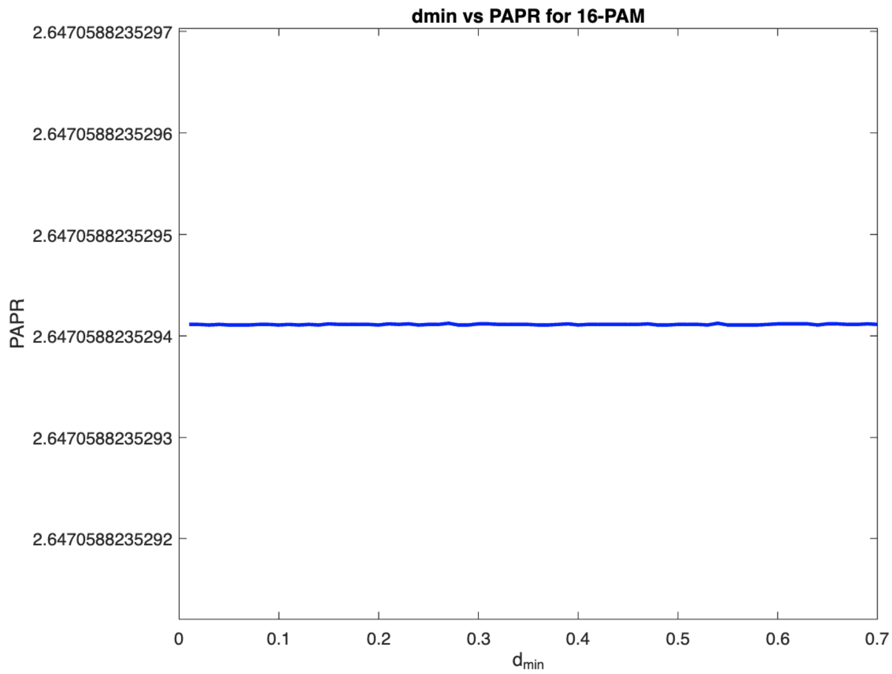

Figure 1: PAPR vs $d_{\min}$ for 16-PAM

1.2 16-PSK

We know that a PSK constellation has as it's attribute that all it's symbols have the same energy, since they are placed at a circle of radius $\sqrt{E_s}$. So the relationship between $E_{\max}$ and E_{avrg} will be:

$$E_{\max} = E_{avrg} \tag{5}$$

Thus, $PAPR = 1$, independent of $d_{\min}$, which is given by equation (6):

$$d_{\min} = 2 \cdot \sqrt{E_s} \cdot \sin(\frac{\pi}{M}) \tag{6}$$

The above theoretical analysis can be confirmed through simulation, whose results can be seen in Figure 2.

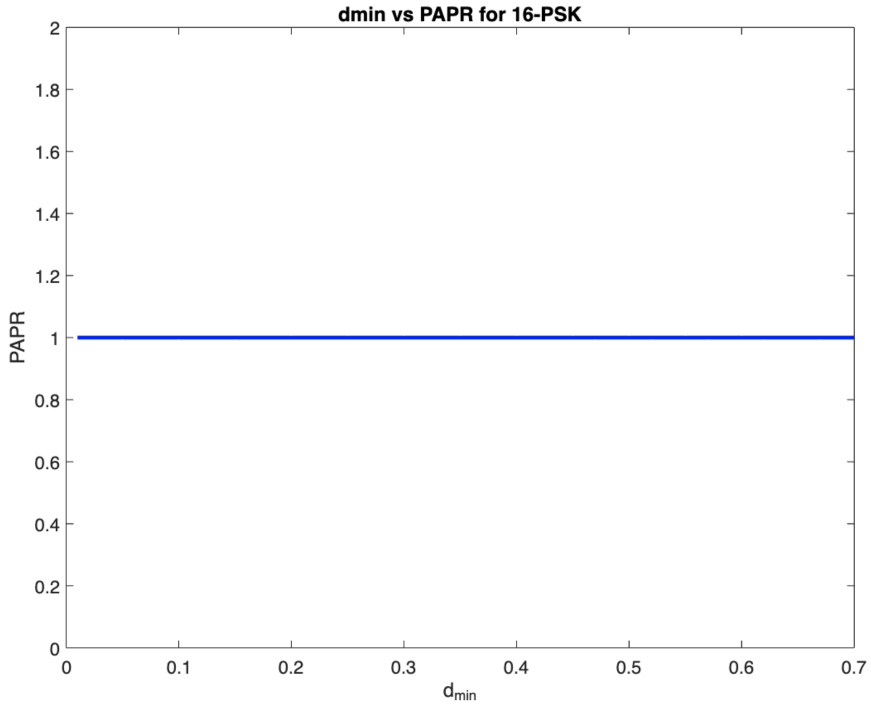

Figure 2: PAPR vs $d_{\min}$ for 16-PSK

1.3 16-QAM

We considered a $4x4$ squared QAM constellation which can be seen in Figure 3 and whose relationship between $d_{\min}$ and E_s can be found in equation (7).

$$d_{\min} = \sqrt{\frac{6 \cdot E_s}{M-1}} \tag{7}$$

Thus, given $d_{\min}$ we can compute the average energy $E_s = \frac{d_{\min}^2 \cdot (M-1)}{6}$.

Moreover $E_{\max}$ can be found from the symbol with the greater euclidean distance from the origin which are the symbols A, D, M, P in Figure 3 and can be seen in equation (8):

$$E_{\max} = x_{\max}^2 + y_{\max}^2 = \left(\frac{3d_{\min}}{2}\right)^2 + \left(\frac{3d_{\min}}{2}\right)^2 = \frac{9 \cdot d_{\min}^2}{2} \tag{8}$$

Thus, $PAPR$ will be:

$$PAPR = \frac{E_{\max}}{E_s} = \frac{27}{M-1} \tag{9}$$

We can notice that $PAPR$ is again independent of $d_{\min}$. For $M = 16$, we will have $PAPR = 1.8$, which can be confirmed through simulation, as shown in Figure 4.

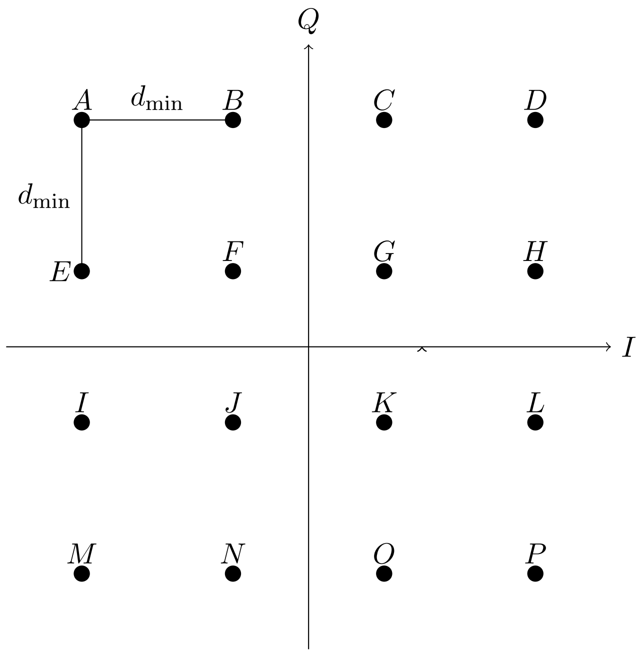

Figure 3: 16-QAM Squared Constellation

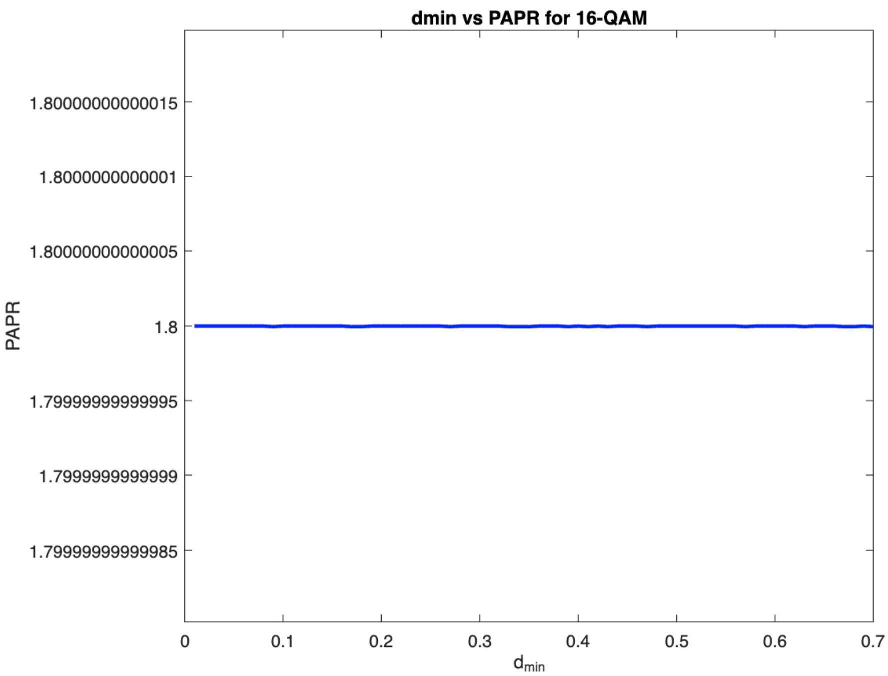

Figure 4: PAPR vs $d_{\min}$ for 16-QAM

1.4 16-CQAM

1.4.1 $M = 16, N = 4$

A 16-QAM constellation with $M = 16$ symbols and $N = 4$ circles , meaning $n = \frac{M}{N} = 4$ symbols per circle is presented in Figure 5

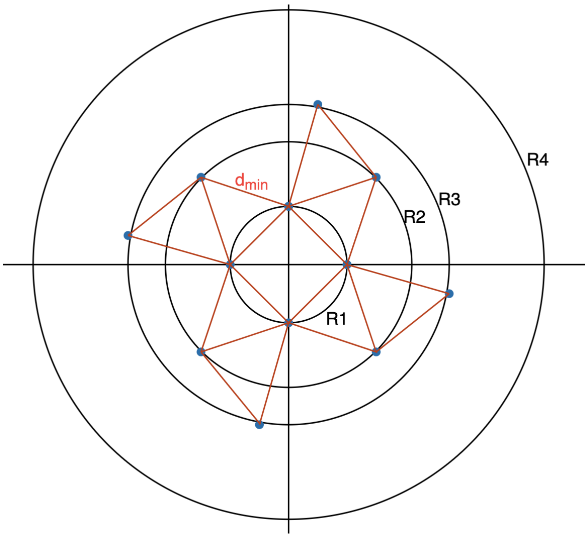

Figure 5: 16-CQAM Constellation with N = 4 circles

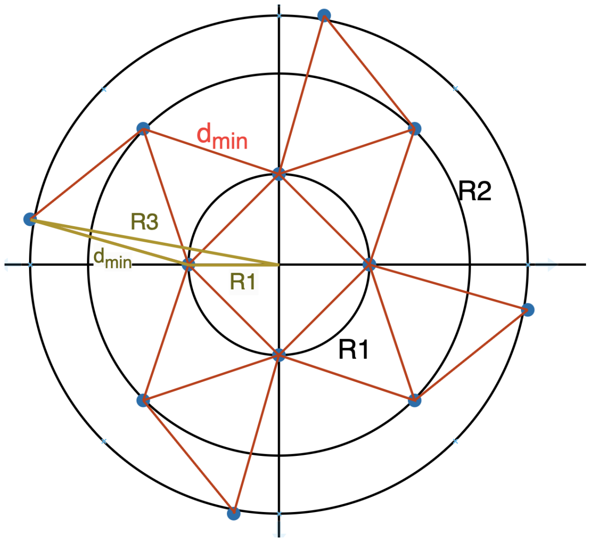

Figure 6: Calculation of radii R_3 for 16-CQAM

Each symbol is distanced from an adjacent symbol distance d_{min}.
In order to calculate the radii R_1, R_2, R_3 we used the following formulas:

$$R_1 = \frac{d_{\text{min}}}{2\sin(\frac{\pi}{n})} \tag{10}$$

$$R_2 = R_1 \cos\left(\frac{\pi}{n}\right) + \frac{\sqrt{3}d_{\text{min}}}{2} \tag{11}$$

$$R_3 = \sqrt{d_{\text{min}} + R_1^2 - 2d_{\text{min}}R_1 \cos\left(\frac{11\pi}{12}\right)} \tag{12}$$

Calculation of R_3 can be seen in Figure 6 where we use the law of cosines. The angle opposite of the R_3 side can be easily calculated as $\theta_3 = 60^\circ + 60^\circ + 45^\circ = \frac{11\pi}{12}$ since we have two equilateral triangles and one isosceles triangle, whose angles make angle θ_3.

Calculation of R_2 can be seen in Figure 7 where $x = R_1 \cos(\frac{\pi}{4})$ and $y = \sqrt{d_{\text{min}}^2 - \frac{d_{\text{min}}^2}{4}} = \frac{\sqrt{3}d_{\text{min}}}{2}$. So $R_2 = x + y$.

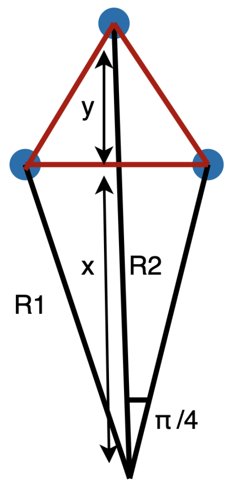

Figure 7: Calculation of radii R_2 for 16-CQAM

In order to calculate radii R_4 we should first find the energy of each circle (or enegy level) with radii R_1, R_2, R_3, where $E_1 = R_1^2$, $E_2 = R_2^2$, $E_3 = R_3^2$. Since we want average energy equal to one we can easily find E_4 by using the following formula:

$$E_4 = R_4^2 = 4 - E_1 - E_2 - E_3 \tag{13}$$

Thus, we can find: $PAPR = E_4$. The correspoding plot of PAPR vs $d_{\min}$ can be seen in Figure 8

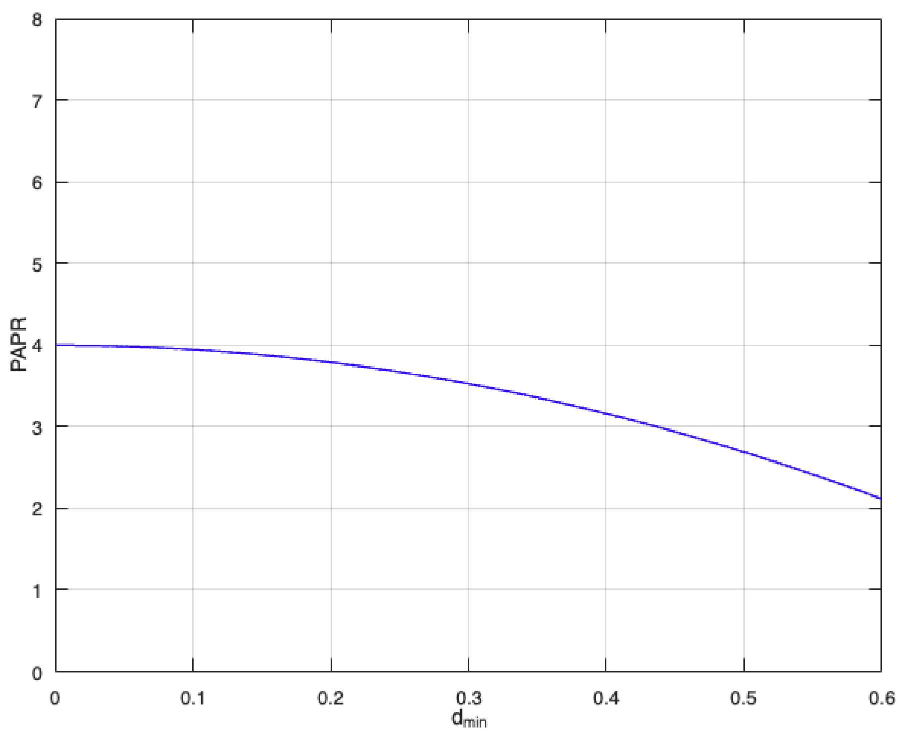

Figure 8: PAPR vs $d_{\min}$ plot for 16-CQAM with $M = 16$ and $N = 4$

1.4.2 $M = 16, N = 8$

A 16-QAM constellation with $M = 16$ symbols and $N = 8$ circles , meaning $n = \frac{M}{N} = 2$ symbols per circle is presented in Figure 9

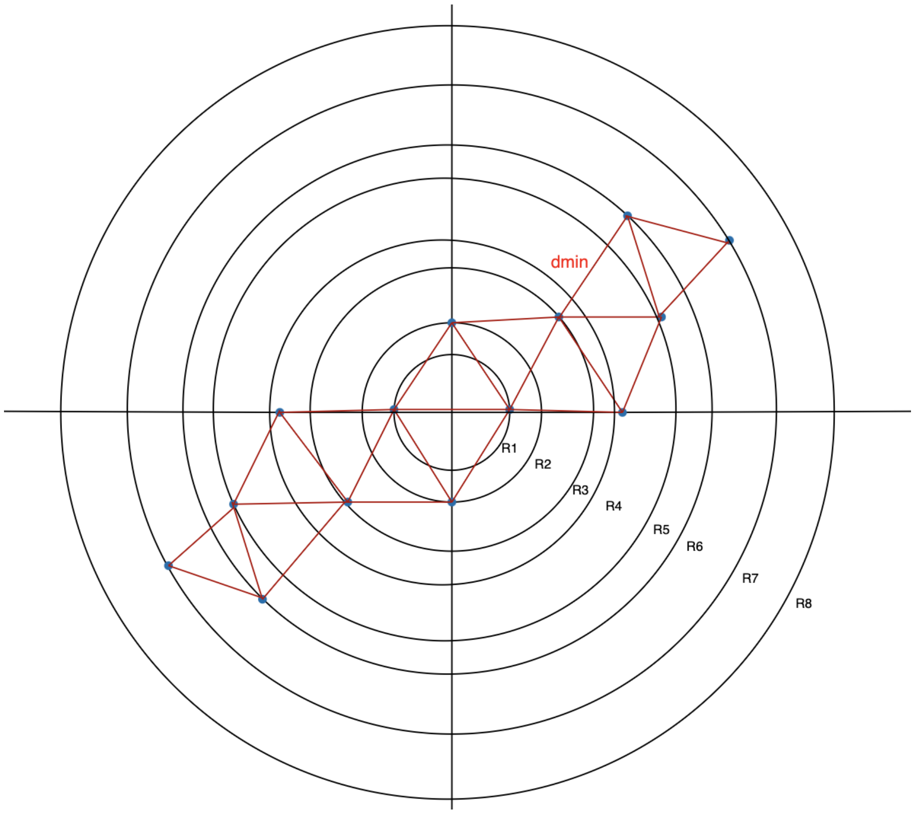

Figure 9: 16-CQAM Constellation with N = 8 circles

Each symbol is distanced from an adjacent symbol distance $d_{\min}$.
In order to calculate the radii R_1, R_2, R_3, R_4, R_5, R_6, R_7 we used the following formulas:

$$R_1 = \frac{d_{\min}}{2} \tag{14}$$

$$R_2 = \frac{\sqrt{3}d_{\min}}{2} \tag{15}$$

$$R_3 = \sqrt{d_{\min}^2 + R_1^2 + -2d_{\min}R_1 \cos\left(\frac{2\pi}{3}\right)} \tag{16}$$

$$R_4 = d_{\min} + R1 \tag{17}$$

$$R_5 = \sqrt{4d_{\min}^2 + R_2^2} \tag{18}$$

$$R_6 = \sqrt{(2d_{\min})^2 + R_1^2 + -2(2d_{\min})R_1 \cos\left(\frac{2\pi}{3}\right)} \tag{19}$$

$$R_7 = 2d_{\min} + R_1 \tag{20}$$

Radii R_i are calculated using the same intuition as in 16-CQAM with $N = 4$. For example, the calculations for radii R_3, R_4, R_5 can be seen in Figure 10

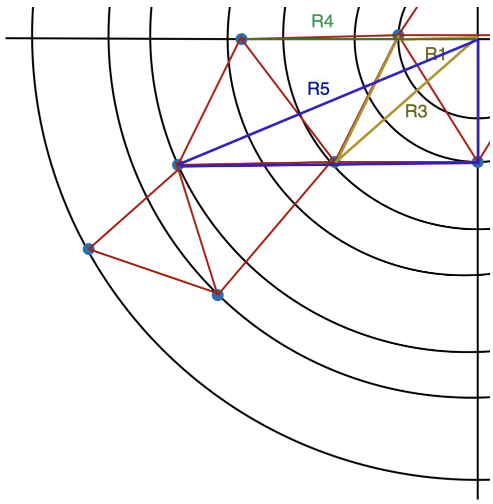

Figure 10: PAPR vs $d_{\min}$ plot for 16-CQAM with $M = 16$ and $N = 8$

In order to calculate radii R_8 we should first find the energy of each circle (or enegy level) with radii R_i, where $E_i = R_i^2$. Since we want average energy equal to one we can easily find E_8 by using the following formula:

$$E_8 = R_8^2 = 8 - \sum_{i=1}^{7} E_i \tag{21}$$

Thus, we can find: $PAPR = E_8$. The correspoding plot of PAPR vs $d_{\min}$ can be seen in Figure 11

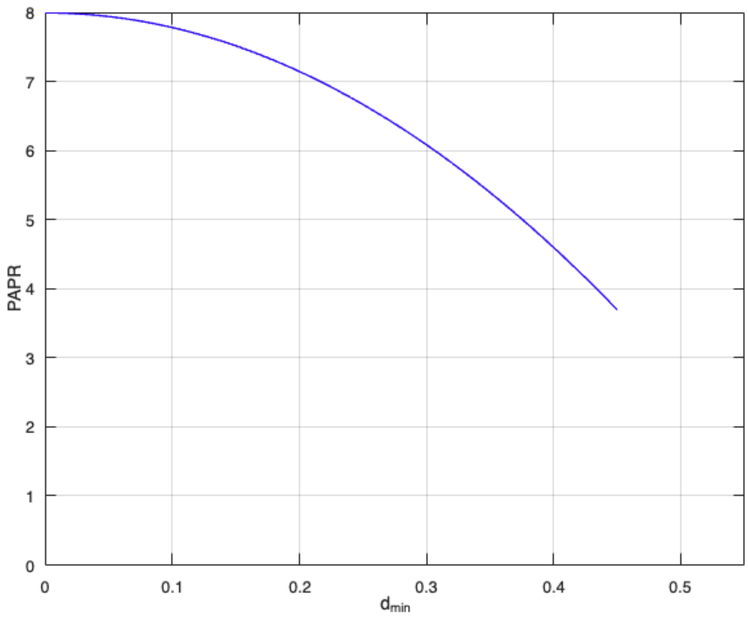

Figure 11: PAPR vs $d_{\min}$ plot for 16-CQAM with $M = 16$ and $N = 8$

2 Task 2

In order to illustrate SEP vs SNR for 16-PAM and 16-QAM constellation we will use the theoretical formulas that associate the error probability with the signal to noise ratio (SNR).

The formula for the 16-PAM constellation is:

$$P_{s,16-PAM} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6E_s}{(M^2-1)N_0}}\right) \tag{22}$$

However, $E_b = \frac{E_s}{\log_2 M} \Rightarrow E_s = \log_2 M E_b \Rightarrow E_s = kE_b$, where $k = \log_2 M$ is thenumber of bits per symbol. Thus fomula (22) becomes:

$$P_{s,16-PAM} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6kE_b}{(M^2-1)N_0}}\right) \tag{23}$$

We know that $SNR = \frac{E_b}{N_0}$ so we have:

$$P_{s,16-PAM} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6kSNR}{(M^2-1)}}\right) \tag{24}$$

We convert SNR from dB and so we have $SNR = 10^{\frac{SNR}{10}}$.

The formula for the 16-QAM constellation is the following:

$$P_{s,16-QAM} = 1 - (1 - P_{s,\sqrt{M}})^2 \tag{25}$$

where:

$$P_{s,\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{E_s}{N_0}}\right) \tag{26}$$

And by using $\frac{E_s}{N_0} = \frac{kE_b}{N_0} = kSNR$

$$P_{s,\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3kSNR}{M-1}}\right) \tag{27}$$

The related plots can be seen in Figure 12.

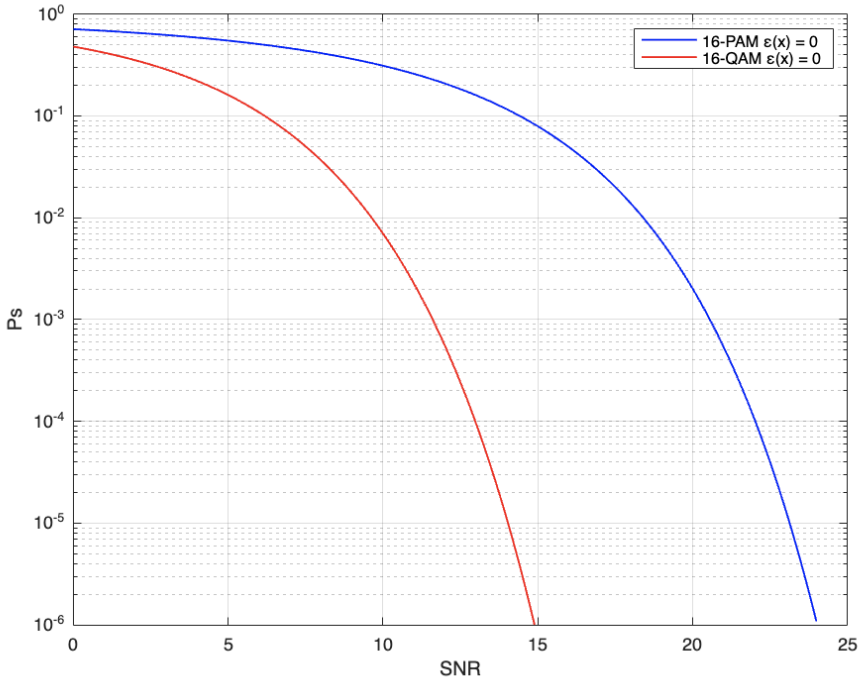

Figure 12: PAPR vs $d_{\min}$ plot for 16-CQAM with $M = 16$ and $N = 8$

References

[1] G. M. Kraidy, C. Psomas and I. Krikidis, "Fundamentals of Circular QAM for Wireless Information and Power Transfer," 2021 IEEE 22nd International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), Lucca, Italy, 2021

[2] M. J. L. Morales, K. Chen-Hu and A. G. Armada, "Optimum Constellation for Symbol-Error-Rate to PAPR Ratio Minimization in SWIPT," 2022 IEEE 95th Vehicular Technology Conference: (VTC2022-Spring), Helsinki, Finland, 2022