Denoising of EEG Signals Using a Multichannel Wiener Filter

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***Abstract*—This work addresses the denoising of EEG signals through the use of a multichannel Wiener filter. Based on covariance estimation between clean brain activity and artifacts (e.g., eye blinks), the method is applied to 19-channel EEG recordings. The filter is trained on data where the time instances of artifacts are known and is then applied to both training and test sets. The method, its results, and a discussion of the assumptions made for its implementation are presented below.**

I. Introduction

Denoising of EEG signals is essential for accurate processing of neurological data, as unwanted signals (artifacts), such as those caused by eye blinks, are often recorded. In this work, a Wiener filter based on estimation and detection theory is applied to remove such interferences.

II. Signal Modeling

The processing is based on the observation model:

$$y[n] = v[n] + d[n], \quad n = 0, 1, \ldots, N-1$$

where:

- $y[n]$: the observed signal (here, the EEG signal),
- $v[n]$: the unknown clean brain activity,
- $d[n]$: additive noise or artifact (e.g., eye blinks).

The objective is to estimate the signal $v[n]$ from the observed samples $y[n]$.

There are three main cases of estimation:

- **Smoothing:** The entire signal $v = [v[0], \ldots, v[N-1]]$ is estimated based on all available samples $y = [y[0], \ldots, y[N-1]]$.
- **Filtering:** The value $v[n]$ is estimated in real time based on samples up to $y[n]$.
- **Prediction:** The value $y[N-1+\ell]$ is estimated based on past data.

In this work, the **smoothing technique** was applied, since the complete EEG recordings are available in both the train and test sets. This allows the use of all time samples for estimating $v[n]$ at each instant.
The choice of smoothing over filtering was made because, in this case, we have access to the full recording for the entire duration, both in training and test data.

Filtering provides optimal estimation in real-time conditions, i.e., when the signal must be estimated as it unfolds, using only past and present data. However, in the present application, denoising is performed offline, making it possible to exploit future information (e.g., samples $y[n+1], y[n+2], \ldots$) to estimate $v[n]$.

Smoothing therefore offers more accurate estimation than filtering, as it uses **all** available input signal samples. This is crucial in EEG denoising, where small deviations can significantly affect the interpretation of the signal.

Moreover, the requirement of the task is to remove artifacts from an already recorded signal—without computational time constraints. Therefore, the smoothing technique is more suitable and leads to better performance than filtering.

III. Computation of the Wiener Filter

We now assume that $\mathbf{v}[k]$ and $\mathbf{d}[k]$ are uncorrelated and zero-mean signals:

$$\mathbb{E}[\mathbf{v}[k]\mathbf{d}^T[k]] = \mathbf{0}$$

Thus:

$$\mathbf{R}_{yy} = \mathbf{R}_{vv} + \mathbf{R}_{dd}$$

The estimation of the clean signal is based on minimizing the mean squared error between the true signal $\mathbf{v}$ and its estimate $\hat{\mathbf{v}}$. Assuming the observed signal is $\mathbf{y} = \mathbf{v} + \mathbf{d}$ and the two signals are uncorrelated, the Wiener filter arises as the optimal LMMSE solution:

$$\hat{\mathbf{v}} = \mathbf{R}_{vv}(\mathbf{R}_{vv} + \mathbf{R}_{dd})^{-1}\mathbf{y} = \mathbf{W} \cdot \mathbf{y}$$

where:

- $\mathbf{R}_{vv}$: covariance matrix of the clean activity. Computed from samples without artifacts (using the mask derived from the blinks matrix).
- $\mathbf{R}_{dd}$: covariance matrix of the noise/artifacts. Computed from samples containing only artifacts.
- $\mathbf{W}$: the Wiener smoothing matrix.

The filter was applied to the entire observed EEG signal, yielding its denoised version.

Finally, the error matrix is:

$$\mathrm{M}(\hat{\mathbf{v}}) = \mathbf{R}_{vv} - \mathbf{W}\mathbf{R}_{vv} = (\mathbf{I} - \mathbf{W})\mathbf{R}_{vv}$$

IV. Assumptions for the Implementation

The following assumptions were made for implementing the method:

- The signals $\mathbf{v}$ and $\mathbf{d}$ are uncorrelated and have zero mean.
- Covariance matrices can be reliably estimated from the available training samples.
- The statistical behavior of the artifacts is the same in the test set as in the training set.
- The multichannel filter exploits the correlation among the 19 channels.
- The signal is assumed to be stationary so that the covariance matrices remain constant over time.

V. Filter Implementation Procedure

The Wiener filter was applied using the smoothing technique, estimating the clean signal at each time instant using the set of observed samples.

Specifically:

- The files `train.mat` and `test.mat`, containing 19-channel EEG signals, were loaded.
- From `train.mat`, the `blinks` matrix was used to create a mask separating artifact-contaminated samples from clean ones.
- Thus, two datasets were created:
  - `clean_data`: samples without artifacts
  - `artifact_data`: samples with blinks
- The covariance matrices of the clean data ($\mathbf{R}_{vv}$) and the artifacts ($\mathbf{R}_{dd}$) were then computed as:

$$\mathbf{R}_{vv} = \text{cov(clean\_data')} \quad \mathbf{R}_{dd} = \text{cov(artifact\_data')}$$

- Based on these matrices, the matrix $\mathbf{W}$ was computed.
- The filter was applied to both the training and test data, assuming that the statistical properties of the artifacts are the same across both sets.

This application reduces to a simple matrix multiplication:

$$\hat{\mathbf{v}} = \mathbf{W} \cdot \mathbf{y}$$

VI. Filter Performance Evaluation

The total variance of the observed signal in the training set was computed as:

$$\text{Var}_{\text{noisy}} = \mathrm{tr}\left(\mathrm{cov}(\mathbf{y})\right)$$

where $\mathbf{y} \in \mathbb{R}^{N \times T}$ is the multichannel noisy EEG signal (with $N = 19$ channels and T time samples), and $\mathrm{tr}(\cdot)$ denotes the trace of a matrix, i.e., the sum of the diagonal elements of the covariance matrix. The resulting value was:

$$\text{Var}_{\text{noisy}} = 8548.33$$

Similarly, the total mean squared error (MSE) of the clean signal estimate was computed from the error matrix:

$$\mathbf{M} = (\mathbf{I} - \mathbf{W})\mathbf{R}_{vv}$$

where $\mathbf{W}$ is the Wiener filter matrix and $\mathbf{R}_{vv}$ the covariance matrix of the clean signal. The total MSE also results as the trace of matrix $\mathbf{M}$:

$$\text{MSE}_{\text{total}} = \mathrm{tr}(\mathbf{M}) = 2057.52$$

This corresponds to an explained variance of:

$$\text{Explained Variance} = 100 \cdot \left(1 - \frac{\text{MSE}}{\text{Var}_{\text{noisy}}}\right) \approx 75.93\%$$

The Wiener filter therefore succeeds in removing a significant portion of the noise, preserving about 76% of the brain information. This demonstrates that the smoothing approach is effective for EEG denoising, particularly when all time samples are available.

Although the explained variance (75.93%) may at first appear moderate, it is in fact a **realistic and satisfactory result** for EEG signals.

It should be noted that artifacts such as eye blinks often have much higher amplitude than the brain activity itself. Thus, the filter must reject a significant portion of the energy to separate the clean neurological signals from the noise.

Furthermore, the smoothing technique used is optimal in the sense of mean squared error (LMMSE), under the assumptions of linearity and stationarity.

Finally, it should be mentioned that explained variance values in the range of 70–80% are typically considered acceptable in EEG denoising, as they achieve a good balance between information preservation and artifact reduction.

Although the mean squared error (MSE) can be precisely computed only in the training set, where artifact time instants are available, a qualitative evaluation was also attempted on the test set. Specifically, the training-set MSE was compared with the total variance of the test signal. The result indicates that the filter generalizes satisfactorily to new, unseen signals, as the percentage of preserved information (explained variance) remained high ($\approx 90.42\%$).

VII. Estimation Error Per Channel

For each channel, the variance of the estimation error produced by the Wiener filter was computed. The diagonal of the error matrix $M = (I - W)R_{vv}$ reflects the estimation uncertainty for each channel:

- Low values indicate that the filter was able to effectively isolate brain activity from noise.
- High values indicate increased uncertainty, either due to strong artifacts or due to poor estimation of the channel statistics.

The following diagram shows the variability of the MSE per channel for the test data, providing additional information for evaluating the filter's performance:

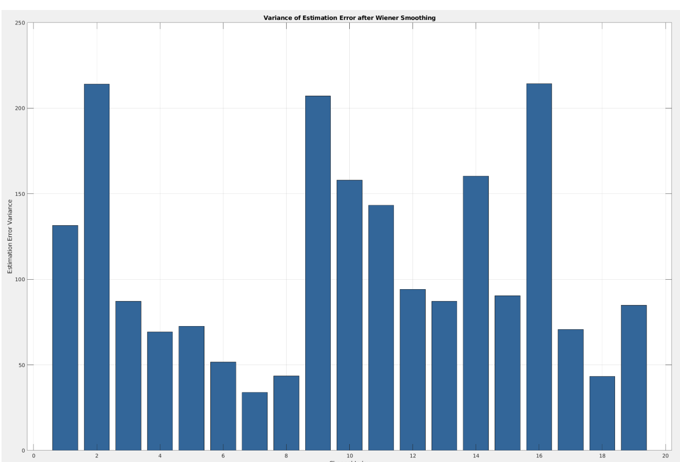

Figure 1: Variance of Estimation Error after applying the Wiener filter

VIII. Results

Below, the signals for all 19 channels are presented graphically. Since the test data were quite extended in time, they were displayed within the same time window as the training data, i.e., the interval 0–25 sec.

- **Channel 1:** Strong artifacts due to blinks, significantly reduced by the filter. The signal acquires a smoother form without abrupt peaks.
- **Channel 2:** Similar behavior to Channel 1 — denoising is very effective, especially at the initial time instants.
- **Channel 3:** Medium-intensity artifacts are observed. The filtered signal shows substantial noise reduction without information loss.
- **Channel 4:** The original signal contains slightly elevated noise, which is smoothed after filtering.
- **Channel 5:** Denoising removes sharp peaks, preserving the waveform almost intact. Very good performance.
- **Channel 6:** The filter successfully handles medium-intensity artifacts. The filtered signal is smoother.
- **Channel 7:** Strong artifacts similar to Channels 3 and 2. The filter achieves substantial denoising.
- **Channel 8:** Quite clean channel — the filter's effect is minimal, which is positive as it does not distort the signal.
- **Channel 9:** Moderate noise presence. The filter removes mild fluctuations and smooths the signal without overcorrection.
- **Channel 10:** The filter preserves signal integrity. Its effect is limited, as expected.
- **Channel 11:** Similar behavior to Channel 9. Denoising is mild and targeted.
- **Channel 12:** Relatively clean channel. The filter acts supportively without introducing distortions.
- **Channel 13:** Low-intensity artifacts. The filter's intervention is mild but beneficial.
- **Channel 14:** The filter smooths slight oscillations without significantly distorting the waveform.
- **Channel 15:** Clean channel — the result is almost identical to the original, demonstrating correct filter behavior.
- **Channel 16:** Similar to Channel 15, denoising has minimal but positive effect.
- **Channel 17:** Artifacts are small, and the filtered signal is mildly smoothed.
- **Channel 18:** Nearly noise-free channel — the filter leaves the information intact.
- **Channel 19:** Similar to Channel 18, the filter leaves the information essentially untouched.

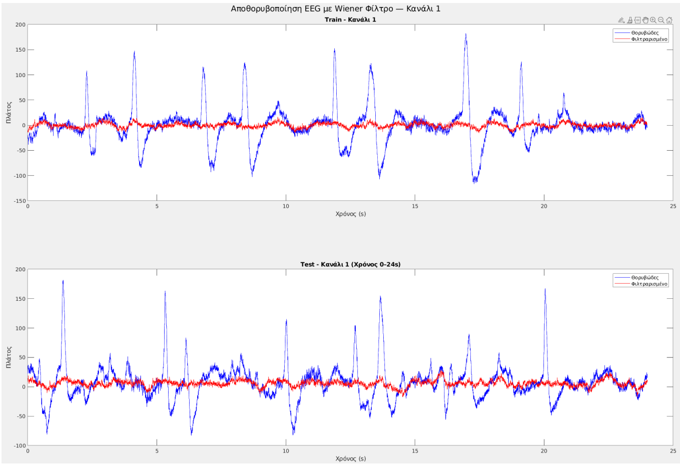

Figure 2: Channel 1

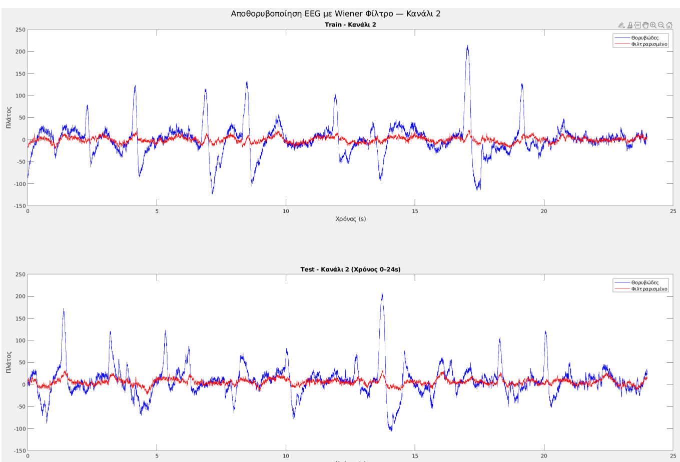

Figure 3: Channel 2

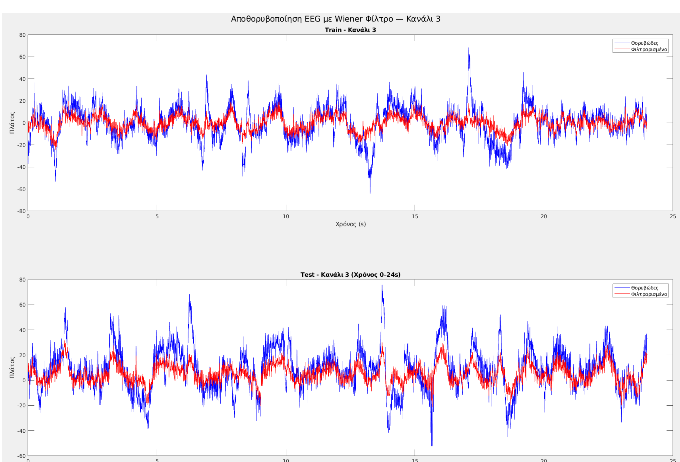

Figure 4: Channel 3

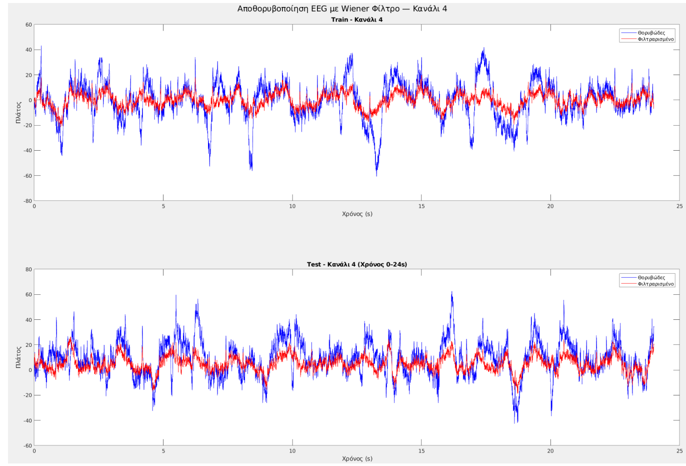

Figure 5: Channel 4

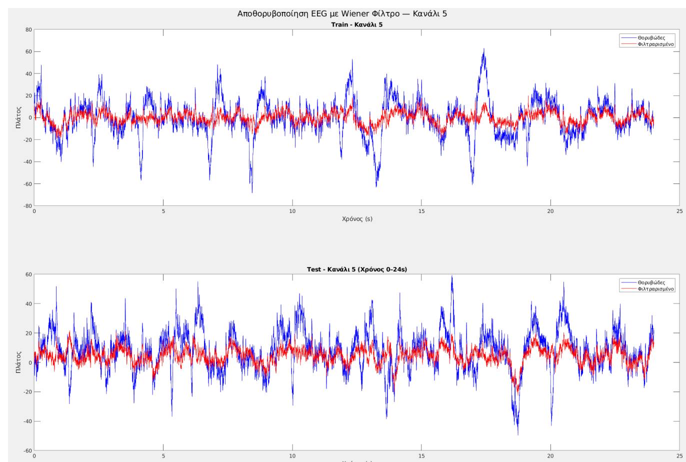

Figure 6: Channel 5

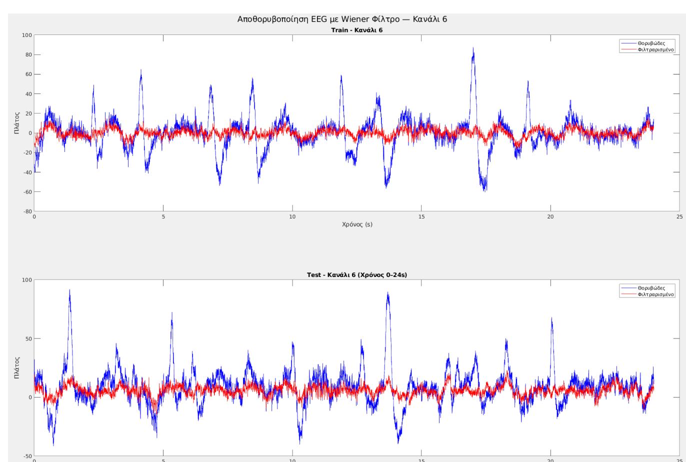

Figure 7: Channel 6

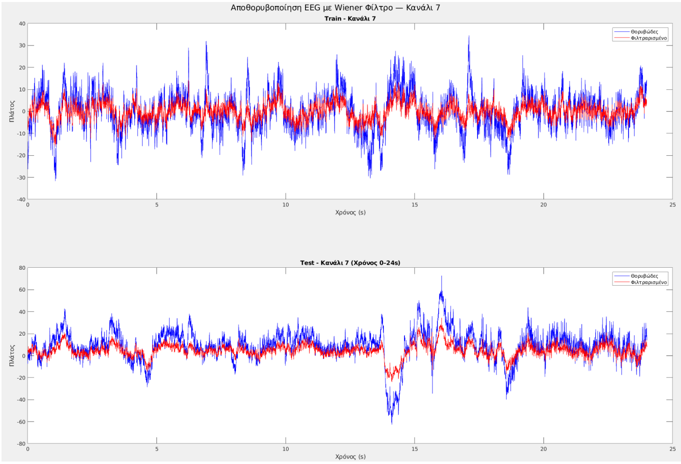

Figure 8: Channel 7

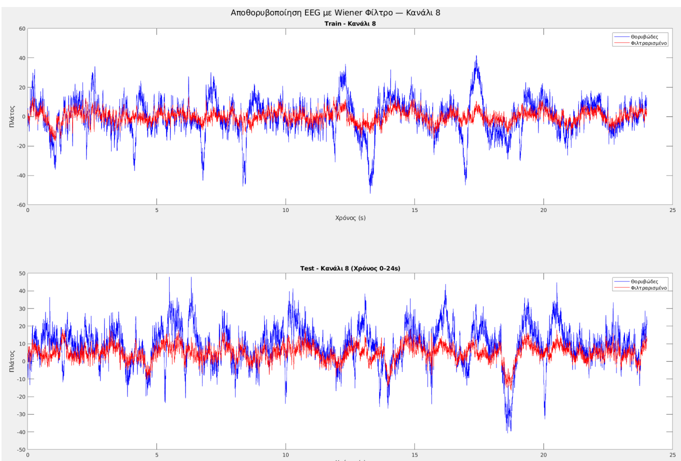

Figure 9: Channel 8

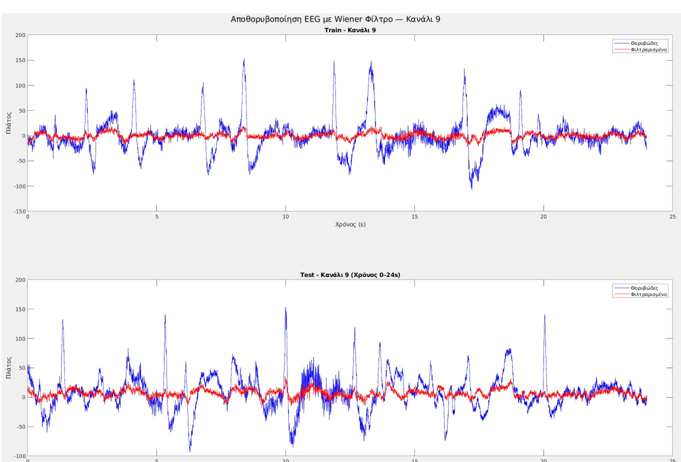

Figure 10: Channel 9

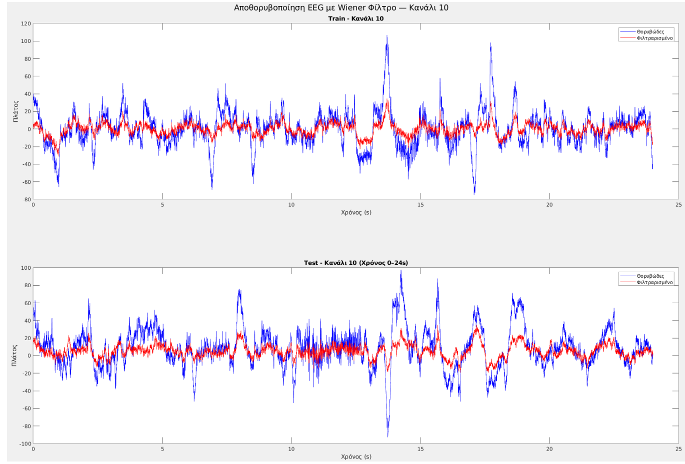

Figure 11: 10

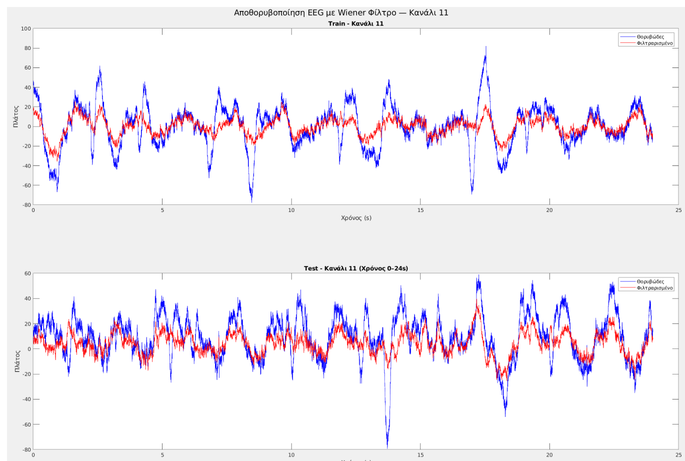

Figure 12: Channel 11

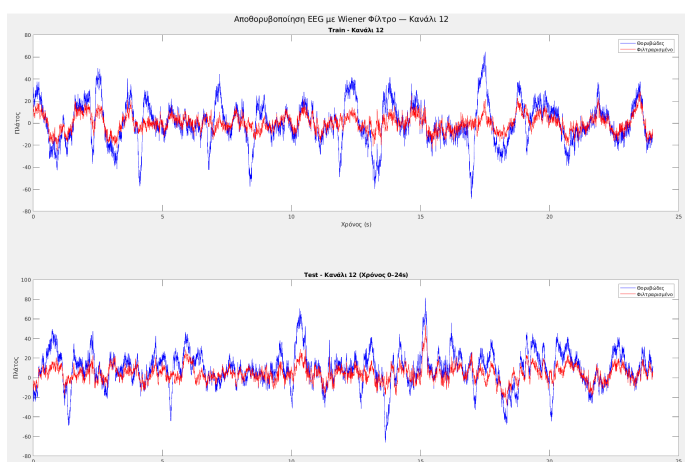

Figure 13: Channel 12

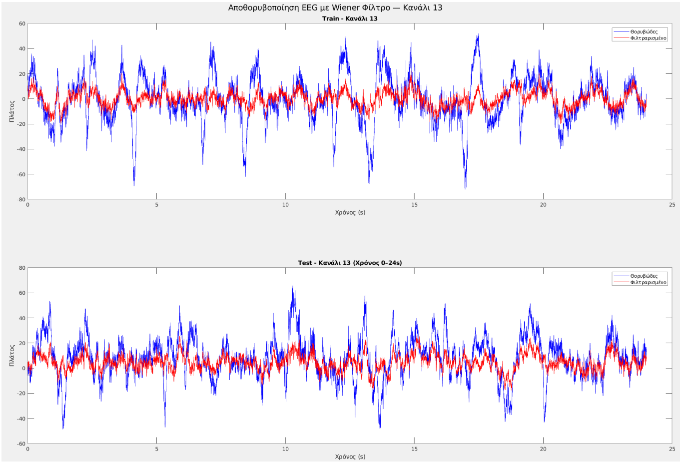

Figure 14: Channel 13

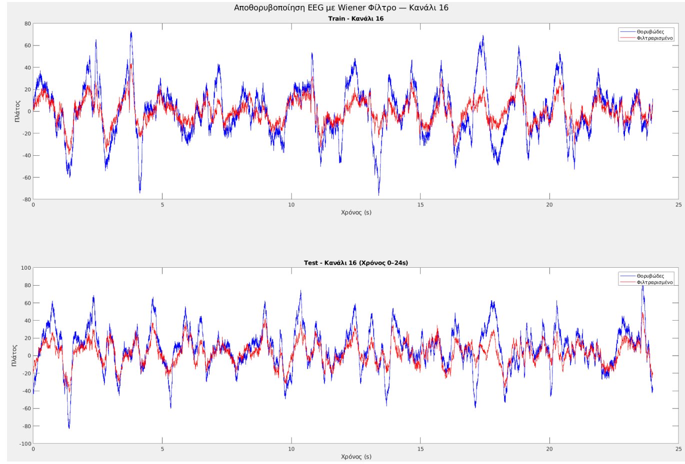

Figure 17: Channel 16

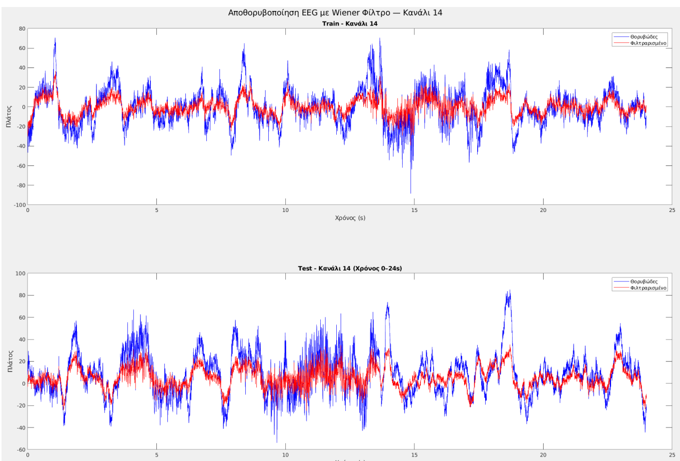

Figure 15: Channel 14

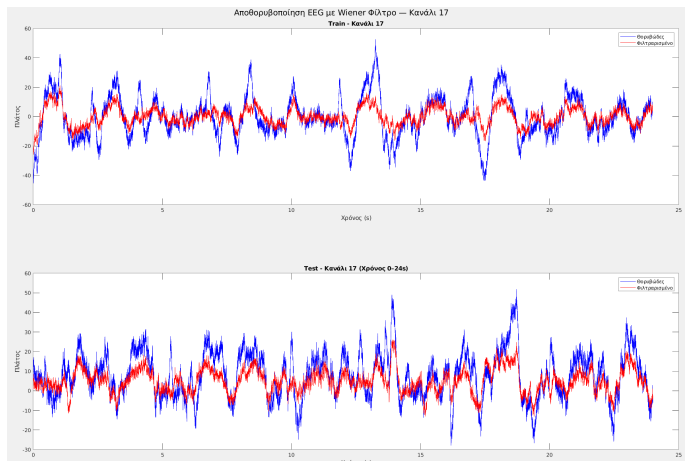

Figure 18: Channel 17.

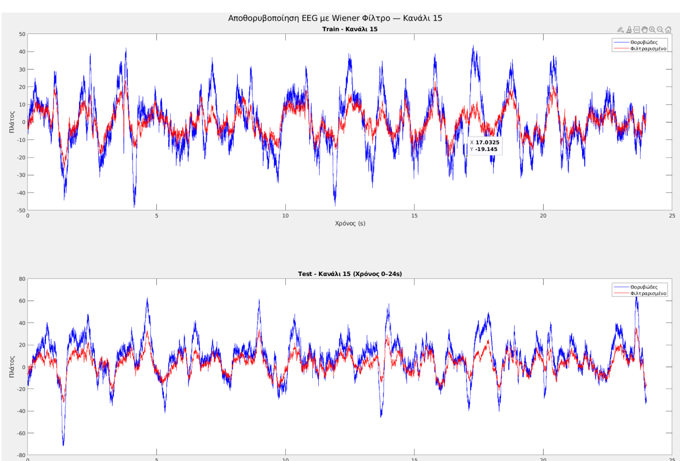

Figure 16: Channel 15

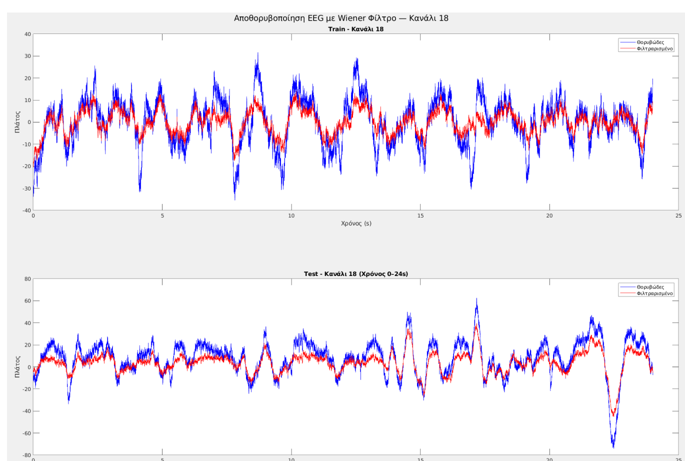

Figure 19: Channel 18

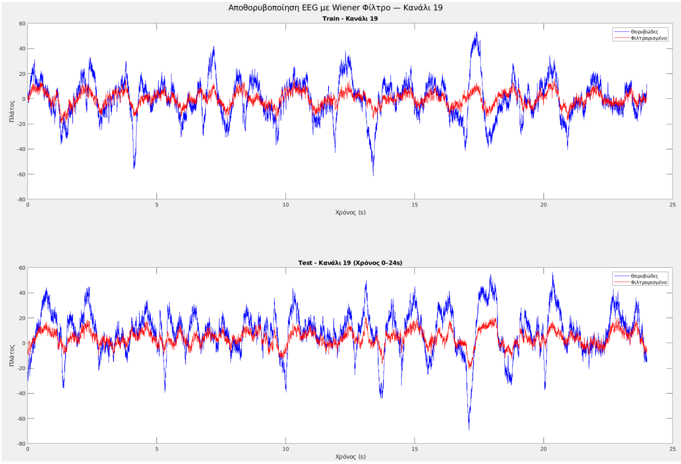

Figure 20: Channel 19

IX. Conclusion

The multichannel Wiener filter provided effective denoising of EEG signals, significantly reducing artifacts without distorting useful brain activity. The assumptions made were realistic, and the method can be applied in real EEG analysis scenarios.