Door and Handle Motion Simulation (PART A)

Dimitrios Karatis 10775, *Electrical and Computer Engineering, AUTH*

***Abstract*—This project presents a complete simulation of a door and handle opening sequence, applying advanced kinematic modeling and trajectory planning techniques. The simulation models the door and handle using homogeneous transformation matrices and performs position and orientation interpolation leveraging spherical linear interpolation (SLERP) combined with quintic time-scaling for smooth motion. The system ensures zero initial and final velocities and accelerations for all phases of motion, adhering to robotic motion planning standards. The simulation is implemented in MATLAB using Peter Corke's Robotics Toolbox and visualizes both the position trajectory and the quaternion-based orientation trajectory of the handle. The project demonstrates the application of robotics kinematics, smooth trajectory generation, and 3D motion visualization.**

***Index Terms*—robotics, kinematics, transformations, SLERP, time-scaling, trajectory planning, handle motion, door mechanism, smooth interpolation, MATLAB simulation**

I. Introduction

The purpose of this work is to simulate the opening motion of a door and its handle, ensuring compliance with specific kinematic constraints:

- The handle rotates $-45°$ around its local X-axis. **(Phase 1)**
- The door then rotates $-30°$ around its Z-axis while the handle maintains its orientation. **(Phase 2)**
- Finally, the handle rotates $+45°$ to restore its initial relative pose. **(Phase 3)**

The entire motion occurs over $T = 5$ seconds with **zero initial and final velocity and acceleration** for both position and orientation.

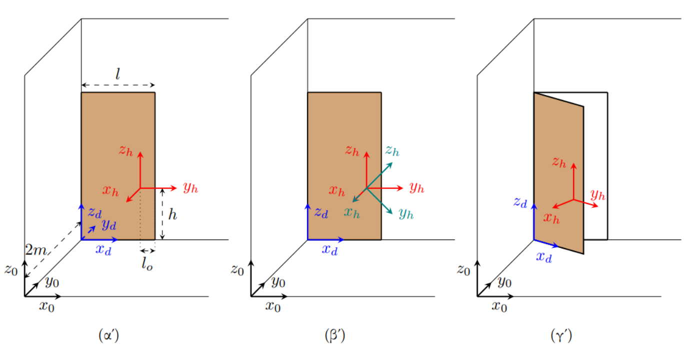

Fig. 1: The 3 Poses

II. Theoretical Background

A. Homogeneous Transformations

The position and orientation of the door and handle are represented using homogeneous transformation matrices, which combine rotation and translation.

This document was prepared by Dimitrios Karatis as part of the first assignment for the Robotics course at Aristotle University of Thessaloniki.

Definition of Transformation Matrices

To describe the motion of the door and handle throughout the motion phases, we define the following homogeneous transformation matrices:

- The pose of the door frame $\{D\}$ with respect to the world frame $\{O\}$ at configuration A is given by:

$$g_{OD}^{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  This corresponds to a translation of 2 meters along the Y-axis.
- The pose of the handle frame $\{H\}$ relative to the door frame at configuration A is:

$$g_{DH}^{A} = \begin{bmatrix} 0 & 1 & 0 & l - l_0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  where l, l_0, and h are fixed geometric parameters defining the handle's location.
- The global pose of the handle at configuration A is computed via composition:

$$g_{OH}^{A} = g_{OD}^{A} \cdot g_{DH}^{A}$$

- At configuration B, the door frame remains unchanged, i.e., $g_{OD}^{B} = g_{OD}^{A}$. The handle is rotated by $-45°$ around the X-axis, resulting in the new orientation:

$$R_{DH}^{B} = R_{DH}^{A} \cdot R_x\left(-\frac{\pi}{4}\right)$$

  and the new transformation:

$$g_{DH}^{B} = \begin{bmatrix} R_{DH}^{B} & \begin{bmatrix} l - l_0 \\ 0 \\ h \end{bmatrix} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix}$$

  Hence, the pose of the handle in the world frame becomes:

$$g_{OH}^{B} = g_{OD}^{B} \cdot g_{DH}^{B}$$

- At configuration C, the door rotates by $-30°$ about the Z-axis. The new door frame pose is:

$$R_{OD}^{C} = R_{OD}^{A} \cdot R_z\left(-\frac{\pi}{6}\right), \quad p_{OD}^{C} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$g^C_{OD} = \begin{bmatrix} R^C_{OD} & p^C_{OD} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix}$$

The handle pose relative to the door remains the same as in configuration A, i.e., $g^C_{DH} = g^A_{DH}$. Thus, the global handle pose becomes:

$$g^C_{OH} = g^C_{OD} \cdot g^C_{DH}$$

These transformations provide a complete description of the spatial configurations of the door and handle at key positions (A, B, and C).

B. SLERP (Spherical Linear Interpolation)

For rotation interpolation, SLERP ensures:

- Constant angular velocity.
- Shortest rotation path on the unit quaternion sphere.

C. Time-Scaling Function

To ensure a smooth trajectory with zero initial and final velocity and acceleration, we use a quintic polynomial of the form:

$$s(t) = k_0 + k_1 t + k_2 t^2 + k_3 t^3 + k_4 t^4 + k_5 t^5$$

where the coefficients k_i are chosen such that:

$$\begin{aligned} s(0) &= 0, & s(T) &= 1 \\ \dot{s}(0) &= 0, & \dot{s}(T) &= 0 \\ \ddot{s}(0) &= 0, & \ddot{s}(T) &= 0 \end{aligned}$$

These boundary conditions ensure that the position $s(t)$, as well as its first and second derivatives (velocity and acceleration), are zero at both ends of the interval $[0, T]$, and the motion smoothly interpolates from start to finish.

We introduce the normalized time parameter:

$$\tau = \frac{t}{T}, \quad \text{with } \tau \in [0, 1]$$

and rewrite the polynomial as:

$$s(\tau) = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3 + a_4\tau^4 + a_5\tau^5$$

Applying the boundary conditions at $\tau = 0$ and $\tau = 1$:

$$\begin{aligned} s(0) &= a_0 = 0 \\ \dot{s}(0) &= a_1 = 0 \\ \ddot{s}(0) &= 2a_2 = 0 \Rightarrow a_2 = 0 \\ s(1) &= a_0 + a_1 + a_2 + a_3 + a_4 + a_5 = 1 \\ \dot{s}(1) &= a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 = 0 \\ \ddot{s}(1) &= 2a_2 + 6a_3 + 12a_4 + 20a_5 = 0 \end{aligned}$$

Solving this linear system (with $a_0 = a_1 = a_2 = 0$), we obtain:

$$\begin{aligned} a_3 &= 10 \\ a_4 &= -15 \\ a_5 &= 6 \end{aligned}$$

Thus, the time-scaling function becomes:

$$s(\tau) = 10\tau^3 - 15\tau^4 + 6\tau^5, \quad \text{where } \tau = \frac{t}{T}$$

III. Implementation Approach

A. Phases

The motion is divided into three phases:

1) **Pose A to B:** Handle rotation. Duration of $T/4$.
2) **Pose B to B':** Door rotation with rotated handle. Duration of $T/2$.
3) **Pose B' to C:** Handle restoration while door is fixed. Duration of $T/4$.

B. Interpolation Strategy

- `trinterp` is used for SE(3) interpolation, internally applying SLERP for rotation and linear interpolation for translation.
- The time scaling function is applied to the interpolation parameter to ensure smooth profiles.

C. Trajectory Planning and Interpolation

The motion is divided into three distinct phases: (1) rotating the handle downward, (2) opening the door while keeping the handle fixed relative to the door, and (3) returning the handle to its original orientation. Each phase is assigned a specific duration, and the motion is discretized using a fixed time step of 0.01 seconds.

To generate continuous and physically realistic trajectories, the code interpolates the pose of the door and handle through all three phases. This is achieved using the `trinterp` function, which performs smooth interpolation between two homogeneous transformation matrices. The interpolation is not linear in time but is instead guided by a time-scaling function $s(t)$, defined as:

$$s(t) = 10\tau^3 - 15\tau^4 + 6\tau^5, \quad \text{where } \tau = \frac{t}{T}$$

This cubic polynomial ensures that both the velocity and acceleration profiles of the motion are continuous and start and end at zero, eliminating abrupt changes in movement. This property is especially important for generating smooth robotic motion and avoiding unrealistic discontinuities in the simulation.

In **Phase 1**, the door frame remains fixed while the handle rotates downward relative to it. In **Phase 2**, the door begins to rotate (as if opening), and the handle remains fixed with respect to the door frame. Finally, in **Phase 3**, the door stays open while the handle returns to its original orientation. During all phases, the world-relative pose of the handle is computed by chaining the door's and handle's local transformations.

For each time step i:

- The scaled time $s(t)$ is evaluated.
- The interpolated transformation $g(t)$ is computed using `trinterp`.
- The global pose of the handle is computed as:

  $$g_{OH}(t) = g_{OD}(t) \cdot g_{DH}(t)$$

  where the composition reflects the kinematic chain from world to door to handle.

- The resulting transformation is stored in a 3D array, along with:
  - The translation vector $\mathbf{p}(t) = \texttt{transl}(g_{OH}(t))$
  - The unit quaternion $q(t) = \texttt{UnitQuaternion}(g_{OH}(t))$

The trajectory data is stored at each time step in the form of homogeneous transformation matrices for both the handle and the door, as well as in position vectors and unit quaternions for further analysis and use in Part B of the assignment.

D. Animation and Visualization

Although the total simulated time is 5 seconds, it was observed during testing that the real-time duration of the animation in MATLAB was significantly longer. This is due to computational delays caused by repeated 3D rendering operations (such as `trplot` and `delete`). To address this, only every 4th frame was visualized in the animation loop, significantly improving performance and making the animation smoother and more responsive.

The function `trplot`, part of the Robotics Toolbox for MATLAB by Peter Corke, was used extensively for plotting coordinate frames of the world, door, and handle. Additionally, the script includes an optional feature for rendering a rectangular door object using the `plotDoor` helper function, which visualizes the door as a solid black patch. This optional visualization is commented out by default for performance reasons but can be enabled for a more complete graphical representation of the environment.

IV. Results

A. Handle Position Trajectory

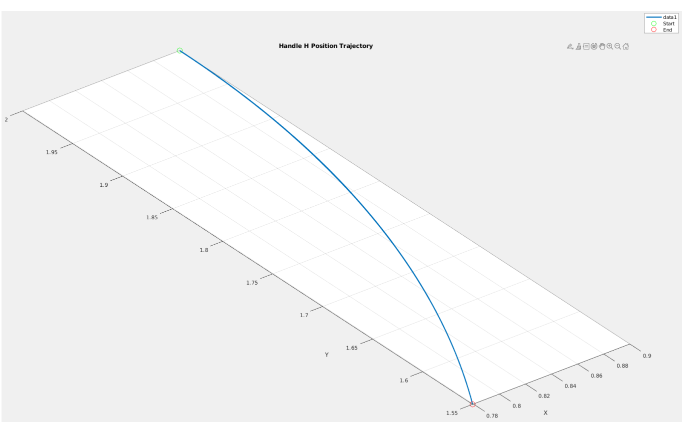

Fig. 2: Handle Position Trajectory

The figure above illustrates the three-dimensional trajectory of the door handle frame $\{H\}$ throughout the complete motion sequence, composed of the three phases described earlier. The trajectory is computed by composing the time-varying transformations of the door frame and the handle frame, i.e., $g_{OH}(t) = g_{OD}(t) \cdot g_{DH}(t)$, at each discrete time step.

The smooth curve demonstrates the continuous evolution of the handle's position in space, as it moves from its initial configuration (marked in green) to the final configuration (marked in red).

The smoothness and curvature of the trajectory validate the effectiveness of the cubic time-scaling function used in the interpolation process, which ensures physically realistic motion by avoiding sudden changes in velocity or acceleration. This trajectory not only serves as a visual verification of the motion but also provides the required data for trajectory tracking in Part B of the assignment.

B. Handle Orientation Trajectory (Unit Quaternion)

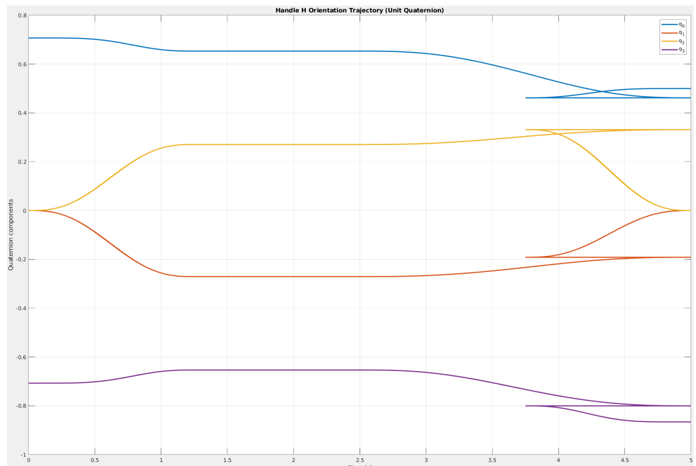

Fig. 3: Quaternion components of handle orientation over time

The figure above displays the time evolution of the door handle's orientation, represented using unit quaternions. Each curve corresponds to one of the four components (q_0, q_1, q_2, q_3) of the quaternion $q(t)$, which is extracted from the handle's homogeneous transformation matrix $g_{OH}(t)$ at each time step. Unit quaternions are used instead of Euler angles or rotation matrices to represent orientation, as they avoid singularities, allow for smooth interpolation, and ensure numerical stability.

The trajectory is continuous and smooth, as expected due to the use of the cubic time-scaling function $s(t)$ and the SLERP-based interpolation via the `trinterp` function. The graph clearly shows the three motion phases of the handle:

- During **Phase 1** (0 to 1.25 seconds), we observe a significant change in the orientation as the handle rotates downward.
- In **Phase 2** (1.25 to 3.75 seconds), the orientation remains relatively constant, since the handle is rigidly attached to the rotating door and does not rotate independently.
- In **Phase 3** (3.75 to 5 seconds), the quaternion components change again as the handle is released and returns to its original orientation.

The smooth transitions in each component demonstrate that the motion has been interpolated with continuous derivatives. The trajectory not only verifies the correctness of the rotational interpolation but also serves as essential input for inverse kinematics and velocity control in Part B of the assignment.

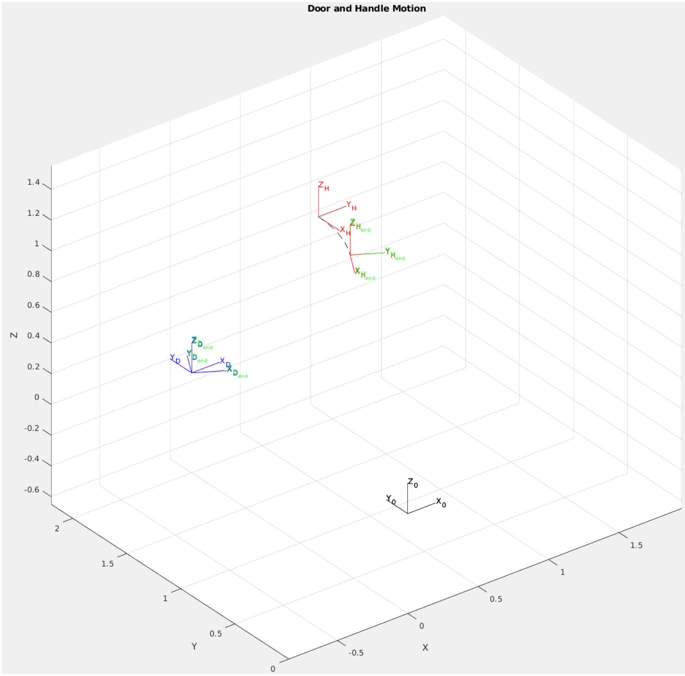

Fig. 4: Initial and final poses of the door and handle in the world frame

C. Animation Screenshot

The figure above illustrates the final configuration of the door and handle motion after executing the complete trajectory defined in this part of the project. The black frame $\{0\}$ denotes the fixed world coordinate system, while the blue and red frames represent the door $\{D\}$ and handle $\{H\}$ at the initial pose (configuration A), respectively. The green frames, labeled $\{D_{end}\}$ and $\{H_{end}\}$, correspond to the final positions of the door and handle after completing the full three-phase trajectory (A $\rightarrow$ B $\rightarrow$ B$'$ $\rightarrow$ C).

V. Conclusion

The simulation successfully achieved the desired door and handle motion, ensuring:

- Correct pose transitions using SE(3) transformations.
- SLERP-based rotation interpolation combined with cubic time scaling.
- Zero initial and final velocity and acceleration, as required.

The generated trajectory plots and the animation confirm the correctness and smoothness of the implemented motion.

References

1) P. Corke, *Robotics Toolbox for MATLAB*.