A Bayesian Analysis of the Fama-French 5 Factor Model

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1 Motivation of Study

The Fama-French 5-factor model is an asset pricing model that was developed by Eugene Fama and Kenneth French in 2014. It is an extension of the Fama-French 3-factor model, which was introduced in 1993. The Fama-French 5-factor model attempts to explain the excess returns of a portfolio or individual security by incorporating additional factors beyond the market risk premium and size and value factors.

The model is given as:

$$R_{it} - R_{ft} = a_i + b_1(R_{Mt} - R_{ft}) + b_2 SMB_t + b_3 HML_t + b_4 RMW_t + b_5 CMA_t$$

where R_{it} is the return on security or portfolio i for period t, R_{ft} is the risk-free return for period t, a_i captures the return not explained by the five factors and:

- Market Risk Premium: $R_{Mt} - R_{ft}$: is the excess return above the risk-free rate
- Size Premium SMB_t: This factor captures the idea that smaller companies tend to outperform larger companies over time. It is measured by the difference in returns between a portfolio of small-cap stocks and a portfolio of large-cap stocks.
- Value Premium HML_t: This factor captures the idea that value stocks (those with low price-to-book ratios) tend to outperform growth stocks (those with high price-to-book ratios) over time. It is measured by the difference in returns between a portfolio of high book-to-market (value) stocks and a portfolio of low book-to-market (growth) stocks.
- Profitability RMW_t: This factor captures the idea that companies with high profitability tend to outperform those with low profitability over time. It is measured by the difference in returns between a portfolio of high profitability stocks and a portfolio of low profitability stocks
- Investment CMA_t: This factor captures the idea that companies that invest more aggressively tend to underperform those that invest less. It is measured by the difference in returns between a portfolio of low-investment stocks and a portfolio of high-investment stocks.

By incorporating these additional factors, the Fama-French 5-factor model seeks to provide a more complete picture of the relationship between risk and return in the stock market, while it has been found to provide a better explanation of returns than earlier models. Since its inception, the model has been mainly estimated using frequentist methods for fitting multivariate regressions. In this paper, we estimate the FF 5-factor model following a Bayesian approach.

2 Data and Models

The data for the research are obtained from two sources: yahoo!finance and the Bloomberg platforms. We analyze nine different stocks listed in the NYSE for three different market sizes: Large-Cap, Mid-Cap, and Small-Cap. We use three stocks for each market size to maintain sufficient diversification. The data comprises 398 monthly returns for each individual stock starting from January 1990 until February 2023. The five factors are obtained from Dr. Kenneth R. French's data library. Each of the five factors is computed on a monthly and daily frequency. We use monthly frequency to match the frequency of the observed asset returns. The size and value factors use independent sorts of stocks into two size groups and three B/M groups. The size breakpoint is the NYSE median market cap, and the B/M breakpoints are the 30th and 70th percentiles of B/M for NYSE stocks. The intersections of the sorts produce six portfolios. The size factor SMB, is the average of the three small stock portfolio returns minus the average of the three big stock portfolio returns. The value factor HML is the average of the two high B/M portfolio returns minus the average of the two low B/M portfolio returns. The profitability and investment factors of the RMW and CMA, are constructed in the same way as HML except the second sort is either on profitability (robust minus weak) or investment (conservative minus aggressive). Like HML, RMW, and CMA can be interpreted as averages of profitability and investment factors for small and big stocks. The 5 factors computed in the way described above constitute the predictors of an asset's return in the Fama-French 5-factor regression model.

Large Cap	Mid Cap	Small Cap
IBM	Whirlpool Corporation	La-Z-Boy Incorporated
Johnson & Johnson	Xerox Holdings Corporation	Brunswick Corporation
The Coca-Cola Company	Intel Corporation	Dillard's, Inc.

Table 1: Stocks by Market Capitalization

Our analysis consists mainly of two parts. In the first part, we fit a random slopes model allowing each individual stock to follow its own Fama-French 5-factor regression line and borrow strength across all nine stocks to estimate the factors' coefficients. In the second part, we group all the data together by creating a portfolio consisting of the nine stocks adjusted by their weights. The weights are adjusted on a monthly basis according to the capitalization of each company at the end of the given month. The portfolio's return at a given month i will then be $R_{ip} = \Sigma_{j=1}^{9} w_j R_{ij}$,where R_{ij} is the return of stock j at month i and the weights are given as $w_j = \frac{\text{Capitalization}_{ji}}{\text{Total Capitalization}_i}$.

We consider the following three models:

- Model 1: Random Slopes with Constant Variance
  Let $Y_{ij} = R_{ij} - R_{if}$ denote the excess return over the risk-free rate for stock j at month i. Then:

$$Y_{ij} \sim N(a_j + b_{1j}(R_{Mi} - R_{fi}) + b_{2j}SMB_i + b_{3j}HML_i + b_{4j}RMW_i + b_{5j}CMA_i, \sigma_j^2)$$

  for $i = 1:398,\ j = 1:9$ with priors: $\alpha_j \sim N(\mu_1, \sigma_1^2),\ b_{\kappa j} \sim N(\mu_\kappa, \sigma_\kappa^2),\ \sigma_\kappa^2 \sim Inverse\Gamma(0.1.0.1)$ and $\mu_\kappa \sim N(0, 100)$ for $k = 1:5$

- Model 2: FF 5-Factor Multivariate Regression with Constant Variance
  Putting all the data together into the diversified portfolio $Y_i = \Sigma_{j=1}^{9} w_j R_{ij} - R_{if}$ denotes the excess return of the portfolio over the risk-free rate at month i. Then:

$$Y_i \sim N(a + b_1(R_{Mi} - R_{fi}) + b_2SMB_i + b_3HML_i + b_4RMW_i + b_5CMA_i, \sigma^2)$$

  with uninformative priors: $\alpha \sim N(0, 100)$, $b_k \sim N(0, 100)$ for $k = 1:5, \sigma^2 \sim Inverse\Gamma(0.1, 0.1)$

- Model 3: FF 5-Factor Multivariate Regression with GARCH Volatility
  Lastly, we attempt to fit the same FF 5-factor multivariate regression ,but now we incorporate a GARCH model for the volatility of the portfolio's return. The GARCH (Generalized Autoregressive Conditional Heteroscedasticity) model is used to analyze the variance of financial assets by allowing it to vary over time as a function of past observations of the asset's return and variance. Essentially, the GARCH model accounts for the volatility clustering and time-varying nature of financial returns and allows for the estimation of conditional variance that changes over time based on past information. The likelihood of the portfolio's returns is:

$$Y_i \sim N(a + b_1(R_{Mi} - R_{fi}) + b_2SMB_i + b_3HML_i + b_4RMW_i + b_5CMA_i, \sigma_i^2)$$

  with the volatility given by the GARCH(1,1) model as:

$$\sigma_i^2 = \omega + a_0 Y_{i-1}^2 + b_0 \sigma_{i-1}^2$$

  where ω denotes the long-term variance, α_0 captures the dependence on previous months' returns and β_0 captures the dependence on previous months' variance. We fit both the FF 5-factor multivariate model and the GARCH(1,1) model within the same JAGS loops. The priors are uninformative $\alpha \sim N(0, 100)$, and $b_k \sim N(0, 100)$ for $k = 1:5$
  $\alpha_0 \sim Beta(1, 1), b_0 \sim Beta(1, 1),\ \omega = \gamma V_L$;and $\gamma = 1 - a_0 - b_0$ where $V_L = (0.00622426)$
  is the long-term variance of the S &P500 index.

3 Model Fitting Results

Model 1

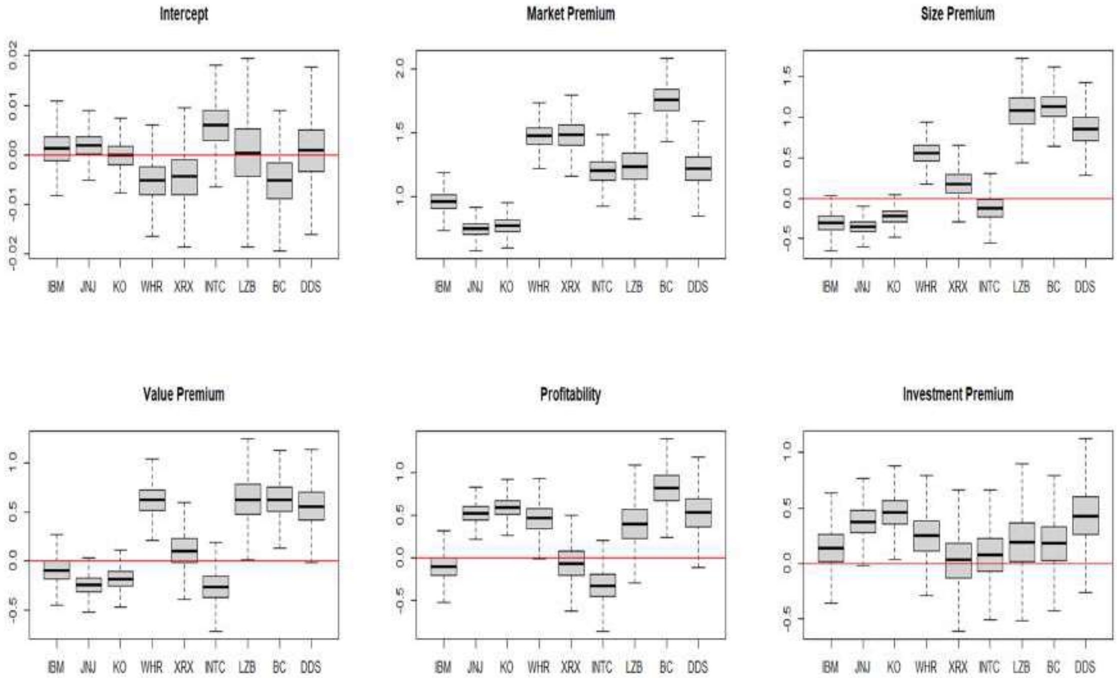


Figure 1: Posteriors of the FF 5 Factors Coefficients

Based on the fit of Model 1 we see that:

- The intercept for all the stocks is insignificant, something that implies that the FF 5-factor model adequately explains the excess returns across equities of all capitalizations
- The market premium is significant for all stocks. For small-cap equities, this premium is larger, implying that small-cap equities encapsulate greater risk and track aggressively the market.
- High-cap stocks have a negative exposure to the size factor, while small-cap stocks have a positive exposure to the size factor. That is, small-cap equities in general tend to outperform small-cap equities in the long term. For mid-cap equities, the exposure to the size factor is not that obvious.
- The effect of the value premium is not that clear for all the stocks. For growth equities with high Book-to-Market ratios like WHR, LZB, and BC a positive relationship arises, confirming the theoretical view that value companies with high B/M ratios tend to outperform growth companies with low B/M ratios. However, because in our equity universe, we have not included high-growth equities (like TESLA), the negative exposure to the factor is not obvious.
- The profitability factor is insignificant for the majority of the equities. However, companies with strong robust earnings such as Coca-Cola and Johnson & Johnson display positive excess returns.
- The investment factor does not seem to adequately capture excess returns across stocks. For Coca-Cola (KO), a company that historically follows a conservative investment approach a positive relationship arises between excess returns and the investment factor. For the rest of the equities, there seems to be no statistically significant relationship.

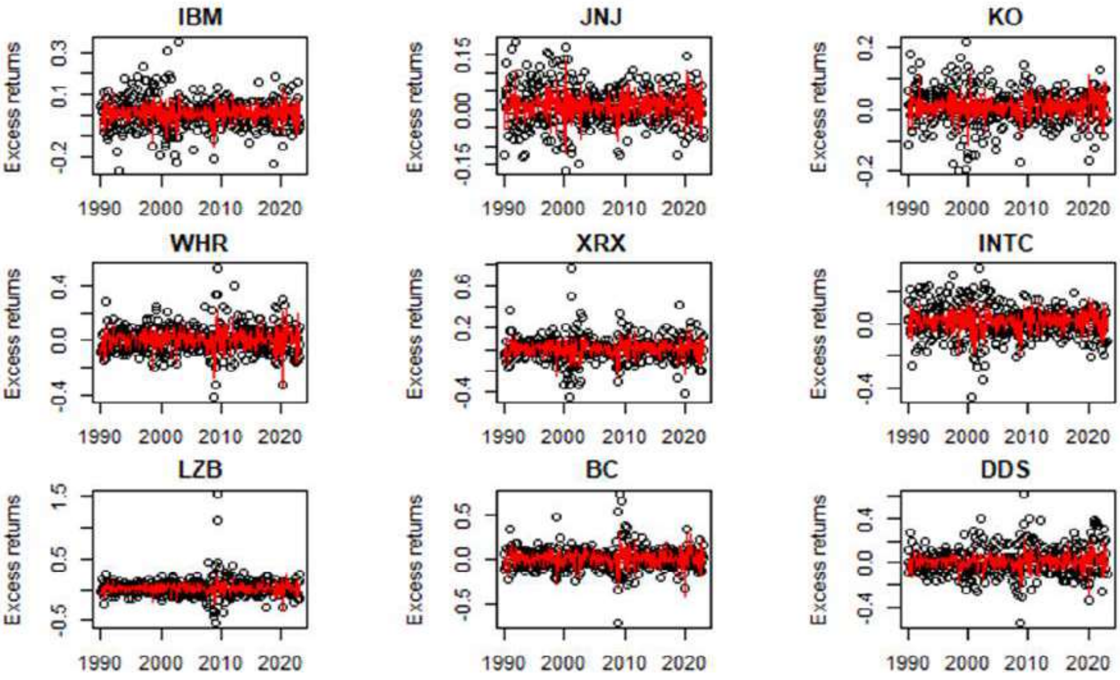


Figure 2: Fitted Curves using the posterior median

To examine the predictive ability of the estimated 5-factor model, the model is refitted by excluding the returns of the last 10 months from May 2022 until February 2023, and then out-of-sample predictions are computed for the corresponding values by drawing samples from the PPD. Figure 3 shows that the majority of the predicted returns follow the same market direction as the actual observed returns. This means that the FF 5-factor model can offer reliable intuition about the trend of a particular stock.

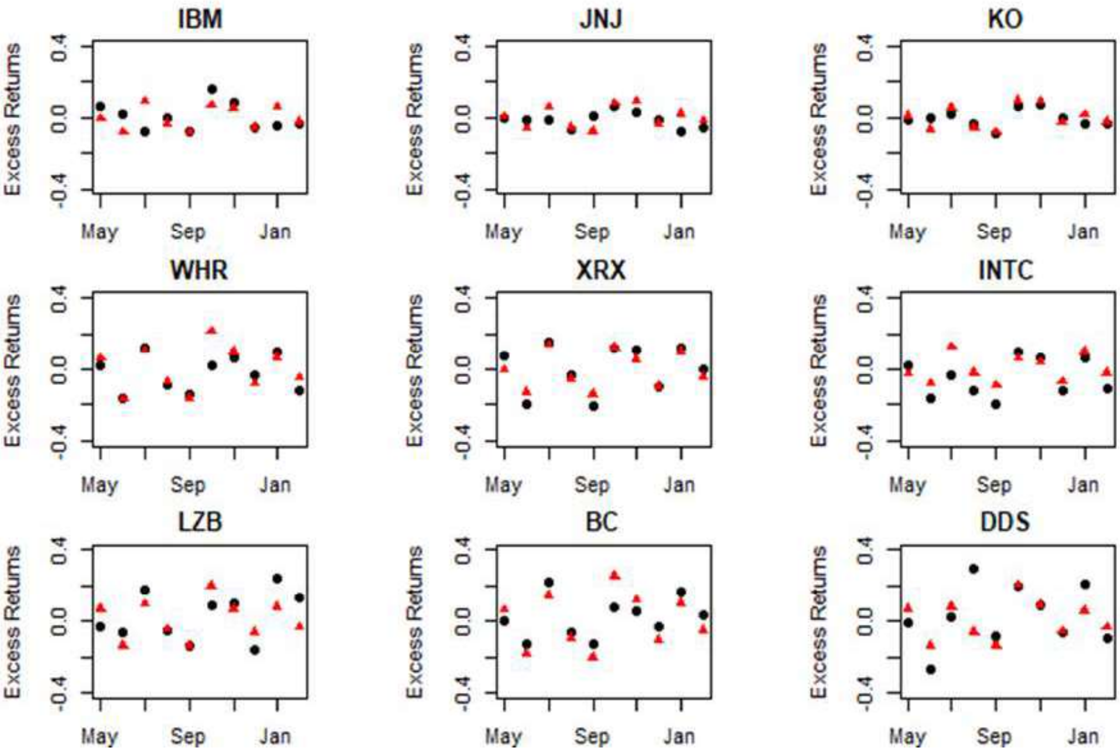
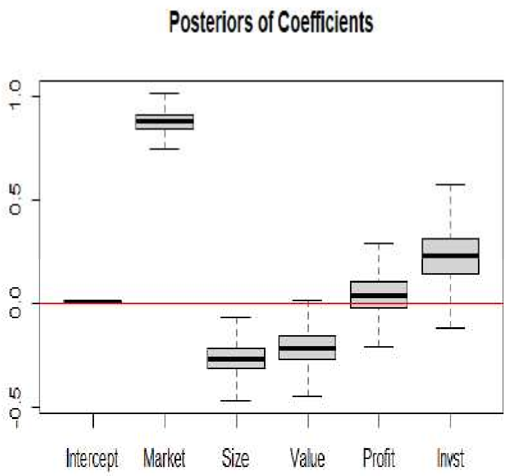


Figure 3: Out of sample model predictions from 05/2022-02/2023: The black bullet points correspond to the observed returns on a given month. The red triangle points correspond to the predicted returns

Model 2



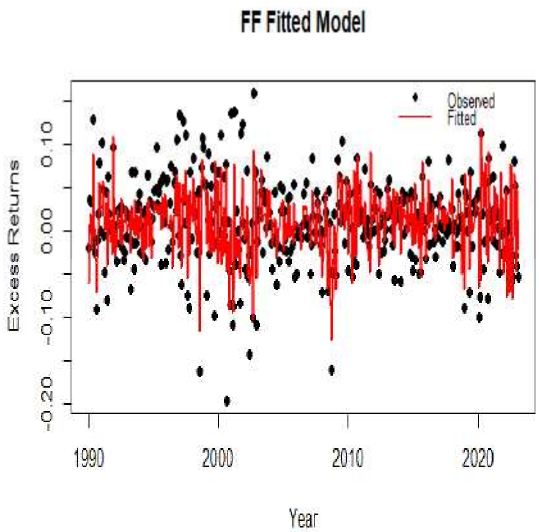


Figure 4: Posteriors of the FF 5 Factors Coefficients (Left Plots) & Fitted model using the posterior medians (Right Plot)

Based on the fit of the FF 5-factor model to the cap-weighted portfolio consisting of the nine stocks we see that:

- The intercept term is 0, thus the model captures the portfolio's returns sufficiently
- The portfolio has a strong positive exposure to the market risk premium
- The portfolio has a negative exposure to the size factor. This happens because high-capitalized equities that tend to underperform small-capitalized equities dominate the portfolio weights
- The portfolio's exposure to the value factor is also negative.
- The exposure of the portfolio to the Profitability premium is insignificant.
- The investment factor also does not seem to offer significant predictive power regarding the portfolio's asset returns.

Following the same out-of-sample prediction procedure as in Model 1, in Figure 5 we see the model-predicted portfolio returns from their posterior predictive distributions. The model captures sufficiently the direction of the observed portfolio's return.

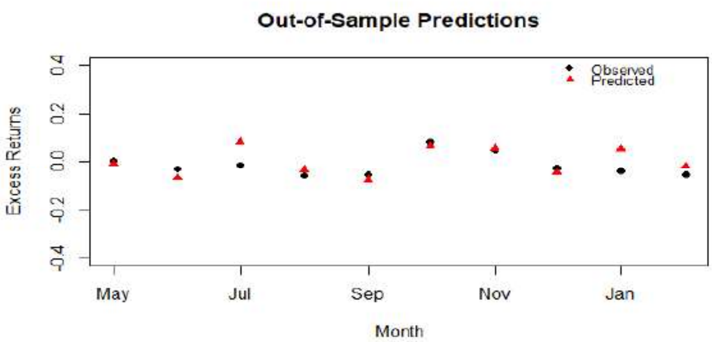


Figure 5: Out of sample model predictions from 05/2022-02/2023

Model 3

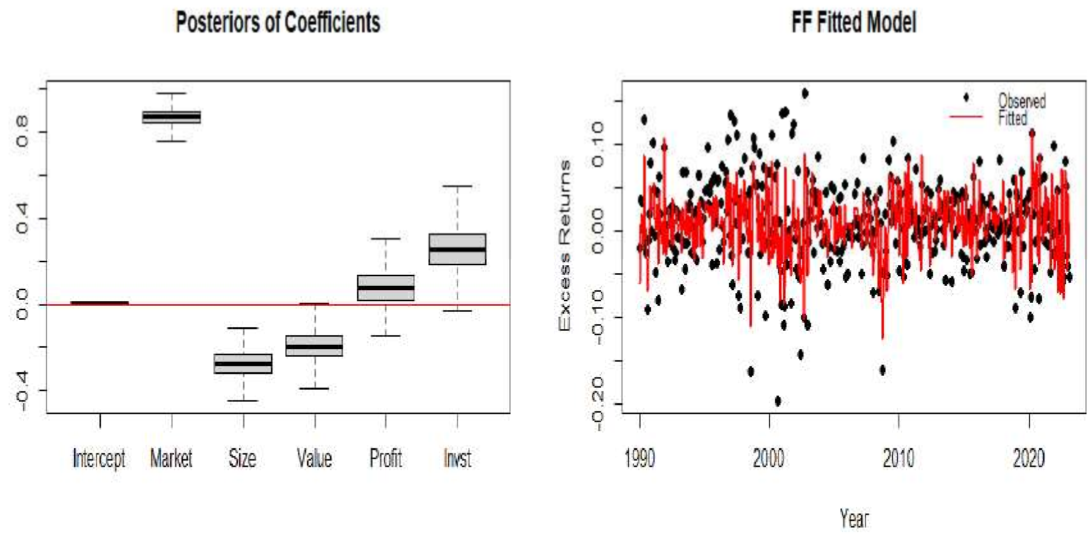


Figure 6: Posteriors of the FF 5-Factor with GARCH Variance and fitted curve using the posterior medians of the coefficients

The estimated FF 5-factor model enhanced with a GARCH(1,1) variance does not change significantly compared to its specification as in Model 2. The profitability factor remains insignificant, while the investment factor is on the edge. The interpretation of the estimated coefficients of the five factors is the same as in Model 2. In the left plot of Figure 7, we see the GARCH(1,1) model fitted to the portfolio's variance estimated through two methods. In method 1 the GARCH is fitted using the frequentist package *rugarch*. In method 2 the GARCH is fitted using the posterior medians of its coefficients.

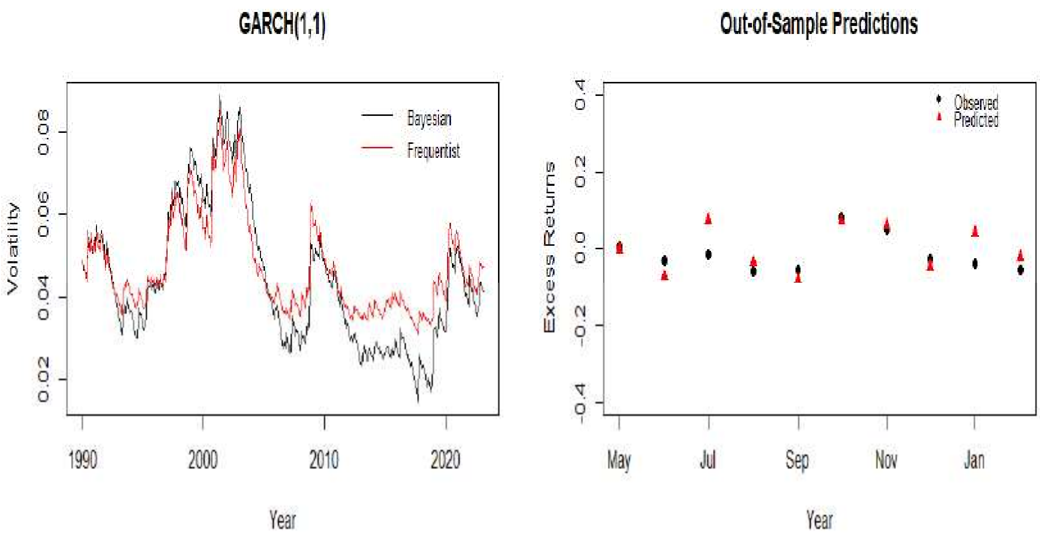


Figure 7: Left Plot: GARCH(1,1) model using the frequentist *rugarch* package and the Bayesian paradigm in JAGS. Right Plot: Out-of-sample model predictions for portfolio returns from 05/2022-02/2023

4 Implications of the study

Overall, the Fama-French 5-factor model with its three specifications described by Model 1, Model 2, and Model 3 and estimated using the Bayesian paradigm offers strong intuition about the factors that drive asset returns. The empirical findings for the case of the particular nine stocks examined in this paper do not diverge much from the theoretical framework suggested by the financial literature. More specifically, in all three alternative specifications of the FF 5-factor model, the findings for the examined equity universe are similar. In particular:

- The model intercept both for the random slopes specification as well as for the grouped portfolio fit is 0. This means that the model is able to capture well the factors that drive asset returns and does not leave some portion of these returns unexplained.
- The market premium factor is significant in explaining both individual asset excess returns as well as portfolios' excess returns. Small-cap equities tend to track the market more aggressively and overreact in market movements.
- The size premium/factor has also significant predictive power in explaining asset returns. For the individual study, it is confirmed that small-cap equities have a positive exposure to the size factor and so tend to outperform large-cap equities in the long run which have a negative exposure to the size factor.
- When considering individual stocks, the value factor seems to have a moderate predictive power for predicting asset returns. Value equities with high Book-to-Market ratios $\frac{B}{M}$ tend to perform better than growth stocks with low Book-to-Market ratios. Equities that maintain average $\frac{B}{M}$ ratios and cannot be classified as growth or value, their exposure to the value factor is insignificant. Furthermore, the value factor is also able to explain well the excess returns of the portfolio. The portfolio contains growth stocks with high weights (J&J, INTC) and negative exposure to the value factor so its exposure to the value premium is also negative.
- For individual equities with robust profitability, the profitability factor can capture excess returns well. However, for the combined portfolio, it loses its predictive power.
- For individual stocks, equities that follow a conservative investment approach such as JNJ & KO have a positive exposure to the investment factor. For the overall portfolio, when we assume a constant variance the investment factor is insignificant, while when a GARCH(1,1) variance is employed, the investment factor tends to be significant.

Comparison with the frequentist approach
In Table 2 the FF 5-factor model is fitted for the portfolio using an OLS regression. The direction of the portfolio exposure to the factors is the same, while the profitability factor is again insignificant.

Variable	**Est.**	**S.E.**	**val**	**p**
Intercept	0	0	2.57	0.01
Market Premium	0.88	0.04	21.45	0
Size Premium	-0.27	0.06	-4.48	0
Value Premium	-0.22	0.07	-3.11	0
Profitability Premium	0.04	0.08	0.5	0.62
Investment Premium	0.23	0.1	2.17	0.03

Table 2: Frequentist OLS fit of the FF 5-factor model

5 Further Questions

Some important topics for further research on the FF 5-factor model would be:

1. Examine the correlation structure between the 5 factors
2. Experiment with more skewed distributions for asset returns
3. Experiment with different equity universes and examine why mid-cap equities seem to have the least sensitivity to the 5-factors