A 6-step commutation, surface mounted permanent magnet, Brushless Direct Current Electric Motor (BLDC EM) simulation model with speed control for driving transient load cycles.

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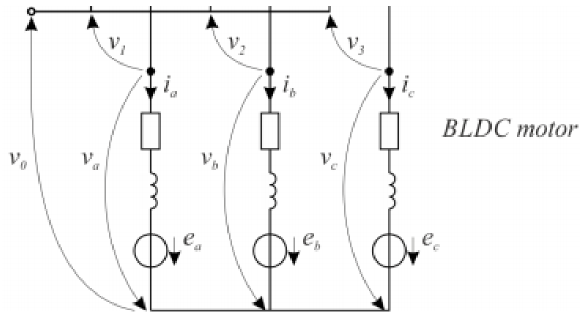


Equivalent circuit diagram

Electric Motor model

Selection of motor design parameters - Inputs:

- K_t: Torque Constant. Defines the torque capabilities of motor. Is directly proportional to current. $K_t = \frac{Max(T)}{Max(i)}$
- $L_s = L_q = L_d$: Defines the stator winding inductance
- M: mutual stator windings inductance, here we assume $M = 0$
- R_s: Defines the stator winding resistance
- J: The Inertia of the rotor
- P: The number of pole pairs
- Phase Connection: The stator 3 phases can be connected only in Star(Y)

Other variables:

- u_i = phase voltage
- u_{ij} = phase voltage difference
- i_i = phase current
- i_{ij} = phase current difference
- e_i = phase back electromotive force (BEMF)
- e_{ij} = back electromotive force difference (BEMF)
- λ = flux linkage
- P_{loss} = Resistive power losses on BLDC

where $i = a, b, c$

Voltage equations:

$$u_{ab} = R_s * i_{ab} + L_s * \dot{i_{ab}} + e_{ab}$$

$$u_{bc} = R_s * i_{bc} + L_s * \dot{i_{bc}} + e_{bc}$$

Assuming $i_a + i_b + i_c = 0, Ls = Ls - M, (M = 0)$

Flux linkage equations:

The flux linkage dependency to the torque constant.

$$\lambda_{PM} = \frac{2}{p * 3} * K_t$$

The *BEMF* relation with flux linkage derivative is:

$$e_i = \dot{\varphi_{el}} * \frac{d\lambda_{PMi}}{d\varphi_{el}}$$

Torque equation:

$$T = \frac{1}{3} * p * [\lambda_{PMab} \quad \lambda_{PMbc}] * \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} * \begin{bmatrix} i_{ab} \\ i_{bc} \end{bmatrix}$$

Resistive Power Losses:

Simplified calculation of iron losses in the stator windings of the motor.

$$P_{losses} = \left({i_a}^2 + {i_b}^2 + {i_c}^2\right) * Rs$$

Mechanical equations of a flexible shaft:

The mechanical system is modelled as a multibody system with two elastically coupled rigid bodies. In one side is the electric motor (*em*) and in the other the load.

$$J_{em} * \frac{d\omega_1}{dt} = T_{em} - c * d\varphi - k * d\omega$$

$$J_{load} * \frac{d\omega_2}{dt} = T_{load} - c * d\varphi - k * d\omega$$

Control & Power Electronics model

Variables:

- i_i = phase current [A], where $i = a, b, c$

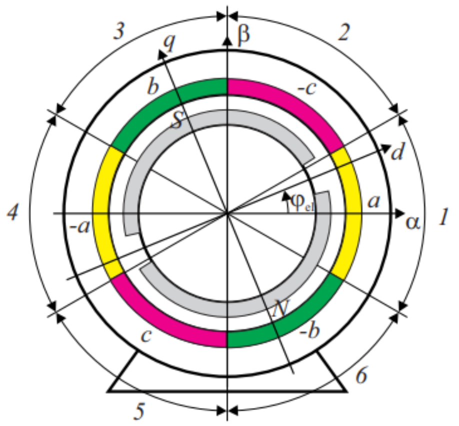
- i_{low_i}: current flowing at the lower switch of each rectifier [A]
- u_i: phase voltage [V], where $i = a, b, c$
- V: Instant battery terminal voltage [V]
- d: duty cycle [-]
- $t_{i_{open}}$: Time period when switch is open [s]
- T_s: Sample time period of PWM signal [s]
- R_{off}: diode reverse direction blocking resistance of a switch [Ω]

Model explanation and equations:

The speed sensor (hall sensor or encoder) is measuring the speed of the shaft and feeds the controller block with the feedback loop signal. The controller calculates the difference between commanded and measured speed and with a PI algorithm sets a correction command, the duty cycle. This duty cycle can take any value from -1 to +1.

The duty cycle signal then has to switch 3 full bridge rectifiers with total 6 switches (MOSFETs) in order to create the magnetic field on the motor's windings. As these switches can only switch between two positions – ON or OFF –with a very low switching frequency, the duty cycle signal is converted into a PWM signal with a constant period sample time. So, the average PWM signal is equal with the average duty cycle signal for every period.

This PWM signal is then fed into six switching patterns, one for each switch of the three rectifiers, according to the commutation mode. The commutation is done only 6 times during a full electrical rotation or 360 electrical degrees according to the motor schematic and each time a different combination of switching is followed as it can be seen on the table below:



Sector	HBhi	HBlo	HBoff
1	2	3	1
2	2	1	3
3	3	1	2
4	3	2	1
5	1	2	3
6	1	3	2

The six MOSFETS are simulated with switches and free-diodes that are parallel to each switch. The diodes are assumed to have a constant reverse direction resistance R_{off}. Each full-bridge is consisted of two ideal switches that switch in opposite directions with each other and without delay.

For every bridge, the mean value of the output phase voltage is:

$$u_i = d_i * V$$

While the duty cycle converted PWM signal is calculated by:

$$d_i = \frac{t_{i_{open}}}{T_s} \text{ with } i = 1,2,3.$$

In order to find the output phase voltage u_i and the current consumption of the battery I_{dc}, we need to calculate the currents flowing at each lower switch i_{low_i} based on the voltage difference while knowing the motor phase currents i_i, the instant battery terminal voltage V and the diode reverse direction resistance R_{off}.

We separate four different conditions:

- When $higher\ switch = 0$ and $lower\ switch = 1$

$$u_i = 0$$

$$i_{low_i} = -i_i + \frac{V}{R_{off}}$$

- When $higher\ switch = 1$ and $lower\ switch = 0$

$$u_i = V$$

$$i_{low_i} = \frac{V}{R_{off}}$$

- When $higher\ switch = 0$ and $lower\ switch = 0$

$$u_i = \min\left(\max\left(\frac{V - i_i * R_{off}}{2}\right), V\right)$$

  - When $i_i \leq 0$

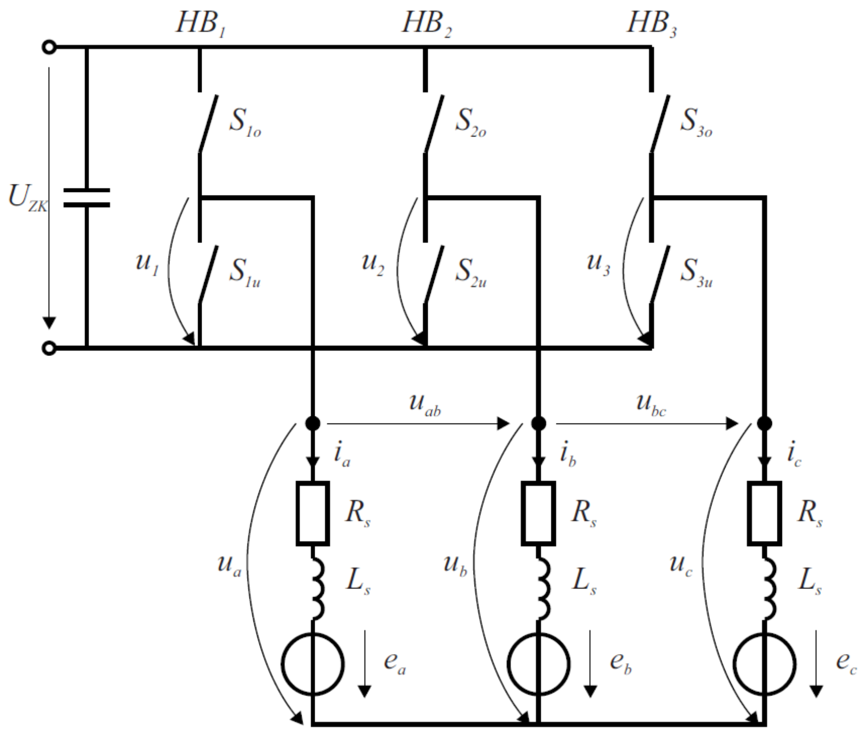
$$i_{low_i} = \frac{V}{R_{off}}$$

  - When $i_i > 0$

$$i_{low_i} = \frac{V - u_i}{R_{off}} - i_i$$

- When $higher\ switch = 1$ and $lower\ switch = 1$

$$u_i = 0, \qquad i_{low_i} = 0$$



These 3 lower bridge currents i_{low_i} will be summed up and sent to the battery block for the current consumption calculation.

Battery model

Selection of battery design parameters - Inputs:

- c : energy capacity [Wh]
- R_{int}: Internal resistance [Ω]
- SoC_{int}: Initial state of charge of battery [%]
- V_{link} : Open circuit voltage of battery [V]

Other variables:

- $\Sigma(i_{low})$: Sum of lower bridge currents of each rectifier [A]
- I_{dc}: DC battery current [A]
- V: Instant battery terminal voltage [V]
- P_{ideal}: Ideal battery power to be delivered [W]
- P_{actual}: Actual battery power to be delivered [W]
- P_{loss}: battery power losses due to internal resistance losses [W]
- E_{int}: Initial energy [W] of battery
- E: Maximum energy [W] of battery

Model explanation and equations:

The battery model is simulated by an equivalent circuit model with a constant internal resistance and open circuit voltage.

The current consumption calculation I_{dc} is calculated by:

$$I_{dc} = -\left(\frac{1}{c} * \int V dt + \Sigma(i_{low})\right) \textit{ but } I_{dc} = \frac{V - V_{link}}{R_{int}}$$

$$\textit{So:} \quad I_{dc} = \left[\frac{R_{int}}{c * V_{link}}\right] * \int I_{dc} dt - \Sigma(i_{low_i})$$

Following, the Power equations of the battery are calculated below:

$$P_{ideal} = P_{actual} + P_{loss}$$

$$P_{ideal} = I * V_{link}$$

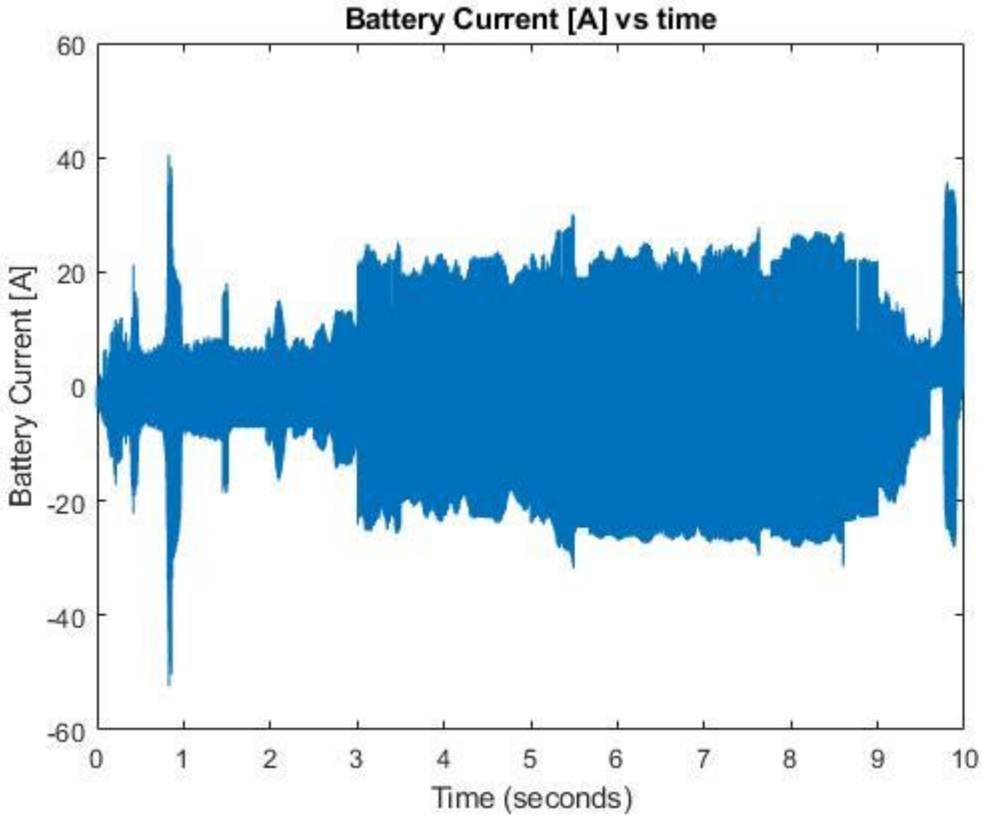
$$P_{loss} = I^2 * R_{int}$$

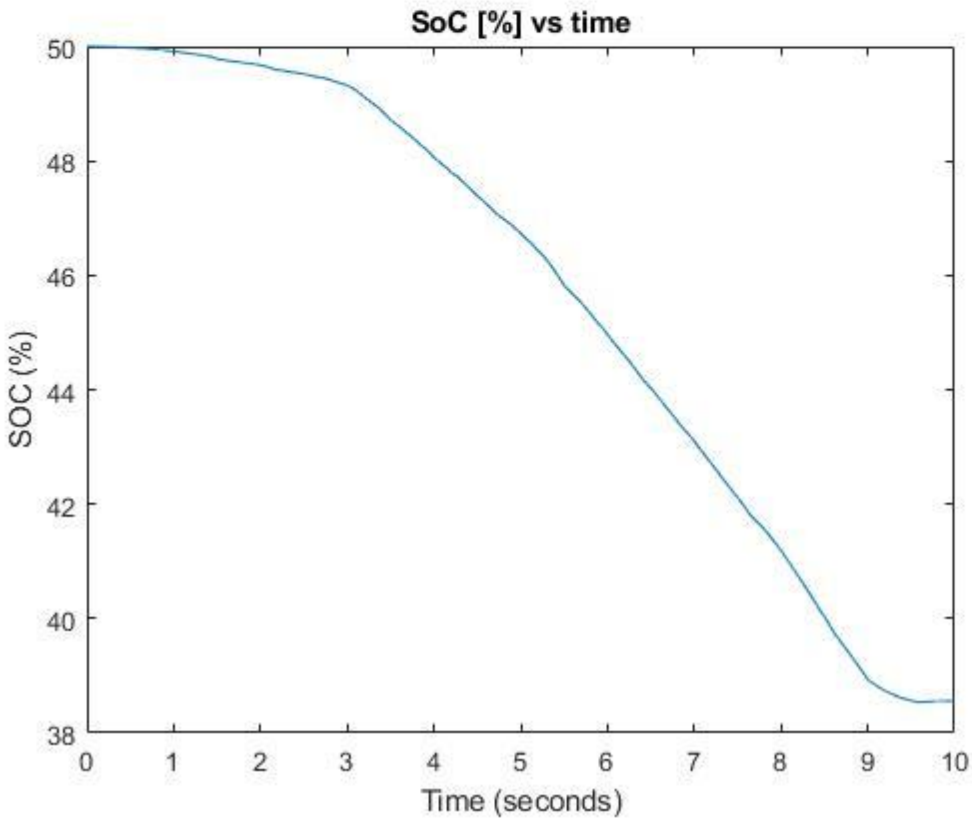
$$P_{actual} = I_{dc} * V_{link} - I_{dc}^2 * R_{int}$$

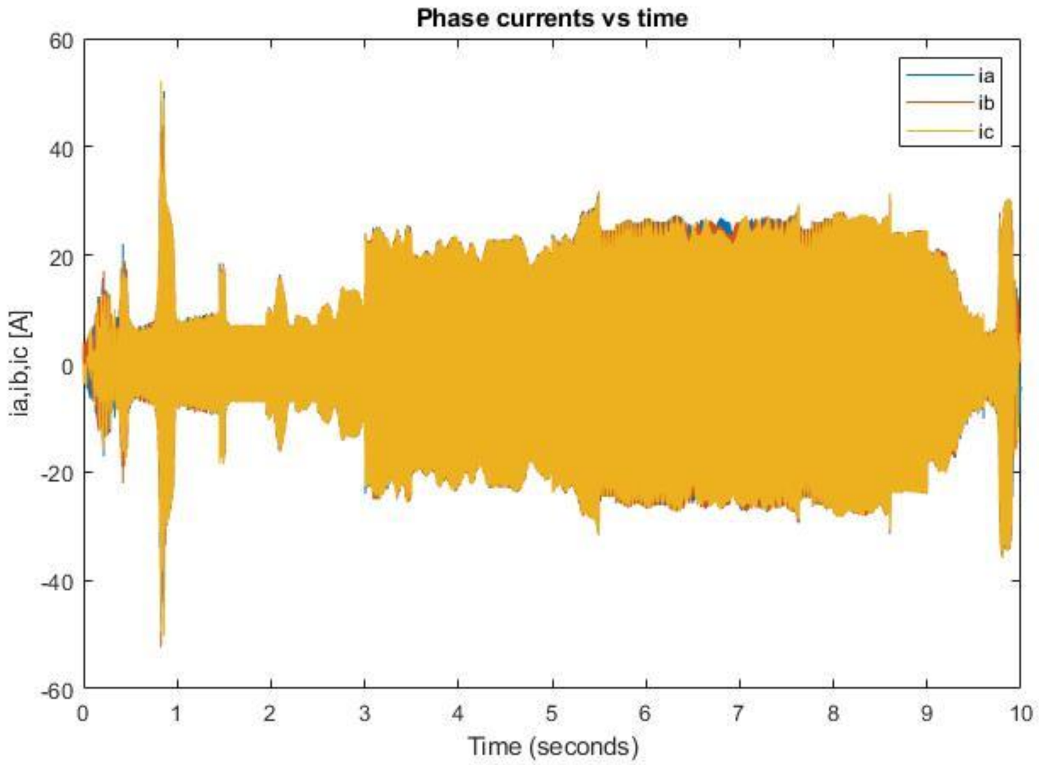
Finally, the state of charge *(SoC)* of the battery is calculated:

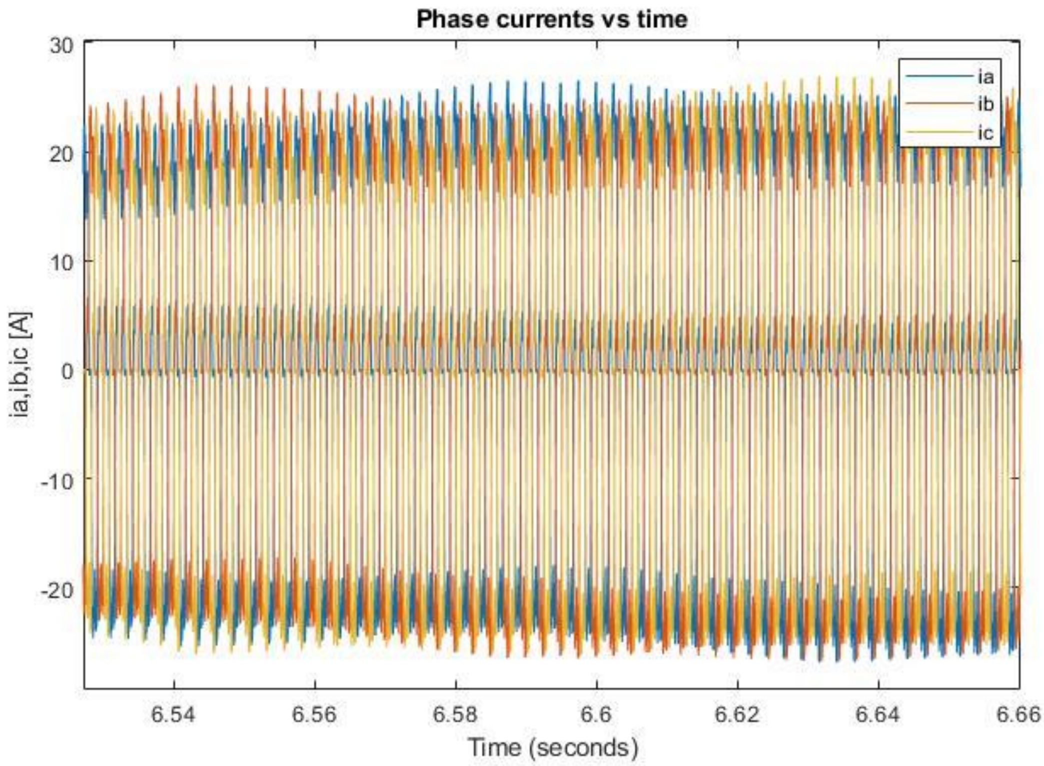
$$SoC = SoC_{old} + 100 * \frac{d(E_{int})}{E}$$

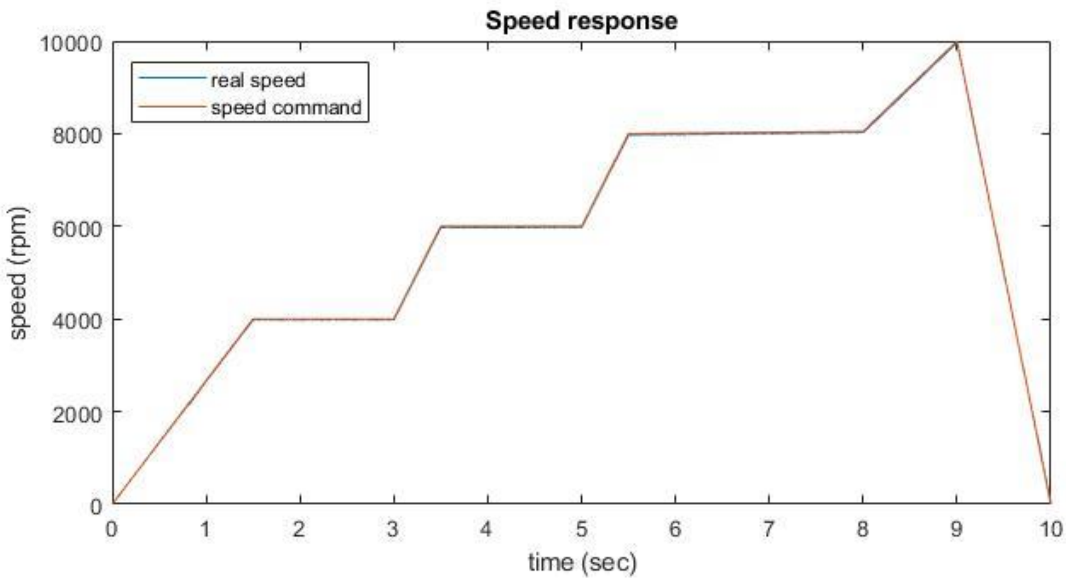
Results

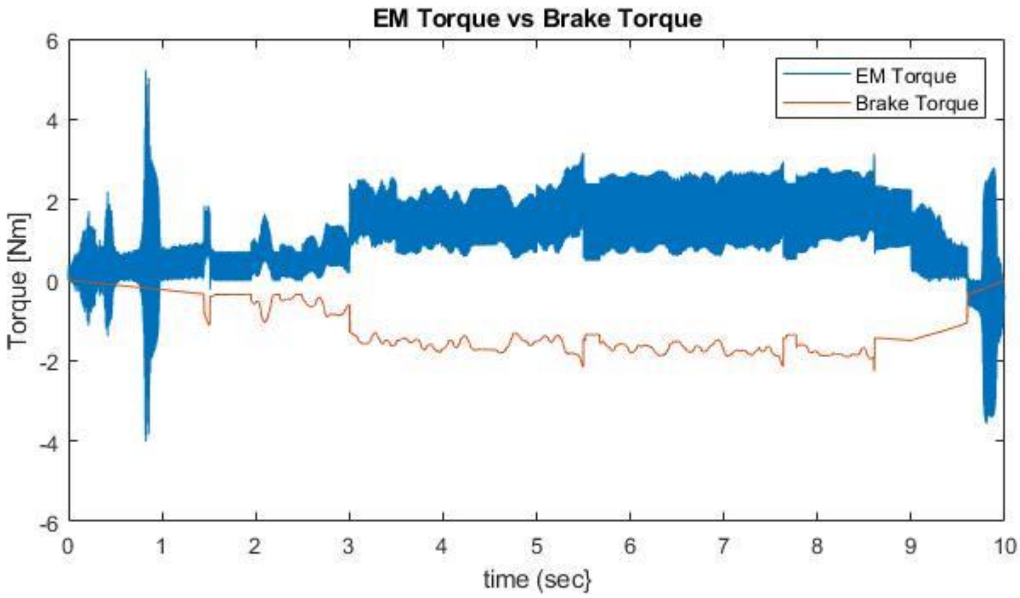

Battery Current [A] vs time
Battery Current [A]
Time (seconds)
60
40
20
0
-20
-40
-60
0 1 2 3 4 5 6 7 8 9 10

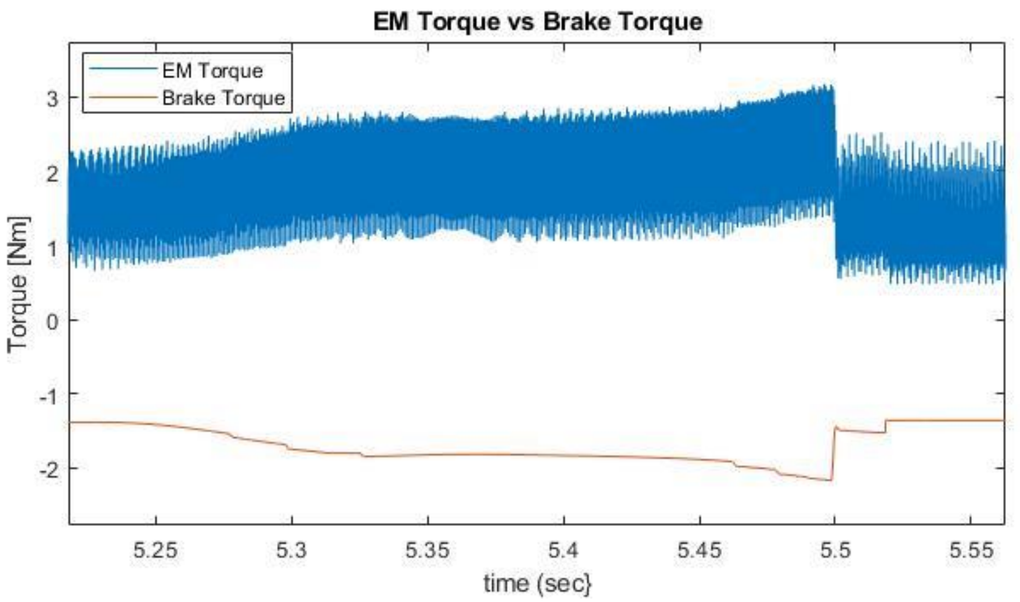

SoC [%] vs time
SOC (%)
Time (seconds)
50
48
46
44
42
40
38
0 1 2 3 4 5 6 7 8 9 10


Phase currents vs time
ia
ib
ic
ia,ib,ic [A]
Time (seconds)


Phase currents vs time
ia
ib
ic
ia,ib,ic [A]
Time (seconds)


Speed response
real speed
speed command
speed (rpm)
time (sec)


EM Torque vs Brake Torque
EM Torque
Brake Torque
Torque [Nm]
time (sec}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