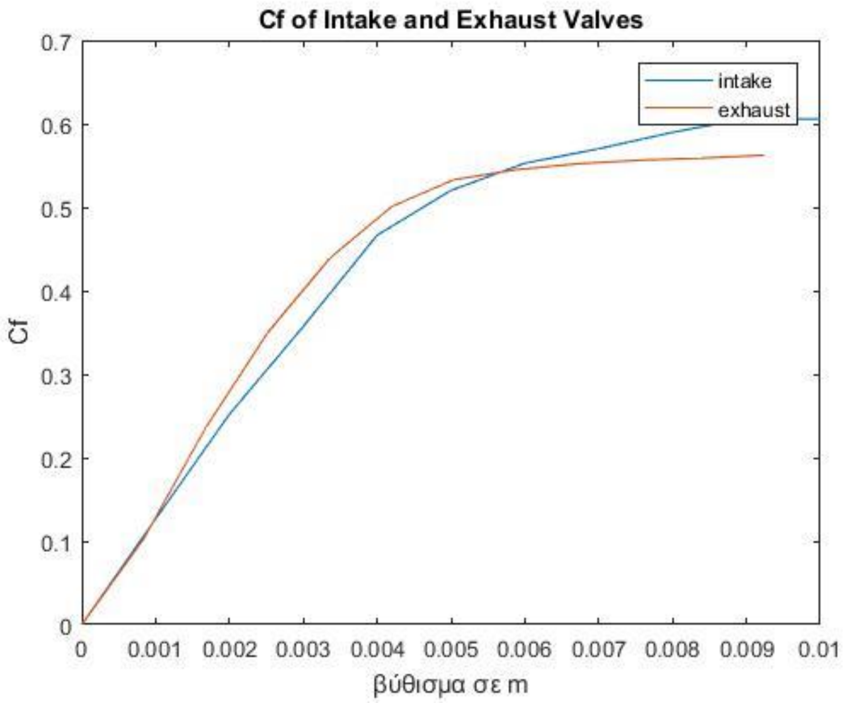
Detailed model explanation

Data Input

- Introduce of the geometric data of the valves,
- Transfer of the profile of the valve flow rate from measured data:



- Introduce the boundary conditions and combustion parameters.
- Consider calorific fuel power LHV = 43400000 J / kg
- Calculate / assume rest conditions for intake & exhaust:

$$dens_{in} = \frac{P_{in}}{287 * T_{in}},\ P_{ex} = \frac{P_{ex}}{P_{in}} * P_{in}\ ,\ dens_{ex} = \frac{P_{ex}}{287 * T_{ex}},$$

$$h_{in} = U_{in} + \frac{P_{in}}{dens_{in}},\ h_{ex} = 660000\ J/kg.$$

Also set the closing of the exhaust valve as a zero point and solve for the 720 degrees of the complete cycle.

- Calculate Imep from the given bmep assuming mfmep as follows: $Imep = bmep + fmep$

$$fmep = mfmep + pmep + amep$$

Where: mfmep = 1.2 , pmep = Pex-Pin , $amep = 2.69 * \frac{N}{1000}^{\frac{3}{2}} * 10^{-2}$.

Initialization

Initialize all arrays with zeros in 720 rows for each crank angle (c.a.). Especially for the tables of the masses I have the following tables:

- m_{in} → indicated how much air flow enters the cylinder from the intake each c.a. [kg/rad]
- m_{ex}→ indicated how much air flow exits the cylinder from the exhaust each c.a. [kg/rad]

- m_{tot} → indicated how much mass is inside the cylinder each c.a. [kg]
- m_{air} και m_{fuel} → indicated how much mass of air and fuel is inside the cylinder each c.a. [kg]
- $\frac{dma}{d\theta}, \frac{dmf}{d\theta}$ και $\frac{dmt}{d\theta}$ → indicates the rate of change for air, fuel and total mass inside the cylinder for each c.a. *[kg/rad].*

  The logic is that the mass m of degree i, is calculated as the mass plus the change dm / dθ of the previous c.a. (i-1). This logic was applied to each mass, ie air, fuel and total.

  Based on the parameters P(IVC) and T (IVC) with IVC = Intake Valve Close, the mass of the remaining exhaust gas is calculated. Moreover, assuming that the fuel mass inside the cylinder at the closing of intake is zero, we know the air mass and combining the assumed λ we know how much fuel we have to inject. So, the mass of residual gass, the total mass but also the air anf fuel mass that enters the cylinder are:

- $m_{rg_{tot}} = f * m_{mix\_tot}$
- Όπου, $m_{mix\_tot} = \frac{P(IVC)*V(IVC)}{R_{tot}*T(IVC)}$ Με $R_{tot} = (1-f)*(287-20*F(IVC)) + f*\left(287-\frac{20}{lambda}\right)$
- Επίσης, $m_{air\_tot} = (1-f)*m_{tot}$
- Και $m_{fuel_{tot}} = \frac{m_{air_{tot}}}{lambda*14.7}$

Also, initialize the *v* for the heat transfer model because it stays constant for all the other c.a. outside of the combustion duration. So:

$$v = C_1 * c_m \text{ όπου } C_1 = \begin{cases} 6.18 + 0.417 * \frac{cu}{cm} & \text{for mixture exchange} \\ 2.28 + 0.308\,\frac{cu}{cm} & \text{for compression expansion.} \end{cases}$$

Where $c_m = 2*\pi*n$, *in the mean piston velocity* $[RPS]$.

Cylinder – Valves

Determining the volume of the cylinder as a function of the angle ϕ is done taking into account the geometry of the cylinder. That is, it applies:

$$V(\varphi) = V_C + A_{CYL}\cdot s(\phi)$$

$$A_{CYL} = \frac{\pi}{4}\cdot b^2$$

$$s(\phi) = r + l - \sqrt{l^2 - r^2\cdot\sin^2(\phi)} - r\cdot\cos(\phi)$$

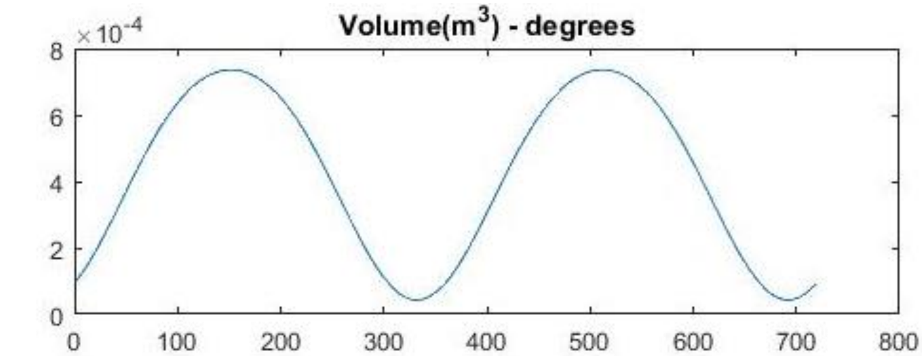
$$\frac{ds}{d\phi} = r\cdot\left[\sin(\phi) + \frac{\lambda_s}{2}\cdot\frac{\sin(2\phi)}{\sqrt{1-\lambda_s^2\cdot\sin^2(\phi)}}\right], \qquad \lambda_s = r/l$$

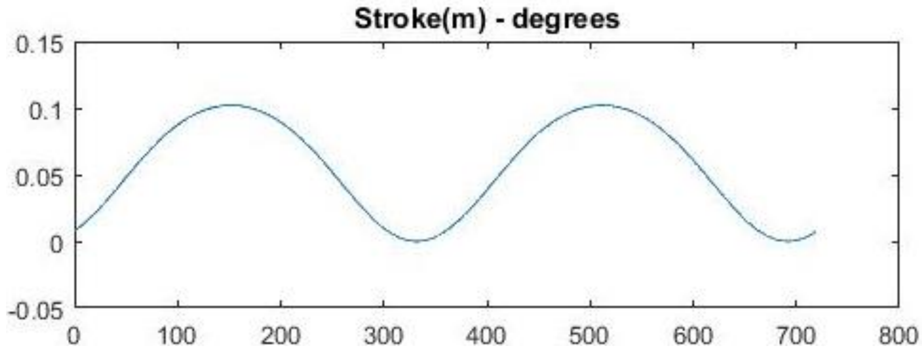
$$\frac{dV}{d\phi} = A_{CYL}\cdot\frac{ds}{d\phi} \quad \text{και} \quad \frac{dV}{dt} = \omega\cdot\frac{dV}{d\phi}$$

$s = Max[s(\varphi)] - Min[s(\varphi)]$ $\qquad A_{CYL}*s = V_1 - V_c = V_d = V_c(\varepsilon - 1)$

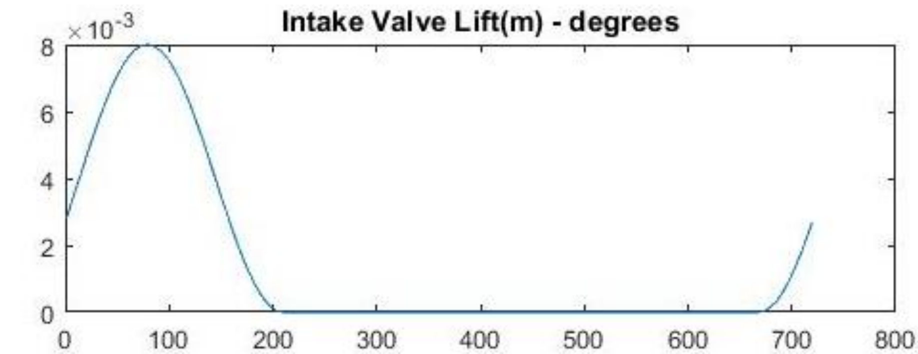
Thus, following the formulas, the volume (V) for each crank degree is obtained, the stroke (s) for each crank degree as well as the $\frac{dV}{d\theta}$ and $\frac{ds}{d\theta}$ respectively.

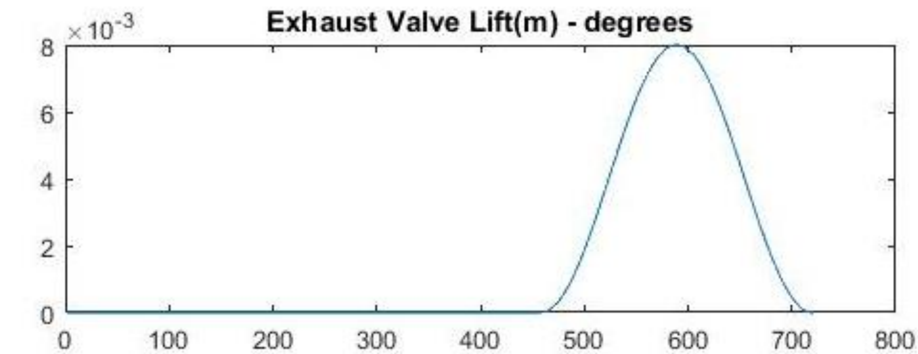
Below are the tables of volume and stroke as a function of crankshaft angle.



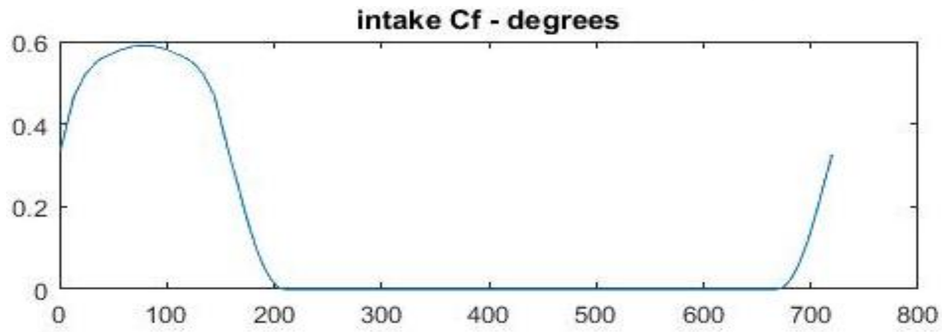


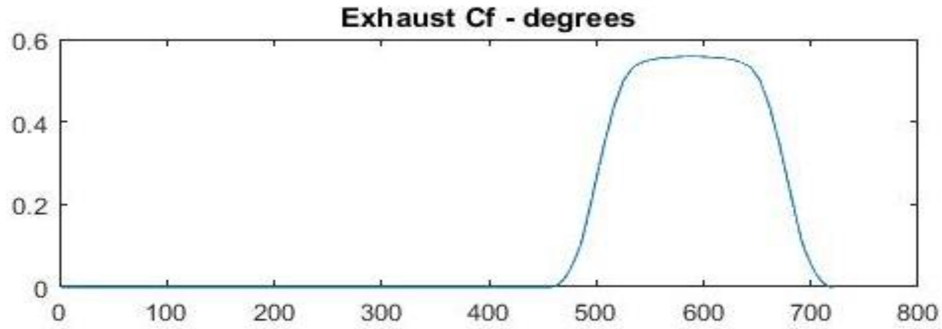
Regarding the Lift of the valves, taking the relation of the lift as a function of θ, the following diagrams are created:





While multiplying the Cf of intake and exhaust valves accordingly, the valve Lift is calculated:





Assumptions

The models solves with a timestep of 0.1 crank-angle degrees one cycle of 720 ca degrees. The calculation starts with the closing of the exhaust valve, where we have clean intake flow and ends up with the end of valve overlap duration.

In every point of operation, five assumptions have to be made. These are for:

- air-fuel ratio "λ",
- Cylinder Pressure at start of the loop "P(1)",
- Cylinder Temperature at start of the loop "T(1)",
- Cylinder Pressure at start of compression "P(Start of Compression)" and
- Cylinder Temperature at start of compression "T(Start of Compression)".

Based on the last two assumptions, the mass of residual gas inside the cylinder is calculated. Moreover, assuming that the fuel mass inside the cylinder at the closing of intake is zero, we know the air mass and combining the assumed λ we know how much fuel we have to inject. From all these values, we assume the rest of values:

$$U(1),\quad Dens(1),\quad H(1),\quad m_{air}(1),\ m_{fuel}(1),\ m_{tot}(1),\ \frac{dU}{dT}(1),\quad Cp(1),\quad Cv(1),\quad k(1)$$

To calculate the mass flow during the opening period of the inlet valve we first have to calculate the active surface area Ain multiplied by the factor Cf. We also need to know the pressure ratio $\pi = \frac{\mathrm{P(i)}}{\mathrm{Pin}}$ but also the critical pressure ratio $\pi,\mathrm{crit} = \frac{2}{(\mathrm{k(i)}+1)^{\frac{\mathrm{k(i)}}{\mathrm{k(i)}-1}}}$, to take into account in which pressure difference the flow is choked. Thus, depending on the value of π I have the following cases:

- If π > 1 then we have backflow and the relations are modified as:

$$\Psi_{in} = \sqrt{2 * \frac{k(i)}{k(i)-1} * (-\pi^{2*k(i)} + \pi^{(k(i)+1)/k(i)})}$$

$$m_{in} = -\frac{2 * A_{in(i)} * Cf(i) * \sqrt{Pin * dens_{in}} * \Psi in}{\omega}$$

- If π<1 and π ≥ π, crit, then :

$$\Psi_{in} = \sqrt{2 * \frac{k(i)}{k(i)-1} * (+\pi^{2*k(i)} - \pi^{(k(i)+1)/k(i)})}$$

$$m_{in} = \frac{2 * A_{in(i)} * Cf(i) * \sqrt{Pin * dens_{in}} * \Psi in}{\omega}$$

- Otherwise:

$$m_{in} = \frac{Pin}{(Rin * Tin)} * \sqrt{k(i) * R(i) * Tin} * Ain(i) * Cf(i) * \frac{2}{k(i)+1}^{\frac{k(i)+1}{2*(k(i)-1)}}$$

$$with\ Ain(i) = \frac{\pi}{4} * din^2$$

So, $m_{air}(i+1) = m_{air}(i) + m_{in}(i)\ and\ \ m_{tot}(i+1) = m_{tot}(i) + m_{in}(i)$.

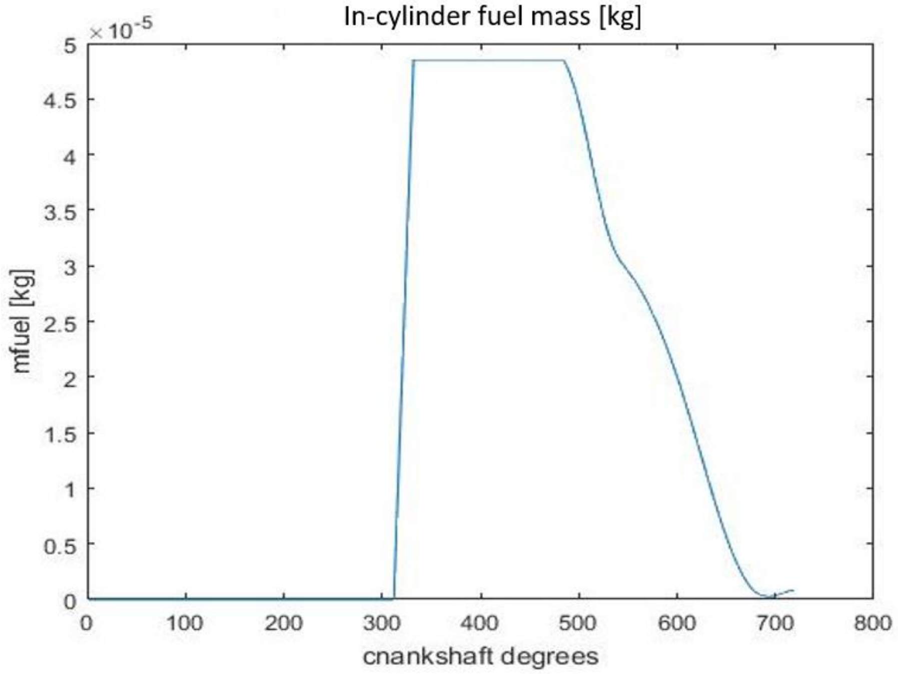
Fuel injection

As for the fuel injection, we should form the table F for each crank angle step. Initially we have F(i) = 0 until the injector starts to spray. From the beginning of the ignition duration until the end of it, F increases linearly from zero to the desired value and then remains constant.

Also, with the start of the injection, the fuel mass in the chamber changes, so for those degrees, respectively, the fuel mass will increase linearly. We have the following relations:

- F (i) = $\frac{1}{lambda(i)}$,
- $m_{fuel}(i+1) = m_{fuel}(i) + m_{air}(i+1) * \frac{F(i+1)}{14.7}$
  - $\frac{dF}{d\theta}(i) = F(i+1) - F(i)$

Below we can see the in-cylinder fuel mass vs crank-angle:



Combustion

Simulating the combustion process with a double Wiebe function, we have the following relations:

$$\frac{dQ_{B,1}(\phi)}{d\phi} = Q_{B,1}\alpha(m_1+1)\left(\frac{\phi-\phi_{BB,1}}{\Delta\phi_{BD,1}}\right)^{m_1} e^{-a\left(\frac{\phi-\phi_{BB,1}}{\Delta\phi_{BD,1}}\right)^{m+1}} \qquad \varphi_{BB,1} < \varphi < \varphi_{BB,1} + \Delta\varphi_{BD,1}$$

$$\frac{dQ_{B,2}(\phi)}{d\phi} = Q_{B,2}\alpha(m_2+1)\left(\frac{\phi-\phi_{BB,2}}{\Delta\phi_{BD,2}}\right)^{m_1} e^{-a\left(\frac{\phi-\phi_{BB,2}}{\Delta\phi_{BD,2}}\right)^{m+1}} \qquad \varphi_{BB,2} < \varphi < \varphi_{BB,2} + \Delta\varphi_{BD,2}$$

$$Q_{B,1} = x \cdot Q_{B,ges} \qquad Q_{B,2} = (1-x) \cdot Q_{B,ges}$$

- Where the constant α is equal with 4.605 for a passenger car.

- In order to calculate the φ_{BB1} $(1st\ combustion\ starting\ angle)$ and the φ_{BB2} $(2nd\ combustion\ starting\ angle)$

We have to know the φ_{FB} (injection starting angle) and the duration of combustion delay, $d\varphi_{ZV}$. As long as $d\varphi_{ZV}$ we calculate the *Pm, Tm, Rm* before combustion:

- $Pm = mean(P(\varphi_{FB}) + P(\varphi_{FB+1}) + P(\varphi_{FB+2}) + P(\varphi_{FB+3}) + P(\varphi_{FB+4}) + P(\varphi_{FB+5}))$
- $Tm = mean(T(\varphi_{FB}) + T(\varphi_{FB+1}) + T(\varphi_{FB+2}) + T(\varphi_{FB+3}) + T(\varphi_{FB+4}) + T(\varphi_{FB+5}))$
- $Rm = mean(R(\varphi_{FB}) + R(\varphi_{FB+1}) + R(\varphi_{FB+2}) + R(\varphi_{FB+3}) + R(\varphi_{FB+4}) + R(\varphi_{FB+5}))$

And we introduce them in Sitkei formula:

$$d\varphi_{ZV} = \frac{6N}{1000}\left(0.1 + 0.135 * \left(1.0197 * Pm^{-0.97}\right) * e^{\frac{7800}{6.9167*Rm*Tm}} + \right.$$

$$\left. 4.8 * \left(1.0197 * Pm^{-1.8}\right) * e^{\frac{7800}{6.9167*Rm*Tm}}\right)$$

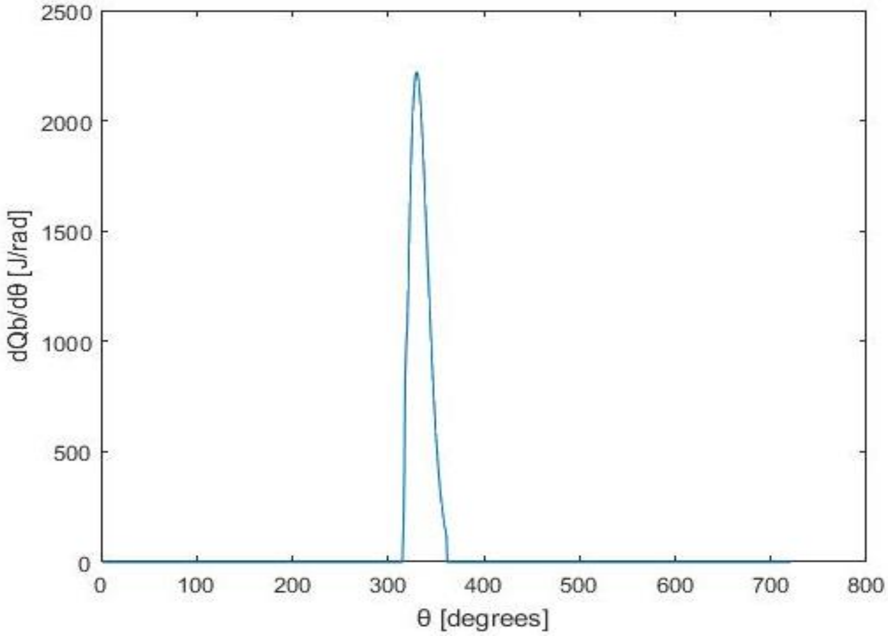
While $a_{ZV} = 0.1$, $b_{ZV} = 0.135$, $c_{ZV} = 4.8$.

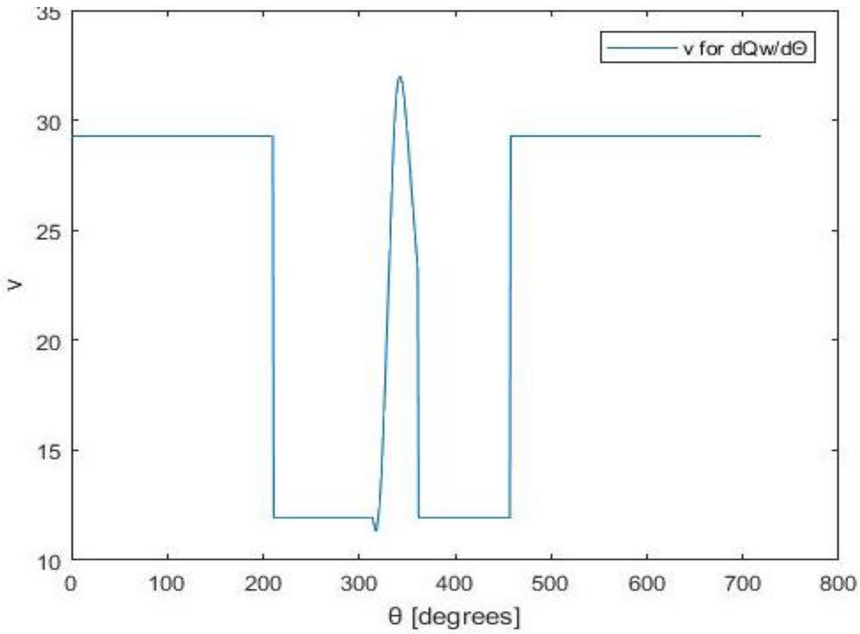
- Furthermore, during combustion, the number v from the heat transfer model of Wosni changes also according to the relation:

$$\mathrm{v} = \mathrm{C_1} * \mathrm{cm} + \mathrm{C_2} * \frac{\mathrm{Vd}*\mathrm{Tsoc}}{\mathrm{Psoc}*\mathrm{Vsoc}} * (\mathrm{P(i)} - \mathrm{Po(i)})$$

where $\mathrm{Po(i)} = Pbc * (Vbc/V(i))^{1.36}$.

Therefore, the heat release diagram from the double Wiebe and the v variation are presented below for the third operating point:





Exhaust

To calculate the mass flow during the opening period of the exhaust valve we first calculate the active surface area Aex multiplied by the factor Cf. We also need to know the pressure ratio $\pi = \frac{\mathrm{Pex}}{\mathrm{P}(i)}$ and the critical ratio $\pi,\mathrm{crit} = \frac{2}{(\mathrm{k(i)+1})^{\frac{\mathrm{k(i)}}{\mathrm{k(i)-1}}}}$ in order to find where the flow is choked. So, depending on the π we have the following relations:

- if π>1 then we have backflow and the relations are modified to:

$$\Psi_{ex}(i) = \sqrt{2 * \frac{k(i)}{k(i)-1} * (-\pi(i)^{2*k(i)} + \pi(i)^{(k(i)+1)/k(i)})}$$

$$m_{ex}(i) = \frac{2 * A_{ex}(i) * Cf(i) * \sqrt{P(i) * dens(i)} * \Psi ex(i)}{\omega}$$

- if $\pi<1$ and $\pi \geq \pi,\mathrm{crit}$, then:

$$\Psi_{ex}(i) = \sqrt{2 * \frac{k(i)}{k(i)-1} * (+\pi(i)^{2*k(i)} - \pi(i)^{(k(i)+1)/k(i)})}$$

$$m_{ex}(i) = -\frac{2 * A_{ex}(i) * Cf(i) * \sqrt{P(i) * dens(i)} * \Psi ex(i)}{\omega}$$

- Otherwise:

$$m_{ex} = -\frac{P(i)}{(R(i) * T(i))} * \sqrt{k(i) * R(i) * T(i)} * Aex(i) * Cf(i) * \frac{2}{k(i)+1}^{\frac{k(i)+1}{2*(k(i)-1)}}$$

$$with\ Aex(i) = \frac{\pi}{4} * dex^2$$

So,

$m_{tot}(i+1) = m_{tot}(i) + m_{ex}(i)$,

$m_{air}(i+1) = 14.7 * (m_{tot}(i+1) - m_{rg-tot})/(14.7 + F(i+1))$ and

$m_{fuel}(i+1) = m_{air}(i+1) * F(i+1)/(14.7)$.

Overlap

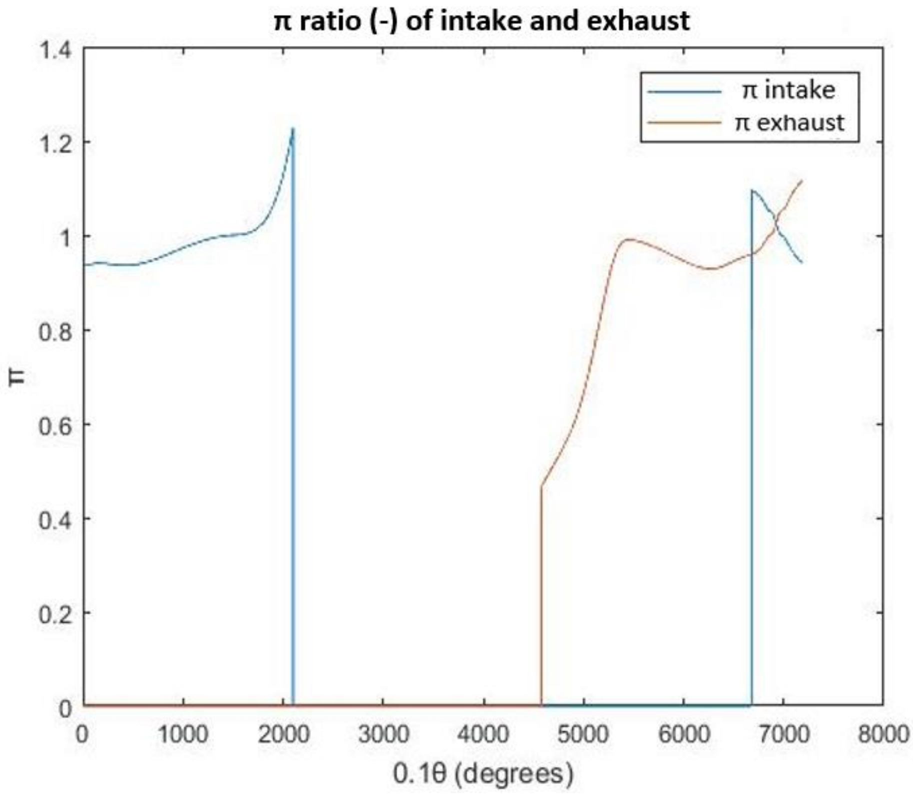
During the overlap period of the valves, we can calculate in the same way the mass flows m_{ex} and m_{in} but now the mass relations will be:

$m_{tot}(i+1) = m_{tot}(i) + m_{ex}(i) + m_{in}(i)$,

$$m_{air}(i+1) = m_{air}(i) + m_{in}(i) + m_{ex}(i) * (\frac{1}{1+\frac{F(i)}{14.7}})$$

$$m_{fuel}(i+1) = m_{fuel}(i) + m_{in}(i) + m_{ex}(i) * \left(\frac{1}{1 + \frac{F(i)}{14.7}} \right) * \frac{F(i)}{14.7}$$

The diagram of the pressure ratios for the first point is presented to show the area of overlap.



Calculations

Having assumed the first crank-degree (i = 1) and having calculated all the stages (intake, fuel injection, combustion, exhaust and valve cover) for the next crank-degree (i = 2) as presented above, we can now compose the iterative procedure for calculating all quantities in state i + 1 since we know state i.

Therefore, by connecting these quantities in the equilibriums of mass, momentum and energy we can solve my problem. For each crank degree (except the first one we have assumed) we calculate in the following order the 18 relations:

- Heat transfer losses from Wosni model:

(1) $a(i) = 127.93 * b^{-0.2} * \frac{P(i)}{100000}^{0.8} * v^{0.8} * T(i)^{-0.53}$

(2) $A(i) = 2 * Acyl + Ahead + Ac + b * \pi * s(i)$

(3) $\frac{dQw}{d\theta}(i) = a(i) * A(i) * \frac{Twall - T(i)}{\omega}$

- Solve the energy balance to find the in-cylinder Temperature changes.

(4) $$\frac{dU}{dF}(i) = (T(i) - 273.15) * (0.0388416 * (T(i) - 273.15) * F(i)^{-0.2} - 5.25619 * 10^{-6} * (T(i) - 273.15)^2 * F(i)^{-0.25} + 62.3546 ** F(i)^{-0.07})$$

- if $m_{in}(i) \leq 0$ then $\Delta H_{in}(i) = m_{in}(i) * h(i)$
- if $m_{in}(i) > 0$ then $\Delta H_{in}(i) = m_{in}(i) * h_{in}$ (5)

Accordingly,

- *if* $m_{ex}(i) \leq 0$ then $\Delta H_{ex}(i) = m_{ex}(i) * h_{ex}$
- if $m_{ex}(i) > 0$ then $\Delta H_{ex}(i) = m_{ex}(i) * h(i)$ (6)

So,

$$(7)\quad \frac{dU}{d\theta}(i) = \frac{dQw}{d\theta}(i) + \frac{dQb}{d\theta}(i) + \Delta H_{in}(i) + \Delta H_{ex}(i) - U(i) * \frac{dmt}{d\Theta}(i)$$

$$(8)\quad \frac{dT}{d\theta}(i) = \frac{-R(i) * T(i) * \frac{\frac{dV}{d\theta}(i)}{V(i)} + \frac{\frac{dU}{d\theta}(i)}{m_{tot}(i)} - \frac{dU}{dF}(i) * \frac{dF}{d\theta}(i)}{\frac{dU}{dT}(i)}$$

Having the rate of change of temperature and knowing the quantities we assumed in the first calculation angle, or in the i degree, we can calculate all the magnitudes in the next degree, or in the i + 1 degree.
So, in the next repetition we will calculate accordingly:

$$(9)\quad R(i) = (m_{air}(i) * \left(287 - 20F(i)\right) + m_{rg_{tot}} * (287 - \frac{20}{lambda}))/m_{tot}(i)$$

$$(10)\quad T(i) = T(i-1) + \frac{dT}{dtheta}(i-1)$$

$$(11)\quad P(i) = m_{tot}(i) * R(i) * T(i)/V(i)$$

$$(12)\quad dens(i) = \frac{P(i)}{R(i)*T(i)}$$

$$(13)\quad \frac{\mathrm{dU}}{\mathrm{dT}}(\mathrm{i}) = \mathrm{T(i)}^2 * \left(-0.0000210248 * \mathrm{F(i)}^{0.75} - 0.0000422662\right) + \mathrm{T(i)} *$$

$$\left(0.0114858 * \mathrm{F(i)}^{0.75} + 0.097104 * \mathrm{F(i)}^{0.8} + 0.247585\right) + 67.048 * (\mathrm{F(i)}^{0.93} - 1.56868 *$$

$$\mathrm{F(i)}^{0.75} - 26.524 * \mathrm{F(i)}^{0.8}) + 642.998$$

$$(14)\quad \mathrm{Cv(i)} = \frac{\mathrm{dU}}{\mathrm{dT}}(\mathrm{i})$$

$$(15)\quad Cp(i) = Cv(i) + R(i)$$

$$(16)\quad k(i) = \frac{Cp(i)}{Cv(i)}$$

$$(17)\quad U(i) = 144.5 * \big(1356.8 + \big(489.6 + 46.4 * F(i)^{0.93}\big) * (T(i) - 273.15) * 0.01\big) + \big(7.768 + 3.36 * F(i)^{0.8} * (T(i) - 273.15)^2 * 10^{-4}\big) - (0.0975 + 0.0485 * F(i)^{0.75} * (T(i) - 273.15)^3 * 10^{-6}))$$

$$(18)\quad h(i) = U(i) + \frac{P(i)}{dens(i)}.$$

Results

- Calculate the total work from volume change according to the P-V diagram:

$$work_{imp} = 4 * sum(P(i) * \frac{dV}{d\theta}(i))$$

- Find the real Imep και Bmep and all their depended values:

$$imep_{imp} = \frac{work_{imp}}{4 * Vd} * 10^{-5}$$

$$bmep_{imp} = imep_{imp} - fmep$$

$$P_i = imep_{imp} * 10^{-5} * 4 * Vd * \frac{N}{120000}$$

$$P_{hp} = P_i * 1{,}38$$

$$eff = 100 * \frac{work_{imp}}{4 * Q_B}$$

$$ev = \frac{m_{air_{tot}}}{dens_{in} * Vd}$$

$$bsfc = \frac{3600}{\frac{eff}{100} * LHV} * 10^6$$

Convergence

When completing the calculations for each crank degree, our initial assumptions for the first degree must converge with the values of the last degree of the cycle in order for the problem to be solved correctly.

The 4 convergence criteria are the following:

- $error1 = P(720) - P(1)$
- $error2 = T(720) - T(1) + \frac{dT}{d\theta}(720)$
- $error3 = R(720) - R(1)$
- $error4 = bmep_{imp} - bmep$

To achieve this, we have to change the user's 5 initial assumptions, in order of priority, depending on their influence on the above convergence criteria. This series is:

1. The fuel-air ratio λ, since it has the greatest influence on any magnitude calculated. The change of λ is determined by the convergence criterion of bmep.
2. The pressure P(1), the gas constant R(1) and the temperature T(1) in the first calculation angle. These are determined by the mass and temperature convergence criteria mentioned above.
3. Pressure and temperature at the closing of the inlet valve (P(IVC) and T (IVC) respectively). These assumptions are of secondary importance to the previous ones, as their value mainly changes the mass of residual exhaust gas and the total mass of fuel present in the cylinder at the end of the injection. Their change is not determined by a convergence criterion, but by the values of P (1) and T (1) mentioned previously.

References

1. Notes from **Advanced Internal Combustion Engines course** of Pr. Koltsakis, Aristotle University of Thessaloniki, Department of Mechanical Engineering, 2017.
2. **Internal Combustion Engines: Applied Thermosciences**, [Book] / auth. Colin R. Ferguson - Alant T. Kirkpatrick.
3. **Internal Combustion Engine Fundamentals** [Book] / auth. Heywood J. - USA : McGraw Hill USA, 1988