

Quantum Information Project

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```
11 #####
12 # FUNCTIONS
13
14 # Lane emden function
15 def lane_emden(t, x, g):
16     theta, z = x
17     return [z, -(2/t)*z-(theta**(1/(g-1)))]
18
19 # create loading bar during the execution of the program
20 def loadbar(iterations, total, prefix='', suffix='', decimals=1, length=100, fill='\u25AE'):
21     percentage = ('{0:. ' + str(decimals) + 'f}').format(100*(iterations/total))
22     filled = int(length*iterations//total)
23     bar = fill * filled + '-' * (length-filled)
24     print(f'\r{prefix} |{bar}| {percentage}% {suffix}', end='\r')
25     if iterations == total:
26         print()
```



Main Code

Constants and Initialization



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Figures

```
40 #####
41 # Initialize arrays and constant variables and arrays
42
43 # Constants
44 gamma = np.linspace(1.2, 1.9, 169) # gamma values
45 k_min_coef = np.array([0.95, 1.00, 1.05]) # coefficients of k_min for fig 3.
46 coef_calc = 1 # coefficient for which figures 1 and 2 are drawn
47 t_init = 10e-4 # initial value of xi for solving the differential
    equation
48 t_end = 30 # final value of xi for solving the differential equation
49 M_coef = 200 # Mass coefficient to scale fig 2.
50 d = 10000 # number of values returned from solve_ivp and simpson
51 pg = np.linspace(1.2, 1.9, 8) # values that will be plotted in figure of lane emden
    solutions
52
53 # Initialization of empty arrays
54 S = np.zeros(len(gamma)) # array where the configurational entropy values will be
    stored
55 S3 = np.zeros((3, len(gamma))) # array where the configurational entropy times a^(3)
    will be stored
56 M = np.zeros(len(gamma)) # array where the mass values will be stored
57
58 # Initial values
59 w0 = [1., 0.] # boundary conditions with values of theta and the
    derivative of theta
```



Initialization of Lane Emden figure and figure1

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Figures

```
61 #####
62 # Initialize the figures
63
64 # Figure of the solutions of lane emden
65 plt.figure(0)
66 plt.title("Lane Emden")
67 plt.ylabel(r"$\theta(\xi)$")
68 plt.xlabel(r"$\xi$")
69 plt.ylim([-1.3, 1.1])
70 plt.xlim([0, 10])
71 plt.grid()
72
73 # Figure 1 Normalized modal fraction f(|k|) for sample values of polytropic index gamma (figure 1
    of paper)
74 plt.figure(1)
75 plt.tick_params(
76     bottom=False, top=True,
77     left=True, right=True)
78 plt.title("Fig 1.")
79 plt.ylabel(r"$\bar{f}(|k|)$")
80 plt.xlabel(r"$k/\sqrt{4\pi G/K\rho_0^{\gamma-2}}$")
81 plt.xlim([0, 1.5])
82 plt.ylim([0, 1.1])
83 plt.grid()
```



Initialization of figures 2 and 3

```
85 # Figure 2 Configurational entropy times  $\rho^{-1}$  (continuous line) and mass (dotted line) versus
    polytropic
86 # index gamma (figure 2 of paper)
87 plt.figure(2)
88 plt.title("Fig 2.")
89 plt.xlabel(r"$\gamma$")
90 plt.xlim([1.25, 1.7])
91 plt.ylim([0.4, 1.3])
92 plt.grid()
93
94 # Figure 3 Configurational entropy versus polytropic index gamma for polytropes. We display results
    for several
95 # choices of cutoff for k_min (figure 5 of paper)
96 plt.figure(3)
97 plt.title("Fig 3.")
98 plt.xlabel(r"$\gamma$")
99 plt.ylabel(r"$S a^3$")
100 plt.xlim([1.25, 1.75])
101 plt.ylim([4.6, 5.6])
102 plt.axvline(x=4/3, color='k', ls='—')
103 plt.text(4/3, 4.63, "4/3", rotation=0)
104 plt.axvline(x=5/3, color='k', ls='—')
105 plt.text(5/3, 4.63, "5/3", rotation=0)
106 plt.grid()
```

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Solving Lane Emden



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```
108 #####
109 # Main for loop to compute the solutions of lane emden and the integrals
110
111 # initiate loading bar
112 loadbar(0, len(gamma), prefix='Progress:', suffix='Complete', decimals=2, length=40)
113
114 # FOR LOOP
115 # Loops over all values of polytropic index gamma
116 for ind, g in enumerate(gamma):
117
118     # SOLVING THE DIFFERANTIAL EQUATION
119     # xi arrays holds the values of xi for the evaluation of theta
120     # solve_ivp is used to solve the differential equation with method Runge–Kutta of 4th and 5th
121     # order
122     # values of the solutions for theta and xi are stored indside the arrays the and ti_n
123     xi = np.linspace(t_init, t_end, d)
124     sol = inte.solve_ivp(fun=lane_emden, t_span=(t_init, t_end), y0=w0, method='RK45', args=(g, ), t_eval
125                          =xi)
126     the = sol.y
127     ti_n = sol.t
```

Plot Lane Emden solutions



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```
127 # theta(xi)==0 corresponds to rho(R)==0 hence negative values of theta have no physical
    meaning
128 # in the next three line we find the index of positive theta and we store the desired values of
    theta and xi
129 # inside the arrays x_n for theta and ti for xi
130 indexx, = np.where(the[0] >= 0)
131 x_n = the[0][indexx]
132 ti = ti_n[indexx]
133
134 # with this if statement we choose for which polytropic index gamma the solution of lane—emden
    will be plotted
135 if g in pg:
136     plt.figure(0)
137     plt.plot(ti_n, the[0], label=f'\u03B3 = {g:.2f}')
```

Calculation of Normalized modal fraction f



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```
139 # in order to plot figure 3 the calculations have to be executed for three different values of
      kappa_min=ak
140 # in array min_k the three values of kappa_min are stored
141 min_k = (np.pi/ti[-1])/k_min_coef
142
143 # Calculation for all values of k_min to compute the product  $Sa^{(3)}$  for each k_min
144
145 # values of kappa for which the integral is computed
146 max_k = 100*min_k
147 k = np.linspace(min_k, max_k, len(ti))
148
149 # using simpson function the h_min is computed for the value k_min
150 y_min = (x_n**(1/(g-1)))*np.sin(np.multiply.outer(min_k, ti))*ti
151 h_min = (inte.simpson(y_min, ti, dx=0.001, axis=1)*(1/min_k))**2
152
153 # For all values of kappa the h(ak) is computed
154 y_data = (x_n**(1/(g-1)))*np.sin(np.multiply.outer(k, ti))*ti
155 h = (inte.simpson(y_data, ti, dx=0.001, axis=2)*(1/k))**2
156
157 # now that both h(kappa) and h(kappa_min) have been computed the Normalized modal fraction  $f(|k|)$ 
158 f = (h/h_min)
```

Calculation of Configurational entropy and Mass



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```
160 # using simpson the Mass divided by a constant factor is computed
161 m = (x_n**(1/(g-1)))*ti**2
162 M[ind] = (4*np.pi*((g/(g-1))**((3/2)))*inte.simpson(m,ti,dx=0.001))/M_coef
163
164 # finally  $Sa^3/((g/(g-1))^{-(3/2)})$  is computed only for  $\pi/(1.00R)$  and  $Sa^3$  for each
    k_min
165 s = f*np.log(f)*(k)**2
166 S3[:,ind] = -4*np.pi*inte.simpson(s,k,dx=0.0001,axis=0)
167 S[ind] = ((g/(g-1))**(-(3/2)))*S3[1,ind]
```



Plot Figure 1

```
169 # here only for value pi/(1.00R) the figure 1 is plotted only for smaller than 1 f(|k|)
170 # also in figure 1 using plt.scatter the points for k_min are added
171 plt.figure(1)
172 index, = np.where(f[:, coef_calc] <= 1)
173 if g==1.2:
174     plt.scatter(k[index[0], coef_calc]/np.sqrt(g/(g-1)), f[index[0], coef_calc], marker='v', color='
    r')
175     plt.plot(k[index[0]:, coef_calc]/np.sqrt(g/(g-1)), f[index[0]:, coef_calc], '—', color='r',
    label=f'\u03B3 = {g}')
176 elif g==1.4:
177     plt.scatter(k[index[0], coef_calc]/np.sqrt(g/(g-1)), f[index[0], coef_calc], marker='v', color='
    g')
178     plt.plot(k[index[0]:, coef_calc]/np.sqrt(g/(g-1)), f[index[0]:, coef_calc], color='k', label=f'\
    u03B3 = {g}')
179 elif g==1.7:
180     plt.scatter(k[index[0], coef_calc]/np.sqrt(g/(g-1)), f[index[0], coef_calc], marker='v', color='
    c')
181     plt.plot(k[index[0]:, coef_calc]/np.sqrt(g/(g-1)), f[index[0]:, coef_calc], '—.', color='c',
    label=f'\u03B3 = {g}')
182
183 # update loading bar
184 loadbar(ind+1, len(gamma), prefix='Progress: ', suffix='Complete', decimals=2, length=40)
```

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Figures

Completion of figures 1 and 2



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Figures

```
186 #####
187 # Finalize and print the figures with legends
188
189 # added legend in plot
190 plt.figure(0)
191 plt.legend()
192
193 # added legend in plot
194 plt.figure(1)
195 plt.legend()
196
197 # plot Mass and configurational entropy S and add legends
198 plt.figure(2)
199 plt.plot(gamma,S,color='r',label=r'$S\rho_0^{-1}/\left(\left(\frac{K}{4\pi G}\right)^{-\frac{3}{2}}\right)\rho_c^{2-\frac{3}{2}\gamma}$')
200 plt.plot(gamma,M,'—',color='g',label=r'$M/\left(200\left(\frac{K}{4\pi G}\right)^{\frac{3}{2}}\right)\rho_c^{\frac{3}{2}\gamma-2}$')
201 plt.legend()
```

Completion of figure 3



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Main Code

Figures

```
203 # plot configurational entropy S times a^(3), points for max and min and legends
204 plt.figure(3)
205 plt.scatter(gamma[np.argmax(S3[0])], np.amax(S3[0]), marker='v', color='r')
206 plt.scatter(gamma[np.argmax(S3[1])], np.amax(S3[1]), marker='v', color='r')
207 plt.scatter(gamma[np.argmax(S3[2])], np.amax(S3[2]), marker='v', color='r')
208 plt.scatter(gamma[np.argmax(S3[0])], np.amin(S3[0]), marker='o', color='b')
209 plt.scatter(gamma[np.argmax(S3[1])], np.amin(S3[1]), marker='o', color='b')
210 plt.scatter(gamma[np.argmax(S3[2])], np.amin(S3[2]), marker='o', color='b')
211 plt.plot(gamma, S3[0], '—', color='b', label=r'$\pi/(0.95R)$')
212 plt.plot(gamma, S3[1], '—', color='k', label=r'$\pi/(1.00R)$')
213 plt.plot(gamma, S3[2], '—', color='r', label=r'$\pi/(1.05R)$')
214 plt.legend()
215
216 #####
217 # Stop timing and printed
218
219 print(f'Execution Time: {datetime.now() - start}')
```



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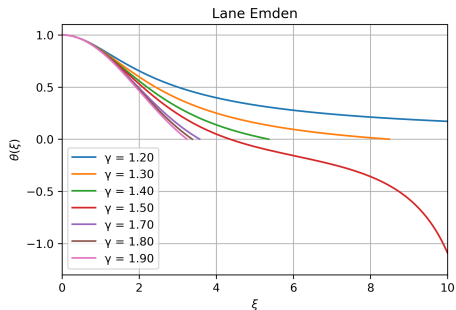
Main Code

Figures

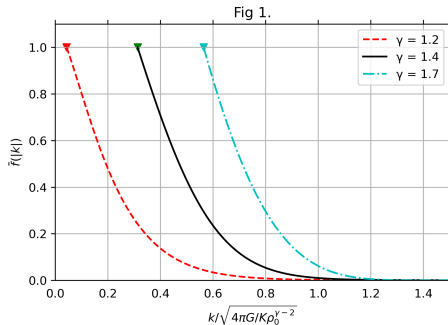
Figures



Solutions of Lane Emden and Figure 1



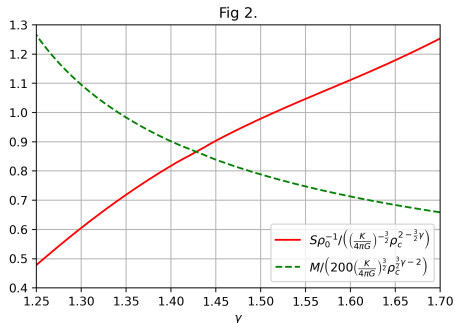
(a) Lane Emden Solutions



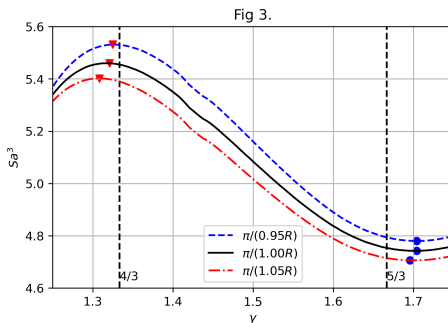
(b) Normalized modal fraction for sample values of the polytropic index γ . From left to right, $\gamma = 1.2$, $\gamma = 1.4$, and $\gamma = 1.7$.



Figures 2 and 3



(c) Configurational entropy times ρ_0^{-1} (continuous line) and mass (dotted line) versus polytropic index γ for $\rho_0 = \rho_c$.



(d) Configurational entropy versus polytropic index γ for polytropes. We display results for several choices of cutoff for k_{min} .