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May 24, 2019



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#### Plan

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#### Intro

#### What is

Pattern recognition is concerned with the automatic identification of similarities in data through the use of computer algorithms. With the use of these similarities, pattern recognition classifies the data into different categories.

In our project, we will study some interesting mathematical tools and algorithms that are used for pattern recognition in images.





### Fundamental problems

The two fundamental problems in a pattern recognition system are feature extraction (shape measurement) and classification. The problem of extracting a vector of shape measurements from a digital image can be further decomposed into three subproblems:

- The first is the image segmentation problem, i.e., the separation of objects of interest from their background.
- **The second** subproblem is that of finding the objects in the segmented image.
- The final subproblem is extracting the shape information from the objects detected.



#### Our interest

In our study we focus on the first and the third subproblem of the shape measurement that seems to us as the most challenging aspects.

In the third subproblem there are many tools available depending on the properties of the objects that are to be classified. We will describe two of them: The Hough transform and the polygonal approximation. The last one will be examined carefully.

Some new techniques that are applied at this problem are based on neural networks an deep geometric learning. That will be our final section.



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### Cluster and Classification analysis

Image segmentation problem

Image segmentation problem: Partitioning the pixels in an image into "meaningful" regions, usually such that each region is associated with one physical object.

Cluster analysis problem: Partitioning a collection of n points in some fixed-dimensional space into m < n groups that are "natural" in some sense. Here m is usually much smaller than n.



A fundamental problem in pattern recognition of images is the segmentation problem: distinguishing the figure from the background.

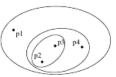
Clustering is one of the most powerful approaches to image segmentation, applicable even to complicated images such as those of outdoor scenes.

Each pixel is mapped into a point in k-dimensional pixel-space. Performing a cluster analysis of all the resulting N  $\times$  N points in pixel-space yields the desired partitioning of the pixels into categories. Labeling each category of pixels with a different color then produces the segmentation.



### Hierarchical Clustering

There is no special number m that we want to discover; rather the goal is the production of a dendrogram (tree) that grows all the way from one cluster to n clusters and shows us at once how good a partitioning is obtained for any number of clusters between one and n. Such methods are referred to as hierarchical methods. They fall into two groups: agglomerative (bottom-up, merging) and divisive (top-down, splitting).



Traditional Hierarchical Clustering

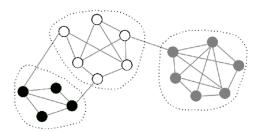


Traditional Dendrogram



## Graph-Theoritic Clustering

The idea is simple: Compute some proximity graph (such as the minimum spanning tree) of the original points. Then delete (in parallel) any edge in the graph that is much longer (according to some criterion) than its neighbors. The resulting forest is the clustering (each tree in this forest is a cluster).





### K-means Clustering

The k-means algorithm searches for k cluster centroids in  $\mathbb{R}^d$  with the property that the mean squared (Euclidean) distance between each of the n points and its nearest centroid ("mean") is minimized.

A typical heuristic starts with an initial partition, computes centers, assigns data points to their nearest center, recomputes the centroids, and iterates until convergence is achieved according to some criterion.

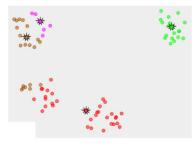


Figure: k-means (Update:PAM)



### K-means Clustering

The K-means algorithm is an iterative technique that is used to partition an image into K clusters. The basic algorithm is

- Pick K cluster centers, either randomly or based on some heuristic method, for example K-means++
- Assign each pixel in the image to the cluster that minimizes the distance between the pixel and the cluster center
- Re-compute the cluster centers by averaging all of the pixels in the cluster
- Repeat steps 2 and 3 until convergence is attained (i.e. no pixels change clusters)

In this case, distance is the squared or absolute difference between a pixel and a cluster center. The difference is typically based on pixel color, intensity, texture, and location, or a weighted combination of these factors. K can be selected manually, randomly, or by a heuristic. This algorithm is guaranteed to converge, but it may not return the optimal solution. The quality of the solution depends on the initial set of clusters and the value of K.



K-means Clustering

### K-means Clustering





- Left Image: Source image.
- **Right Image:** Image after running k-means with k = 16. Note that a common technique to improve performance for large images is to downsample the image, compute the clusters, and then reassign the values to the larger image if necessary.



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### Hough transform

The Hough transform was originally proposed (and patented) as an algorithm to detect straight lines in digital images. The method may be used to detect any parametrizable pattern, and has been generalized to locate arbitrary shapes (e.g lines,circles,object location) in images. The basic idea is to let each above-threshold pixel in the image vote for the points in the parameter space that could generate it.

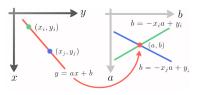


Figure: Dual: b= -ax+v

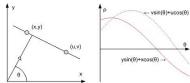
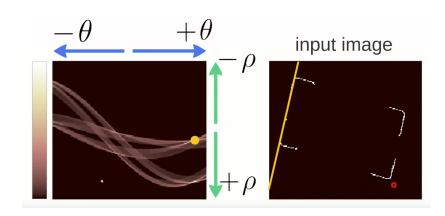


Figure: polar representation  $\rho = x \cos(\theta) + y \sin(\theta)$ 

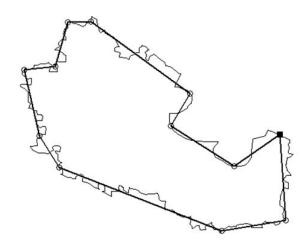


### Hough transform





# Polygonal Approximation





# Polygonal Approximation

In order to reduce the complexity of costly processing operations, it is often desirable to approximate a curve *P* with one that is composed of far fewer segments, yet is a close enough replica of *P* for the intended application.

Let  $P = (p_1, p_2, ..., p_n)$  be a polygonal curve or chain in the plane, consisting of n points  $p_i$  joined by line segments  $p_i p_{i+1}$ . We want to determine a new curve  $Q = (q_1, q_2, ..., q_m)$  such that

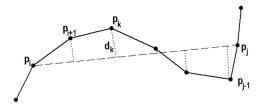
- $\blacksquare$  m is significantly smaller than n
- $\blacksquare$  the  $q_i$  are selected from among the  $p_i$
- any segment  $q_jq_{j+1}$  that replaces the chain  $q_j=p_r,...,p_s=q_{j+1}$  is such that the distance between  $q_jq_{j+1}$  and each  $p_k, r \le k \le s$ , is less than some predetermined error tolerance  $\omega$



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### The problem



#### Our Problem

**Min-**#. Given the error tolerance  $\omega$ , find a curve  $Q = (q_1, q_2, ..., q_m)$  satisfying the constraint such that m is minimum.



### Douglas-Peucker pseudocode

A very easily implemented algorithms is the Ramer–Douglas–Peucker algorithm.

#### Algorithm **DOUGLAS-PEUCKER** $(P, \epsilon)$

Input: An ordered set of points P and the distance dimension  $\epsilon$ 

Output: An ordered set  $Q \subseteq P$  with the minimum number of points such that the  $\epsilon$ -criterion is satisfied

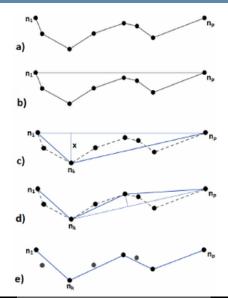
- $Q \leftarrow \emptyset$
- Assume  $p_1, p_n$  is the first and the last point of P, respectively. Insert  $p_1, p_n$  in Q.
- **III find** in *P* the farthest point from the line segment  $\overline{p_1p_n}$ . Let that point be  $p_i$ .
- | | **if**  $dist(p_i, \overline{p_1p_n}) \ge \epsilon$ :
  - $||P_1 \leftarrow \{p_1, ..., p_i\}, P_2 \leftarrow \{p_i, ..., p_n\}$
  - $| \mid Q_1 \leftarrow DOUGLAS-PEUCKER(P_1, \epsilon)$
  - $| | Q_2 \leftarrow DOUGLAS-PEUCKER(P_2, \epsilon)|$
  - | Insert the points of  $Q_1$ ,  $Q_2$  in the right position of Q (between  $p_1$  and  $p_n$ ).
- Return Q



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# Douglas-Peucker Animated





### Complexity

Introduction

#### Complexity of Douglas-Peucker algorithm

The algorithm does not guarantee performance and could be quadratic in the worst case.

Now near-linear algorithms have been introduced. They will be studied in our final project.



#### References



J Oseph O'ROURKE and Godfried T. Toussaint. "54 PATTERN RECOGNITION".



https://en.wikipedia.org/wiki/Image\_segmentation



The end

# **Thank You!**

