Assignment 1:

Homography & Panorama

1. While an affine transformation is used for scaling, skewing and rotation, a projective transformation is used to express how the perceived objects coordinates change when the view point of the observer changes.

A projective transformation of coordinates (x,y) in one vector space to (x’,y’) on another vector space, can be expressed by the following equation:

Where H is a 3x3 matrix, and the sign represents an equality up to scale.

Let’s denote coordinates, and their projections with a subindex i.

Thus, we can write the following equations for each pair:

Where holds a degree of freedom since we only demand equality up to scale. Therefore, each pair (and their projection) of coordinates only sum up to 2 constraints on the values of H which can be expressed with the following equations:

Suppose we have n pairs of coordinates. Then we can express the equations as follows:

We denote this matrix as A – a matrix of size 2n x 9, and get a matrix of the form where the solution for x, describe H.

Since the source equality is up to scale, there are actually 8 degrees of freedom for H. Thus only 4 pairs of coordinates are sufficient to find H (We assume that A is not defective).

If n is bigger than 4, equality may not be possible (due to some error in the projective input). In such case, we strive to minimize .

One can notice that:

We denote – a PSD matrix of size 9x9. Thus, our objective is to find x such that is minimized.

Assuming M has 9 linearly independent eigenvectors, we can define – a matrix whose columns are the linearly independent eigenvectors of M, and – a diagonal matrix whose diagonal elements are the corresponding eigenvalues of M.

So:

Each vector x can be described as a weighted sum of the eigenvectors of M. From here, it is trivial to see that the desired x that minimizes is the eigenvector corresponding to the smallest eigenvalue of M (which is necessarily not negative since M is a PSD matrix).