Optimization – Programming Assignment – Part II - Solution

Objective function:

$$f(X,Y)=\sum_{i,j}\frac{1}{n}\left(x_{ij}-y_{ij}\right)^2+\mu\sqrt{\left(y_{i+1,j}-y_{ij}\right)^2+\left(y_{i,j+1}-y_{ij}\right)^2+\epsilon}$$

Assuming Neumann boundary conditions

Gradient:

$$\frac{\partial f}{\partial y_{kl}}=-\frac{2}{n}(x_{kl}-y_{kl})+\mu\sum_{i,j}\frac{1}{\sqrt{\left(y_{i+1,j}-y_{ij}\right)^2+\left(y_{i,j+1}-y_{ij}\right)^2+\epsilon}}\left[\left(y_{kl}-y_{k-1,l}\right)\delta_{i+1,k}\delta_{jl}-\left(y_{k+1,l}+y_{k,l+1}-2y_{kl}\right)\delta_{ik}\delta_{jl}+\left(y_{k,l}-y_{k,l-1}\right)\delta_{ik}\delta_{j+1,l}\right]$$

$$=-\frac{2}{n}(x_{kl}-y_{kl})+\mu\left[\frac{y_{kl}-y_{k-1,l}}{\sqrt{\left(y_{kl}-y_{k-1,l}\right)^2+\left(y_{k-1,l+1}-y_{k-1,l}\right)^2+\epsilon}}-\frac{\left(y_{k+1,l}-y_{kl}\right)+\left(y_{k,l+1}-y_{kl}\right)}{\sqrt{\left(y_{k+1,l}-y_{kl}\right)^2+\left(y_{k,l+1}-y_{kl}\right)^2+\epsilon}}+\frac{y_{kl}-y_{k,l-1}}{\sqrt{\left(y_{k+1,l-1}-y_{k,l-1}\right)^2+\left(y_{kl}-y_{k,l-1}\right)^2+\epsilon}}\right]$$